A Note for Master Project

Zhi-Zhen WANG

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1 Kachru–Pearson–Verlinde Mechanism

In [1], Klebanov and Strassler (KS) discovered a possible gravitational theory dual to $\mathcal{N}=1$ supersymmetric Yang–Mills theory (SYM), known as KS geometry. This dual starts with the theory of N D3 branes probing M "fractional" wrapped D5 branes in a conifold geometry, and finally end with all branes replaced by RR and NS three form fluxes through two intersecting three-cycles in the deformed conifold. The KS geometry can be embedded in a compact geometry [2], based on this fact, Kachru, Pearson and Verlinde (KPV) proposed a mechanism by introducing some number of anti-D3 branes which could break the supersymmetry (SUSY) around the compact region—which is at the tip of the deformed conifold [3]. In this section, we briefly introduce the KPV mechanism and the meta- and un-stable states it derived.

1.1 Klebanov–Strassler Geometry and $\overline{\text{D3}}$-NS5 System

The KS geometry is induced from F theory compactification on the fourfold X, which can be viewed as an orientifold of IIB string theory on a Calabi–Yau (CY) threefold Y. Consider the insertion of N_3 D3-branes and/or $\bar{N}_3$ $\overline{\text{D}}$3-branes on the KS geometry, the net D3 charge is denoted as $Q_3 \equiv N_3 - \bar{N}_3$, which is fixed by the global tadpole condition

$$\frac{\chi(X)}{24} = Q_3 + \frac{1}{2\kappa_{10}^2 T_3}\int_Y H_3 \wedge F_3, \tag{1}$$

where $\chi(X)$ is the Euler characteristic of the CY fourfold that specifies the F theory compactification, T_3 is the D3-brane tension.

At the singular point in moduli space where Y has developed a conifold singularity. The KS solution corresponds to the placing of M units of F_3 flux through the A-cycle of the conifold and K units of H_3 through the dual B-cycle:

$$\begin{aligned} \frac{1}{4\pi^2}\int_A F_3 &= M, \\ \frac{1}{4\pi^2}\int_B H_3 &= -K \end{aligned} \tag{2}$$

In general, to keep D3 charge conserved, we could fix $\frac{\chi}{24} = MK$ and so that no extra D3-brane insertions needed. As [2] noted, these fluxes will induce a non-trivial superpotential for the complex structure moduli $z_A = \int_A \Omega$ that controls the relative size of the A-cycle, stabilizing it at an exponentially small value

$$z_A \sim e^{-\frac{2\pi K}{M g_s}}, \tag{3}$$

where g_s denotes the string coupling constant. Therefore, the compactified geometry grows a smooth conical region which can be described by the warped geometry of the deformed conifold

$$\sum_{i=1}^{4} z_i^2 = \varepsilon^2, \tag{4}$$

with $\varepsilon \sim z_A$ exponentially small. This geometry is dual to an $\mathrm{SU}(N+M) \times \mathrm{SU}(N)$ gauge theory.

Now let us put $\bar{N}_3 = p$ $\overline{\mathrm{D}}$3-branes probing the tip of KS throat and discuss the transition of vacuum states. Note that $N_3 = 0$ and $p \ll K, M$. Thus from 1, 2 we have

$$\frac{\chi}{24} = KM - p. \tag{5}$$

Note that this theory is non-supersymmetric because the SUSY preserved by the $\overline{\mathrm{D}}$3-branes is incompatible with the global symmetry preserved by the imaginary self-dual three-form flux of the background geometry. Consider such case from the dual field theory perspective. It is natural to identify the total D3 charge carried by the compactified manifold, $\chi/24$, with the total rank N of the $\mathrm{SU}(N+M) \times \mathrm{SU}(N)$ dual gauge theory. If we took $N = KM + p$ with $p \ll M$, the duality cascade would proceed in K steps until we were left in the infrared with an $\mathrm{SU}(M+p) \times \mathrm{SU}(p)$ gauge theory. This process could be matched with the moduli space of p D3 branes moving on the deformed conifold geometry. This is what we expect in the case $\frac{\chi}{24} = KM + p$, after the correspondence, it is natural to guess that our situation 5 should be related to the KS gauge theory with $N = KM - p$.

However, in this case, the RG cascade can only involve $K - 1$ S-dualities, leaving supersymmetric $\mathrm{SU}(2M - p) \times \mathrm{SU}(M - p)$ gauge theory in the infrared, which violates the statement that the situation 5 should be non-supersymmetric. Therefore, it actually corresponds to the supergravity (SUGRA) solution with $M-p$ D3 branes and only $K-1$ unis of H_{NS} flux through the B-cycle:

$$N = M - p, \quad \bar{N} = 0, \quad \frac{\chi}{24} = (K-1)M + (M-p) = KM - p \tag{6}$$

One natural hypothesis is that this supersymmetric situation should exhibit an endpoint of a decay process initiated from situation 5. It is true that the IIB string theory admits domain walls across which the flux of H_3 through the B-cycle drops by one unit, described by NS5 branes wrapping the A-cycle of the conifold. The situation5 can decay via nucleation of a bubble of supersymmetric vacuum6 surrounded by a spherical NS5 domain wall, which is exponentially suppressed by the drop in the vacuum energy.

