M. C. Lin

B. Ravani

S. A. Velinsky

Department of Mechanical and Aeronautical Engineering, University of California-Davis, Davis, CA 95616

Kinematics of the Ball Screw Mechanism

This paper studies the kinematics of the Ball Screw Mechanism (BSM) with the aim of developing a foundation for understanding the motion of the balls and their contact patterns with the contacting elements. It is shown that there is always slip between the balls and the nut or screw, and therefore, the no-slip condition assumed in the BSM literature is not attainable. The effect of contact deformation on the motion of the balls is also studied and is used to develop the pattern of the constant sliding lines of contact between the ball and the screw or the nut. The results have applications in efficiency analysis, design, wear evaluation and finite element modeling of the BSM.

1 Introduction

The reciprocating ball screw mechanism is a force and motion transfer device belonging to the family of power transmission screws (Fig. 1). Two of the most important features of the mechanism are its positional accuracy and load carrying capacity making it suitable as the drive mechanism for robot manipulators or the feed-drive mechanism of machine tools. The utilization of bearing balls in the mechanism replaces the sliding friction of the conventional power screw with the rolling friction of the balls. This results in minimal friction during force and motion transmission and eliminates slipstick with minimal wear.

This paper provides a theoretical study of the kinematics of the ball screw mechanism. It derives relationships describing the motion of the ball and shows that slipping takes place between the ball and the nut (or the screw) at all times. This means that the no-slip condition assumed in the literature (Levit, 1963; Drozdov, 1984) is unattainable. The proper slip conditions are derived and the ball motion is studied.

In addition, the effect of elastic deformation at contact areas between the ball and the nut (or the screw) on the kinematics of the mechanism is analyzed. The analysis is used to determine the pattern of constant sliding lines of the ball in the contact area. The contact line pattern is useful in wear and finite element analyses of the ball screw mechanism. The work presented provides a theoretical framework for understanding the motion of the ball in the BSM and sets the basis for efficiency and design analysis of the mechanism. The application of the results of this paper to efficiency and friction analysis of the BSM is given in a companion paper (Lin, Velinsky, and Ravani, 1994).

In all the analyses presented right-hand screw threads and a single nut are assumed.

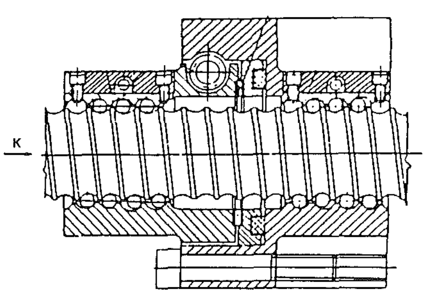

Fig. 1 A ball screw and nut mechanism

2 Motion of the Ball

In this section, we study the motion of the ball by affixing a Frenet coordinate system to the path of the center of the ball. This will enable us to study the kinematics of the ball motion and derive the slip conditions.

In a BSM, the center of the ball moves along the helical groove of the screw. We introduce three sets of coordinate systems. The first (world) coordinate system, $ox'y'z'$, is fixed with its z' axis coincident with the axis of the screw. The second (rotating) coordinate system, $oxyz$, also has its z axis coincident with the screw axis (Fig. 2) but it rotates with the screw. The third coordinate system is the Frenet frame moving with the center of the ball along the trajectory of the ball center. This trajectory, with respect to the frame $oxyz$, is a circular helical line along a circular cylindrical surface with a mean radius r_m (Fig. 2). The coordinate transformation between the first two coordinate systems can be written as

$$\mathbf{X}' = \mathbf{T}_1\mathbf{X} \tag{1}$$

where $\mathbf{X}' = [i'\ j'\ k']^T$, $\mathbf{X} = [i\ j\ k]^T$,

$$\mathbf{T}_1 \equiv \begin{bmatrix} C_\Omega & -S_\Omega & 0 \\ S_\Omega & C_\Omega & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Contributed by the Mechanisms Committee for publication in the Journal of Mechanical Design. Manuscript received March 1991; revised Feb. 1994. Associate Technical Editor: G. L. Kinzel.

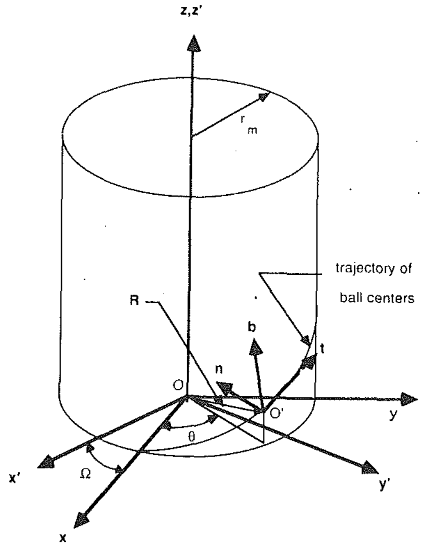

Fig. 2 The position of the ball center, O′, in Cartesian coordinates and Frenet coordinates

and $C_\Omega \equiv \cos(\Omega)$, $S_\Omega \equiv \sin(\Omega)$, and Ω denotes the angular displacement between the two coordinate systems.

