

data structures and algorithms analysis

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1 Graph

1.1 Definition

Definition 1.1 (graph). *A graph $G = (V, E)$ consists of a set of vertices, V , and a set of edges, E .*

Each edge is a pair (v, w) , where $(v, w) \in V$. Edge are sometimes referred to as *arcs*.

Definition 1.2 (directed). *Pair is ordered.*

Directed graphs are sometimes referred to as *digraphs*.

Vertex w is adjacent to v if and only if $(v, w) \in E$.

In an undirected graph with edge (v, w) , and hence (w, v) , w is adjacent to v and v is adjacent to w .

Sometimes an edge has a third component, known as either a *weight* and a *cost*.

Definition 1.3 (path). *A path in a graph is a sequence of vertices $w_1, w_2, w_3, \dots, w_N$ such that $(w_i, w_{i+1}) \in E$ for $i \leq i < N$.*

A *length* of such a path is the number of edges on the path, which is equal to $N - 1$. We allow a path from a vertex to itself; if this path contains no edges, then the path length is 0. The (v, v) is sometimes referred to as a *loop*.

Definition 1.4 (simple path). *A simple path is a path such that all vertices are distinct, except that the first and last could be the same.*

Definition 1.5 (cycle). *A cycle in a directed graph is a path of length at least 1 such that $w_1 = w_N$.*

A directed graph is *acyclic* if it has no cycles. A directed acyclic graph is sometimes referred to by its abbreviation, *DAG*.