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# Recursive Dynamic Compressive Sensing in Smart Distribution Systems

Hazhar Sufi Karimi, *Student Member, IEEE*, and Balasubramaniam Natarajan, *Senior Member, IEEE*

**Abstract**—With a transition to a smarter grid, we are witnessing a significant growth in smart metering infrastructure and sensor deployment in the distribution system. The underlying communication infrastructure is stressed due to the large volume of data that is generated by the smart meters/sensors. Furthermore, real time operations such as state estimation and control are impaired due to the lack of reliable aggregation of the data. In this paper, we exploit the underlying sparsity in grid data to implement two recursive dynamic compressive sensing (CS) approaches—streaming modified weighted- $\ell_1$  CS and Kalman filtered CS. These approaches aim to reconstruct the sparse signal using the current underdetermined measurements and the prior information about the sparse signal and its support set. Slow signal and support change is in distribution grid data is validated using Pecan Street data. Both the IEEE 34 node test feeder system and PecanStreet data are considered as two examples to validate the superior performance of the two recursive CS techniques relative to classic CS.

**Index Terms**—Smart grid, Compressive Sensing, Recursive Dynamic CS, Streaming modified weighted- $\ell_1$ , Kalman filtered CS

## I. INTRODUCTION

SMART distribution grids include different components such as distributed generation (e.g., solar, wind), advanced metering infrastructure (AMI) and sensors. The sensing and smart metering infrastructure provides previously unavailable insights into the distribution grid all the way to the home level. In addition to situational awareness, data from all the sensing and smart metering infrastructure could be used for real time operations. However, the physical scale of the system and sampling rate (varies from a few seconds to the order of minutes) contribute to a data deluge. Obviously, the underlying communication network struggles with the large volume and high velocity of information flowing from the smart meters and sensors which results in unreliable data aggregation [1]. Therefore, the current smart meter data is mainly used for billing purposes rather than for real time operations. Optimal control and situational awareness of a distribution grid critically depends on accuracy of the estimated system states. Over many decades, state estimation in power systems has received a lot of attention. Various estimation strategies including their advantages and the key challenges are summarized in [2] and [3]. The conventional state estimation methods for distribution grids require a large number of measurements to ensure observability of the system. However, among the existing approaches, compressive sensing (CS) has been an efficient

method in aggregation of large volumes of data over an unreliable communication infrastructure. Existing correlation among loads with the increasing penetration of rooftop photovoltaic (PV) generation results in a strong correlation between different components of a smart grid. This underlying correlation normally leads to a significant sparsity in a transformation basis (e.g., wavelet basis) as shown in [4]. Obviously, since the CS strategies are based on existence of sparsity in the signal of interest, one can reconstruct a high dimensional signal using a smaller number of measurements [5].

CS is mainly developed for speech, video, or image compression applications, however, there have been some CS studies in the field of smart distribution systems. In [6], the authors summarize the initial efforts involved in the space of compressive sensing for distribution power systems. To compress the signals, [7] presents a fuzzy based transformation with a least-squares method that improves accuracy of signal reconstruction compared to principal component analysis and discrete wavelet transformation approaches (under certain conditions). In [8], the distribution grid data is compressed by a singular value decomposition approach. In the initial efforts, the physical grid model (and the associated power flow equations) is not incorporated as part of their analysis. Spatio-temporal correlation between loads and distributed generation is exploited in [9] that recovers the power measurements across the grid and estimates the voltages using noiseless CS. [10] extends the results presented in [9] for a three-phase unbalanced smart distribution system. In [11], a Gaussian mixture model is proposed for characterization of high frequency smart meter data. Then, an adaptive data reduction scheme is implemented. [12] compresses the load data using a stacked auto-encoder method for market-based applications. To address the problem of power line outage detection, [13] exploits compressive system identification which is well-known for being a time-efficient approach in complex network analysis. Recently, a comprehensive survey on the compression methods for smart meter big data is presented in [14] that evaluates the methods and discusses the existing challenges in data compression of smart grids.

