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Threshold selection by clustering gray levels of boundary

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Abstract

In this paper, threshold selection is considered in the continuous image rather than in digital image. We prove that, for each given object within 2D image, its optimal threshold is determined by the mean of the gray values of the points lying on its continuous boundary. Thus, we try to deduce threshold from the gray values of the boundary rather from the gray values of the given discrete sampling points (pixels or edge pixels). By the scheme, we well overcome some disadvantages existing in the threshold methods based on the histogram of edge pixels. Besides, the proposed method has the ability to well handle the image whose histogram has very unequal peaks and broad valley.

Keywords: Threshold selection; Clustering algorithm; Image segmentation

1. Introduction

A popular tool used in image segmentation is thresholding. Thresholding assumes that image present a number of components, each of a nearly homogeneous value, and that one can separate the components by a proper choice of intensity threshold. Many thresholding techniques are proposed in 2D image processing (Sahoo et al., 1988; Rosenfeld and Kak, 1982), including the thresholding methods selecting threshold by analyzing histogram of whole image (Olivo, 1994; Otsu, 1979; Glasbey, 1993; Kapur et al., 1985), the thresholding methods selecting threshold from histogram of edge pixels (Weszka et al., 1974; Wang and Haralick, 1984; Milgram and Herman, 1979; Katz, 1965; Yanowitz and Bruckstein, 1989), etc. In this paper, we will present a new method on threshold selection.

1.1. The problem of optimal threshold selection

In this paper, 2D image is treated as the discrete sampling of the underlying 2D continuous function represented as $f(x,y)$. Therefore, the boundary of the objects within 2D image actually should be some implicitly defined continuous curves determined by $f(x,y)$. We know that, the boundary usually is such curve on the either side of which gray values have sharp change. Thus, in terms of computer vision theory, each boundary consists of such points that are zero-value points of the Laplacian function of 2D image and have high

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gradient values (Marr and Hildreth, 1980; Haralick, 1984). Mathematically, the boundaries within 2D image could be represented as follows:

$$\begin{cases} l(x,y) = 0 \\ \|\Delta f(x,y)\| \geqslant T \end{cases} \tag{1}$$

where

$$l(x,y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

and

$$\|\Delta f(x,y)\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

represent the Laplacian function and gradient magnitude function of $f(x,y)$, respectively. T is a predefined gradient threshold. Sometimes, it is selected adaptively in different local neighborhood as that in (Peter and David, 1996; Jung and Park, 1988). Each point lying on boundary has an intermediate gray value between object and background gray levels as illustrated in Fig. 1, where, O is a boundary point of 1D continuous function and has an in between gray value. Thus, the boundary of object within 2D image is a continuous curve that separates pixels of the object from pixels of the background, and has the gray level ranges between the object and the background gray levels in the sense of statistics. However, the points lying on boundaries differ from the edge pixels detected by 2D edge detection techniques. Meanwhile, the gray values of boundary points differ from the gray values of the edge pixels. The edge pixels usually have the gray values belonging to object or background.

In principal, for each object within 2D image, its boundary is the exact curve separating the object from background. Thus, we try to deduce the optimal threshold from the object's boundary. It is obvious that, better a threshold approximate the gray values of the points lying on the object's boundary in the sense of least square error, better the threshold separates the pixels of the object from the pixels of the background. Thus, we think that, for each object in 2D image, gray level that approximates the gray values of the points lying on the object's boundary with least square error will determine an optimal threshold for this object. In other words, let $C(x,y)$ represent the boundary of one object within 2D image. Then the optimal threshold for the object is determined by the solution of the following optimization problem:

$$\min_r \int_{C(x,y)} (f(x,y) - r)^2 \, \mathrm{d}(x,y), \quad r \in R \tag{2}$$

where, $\int_{C(x,y)} (f(x,y) - r)^2 \, \mathrm{d}(x,y)$ represents the integration of error function $(f(x,y) - r)^2$ over the boundary curve $C(x,y)$. Thus, the problem of selecting optimal threshold for one object within 2D image is converted into the problem of solving above optimization problem (2) for the object.

In this paper, we will solve the optimization problem (2) and present a new method to select multiple optimal thresholds for different objects within 2D image.

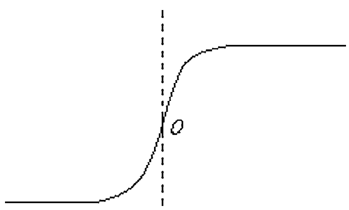


Fig. 1. Step-like edge point (O) of 1D function.

1.2. Related works

Thresholding techniques selecting threshold from the histogram of 2D image assume that gray values of each object are possible to cluster around a peak of the histogram of 2D image and try to directly compute the location of valley or peaks from the histogram (Sahoo et al., 1988; Rosenfeld and Kak, 1982; Olivo, 1994; Otsu, 1979; Glasbey, 1993; Kapur et al., 1985). However, in many cases, interesting structures within 2D image only occupy a small percentage of the whole image, such as bone in CT image, signature in a sheet, and etc. In these cases, histogram of whole image exhibits several peaks of very unequal amplitude separated by a broad valley or contains only one peak and a "shoulder". For images with such histogram, interesting structures cannot be well "seen" or "recognized" directly from the histogram of whole image, and the threshold methods based on the histogram of image are limited.

Thresholding techniques selecting threshold from histogram of edge pixels can overcome the above difficulty to some extent (Weszka et al., 1974; Wang and Haralick, 1984; Milgram and Herman, 1979; Katz, 1965; Yanowitz and Bruckstein, 1989). In many cases, they can handle image whose histogram has very unequal peaks or broad valley very well. They are based on the fact that, no matter how much percentage one object occupies in the whole 2D image, its threshold actually is possible to be deduced from the gray levels of the edge pixels of this object. Katz (1965) pointed out that since the pixels in the neighborhood of an edge have higher edge values, the gray level histogram for these pixels should have a single peak at a gray level between the object and the background gray levels. This gray level is, therefore, a suitable choice of the threshold value. It provides the basis for designing threshold selection method based on histogram of edge pixels.

Weszka et al. (1974) suggested a bi-level thresholding method. They first filter 2D image by a Laplacian operator, and then select the valley of histogram of pixels with high Laplacian value (edge pixels) as threshold.

Wang and Haralick (1984) proposed a multi-threshold selection method based on the histogram of edge pixels. In their methods, edge pixels are first classified, on the basis of their neighborhoods, as being relatively dark or relatively light. Then two histograms of gray level are obtained respectively for these two sets of edge pixels. Threshold is selected as one of the highest peaks of the two histograms. By recursively using the procedure, the multiple thresholds can be obtained.

Milgram and Herman (1979) selected thresholds from images containing several object classes by clustering thinned edge pixels in a 2D histogram whose axes represent gray level value and edge value. Where, each such edge cluster suggests its average gray level as a threshold.

