

We take the first equation to illustrate this problem. Firstly, we can obtain an equation according to the Euclidean distance formulation like equation (1) through considering the only one remote sensor.

$$\sqrt{(x_1 - x)^2 + (y_1 - y)^2 + (z_1 - z)^2} = c(t_1 - t) \quad (1)$$

$$(x_1 - x)^2 + (y_1 - y)^2 + (z_1 - z)^2 = c^2(t_1 - t)^2 \quad (2)$$

$$x_1^2 + y_1^2 + z_1^2 + x^2 - 2x_1x + y^2 - 2y_1y + z^2 - 2z_1z = c^2t_1^2 - 2c^2t_1t + c^2t^2 \quad (3)$$

Put the  $c^2t_1^2$  to the left of the equation, then we can get the equation(4).

$$x_1^2 + y_1^2 + z_1^2 - c^2t_1^2 + x^2 - 2x_1x + y^2 - 2y_1y + z^2 - 2z_1z = -2c^2t_1t + c^2t^2 \quad (4)$$

Then, we can replace the first four items to  $R_1$ , get equation (5)

$$R_1 - c^2t^2 + x^2 + y^2 + z^2 - 2x_1x - 2y_1y - 2z_1z = -2c^2t_1t \quad (5)$$

We replace the  $c^2t^2 - R_1$  to  $T_1$ , the equation (5) will become equation (6).

$$x^2 + y^2 + z^2 - c^2t^2 - 2(x_1x + y_1y + z_1z - c^2t_1t) = -R_1 \quad (6)$$

We list all the formula from sensor 1 to sensor n and get the equation (7).

$$\begin{aligned} x^2 + y^2 + z^2 - c^2t^2 - 2(x_1x + y_1y + z_1z - c^2t_1t) &= -R_1 \\ x^2 + y^2 + z^2 - c^2t^2 - 2(x_2x + y_2y + z_2z - c^2t_2t) &= -R_2 \\ x^2 + y^2 + z^2 - c^2t^2 - 2(x_3x + y_3y + z_3z - c^2t_3t) &= -R_3 \\ x^2 + y^2 + z^2 - c^2t^2 - 2(x_4x + y_4y + z_4z - c^2t_4t) &= -R_4 \\ x^2 + y^2 + z^2 - c^2t^2 - 2(x_5x + y_5y + z_5z - c^2t_5t) &= -R_5 \\ &\dots \\ x^2 + y^2 + z^2 - c^2t^2 - 2(x_nx + y_ny + z_nz - c^2t_nt) &= -R_n \end{aligned} \quad (7)$$

We simplify the equation (7) from the second to the n through minusing the first equation, and get the equation (8).

$$\begin{aligned} x^2 + y^2 + z^2 - c^2t^2 - 2(x_1x + y_1y + z_1z - c^2t_1t) &= -R_1 \\ -2(x_2x + y_2y + z_2z - c^2t_2t) + 2(x_1x + y_1y + z_1z - c^2t_1t) &= -R_2 + R_1 \\ -2(x_3x + y_3y + z_3z - c^2t_3t) + 2(x_1x + y_1y + z_1z - c^2t_1t) &= -R_3 + R_1 \\ -2(x_4x + y_4y + z_4z - c^2t_4t) + 2(x_1x + y_1y + z_1z - c^2t_1t) &= -R_4 + R_1 \\ -2(x_5x + y_5y + z_5z - c^2t_5t) + 2(x_1x + y_1y + z_1z - c^2t_1t) &= -R_5 + R_1 \\ &\dots \\ -2(x_nx + y_ny + z_nz - c^2t_nt) + 2(x_1x + y_1y + z_1z - c^2t_1t) &= -R_n + R_1 \end{aligned} \quad (8)$$

We can further simplify the above equation to equation (9).

$$\begin{aligned}
(x_1 - x_2)x + (y_1 - y_2)y + (z_1 - z_2)z + (t_2 - t_1)c^2t &= \frac{R_1 - R_2}{2} \\
(x_1 - x_3)x + (y_1 - y_3)y + (z_1 - z_3)z + (t_3 - t_1)c^2t &= \frac{R_1 - R_3}{2} \\
(x_1 - x_4)x + (y_1 - y_4)y + (z_1 - z_4)z + (t_4 - t_1)c^2t &= \frac{R_1 - R_4}{2} \\
(x_1 - x_5)x + (y_1 - y_5)y + (z_1 - z_5)z + (t_5 - t_1)c^2t &= \frac{R_1 - R_5}{2} \\
&\dots \\
(x_1 - x_n)x + (y_1 - y_n)y + (z_1 - z_n)z + (t_n - t_1)c^2t &= \frac{R_1 - R_n}{2}
\end{aligned} \tag{9}$$

And we can transform this equation group into a determinant as equation (10).

$$\begin{bmatrix}
(x_1 - x_2) & (y_1 - y_2) & (z_1 - z_2) & (t_2 - t_1)c^2 \\
(x_1 - x_3) & (y_1 - y_3) & (z_1 - z_3) & (t_3 - t_1)c^2 \\
(x_1 - x_4) & (y_1 - y_4) & (z_1 - z_4) & (t_4 - t_1)c^2 \\
(x_1 - x_5) & (y_1 - y_5) & (z_1 - z_5) & (t_5 - t_1)c^2 \\
\vdots & \vdots & \vdots & \vdots \\
(x_1 - x_n) & (y_1 - y_n) & (z_1 - z_n) & (t_n - t_1)c^2
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
t
\end{bmatrix}
=
\begin{bmatrix}
\frac{R_1 - R_2}{2} \\
\frac{R_1 - R_3}{2} \\
\frac{R_1 - R_4}{2} \\
\frac{R_1 - R_5}{2} \\
\vdots \\
\frac{R_1 - R_n}{2}
\end{bmatrix} \tag{10}$$

Then, we can use the theory of the linear algebra to find many consequences of this equation cause when the order of the determinant are not full.