R, G, B
Red, Green, Blue

W → vector   V → Vector space $V_1, V_2, V_3, \ldots, V_n$ in V

Span → Generate a vector

If w is a vector in vector space v then w is said to be a linear combination of vectors $V_1, V_2, V_3 \ldots V_n$. If w can be written as

$$W = a_1v_1 + a_2v_2 + a_3v_3 + \cdots + a_nv_n$$

(spanning w)

Q# $v = (2,-3)$ $V_1 = (1,2)$ $V_2 = (-2,-5)$

$$V = a_1V_1 + a_2V_2 \text{ — (A)}$$

$$(2,-3) = a_1(1,2) + a_2(-2,-5)$$

$$= (a_1, 2a_1) + (-2a_2, -5a_2)$$

$$= (a_1 - 2a_2, 2a_1 - 5a_2)$$

$$2 = a_1 - 2a_2 \text{ — (1)}$$

$$-3 = 2a_1 - 5a_2 \text{ — (2)}$$

$\boxed{a_1 = 16}$ $\boxed{a_2 = 7}$ put in (A) $\boxed{V = 16V_1 + 7V_2}$ linear combination

$$\left[\begin{array}{cc:c} 1 & -2 & 2 \\ 2 & -5 & -3 \end{array}\right]$$

$$A = \begin{bmatrix} 1 & -2 \\ 2 & -5 \end{bmatrix}$$

$|A| = -5+4 = -1$ $|A| \neq 0$ can find a_1 & a_2

$|A| = 0$ we cannot find a_1 & a_2, v cannot be written as as linear combination of V_1 and V_2.

Q# Determine whether these polynomials span P_2 or not.

$$P_1 = 1 + x + x^2$$

$$P_2 = -1 + x + 0x^2$$

$$P_3 = 2 + 2x + x^2$$

$P_2(x) = \{ a+bx+cx^2 \mid a, b, c \in R \}$

let $P = a + bx + cx^2$

$P = a_1P_1 + a_2P_2 + a_3P_3$

$a+bx+cx^2 = a_1(1+x+x^2) + a_2(-1+x+0x^2) + a_3(2+2x+x^2)$

$a = a_1 - a_2 + 2a_3$

$b = a_1 - a_2 + 2a_3$

$c = a_1 + a_3$

$$\begin{bmatrix} 1 & -1 & 2 & a \\ 1 & -1 & 2 & b \\ 1 & 0 & 1 & c \end{bmatrix} \quad R_2-R_1 \quad R_3-R_1$$

$$\begin{bmatrix} 1 & -2 & 2 & a \\ 0 & 0 & 0 & b-a \\ 0 & 1 & -1 & c-a \end{bmatrix} \qquad |A| = \begin{vmatrix} 1 & -1 & 2 \\ 1 & -1 & 2 \\ 1 & 0 & 1 \end{vmatrix} \qquad |A| = 0$$

$\times$ not possible

$0 = b-a \quad b = a$

means not general and are not generators like R, G, B

Q# $P_1 = x + x^2$

$P_2 = x - x^2$

$P_3 = 1 + x$

$P_4 = 1 - x$

let $P = a + bx + cx^2$

$P = P_1a_1 + P_2a_2 + P_3a_3 + P_4a_4$

$(a+bx+cx^2) = a_1(x+x^2) + a_2(x-x^2) + a_3(1+x) + a_4(1-x)$

$$= (a_1 - a_2)x^2 + (a_1 + a_2 + a_3 - a_4)x + (a_3 + a_4)$$

$a = a_3 + a_4$

$b = a_1 + a_2 + a_3 - a_4$

$c = a_1 - a_2$

$$\left[\boxed{\begin{matrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 1 & -1 & 0 & 0 \end{matrix}} \begin{matrix} : & a \\ : & b \\ : & c \end{matrix}\right]$$

not a square matrix, can't find determinant so, change it into row echelon form.

$R_1 \longleftrightarrow R_2$

$$\begin{bmatrix} 1 & 1 & 1 & -1 & b \\ 0 & 0 & 1 & 1 & a \\ 1 & -1 & 0 & 0 & c \end{bmatrix}$$

$R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 & -1 & b \\ 0 & 0 & 1 & 1 & a \\ 0 & -2 & -1 & 1 & c-b \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & -1 & b \\ 0 & 1 & 1/2 & 1/2 & c-b/-2 \\ 0 & 0 & 1 & 1 & a \end{bmatrix}$$

$a_3 + a_4 = a$

$a_2 + \frac{a_3}{2} + \frac{a_4}{2} = \frac{c-b}{-2}$

$$a_1 + a_2 + a_3 - a_4 = b$$

$$a_2 = \frac{c-b}{-2} - \frac{1}{2}(a_4 - t) + \frac{1}{2} t$$

$$a_1 = b - a_2 - a_3 + a_4$$
$$= b - (\quad) - a_3 + t$$