题型：求矩阵的LU分解来解方程

3.1 矩阵的LU分解

例 1 求解线性方程组

$$\begin{cases} x_1 + 2x_2 - x_3 = 0 & ① \\ 3x_1 + x_2 = -1 & ② \\ -x_1 - x_2 - 2x_3 = 1 & ③ \end{cases} \qquad (I)$$

解:

$$(I) \xrightarrow[1\times①+③]{(-3)\times①+②} \begin{cases} x_1 + 2x_2 - x_3 = 0 & ① \\ -5x_2 + 3x_3 = -1 & ② \\ x_2 - 3x_3 = 1 & ③ \end{cases}$$

3.1 矩阵的LU分解

$$\xrightarrow{\frac{1}{5}\times②+③} \begin{cases} x_1 + 2x_2 - x_3 = 0 & ① \\ -5x_2 + 3x_3 = -1 & ② \\ -\frac{12}{5}x_3 = \frac{4}{5} & ③ \end{cases}$$

$$\xrightarrow[\frac{3}{5}\times③+②]{(-\frac{5}{12})\times③} \begin{cases} x_1 + 2x_2 = -1/3 & ① \\ x_2 = 0 & ② \\ x_3 = -1/3 & ③ \end{cases}$$

$$\xrightarrow{(-2)\times②+①} \begin{cases} x_1 = -1/3 & ① \\ x_2 = 0 & ② \\ x_3 = -1/3 & ③ \end{cases} \qquad (II)$$

3.1 矩阵的LU分解

用矩阵形式表示，系数矩阵

$$A=\begin{bmatrix}1&2&-1\\3&1&0\\-1&-1&-2\end{bmatrix}\xrightarrow[r_{13}(1)]{r_{12}(-3)}\begin{bmatrix}1&2&-1\\0&-5&3\\0&1&-3\end{bmatrix}$$

$$\xrightarrow{r_{23}(\frac{1}{5})}\begin{bmatrix}1&2&-1\\0&-5&3\\0&0&-12/5\end{bmatrix}\equiv U$$

3.1 矩阵的LU分解

$$R_{23}(\tfrac{1}{5})R_{13}(1)R_{12}(-3)A=U.$$

$$A=[R_{23}(\tfrac{1}{5})R_{13}(1)R_{12}(-3)]^{-1}U$$

$$=[R_{12}(-3)]^{-1}[R_{13}(1)]^{-1}[R_{23}(\tfrac{1}{5})]^{-1}U$$

$$=R_{12}(3)R_{13}(-1)R_{23}(\tfrac{-1}{5})Q\quad\boxed{\equiv LU}$$

$$=\begin{bmatrix}1&0&0\\3&1&0\\-1&-1/5&1\end{bmatrix}\begin{bmatrix}1&2&-1\\0&-5&3\\0&0&-12/5\end{bmatrix}$$