Assume that a ball has moved through an angle $\theta\mathbf{k}$ along the helical groove of a screw with lead L. The position vector of the ball center, $\mathbf{R}(\theta)$, can be expressed as

$$^r\mathbf{R}(\theta) = \mathbf{R}^T\mathbf{X} \tag{2}$$

where $\mathbf{R}^T = [r_m C_\theta \; r_m S_\theta \; r_m \theta t_\alpha]$, $C_\theta = \cos(\theta)$, $S_\theta = \sin(\theta)$, and the helix angle, α, is defined as $t_\alpha = \tan(\alpha) = L/2\pi r_m$. The superscripts "$r$" and "$W$" are used here to distinguish a vector with respect to the rotational and the world coordinate systems, respectively.

By substituting Eq. (1) into Eq. (2), the position of the ball center can be expressed with respect to the world coordinate system as

$$^W\mathbf{R} = \mathbf{R}^T\mathbf{T}_1^{-1}\mathbf{X}'. \tag{3}$$

By definition (Kreyszig, 1983), the triad of unit vectors describing the Frenet Coordinate system of the ball center with respect to the rotating Cartesian system, $oxyz$, can be expressed as follows (Fig. 2):

(*a*) Unit tangent vector

$$\mathbf{t} = [-kdS_\theta \quad kdC_\theta \quad \tau d]\mathbf{X} \tag{4}$$

(*b*) Unit normal vector

$$\mathbf{n} = [-C_\theta \quad -S_\theta \quad 0]\mathbf{X} \tag{5}$$

(*c*) Unit binormal vector

$$\mathbf{b} = [\tau d S_\theta \quad -\tau d C_\theta \quad kd]\mathbf{X} \tag{6}$$

where

$$d = \frac{r_m}{C_\alpha},$$

curvature

$$k = \frac{C_\alpha^2}{r_m},$$

and torsion

$$\tau = \frac{S_\alpha \quad C_\alpha}{r_m} \tag{7}$$

The coordinate transformation between the Frenet frame of the ball center and the rotational Coordinate system $oxyz$ can be expressed as

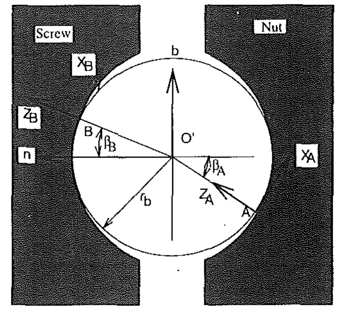

Fig. 3 Location of contact points on the normal plane

$$\mathbf{X} = \mathbf{T}_2\mathbf{Y} \tag{8}$$

where

$$\mathbf{Y} \equiv [\mathbf{t} \;\; \mathbf{n} \;\; \mathbf{b}]^T, \quad \text{and} \quad \mathbf{T}_2 \equiv \begin{bmatrix} -C_\alpha S_\theta & -C_\theta & S_\alpha S_\theta \\ C_\alpha C_\theta & -S_\theta & -S_\alpha C_\theta \\ S_\alpha & 0 & C_\alpha \end{bmatrix}.$$

The terms S_α and C_α denote $\sin(\alpha)$ and $\cos(\alpha)$, respectively. Now, Eq. (2) can be rewritten in terms of the Frenet frame of the ball center as

$$^r\mathbf{R} = \mathbf{R}^T\mathbf{T}_2\mathbf{Y} = r_m[S_\alpha t_\alpha \theta \quad -1 \quad C_\alpha t_\alpha \theta]\mathbf{Y}. \tag{9}$$

It will be shown in the next section, that the ball can only move relative to the screw in the tangential direction of the Frenet frame of the ball center trajectory. This is because the ball is confined along the helical groove in directions parallel to the normal plane of the trajectory of the ball center. Physically, this means that the contact points between the ball and the screw, as well as between the ball and the nut, must be located on this normal plane.

In order to locate these contact points, the contact angle, β, is defined as the angle between the unit normal vector and the contact vector. The contact vector is oriented from the ball center toward the contact point, as shown in Fig. 3. Points A and B represent the instantaneous contact points between the ball and the nut and between the ball and the screw, respectively. Note that β_A (Fig. 3) is considered positive when measured clockwise from the negative side of the normal axis; whereas β_B (Fig. 3) is considered positive when measured clockwise from the positive side of the normal axis. The angle β_A and β_B are always positive for a counterclockwise rotation of the screw (viewed from the end) and negative for a clockwise screw rotation.

We now introduce a pair of new coordinate systems $iX_iY_iZ_i$, with $i = A, B$, between the ball and the raceway such that the X_iY_i plane lies on the plane of contact and the Z_i-axis lies along the common normal of the two contacting bodies (Fig. 3). These coordinate systems are used to describe the position of the contact point between the two contacting bodies. We also assume point contacts along a diagonal line between the ball and the screw and nut. The coordinate transformation between the Frenet frame of the ball center trajectory and the $iX_iY_iZ_i$ coordinate system is

$$\mathbf{X}_i = \mathbf{T}_3\mathbf{Y} \tag{10}$$

where

$$\mathbf{X}_i = [\mathbf{i}_i \quad \mathbf{j}_i \quad \mathbf{k}_i]^T \quad \text{and} \quad \mathbf{T}_3 = \begin{bmatrix} 0 & -S_{\beta i} & C_{\beta i} \\ 1 & 0 & 0 \\ 0 & C_{\beta i} & S_{\beta i} \end{bmatrix}.$$

The position vector of the contact point B with respect to the ball center can thus be expressed as

