Assignment 04: Taylor series and Newton Raphson
Assigned: 7th November 2019
Due: 25th November 2019 at 5 PM

Note: Please upload your solution as an ipynb file to the Canvas page.

The purpose of this assignment is to develop your skills in creating approximate functions using Taylor series and find roots of functions using the Newton's iterations.

1. Write the Taylor's series expansion of the function $f(x) = sin(ax)$ near $x = 0$, where $a \neq 0$ is a known constant. Write a Python function to compute the approximate solution. Compute the relative and absolute errors using $math.sin(ax)$ as the exact answer for 3 and 5 terms. Also compute the truncation error at each iteration.

2. Write the Taylor's series expansion of the function $f(x, y) = sin(x)cos(y)$ near the point $(2, 2)$. Write a Python function to compute the approximate solution. Compute the relative and absolute errors using the $math$ module as the exact answer for the first 3 terms. Also compute the truncation error at each iteration.

3. Determine the root of $f(x) = x - 2e^{-x}$ by:

   (a) Using the bisection method. Start with $a = 0$ and $b = 1$, and compute the error at the end of the first three iterations.

   (b) Using the Newton's method, Start at $x_0 = 1$ and compute the roots at the end of the the first three iterations. Compare the error against the bisection approach.

4. Using the Newton-Raphson iteration find the root of the equation $\sqrt{x}+x^2 = 7$. With an initial guess of $x = 7$ compute the number of iterations required for the error in the root to be below $1.0e^{-6}$.