4-7 Inverse Linear Functions

Find the inverse of each relation.

8. $\{(-5,13),(6,10.8),(3,11.4),(-10,14)\}$

SOLUTION:

To find the inverse, exchange the coordinates of the ordered pairs.

(–5, 13) → (13, –5)

(6, 10.8) → (10.8, 6)

(3, 11.4) → (11.4, 3)

(–10, 14) → (14, –10)

The inverse is {(13, –5), (10.8, 6), (11.4, 3), (14, –10)}.

10.

x	y
–8	–36.4
–2	–15.4
1	–4.9
5	9.1
11	30.1

SOLUTION:

Write the coordinates as ordered pairs. Then exchange the coordinates of each pair.

(–8, –36.4) → (–36.4, –8)

(–2, –15.4) → (–15.4, –2)

(1, –4.9) → (–4.9, 1)

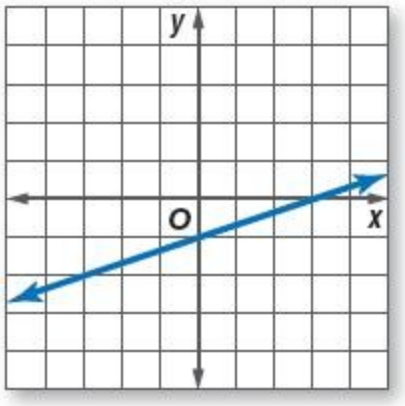
(5, 9.1) → (9.1, 5)

(11, 30.1) → (30.1, 11)

The inverse is {(–36.4, –8), (–15.4, –2), (–4.9, 1), (9.1, 5), (30.1, 11)}.

Graph the inverse of each relation.

12.



SOLUTION:

The graph of the relation passes through the points at (–3, –2), (0, –1), and (3, 0).

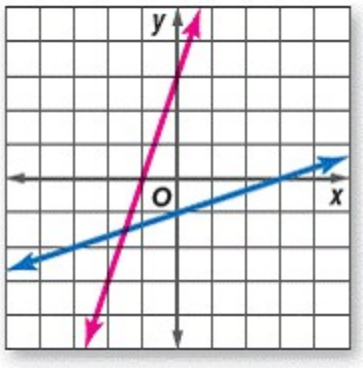
To find points through which the inverse passes, exchange the coordinates of the ordered pairs

(–3, –2) → (–2, –3)

(0, –1) → (–1, 0)

(3, 0) → (0, 3)

Graph these points and then draw a line that passes through them.



4-7 Inverse Linear Functions

Find the inverse of each function.

14. $f(x) = 25 + 4x$

SOLUTION:

$f(x) = 25 + 4x$	Original equation
$y = 25 + 4x$	Replace $f(x)$ with y.
$x = 25 + 4y$	Interchange x and y.
$x - 25 = 4y$	Subtract.
$\frac{x-25}{4} = y$	Divide each side by 4.
$\frac{x-25}{4} = f^{-1}(x)$	Replace y with $f^{-1}(x)$.

Write the final equation in slope-intercept form.

So, $f^{-1}(x) = \frac{1}{4}x - \frac{25}{4}$.

16. $f(x) = 4(x + 17)$

SOLUTION:

$f(x) = 4(x + 17)$	Original equation
$y = 4(x + 17)$	Replace $f(x)$ with y.
$x = 4(y + 17)$	Interchange x and y.
$\frac{x}{4} = y + 17$	Divide each side by 4.
$\frac{x}{4} - 17 = y$	Subtract 17 from each side.
$\frac{x}{4} - 17 = f^{-1}(x)$	Replace y with $f^{-1}(x)$.

Write the final equation in slope-intercept form.

So, $f^{-1}(x) = \frac{1}{4}x - 17$.

18. $f(x) = \frac{2}{5}x + 10$

SOLUTION:

$f(x) = \frac{2}{5}x + 10$	Original equation
$y = \frac{2}{5}x + 10$	Replace $f(x)$with y.
$x = \frac{2}{5}y + 10$	Interchange x and y.
$5x = 2y + 50$	Multiply each side by 5.
$5x - 50 = 2y$	Subtract 50 from each side.
$\frac{5x-50}{2} = y$	Divide each side by 2.
$\frac{5x-50}{2} = f^{-1}(x)$	Replace y with $f(x)$.

Write the final equation in slope-intercept form.

So, $f^{-1}(x) = \frac{5}{2}x - 25$.

21. **LANDSCAPING** At the start of the mowing season, Chuck collects a one-time maintenance fee of $25 from his customers. He charges the Fosters $45 for each cut. The total amount collected from the Fosters in dollars for the season is $C^{-1}(x) = 25 + 45x$, where x is the number of times Chuck mows the Fosters' lawn.

a. Find the inverse function.

b. What do x and $C^{-1}(x)$ represent in the context of the inverse function?

c. How many times did Chuck mow the Fosters' lawn if he collected a total of $1015 from them?

SOLUTION:

a.

