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Conclusion

Does a 6 bit counter exists ?

Can we create a **deterministic** network that will iterate over all 6 bits string – in any order – ?

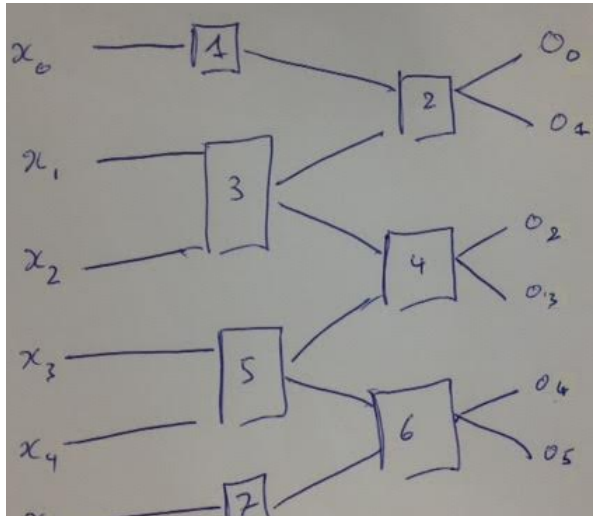
Problem Statement

Computable bijections on $\{0,1\}^6$

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Conclusion

Does a 6 bit counter exists ?



Can we bruteforce that problem ?

There are 2^{44} different networks which is approximately 17000 billions.

We could hope for an answer in a few days.

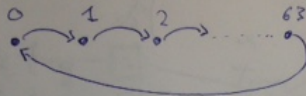
But we can drastically restrict our search space with a **few** observations.

Main idea : we are going to bruteforce on **sub networks** and select those who respect a **necessary** condition.

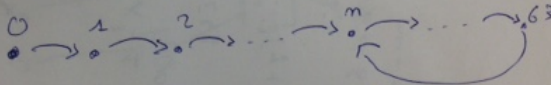
How such a network's dynamic would look like ?

There are only two possibilities (up to any permutation):

Case 1:



Case 2:



Implication on the layer function of the network

The function that our counting network computes on one layer is either:

- ▶ **A bijection**
- ▶ **An almost-bijection** f i.e, $\exists! x_0, y_0 \in \{0, 1\}^6$ such that f' is a bijection where:

$$\begin{aligned}\forall x \neq x_0 \quad f'(x) &= f(x) \\ f'(x_0) &= y_0\end{aligned}$$

We are going to check on both cases.

The $4 * 16$ property

Let get rid of the last 4 bits:

0000000000000000	0000000000000000	1111111111111111	1111111111111111
0000000000000000	1111111111111111	0000000000000000	1111111111111111

We see that each of the 2 bits patterns: **00, 01, 10, 11** occurs **16** times in this enumeration.

It's the **$4 * 16$ property**.

The $4 * 16$ property

Hence the sub network responsible for these 2 bits must have the **4*16** property to be eligible as a being a sub network of a 6 bits bijective counter network.

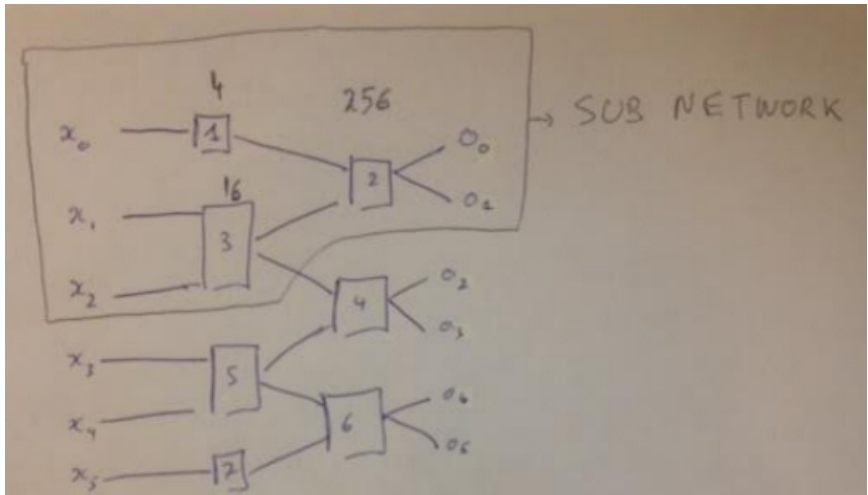
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The 4 * 16 property