Similar method as above introduced ones is applied to select local adaptive threshold (Yanowitz and Bruckstein, 1989). Where, 2D image is partitioned into several non-overlapping sub-images of equal area, and a threshold for each sub-image is selected from histogram of edge pixels of the sub-image by similar method as that in the references (Wang and Haralick, 1984; Milgram and Herman, 1979).

Thresholding techniques based on the histogram of edge pixels try to deduce the threshold from the gray values of edge pixels. We know that, because of the "double responding" phenomenon of edge pixels, the pixels closely distributing both side of the boundary are detected out by edge detector. Generally, the "double responding" edge pixels could be categorized into two classes: one belongs to object and has the gray value of object, and another belongs to background and has the gray value of background. Thus, the histogram of edge pixels of each object has two peaks (clusters) with similar amplitude (see Fig. 2). One peak (cluster) represents the edge pixels in the background and another represents the edge pixels in the object. Thresholding technique in the reference (Weszka et al., 1974) is based on the fact. However, the technique fails for images having several object classes. In reference (Wang and Haralick, 1984), threshold is selected as one of the higher peaks on the histogram of edge pixels. However, selecting directly threshold from the histogram of edge pixels might mistakenly classify some edge pixels and some pixels around these edge pixels. For example, in Fig. 2, selecting the peak of cluster ***A*** as threshold is possible to mistakenly classify some edge pixels in cluster ***A*** and some pixels around these edge pixels (they belong to background) into object. In references (Milgram and Herman, 1979; Yanowitz and Bruckstein, 1989), each edge pixel is assigned a new gray value that is the average value of gray values of two adjacent

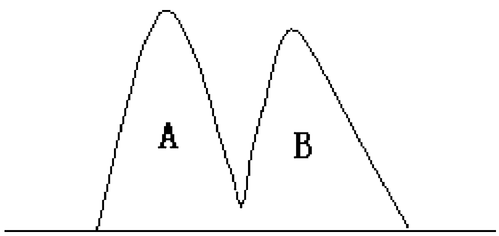


Fig. 2. Each object has its two peaks in the histogram of "double responding" edge points. ***A*** represents edge points in the background and ***B*** represents edge points in the object.

points of this edge pixel. By using the scheme, for each given object, "double-peaks" phenomenon does not appear on the histogram of its edge pixels, and only one peak exists in the histogram of its edge pixels. However, the problem what are the suitable values to be assigned to different edge pixels is still open, and it lacks a clear mathematical explanation.

As we have introduced, thresholding techniques based on the histogram of edge pixels have different drawbacks. In this paper, we will introduce a new threshold method that deduces the optimal threshold from gray values of the points lying on the boundary rather than from histogram of whole 2D image or from histogram of edge pixels. In this way, we well overcome the drawbacks in the thresholding techniques based on the histogram of edge pixels (Weszka et al., 1974; Wang and Haralick, 1984; Milgram and Herman, 1979; Katz, 1965; Yanowitz and Bruckstein, 1989). Meanwhile, comparing with the thresholding techniques based on the histogram of whole 2D image, this method can still well handle image whose histogram has very unequal peaks or broad valley. The proposed method is shown to be effective through lots of examples and by comparing its experimental results with the ones of Otsu's threshold method (Otsu, 1979) and Kapur's threshold method (Kapur et al., 1985).

2. Theoretical analysis on optimal threshold

Let $C(x,y)$ represent a boundary of a given object in 2D image. Recall that, the optimal threshold of this object is the solution of the optimization problem (2). Below, we solve the optimization problem (2). Let $F(r) = \int_{C(x,y)} (f(x,y) - r)^2 \,\mathrm{d}(x,y)$. To find the threshold that minimizes $F(r)$, we differentiate $F(r)$ with respect to r and set the result to zero:

$$F'(r) = \int_{C(x,y)} 2 \cdot f(x,y)\,\mathrm{d}(x,y) - \int_{C(x,y)} 2 \cdot r\,\mathrm{d}(x,y) = 0$$

Then, we have

$$r = \frac{\int_{C(x,y)} f(x,y)\,\mathrm{d}(x,y)}{\int_{C(x,y)} \mathrm{d}(x,y)} \tag{3}$$

It shows that, solution of the optimization problem (2) is the mean of gray values of points lying on the boundary $C(x,y)$. Thus, the optimal threshold of one given object accurately equals to the mean of gray values of points lying on the boundary of this object. In order to compute the optimal threshold for a given object within 2D image, we only need to compute the mean of gray values of points lying on the boundary of this object. Recall that, each point lying on the boundary has an intermediate gray value between object and background gray levels. Thus, statistically, the mean of gray values of points lying on a boundary will determine a gray level that is between object and background gray levels.

For the given object, the histogram of its "double responding" edge pixels has two peaks as shown in Fig. 2. However, the histogram of the points lying on its boundary has the unique cluster as shown in Fig. 3. In other words, for a given object, the gray values of points lying on its boundary will cluster together around their mean. Besides, we notice that, for the same object within 2D image, the cluster formed by the gray values of points lying on its boundary locates between the two clusters formed by the gray values of its "double responding" edge pixels. This is intuitively displayed in Fig. 3, where, solid curve represents the cluster of gray values of points lying on the boundary, and two dashed curves represent clusters of gray values of "double responding" edge pixels of the same object. The fact shows that, for each object within 2D image, "two-peaks" phenomenon existing in histogram of edge pixels is avoided and gray values of points lying on its boundary will manifest as an cluster around their mean.

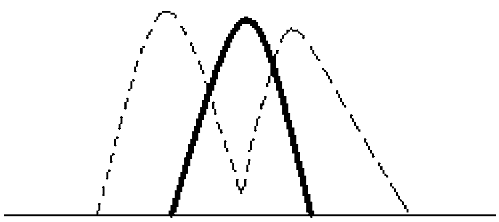


Fig. 3. Histogram of gray levels of true boundary (solid curve) and histogram of "double responding" edge points (two dashed curves) of each object in 2D image.

3. Computation of discrete sampling of gray values of boundaries

Generally, it is impossible to compute the mean of gray values of boundary by analytical method from discrete 2D image. Thus, we will compute discrete sampling of gray values of points lying on the boundaries within 2D image, and estimate the mean from these discrete sampling. We first introduce a method to compute discrete sampling points of the boundaries within 2D image.

In this paper, 2D image is treated as the discrete sampling data sampled from the grid-points of 2D regular grids as shown in Fig. 4, where all squares constitute the continuous region occupied by 2D image. Since boundaries of the objects within 2D image are some continuous curves contained in the continuous region, they will divide the set of all squares into two classes: edge-cells, which are the squares intersected by boundary, and non-edge cells. The boundaries are included in the set of all edge-cells. Thus, we can detect and extract the boundaries from 2D image by first detecting all edge-cells from 2D image and then approximating the boundary in each edge-cell. In order to avoid confusing the meanings of "edge pixel", "edge curve" and "step-like edge" with the meaning of "edge" of square (each square has four edges), we express the "edge" of square with italic, i.e., *edges* of a square.