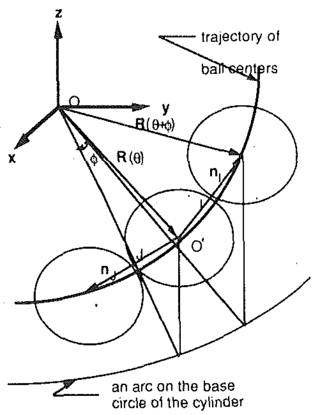

Fig. 4 Phase angle between two consecutive balls

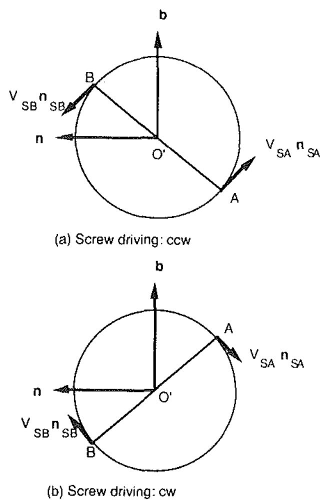

Fig. 5 Slip velocities at steady state with no-slip along the tangential direction, for the conversion of rotary into linear motion

$$\mathbf{R}_{BO}{}' = [0 \quad 0 \quad r_b]\mathbf{X}_B \tag{11}$$

where r_b denote the ball radius. The position vectors of contact points A and B with respect to the rotational coordinate system can be expressed as

$${}^r\mathbf{R}_A = {}^r\mathbf{R} + \mathbf{R}_{AO}{}' \tag{12}$$

and

$${}^r\mathbf{R}_B = {}^r\mathbf{R} + \mathbf{R}_{BO}{}'. \tag{13}$$

There is only one independent variable, θ, which describes the relative position of the center of the ball with respect to the screw in the equation of the circular helical line [Eq. (2)]. The relative position of the two consecutive ball centers can therefore be expressed as a function of a phase angle, ϕ, shown in Fig. 4, as

$$\begin{aligned}\delta\mathbf{R}_I &= {}^r\mathbf{R}(\theta+\phi) - {}^r\mathbf{R}(\theta)\\ &= r_m[C_\alpha S_\phi + S_\alpha t_\alpha \phi \quad 1 - C_\phi \quad S_\alpha(\phi - S_\phi)]\mathbf{Y}.\end{aligned} \tag{14}$$

Furthermore, the minimal central distance between two consecutive ball centers must be equal to the diameter, $2r_b$, of the balls; that is

$$\sqrt{\delta\mathbf{R}_I^T \cdot \delta\mathbf{R}_I} = 2r_b$$

or

$$(\phi t_\alpha)^2 + 2(1 - C_\phi) = 4a^2 \tag{15}$$

where

$$a \equiv \frac{r_b}{r_m}.$$

By solving the above equation for the phase angle, ϕ, the maximum number of balls per revolution, N, can be obtained:

$$N = \frac{2\pi}{\phi}. \tag{16}$$

Furthermore, Eq. (14) can be expressed as

$$\delta\mathbf{R}_I = 2r_b\mathbf{n}_I. \tag{17}$$

Similarly,

$$\delta\mathbf{R}_J = 2r_b\mathbf{n}_J. \tag{18}$$

In these equations, $\mathbf{n}_I$ and $\mathbf{n}_J$ are unit vectors between two consecutive ball centers. Using the Frenet frame, the relative motion between the centers of the balls in contact with each other can be written as:

$$\mathbf{Y}(\theta+\phi) = \begin{bmatrix} C_\alpha^2 C_\phi + S_\alpha^2 & -C_\alpha S_\phi & C_\alpha S_\alpha(1 - C_\phi) \\ C_\alpha S_\phi & C_\phi & -S_\alpha S_\phi \\ C_\alpha S_\alpha(1 - C_\alpha) & S_\alpha S_\phi & S_\alpha^2 C_\phi + C_\alpha^2 \end{bmatrix} \mathbf{Y}(\theta). \tag{19}$$

The above equation is valid for balls which are both ahead (positive ϕ values) and behind (negative ϕ values).

3 Slip Analysis

Determination of the slip conditions between the reciprocating balls and the nut or the screw is important in understanding the motion of the ball in the ball screw mechanism. These conditions are also necessary for efficiency analysis and the design considerations (see Lin, Velinsky, and Ravani, 1994). A complete velocity analysis is necessary for determining slip directions and velocities. This section provides such an analysis and applies the results to a characterization of the slip conditions for the BSM. Previous works (see Levit, 1963; and Drozdov, 1984) have treated the kinematics of the BSM using the previous results from ball bearings (see, e.g., Harris, 1971; and Jones, 1959). This has resulted in incorrect results for the BSM since the angular velocities of different elements, namely the ball, the screw, and the nut are not additive as used in ball bearing analysis.

The velocity of the ball center with respect to the rotational Cartesian Coordinates, $oxyz$, can be obtained by differentiating Eq. (9) with respect to time; i.e.,

$${}^r\dot{\mathbf{R}} = [d\dot{\theta} \quad 0 \quad 0]\mathbf{Y}. \tag{20}$$

Note that in the above equation, the velocity of the ball center relative to the rotating coordinate system has only a tangential component. Physically, the ball cannot move in the normal plane since contacting surfaces would have to separate or crush together for motion in the normal plane to exist. Furthermore, the radius of motion of the ball center is d, which includes both the curvature and the torsion of the helix.