All the mentioned researches employ the static sparse recovery methods where they only apply current measurements for signal recovery purposes. Recently, it has been shown that if the sparse signal satisfies certain conditions, one can superiorly reconstruct the current signal using the current and previous information. [15] presents a survey of recursive dynamic CS strategies which outperform the batch algorithms for having advantages such as on-line, fast operation and a smaller memory requirement. To the best knowledge of the

Hazhar Sufi Karimi and Balasubramaniam Natarajan are with the Department of Electrical and Computer Engineering, Kansas State University, Manhattan, KS, 66502 USA e-mail: hazhars@ksu.edu , bala@ksu.edu.

authors, recursive CS methods have not been employed in smart grids. Therefore, we aim to address this problem for the first time. To this end, we first exploit slow support set change and slow sparse signal change properties of the underlying data. To validate our assumptions, we incorporate practical home-level data from an actual distribution system. Then, we employ two recursive algorithms: 1- Streaming modified weighted- $\ell_1$ , 2- Kalman filtered CS. The simulation results (based on two different data framework) validate the efficiency of the recursive CS techniques where the performance of CS is significantly improved relative to the classic CS approach presented in [9].

#### A. Notation

Let  $t$  denote the discrete time index; for a set  $T$ , we use  $T^c$  to indicate the complement of  $T$ ; the set operations  $\setminus$  and  $\cup$  denote set difference and union of two sets, respectively;  $\mathbf{M}'$  denote transpose of  $\mathbf{M}$ . We use  $|T|$  to denote the cardinality (size or number of elements) of set  $T$ . Also,  $\|\nu\|_k$  denotes the  $l_k$  norm of a vector  $\nu$ . We use  $\text{supp}(\nu) = \{i : \nu_i \neq 0\}$  to denote the support set of a sparse vector  $\nu$ , i.e., the set of indices of its nonzero entries. For a matrix  $\mathbf{A}$ ,  $\mathbf{A}_T$  denotes the sub-matrix obtained by extracting the columns of  $\mathbf{A}$  corresponding to the indices in  $T$ . We use  $\mathbf{I}$  to denote the identity matrix.  $(\mathbf{x}_t)_i$  denote  $i^{\text{th}}$  element of vector  $\mathbf{x}_t$ ,  $N_{i,t}$  is the  $i^{\text{th}}$  member of set  $N_t$ .

### II. COMPRESSIVE SENSING

Traditional state estimation methods for smart grids require aggregation of all available sensor measurements. To this end, a large volume of data should be collected and transmitted over a network in an actual distribution system. Therefore, a communication infrastructure with large bandwidth and high reliability must be provided. However, establishing such a communication network is not feasible or affordable in many cases. Therefore, our goal is to alleviate the burden on the communication network, while still being able to reconstruct power signals with reasonable fidelity. Compressive sensing theory allows us to recover a signal from a smaller number of random measurements. The necessary condition for using such data recovery methods is that the signal of interest exhibits sparsity or must be approximately sparse in a linear transformation basis. The following theorem describes the concept of compressive sensing and its proof can be found in [16]:

**Theorem 1.** Let  $\mathbf{z} \in \mathbb{R}^n$  be the signal of interest, which is sparse in a linear transformation basis  $\Psi \in \mathbb{R}^{n \times n}$  such that,

$$\mathbf{z} = \Psi \mathbf{x}$$

where,  $\mathbf{x}$  has at most  $S \ll n$  significant coefficients i.e.,  $\mathbf{z}$  is  $S$ -sparse in the sparsifying basis  $\Psi$ . If the sensing mechanism is such that:

$$\mathbf{h} = \Phi \mathbf{z} \quad (1)$$

where,  $\mathbf{h} \in \mathbb{R}^m$  is the available measurement vector and  $\Phi \in \mathbb{R}^{m \times n}$  is a random measurement/projection matrix (e.g., matrix elements distributed as i.i.d. Gaussian random

variables with mean 0 and variance  $1/m$  or Bernoulli random variables), then the original signal  $\mathbf{z}$  can be reconstructed by solving the following  $\ell_1$  minimization problem,

$$\begin{aligned} \mathbf{x}^* &= \arg \min_{\mathbf{b}} \|\mathbf{b}\|_1 \text{ subject to } \mathbf{h} = \Phi \Psi \mathbf{b} \\ \mathbf{z}^* &= \Psi \mathbf{x}^* \end{aligned} \quad (2)$$

