3-5 Notes: Arithmetic Sequences as Linear Functions

Recognize Arithmetic Sequences A **sequence** is a set of numbers in a specific order. If the difference between successive terms is constant, then the sequence is called an **arithmetic sequence**.

Arithmetic Sequence	*pattern of numbers that increase or decrease at a constant rate called the common difference.*
Terms of an Arithmetic Sequence	a_1, a_2, a_3 *1st term, 2nd term, 3rd term* *each new term is generated by adding "d" — common difference*
nth Term of an Arithmetic Sequence	$a_n = a_1 + (n-1)d$ ← *common difference* *any term, 1st term, term #*

Example 1: Determine whether the sequence
1, 3, 5, 7, 9, 11, . . . is an arithmetic sequence. Justify your answer.

$+2 \ +2 \ +2$ *yes! common difference = 2*

Example 2: Write an equation for the nth term of the sequence 12, 15, 18, 21, ...

$+3 \ +3 \ +3$ $d = 3$

$a_1 = 12$ $a_n = 3n + 9$

$a_n = a_1 + (n-1)d$

$a_n = 12 + (n-1)(3)$

$a_n = 12 + 3n - 3$

$\quad = 9 + 3n$

Exercises
Find the next three terms of each arithmetic sequence.

4. 9, 13, 17, 21, 25, *29, 33, 37*
$+4 \ +4$

5. 4, 0, −4, −8, −12, *−16, −20, −24*
$-4 \ -4$

Write an equation for the nth term of each arithmetic sequence.

7. 1, 3, 5, 7, . . .

$d = 2$
$a_1 = 1$

$a_n = a_1 + (n-1)d$

$a_n = 1 + (n-1)(2)$

$\quad = 1 + 2n - 2$

$a_n = 2n - 1$

8. −1, −4, −7, −10, . . .

$d = -3$
$a_1 = -1$

$a_n = a_1 + (n-1)d$

$a_n = -1 + (n-1)(-3)$

$a_n = -1 - 3n + 3$

$a_n = 2 - 3n$

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Arithmetic Sequences and Functions An **arithmetic sequence** is a linear function in which n is the independent variable, a_n is the dependent variable, and the common difference d is the slope. The formula can be rewritten as the function $a_n = a_1 + (n-1)d$, where n is a counting number.

Example: SEATING There are 20 seats in the first row of the balcony of the auditorium. There are 22 seats in the second row, and 24 seats in the third row.

a. Write a function to represent this sequence.

$20, 22, 24$

$a_1 = 20$ $\qquad a_n = a_1 + (n-1)d$

$d = 2$ $\qquad\quad = 20 + (n-1)(2)$
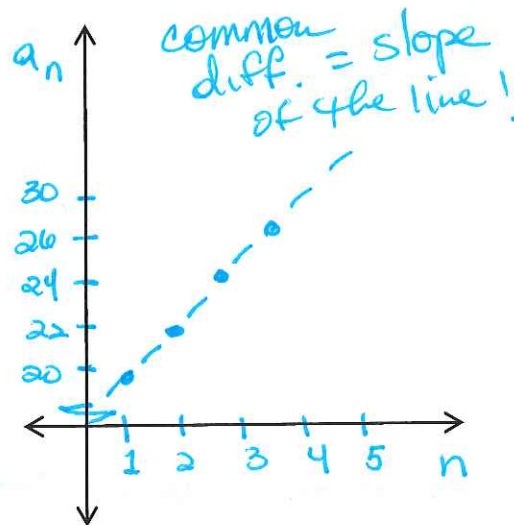
$\qquad\qquad a_n = 20 + 2n - 2$

$\qquad\qquad \boxed{a_n = 2n + 18}$

c. How many seats would be in the 25th row?

$a_{25} = 2(25) + 18$

$a_{25} = 68$

68 Seats in 25th row

b. Graph the function.

X	Y
1	20
2	22
3	24
4	26
5	30

common diff. = slope of the line!

a_n

30, 26, 24, 22, 20

1 2 3 4 5 n

Exercises

1. KNITTING Sarah learns to knit from her grandmother. Two days ago, she measured the length of the scarf she is knitting to be 13 inches. Yesterday, she measured the length of the scarf to be 15.5 inches. Today it measures 18 inches. Write a function to represent the arithmetic sequence.

a. Write a function to represent this sequence.

$13, 15.5, 18$

$a_1 = 13$ $\qquad a_n = a_1 + (n-1)d$

$d = .5$ $\qquad\quad = 13 + (n-1)(2.5)$

$\qquad\qquad a_n = 13 + 2.5n - 2.5$

$\qquad\qquad \boxed{a_n = 2.5n + 10.5}$

b. Graph the function.

X	Y
1	13
2	15.5
3	18
4	19.5

common diff = slope of the line!

18, 15.5, 13

1 2 3
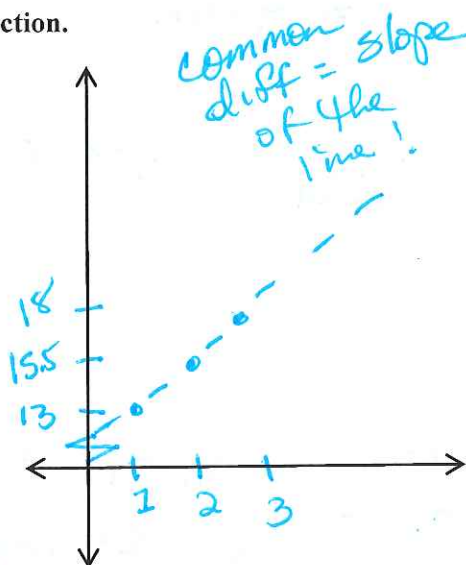
c. What would be the 17th term?

$a_{17} = 2.5(17) + 10.5$

$a_{17} = 53$

After 17 days the scarf would be 53 inches long