The velocity of the ball center with respect to the world coordinate system, $ox'y'z'$, can be obtained by differentiating Eq. (3) with respect to time; i.e.,

$${}^W\dot{\mathbf{R}} = [d\dot{\theta} + r_m C_\alpha \dot{\Omega} \quad 0 \quad -r_m S_\alpha \dot{\Omega}]\mathbf{Y}. \tag{21}$$

The above equation can also be obtained from ${}^W\dot{\mathbf{R}} = {}^r\dot{\mathbf{R}} + \dot{\Omega} \times {}^r\mathbf{R}$.

If we let $\omega = [\omega_t \quad \omega_n \quad \omega_b]\ \mathbf{Y}$ be the angular velocity of the ball, then the instantaneous velocity of the two points A and B (namely V_{Ab} and V_{Bb}) on the ball (which are coincident with the two contact points between the nut and the screw, respectively) can be expressed as

$$\mathbf{V}_{Ab} = {}^W\dot{\mathbf{R}} + \omega \times \mathbf{R}_{AO}{}'$$

$$= \begin{bmatrix} d\dot{\theta} + r_m C_\alpha \dot{\Omega} + r_b(\omega_b C_{\beta A} - \omega_n S_{\beta A}) \\ r_b \omega_t S_{\beta A} \\ -r_m S_\alpha \dot{\Omega} - r_b \omega_t C_{\beta A} \end{bmatrix}^T \mathbf{Y} \quad (22)$$

and

$$\mathbf{V}_{Bb} = {}^W\dot{\mathbf{R}} + \omega \times \mathbf{R}_{BO}$$

$$= \begin{bmatrix} d\dot{\theta} + r_m C_\alpha \dot{\Omega} + r_b(\omega_b C_{\beta B} - \omega_n S_{\beta B}) \\ -r_b \omega_t S_{\beta B} \\ -r_m S_\alpha \dot{\Omega} + r_b \omega_t C_{\beta B} \end{bmatrix}^T \mathbf{Y} \quad (23)$$

We will now determine the velocity of the point on the nut which is coincident with the contact point for the case when the screw is driving. If the nut is driving, similar results can easily be obtained. We will only consider the conversion of rotary into linear motion. The cases involving the conversion of linear into rotary motion are merely the kinematic inversions of the cases presented here.

As in the conventional power screw unit, if the nut is restrained from rotating, it will move axially a distance $\Omega L/2\pi$ along the screw for a screw rotation of angle Ω. Hence, if the angular velocity of the screw is $\dot{\Omega}\mathbf{k}$, then the velocity of any point on the nut with respect to the world coordinate system, $ox'y'z'$ can be represented as

$$\mathbf{V}_{An} = \begin{bmatrix} 0 & 0 & \dfrac{-\dot{\Omega} L}{2\pi} \end{bmatrix} \mathbf{X}'$$

$$= -r_m S_\alpha \dot{\Omega} [t_\alpha \quad 0 \quad 1] \mathbf{Y}. \quad (24)$$

The velocity of the contact point on the screw can similarly be obtained as follows.

If the screw rotates with an angular velocity $\dot{\Omega}\mathbf{k}$, the velocity of point B on the screw (namely V_{BS}) coincident with the contact point will be

$$\mathbf{V}_{BS} = \dot{\Omega}\mathbf{k} \times {}^r\mathbf{R}_B$$

$$= \dot{\Omega} \begin{bmatrix} (r_m - r_b C_{\beta B}) C_\alpha \\ -r_b S_{\beta B} S_\alpha \\ -(r_m - r_b C_{\beta B}) S_\alpha \end{bmatrix}^T \mathbf{Y}. \quad (25)$$

This velocity is with respect to the world coordinate system, $ox'y'z'$.

The slip velocities at contact points A and B (V_{SA} and V_{SB}, respectively) can now be determined as follows.

From Eqs. (22), (23), (24), and (25), the slip velocities at points A and B are, respectively:

$$\mathbf{V}_{SA} = \mathbf{V}_{Ab} - \mathbf{V}_{An}$$

$$= \begin{bmatrix} d(\dot{\theta} + \dot{\Omega}) + r_b(\omega_b C_{\beta A} - \omega_n S_{\beta A}) \\ r_b \omega_t S_{\beta A} \\ -r_b \omega_t C_{\beta A} \end{bmatrix}^T \mathbf{Y}$$

$$= \begin{bmatrix} -r_b \omega_t \\ d(\dot{\theta} + \dot{\Omega}) + r_b(\omega_b C_{\beta A} - \omega_n S_{\beta A}) \\ 0 \end{bmatrix}^T \mathbf{X}_A$$

$$= V_{SA} \mathbf{n}_{SA} \quad (26)$$

and

$$\mathbf{V}_{SB} = \mathbf{V}_{Bb} - \mathbf{V}_{Bn}$$

$$= \begin{bmatrix} d\dot{\theta} - r_b[(\omega_b - \dot{\Omega} C_\alpha) C_{\beta B} - \omega_n S_{\beta B}] \\ -r_b(\omega_t - \dot{\Omega} S_\alpha) S_{\beta B} \\ r_b(\omega_t - \dot{\Omega} S_\alpha) C_{\beta B} \end{bmatrix}^T \mathbf{Y}$$

$$= \begin{bmatrix} r_b(\omega_t - \dot{\Omega} S_\alpha) \\ d\dot{\theta} - r_b[(\omega_b - \dot{\Omega} C_\alpha) C_{\beta B} - \omega_n S_{\beta B}] \\ 0 \end{bmatrix}^T \mathbf{X}_B$$

$$= V_{SB} \mathbf{n}_{SB} \quad (27)$$

where $\mathbf{n}_{SA}$ and $\mathbf{n}_{SB}$ denote the unit vectors in the direction of the slip velocities at the instantaneous contact points A and B, respectively, and $\mathbf{V}_{SA}$ and $\mathbf{V}_{SB}$ are the magnitudes of these velocities (Fig. 5).

