Rate of Change and Slope

Key

A **rate of change** is a ratio that compares the amount of change in a dependent variable to the amount of change in an independent variable.

$$\text{rate of change} = \frac{\text{change in dependent variable}}{\text{change in independent variable}}$$

Before we get into the slope formula that you all know, let's look at some application problems...

we'll review this more tomorrow!

SLOPE

If all of the connected segments have the same rate of change, then they all have the same steepness and together form a straight line. The constant rate of change of a line is called the *slope* of the line.

Slope of a Line

The **rise** is the difference in the y-values of two points on a line.

The **run** is the difference in the x-values of two points on a line.

The **slope** of a line is the ratio of rise to run for any two points on the line.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$$

(Remember that y is the dependent variable and x is the independent variable.)

Rise = 3   Run = 2
Run = 2
−6
Rise = 3
Run = 2   Slope = $\frac{3}{2}$
Rise = 3
0   2   4   6
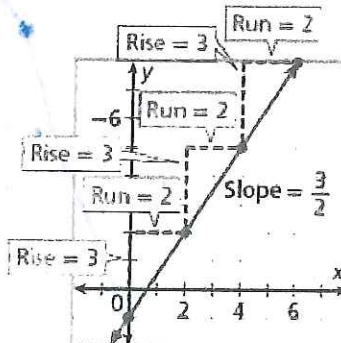
Application

The table shows the average temperature (°F) for five months in a certain city. Find the rate of change for each time period. During which time period did the temperature increase at the fastest rate?

Independent →
dependent →

Month	2	3	5	7	8
Temp. (°F)	56	56	63	91	98

(handwritten annotations: 1, 2, 2, 1 above; 0, 7, 18, 7 below)

Step 1 Identify the dependent and independent variables.

Avg. Temperature depends on time of year (month) so month is the independent variable.

Step 2 Find the rates of change.

2 to 3   change in dependent = 0 / change in independent = 1 $= 0$ |rate|

3 to 5 $\dfrac{7}{2}$ = 3.5 degrees increase per month
* also called the unit rate.

5 to 7 $\dfrac{18}{2}$ = 9 degree increase per month

7 to 8 $\dfrac{7}{1}$ = 7 degree change in 1 month

Graph the data show the rates of change.

Month	2	3	5	7	8
Temp. (°F)	56	56	63	91	98
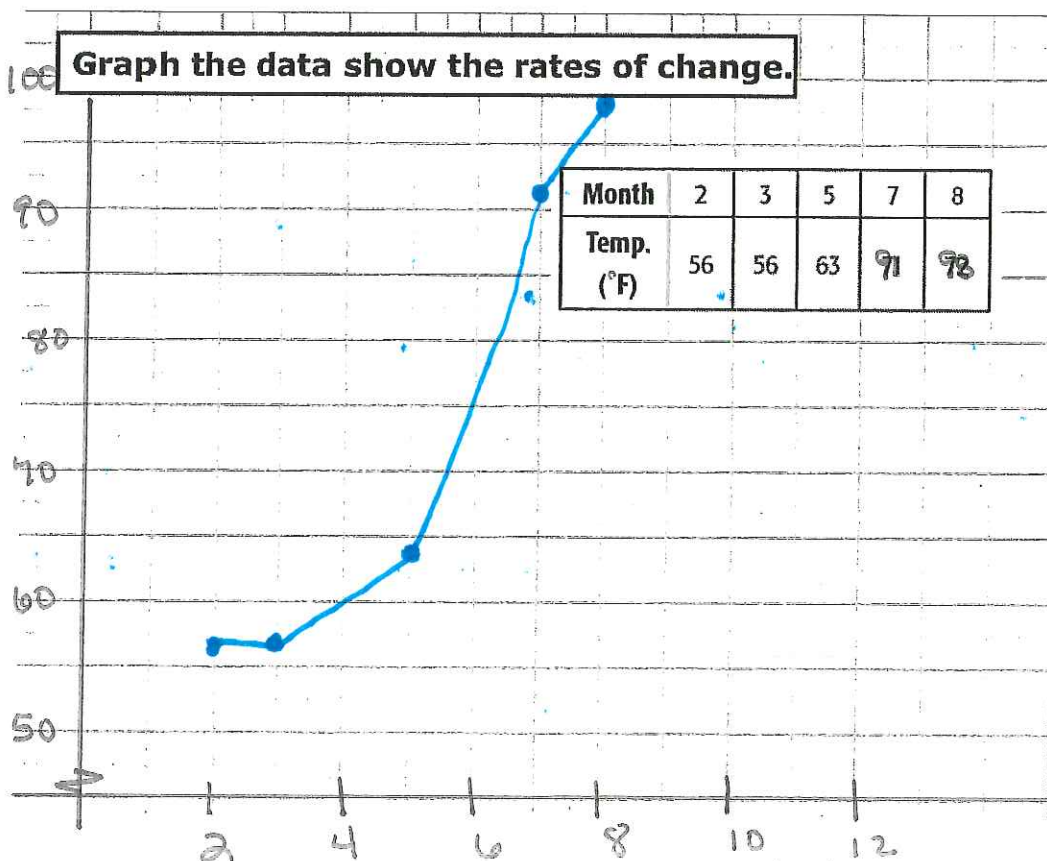
Temp. increased at fastest rate between 5th & 7th month.

