Nome: ______________________________ Turma:

Basic Concepts

1. (1 Ponto) In special relativity the rest energy of the particle is defined $E_0 = m_0c^2$ where c is the velocity of light and m_0 is the rest mass. Write down the expressions for the total energy E and the momentum p in terms of E_0 and kinetic energy T.

Solution: The total energy E is given by:

$$E = T + E_0 = T + m_0c^2$$

The momentum p can be expressed as:

$$p = \frac{1}{c}\sqrt{E^2 - E_0^2} = \frac{1}{c}\sqrt{(T + E_0)^2 - E_0^2}$$

2. (1 Ponto) Using the notation of special relativity $\beta = \frac{v}{c}$ and $\gamma = 1/\sqrt{1-\beta^2}$, show that $\gamma = \frac{E}{E_0}$ and $\beta = \frac{pc}{E}$.

Solution: Starting with the definition of total energy:

$$E = \gamma m_0c^2 = \gamma E_0 \Rightarrow \gamma = \frac{E}{E_0}$$

For β:

$$p = \gamma m_0 v = \gamma m_0 c\beta$$
$$E = \gamma m_0 c^2$$

Dividing the two equations:

$$\frac{p}{E} = \frac{\gamma m_0 c\beta}{\gamma m_0 c^2} = \frac{\beta}{c} \Rightarrow \beta = \frac{pc}{E}$$

3. (1 Ponto) The kinetic energy T of a proton is 1 GeV. If its rest mass m is $0.9383\,\mathrm{GeV/c^2}$, what is its total energy?

Solution: The total energy is:

$$E = T + m_0c^2 = 1\,\text{GeV} + 0.9383\,\text{GeV} = 1.9383\,\text{GeV}$$

4. (1 Ponto) Given that the relation between relativistic momentum and total energy is

$$E^2 = (m_0c^2)^2 + (pc)^2$$

calculate its momentum (in GeV/c).

Solution: Using the energy-momentum relation:

$$E^2 = (m_0c^2)^2 + (pc)^2$$

$$(pc)^2 = E^2 - (m_0c^2)^2 = (1.9383)^2 - (0.9383)^2 = 3.757 - 0.880 = 2.877$$

$$pc = \sqrt{2.877} = 1.696\,\text{GeV}$$

$$p = 1.696\,\text{GeV/c}$$

5. (1 Ponto) A betatron has a beam radius of 0.1 m and is powered from 50 Hz mains. Its peak guide field is 1 T while the flux linking the orbit is twice that which would result from a uniform field of this value. What will be the peak energy of the electrons it accelerates?

Solution: For a betatron, the peak energy is given by:

$$E_{\text{max}} = \frac{e}{2\pi r}\frac{d\Phi}{dt}$$

Given $r = 0.1\,\text{m}$, $f = 50\,\text{Hz}$, $B_{\text{peak}} = 1\,\text{T}$, and flux linking factor of 2:

$$\frac{d\Phi}{dt} = 2 \times \pi r^2 \times 2\pi f B_{\text{peak}} = 4\pi^2 r^2 f B_{\text{peak}}$$

$$E_{\text{max}} = \frac{e}{2\pi r} \times 4\pi^2 r^2 f B_{\text{peak}} = 2\pi e r f B_{\text{peak}}$$

$$E_{\text{max}} = 2\pi \times (1.6 \times 10^{-19}\,\text{C}) \times (0.1\,\text{m}) \times (50\,\text{Hz}) \times (1\,\text{T}) = 5.03 \times 10^{-18}\,\text{J} = 31.4\,\text{eV}$$

6. (1 Ponto) Using classical mechanics show that the angular frequency of revolution of a proton in a cyclotron is equal to $B_z(e/m)$. Calculate this frequency for a field of 1–2 T ($e/m = 1.58 \times 10^7\,\text{C/kg}$).

Solution: The centripetal force is provided by the Lorentz force:

$$\frac{mv^2}{r} = evB_z \Rightarrow \frac{v}{r} = \frac{eB_z}{m}$$

Angular frequency:

$$\omega = \frac{v}{r} = \frac{eB_z}{m}$$

For $B_z = 1\,\mathrm{T}$:

$$\omega = 1.58 \times 10^7\,\mathrm{rad/s}$$

For $B_z = 2\,\mathrm{T}$:

$$\omega = 3.16 \times 10^7\,\mathrm{rad/s}$$

7. (1 Ponto) A synchrotron of 25 m radius accelerates protons from a kinetic energy of 50 to 1000 MeV in 1 s. The dipole magnets saturate at 1000 MeV. What is the maximum energy of deuteron ($Z = 1, A = 2$) that it could accelerate?
Hint: for protons use the expression:

$$B\rho = \frac{p}{e}$$

Solution: Using $B\rho = \frac{p}{e}$: At saturation: $B\rho_{\max} = \frac{p_{\max}}{e}$ for protons For deuterons ($Z = 1,\ A = 2$):

$$p_d = \frac{(B\rho)_{\max} e}{Z} = p_{\max}$$

But $E_d = \sqrt{(m_d c^2)^2 + (p_d c)^2}$ At $1000\,\mathrm{MeV}$ for protons:

$$E_p = T_p + m_p c^2 = 1000\,\mathrm{MeV} + 938.3\,\mathrm{MeV} = 1938.3\,\mathrm{MeV}$$

$$p_p c = \sqrt{E_p^2 - (m_p c^2)^2} = \sqrt{(1938.3)^2 - (938.3)^2} = 1696\,\mathrm{MeV}$$

For deuterons ($m_d c^2 = 1875.6\,\mathrm{MeV}$):

$$E_d = \sqrt{(1875.6)^2 + (1696)^2} = 2528\,\mathrm{MeV}$$

$$T_d = E_d - m_d c^2 = 2528\,\mathrm{MeV} - 1875.6\,\mathrm{MeV} = 652.4\,\mathrm{MeV}$$

8. (1 Ponto) What is the revolution frequency for (a) protons and (b) deuterons?

Solution: Revolution frequency $f = \frac{v}{2\pi r} = \frac{\beta c}{2\pi r}$

For protons at 1000 MeV:

$$E_p = 1938.3\,\mathrm{MeV}$$

$$\gamma = \frac{E_p}{m_p c^2} = \frac{1938.3}{938.3} = 2.066$$

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = 0.877$$

$$f_p = \frac{0.877 \times 3e8}{2\pi \times 25} = 1.68 \times 10^6\,\mathrm{Hz}$$

For deuterons at 652.4 MeV:

$$E_d = 2528\,\mathrm{MeV}$$

$$\gamma = \frac{2528}{1875.6} = 1.348$$

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = 0.695$$

$$f_d = \frac{0.695 \times 3e8}{2\pi \times 25} = 1.33 \times 10^6\,\mathrm{Hz}$$