The result of the optimization problem in (2) provides an exact reconstruction with overwhelming probability if there exists a  $\delta \in (0, 1)$  such that,

$$(1 - \delta) \|\mathbf{x}\|_2^2 \leq \|\Phi \Psi \mathbf{x}\|_2^2 \leq (1 + \delta) \|\mathbf{x}\|_2^2,$$

holds for all  $K$ -sparse signal  $\mathbf{x}$ . This is called the Restricted Isometry Property (RIP) of order  $K$ . Correspondingly, the measurement dimension  $m$  is bounded in the following order [5],

$$\begin{aligned} m &= \mathcal{O}\left(\frac{1}{\delta^2} K \log\left(\frac{n}{K}\right)\right) \\ \Rightarrow \left(\frac{m}{n}\right) &= \mathcal{O}\left(\frac{1}{\delta^2} \frac{K}{n} \log\left(\frac{n}{K}\right)\right) \\ \Rightarrow \text{CMR} &= \mathcal{O}\left(\frac{1}{\delta^2} \text{SR} \log\left(\frac{1}{\text{SR}}\right)\right) \end{aligned} \quad (3)$$

where CMR and SR are compressed measurement and sparsity ratio, respectively.

Obviously, to guarantee a certain signal recovery fidelity, the number of measurements can not be arbitrarily reduced (i.e.,  $m$  must be large enough to achieve an acceptable level of reconstruction). However, the number of required measurements can be further reduced if the underlying sparse signal satisfies certain conditions. Let  $N_t$  denote the support set of a sparse signal  $\mathbf{x}_t$ . That is,  $N_t = \{i_1, i_2, \dots, i_{S_t}\}$  where  $i_k$  are the non-zero coordinates of  $\mathbf{x}_t$  and  $S_t = |N_t|$ , i.e.,

$$N_t := \text{supp}(\mathbf{x}_t) = \{i : (\mathbf{x}_t)_i \neq 0\}.$$

Definitely, signal sparsity implies that  $S_t \ll n$ . Let,  $T_t = \hat{N}_t$  denotes an estimate of the actual support set.

**Assumption 1: Sparsity patterns are assumed to be changing slowly over time.** Slow support changes are practically observed in many applications such as [15]. Here, we assume that the maximum change in the support set size at time  $t$  is small relative to the size of the actual support set. In other words if  $S_t^m$  denotes the size of maximum changes in the support set,  $S_t^m = \max[|N_t \setminus N_{t-1}| + |N_{t-1} \setminus N_t|] < \min S_t$  must hold for all times  $t$ . If this assumption is valid for a dynamical system, intuitively, one can employ the knowledge about the previous support set to estimate current support set and the sparse signal, simultaneously. For example a new version of basis pursuit denoising (BPDN) is introduced in [17], prior support set information is used to recover the current signal.

**Assumption 2: Similar to slow support changes, sparse signals can be slowly changing over time.** Here, we assume that the difference between current signal  $\mathbf{x}_t$  and the previous signal  $\mathbf{x}_{t-1}$  is significantly smaller than the signal  $\mathbf{x}_t$ , i.e.,  $\|\mathbf{x}_t - \mathbf{x}_{t-1}\|_2 \ll \|\mathbf{x}_t\|_2$ . Using this information, it is

