6-4 Notes: Elimination Using Multiplication

Elimination Using Multiplication Some systems of equations cannot be solved simply by adding or subtracting the equations. In such cases, one or both equations must first be multiplied by a number before the system can be solved by elimination.

Example 1:

$4\left(\begin{array}{l}x + 10y = 3\\4x + 5y = 5\end{array}\right) = -4x - 40y = -12$

$+\ 4x + 5y = 5$

$-35y = -7$
$\dfrac{-35y}{-35} = \dfrac{-7}{-35}$

$\boxed{y = \dfrac{1}{5}}$ $\boxed{\left(1, \dfrac{1}{5}\right)}$ ✱Solution

$x + 10y = 3$

$x + 10\left(\dfrac{1}{5}\right) = 3$ $\boxed{x = 1}$

$x + 2 =$

Example 2:

$2\left(\begin{array}{l}3x - 2y = -7\\2x - 5y = 10\end{array}\right) \Rightarrow 6x - 4y = -14$

$-3 + \ -6x + 15y = -30$

$11y = -44$
$\boxed{y = -4}$

$3x - 2y = -7$
$3x - 2(-4) = -7$
$3x + 8 = -7$
$3x = -15$
$\boxed{x = -5}$ $\boxed{(-5, -4)}$ ✱Solution

Exercises

Use elimination to solve each system of equations.

1. $-2\left(\begin{array}{l}2x + 3y = 6\\x + 2y = 5\end{array}\right) \Rightarrow 2x + 3y = 6$

$+\ -2x - 4y = -10$

$-y = -4$
$\boxed{y = 4}$ $\boxed{(-3, 4)}$ ✱

$2x + 3(4) = 6$
$2x + 12 = 6$ $\boxed{x = -3}$
$2x = -6$

2. $2\left(\begin{array}{l}2m + 3n = 4\\-m + 2n = 5\end{array}\right)$ $2m + 3n = 4$

$+\ -2m + 4n = 10$

$7n = 14$
$\boxed{n = 2}$ $\boxed{(-1, 2)}$ ✱

$2m + 3n = 4$
$2m + 3(2) = 4$ $\boxed{m = -1}$
$2m + 6 = 4$

$2m = -2$
$m = -1$

3. $3(3a - b = 2) \Rightarrow 6a - 2b = 4$
$a + 2b = 3$

$+\ a + 2b = 3$

$7a = 7$
$\boxed{a = 1}$

$a + 2b = 3$
$1 + 2b = 3$ $\boxed{(1, 1)}$ ✱
$2b = 2$
$\boxed{b = 1}$

4. $3\left(\begin{array}{l}4x + 2y = -4\\-2(5x + 3y = -17)\end{array}\right) \Rightarrow 12x + 6y = -12$

$+\ -10x - 6y = 34$

$2x = 22$
$\boxed{x = 11}$

$4(11) + 2y = -4$
$2y = -4 - 44$ $\boxed{(11, -24)}$ ✱
$2y = -48$
$\boxed{y = -24}$

Solve Real-World Problems Sometimes it is necessary to use multiplication before elimination in real-world problems.

Example : During a canoeing trip, it takes Raymond 4 hours to paddle 12 miles upstream. It takes him 3 hours to make the return trip paddling downstream. Find the speed of the canoe in still water.

Read   You are asked to find the speed of the canoe

Solve Let $c =$ rate of canoe in still water
Let $w =$ rate of water current

	r	t	d	$r \cdot t = d$
Against the Current	$c - w$	4	12	$4(c-w) = 12$
With the Current	$c + w$	3	12	$3(c+w) = 12$

system $\begin{cases} 4c - 4w = 12 \\ 3c + 3w = 12 \end{cases}$ $\Rightarrow$ $\begin{array}{l} 3[4c - 4w = 12] \\ 4[3c + 3w = 12] \end{array}$ $\Rightarrow$ $\begin{array}{l} 12c - 12w = 36 \\ +12c + 12w = 48 \end{array}$

$$\frac{24c}{24} = \frac{84}{24} \qquad c = 3.5$$

Rate of canoe in still water
$$= 3.5 \text{ miles per hour}$$

Exercises

1. An airplane traveling with the wind flies 450 miles in 2 hours. On the return trip, the plane takes 3 hours to travel the same distance. Find the speed of the airplane if the wind is still.

	r	t	d	$r \cdot t = d$
With the wind	$p + w$	2	450	$2(p+w) = 450$
against the wind	$p - w$	3	450	$3(p-w) = 450$

let $p =$ rate of plane w/ no wind   system $\begin{cases} 2(p+w) = 450 \\ 3(p-w) = 450 \end{cases}$
let $w =$ rate of wind

$(2p + 2w = 450)^3 \Rightarrow 6p + 6w = 1350$
$(3p - 3w = 450)^2 \Rightarrow +6p - 6w = 900$

$$\frac{12p}{12} = \frac{2250}{12} \qquad \begin{array}{l} p = 187.50 \text{ miles/hr} \\ \text{w/ no wind} \end{array}$$