Among four *edges* of each edge-cell, there are at least two *edges* intersected by the boundary. Thus, all edge-cells are possible to be recognized by examining whether there are at least two interacted *edges* in each square.

Let p_1, p_2 represent two vertices of one *edge* in a square, and $g(p_i)$, $l(p_i)$, $i = 1, 2$ represent gradient magnitudes and Laplacian values of the two vertices, respectively. We know that, if this *edge* is intersected by a boundary, then it has the following properties:

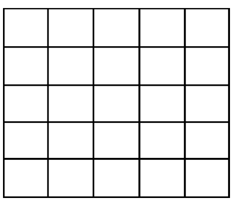


Fig. 4. Regular grid of 2D image.

(1) Its two vertices p_1, p_2 are a pair of zero-crossing points, namely, $l(p_1) \cdot l(p_2) < 0$.
(2) Its two vertices p_1, p_2 both have high gradient values. Namely, for the predefined gradient threshold $\widetilde{T}, g(p_1) + g(p_2) \geqslant 2 \cdot \widetilde{T}$.

Therefore, based on these two rules, we can recognize all *edges* intersected by the boundary from 2D image. Here, gradient threshold $\widetilde{T}$ may be selected as that in edge detection (Rosenfeld and Kak, 1982). However, it is a bit small than the gradient threshold selected for detecting thinned edge pixels, since the two vertices p_1, p_2 might have different gradient values. Certainly, it is better to select $\widetilde{T}$ adaptively in different local region.

After marking all *edges* intersected by the boundary in 2D image, by marching all squares, all edge-cells can be found out.

Recall that, in each edge-cell, the boundary actually is the zero-value isoline of Laplacian function of 2D image. Thus, in each edge-cell, the intersecting points between the four *edges* and the boundary are some zero-value points of Laplacian function of 2D image. Their positions at the *edges* and their gray values can be computed by interpolation. The simplest method to compute the position and the gray value of an intersecting point is to linearly interpolate the positions and the gray values of two vertices of the *edge* at which this intersecting point locates (Lorensen and Cline, 1987; Sabin, 1986). However, good interpolation methods are possible to decrease the error. By using the above scheme, eventually, we obtain the discrete sampling points of the boundaries within 2D image and the gray values of these discrete sampling points.

The computed discrete sampling points usually are valid discrete sampling points of boundary for estimating the mean of the gray values of points lying on the boundary. We know that, because of noise in 2D image, few possible discrete sampling points of boundary might not be computed. However, the lost sampling points usually have a too small ratio comparing with the computed discrete sampling points to change the validity of the computed discrete sampling points.

4. Threshold selection method

The discussion above has demonstrated that, for each object within 2D image, the optimal threshold is determined by the mean of the gray values of points lying on its boundary. Besides, the ideal mean could be estimated or deduced from the discrete sampling of gray values of the boundary that are computed by the method introduced in Section 3. In what follows, these results are used in the selection of bi-level threshold or multi-threshold from 2D image.

4.1. Bi-level threshold selection

In the case of 2D image containing only one object class and one background class, the unique cluster exists in the histogram of discrete sampling points of the boundary. Thus, threshold can be selected as the average value of gray values of the discrete sampling points of the boundary. However, if there is much noise or other small objects in 2D image, it is better to select threshold at the main peak of histogram of all discrete sampling points. In this case, some computed discrete sampling points might belong to noise or the other small objects. Selecting threshold at the main peak may decrease the affection of noise or small object. Usually, in the case that 2D image contains one main object and many other very small objects, the discrete sampling points of the boundary of the main object would manifest itself as the main peak in the histogram of the computed discrete sampling points.

In some situations, 2D image contains more than one object class. But we are only interested in one main object, and try to segment this object by a global threshold. In the case, if possible, we could compute solely discrete sampling points of the boundary of the main object by suitably selecting gradient threshold T in Eq. (1), and then deducing solely the threshold of the main object from the gray values of the computed discrete sampling points.

In Figs. 5 and 6, six different gray images are shown. Correspondingly, their histograms are shown in Figs. 7 and 8, respectively. We notice that, these histograms all are not bimodal mode. The three images shown in Fig. 5 have complex histograms and the three images shown in Fig. 6 have such histograms that exhibit the peaks of very unequal amplitude separated by a broad valley or contains only one peak and a "shoulder". However, by suitably selecting gradient threshold T, the discrete sampling points of the boundary of the main object contained within these images could be extracted solely, and their histograms all exhibit one main peak or even the unique and obvious cluster. See Figs. 9 and 10, where the histograms of discrete sampling points of the boundary of the main object contained within the six images are shown. Thus, the bi-level thresholds are easily computed from these images based on the threshold method we propose above. The segmentation results of the six images are shown in Figs. 11(c), 12(c), 13(c), 14(c), 15(c) and 16(c), respectively. We notice that, the main and interesting features within the six images are well segmented.

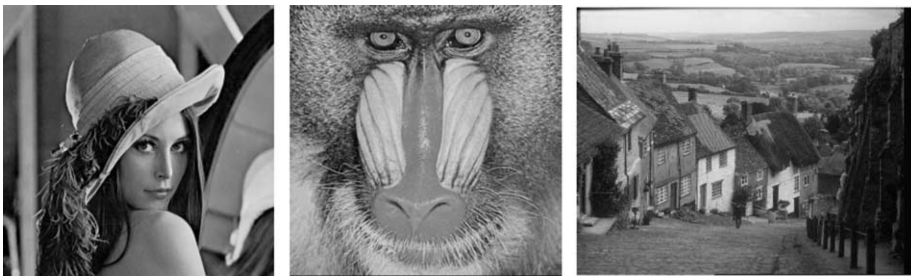


Fig. 5. 256-level gray images of the girl, the baboon and the goldhill.

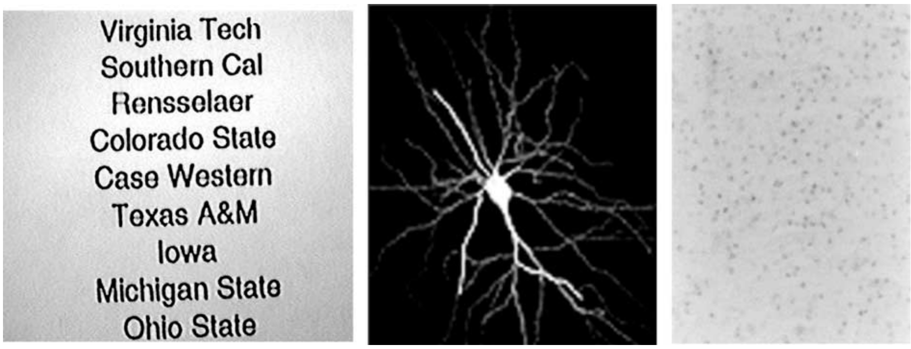


Fig. 6. 256-level gray images of the characters, the nerve cell and the mouse nervous tissue.