1.2 Dirac–Born–Infeld (BDI) Action and Effective Potential

Assume that all interesting physics we care will take place around the tip region of the conifold, we expect the $\overline{\text{D}}$3-branes accumulate quickly near the apex. The metric near the apex can be written as [4]:

$$ds^2 = a_0^2 dx_\mu dx_\mu + g_s M b_0^2 \left(\frac{1}{2} dr^2 + d\Omega_3^2 + r^2 d\tilde{\Omega}_2^2 \right),$$

$$a_0^2 \simeq \frac{\varepsilon^{4/3}}{g_s M} \quad b_0^2 \approx 0.93266. \tag{7}$$

Note that here the string tension constant $\alpha' = 1$. Near $r = 0$, the spacetime has the topology $\mathbb{R} \times S^3$. The RR field $F_3 = dC_2$ has a quantized flux around the S^3

$$\int_{S^3} F_3 = 4\pi^2 M, \tag{8}$$

while in the supersymmetric background dictates $\star_6 H_3 = -g_s F_3$, so that

$$dB_6 = \frac{1}{g_s^2} \star_{10} H_3 = -\frac{1}{g_s} dV_4 \wedge F_3, \tag{9}$$

where dV_4 the 4-dimensional Euclidean volume element, $dV_4 = a_0^4 d^4 x$. The dilaton field is constant and the self-dual five-form field vanishes at the tip.

Without considering the backreaction due to the $\overline{\text{D}}$3-branes, we can understand the picture as following after introducing p number of $\overline{\text{D}}$3-branes.

On the NS5-brane worldvolume, there exists a three-form magnetic flux out of which delocalized the D3 charges, which admits those $\overline{\text{D}}$3-branes annihilate or nucleate with D3-branes. The decay of $\overline{\text{D}}$3 drives a brane polarization and the NS5-brane is forced to form an internal S^2 over S^3, which means that the worldvolume geometry piercing from $\mathbb{R}^4 \times S^5$ to $\mathbb{R}^4 \times S^3 \times S^2$. Besides, to coincide with the compactification, the spherical NS5 also guarantees the mediation of brane-flux decay so that the D3 charge changes induced from $\overline{\text{D}}$3 charge dissolving can be realized. The validity holds for $R \simeq \sqrt{g_s M} \gg r \simeq \sqrt{g_s p}$, where R the characteristic scale of the geometry and r the scale of backreaction effect. Thus $p/M \ll 1$ should always be considered as a restriction for this mechanism.

Then consider an NS5-brane of type IIB string theory located at an S^2 inside S^3 with radius specified by a polar angle ψ. The bosonic worldvolume action is written as [6]:

$$S = \frac{\mu_5}{g_s^2} \int d^6\xi \left[-\det\left(G_\parallel\right) \det\left(G_\perp + 2\pi g_s \mathcal{F}\right) \right]^{1/2} + \mu_5 \int B_6, \tag{10}$$

$$2\pi \mathcal{F}_2 = 2\pi F_2 - C_2. \tag{11}$$

Here $F_2 = dA$ is the two-form field strength of the worldvolume gauge field, $G_\perp$ denotes the induced metric along the internal S^2 while $G_\parallel$ encodes the remaining components along the $d\psi$ and $\mathbb{R}^4$ directions. Inserting 7, we have

$$\begin{aligned} ds^2_{induced} &= b_0^2 g_s M \left[dx_\mu dx^\mu + d\psi^2 + \sin^2\psi d\Omega_2^2\right] \\ &= ds_\parallel^2 + ds_\perp^2. \end{aligned} \tag{12}$$

Besides, there are some useful integrals over S^2:

$$\int_{S^2} \sqrt{\det G_\perp} = 4\pi b_0^2 g_s M \sin^2\psi, \tag{13}$$

$$\int_{S^2} C_2(\psi) = 4\pi M \left(\psi - \frac{1}{2}\sin(2\psi)\right), \tag{14}$$

$$2\pi \int_{S^2} F_2 = 4\pi^2 p. \tag{15}$$

The last equation exhibits the conserved number p of $\overline{\text{D}}$3-branes. Those three give:

$$\int_{S^2} \sqrt{\det\left(G_\perp + 2\pi g_s \mathcal{F}\right)} = 4\pi^2 M g_s V_2(\psi), \tag{16}$$

$$V_2(\psi) = \frac{1}{\pi}\sqrt{b_0^4 \sin^4\psi + \left(\pi\frac{p}{M} - \psi + \frac{1}{2}\sin(2\psi)\right)^2}. \tag{17}$$

From 9, a total NS5-brane action is

$$S = \int d^4x \sqrt{-\det G_\parallel}\mathcal{L}(\psi), \tag{18}$$

$$\mathcal{L}(\psi) = A_0\left(V_2(\psi)\sqrt{1-\dot{\psi}^2} - \frac{1}{2\pi}(2\psi - \sin 2\psi)\right), \tag{19}$$

with $A_0 = \frac{4\pi^2\mu_5 M}{g_s} = \frac{\mu_3 M}{g_s}$. From the Lagrangian, it is easy to extract the Hamiltonian density $\mathcal{H}$ as

$$\mathcal{H}(\psi, P_\psi) = -\frac{A_0}{2\pi}(2\psi - \sin 2\psi) + \sqrt{A_0^2\left(V_2(\psi)\right)^2 + P_\psi^2}, \tag{20}$$

which generates the time evolution of $\psi(t)$. Set $P_\psi = 0$, we obtain the effective potential

$$\begin{aligned} V_{\text{eff}}(\psi) &\equiv \mathcal{H}(\psi, P_\psi = 0) \\ &= A_0\left(V_2(\psi) - \frac{1}{2\pi}(2\psi - \sin 2\psi)\right). \end{aligned} \tag{21}$$

[3] illustrated the potential as a function of ψ (see Fig. 1). It shows that if we hope to obtain an at-least-meta-stable state, the validity regime holds for $\frac{p}{M} \lesssim 0.08$, but the quantum tunneling is possible so that it can still be decayed. If $\frac{p}{M}$ is much larger than 0.08, the effective potential will be negative everywhere, which should not be the

physical wanted case, however, it shows a relaxation to the supersymmetric minimum via a classical process: The anti-branes cluster together to form the maximal size fuzzy NS5-brane which rolls down the potential until it reduces to $M - p$ D3-branes located at the north pole ($\psi = \pi$). In both cases, the result of the process is $M - p$ D3-branes in place of p $\overline{\text{D}}$3-branes with the H_3 flux around the B-cycle changed from K to $K - 1$.

1.3 Vacuum Tunneling

To be completed.

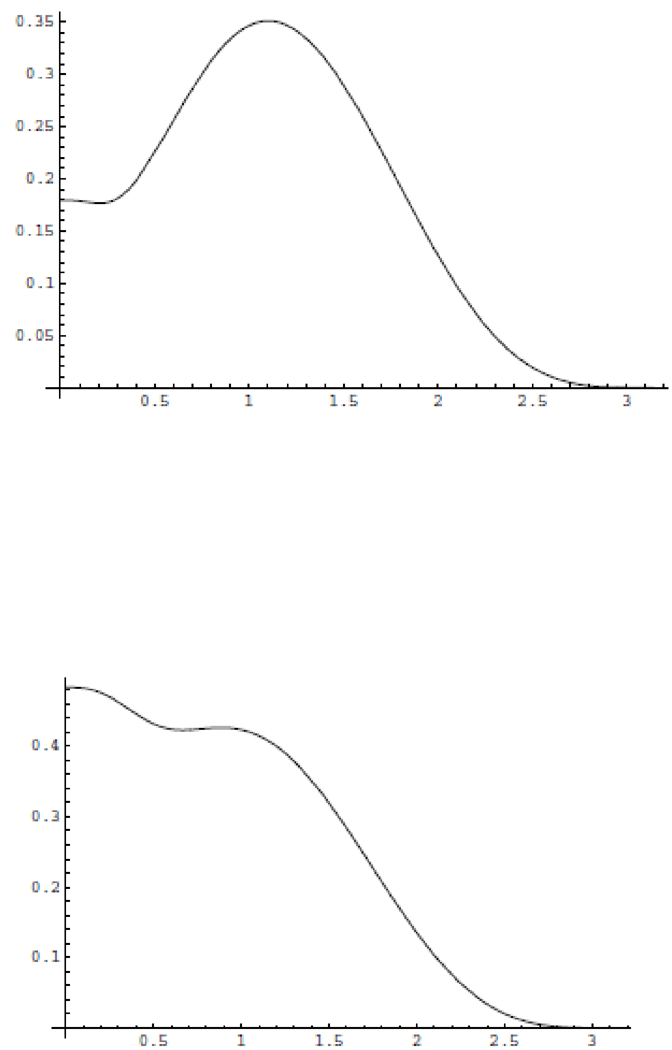

图 1: The effective potential $V_{\text{eff}}(\psi)$ for $\frac{p}{M} \simeq 3\%$, showing the stable false vacuum, and for $\frac{p}{M} \simeq 8\%$ with only a marginally stable vacuum. (from [3] Fig. 2).

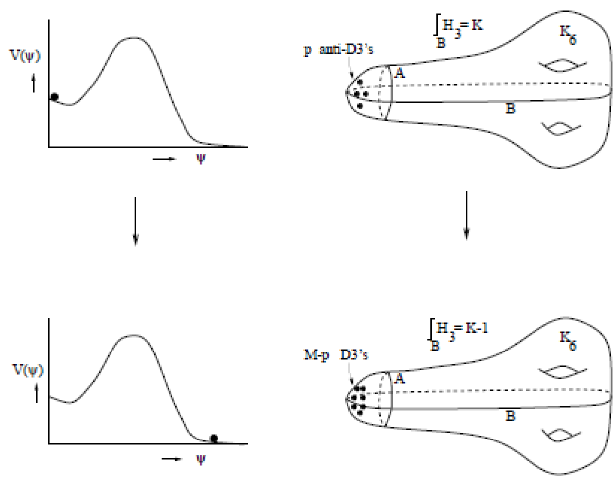

图 2: The decay takes place between an initial non-supersymmetric situation with p $\overline{\text{D}}3$-branes near the tip of the conifold, to a final supersymmetric situation with $M - p$ S3-branes. The total D3 charge is preserved via the simultaneous jump in the H_3 flux around the B-cycle by one unit, from K to $K - 1$. (from [3] Fig. 1).

2 Non-extremal Anti-branes

Although we have determined a reasonable mechanism to describe a string-induced cosmology. It has been pointed out that once backreaction of $\overline{\text{D}}3$ introduced, some unphysical singularities will appear [6]. However, other researchers argued that the observed singularities cannot be proven to exist once one backreacts spherical NS5 ones instead of $\overline{\text{D}}3$ ones, and thus the mata-stability can still be kept even when $p \gg 1$ [7, 8].

Besides, to understand phase transitions in the holographic dual and test the conjectured stability of the KPV state, it is necessary to investigate the finite temperature $\overline{\text{D}}3$-NS5 system. It is also believed that the finite temperature system, which corresponds to non-extremal effect, is an useful way to determine if the singularities can be cloaked by a horizon [9].

Recently, one investigation on non-extremal $\overline{\text{D}}3$-NS5 system has been proposed based on blackfold formalism and provide perfect coincidence with that derived from KPV mechanism [10]. In this section, we give a very short review for the blackfold formalism and highlight some remarkable results from [10].

2.1 Blackfold Formalism

In blackfold generalizations of the fluid/gravity correspondence the long wavelength expansion of the (super)gravity equations around black brane solutions is an affair that combines the fluid dynamical nature of black hole physics with the extrinsic dynamics of hypersurfaces in ambient spacetimes that is characteristic of D-brane physics. Here we omit the motivations and the constructions, with only the essential definition for blackfold equations explicitly displayed. One can check detailed reviews such as [11, 12].

One who have followed the GR lecture knows that for a n-dimensional manifold equipped with metric $g_{\mu\nu}$, the induced metric can be written as

$$\gamma_{ab} = g_{\mu\nu}\partial_a X^\mu \partial_b X^\nu. \tag{22}$$

For a general p-brane, denote the worldvolume as $\mathfrak{W}_{p+1}$, given the induced metric on $\mathfrak{W}_{p+1}$, the first fundamental form of the submanifold is

$$h^{\mu\nu} = \partial_a X^\mu \partial_b X^\nu \gamma^{ab}. \tag{23}$$

Defining $\perp_{\mu\nu} \equiv g_{\mu\nu} - h_{\mu\nu}$, it is easy to see that the tensor $h^\mu{}_\nu$ acts as a projector onto $\mathfrak{W}_{p+1}$, and $\perp^\mu{}_\nu$ along directions orthogonal to $\mathfrak{W}_{p+1}$. The first fundamental form can be used to define the extrinsic equation, at first, we define extrinsic curvature tensor as:

$$K_{\mu\nu}{}^\rho = h_\mu{}^\sigma \overline{\nabla}_\nu h_\sigma{}^\rho, \tag{24}$$

where $\overline{\nabla}_\mu = h_\mu{}^\nu \nabla_\nu$, since the covariant differentiation of tensors that live in the worldvolume is well defined only along tangential directions. Thus $K_{\mu\nu}{}^\rho$ is tangent to $\mathfrak{W}_{p+1}$ along indices μ, ν. Similar to the definition of Riemann curvature, a mean curvature vector can be contractedly defined:

$$K^\rho = h^{\mu\nu} K_{\mu\nu}{}^\rho = \overline{\nabla}_\mu h^{\mu\rho}. \tag{25}$$

Then we can write down the blackfold equations:

$$\perp^\rho{}_\mu T^{\mu\nu} = 0, \tag{26}$$

$$\overline{\nabla}_\mu T^{\mu\rho} = 0. \tag{27}$$

They are equivalent to

$$T^{\mu\nu} K_{\mu\nu}{}^\rho = 0 \quad (\textit{extrinsic equations}), \tag{28}$$

$$D_a T^{ab} = 0 \quad (\textit{intrinsic equations}). \tag{29}$$

Specifically, when the extrinsic equations be written explicitly in terms of the embedding $X^\mu(\sigma^a)$ become

$$T^{ab}\perp_\sigma^{\ \rho}\left(\partial_a\partial_b X^\sigma+\Gamma^\sigma_{\mu\nu}\partial_a X^\mu\partial_b X^\nu\right)=0 \tag{30}$$

$$\Leftrightarrow T^{ab}\left(D_a\partial_b X^\rho+\Gamma^\rho_{\mu\nu}\partial_a X^\mu\partial_b X^\nu\right)=0. \tag{31}$$

Note that all the $T^{\mu\nu}$ are perfect tensor

$$T^{\mu\nu}=(\varepsilon+P)u^\mu u^\nu+Ph^{\mu\nu}, \tag{32}$$

where u^μ the timelike unit velocity-like vector.

2.2 Blackfold Equations for $\overline{\text{D}}$3-NS5 branes

At the tip of the KS throat, the metric 7 can still be taken, but we can choose appropriate units so that the string frame metric is

$$ds^2=g_sMb_0^2\ell_s^2\left(dx^\lambda dx_\lambda+d\Omega_3^2+dr^2+r^2d\Omega_2^2\right), \tag{33}$$

where λ runs over 0 to 3 and α' is not identity any more. In the Einstein frame, the energy-momentum tensor can be computed as

$$T_{ab}=\mathcal{C}\left[r_0^2\left(u_au_b-\frac{1}{2}\gamma_{ab}\right)-r_0^2\sin^2\theta\sinh^2\alpha\hat{h}_{ab}-r_0^2\cos^2\theta\sinh^2\alpha\gamma_{ab}\right], \tag{34}$$

where $\mathbb{C}=\frac{1}{(2\pi)^5g_s^2\ell_s^8}$, r_0 is the Schwarzschild radius, $\hat{h}_{ab}$ obviously is the projector similarly defined in the last subsection, with v^a, w^a the spacelike orthonormal vectors, it will project things onto the directions of the dissolved $\overline{\text{D}}$3-brane charge inside the five-brane. Besides, we should have conserved currents correspond to D3, NS5 and non-zero C_2-field (F_3) charge. In this frame, they obtained as

$$J_2=\mathcal{C}r_0^2\sinh^2\alpha\sin\theta\cos\theta\ v\wedge w, \tag{35}$$

$$J_4=\mathcal{C}r_0^2\sinh\alpha\cosh\alpha\sin\theta*(v\wedge w), \tag{36}$$

$$j_6=\mathcal{C}r_0^2\sinh\alpha\cosh\alpha\cos\theta*1, \tag{37}$$

where J_4, j_6 the electric currents express the D3, NS5 currents of the solution while J_2 contributed by C_2-field, and $*$ the Hodge dual inside five-brane. The parameter θ controls how much $\overline{\text{D}}$3-brane charge is dissolved inside the NS5-brane.

Therefore, the general effective blackfold equations of the $\overline{\text{D}}$3-NS5 brane are

$$\nabla_aT^{a\mu}=\frac{g_s^{-1}}{6!}H_7^{\mu a_1\cdots a_6}j_{6a_1\cdots a_6}+\frac{1}{2!}F_3^{\mu a_1a_2}J_{2a_1a_2}+\frac{3}{4!}H_3^{\mu a_1a_2}C_2^{a_3a_4}J_{4a_1\cdots a_4}, \tag{38}$$

$$d\star J_2+H_3\wedge\star J_4=0, \tag{39}$$

$$d\star J_4-\star \mathrm{j}_6\wedge F_3=0,\quad d\star \mathrm{j}_6=0 \tag{40}$$

Here μ the index in the ten dimensional ambient KS background metric and correspondingly, $\star$ is the Hodge dual for 10d KS metric. Note that the dilaton field is constant and both one-form and five-form RR fluxes are vanished, the three-form NSNS/RR fluxes are the dominating contributions.

2.2.1 Recovering KPV at Extremality

The extremal limit implies $r \to 0$ and $\alpha \to \infty$ with $r^2 e^{2\alpha}$ fixed. We focus on a possibly time-dependent configuration where the five-brane bound state at $r = 0$ wraps an S^2 inside the S^3 of the KS background. Set $v^a\partial_a = (\sqrt{g_s M} l_s b_0 \sin\psi)^{-1}\partial_\omega$, $w^a\partial_a = (\sqrt{g_s M} l_s b_0 \sin\psi \sin\omega)^{-1}\partial_\varphi$, $u^a\partial_a = (\sqrt{g_s M} l_s b_0 \sqrt{1-\psi'^2})^{-1}\partial_t$ with ($t = \sigma^0$, $x^i = \sigma^i$, $\omega = \sigma^4$, $\varphi = \sigma^5$) the static gauge. Then the equations(38)-(40) reduces to

$$\tan\theta = \frac{1}{b_0^2 \sin^2\psi}\left(\frac{\pi p}{M} - \psi + \frac{1}{2}\sin(2\psi)\right), \tag{41}$$

$$\cot\psi = \frac{1}{b_0^2}\sqrt{1-\psi'^2}\sqrt{1+\tan^2\theta} + \frac{1}{b_0^2}\tan\theta - \frac{1}{2}(1+\tan^2\theta)\frac{\psi''}{1-\psi'^2}. \tag{42}$$

Combining these two and eliminate $\tan\theta$ gives

$$b_0^2(b_0^4\sin^4\psi + P^2)\psi'' - 2b_0^2\sin^2\psi P(1-\psi'^2) - 2(b_0^4\sin^4\psi + P^2)(1-\psi'^2)^{\frac{3}{2}} + 2b_0^6\sin^3\psi\cos\psi = 0, \tag{43}$$

where $P = \frac{\pi p}{M} - \psi + \frac{1}{2}\sin\psi = \pi^2 V_2^2$. This equation coincides with the Euler–Lagrange equations derived from KPV DBI action.

In the last section, the maximal p value is given by the marginally merged unstable and metastable vacuum, here we take use of blackfold language to give a different description. Note that the maximal p (say p^*) implies that the $\overline{\text{D}}3$ charges has been maximally dissolved and so that maximal number of NS5-brane would be produced. At extremality, NS5-branes have a non-vanishing Hagedorn local temperature $T_{\text{H}} = \frac{1}{1\pi}\sqrt{\frac{\mathbb{C}}{Q_5}}\sqrt{|\cos\theta|}$. If maximal number of NS5-branes have been produced, the temperature should have a maximum value, in which $\cos\theta = 1$ required. This is also consistent to the maximally dissolved $\overline{\text{D}}3$ charges. Then from 42 it is natural to deduce that

$$\cot\psi^* = \frac{1}{b_0^2} \simeq 0.7506. \tag{44}$$

The derivatives automatically vanished since we are considering the situation near metastable vacuum. Then 41 gives

$$\frac{p^*}{M} = \psi^* - \frac{1}{2}\sin(2\psi^*) \simeq 0.08, \tag{45}$$

that is a satisfactory result as coincided with [3].

2.2.2 Non-extremal Static Configurations

Now consider the finite α and restricting to time-independent profiles. The main difference between extremal and non-extremal case can be understood by considering that of currents.

$$J_2 \propto r_0^2 \sinh^2\alpha \sim e^{2x} \text{ term} + \frac{1}{2} r_0^2 \text{ term} + e^{-2x} \text{ term}, \tag{46}$$

$$J_4 \sim j_6 \propto r_0^2 \sinh\alpha \cosh\alpha \sim e^{2x} \text{ term} + e^{-2x} \text{ term}. \tag{47}$$

Thus 41 will not change while 42 becomes

$$\cot\psi = \frac{1}{b_0^2}\left(\frac{\coth\alpha}{\cos\theta} + \tan\theta\right) \frac{2\cos^2\theta}{2\cos^{2\theta} + (\sinh\alpha)^{-2}}. \tag{48}$$

Now we can analyze the effective potential, which could have dependence on the local temperature $T = \frac{1}{2\pi r_0 \cosh\alpha}$, or the local entropy density $s = 2\pi \mathbb{C} r_0^3 \cosh\alpha$, in non-extremal case. Define the global entropy as

$$S = \int_{\mathcal{B}_5} \frac{\sqrt{-\gamma} s}{\mathbf{k}} = 8\pi^2 (g_s M b_0^2)^{5/2} \mathbb{C} r_0^3 \cosh\alpha \sin^2\psi, \tag{49}$$

where $\mathcal{B}_5$ is the spatial part of $\mathfrak{W}_6$, $\mathbf{k}$ the Killing vector in the direction of u. The potential at fixed global entropy is

$$V_S[\psi] = -\int_{\mathcal{M}_6} d^6\sigma \sqrt{-\gamma}\varepsilon + Q_5 \int_{\mathcal{M}_6} \mathbb{P}\left[B_6\right], \tag{50}$$

with $\varepsilon = \frac{3}{2}\mathbb{C} r_0^2 + |Q_5\sqrt{1+\tan^2\theta}| \tanh\alpha$ the local energy density and $\mathbb{P}[B_6]$ the pullback of the background B_6-field. $V_S[\psi]$ can be understood as the total energy which differ to the Euclidean on-shell action of the $\overline{\text{D}}$3-NS5 bound state by a Legendre transformation.

2.3 Results for Non-extremal Configurations

3 illustrates the effective potential at different fixed entropy while the right side graph is the zooming-in of the left-side one near north pole ($\psi \simeq 0$). It is obviously that, except the metastable one (blue points) and a global unstable vacuum, there are new unstable vacuum (black points) appear near the north pole in the non-extremal case. Remarkably, the new unstable vacuum will coincide with the metastable one and merge into one point with a critical value $S^* \simeq 0.01$. If S becomes larger, the metastable vacuum will vanish. The new unstable state represents a fat black NS5 with a highly pinched $\mathbb{R}^3 \times S^5$ horizon geometry that resembles a black $\overline{\text{D}}$3. While the metastable state implies a thin black NS5 with $\mathbb{R}^3 \times S^2 \times S^3$ horizon topology. At the merger point,

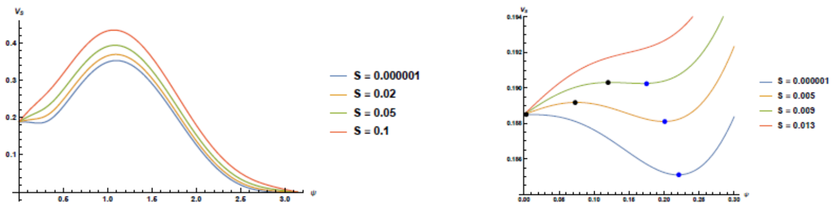

图 3: Plots of the effective potential at fixed entropy, V_S, as a function of the angle ψ on S^3. Both figures represent plots at $p/M = 0.03$. The tight plot zooms into the region near the North pole of the S^3. As we increase the entropy we encounter a critical value S^*, where the metastable vacuum of KPV (blue dots on the right) merges with a new unstable vacuum (black dots on the left). (from [10] Fig. 1).

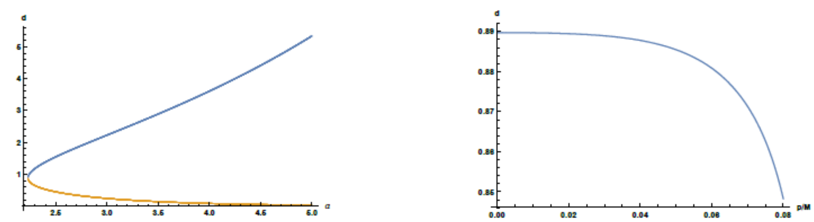

图 4: Plots of the ratio d that express how 'fat' a non-extremal $\overline{\text{D}}$3-NS5 bound state is. On the left plot we depict the dependence of d on the non-extremality parameter α for the unstable (blue) and metastable (orange) branches for $p/M = 0.03$. On the right plot, we depict d at the critical merger point as a function of p/M. (from [10] Fig. 2).

the metastable black NS5 turns effectively into a black $\overline{\text{D}}$3. Both illustrations are fixed with $p/M = 0.03$.

We could define a fatness ratio to measure the S^2 wrapping magnitude inside NS5. $d = \frac{2\sqrt{p}\hat{r}_0}{\sqrt{M}\sin\psi}$, where $\hat{r}_0 \equiv \sqrt{\frac{\mathbb{C}}{Q_5}}r_0$. Also the function between d and α can be depicted as 4. Obviously, if we take $\alpha \to \infty$, which is the extremal case, the unstable vacuum will exclusively dominate because of the tiny r_0. The merger of both two branches occurs at $d \simeq 1$.

The right-side plot is the relation between d and p/M. As expected, d is nearly a constant which is only slightly changed with $p/M \lesssim 0.08$. This fact strongly implies that the properties of the merger point are closely tied to the properties of the horizon geometry.

2.4 Regimes of Validity

The blackfold approach can be applied to D3-NS5 branes in a far-zone background KS geometry only when the characteristic length scales of the near-zone solution, denoted collectively by r_b, are hierarchically smaller than the characteristic length scales $\mathcal{R}$ of worldvolume inhomogeneities, and that of the background, denoted by L. The validity regime is $r_b \ll \mathcal{R}, L$, which corresponds to $p \ll M, K$ in section 1.

According to the analysis of the last few subsections, r_b is the largest scale among the energy density scales $r_\varepsilon \sim r_0 \sinh\alpha$ and that of $\overline{\mathrm{D}}3$/NS5 charges. And $\mathcal{R}$ is controlled by the magnitude of internal S^2 wrapping inside spherical NS5 while L is certainly the largest one since it corresponds to the scale of S^3. Thus we only need to consider $r_h^{(NS5)} \ll \mathcal{R}$ and $r_h^{(\overline{\mathrm{D}}3)} \ll \mathcal{R}$, these give

$$N_5 \ll g_s M \sin^2\psi, \tag{51}$$

$$\sqrt{\frac{p}{M}} \ll \sqrt{g_s M} \sin^2\psi. \tag{52}$$

Obviously, the blackfold formalism will be ineffective around the north pole, at which ψ becomes too tiny to keep the validity. However, for sufficiently large M, everywhere except the north pole could be perfectly described by this new approach.

Besides, $r_\varepsilon \ll \mathcal{R}$ is also a constraint which leads to

$$d \ll \frac{\sqrt{g_s p}}{\sqrt{N_5 \sinh\alpha}}. \tag{53}$$

This restricts the regime of validity of the unstable branch close to extremality, at large values of α.

3 Ideas and Questions

3.1 $\overline{\text{D}}$3 Backreaction

As [10] showed, the singularities and the corresponding unstable vacua can be treated as horizon geometry in non-extremal case. One natural idea is to further explore the instabilities in different non-extremal situations with full backreaction considered. To describe the physical world, one may hope to understand how the non-extremal picture constructed when the S-duality-induced fermionic NS5-brane be treated. The action up to quadratic order in fermions is given by [13] (still, the dilaton field is constant and one-form and five-form fluxes vanished):

$$S_{NS5} = \frac{1}{2}\frac{\mu_5}{g_s^2}\int d^6\xi\sqrt{-\det(g+2\pi g_s\mathcal{F}_2)}\bar{\theta}(1-\Gamma_{NS5})\left[(\bar{M}^{-1})^{\alpha\beta}\hat{\Gamma}_\beta D_\alpha - \Delta\right]\theta, \tag{54}$$

where

$$\begin{aligned}
\bar{M}_{\alpha\beta} &= g_{\alpha\beta} + 2\pi g_s\sigma_3\mathcal{F}_{\alpha\beta},\\
D_\alpha &= \nabla_\alpha + W_\alpha,\\
W_\alpha &= \frac{1}{8}\left(-F_{\alpha np}\Gamma^{np}\sigma_3 + \frac{1}{3!}g_s^{-1}H_{mnp}\Gamma^{mnp}\hat{\Gamma}_\alpha\sigma_1\right),\\
\Delta &= \frac{1}{24}\left(-F_{mnp}\sigma_3 - g_s^{-1}H_{mnp}\sigma_1\right)\Gamma^{mnp}.
\end{aligned} \tag{55}$$

Here the indices m, n are ten-dimensional curved indices while α, β are worldvolume ones. $\hat{\Gamma}_\alpha \equiv \Gamma_{\underline{m}} e_m^{\underline{m}}\partial_\alpha x^m$ with underline indices denote that of tangent space ones. θ is obviously the fermion doublet, in this action, it is considered as a Majorana–Weyl spinor with positive chirality.