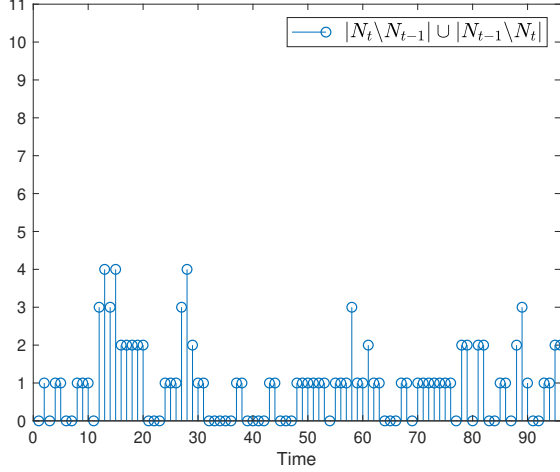


Fig. 1. Changes in a support set over time in a smart distribution system

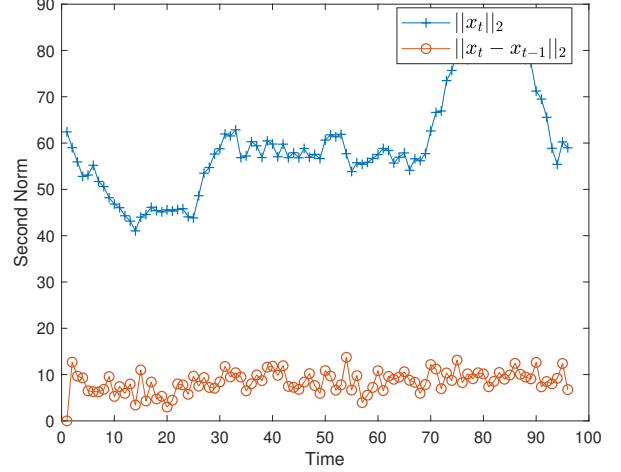


Fig. 2. Changes in a sparse signal over time in a smart distribution system

possible to fit a dynamical model for the sparse signal  $\mathbf{x}_t$ . For example, one may consider a Gaussian random walk model with constant variance  $\sigma_{sys}^2$  in all directions [18].

To evaluate the validity of the above assumptions, we use the load data of 272 homes in an actual distribution system [19]. This data is publicly available which is collected and cleaned by Pecan Street research group for off-line analysis. Specifically, we consider 24 hours load data averaged in 15 minutes intervals on March 1<sup>st</sup> 2018. Here, we cluster the loads of every 17 homes together, therefore, we have 16 lumped load data at each time  $t \in \{1, 2, \dots, 96\}$ . We randomly select 75% of data without any measurement noises. Hence, the dimensions are  $\mathbf{x}_t \in \mathbb{R}^{16}$ ,  $\mathbf{h}_t \in \mathbb{R}^{12}$  and  $\Phi \in \mathbb{R}^{12 \times 16}$ . Then, we reconstruct the spatial signal using the simple compressive sensing approach described by Theorem 1. At current time  $t_1$ , both the sparse signal and its support set are estimated without considering any temporal correlations with the previous times  $t < t_1$ . Fig 1 demonstrates the changes in the support set during the entire day. This figure shows both addition and deletion in the support set ( $|N_t \setminus N_{t-1}| + |N_{t-1} \setminus N_t|$ ). It is evident that these support set changes are slow for this distribution system. Furthermore, Fig 2 shows the second norm of the sparse signal and its variations over a day. Fig 2 suggests that slow signal changes are typical in a distribution system. Therefore, Assumption 1 and 2 are reasonable and can help motivate the design of a new state estimator that exploit these properties.

### III. DYNAMIC RECURSIVE COMPRESSIVE SENSING

Dynamic CS refers to the techniques that aim to recover a time sequence of sparse signals. Although the batch algorithms can deal with some dynamic CS problems, they suffer from serious drawbacks such as being offline, slow operation and a huge memory requirement. Dynamic recursive CS techniques overcome these drawbacks and provide superior performance relative to classic CS approaches with equal number of measurements. Recursive methods rely on one or both the Assumptions 1 and 2 stated in section II. However, the problem

of dynamic CS is significantly more difficult than the static set up because the support set  $N_t$  changes with time and is unknown. Furthermore, large errors in signal estimation may propagate and lead to poor performance of future signal recovery. Therefore, all recursive techniques typically assume that the initial estimates of support set and sparse signal are accurate.

In this paper, we assume that the sparse signal changes slowly and its dynamic corresponds to:

$$(\mathbf{x}_t)_i = \begin{cases} (\mathbf{x}_{t-1})_i + (\omega_t)_i, & \text{if } i \in N_t \\ (\mathbf{x}_{t-1})_i, & \text{otherwise} \end{cases} \quad (4)$$

where,  $\omega_t \in \mathbb{R}^n$  is a white Gaussian noise with covariance matrix  $\mathbf{Q}_t$ , i.e.,  $\omega_t \sim \mathcal{N}(0, \mathbf{Q}_t)$ . Covariance matrix  $\mathbf{Q}_t$  plays an important role in accuracy of reconstruction when we employ Kalman filter (KF) based methods. However, historical data can be a used source to estimate covariance matrix  $\mathbf{Q}$ .

A majority of existing CS approaches for smart grids assume that the measured signal is noiseless. However, this assumption is unrealistic and degrades the accuracy of signal recovery. Therefore, in this paper, we assume that an additive noise vector is associated with measurements,

$$\mathbf{y}_t = \mathbf{C}\mathbf{x}_t + \nu_t \quad (5)$$

where,  $\mathbf{y}_t \in \mathbb{R}^m$  ( $m < n$ ) is the available measurement at time  $t$ ,  $\mathbf{C} = \Phi\Psi$ , and  $\nu_t \in \mathbb{R}^m$  is a white Gaussian noise with covariance matrix  $\mathbf{R}_t$ , i.e.,  $\nu_t \sim \mathcal{N}(0, \mathbf{R}_t)$  and  $\mathbb{E}[\omega_t \nu_j'] = 0; \forall j, t$ . The objective of dynamic recursive CS is to observe  $\mathbf{y}_t$ ,  $t \in \{1, 2, \dots, t\}$  and estimate  $\mathbf{x}_t$ . In the following subsections, we describe dynamic recursive CS approaches to estimate  $\mathbf{x}_t$ .

#### A. Weighted BPDN

Streaming modified weighted- $\ell_1$  method is summarized in Algorithm 1. As a generalization of the modified BPDN idea, a weighting scheme is introduced in [20]. Instead of 0 or 1 weighting policy, the weights can be selected to be any value

between 0 and 1. If  $i$  does not belong to support set (i.e.,  $(\hat{\mathbf{x}}_{t-1})_i \approx 0$ ), the corresponding weight  $(\mathbf{W}_t)_{ii}$  tends to be  $\gamma$ ; otherwise, it takes smaller values. The weights are updated such that they become inversely proportional to the magnitude of the entries of  $\hat{\mathbf{x}}_{t-1}$ . Therefore, this approach incorporates the slow support change assumption while using the signal values to update the weights. It should be noted that Step 1 in Algorithm 1 can use any CS approach as long as it returns an accurate estimate of the initial signal.

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**Algorithm 1** Streaming modified weighted- $\ell_1$  (SMW- $\ell_1$ ) [15]

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Step 1: At  $t = 1$ : Solve BPDN with sufficient measurements, i.e., compute  $\hat{\mathbf{x}}_0$  as the solution of  $\min_{\mathbf{b}} \gamma \|\mathbf{b}\|_1 + \|\mathbf{y}_1 - \mathbf{C}\mathbf{b}\|_2^2$

Step 2: For  $t > 1$ , set,

$$(\mathbf{W}_t)_{ii} = \frac{\gamma}{\beta |(\hat{\mathbf{x}}_{t-1})_i| + 1}$$

where  $\beta = n \frac{\|\hat{\mathbf{x}}_{t-1}\|_2^2}{\|\hat{\mathbf{x}}_{t-1}\|_1^2}$ ,

Step 3: Compute  $\hat{\mathbf{x}}_t$  as the solution of

$$\argmin_{\mathbf{b}} \|\mathbf{W}_t \mathbf{b}\|_1 + \|\mathbf{y}_t - \mathbf{C}\mathbf{b}\|_2^2$$

Increment  $t$  and go to step 2

---

### B. Kalman Filter CS

As a first solution to recursive dynamic CS problem, Kalman Filtered CS (KFCS) was introduced in [18]. Recently, [15] implements the Mod-BPDN-residual to capture possible changes in support set with the sparse signal being estimated by a regular KF. This method exploits both Assumption 1 and Assumption 2. Optimality and convergence analyses of the KFCS approaches remain as open questions. Recently, [21] provide statistical error analysis of KF-Mod-CS in presence of lossy measurements. When accurate prior knowledge of the signal values is available, KF-Mod-CS outperforms the modified-BPDN. Algorithm 2 summarizes Kf-ModCS. It should be noted that the parameters including  $\mathbf{P}_0$ ,  $\sigma_{sys}$  and  $\mathbf{R}$  matrices are extremely critical to find an accurate solution; otherwise, Algorithm 2 fails to converge.

## IV. RESULTS AND DISCUSSION

In this section, we evaluate the performance of the algorithms introduced in section III. We characterize performance using the instantaneous *Integrated Normalized Absolute Error* (INAE) metric [9], which is defined as,

$$INAE = \frac{\sum_{j=1}^N |\mathbf{z}_j - \hat{\mathbf{z}}_j|}{\sum_{j=1}^N |\mathbf{z}_j|} \times 100 \quad (6)$$

where,  $\mathbf{z}_j$  is the actual real power at  $j^{th}$  node and  $\hat{\mathbf{z}}_j$  is the corresponding estimate of  $\mathbf{z}_j$  recovered by the compressive sensing approaches. As described in section II, the signal of interest  $\mathbf{z}$  is not a sparse vector itself. However, we transform it to a sparse signal  $\mathbf{x}$  by linear transformation  $\Psi$ . Here, we apply

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**Algorithm 2** Kalman Filtered modified Compressive Sensing [15]

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**Initialization:** Set  $\mathbf{x}_0 = 0$ ,  $\mathbf{P}_0$ ,  $N_0$  =empty (if unknown) or equal to the known/partially known support.

For  $t > 0$ , do,

Step 1: Set  $T = \hat{N}_{t-1}$

Step 2: Mod-BPDN residual:

$$\hat{\mathbf{x}}_{t,mod} = \hat{\mathbf{x}}_{t-1} + \left[ \argmin_{\mathbf{b}} \gamma \|\mathbf{b}^{T^c}\|_1 + \|\mathbf{y}_t - \mathbf{C}\hat{\mathbf{x}}_{t-1} - \mathbf{C}\mathbf{b}\|_2^2 \right]$$

Step 3: Support Estimation - Simple thresholding:

$$\hat{N}_t = \{i : |(\hat{\mathbf{x}}_{t,mod})_i| > \alpha_a\}$$

Step 4: Modified Kalman Filter:

$$\hat{\mathbf{Q}}_t = \sigma_{sys}^2 \mathbf{I}_{\hat{N}_t} \mathbf{I}_{\hat{N}_t}'$$

$$\mathbf{K}_t = (\mathbf{P}_{t-1} + \hat{\mathbf{Q}}_t) \mathbf{C}' (\mathbf{C}(\mathbf{P}_{t-1} + \hat{\mathbf{Q}}_t) \mathbf{C}' + \mathbf{R})^{-1}$$

$$\mathbf{P}_t = (\mathbf{I} - \mathbf{K}_t \mathbf{C}) (\mathbf{P}_{t-1} + \hat{\mathbf{Q}}_t)$$

$$\hat{\mathbf{x}}_t = (\mathbf{I} - \mathbf{K}_t \mathbf{C}) \hat{\mathbf{x}}_{t-1} + \mathbf{K}_t \mathbf{y}_t$$

Increment  $t$  and go to step 1

---

the ‘‘Haar’’ mother wavelet as the sparsifying basis  $\Psi$  as it has been shown to outperform other basis choices [10]. A random measurement matrix  $\Phi$  consists of random Bernoulli entries which selects  $\frac{m}{n} \times 100\%$  of the data is used. We establish our experiments based on two data frameworks: 1-IEEE 34-test feeder system; 2-Pecan Street data.

### A. IEEE 34-bus system

[9] and [10] provides details of the IEEE 34 node distribution test feeder system considered. Here, we assume that every node has a stochastic trend  $D_t = 0.05 \sin(\frac{\pi}{12}t) + 0.3\mathbf{w}_t$  for load, where  $\mathbf{w}_t \sim \mathcal{N}(0, \mathbf{I})$ . The measurements are also associated with white noises where  $\nu_t \sim \mathcal{N}(0, 0.4\mathbf{I}_{m \times m})$ . In Algorithm 1, we consider  $\gamma = 1.5$ . In Algorithm 2, we set  $\gamma = 0.075$  and  $\alpha_a = 0.25$ . Then, we recover the data using half ( $m = 16$ ,  $n = 32$ ) of the measurements. As it is shown by Fig 3, the Algorithms 1 and 2 (the recursive approaches) which rely on Assumption 1 and 2 outperform the conventional CS method suggested by Theorem 1. This improvement in performance can be justified by the facts that the underlying sparse power signals and their supports slowly vary over time.