$C(x) = 25 + 45x$	Original equation
$y = 25 + 45x$	Replace $C(x)$ with y.
$x = 25 + 45y$	Interchange x and y.
$x - 25 = 45y$	Subtract 25 from each side.
$\frac{x-25}{45} = y$	Divide each side by 45.
$y = \frac{x-25}{45}$	
$C^{-1}(x) = \frac{x-25}{45}$	Replace y with $C^{-1}(x)$.
$C^{-1}(x) = \frac{1}{45}x - \frac{5}{9}$	Slope-intercept form

b. x is the total amount collected from the Fosters, and $C^{-1}(x)$ is the number of times Chuck mowed the Fosters' lawn.

c. Evaluate $C^{-1}(1015)$.

$$C^{-1}(x) = \frac{1}{45}x - \frac{5}{9}$$
$$C^{-1}(1015) = \frac{1}{45}(1015) - \frac{5}{9}$$
$$C^{-1}(1015) = \frac{1015}{45} - \frac{5}{9}$$
$$C^{-1}(1015) = \frac{1015}{45} - \frac{25}{45}$$
$$C^{-1}(1015) = \frac{990}{45}$$
$$C^{-1}(1015) = 22$$

So, Chuck mowed the Foster's lawn 22 times.

Write the inverse of each equation in $f^{-1}(x)$ notation.

22. $3y - 12x = -72$

SOLUTION:

$3y - 12x = -72$	Original equation
$3x - 12y = -72$	Interchange x and y.
$-12y = -3x - 72$	Subtract $3x$ from each side.
$y = \frac{1}{4}x + 6$	Divide each side by -12.
$f^{-1}(x) = \frac{1}{4}x + 6$	Replace y with $f^{-1}(x)$.

24. $-42 + 6y = x$

SOLUTION:

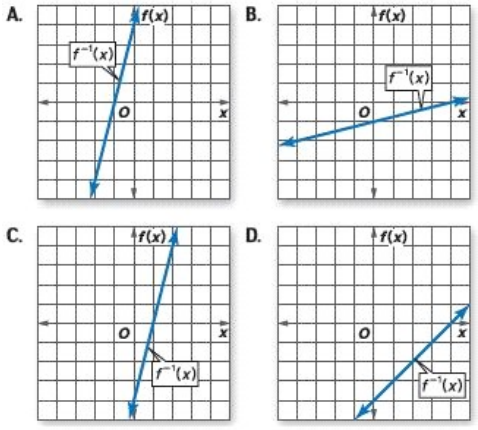
$-42 + 6y = x$	Original equation
$-42 + 6x = y$	Interchange x and y.
$y = 6x - 42$	
$f^{-1}(x) = 6x - 42$	Replace y with $f^{-1}(x)$.

26. $-7y + 2x = -28$

SOLUTION:

$-7y + 2x = -28$	Original equation
$-7x + 2y = -28$	Interchange x and y.
$2y = 7x - 28$	Add $7x$ to each side.
$y = \frac{7}{2}x - 14$	Divide each side by 2.
$f^{-1}(x) = \frac{7}{2}x - 14$	Replace y with $f^{-1}(x)$.

TOOLS & TECHNIQUES Match each function with the graph of its inverse.



28. $f(x) = x + 4$

SOLUTION:

$$f(x) = x + 4$$
$$y = x + 4$$
$$x = y + 4$$
$$x - 4 = y$$
$$f^{-1}(x) = x - 4$$

This equation is shown on graph D.

29. $f(x) = 4x + 4$

SOLUTION:

$$f(x) = 4x + 4$$
$$y = 4x + 4$$
$$x = 4y + 4$$
$$x - 4 = 4y$$
$$\frac{1}{4}x - 1 = y$$
$$f^{-1}(x) = \frac{1}{4}x - 1$$

This equation is shown on graph B.

30. $f(x) = \frac{1}{4}x + 1$

SOLUTION:

$$f(x) = \frac{1}{4}x + 1$$
$$y = \frac{1}{4}x + 1$$
$$x = \frac{1}{4}y + 1$$
$$x - 1 = \frac{1}{4}y$$
$$4x - 4 = y$$
$$f^{-1}(x) = 4x - 4$$

This equation is shown on graph C.

31. $f(x) = \frac{1}{4}x - 1$

SOLUTION:

$$f(x) = \frac{1}{4}x - 1$$
$$y = \frac{1}{4}x - 1$$
$$x = \frac{1}{4}y - 1$$
$$x + 1 = \frac{1}{4}y$$
$$4x + 4 = y$$
$$f^{-1}(x) = 4x + 4$$

This equation is shown on graph A.

Write an equation for the inverse function $f^{-1}(x)$ that satisfies the given conditions.

33. graph of $f(x)$ contains the points (−3, 6) and (6, 12)

SOLUTION:

If the graph of $f(x)$ contains the points (−3, 6) and (6, 12), then the graph of $f^{-1}(x)$ contains the points (6, −3) and (12, 6). Find the slope of the line that passes through these points.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{6-(-3)}{12-6}$$
$$= \frac{9}{6} \text{ or } \frac{3}{2}$$

Choose (12, 6) and find the y-intercept of the line.

$$y = mx + b$$
$$6 = \frac{3}{2}(12) + b$$
$$6 = 18 + b$$
$$-12 = b$$

The line that passes through (6, −3) and (12, 6) is $y = \frac{3}{2}x - 12$. An equation for $f^{-1}(x)$ is $f^{-1}(x) = \frac{3}{2}x - 12$.