A note . "slope" in each section

The table shows the balance of a bank account on different days of the month. Find the rate of change during each time interval. During which time interval did the balance decrease at the greatest rate?

indep.
dependent

Day	1	6	16	22	30
Balance ($)	550	285	210	210	175

(handwritten annotations above table: 5↑ 10↑ 6↑ 8↑; below table: 265 75 0 35)

Step 1 Identify the dependent and independent variables.

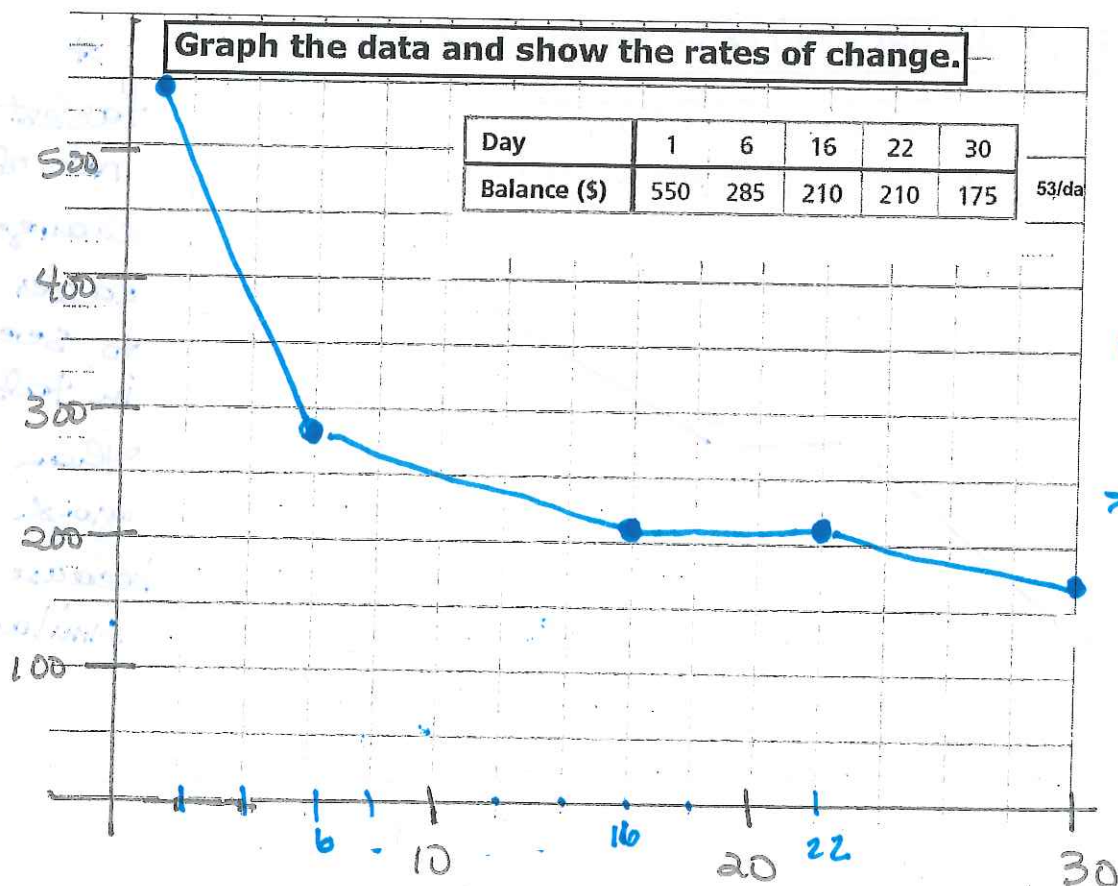How much money you have depends on what day you check your balance!

Step 2 Find the rates of change.