Physically, contacting surfaces should have common normals at their instantaneous points of contact. This means that the slip velocities should be perpendicular to the common normal to the two surfaces, namely $\mathbf{R}_{AO'} \cdot \mathbf{V}_{SA} = 0$ and $\mathbf{R}_{BO'} \cdot \mathbf{V}_{SB} = 0$. These conditions on slip velocities represent the physical contact requirement for the contacting surfaces not to separate or crush together. We should note that the unit vectors $\mathbf{n}_{SA}$ and $\mathbf{n}_{SB}$ are opposite in direction to the frictional forces on the ball at the contact points.

The location of all contact points can be determined using these physical contact conditions. Suppose that the contact points are not located in the central normal plane. Since the balls are spherical, only two angles are necessary to define the location of a contact point on a ball. Suppose these two angles are β_B and an angle Ψ off the central normal plane. Consider point B as an example to simplify the analysis. The position vector $\mathbf{R}_{BO}{}'$ can be determined from geometry as

$$\mathbf{R}_{BO}{}' = [-r_b S_\Psi \quad r_b C_\Psi C_{\beta B} \quad r_b C_\Psi S_{\beta B}] \mathbf{Y}.$$

Substituting this last equation into Eqs. (11), (13), (23), and (25), we can obtain $\mathbf{V}_{SB}$, which when substituted into the contact conditions result in:

$$\mathbf{R}_{BO}{}' \cdot \mathbf{V}_{SB} = -r_b d\dot{\theta} S_\Psi = 0.$$

Since $r_b d$ cannot be zero and $\dot{\theta}$ is not a geometric parameter, the offset angle, Ψ, has to be zero to satisfy the contact condition. In other words, all the contact points must lie on the normal plane.

Let us decompose the slip-velocity into two parts: (*1*) on the central normal plane, and (*2*) along the tangential direction of the Frenet frame of the ball center trajectory. Then, as we can see from the Eqs. (26) and (27), the magnitudes of the resultant slip-velocities on the central normal plane at the ball/nut and the ball/screw contact points are $r_b(\omega_t - S_\alpha \dot{\Omega})$ and $r_b \omega_t$, respectively. Accordingly, no ω_t value exists which causes the slip-velocities on the central normal plane at both contact points to vanish simultaneously. In other words, friction can never vanish at both contact points on the central normal plane unless the helix angle, α, equals zero. A method for calculating the frictional losses, which has been used as the basis for many subsequent studies, was given by Levit (1963) who defined the BSM friction angle to lie along the tangential direction of the Frenet frame of the ball center trajectory. This produces an inaccuracy due to the fact that it does not account for the effects of the torsion of the helix.

4 Acceleration Analysis

The acceleration of the ball center with respect to the rotational Cartesian coordinate system, $oxyz$, can be expressed as

$$ {}^r\ddot{\mathbf{R}} = [d\ddot{\theta} \quad r_m\dot{\theta}^2 \quad 0]\mathbf{Y} \tag{28} $$

where $d\ddot{\theta}$ and $r_m\dot{\theta}^2$ represent components of tangential and centripetal acceleration of the ball center with respect to the rotational Coordinate System $oxyz$, respectively. The acceleration of the ball center in the world coordinate system is expressed as

$$ {}^W\ddot{\mathbf{R}} = {}^r\ddot{\mathbf{R}} + \ddot{\Omega}\times{}^r\mathbf{R} + \dot{\Omega}\times\dot{\Omega}\times{}^r\mathbf{R} + 2\dot{\Omega}\times{}^r\dot{\mathbf{R}} $$

$$ = [d(\ddot{\theta} + C_\alpha^2\ddot{\Omega}) \quad r_m(\dot{\theta}+\dot{\Omega})^2 \quad -r_m S_\alpha\ddot{\Omega}]\mathbf{Y} \tag{29} $$

where the component along the normal direction, $r_m(\dot{\theta}+\dot{\Omega})^2$ includes both the centripetal and Coriolis effects. The angular acceleration of the ball is given by the symbol $\mathbf{a}$:

$$ \mathbf{a} = \begin{bmatrix} \dot{\omega}_t - \omega_n C_\alpha(\dot{\theta}+\dot{\Omega}) + S_\alpha(\ddot{\theta}+\ddot{\Omega}) \\ \dot{\omega}_n + (\omega_t C_\alpha - \omega_b S_\alpha)(\dot{\theta}+\dot{\Omega}) \\ \dot{\omega}_b + \omega_n + S_\alpha(\dot{\theta}+\dot{\Omega}) + C_\alpha(\ddot{\theta}+\ddot{\Omega}) \end{bmatrix}^T \mathbf{Y} \tag{30} $$

where the first term in each component represents the change in magnitude and the second term represents the gyroscopic motion.

5 The Kinematics of the BSM with Elastic Deformation

Loads acting between the balls and raceways in the BSM develop only small areas of contact between the mating members. Consequently, although the elemental loading may only be moderate, stresses induced on the surfaces of the balls and raceways are usually large. Contact deformations are caused by contact stresses. Because of the rigid nature of the balls, these deformations are generally of a low order of magnitude. The classical solution for the local stress and deformation of two elastic bodies contacting at a single point is that of Hertz (1881) which has been applied to ball-bearing problems. Levit (1963) introduced the theory to the BSM. However, the kinematics of the BSM regarding elastic deformations has never been solved completely. It is the purpose of this section to develop the internal motions of the BSM, the relative slip-velocity between the ball and the nut/screw and the pattern of sliding lines of contact and thus set the foundation for further investigations on friction, wear and finite element analyses.

5.1 Position of the Contact Point. To specify the position of a contact point between two deformed bodies, a coordinate system, $iX_iY_iZ_i, i = A, B$, is introduced between the ball and the raceway (at each contact point as before). In this coordinate system, the X_iY_i plane lies on the plane of contact and the Z_i-axis is the common normal of the two contacting bodies. Figure 6 shows a ball contacting the screw raceway such that the normal force between the ball and the screw is distributed over an elliptical surface defined by the projected major and minor semi-axes, a_B and b_B, respectively. The coordinate transformation between the Frenet frame and the $iX_iY_iZ_i$ system is

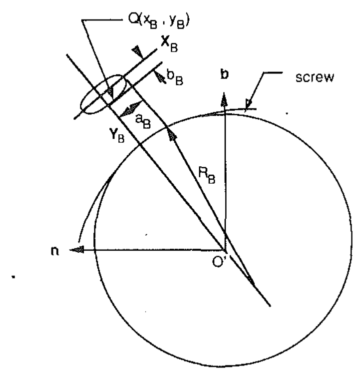

Fig. 6 Ball-screw contact

$$ \mathbf{X}_i = \mathbf{T}_3\mathbf{Y} \tag{31} $$

where

$$ \mathbf{X}_i = [i_i \quad j_i \quad k_i]^T \quad \text{and} \quad \mathbf{T}_3 = \begin{bmatrix} 0 & -S_{\beta i} & C_{\beta i} \\ 1 & 0 & 0 \\ 0 & C_{\beta i} & S_{\beta i} \end{bmatrix}. $$

The position vector of an arbitrary point Q, on the X_BY_B plane with respect to the origin B, can be expressed as $\mathbf{R}_{QB} = [x_B \quad y_B \quad 0]\,\mathbf{X}_B$ and the position vector of point B with respect to the ball center is $\mathbf{R}_{BO}' = [0 \quad 0 \quad r_b]\,\mathbf{X}_B$. The radius of curvature of the screw raceway groove is

$$ r_B = \sqrt{R_B^2 - x_B^2} - \sqrt{R_B^2 - a_B^2} + \sqrt{r_b^2 - a_B^2}, \tag{32} $$

and the radius of curvature of the deformed surface as defined by Hertz is

$$ R_i = \frac{2f_i r_b}{f_i + 1} \tag{33} $$

where

$$ f_i = \frac{r_i}{r_b}, \; i = A, B. $$

Therefore, the position vector of point Q with respect to the world coordinates is

$$ \mathbf{R}_{QO}' = \mathbf{R}_{QB} + \mathbf{R}_{BO}' + {}^W\mathbf{R}. \tag{34} $$

5.2 Velocity of BSM with Deformation. The derivation of the velocity of the BSM with deformation is similar to the procedure in Section 3, from which, the following equations can be determined:

5.2.1 Velocity of Any Contact Point P/Q on the Ball

$$ \mathbf{V}_{Pb} = {}^W\dot{\mathbf{R}} + \omega\times\mathbf{R}_{PO}' $$

$$ = \begin{bmatrix} d(\dot{\theta} + C_\alpha^2\dot{\Omega}) + (x_A S_{\beta A} + r_A C_{\beta A})\omega_b + (x_A C_{\beta A} - r_A S_{\beta A})\omega_n \\ -(x_A C_{\beta A} - r_A S_{\beta A})\omega_t + y_A\omega_b \\ -r_m S_\alpha\dot{\Omega} - (x_A S_{\beta A} + r_A C_{\beta A})\omega_t - y_A\omega_n \end{bmatrix}^T \mathbf{Y} \tag{35} $$

$$ \mathbf{V}_{Qb} = {}^W\dot{\mathbf{R}} + \omega\times\mathbf{R}_{QO}' $$

$$ = \begin{bmatrix} d(\dot{\theta} + C_\alpha^2\dot{\Omega}) + (x_B S_{\beta B} - r_B C_{\beta B})\omega_b + (x_B C_{\beta B} + r_B S_{\beta B})\omega_n \\ -(x_B C_{\beta B} + r_B S_{\beta B})\omega_t + y_B\omega_b \\ -r_m S_\alpha\dot{\Omega} - (x_B S_{\beta B} + r_B C_{\beta B})\omega_t + y_B\omega_n \end{bmatrix}^T \mathbf{Y} \tag{36} $$

5.2.2 Velocity of Any Contact Point P on the Nut

$$ \mathbf{V}_{Pn} = -r_m S_\alpha\dot{\Omega}\,[t_\alpha \quad 0 \quad 1]\mathbf{Y} \tag{37} $$

5.2.3 Velocity of Any Contact Point Q on the Screw

$$\mathbf{V}_{QS}=\dot{\Omega}\mathbf{K}\times\mathbf{R}_{QO}$$

$$=\begin{bmatrix} (r_m-r_B\,C_{\beta B}+x_B\,S_{\beta B})\,C_\alpha\dot{\Omega} \\ y_B\,C_\alpha\dot{\Omega}-(r_B\,S_{\beta B}+x_B\,C_{\beta B})S_\alpha\,\dot{\Omega} \\ -(r_m-r_B\,C_{\beta B}+x_B\,S_{\beta B})\,S_\alpha\,\dot{\Omega} \end{bmatrix}^T \mathbf{Y} \tag{38}$$

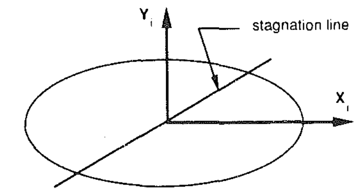

Fig. 7 Stagnation line, line of no-slip along the common normal of the contact surfaces

5.3 Slip Velocities at Contact Points P and Q

$$\mathbf{V}_{SP}=\mathbf{V}_{Pb}-\mathbf{V}_{Pn}$$

$$=\begin{bmatrix} d(\dot{\theta}+\dot{\Omega})+(x_A\,S_{\beta A}+r_A\,C_{\beta A})\omega_b+(x_A\,C_{\beta A}-r_A\,S_{\beta A})\omega_n \\ -(x_A\,C_{\beta A}-r_A\,S_{\beta A})\omega_t+y_A\,\omega_b \\ -(x_A\,S_{\beta A}+r_A\,C_{\beta A})\omega_t-y_A\,\omega_n \end{bmatrix}^T \mathbf{Y}$$

$$=\begin{bmatrix} -y_A\,(S_{\beta A}\,\omega_b+C_{\beta A}\,\omega_n)-r_A\,\omega_t \\ x_A\,(S_{\beta A}\,\omega_b+C_{\beta A}\,\omega_n)+d(\dot{\theta}+\dot{\Omega})+r_A\,(C_{\beta A}\,\omega_b-S_{\beta A}\omega_n) \\ -x_A\,\omega_t+y_A\,(C_{\beta A}\,\omega_b-S_{\beta A}\,\omega_n) \end{bmatrix}^T \mathbf{X}_A \tag{39}$$

$$\mathbf{V}_{SQ}=\mathbf{V}_{Qb}-\mathbf{V}_{QS}$$

$$=\begin{bmatrix} d\dot{\theta}+(x_B\,S_{\beta B}-r_B\,C_{\beta B})\,(\omega_b-C_\alpha\dot{\Omega})+(x_B\,C_{\beta B}+r_B\,S_{\beta B})\omega_n \\ (x_B\,C_{\beta B}+r_B\,S_{\beta B})\,(S_\alpha\dot{\Omega}-\omega_t)+y_B\,(\omega_b-C_\alpha\dot{\Omega}) \\ (x_B\,S_{\beta B}-r_B\,C_{\beta B})\,(S_\alpha\dot{\Omega}-\omega_t)-y_B\,\omega_n \end{bmatrix}^T \mathbf{Y}$$

$$=\begin{bmatrix} -y_B[S_{\beta B}(\omega_b-C_\alpha\dot{\Omega})+C_{\beta B}\,\omega_n]-r_B(S_\alpha\dot{\Omega}-\omega_t) \\ x_B[S_{\beta B}(\omega_b-C_\alpha\dot{\Omega})+C_{\beta B}\,\omega_n]+d\dot{\theta}-r_B[C_{\beta B}(\omega_b-C_\alpha\dot{\Omega})-S_{\beta B}\,\omega_n] \\ x_B(S_\alpha\,\dot{\omega}-\omega_t)+y_B[C_{\beta B}(\omega_b-C_\alpha\dot{\Omega})-S_{\beta B}\,\omega_n \end{bmatrix}^T \mathbf{X}_B \tag{40}$$

Note that the equations derived in this section can be reduced to the corresponding equations in Section 3, without considering elastic deformations, where $r_i=r_b$ and $x_i=y_i=0$.

5.4 Slip Velocity and Pattern of Constant Sliding Lines. The pattern of constant sliding lines of ball bearings in the elliptical contact area was investigated by Lundberg (1954) and presented by Harris (1984) in his book. Here a pattern of sliding lines are derived for the BSM directly from the equations of slip-velocity obtained from the previous section. Since the pattern of constant sliding lines are similar for different types of motions, only the case with screw driving and conversion of rotary into linear motion is shown as an example. In accordance with the Hertzian radius of contact in the direction transverse to the motion, the contact surface has a harmonic mean profile radius. This implies that the contact surface is not straight but generally curved. The no-slip condition along the common normal of the contact surfaces between the ball and the nut is obtained from the Z_A-component of Eq. (39) as follows:

$$-x_A\,\omega_t+y_A\,(\omega_b\,C_{\beta A}-\omega_n\,S_{\beta A})=0. \tag{41}$$

This equation represents a straight line, referred to as the stagnation line, on the $X_A\,Y_A$ plane. This line passes through the origin A, with a slip of $\omega_t/(\omega_b\,C_{\beta A}-\omega_n\,S_{\beta A})$, as shown in Fig. 7.