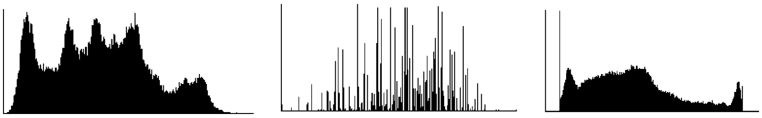


Fig. 7. Histograms of images of the girl, the baboon and the goldhill, shown in Fig. 5.

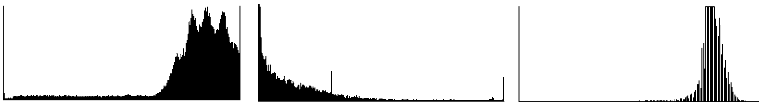


Fig. 8. Histograms of images of the characters, the nerve cell and the mouse nervous tissue, shown in Fig. 6.

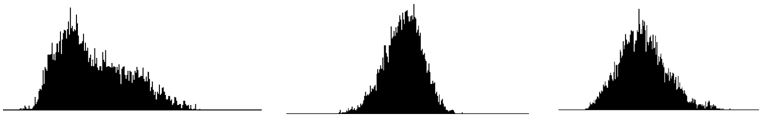


Fig. 9. Histograms of the discrete sampling points of the boundaries within images of the girl, the baboon and the goldhill, shown in Fig. 5.

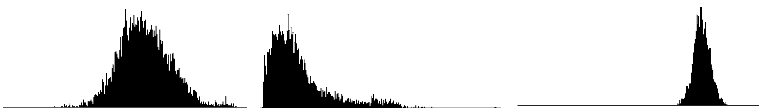


Fig. 10. Histograms of the discrete sampling points of the boundaries within images of the characters, the nerve cell and the mouse nervous tissue, shown in Fig. 6.

In order to compare the proposed threshold method with the conventional threshold methods, the segmentation results of the same six images by Otsu's threshold method (Otsu, 1979) and Kapur's threshold method (Kapur et al., 1985) are provided as well, which are shown in Figs. 11(a), 12(a), 13(a), 14(a), 15(a) and 16(a) and in Figs. 11(b), 12(b), 13(b), 14(b), 15(b) and 16(b), respectively. We observe that, the proposed method and the Otsu's threshold method obtain the much similar binary images when thresholding the images of girl, baboon, goldhill and character (see

Fig. 11. Binary images of the girl: (a) Otsu method ($t = 101$); (b) Kapur, Sahoo and Wong method ($t = 139$); (c) our method ($t = 90$).

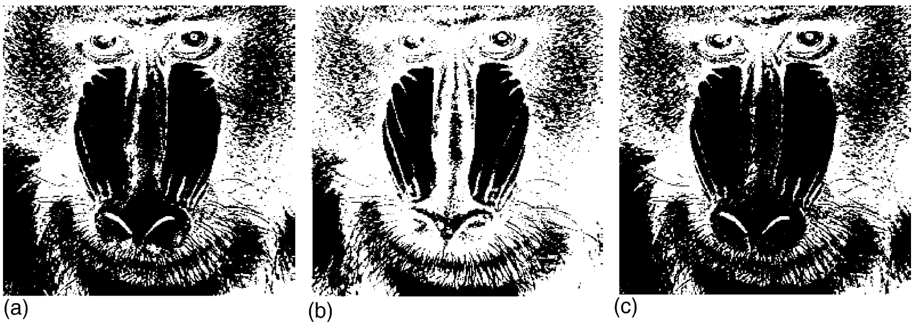


Fig. 12. Binary images of the baboon: (a) Otsu method ($t = 125$); (b) Kapur, Sahoo and Wong method ($t = 142$); (c) our method ($t = 122$).

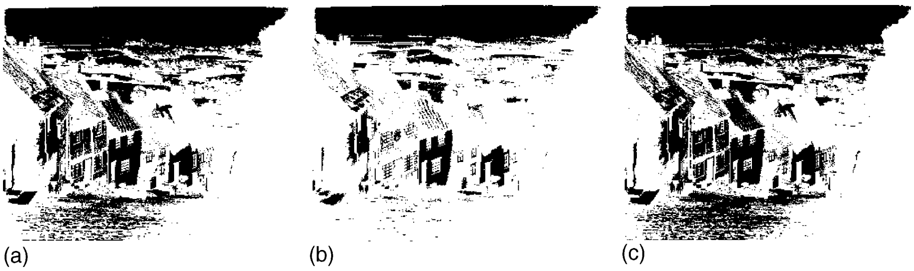


Fig. 13. Binary images of the goldhill: (a) Otsu method ($t = 113$); (b) Kapur, Sahoo and Wong method ($t = 133$); (c) our method ($t = 107$).

Figs. 11–14). However, the proposed threshold method excels the Otsu's threshold method when thresholding the images of nerve cell and mouse nervous tissue (see Figs. 15 and 16). The histograms of these two images exhibit the peaks of very unequal amplitude separated by a broad valley or contains only one peak and a "shoulder" (see Fig. 8). When thresholding the images of girl, baboon, goldhill, character and nerve cell, the proposed threshold method excels the Kapur's threshold method (see Figs. 11–15). When segmenting the image of mouse nervous tissue, the proposed threshold method also provides a good result, which is better than that of the Otsu's threshold method, but is not as good as that of the Kapur's threshold method. Thus, from the visual analysis and the comparison, the proposed threshold method is shown to be an effective

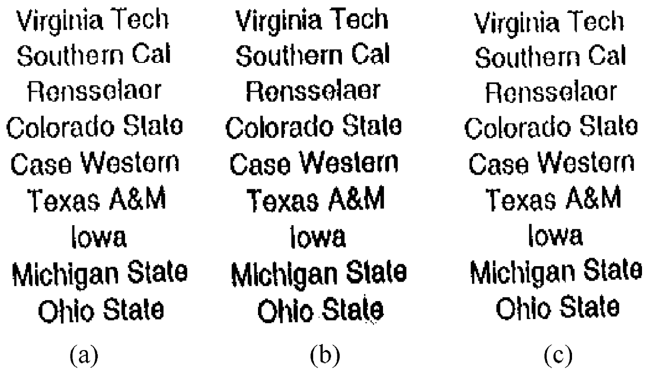


Fig. 14. Binary images of the character: (a) Otsu method ($t = 147$); (b) Kapur, Sahoo and Wong method ($t = 167$); (c) our method ($t = 148$).

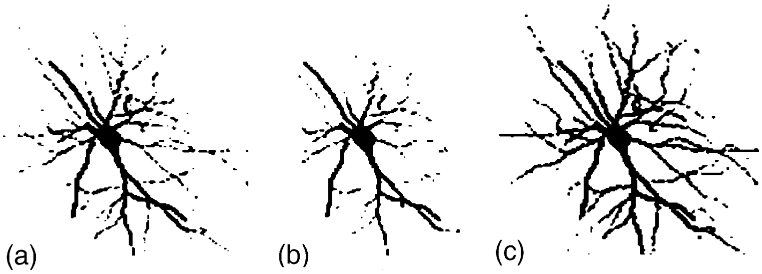


Fig. 15. Binary images of the nerve cell: (a) Otsu method ($t = 67$); (b) Kapur, Sahoo and Wong method ($t = 88$); (c) our method ($t = 42$).