### B. Pecan Street Data

Pecan Street Inc provides electricity and water consumptions of more than 1000 volunteer customers. Furthermore, different types of the load data is available (e.g., HVAC use, refrigerator use, solar generation, etc.). The data is recorded at two time scales 1 minute and 1 hour. Here, we employ the one-minute load data relevant to 272 homes collected by Pecan Street Inc [19]. Specifically, we consider 24 hours load data averaged in 15 minutes intervals on March 2<sup>nd</sup> 2018. As the loads associated 17 homes are lumped together, we aim to recover 16 data points at each time  $t \in \{1, 2, \dots, 96\}$ . Hence,

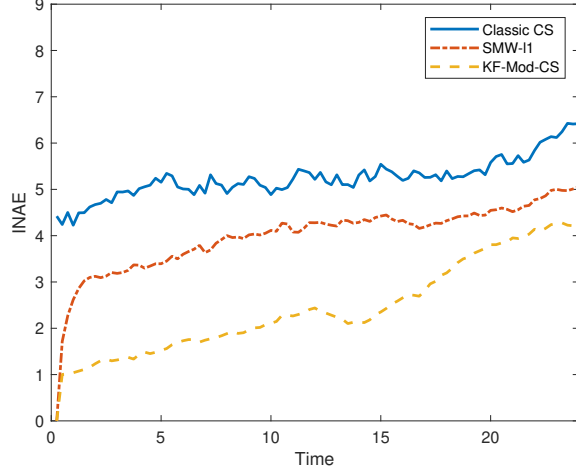


Fig. 3. INAE of Algorithms 1 and 2 versus classic CS (Theorem 1)

TABLE I  
AVERAGE INAE OF DIFFERENT CS APPROACHES FOR PECAN STREET DATA

Method	INAE (CR=75%)	INAE (CR=50%)
Classic CS	17.48	24.61
SMW- $\ell_1$ (Algorithm 1)	14.74	20.23
KF-Mod-CS (Algorithm 2)	12.38	17.54

the dimensions are  $\mathbf{x}_t \in \mathbb{R}^{16}$ ,  $\mathbf{h}_t \in \mathbb{R}^8$  and  $\Phi \in \mathbb{R}^{8 \times 16}$ . We set  $\gamma = 2$  in Algorithm 1 and  $\gamma = 0.3$ ,  $\alpha_a = 0.2$  in Algorithm 2. Since, we don't have the exact information about the underlying sparse signal dynamics, we approximate the covariance matrix  $\mathbf{Q}$  using prior data collected on March 1<sup>st</sup> 2018. Table I summarizes the averages of INAE over the entire day resulting from the use of classic CS and the algorithms introduced in section III. In this table, we consider different levels of compression ratio. Similar to the previous results for IEEE-34 test feeder system, the results are significantly improved by the recursive approaches which exploit slow signal and support changes.

## V. CONCLUSION

In this paper, we aim at dynamically reconstructing a power signal from a fewer number of measurements. To do so, we employ two recursive dynamic CS techniques known as streaming modified weighted- $\ell_1$  CS and Kalman filtered CS. Using a practical power data from an actual distribution system, we validate the underlying assumptions (slow signal and support change) required by dynamic recursive CS approaches. Since the recursive methods use the prior information about the sparse signal and its support set, they could improve the previous results obtained by classic CS. The superior performance of the two recursive CS methods are validated via two examples (the IEEE 34 node test feeder system and PecanStreet data).

