Tutorial #5

Rules

Recall

* mean waiting time

(43) $$\boxed{T_W = \frac{\rho(1+CV_s^2)}{2(1-\rho)} T_s}$$

* Coefficient of Variation of RV s (service time) CV_s

$$\boxed{CV_s = \frac{\sigma_s}{T_s}} \quad (44)$$

(45) $$\hookrightarrow \boxed{T_s = E(s) = \sum_{\forall s} s P(s)}$$

(46) $$\hookrightarrow \boxed{\sigma^2 = E(s^2) - T_s^2 = \sum_{\forall s} s^2 P(s) - \left(\sum_{\forall s} s P(s)\right)^2}$$

* CV_s for some distributions

(47)
- deterministic time (D)
  - $\boxed{CV_s = 0}$ ($\sigma_s = 0$)
- Exponential (M)
  - $\boxed{CV_s = 1}$ ($\sigma_s = T_s$)
- Erlang-k (E_k)
  - $\boxed{CV_s = \frac{1}{\sqrt{k}}}$

(43) $$\boxed{T_W = \frac{\lambda E(s^2)}{2(1-\rho)}}$$ (RR)

* Avg response time of a specific task

$$\boxed{E(r|s=t) = \frac{t}{1-\rho}} \quad (48)$$

t → schedule time

$E(r|s=t)$ → T_Q for RR

sheet #6(2) M/G/1

Examples

Sheet#6.2(a)

1) Suppose that ATM streams are multiplexed at an output link with speed μ 155 Mbps. An ATM cell has a fixed size (53 bytes) and thus the transmission time is constant ($CV_s = 0$). What is the mean time for cell to
b) traverse the link when the avg info arrival rate is λ 124 Mbps?
& the mean # of cells in the
a) buffer (including the one being transmitted)

$\hookrightarrow T_Q = T_W + T_s$

$\lambda_{per\,cell} = 124/(53\times8) = 0.292$ cell/μs

$T_s = \frac{1}{155} \times 53 \times 8 = 2.735\ \mu s$

$$T_W = \frac{\rho(1+CV_s^2)T_s}{2(1-\rho)}$$

$$= \frac{\frac{124}{155}(2.735)}{2(1-\frac{124}{155})} = 5.47\ \mu s$$

$$T_Q = \underset{5.47}{T_W} + \underset{2.735}{T_s} = \boxed{8.205\ \mu s}$$ (b)

$$L_Q = \lambda T_Q = 0.292(8.205) = 2.39586 \text{ cells}$$ (a)

6.2(b)

2) Suppose that email messages arrive at an email server according to poisson process at a mean rate of λ 1.2 msg/s. Suppose that 30% of the msg are processed in 0.1 sec, 50% in 0.3 sec. & 20% in 2 sec

Except for (48), M/M/1 rules are applied

$A/B/m/\overset{\infty}{K}/\overset{\infty}{P}/Z$ FCFS

A: Inter-arrival times prob dist

B: Service times probability dist

m: # of parallel servers

K: Total sys capacity → m in service, ↳ k−m waiting

P: calling population size

Z: Query/service discipline

M = Exponential dist D = Deterministic

E_k = Erlang-k dist G = General dist. (σ, T_s)

6.2(c)

4) Suppose a processor sends on avg λ 10 disk I/O requests/sec. The disk service time has a $CV_s^2 = 1.5$ & $T_s = 20$ ms

(i) mean # requests waiting

$$T_W = \frac{\rho(1+CV_s^2)T_s}{2(1-\rho)} = \frac{0.2(1+1.5)(0.02)}{2(1-0.2)}$$
$$= 6.25 \text{ ms}$$

$$T_Q = T_W + T_s = 26.25 \text{ ms}$$

$$L_W = \lambda T_W = (10 \times 6.25 \times 10^{-3}) = \frac{1}{16}$$

(ii) mean response time

(iii) repeat with Erlang-2

$$CV_s = \frac{1}{\sqrt{2}}, \quad T_W = \frac{0.2(1+\frac{1}{2})(0.02)}{2(1-0.2)}$$
$$= 3.75 \text{ ms}$$

$$L_W = \lambda T_W = 3.75 \times 10^{-2} = 0.0375$$

$$T_Q = T_W + T_s = 3.75 + 20$$
$$= 23.75 \text{ ms}$$

i) what's the avg time to process a message? T_s

P(S)	0.3	0.5	0.2	
S	0.1	0.3	2	sec

$$T_s = E(S) = \sum_{\forall s} S P(S) =$$

$$0.3 \times 0.1 + 0.5 \times 0.3 + 0.2 \times 2 = 0.58 \text{ sec}$$

ii) what is the mean waiting time in the queue? T_W

$$T_W = \frac{\lambda E(S^2)}{2(1-\rho)}$$

$$E(S^2) = \sum_{\forall s} S^2 P(S) = (0.1)^2(0.3) + (0.3)^2(0.5) + (2)^2(0.2)$$
$$= 0.848 \text{ sec}^2$$

$$\therefore T_W = \frac{1.2(0.848)}{2(1-1.2(0.58))} = 1.67 \text{ sec}$$

iii) what is the avg response time?

$$T_Q = T_s + T_W = 0.58 + 1.67$$
$$= 2.25 \text{ sec}$$

3) A processor is multiplexed at infinite speed among all processes present in a ready Q with no overhead. Round-Robin scheduling $T_s = 9$ min, inter-arrival time exponentially distributed with mean of 18 mins $\frac{1}{\lambda}$

i) What is the mean turnaround time? T_Q of a process having a run time of 3 mins?

$$E(r|s=3) = \frac{t}{1-\rho} = \frac{3}{1-\frac{9}{18}} = 6 \text{ mins}$$

ii) compare with M/M/1 model

$$E(r|s=3) = T_s + T_W$$
$$= 9 + \rho T_Q = 9 + 0.5 \frac{3}{1-0.5} = 12 \text{ mins}$$