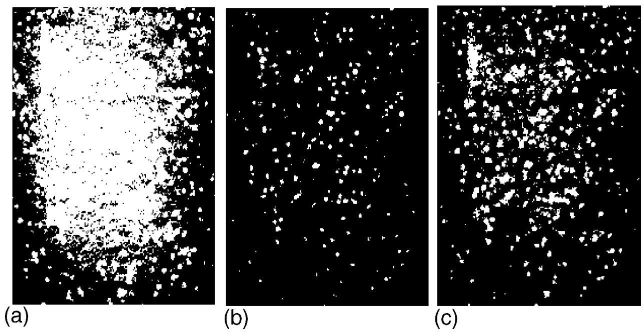


Fig. 16. Binary images of the mouse nervous tissue: (a) Otsu method ($t = 205$); (b) Kapur, Sahoo and Wong method ($t = 185$); (c) our method ($t = 194$).

threshold method and has its own advantage in processing specific class of images. Additionally, comparing with the Otsu's threshold method and the Kapur's threshold method, the proposed threshold method could be easily extended to the selection of multilevel threshold as shown in Section 4.2.

4.2. Multilevel threshold selection

For 2D image containing more than one interesting object class, multilevel thresholds are needed to select. In the case, gray values of discrete sampling points of different boundaries are mixed together. However, since gray values of discrete

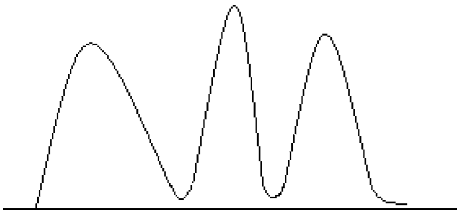


Fig. 17. Histogram of discrete sampling points of boundaries within 2D image including three objects.

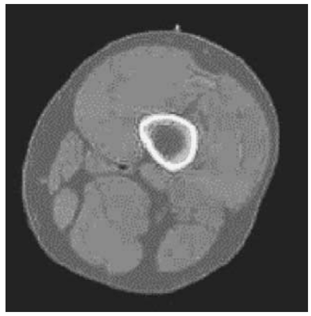


Fig. 18. CT image of leg with three different objects.

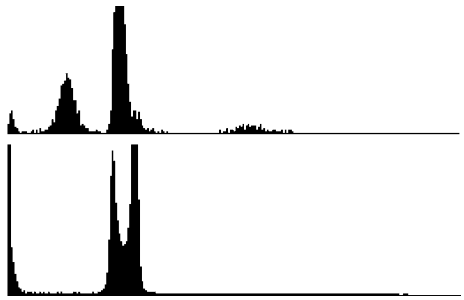


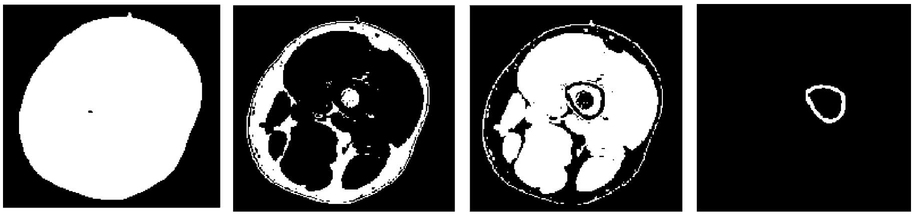
Fig. 19. Histogram of discrete sampling points of boundaries (above), histogram of 2D image (below).

sampling points of each boundary will cluster together around their mean, gray values of discrete sampling points of each boundary will display themselves implicitly as a different cluster in the histogram of discrete sampling points of all boundaries. See Fig. 17, where, histogram of discrete sampling points of the boundaries of 2D image containing three different objects is shown. By identifying appropriate clusters from the histogram of discrete sampling points of all boundaries, clusters corresponding to different boundaries are found out. Their means are approximation of the means of gray values of different boundaries. Thus, by computing the mean of each cluster, the means of gray values of different boundary are estimated, and therefore, multilevel thresholds corresponding to different objects are obtained.

We know that, the clusters corresponding to different object boundaries usually manifest as the main peaks in the histogram of discrete sampling points of all boundaries. Therefore, similarly, if there is noise or other small object in 2D image, it is better to select the gray level at the peak of each main cluster as threshold. In other words, we can select thresholds from the histogram of discrete sampling points of all boundaries by peak detection methods (Olivo, 1994; Papamarkos and Gatos, 1994) as well.

In Fig. 18, we consider the multilevel threshold selection from a 2D CT (leg section) image containing three different objects (bone, muscle and connective tissue). The histogram of the image is shown in Fig. 19 (below). It has a broad valley and very unequal peaks (the peak of bone can hardly be "seen" from the histogram of the image). However, in the histogram of discrete sampling points of the boundaries within the image, which is shown in Fig. 19 (above), three obvious clusters, which correspond to boundaries of bone, muscle and connective tissue, respectively, exist. The means of the three clusters correspond to thresholds of bone, muscle and connective tissue, respectively. Segmentation result of the image based on the selected thresholds is displayed in Fig. 20, where background and each object segmented from 2D image are displayed, respectively.

In Fig. 21, we consider the multilevel threshold selection from 2D CT head image containing two different objects (background, bone, and soft tissue). The histogram of the image is shown in Fig. 22 (below). It has a broad valley. However, in the histogram of discrete sampling points of the boundaries computed from the image, which is shown in Fig. 22 (above), two obvious clusters exist. They correspond to the boundaries of soft issue and bone, respectively. By computing the mean of the two clusters, the thresholds of bone and soft tissue are obtained. The corresponding segmentation result is displayed in Fig. 23, where each structure in the image, including background, soft tissue and bone, is displayed, respectively.



(a) (b) (c) (d)

Fig. 20. Different structures (white area) segmented from 2D CT image of leg by selecting multi-thresholds. (a) Background, (b) connective tissue, (c) muscle, (d) bone.

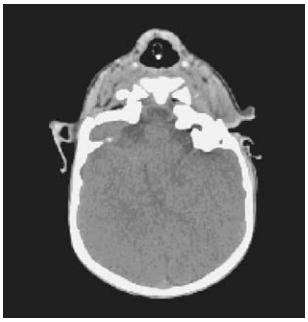


Fig. 21. CT image of head with three different structures.

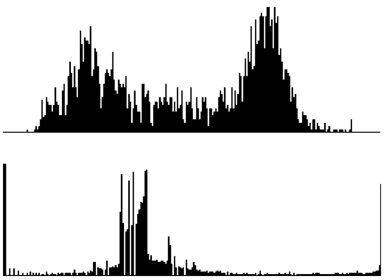


Fig. 22. Histogram of discrete sampling points of boundaries (above), histogram of 2D image (below).

