

6-2 Notes

Substitution

Solve by Substitution One method of solving systems of equations is **substitution**.

Example 1: Use substitution to solve the system of equations.

$$\begin{aligned} y &= 2x \\ 4x - y &= -4 \end{aligned}$$

$$\begin{aligned} 4x - 2x &= -4 \\ 2x &= -4 \\ \boxed{x = -2} \end{aligned}$$

$$\begin{aligned} y &= 2x \\ y &= 2(-2) \\ \boxed{y = -4} \end{aligned}$$

solution:
 $(-2, -4)$

Example 2: Solve for one variable, then substitute.

$$\begin{aligned} x + 3y &= 7 \\ 2x - 4y &= -6 \end{aligned}$$

$$\begin{aligned} x &= 7 - 3y \\ 2(7 - 3y) - 4y &= -6 \\ 14 - 6y - 4y &= -6 \\ -10y &= -20 \\ \boxed{y = 2} \end{aligned}$$

$$\begin{aligned} x &= 7 - 3y \\ x &= 7 - 3(2) \\ x &= 7 - 6 \\ \boxed{x = 1} \end{aligned}$$

(1, 2) = solution

Exercises:

Use substitution to solve each system of equations.

$$\begin{aligned} y &= 4x \\ 3x - y &= 1 \end{aligned}$$

$$\begin{aligned} 3x - 4x &= 1 \\ -x &= 1 \\ \boxed{x = -1} \end{aligned}$$

$$\begin{aligned} y &= 4x \\ y &= 4(-1) \\ \boxed{y = -4} \end{aligned}$$

solution: $(-1, -4)$

$$\begin{aligned} x &= 2y \\ y &= x - 2 \end{aligned}$$

$$\begin{aligned} y &= 2y - 2 \\ -y &= -2 \\ \boxed{y = 2} \end{aligned}$$

$$\begin{aligned} x &= 2y \\ x &= 2(2) \\ \boxed{x = 4} \end{aligned}$$

solution: $(4, 2)$

$$\begin{aligned} x &= 2y - 3 \\ x &= 2y + 4 \end{aligned}$$

$$\begin{aligned} 2y - 3 &= 2y + 4 \\ -3 &\neq 4 \end{aligned}$$

no solution!

$$\begin{aligned} y &= 3x + 2 \\ y &= -x - 6 \end{aligned}$$

$$\begin{aligned} 3x + 2 &= -x - 6 \\ +x & \quad +x \\ 4x + 2 &= -6 \\ 4x &= -8 \\ \boxed{x = -2} \end{aligned}$$

$$\begin{aligned} y &= 3x + 2 \\ y &= 3(-2) + 2 \\ \boxed{y = -4} \end{aligned}$$

solution: $(-2, -4)$

Solve Real-World Problems Substitution can also be used to solve real-world problems involving systems of equations. It may be helpful to use tables, charts, diagrams, or graphs to help you organize data.

Example: CHEMISTRY How much of a 10% saline solution should be mixed with a 20% saline solution to obtain 1000 milliliters of a 12% saline solution?

Let s = # ml. of 10% solution

Let t = # ml. of 20% solution

Use a table to organize the information.

	10% saline	20% saline	12% saline
Total milliliters	s	t	1000
Milliliters of saline	$0.10s$	$0.20t$	$0.12(1000)$

Write a system of equations.

$$\begin{cases} s + t = 1,000 \\ 0.1s + 0.2t = 0.12(1,000) \end{cases}$$

step 1: $s = 1,000 - t$

step 2: $0.1(1,000 - t) + 0.2t = 120$

$$100 - 0.1t + 0.2t = 120$$

$$\begin{array}{r} 100 + 0.1t = 120 \\ -100 \quad -100 \end{array}$$

$$\frac{0.1t}{0.1} = \frac{20}{0.1}$$

$$t = 200 \text{ ml}$$

$$s + t = 1000$$

$$\begin{array}{r} s + 200 = 1000 \\ -200 \quad -200 \end{array}$$

$$s = 800 \text{ ml}$$

Exercises

1. **SPORTS** At the end of the 2007–2008 football season, 38 Super Bowl games had been played with the current two football leagues, the American Football Conference (AFC) and the National Football Conference (NFC). The NFC won two more games than the AFC. How many games did each conference win?

Let x = AFC

Let y = NFC

$$x + y = 38 \text{ games}$$

$$y = x + 2$$

$$x + (x + 2) = 38$$

$$2x + 2 = 38$$

$$\begin{array}{r} 2x = 36 \\ \hline x = 18 \text{ games} \\ y = 20 \text{ games} \end{array}$$

2. **CHEMISTRY** A lab needs to make 100 gallons of an 18% acid solution by mixing a 12% acid solution with a 20% solution. How many gallons of each solution are needed?

Let x = 12%

Let y = 20%

use 80 gallons 20%
w/ 20 gallons 10%
to equal 100 gallons
of 18% saline

10%	20%	18% saline
x	y	$= 100$
$.1x$	$.2y$	$= .18(100)$

$$\begin{array}{l} x + y = 100 \\ x = 100 - y \end{array}$$

$$\begin{array}{l} .1(100 - y) + .2y = 18 \\ 10 - .1y + .2y = 18 \end{array}$$

$$\begin{array}{l} .1y = 8 \\ y = 80 \text{ gallons} \\ x = 20 \text{ gallons} \end{array}$$