Problem 1.1

$$\frac{x^{28}}{x^{9} \cdot x^{21}} \cdot \frac{x^{7}}{x^{21}} = \frac{x^{35}}{x^{9}} = x^{26}$$

Problem 1.2

$$8^2 \cdot 4^2 \cdot 2^x = 8^4$$
$$(2^3)^2 \cdot (2^2)^2 \cdot 2^x = (2^3)^4$$
$$2^6 \cdot 2^{2x} \cdot 2^x = 2^{12}$$
$$6 + 2x + x = 12$$
$$3x = 6$$
$$x = 2$$

Problem 1.3

If $\frac{x}{y}$ is 3, then $x^{-4} y^4 = ?$

$$\frac{x}{y} = 3, \quad \frac{y^4}{x^4} = ?$$
$$\frac{y^4}{x^4} = \frac{y}{x} = \frac{1}{3} \qquad \frac{y^4}{x^4} = \left[\frac{1}{3}\right]^4 = \frac{1}{81}$$

Problem 1.4

$$\frac{\sqrt{4^{15}}}{\sqrt{16^7}} = \frac{\sqrt{(2^2)^{15}}}{\sqrt{(2^4)^7}} = \frac{2^{\frac{30}{2}}}{2^{\frac{28}{2}}} = 2^1 = 2$$

Problem 1.5

a) $x + (y+z) = (y+x) \neq z \rightarrow$ True

b) $y(x+z) = xy + zy \rightarrow$ True.

c) $x^{y+z} = x^z + x^y \rightarrow$ True

d) $\frac{x^z}{x^y} = x^{y-z} \rightarrow$ True.

Problem 1.6

$$\ln(x) \geq e$$

$$[e^e, \infty) \therefore x \geq e^e \text{ y}$$

Problem 2.1

$$0^\circ C = 32^\circ F$$
$$100^\circ C = 212^\circ F$$
$$\therefore 100x = 180$$
$$x = 1.8$$

$$\frac{32}{1.8} = 17.\overline{7} \text{ y.}$$

Problem 2.2

$$f(x) = 3x - 12$$
$$3x - 12 = 0$$
$$x = 4 \text{ y.}$$

Problem 2.3

$$9^{x^2-6x+2} = 81$$
$$9^{x^2-6x+2} = 9^2$$
$$x^2 - 6x + 2 = 2$$

$$x^2 - 6x = 0$$
$$x(x-6) = 0$$
$$x = 0, \; x = 6 \text{ y}$$

Problem 2-4

Assuming current GDP to be 'x'

equation $= (1 + 0.03)^x = 3$

$(1.03)^x = 3$

$x \ln(1.03) = \ln 3$

$x = \frac{\ln 3}{\ln 1.03}$

$x = 37_4$

∴ In 37 years the GDP of a country will be three times given it increase by 3% annualy.

Problem 2-5

$$\log_{\bar{\pi}} \left(\frac{1}{\bar{\pi}^5} \right)$$

$$\bar{\pi} \log \left[\frac{1}{\bar{\pi}^5} \right] = \bar{\pi}^{-5}$$

$$\log_{\bar{\pi}} \left[\frac{1}{\bar{\pi}^5} \right] = -5_4.$$

Problem 3-1

$$\sum_{i=0}^{\infty} \left[\frac{1}{5^i} + 0.3^i \right]$$

As the value of i increase the total value of a function more closer to 0 ∴ $\frac{1}{5^\infty} + 0.3^\infty$

$0 + 0 = 0_4.$

Problem 3.2

$$\lim_{x \to 5} \frac{x^2 - 25}{x - 5} = \frac{5^2 - 25}{5 - 5} = 0$$

$$\lim_{x \to 4} \frac{4^2 - 25}{4 - 5} = 9$$

$$\lim_{x \to 6} \frac{6^2 - 25}{6 - 5} = 11$$

∴ function converging towards 10.
limit is 10.

Problem 3.3

$f(x)$ $x^3 - 4$ at $(-2, -12)$

$$\frac{df(x)}{dx} = 3x^2$$

$$3(-2)^2 = 12.$$

Problem 3.4

$$f(x) = \frac{(x^2 - 1)(5x^4) - (x^5 + 3)(2x)}{(x^2 - 1)^2}$$

$$= \frac{3x^6 - 5x^4 - 6x}{(5x^2 - 1)^2}$$

Problem 3.6

$$f(x) = \frac{1}{x}$$

$1/x$ is a continuous function on $(0, \infty)$ and $(-\infty, 0)$ in every point in its domain. However, when the value of $x = 0$, it has an infinite discontinuity.