5. Analysis of method

In the proposed threshold method, threshold is deduced from the histogram of the discrete sampling points of boundary. Thus, it is useful to enhance the quality of the computed discrete sampling points of boundary. Recall that, nonlinear diffusion methods allow a denoising and smoothing of image intensities while retaining and enhancing edges (Weickert, 1998). Thus, in order to enhance the quality of the poor discrete sampling points of boundary, we suggest using nonlinear diffusion methods as the preprocessing before the discrete sampling points of boundary are computed. By the scheme, usually the poor quality of the discrete sampling points of boundary could be well enhanced. However, we observe another important fact that, in many cases (perhaps not in all cases), even if the quality of the discrete sampling points of boundary could not be well enhanced, it does not necessarily and greatly affect the computed threshold. In other words, the computed threshold sometimes could keep stable to some extent. In order to demonstrate the fact, in

(a) (b) (c)

Fig. 23. Different structures (white area) segmented from 2D CT image of head by selecting multi-thresholds. (a) Background, (b) soft tissue, (c) bone.

what follows we will discuss the sensitivity of the proposed threshold method to noise and to the selection of different gradient thresholds T. The noise and the different selection of the gradient threshold T in Eq. (1) are main factors affecting the quality of discrete sampling points of boundary.

5.1. Analysis of the sensitivity to noise

In the proposed threshold method, the computation of the discrete sampling points of the boundaries is mainly determined by the values of Laplacian function of 2D image. Therefore, the affection of the proposed method by noise is mainly determined by the sensitivity of the sign of Laplacian operator to noise. Generally, the sensitivity of sign of Laplacian operator to noise is the main drawback of zero-crossing operator. However, this drawback is overcome to some extent in the proposed method, since the discrete sampling points of boundary are computed (by interpolation) only from zero-crossing points with high gradient values rather than from all zero-crossing points. The detailed explanation is as below.

Suppose that, the noise added to a given binary image is an independent normal having mean 0 and variance σ^2, and let η denote the noise function. Then the expression of digital Laplacian operator is rewritten as follows:

$$l(q_0) = \sum_{q \in V_8} (f(q) + \eta(q)) - 8 \cdot (f(q_0) + \eta(q_0))$$

where, V_8 represent 8-neighborhood of pixel q_0. Below, we suppose that q_0 is an edge pixel. Then, V_8 is separated into two subsets A and B by the boundary. Here, we assume that pixels in A belong to the object and pixels in B and q_0 belong to the background. Then, we have

$$l(q_0) = \sum_{q \in A} (f(q) - f(q_0)) + \sum_{q \in V_8} \eta(q) - 8 \cdot \eta(q_0)$$

The mean and variance of digital Laplacian $l(q_0)$ is as follows:

$$E(l(q_0)) = \sum_{q \in A} (f(q) - f(q_0)), \quad \sigma(l(q_0)) = 72 \cdot \sigma^2$$

Since the value of probability $p(|\sum_{q \in V_8} \eta(q) - 8 \cdot \eta(q_0)| \geqslant E(l(q_0)))$ equals to the value of probability $p(|l(q_0) - E(l(q_0))| \geqslant E(l(q_0)))$, by Chebyshev inequality,

$$p(|l(q_0) - E(l(q_0))| \geqslant E(l(q_0))) \leqslant \frac{72}{E(l(q_0))^2} \cdot \sigma^2 \leqslant \frac{72}{\left(\sum_{q \in A} (f(q) - f(q_0))\right)^2} \cdot \sigma^2$$

Let $\overline{M}$ represent the statistical difference between object and background gray levels. Since in V_8, there are at least three pixels belonging to object, namely, at least three pixels in A, the probability of "Laplacian value changing sign" at edge pixels q_0 is less than $(8 \cdot \sigma^2)/\overline{M}^2$. It concludes that, although the noise is enlarged in the digital Laplacian operator, but the probability of "Laplacian value changing sign" could be small when noise is small (with small variance σ^2) or $\overline{M}$ has a large value (have high edge magnitude). Therefore, our approach can keep the stable "Laplacian sign" with large probability when edge pixels have high edge magnitude or only little perturbation occurs in gray values of 2D image.

As an example, let's consider a binary image consisting of gray levels of black (0) and white (255) (see Fig. 24(a)). It is added the enlarged gaussian noise in turn as shown in Fig. 24(b)–(d). Histograms of these images are shown in Fig. 25, and histograms of discrete sampling points of the boundaries within these images are shown in Fig. 26, respectively. Thresholds computed from the histogram of discrete sampling point of the boundary within the four images are 126, 127, 124 and 128, respectively. We notice that, even if different noise is added to the image, but discrete sampling points always keep clustering around some nearly equal numbers—they all are approximation of the mean of gray values of points lying on the boundary. Thus, this experimental result shows that the threshold computed by our method could keep stable to some extent when noise exist in image.

5.2. Analysis of the sensitivity to the gradient thresholds T

Usually different gradient threshold T might produce different discrete sampling points of

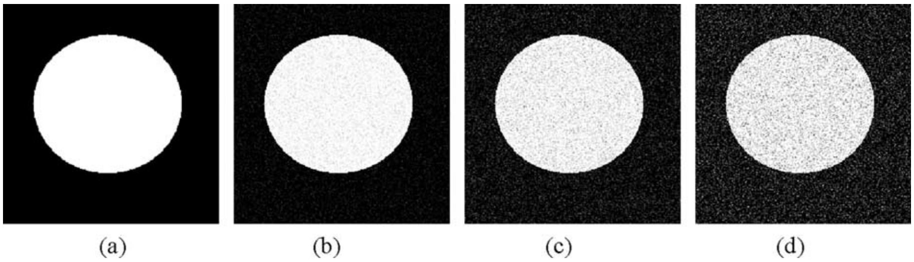


Fig. 24. Binary image and its changed versions added the enlarged gaussian noise in turn.

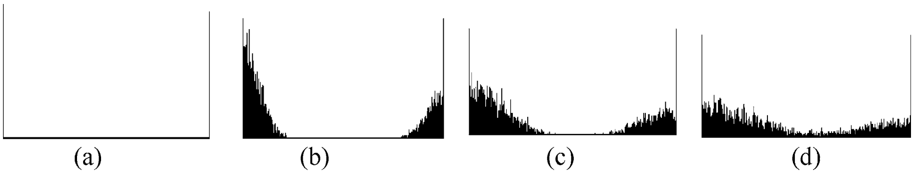


Fig. 25. Histograms of the images shown in Fig. 24.

