MARCH 12TH, 2025

Differentiation

CE 311K - L26

Numerical Differentiation

Numerical differentiation approximates derivatives using discrete data points

Often required when analytical derivatives are difficult or impossible to compute

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

For numerical differentiation, we use finite values for *h*

Accuracy depends on *h*: too small leads to round-off errors while too large can lead to truncation errors

Forward Difference

Forward Difference approximates the derivative by calculating the slope of a line between the current point and a point slightly *ahead*

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$

Increased accuracy can be achieved by including more points/terms

$$f'(x) \approx \frac{-3f(x) + 4f(x+h) - f(x+2h)}{2h}$$

Backward Difference

Backward Difference approximates the derivative by calculating the slope of a line between the current point and a point slightly *behind*

$$f'(x) \approx \frac{f(x) - f(x-h)}{h}$$

And for **second-order** accuracy:

$$f'(x) \approx \frac{3f(x) - 4f(x-h) + f(x-2h)}{2h}$$

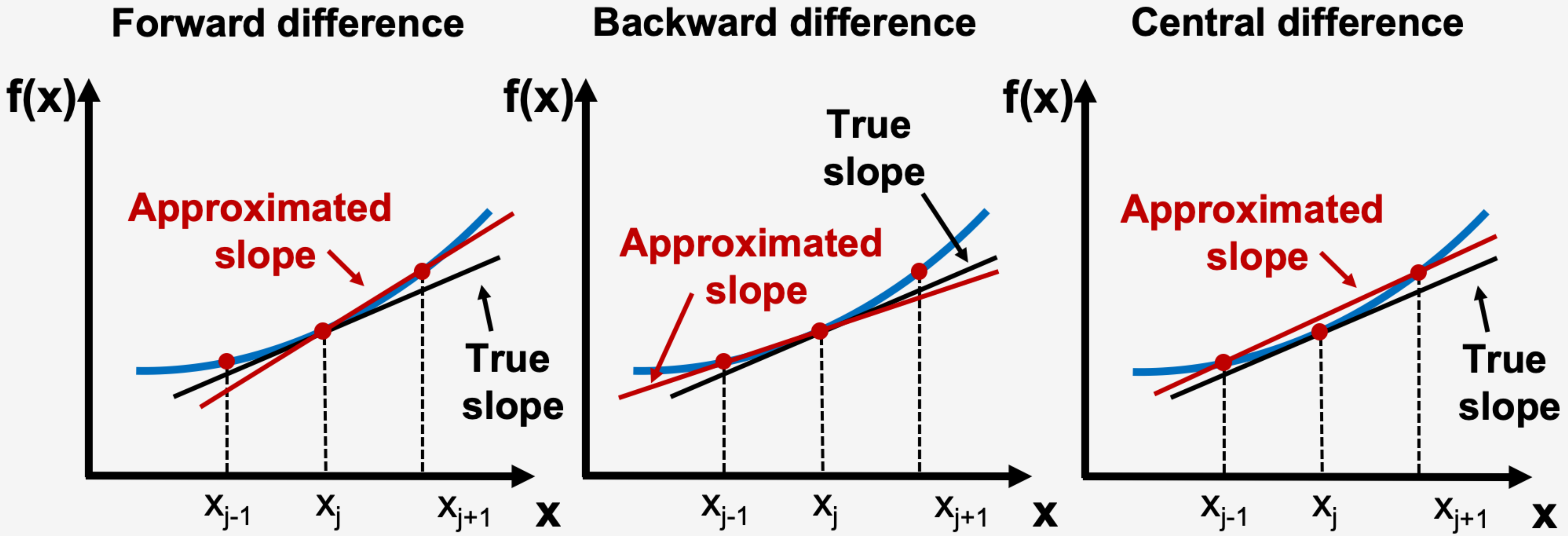
Central Difference

Backward Difference approximates the derivative by calculating the slope of a line between the current point and a point slightly *behind*

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$$

And for **second-order** accuracy:

$$f'(x) \approx \frac{-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)}{12h}$$


Forward difference
Backward difference
Central difference
f(x)
Approximated slope
True slope
x_{j-1}
x_j
x_{j+1}
x