35. slope of $f(x)$ is 4; $f^{-1}(5) = 2$

SOLUTION:

Write an equation for $f(x)$ in terms of x and y.

$(y - y_1) = m(x - x_1)$ Point – slope form
$(y - y_1) = 4(x - x_1)$ Replace m with 4.

Find the inverse of $f(x)$.

$(y - y_1) = 4(x - x_1)$ Original equation
$(x - x_1) = 4(y - y_1)$ Interchange x and y.
$\frac{1}{4}(x - x_1) = (y - y_1)$ Multiply each side by $\frac{1}{4}$.
$\frac{1}{4}(x - 5) = (y - 2)$ $x_1 = 5, y_1 = 2$
$\frac{1}{4}(x - 5) + 2 = y$ Add 2 to each side.
$y = \frac{1}{4}(x - 5) + 2$
$y = \frac{1}{4}x - \frac{5}{4} + 2$ Distributive Property
$y = \frac{1}{4}x + \frac{3}{4}$ Add.
$f^{-1}(x) = \frac{1}{4}x + \frac{3}{4}$ Replace y with $f^{-1}(x)$.

39. **PROBLEM SOLVING** If $f(x) = \frac{1}{a}x + 7$ and $f^{-1}(x) = 2x - b$, find a and b.

SOLUTION:

Write the inverse of $f(x) = \frac{1}{a}x + 7$.

$$f(x) = \frac{1}{a}x + 7$$
$$y = \frac{1}{a}x + 7$$
$$x = \frac{1}{a}y + 7$$
$$x - 7 = \frac{1}{a}y$$
$$ax - 7a = y$$

So, $f^{-1}(x) = ax - 7a$. Compare this equation to the given equation, $f^{-1}(x) = 2x - b$. Because both equations represent the same line, the values for slope m are equal. Thus, $a = 2$. Like slope, the values for the y-intercept b are equal. So, $7a = b$. Substitute $a = 2$ into this equation. Thus, $b = 14$.

44. The table shows some values of a linear function.

x	–2	0	3	7
y	0	1	2.5	?

What is the missing value in the table?

A 1
B 4
C 4.5
D 5.5

SOLUTION:

Find the slope of the line given by the table. The line passes through the points (–2, 0) and (0, 1). Find the slope of the line.

$$m = \frac{x_2 - x_1}{y_2 - y_1} = \frac{1-0}{0-(-2)} = \frac{1}{2}$$

The slope is $\frac{1}{2}$ and the y-intercept is 1. So the equation of the line is $y = \frac{1}{2}x + 1$.
Find y when x is 7.

$$y = \frac{1}{2}x + 1$$
$$y = \frac{1}{2}(7) + 1$$
$$y = \frac{7}{2} + \frac{2}{2} = \frac{9}{2} \text{ or } 4.5$$

So, the correct choice is C.

45. For the function $f(x) = -\frac{1}{3}x - 3$, what is the value of x when $f(x) = –6$?

F –5
G –1
H 3
J 9

SOLUTION:

Substitute –6 for $f(x)$ and solve for x.

$$f(x) = -\frac{1}{3}x - 3$$
$$-6 = -\frac{1}{3}x - 3$$
$$-6 + 3 = -\frac{1}{3}x - 3 + 3$$
$$-3 = -\frac{1}{3}x$$
$$-3(-3) = -3\left(-\frac{1}{3}x\right)$$
$$9 = x$$

So, the correct choice is J.

46. **GRIDDABLE** For what value of the domain does $x = f(x)$ if $f(x) = \frac{1}{2}x + 5$?

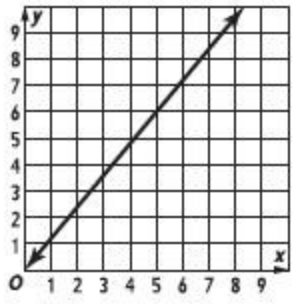
SOLUTION:

Find x when $f(x)$ is x.

$$f(x) = \frac{1}{2}x + 5$$
$$x = \frac{1}{2}x + 5$$
$$x - \frac{1}{2}x = \frac{1}{2}x - \frac{1}{2}x + 5$$
$$\frac{1}{2}x = 5$$
$$2\left(\frac{1}{2}x\right) = 2(5)$$
$$x = 10$$

When x is 10, then $f(x) = 10$.

47. The graph shows the adjusted price y for an item with an original price of x after a certain percent increase.



What is the slope of the line that takes the increased price as an input x, and returns the original price as the output y?

A $\frac{1}{5}$
B $\frac{6}{5}$
C $\frac{5}{6}$
D 5

SOLUTION:
Find the slope of the line with points (0, 0) and (5, 6).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{6 - 0}{5 - 0}$$
$$= \frac{6}{5}$$

The correct choice is B.