We first assume that the nut is fixed in space. Thus, it tends to "cut-in" the contact surface for every contact point on the ball lying on one side of the stagnation line in the contact ellipse and "leave-from" the contact surface for every point lying on the opposite side of the line. There is a special case, in which $\omega_b\,C_{\beta A}-\omega_n\,S_{\beta A}=0$, which indicates that the resultant spinning velocity on the normal plane is along the common normal of the contact surfaces. In this case, the common normal has been referred to as the "spinning axis" in the ball-bearing literature. Additionally, the "stagnation line" becomes the Y_A-axis ($x_A=0$) which is true because pure spin will not change the depth of "cut-in" of a certain point. Similarly, if $\omega_t=0$, the "stagnation line" becomes the X_A-axis ($y_A=0$) because the rolling motion changes the depth of "cut-in."

The equation of constant sliding lines can now be obtained from the X_A- and Y_A-components of Eq. (39); that is

$$(x_A-p_A)^2+(y_A-q_A)^2=C \tag{42}$$

where

$$p_A=-\frac{d(\dot{\theta}+\dot{\Omega})+r_A\,(\omega_b\,C_{\beta A}-\omega_n\,S_{\beta A})}{\omega_b\,S_{\beta A}+\omega_n\,C_{\beta A}},$$

$$q_A=-\frac{r_A\,\omega_t}{\omega_b\,S_{\beta A}+\omega_n\,C_{\beta A}},$$

and

$$C=\frac{\text{constant}}{(\omega_b\,S_{\beta A}+\omega_n\,C_{\beta A})^2}.$$

Equation (42) represents a family of circles on the $X_A\,Y_A$-plane with center located at $(p_A,\,q_A)$, where the no-slip condition on the $X_A\,Y_A$-plane occurs, as shown in Fig. 8. It is interesting that the point, $(p_A,\,q_A)$, will not lie on the stagnation line [Eq. (41)] unless the term, $\dot{\theta}+\dot{\Omega}$, vanishes which is impossible except for the static situation. In other words, a point with no-slip is not possible within the contact ellipse. Note that r_A can also be defined as a variable from Hertzian contact theory and Eq. (42) remains valid. In such a case, Eq.

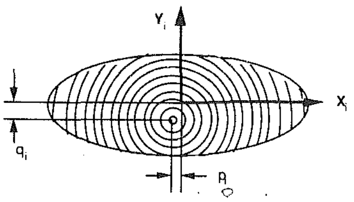

Fig. 8 Pattern of constant sliding lines at constant surfaces

(42) results in a family of non-coplanar circles representative of the true contact surface shape.

As one can see from Eq. (42), under the special case where

$$\omega_b S_{\beta A} + \omega_n C_{\beta A} = 0, \tag{43}$$

the slip-velocity on the $X_A\, Y_A$-plane is an invariant which is exactly the same as the slip-velocity at point A without elastic deformation if $r_i = r_b$. The line defined by Eq. (43), which is a line on the normal plane perpendicular to the common normal of the contact surfaces, has been called the "rolling axis" in the case of ball-bearings. It is obvious from Eq. (42) that the no-slip condition is not possible even when spinning is absent.

A similar analysis can be applied to the contact area between the ball and the screw, and the corresponding resultant equations are:

Equation of stagnation line:

$$x_B(S_\alpha \dot{\Omega} - \omega_t) + y_B[(\omega_b - C_\alpha \dot{\Omega}) C_{\beta B} - \omega_n S_{\beta B}] = 0 \tag{44}$$

Equation of constant sliding lines:

$$(x_B - p_B)^2 + (y_B - q_B)^2 = C \tag{45}$$

where

$$p_B = -\frac{d\dot{\theta} - r_b[(\theta_b - C_\alpha \dot{\Omega}) C_{\beta B} - \omega_n S_{\beta B}]}{(\omega_B - C_\alpha \dot{\Omega})\, S_{\beta B} + \omega_n C_{\beta B}},$$

$$q_B = -\frac{r_B(s_\alpha \dot{\Omega} - \omega_t)}{(\omega_b - C_\alpha \dot{\Omega}) S_{\beta B} + \omega_n C_{\beta B}},$$

and

$$C = \frac{\text{constant}}{[(\omega_b - C_\alpha \dot{\Omega}) S_{\beta B} + \omega_n C_{\beta B}]^2}.$$

Equation of "pure rolling":

$$(\omega_b - C_\alpha \dot{\Omega})\, S_{\beta B} + \omega_n C_{\beta B} = 0 \tag{46}$$

Equation of "pure spinning":

$$(\omega_b - C_\alpha \dot{\Omega})\, C_{\beta B} - \omega_n S_{\beta B} = 0 \tag{47}$$

Note that the corresponding equations at the ball-nut contact area are independent of the helix angle of the BSM, whereas those at the ball-screw contact area are dependent on the helix angle.

Conclusions

In this paper, we have studied the kinematics of the ball screw mechanism with the aim of understanding the slip conditions and pattern of contact points between the elements. We have derived the general slip conditions and have shown that the condition of no-slip in the central normal plane assumed in the previous literature is theoretically unattainable. The effect of contact deformation on the motion of the balls was also studied and used to determine the pattern of sliding lines of the balls in contact areas.

The results, in addition to their theoretical interest, provide the basis for efficiency analysis, design, wear analysis, and finite element modeling of the ball screw mechanism.

Acknowledgment

This research was supported in part by US Army Research Office grant number DAAL03-90-G-0005 and California Department of Transportation through the AHMCT program.

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