1. For $n=2$:

$x_1 \backslash x_2$	0	1
0	$(1-p)^2$	$p(1-p)$
1	$p(1-p)$	p^2

We know that samplemean $\bar{X} = \frac{1}{2}(X_1+X_2)$.

Hence $\bar{X}$ has distribution

$\bar{X}$	0	$\frac{1}{2}$	1
$p(\bar{x})$	$(1-p)^2$	$2p(1-p)$	p^2

For general n: we know that, $n\bar{X} = X_1 + \cdots + X_n$ follows Binomial (n, p) distribution.

$\bar{X}$	0	$\frac{1}{n}$	$\cdots$	$\frac{k}{n}$	$\cdots$	1
$p(\bar{x})$	$(1-p)^n$	$\binom{n}{1}(1-p)^{n-1}p^1$	$\cdots$	$\binom{n}{k}(1-p)^{n-k}p^k$	$\cdots$	p^n

2. (1) $X \sim N(60, 10^2)$, Hence $X = 10Z + 60$,

where $Z \sim N(0,1)$.

$$
\begin{aligned}
P(55 < X < 75) &= P(55 < 10Z + 60 < 75) \\
&= P(-5 < 10Z < 15) \\
&= P(-0.5 < Z < 1.5) \\
&= P(-0.5 < Z < 0) + P(0 < Z < 1.5) \\
&= P(0 < Z < 0.5) + P(0 < Z < 1.5)
\end{aligned}
$$

From the normal table, we know that

$$P(0<Z<0.5)+P(0<Z<1.5)$$
$$= 0.1915 + 0.4332 = 0.6247$$

(2) X not be the top 5% or the bottom 20%, then we should use the quantile q_1, q_2 such that

$$P(X>q_1) = 0.05, \quad P(X<q_2) = 0.2$$

Let

$$q_1 = 60+10\cdot r_1 \qquad \cdots \qquad (*)$$
$$q_2 = 60+10\cdot r_2$$

then $P(X>q_1) = P(60+10Z > 60+10r_1)$
$= P(Z>r_1)$

Similarly $P(X<q_2) = P(Z<r_2)$.

Then, we should find in the table:

$$P(0<Z<r_1) = 0.5-0.05 = 0.45$$
$$P(0<Z<(-r_2)) = 0.5-0.2 = 0.30$$

From the table, we know that

$$r_1 = 1.64, \quad -r_2 = 0.84$$

plug in to (*) we get

$$q_1 = 76.4, \quad q_2 = 51.6,$$

Hence score X should satisfy

$$51.6 < X < 76.4.$$