Problem 3.5

$$f(x) = x^9 + 3$$

$$\frac{df(x)}{dx} = 9x^8$$

$$\frac{d^2 f(x)}{dx^2} = 72x^7$$

Problem 3.7

$$f(x) = 4x^3 - 12x$$

$$\frac{df(x)}{dx} = 12x^2 - 12$$

$$12x^2 - 12 = 0$$

$$12x^2 = 12$$

$$x = \pm 2$$

Problem 3.8

Let $f(x, y) = x^3 - y^2$

$$f(2, 3) = 2^3 - 3^2$$

$$= 8 - 9$$

$$= -1$$

Problem 3.10.

$$\frac{d}{dx}\left[x^5y^7 + \frac{x^2}{y^3}\right] = 5x^4y^7 + \frac{2x}{y^3}$$ Ans.

Problem 3.9

$$f(x,y) = \ln(x-3y)$$

for values of $x > 3y$

for values of y being +ve.

Problem 4.1

$$\begin{bmatrix} 1 & 0 & 1 \\ 9 & 1 & 8 \end{bmatrix} \cdot \begin{bmatrix} 2 & 5 \\ 2 & 1 \\ 7 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 2+0+7 & 8+0+6 \\ 18+2+35 & 45+1+30 \end{bmatrix}$$

$$\begin{bmatrix} 9 & 14 \\ 55 & 76 \end{bmatrix}$$ Ans.

Problem 4.2

$$A = \begin{bmatrix} 5 & 3 \\ 0 & 1 \\ 1 & 2 \end{bmatrix} \quad 3 \times 2$$

$$B = \begin{bmatrix} 8 & 4 & 0 \\ 2 & 1 & 2 \end{bmatrix} \quad 2 \times 3$$

$$\begin{bmatrix} (5\times8 + 3\times2) & (5\times4 + 1\times3) & (5\times0 + 3\times2) \\ (0\times8 + 1\times2) & (0\times4 + 1\times1) & (0\times0 + 1\times2) \\ (1\times8 + 2\times2) & (1\times4 + 2\times1) & (1\times0 + 2\times2) \end{bmatrix}$$

$$\begin{bmatrix} 46 & 23 & 6 \\ 2 & 1 & 2 \\ 12 & 6 & 4 \end{bmatrix}$$

Problem 4.3

$$\begin{bmatrix} e & 93 & 4.7 \\ 2 & 6.1 & 4.22 \\ 4 & \pi & 0 \end{bmatrix}$$

Transpose =

$$\begin{bmatrix} e & 2 & 4 \\ 93 & 6.1 & \pi \\ 4.7 & 4.22 & 0 \end{bmatrix}$$

Problem 4.4

$$\begin{bmatrix} 2 & 6 \\ 2 & 8 \end{bmatrix}$$

determinant = $ad - bc$

$= (2 \times 8) - (6 \times 2)$

$= 16 - 12$

$= 4$ #.

Problem 5.1

Sample Space = 6^{36} #.

Problem 5.2

$$P(\text{Positive} \mid \text{Drug User}) = \frac{P(\text{Positive} \cap \text{Drug User})}{P(\text{Positive})}$$

$$= \frac{0.007 \times 0.98}{0.007}$$

$= 0.98$ #.

Problem 5.3

n = no. of events

Expected Value = $P(x) * n$.

$= \frac{1}{6} \times 20$

$= 3.33$ %.

Problem 3.11

$$f(x,y) = \sqrt{xy} - x - y$$

$$\frac{df(x)}{dx} = \frac{1}{2\sqrt{xy}} - 1 - 1 = 0.$$

$$\frac{1}{2\sqrt{xy}} = 2$$

$$xy = \left[\frac{1}{4}\right]^2 = \frac{1}{16}.$$

$$f(x,y) = \sqrt{x \cdot x/16} - x - \frac{x}{16} = 0.$$

$$\frac{x}{4} - x - \frac{x}{16} = 0$$

$$\frac{4x - 16 - x}{16} = 0.$$

$$3x = 16$$

$$x = 5.33$$

$$y = 0.33$$

$\therefore$ value is in -ve $\therefore$ it is a maxima.

$$\frac{df(x)}{dx} = \frac{(x \cdot y)^{-1/2}}{2} = 0.$$

$$-\frac{1}{4}(xy)^{-3/2} = 0.$$