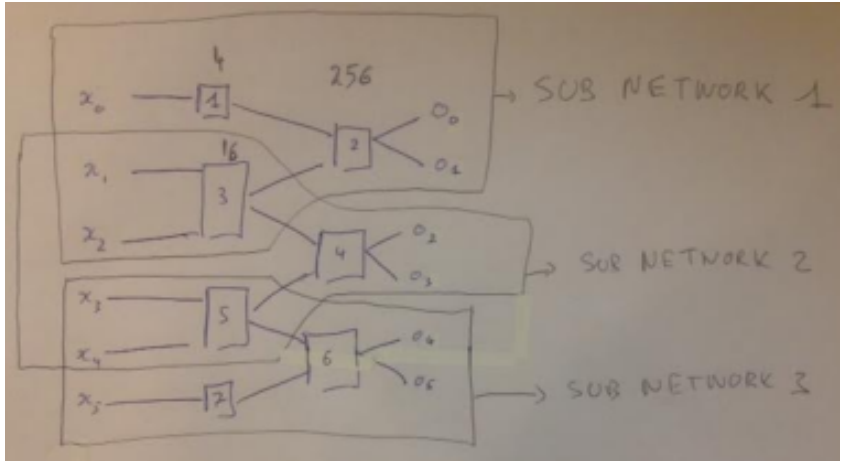


Compute all the $4 * 16$ sub networks

By exhaustive search we find **288** (over $4 * 16 * 256 = 16384$) sub networks with this property.

By removing equivalent networks we are left with **72 circuits for the first 2 bits**.

4*16 holds for middle and ending bits



4*16 holds for middle and ending bits

- ▶ **72** networks for the first 2 bits
- ▶ **216** networks for the middle 2 bits
- ▶ **72** networks for the last 2 bits

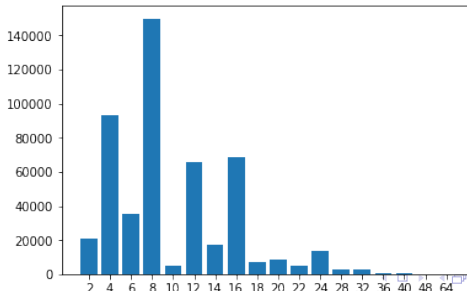
How to conclude I ? Combine it !

Hence by combining these we have $72 * 216 * 72 \simeq 10^6$
6 bits networks to test.

How to conclude II ? Count orbits !

Over all these potential networks we count **497664** bijections.
For each of these bijections we have to **count their orbits**, we
have a winner **iff it has only 1 orbit**.

We do not find such a network, here there's the histogram of
orbits:



Conclusion

Conclusion: There are no bijective 6bits counters.

2*161517 property

If we have an almost-bijection **all 6 bits sequences will be reached by our network but one.**

Also one bit string will be reached **twice.**

It means that **at least one** of our sub network will see:

- ▶ 2 patterns **16** times
- ▶ 1 pattern **15** times
- ▶ 1 pattern **17** times

2*161517 property

We call this the **2*161517 property**.

$2^{*161517}$ does not occur !

By enumeration, **there exist no sub network with the $2^{*161517}$ property.**

Hence, we cannot hope for an **almost-bijective counter**.

No 6bit counter network :'(

By case distinction, **there's no 6bit counter network.**

But there are **0 to 62** counters and this kind of methods helps to exhibit **at least 24**.

Sources ?

All our sources for these computations are available here:

[https://github.com/cosmo-sterin/ER_MolProg_Project2/
tree/master/circuit_sim](https://github.com/cosmo-sterin/ER_MolProg_Project2/tree/master/circuit_sim)