

Key

4-4 Notes: Parallel and Perpendicular Lines

Parallel Lines

2 non-vertical lines are parallel if they have same slope & different y-intercepts. All vertical lines are parallel. All horizontal lines are parallel.

Example: Write an equation in slope-intercept form for the line that passes through $(-1, 6)$ and is parallel to the graph of $y = 2x + 12$.

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 $m = 2$
 $(-1, 6)$
 $x, y,$

Step 1: $y - y_1 = m(x - x_1)$
 $y - 6 = 2(x - (-1))$
 $y - 6 = 2(x + 1)$
 $y - 6 = 2x + 2$
 $+6 \quad +6$

$y = 2x + 8$

Example: Find the slope intercept form of the line parallel to the function $3x - 4y = 9$ and contains the point $(-4, 7)$.

Step 1: Slope

$$3x - 4y = 9$$

$$-4y = -3x + 9$$

$$y = \frac{3}{4}x - \frac{9}{4}$$

$$m = \frac{3}{4}$$

Step 2: $y - y_1 = m(x - x_1)$

$$y - 7 = \frac{3}{4}(x - (-4))$$

$$y - 7 = \frac{3}{4}(x + 4)$$

$$y - 7 = \frac{3}{4}x + 3$$

$+7 \quad +7$

$y = \frac{3}{4}x + 10$

$x, y,$

Example: Given the points A $(-3, 5)$ B $(4, 5)$ C $(1, 1)$ and D $(-6, 1)$ represent quadrilateral ABCD, are any of the sides parallel?

Slope of AB

$$m = \frac{5 - 5}{4 - (-3)} = \frac{0}{7}$$

$$m = 0$$

Slope of BC

$$m = \frac{1 - 5}{1 - 4} = \frac{-4}{-3} = \frac{4}{3}$$

$$m = \frac{4}{3}$$

Slope of CD

$$m = \frac{1 - 1}{-6 - 1} = \frac{0}{-7}$$

$$m = 0$$

Slope of AD

$$m = \frac{1 - 5}{-6 - (-3)} = \frac{-4}{-3} = \frac{4}{3}$$

\overline{AB} is parallel to \overline{CD}
 \overline{BC} is parallel to \overline{AD}

Quadrilateral is a parallelogram

Perpendicular Lines

2 lines are perpendicular if the product of their slopes = -1. Slopes are negative reciprocals of one another.
All vertical & horizontal lines are perpendicular.

Example: Write an equation in slope-intercept form for the line that passes through $(-4, 2)$ and is perpendicular to the graph of $2x - 3y = 9$.

Step 1: Slope

$$2x - 3y = 9$$

$$-3y = -2x + 9$$

$$y = \frac{2}{3}x - 3$$

$$m = \frac{2}{3} \text{ so } \perp m = -\frac{3}{2}$$

Step 2: $y - y_1 = m(x - x_1)$

$$y - 2 = -\frac{3}{2}(x - (-4))$$

$$y - 2 = -\frac{3}{2}(x + 4)$$

$$y - 2 = -\frac{3}{2}x - 6$$

$$y = -\frac{3}{2}x - 4$$

Example: Write the equation of the perpendicular bisector of segment AB if A $(-4, 10)$ and B $(2, -4)$.

1) Perpendicular Slope

Slope of \overline{AB}

$$m = \frac{-4 - 10}{2 - (-4)} = \frac{-14}{6} = -\frac{7}{3}$$

$$\perp m = \frac{3}{7}$$

2) Midpoint

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$(x, y) = \left(\frac{-4 + 2}{2}, \frac{10 + (-4)}{2} \right)$$

$$(x, y) = (-1, 3)$$

3) Substitute the perpendicular slope and midpoint you found into the point-slope formula, then reduce to slope-intercept form.

$$y - y_1 = m(x - x_1)$$

$$y - 3 = \frac{3}{7}(x - (-1))$$

$$y - 3 = \frac{3}{7}(x + 1)$$

$$y - 3 = \frac{3}{7}x + \frac{3}{7}$$

$$+3 \text{ or } \frac{21}{7}$$

$$y = \frac{3}{7}x + \frac{3}{7} + \frac{21}{7}$$

$$y = \frac{3}{7}x + \frac{24}{7}$$