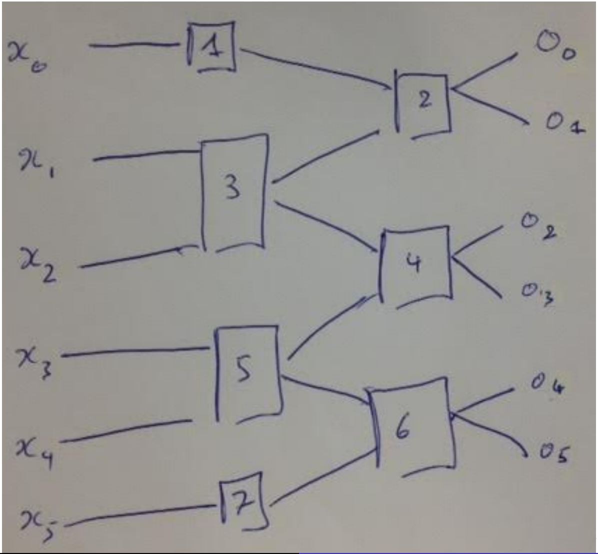
Table of contents

Does a 6 bit counter exists ?

Can we create a **deterministic** network that will iterate over all 6 bits string – in any order – ?

Does a 6 bit counter exists ?

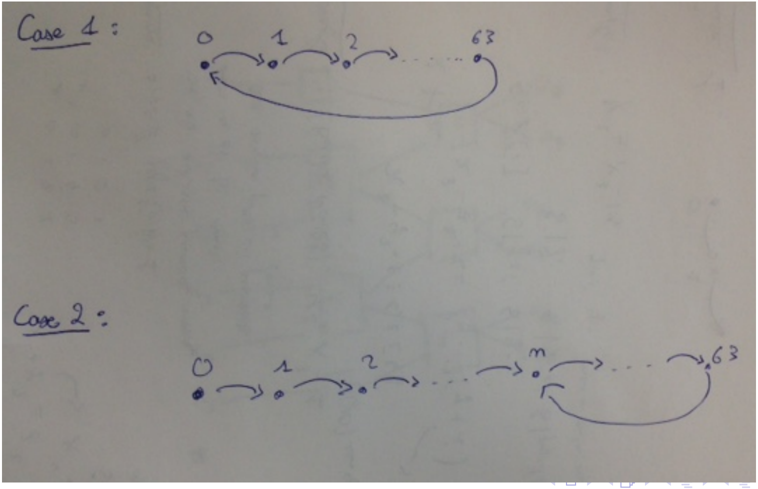


Can we bruteforce that problem ?

There are $\mathbf{2^{44}}$ differents networks which is approximatively 17000 billions.
We could hope for an answer in a few days.
But we can drastically restrict our search space with a **few** observations.

How such a network's dynamic would look like ?

There are only two possibilities (up to any permutation):



Implication on the computed function by the network

The function that our counting network coumputes is either:

- **A bijection**
- **An almost bijection** f i.e, $\exists! x_0, y_0 \in \{0,1\}^6$ such that f' is a bijection where:

$$\begin{aligned} \forall x \neq x_0 \quad f'(x) &= f(x) \\ f'(x_0) &= y_0 \end{aligned}$$

We are going to check on both cases.

The $4 * 16$ property

If we have a bijection it will **enumerate** all strings of $\{0,1\}^6$.
After reordordering it will look like:

```
0000000000000000000000000000000011111111111111111111111111111111
0000000000000000111111111111111100000000000000001111111111111111
0000000011111111000000001111111100000000111111110000000011111111
0000111100001111000011110000111100001111000011110000111100001111
0011001100110011001100110011001100110011001100110011001100110011
0101010101010101010101010101010101010101010101010101010101010101
```

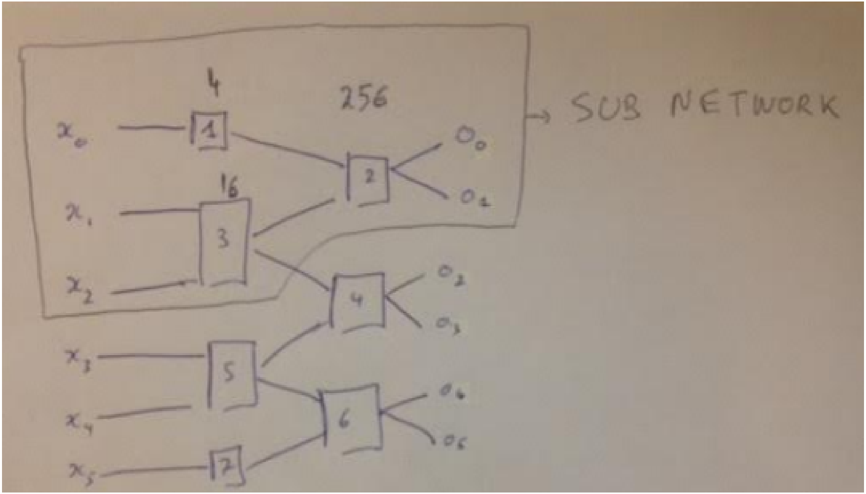
The $4 * 16$ property

Let's focus on the two first bits, we can organise our sequences this way:

0000000000000000	0000000000000000	1111111111111111	1111111111111111
0000000000000000	1111111111111111	0000000000000000	1111111111111111
0000000011111111	0000000011111111	0000000011111111	0000000011111111
0000111100001111	0000111100001111	0000111100001111	0000111100001111
0011001100110011	0011001100110011	0011001100110011	0011001100110011
0101010101010101	0101010101010101	0101010101010101	0101010101010101

We see that **each of the 2 bits pattern happens 16 times**.
It's the **$4 * 16$ property**.

The $4 * 16$ property



Compute all the $4 * 16$ sub networks

By exausthive search we find **288** (over 16384) sub networks with this property.

By removing equivalent networks we are left with **72 circuits for the 2 first bits**.

4*16 holds for middle and ending bits

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How to conclude I ? Combine it !

The work we have done for the 2 first bits can also be done for the 2 middle bits and the 2 last bits **that also have the 4x16 property**.

We count:

- 72 networks for the 2 first bits
- 216 networks for the 2 middle bits
- 72 networks for the 2 last bits

Hence by combining these we have $72 * 216 * 72 \simeq 10^6$ 6 bits network to test.

How to conclude II ? Count orbits !

Over all these potential networks we count **497664** bijections. For each of these bijections we have to **count their orbits**, we have a winner iif it has only 1 orbit.

We do not find such a network, here there's the histogram of orbits: