- ExFT [Olaf, Samtleben, Zwiebach, Emanuel]

ExFT is an extension of 10- and 11-d SUGRA that makes an $E_{d(d)}$ sym. manifest.

D	$E_{d(d)}$	H_d	G_{half}	G_R
7	$SL(5)$	$USp(4)$	$SU(2)$	$SU(2)$
6a	$Spin(5,5)$	$USp(4)\times USp(4)$	$SU(2)\times SU(2)$	$SU(2)\times SU(2)$
6b	$Spin(5,5)$	...	$USp(4)$	$USp(4)$
5	$E_{6(6)}$	$USp(8)$	...	...
4	$E_{7(7)}$	$SU(8)$	$SU(4)$	$SU(4)\times U(1)$

K-K A 11d SUGRA → D ex., d int. } bos. d.o.f. into rep. of $E_{d(d)}$

fixed external legs → R_i of $E_{d(d)}$

internal → y^M → R_1 ...

↓

combined into sym. $E_{d(d)}$ mat.

$\mathcal{M} \in \frac{E_{d(d)}}{H_d}$ (generalized metric)

Rep: D	R_1	R_2	R_3	R_4	R_c
7	10	$\bar{5}$	5	$\overline{10}$	ϕ
6	16	10	$\overline{16}$	N/R	1
5	27	$\overline{27}$	78⊕1	N/R	27
4	56	133	912	N/R	1539

Want: diffm. + gauge : local $E_{d(d)}$ transf. → gen. diffm.

11d SUGRA

gen. → diffm. gauge.

local ...

Want: diffm. + gauge : local $E_{d(d)}$ transf. $\to$ gen. diffm.

$\underbrace{\text{diffm. + gauge}}$ IIA SUGRA

$\overbrace{\text{diffm.} \quad \text{gauge.}}^{\text{gener.}}$

4 56 133 912 N/R 1539

Local sym. :

vect. pform

$\underbrace{\text{vect.} \quad \text{pform}}$

gen. vect. fd. $\to$ R. of $E_{d(d)}$

$$\underset{\text{gen. Lie der.}}{\mathcal{L}_\Lambda} V^M = \Lambda^N \partial_N V^M + (\mathbb{P}_{adj})^M{}_N{}^P{}_Q V^N \partial_P \Lambda^Q + \lambda V^M \partial_N \Lambda^N$$

V - gen. vect. fd. of weight λ

∂_M - $\frac{\partial}{\partial y^M}$

$\mathbb{P}_{adj}$ - proj. onto adj. of $E_{d(d)}$

When $\lambda = \frac{1}{D-2}$, $\mathcal{L}_\Lambda V^M = \Lambda^N \partial_N V^M - V^N \partial_N \Lambda^M + \underset{E_{d(d)} \text{ inv. tensor}}{Y^{MN}_{PQ}} V^P \partial_N \Lambda^Q$

closed gen. alg. : $Y^{MN}_{PQ} \partial_M \otimes \partial_N = 0$ - Section constr. $\begin{cases} \text{IIB SUGRA} \\ \text{11d} \ \ldots \\ \text{ExFT} \end{cases}$

$\to$ inv. action ______ [sambtleben]

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↳ Half-maximal SUGRA

int. mfd. admit $\frac{1}{2}$-max. # of spinors. (e.g. no flux, spinors of $SO(d)$)

Flux → H_d action on spinors.

$$\begin{array}{c} H_d \\ \cup \\ SO(d) \end{array}$$

G-structure:

$$\begin{array}{ccccccc} E_{d(d)} & \to & H_d & \to & G_{half} & \to & G_p \\ & & \cup & & & & \\ GL(d) & \to & SO(d) & \to & SO(d-1) & & \end{array}$$

R-sym. of $\frac{1}{2}$-max. ⟺ max. commutant of G_{half}

} algebraic constr.

max. comp. sub.

stabilizer of $\frac{1}{2}$-max. set of spinors in H_d

To reduce and yield $Mink_D / AdS_D$,

diff. constr. required. ~ integrability ~ holonomy constr.

Generalization: vect. bund. → diff. forms (with Λ)

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Generalization : vect. bund. $\rightarrow$ diff. forms (with $\wedge$)

Def. vect. bund. $\bar{\mathcal{R}}_i$, vect. spaces R_i, base spa. = full ..

Weig. vect. bund. $\mathcal{R}_i = \bar{\mathcal{R}}_i \otimes S^i$

$\qquad\qquad\qquad\qquad$ $\shortparallel$ rank-0 vect. bund. $\cong \det(T^*M)^{\frac{i}{D-2}}$

$\qquad\qquad\qquad\qquad$ sect. = scalar densities of weig. $\frac{i}{D-2}$

Def.

$\wedge : R_i \otimes R_j \rightarrow R_{i+j}$

e.g. $A \wedge B = (A \otimes B)|_{R_{i+j}}$ for $A \in R_i$, $B \in R_j$

$\wedge : \mathcal{R}_i \otimes \mathcal{R}_j \rightarrow \mathcal{R}_{i+j}$ sect. = {vect. fd. + diff. forms.

$\bar{\mathcal{R}}_i \otimes \bar{\mathcal{R}}_j \rightarrow \bar{\mathcal{R}}_{i+j}$

Def. adj. rep. P , adj. bund. $\mathcal{P}$

for $0 < i \leq D-3$, $R_{D-2-i} = R_i^* \Rightarrow \mathcal{R}_{D-2-i} = \mathcal{R}_i^* \otimes S^{D-2}$

Def. $\wedge : R_i \otimes R_{D-2-i} \to 1$

$\wedge_P : R_i \otimes R_{D-2-i} \to P$ } singlet rep.

Def. $\wedge : \mathcal{R}_i \otimes \mathcal{R}_{D-2-i} \to S^{D-2}$

$\wedge_P : \mathcal{R}_i \otimes \mathcal{R}_{D-2-i} \to \mathcal{P} \otimes S^{D-2}$

Def. gen. wedge prod. $(D \geq 5)$

$\bullet : R_i \otimes R_j \to R_{i+j+2-D}$ $\quad$ $(i+j > D-2)$

$\bullet_P : R_i \otimes R_j \to R^P_{i+j+2-D}$

convention: $R_0 = \mathbb{1}$ $\quad$ $R^P_0 = P$, $\quad R^P_i = R_i$

similarly defined on $\mathcal{R}_i$

Def. (diff. oper.) $V \in \Gamma(\mathcal{R}_i)$

$d : \Gamma(\mathcal{R}_i) \to \Gamma(\mathcal{R}_{i-1})$

e.g. $dV = (\partial \otimes V)|$ ...

Def. (diff. oper.) $V \in \Gamma(R_i)$

$$d : \Gamma(R_i) \to \Gamma(R_{i-1})$$

e.g. $dV = (\underset{\partial_M}{\overset{\shortparallel}{\partial}} \otimes V)\Big|_{R_{i-1}} \Rightarrow \underset{(D\geq 6)}{d^2 = 0}$

Fact. ① $\mathcal{L}_A B = A \wedge dB + d(\underset{\Gamma(R_2)}{A\wedge B}) \quad \forall A \in \Gamma(R_1)$

② $\mathcal{L}_{A_1} A_2 + \mathcal{L}_{A_2} A_1 = d(\underset{\Gamma(R_1)}{A_1 \wedge A_2})$

$D \geq 5$ G-structure

Want: gen. $SO(d-1,d-1) \subset E_{d(d)} \times \mathbb{R}^+$

$\Leftrightarrow$ having fields: $\left\{\begin{array}{l} \text{scalar density } \kappa \in \Gamma(S) \text{ of weight } \frac{1}{D-2} \\ \text{sect. } K \in R_2 \\ \quad\;\; \hat{K} \in R_{D-4} \end{array}\right\} = \mathcal{K} = (K, \hat{K}, \kappa)$

$(K \otimes K)\big|_{\dots} = 0 \quad (\hat{K} \otimes \hat{K})\big|_{\dots} = 0$ (point-wise cond.)

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$\Leftrightarrow$ having fields: $\begin{cases} \text{scalar density } k \in \Gamma(S) \text{ of weig. } \frac{1}{D-2} \\ \text{sect. } K \in R_2 \\ \quad\cdot\cdot\ \hat{K} \in R_{D-4} \end{cases}$ $= \mathcal{K} = (k, K, \hat{K})$

$(K \otimes K)\big|_{R_c \otimes S^4} = 0 \qquad (\hat{K} \otimes \hat{K})\big|_{R_c^* \otimes S^{2D-8}} = 0$ (point-wise cond.)

$K \wedge \hat{K} = k^{D-2}$ (compatibility cond.)

Def. $J_u \in \Gamma(R_1)$, $u = 1, \dots, d-1$ labels $SO(d-1)$ R-sym. $\rightarrow \begin{cases} J_u \wedge K = 0 & \text{— } J_u \text{ in vect. rep. } V_{d-1,d-1} \text{ of } SO(d-1, d-1) \\ J_u \wedge J_v = \delta_{uv} K & \text{— } J_u \text{ as vect. of } SO(d-1)_R \end{cases}$

(comp. cond.)

$V_{d-1,d-1} \downarrow V_S \oplus V_R$ of $SO(d-1,d-1) \downarrow SO(d-1)$

V_S: struc.; V_R: R-sym.

$V_S \oplus V_R$ enter this equ. with opposite signs

Rk. J_u, K & $\hat{K}$ $\rightarrow$ generalization of almost hyper-complex structures on 4-mfd.

Rk. J_u stabilized by $SO(d-1)_S$ (form a basis (d-1))

$\mathcal{K} \Leftrightarrow SO(d-1)_S$

Def. ∇ a G_{half}-connec. s.t.

$\nabla J_u = \nabla \mathcal{K} = 0$

Def. Torsion $\tau^M{}_{NP}$ the tens. part of conn. given by

$$(\mathcal{L}^{\nabla}_{\Lambda} - \mathcal{L}_{\Lambda}) V^M = \tau^M{}_{NP} V^N V^P$$

Rk. τ only take values in certain rep. $\tau \subset W \subset R_1^* \otimes P$

Generally,

$$\begin{cases} \mathcal{L}_{J_{[u}} J_{v]} = W_{J_{uv}} \\ \mathcal{L}_{J_u} \hat{K} = W_{C_u} \\ dK = W_K \\ d\hat{K} = W_{\hat{K}} \quad (D=6,7) \\ \mathcal{L}_{\hat{K}} K = W_{\hat{K}}, \ \mathcal{L}_{\hat{K}} J_u = W_{\hat{K}_u} \quad (D=5) \end{cases}$$

parametrise intrinsic torsion

indep. comp. of intrinsic torsion

RmK. torsion classes transf. in certain irrep.

dep. on D in detail

↪ $\frac{1}{2}$-max. Mink. & AdS vacua

SUSY variations of SUGRA: inter. space cond.

$\rightarrow$ G_{half} has vanishing ∇ tors.

$$\left.\begin{array}{l} \mathcal{L}_{J_u} J_v = \mathcal{L}_{J_u}\hat{K} = \mathcal{L}_{J_u} K = dK = 0 \\ \begin{cases} d\hat{K} = 0 & (D=6,7) \\ \mathcal{L}_{\hat{K}} K = \mathcal{L}_{\hat{K}} J_u = 0 & (D=5) \end{cases} \end{array}\right\} \Longleftrightarrow$$

integrable G_{half} struc. (vanishing of certain moment maps) $\rightarrow$ Mink. flux vac.

$\xRightarrow{\text{weaken}}$ AdS vacua (preserve all SUSY for $\frac{1}{2}$-max)

$$\begin{cases} dK = \mathcal{L}_{J_u}\hat{K} = \mathcal{L}_{J_u} K = 0 \\ \mathcal{L}_{J_u} J_v = \underbrace{\bar{R}_{uvw}}_{\text{encodes c.c.}} J^w \end{cases} \quad \& \quad \begin{cases} d\hat{K} = -\frac{1}{4}\epsilon_{uvw}\bar{R}^{uvw} K & (7D) \\ d\hat{K} = -\frac{1}{18}\epsilon_{uvwx} J^u \bar{R}^{vwx} & (6D) \\ \mathcal{L}_{\hat{K}} J_u = -\frac{1}{3\sqrt{2}}\epsilon_{uvwxy} J^v \bar{R}^{xwy},\ \mathcal{L}_{\hat{K}} K = \mathcal{L}_{\hat{K}} \hat{K} = 0 & (5D) \end{cases}$$

breaks R-sym. $\rightarrow$ lower-dim. SCF. alg.

D	$SO(d-1)_R$	Rep. of $\bar{R}_{uvw}$	Unbroken R-sym.
7	$SU(2)$	1	$SU(2)$
6	$SU(2)\times SU(2)$	(2, 2)	$SU(2)$
5	$USp(4)$	10	$SU(2)\times U(1)$
	Mink.	AdS	

Exp. 1. M-theory on K3

$$\left.\begin{array}{l}\text{vect. fd.} - v \\ \text{p-form.} - \omega_{(p)}\end{array}\right\} \xrightarrow[\text{fluxes}]{\text{vanishing}} \begin{cases} K^4 = \sqrt{g} & \text{- 4d det. metr.} \\ K = \omega_{(4)} + \omega_{(1)} \\ \hat{K} = \omega_{(3)} + \omega_{(0)} \\ J_u = v_u + \omega_{(2)u} \end{cases}$$

① Compat. requirement: $\omega_{(0)} \wedge \omega_{(4)} + \omega_{(1)} \wedge \omega_{(3)} = \mathrm{vol}_4$

(for K & $\hat{K}$) $\quad \omega_{(0)} = 1$, $\quad \omega_{(4)} = \mathrm{vol}_4(g)$ (for K3 taken)

②

(for J_u) : $\iota_{v_u} \omega_{(4)} = 0$, $\quad \omega_{(2)u} \wedge \omega_{(2)v} = \delta_{uv} \mathrm{vol}_4$ (solved by taking 3 2-forms)

Denote $\omega_{(2)u} = \Omega_u$

$$\Rightarrow \begin{cases} K^4 = \sqrt{g} \\ K = \mathrm{vol}_{(4)} \\ \hat{K} = 1 \\ J_u = \Omega_u \end{cases} \xrightarrow[\text{tors.}]{\text{intr.}} \begin{cases} dK = d\hat{K} = 0 & \text{(automatically vanished)} \\ \mathcal{L}_{J_u} J_v = 0 \\ \mathcal{L}_{J_u} \hat{K} \propto d\Omega_u & \rightarrow \text{vanish} \because \text{kähler \& holom. 2-form closed } (d\Omega_u = 0) \\ \mathcal{L}_{J_u} K^4 = 0 \end{cases}$$

2. M-th. on $K3\times S^1$

($\Longleftrightarrow$ IIA SUGRA on $K3$)

$$\begin{cases} \kappa^4 = \sqrt{g} \\ K = \omega_{(1)} + \omega_{(4)} \\ \hat{K} = \hat{\omega}_{(1)} + \hat{\omega}_{(4)} \\ J_u = v_u + \omega_{(1)u} + \omega_{(5)u} \end{cases}$$

$$(K,\hat{K}) \xrightarrow[\text{for dilaton str.}]{\text{comp. cond.}} \begin{cases} \omega_{(4)}\wedge\omega_{(1)} = \hat{\omega}_{(4)}\wedge\hat{\omega}_{(4)} = 0 \\ \omega_{(4)}\wedge\hat{\omega}_{(1)} + \hat{\omega}_{(4)}\wedge\omega_{(1)} = \mathrm{vol}_5 \end{cases}$$

$$(J_u) \longrightarrow \begin{cases} L_\zeta \omega_{(1)u} = 0 \qquad \zeta \equiv *\,\mathrm{vol}_4 \\ L_{v_u}\mathrm{vol}_4 = 0 \end{cases}$$

vect. fd. on S^1 ($\iota_\zeta \sigma = 1$)

$$\omega_{(1)u}\wedge\omega_{(1)v} + \tfrac{1}{2}\left(L_{v_u}\omega_{(5)v} + L_{v_v}\omega_{(5)u}\right) = \delta_{uv}\,\mathrm{vol}_4$$

↘ solve $J_u = \Omega_u$ – hyperkähler str.

by taking $J_4 = \zeta + \mathrm{vol}_5$

$$\xrightarrow[\omega_1 = \hat{\omega}_{(4)} = 0]{\text{taking } K = \omega_{(4)},\ \hat{K} = \hat{\omega}_{(1)}} \underbrace{\begin{matrix} K = \mathrm{vol}_4 \\ \hat{K} = \sigma \end{matrix}}_{\text{vanished intr. tors.}}$$

$\hat{K} = \sigma$ – conn. 1-form on S^1

3. M-th. on $K3\times T^2$

(type II str. on $K3\times S^1$)

dilton str. $\begin{cases} k^3=\sqrt{g} \\ k=\sqrt{g}\,\omega_{(1)}+\omega_{(4)}+\omega_{(1)} \\ \hat{k}=\hat{v}+\omega_{(2)}+\omega_{(5)} \end{cases}$ $\xrightarrow[\text{coord. }(K,\hat{K})]{\text{comp.}}$ $\begin{array}{l} K=\mathrm{vol}_4\,(K3) \\ \hat{K}=\mathrm{vol}_2\,(T^2) \end{array}$

$\xrightarrow{(J_u)}$ $J_u=v_u+\omega_{(2)u}+\omega_{(5)u}$ $\rightarrow \begin{cases} \mathrm{vol}_{(4)}\wedge\omega_{(2)u}=0 \\ \iota_{v_u}\mathrm{vol}_4=0 \\ \iota_{\tilde{v}_u}\mathrm{vol}_4=0 \end{cases}$ $\tilde{v}_u\equiv *\omega_{(5)u}$ & $\begin{cases} \iota_{v_u}\omega_{(2)v}+\iota_{v_v}\omega_{(2)u}=0 \\ \iota_{\tilde{v}_u}\omega_{(2)v}+\iota_{v_u}\omega_{(5)v}=0 \\ \omega_{(2)u}\wedge\omega_{(2)v}+\iota_{v_u}\iota_{\tilde{v}_v}\mathrm{vol}_6+\iota_{v_v}\iota_{\tilde{v}_u}\mathrm{vol}_6=\delta_{uv}\mathrm{vol}_4 \end{cases}$

$\Rightarrow$ solve by taking $J_u=\Omega_u$ - holom. & kähler

vect. fd. in T^2

$J_4=\cancel{6}+\tilde{6}\wedge\mathrm{vol}_4$ (closed) 2-form

$J_5=6'+\tilde{6}'\wedge\mathrm{vol}_4$

$\tilde{6},\tilde{6}'$ dual one-form

$\longrightarrow$ vanishing intr. tors.