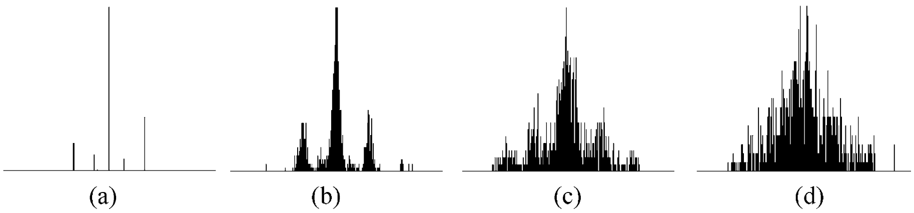


Fig. 26. Histograms of discrete sampling points of boundaries within the images in Fig. 24.

boundary. Thus, it is important to explore the affection of gradient threshold T to the final threshold computed by our threshold method. We do many experiments in which the edge map (the edge pixels, here they are detected by Prewitt detector, Rosenfeld and Kak (1982)), the histogram of the discrete sampling points of the boundary and the global threshold are computed when different gradient threshold T is selected. Through these experiments, we observe that, the edge map and histogram of the discrete sampling points of the boundary are comparatively apt to change when different gradient threshold T is selected, but the global threshold computed by the proposed threshold method is comparatively stable. In other words, when gradient threshold T selects different values, even if the edge map and the histogram of the discrete sampling points of the boundary have great change, the global threshold computed by the proposed threshold method might has little change. Let us see some examples shown in Figs. 27 and 28. In Fig. 27, the edge map, the histogram of the discrete sampling points of the boundary and the bi-level threshold are computed from the image of girl when gradient threshold T selects three different values 40, 100 and 160. Three different edge maps are shown in Fig. 27(a)–(c), respectively. Three different histograms of the discrete sampling points of the boundary are shown in Fig. 27(a1)–(c1), respectively. Three bi-level thresholds are 90, 87 and 92, respectively. The corresponding binary images of the three bi-level thresholds are shown in Fig. 27(a2)–(c2), respectively. In Fig. 28, the edge map, the histogram of the discrete sampling points of the boundary and the global threshold are computed from the image of baboon when gradient threshold T selects three different values 40, 100 and 200. Three different

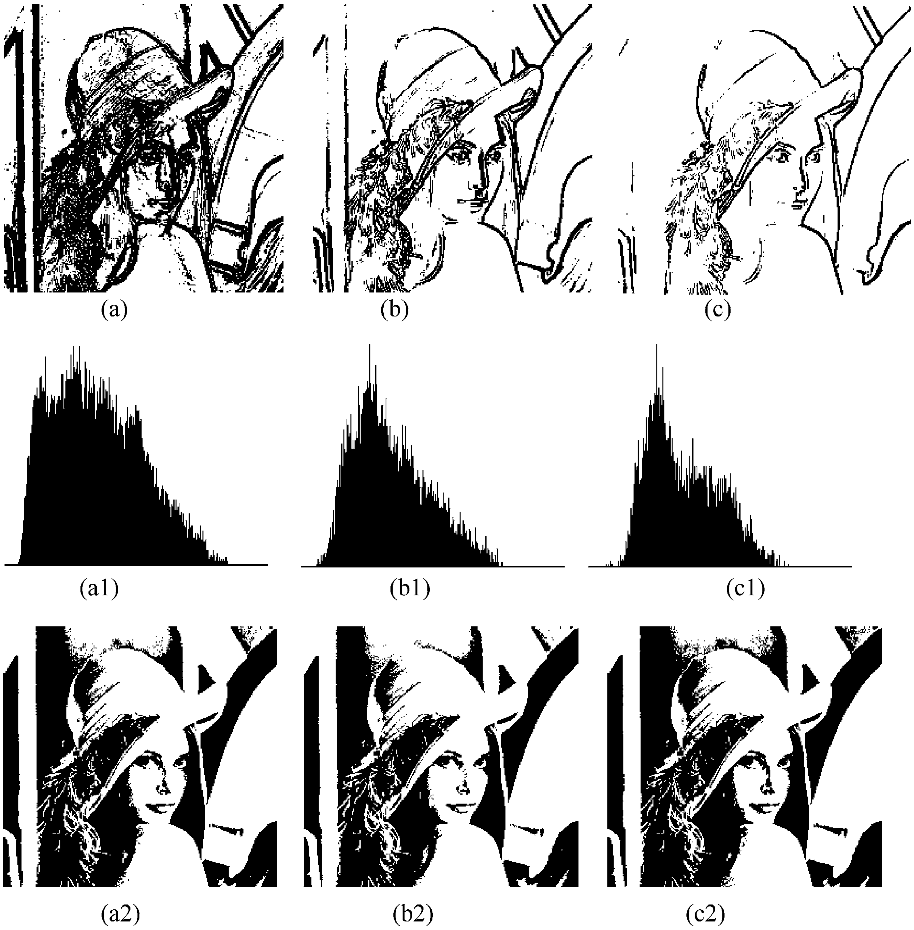


Fig. 27. Corresponding edge maps (a)–(c), histograms of the discrete sampling points of boundary (a1)–(c1), and segmentation results (a2)–(c2) of the image of girl when gradient threshold T selects three different values from low to high (from left-to-right), respectively.

edge maps are shown in Fig. 28(a)–(c), respectively. Three different histograms of the discrete sampling points of the boundary are shown in Fig. 28(a1)–(c1), respectively. Three bi-level thresholds are 129, 124 and 122, respectively. The corresponding binary images of the three bi-level thresholds are shown in Fig. 28(a2)–(c2), respectively. We can see that, in Figs. 27 and 28, when gradient threshold T selects three different values, the edge maps have great change, and the histograms of the discrete sampling points of the boundary are changed. However, the bi-level thresholds computed by the proposed threshold method have only small change. Specially, their corresponding binary images have little difference. The similar results are observed in the experiments we have done on many other images. Thus, these examples show that, in many cases (perhaps not in all cases), the proposed threshold method could keep stable to some extent. It is not very sensitive to the different selection of gradient thresholds T as long as most discrete sampling points of the boundary of the main object within image could be computed from the image.