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## REFERENCES

- [1] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Transactions on information theory*, vol. 46, no. 2, pp. 388–404, 2000.
- [2] A. Primadianto and C.-N. Lu, "A review on distribution system state estimation," *IEEE Transactions on Power Systems*, vol. 32, no. 5, pp. 3875–3883, 2016.
- [3] K. Dehghanpour, Z. Wang, J. Wang, Y. Yuan, and F. Bu, "A survey on state estimation techniques and challenges in smart distribution systems," *IEEE Transactions on Smart Grid*, vol. 10, no. 2, pp. 2312–2322, 2018.
- [4] A. Ciancio, S. Pattem, A. Ortega, and B. Krishnamachari, "Energy-efficient data representation and routing for wireless sensor networks based on a distributed wavelet compression algorithm," in *Proceedings of the 5th international conference on Information processing in sensor networks*. ACM, 2006, pp. 309–316.
- [5] M. F. Duarte and Y. C. Eldar, "Structured compressed sensing: From theory to applications," *IEEE Transactions on signal processing*, vol. 59, no. 9, pp. 4053–4085, 2011.
- [6] M. P. Tcheou, L. Lovisolo, M. V. Ribeiro, E. A. Da Silva, M. A. Rodrigues, J. M. Romano, and P. S. Diniz, "The compression of electric signal waveforms for smart grids: State of the art and future trends," *IEEE Transactions on Smart Grid*, vol. 5, no. 1, pp. 291–302, 2013.
- [7] V. Loia, S. Tomasiello, and A. Vaccaro, "Fuzzy transform based compression of electric signal waveforms for smart grids," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 47, no. 1, pp. 121–132, 2016.
- [8] J. C. S. de Souza, T. M. L. Assis, and B. C. Pal, "Data compression in smart distribution systems via singular value decomposition," *IEEE Transactions on Smart Grid*, vol. 8, no. 1, pp. 275–284, 2015.
- [9] S. S. Alam, B. Natarajan, and A. Pahwa, "Distribution grid state estimation from compressed measurements," *IEEE Transactions on Smart Grid*, vol. 5, no. 4, pp. 1631–1642, 2014.
- [10] H. S. Karimi and B. Natarajan, "Compressive sensing based state estimation for three phase unbalanced distribution grid," in *GLOBECOM 2017-2017 IEEE Global Communications Conference*. IEEE, 2017, pp. 1–6.
- [11] S. Tripathi and S. De, "An efficient data characterization and reduction scheme for smart metering infrastructure," *IEEE Transactions on Industrial Informatics*, vol. 14, no. 10, pp. 4300–4308, 2018.
- [12] X. Huang, T. Hu, C. Ye, G. Xu, X. Wang, and L. Chen, "Electric load data compression and classification based on deep stacked auto-encoders," *Energies*, vol. 12, no. 4, p. 653, 2019.
- [13] M. Babakmehr, F. Harirchi, A. Al Durra, S. Muyeen, and M. Simoes, "Compressive system identification for multiple line outage detection in smart grids," *IEEE Transactions on Industry Applications*, 2019.
- [14] L. Wen, K. Zhou, S. Yang, and L. Li, "Compression of smart meter big data: A survey," *Renewable and Sustainable Energy Reviews*, vol. 91, pp. 59–69, 2018.
- [15] N. Vaswani and J. Zhan, "Recursive recovery of sparse signal sequences from compressive measurements: A review," *IEEE Transactions on Signal Processing*, vol. 64, no. 13, pp. 3523–3549, 2016.
- [16] E. Candes and T. Tao, "Decoding by linear programming," *arXiv preprint math/0502327*, 2005.
- [17] W. Lu and N. Vaswani, "Regularized modified bpdn for noisy sparse reconstruction with partial erroneous support and signal value knowledge," *IEEE Transactions on Signal Processing*, vol. 60, no. 1, pp. 182–196, 2011.
- [18] N. Vaswani, "Kalman filtered compressed sensing," in *2008 15th IEEE International Conference on Image Processing*. IEEE, 2008, pp. 893–896.
- [19] "Pecan street." [Online]. Available: <https://www.pecanstreet.org/>
- [20] M. S. Asif and J. Romberg, "Sparse recovery of streaming signals using  $\ell_1$ -homotopy," *IEEE Transactions on Signal Processing*, vol. 62, no. 16, pp. 4209–4223, 2014.
- [21] H. S. Karimi and B. Natarajan, "Kalman filtered compressive sensing with intermittent observations," *Signal Processing*, vol. 163, pp. 49–58, 2019.