Day 1 to 6 : $\dfrac{-265}{5}$ = $53 per day decrease

6 to 16 : $\dfrac{-75}{10}$ = $ 7.50 per day decrease

16 to 22 : $\dfrac{0}{6}$ = $ 0 per day

22 to 30 = $-\dfrac{35}{8}$ = ≈ $ 4.38 per day decrease

Graph the data and show the rates of change.

Day	1	6	16	22	30
Balance ($)	550	285	210	210	175

53/da

fastest
rate of change
(steepness of
line)
between
day 1 &
day 6

* Decreasing! *

(Graph with y-axis labeled 500, 400, 300, 200, 100 and x-axis labeled 6, 10, 16, 20, 22, 30)

The table shows the number of bikes made by a company for certain years. Find the rate of change for each time period. During which time period did the number of bikes increase at the fastest rate?

Independent →
Dependent →

Year	1	2	5	7	11
Bikes	32	35	47	47	61

(annotations above: 1 3 2 4)
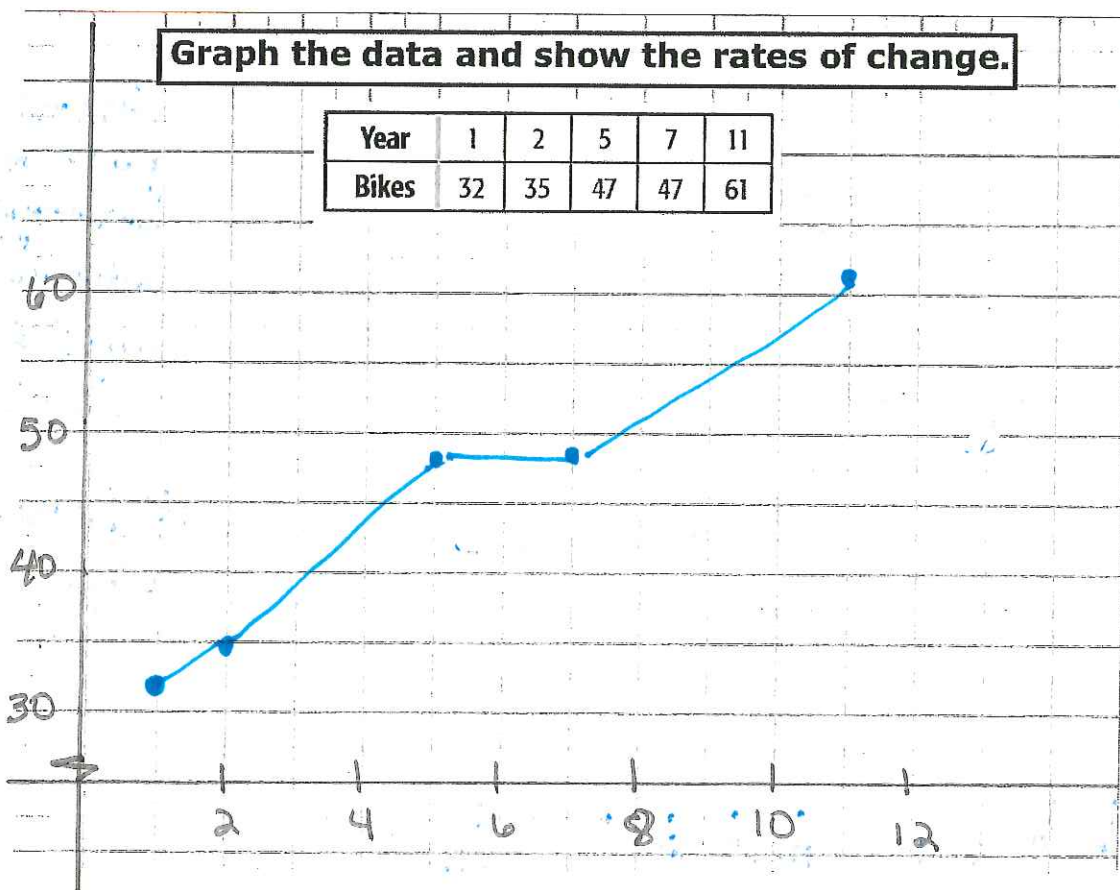*(annotations below: 3 12 0 14)*

1 to 2: $\frac{3}{1}$ = 3 bikes per year increase

2 to 5: $\frac{12}{3}$ = 4 bikes per year increase

5 to 7: $\frac{0}{2}$ = 0 increase

7 to 11: $\frac{14}{4}$ = 3.5 bikes per year

} fastest rate of change b/t years 2 & 5

Graph the data and show the rates of change.

Year	1	2	5	7	11
Bikes	32	35	47	47	61

★ fastest rate of change easier to see in table than on graph because all similar.