6. Discussion and conclusion

In threshold techniques, there are two classes of important methods: the threshold techniques

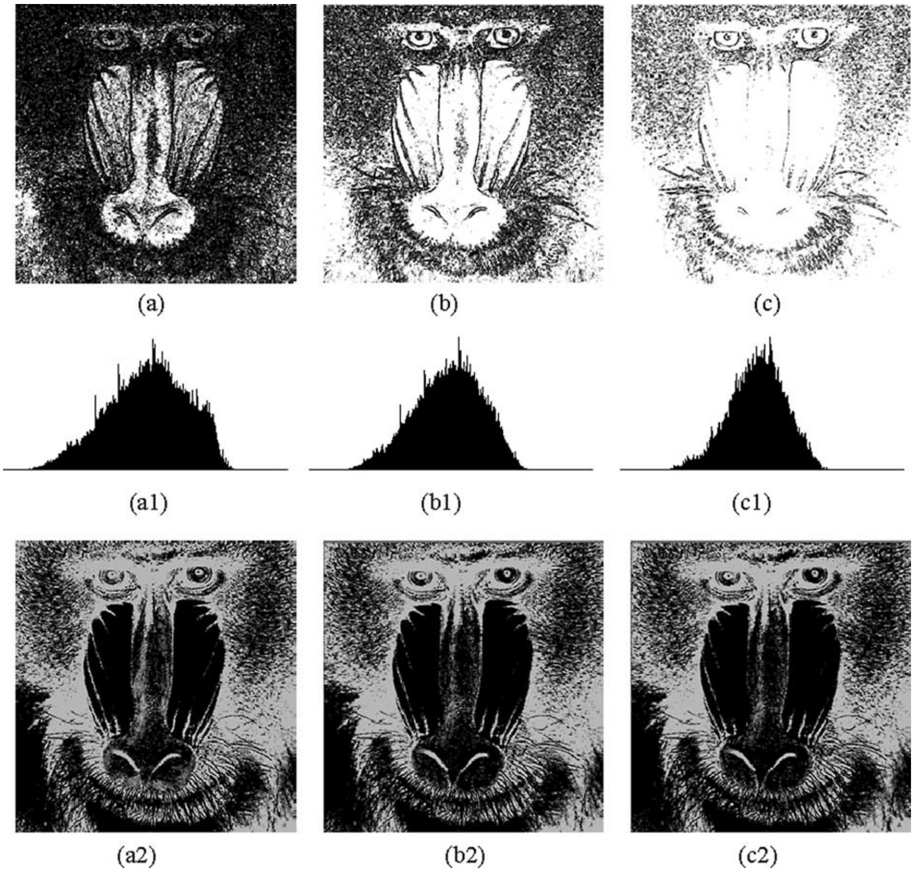


Fig. 28. Corresponding edge maps (a)–(c), histograms of the discrete sampling points of boundary (a1)–(c1), and segmentation results (a2)–(c2) of the image of baboon when gradient threshold T selects three different values from low to high (from left-to-right), respectively.

based on the histogram of whole image and the threshold techniques based on the histogram of edge pixels. The former is widely used in image processing. However, they cannot well deal with such images whose histograms exhibit several peaks of very unequal amplitude separated by a broad valley or contain only one peak and a "shoulder". The later overcomes the mentioned difficulty to some extent and could process such images. However, they have other various drawbacks as pointed out in Section 1.2. We notice that, those drawbacks of the threshold techniques based on the histogram of edge pixels are actually brought by two factors. One is the "double-responding" phenomenon of edge pixels—an important phenomenon in edge detection of 2D discrete image, and another is the fact that edge pixels have gray values either belonging to object or belonging to background.

In order to overcome the drawbacks of the threshold techniques based on the histogram of edge pixels, this paper considers the threshold in continuous image rather in discrete image. For the purpose, 2D image is implicitly reconstructed and the discrete sampling points (and their gray values) of the continuous boundaries are computed from the reconstructed continuous image by interpolation. Here, the boundaries of objects within image are referred to the continuous curve determined by Eq. (1). Usually, the points on the boundary differ from edge pixels. They locate between each pair of adjacent "double-responding" edge pixels, and have gray values between object and background gray values, differing from the gray values of edge pixels. Since we prove that, for each object within image, its optimal threshold is determined by the mean of the gray values of the points lying on its boundary, in this paper, thresholds corresponding to different objects are deduced from the histogram of discrete sampling points of the boundaries. By the scheme, the phenomenon that each object has "two-peaks" in the histogram of its edge pixels is eliminated, and therefore, many drawbacks of the threshold techniques based on the histogram of edge pixels are well overcome.

We think that, selecting threshold from the discrete sampling points of boundaries not only has a clear mathematical explanation, but also is more reasonable than selecting threshold from histogram of edge pixels. The reason is that the boundary is the continuous curve separating pixels of object from pixels of background and has gray values range between object and background gray levels. However, edge pixels might have gray values either belonging to object or belonging to background.

Lots of examples and the experiment comparing the proposed threshold method with the Otsu's threshold method and Kapur's threshold method show that, the proposed threshold method is an effective threshold method. It can well handle image whose histogram has very unequal peaks or broad valley, and can select threshold for those structures that only occupy a small percentage of the whole 2D image. Generally, such structures cannot be well "recognized" from the histogram of 2D image, and therefore are not easy to select corresponding threshold by the threshold techniques based on the histogram of whole image. Comparing with the conventional Otsu's threshold method and Kapur's threshold method, the proposed threshold not only has its own advantage in processing the specific class of images, but also is easy to be extended to the selection of multilevel threshold. In addition, the proposed threshold method has obvious geometric meaning—it computes such threshold that approximates the gray values of points lying on the object's boundary with least square error.

In this paper, threshold is deduced from the histogram of the discrete sampling points of boundary. It is selected as the mean of the gray values of the discrete sampling points of boundary. Such a statistical method ensures that, the computed threshold could keep stable to some extent as long as most discrete sampling points of the boundary of the main object within image could be computed from the image. In Section 5.1, we show that, although different enlarged gaussian noise are added to the image, but the computed threshold is possible to keep stable to some extent. In Section 5.2, we show that, in many cases (perhaps not in all cases), when different gradient thresholds T is selected, the edge map (or discrete sampling points of boundary) is apt to change. However, the threshold computed by the proposed threshold method is possible to have only little change. Thus, in many cases (perhaps not in all cases), even if the poor quality of edge map or the discrete sampling points of boundary could not be well enhanced, it does not necessarily and greatly affect the computed threshold.

Generally, thresholding assumes that image present a number of components with nearly homogeneous value. Thus, the gray values of the points lying on the boundary usually exhibit obvious cluster around their mean, and we could estimate the mean by compute the average value of the discrete sampling points of the boundary. Here, even if the discrete sampling points cannot be computed in few parts of boundary (i.e., closed boundary could not be detected by the method), but they usually only occupy a small percentage of the all computed discrete sampling points. Thus, the mean usually can still keep stable to some extent, see Section 5.2 and the discussion above. We note that, in Section 2, the result in Eq. (3) yields even if $C(x, y)$ is only a piece of boundary rather than a closed boundary. Thus, the proposed threshold method could be used to compute local threshold. In other words, in the local region containing $C(x, y)$ (here $C(x, y)$ is assumed to be a piece of boundary), the mean of gray values of points lying on the boundary $C(x, y)$ determines an optimal local threshold of the local region.

Weszka et al. (1974) proposed a bi-level threshold method that selects the valley of histogram of pixels with high Laplacian value (edge pixels) as threshold. The method is based on the phenomenon that each object has "two-peaks" in the histogram of its edge pixels. However, because of the same phenomenon, the method is limited and cannot be extended to the selection of multilevel threshold. By avoiding the "two-peaks" phenomenon existing in the histogram of edge pixels, the proposed method can be easily extended to multilevel threshold selection, while it could well compute bi-level threshold.

The proposed method has been used to many images, and satisfactory results are obtained.

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