In [14], the authors analyzed the supersymmetry breaking mechanism of this model without coupling gravity. The mass matrix they derived is:

$$\mathcal{M} = \frac{1}{24}\left(\cos(2\alpha)F_{mnp} - g_s^{-1}\cos(\alpha)H_{mnp}\Gamma_{\underline{0123}}\right)\Gamma^{mnp}. \tag{56}$$

This is the mass matrix on the six-dimensional worldvolume. With the imaginary self-dual condition, it can be reduced to four-dimensional complex three-form:

$$\mathcal{M} = \frac{1}{48}(\cos(2\alpha)+\cos(\alpha))\left(G_3+\bar{G}_3\right)_{mnp}\Gamma^{mnp}. \tag{57}$$

This is the known mass term for $\overline{\text{D}}$3-branes in a supersymmetric background with fluxes that carry on D3-brane charges. Then the dependence to angular coordinate ψ can be treated for mass matrix, as [14] showed,

$$\psi = 0: \quad \mathcal{M} = \frac{1}{24}(G_3 + \bar{G}_3)_{mnp}\Gamma^{mnp}, \tag{58}$$

$$\psi = \pi: \quad \mathcal{M} = 0, \tag{59}$$

$$\psi = \psi_{min}: \quad \mathcal{M} = \left(\frac{1}{24} - \frac{5}{6}\frac{\pi^2}{b_0^{12}}\frac{p^2}{M^2}\right)(G_3 + \bar{G}_3)_{mnp}\Gamma^{mnp}, \tag{60}$$

where the ψ_{min} is similar to ψ^* as we noted in bosonic action, it denotes the meta- or un- stable point of fermionic effective potential, satisfies: $\alpha(\psi_{min}) = \frac{4\pi}{b_0^6}\frac{p}{M}$.

After nilpotent superfields, which can be decomposed to a massless goldstino λ^0 and the triplet λ^i under the SU(3) holonomy, introduced, the authors of [14] discussed the SUSY of $\overline{\text{D}}$3-branes at both South pole, North pole and metastable minimum, respectively. And the SUSY is realized linearly and non-linearly at the South pole and North pole, respectively. This also coincides with the fact that the North pole always induce an unstable vacuum for p/M no longer small. As section 2 showed, the similar phenomena should be restored even for non-extremal case.

Besides, the treatment near metastable minimum implies that the Kaluza–Klein (KK) tower of fields involved in S^2 has to be considered to restore a linearly realized SUSY. To discover if the similar things happen in non-extremal case could be an evidence that the finite temperature field theory can be equipped with compactification.

Furthermore, though not explicitly discussed in this note, to determine the decay rate of the $\overline{\text{D}}$3-NS5 branes system embedded into GKP background and confirm the exponentially suppression is necessary for both extremal and non-extremal case. With blackfold formalism, the tunneling amplitude can be associated better to the reality.

3.2 Ten-dimensional Uplifting

Inspired by the fact that blackfold approach alleviates the instabilities of singularities, one may wonder more explicitly that how to build a general mechanism which could restore de Sitter vacua from 10d, beyond the Kachru–Kallosh–Linde–Trivedi (KKLT) scenario [15].

Although I have not formed a clear idea about this question, there are a huge amount of references recently raised [16, 17, 18, 19].

[16] pointed out that the simplest KKLT vacua cannot be uplifted to de Sitter and the strong backreaction-induced uplift will always result in metastable, SUSY breaking, but AdS vacua. By racetrack stabilization, they showed that the cosmological constant of supersymmetric AdS vucua can be tuned to zero with finite moduli stabilization retained. This implies that the de Sitter uplifts are possible with sufficiently small

backreaction. One may wonder if the non-extremal backreaction also satisfies this conclusion[1].

[19] set up from fermionic couplings in the D7-brane action and derived the ten-dimensional energy-momentum tensor due to gaugino condensation, argued in [17], on D7-branes. After such tensor combined with that of $\overline{\text{D}}$3-branes, the effects of them can be computed either in ten dimensions or in the four-dimensional effective theory. That is a great generalization from 10d to 4d for KKLT scenario.

3.3 More Fundamental Things

One thing I noticed is that all moduli stabilization and the relative dS vacua uplifting, more specifically, either KKLT or Large Volume Scenario (LVS), are focusing on IIB string or IIB SUGRA foundation. Since the IIA string theory only differ by a T-duality to IIB, why there are only a few papers consider such mechanisms induced from IIA string or SUGRA? One recent paper [20] discussed such uplifting for $\overline{\text{D}}$6-branes, it has been pointed out that, by adding pseudo-calibrated $\overline{\text{D}}$p-branes wrapped on supersymmetric cycles, one can generalize the effective four-dimensional SUGRA derived from string theory in a way that it includes a nilpotent multiplet [21]. However, an explicit KKLT-like construction in IIA string theory has not been presented until now. [20] only tried to obtain the mass matrix and tuned some possible metastable minimum from AdS to dS. The physical mechanism equipped with nilpotent multiplet is still to be discovered. To be honest, this could be the thing most interested me, but it could be pretty hard for a master project.

Another topic I am interested in is how the KS geometry be realized in F theory and if there exists any other way to realize a geometry which could introduce NSp/Dp-branes. As I know, Hanany–Witten transition [22] guarantees that when an NS5-brane and a Dp-brane cross, one necessarily creates or destroys a D$(p-2)$-brane. Therefore, the $\overline{\text{D}}$3-NS5 branes could be generalized to $\overline{\text{D}}$p-NS5 branes with more fruitful geometry and relative consequences[2].

[1] Since in non-extremal case, the unstable vacua are all believed to be treatable as horizon geometry, we would hope that such conclusion can be generalized and de-constrained in the non-extremal case.

[2] I concede that I have not done clearly literature researches on this topic, it is also possible that people have thought about that and only $\overline{\text{D}}$3-NS5 one can be uplifted with a metastable vacuum which coincides with the reality.

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