National University of Computer and Emerging Sciences, Lahore Campus

Course:	Linear Algebra	Course Code:	MT104
Program:	BS (CS , SE , DS)	Semester:	Fall
Duration:	3 hours	Total Marks:	100
Paper Date:	20-1-22	Weight	50-54%
Section:	All	Page(s):	03
Exam:	Final term	Roll No:	20L-1080

Instruction/Notes: Programmable calculators are not allowed. Show complete working in all questions. 54% wtg. is applicable to only those sections who had no quiz-3 due University Closure. **Q#9 is BONUS question.**

Application in Computer Graphics

Question#1[05+05][CLO-1]: Use **Elementary Matrices** to find the **Inverse** of $A = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$. Also verify that $A = E_1^{-1} E_2^{-1} E_3^{-1} \ldots E_{k-1}^{-1} E_k^{-1}$ for some k . Left

Any other method used to evaluate the inverse will not be considered for marking.

Application in Computer Graphics

Question#2[2+5+5+5+3][CLO-1,5]: Discuss the **Geometric Effect** on the **Unit Square** of multiplication by the matrix $A = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ using the following steps:

1. Decompose $A = E_1^{-1} E_2^{-1} E_3^{-1} \ldots E_{k-1}^{-1} E_k^{-1}$ for some k.
2. Show the effect of $E_1^{-1} E_2^{-1} E_3^{-1} \ldots E_{k-1}^{-1} E_k^{-1}$ on the unit square. Also show the action of elementary matrix (each) via diagram separately.
3. Show mathematically action of each elementary matrix on the end points of the edges.
4. Illustrate the geometric effects at each step.

Lines & Planes in R³

Question#3(a) [05][CLO-2]: Find the vector and parametric equation of the plane in R^3 that passes through the origin and is orthogonal to $= (3,1,-6)$.

3(b)[5] [CLO-2]: Find a vector parallel to the line of intersection of two planes $3x - 6y - 2z = 15$ & $2x + y - 2z = 5$.

Gram– Schmidt Process for Orthonormal basis

Question#4[20][CLO-4]: Suppose $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix}$ define the column vectors u_1 , u_2 *and* u_3 as

$$u_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, u_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \ \& \ u_3 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}.$$

$\frac{1}{3}, \frac{-1}{3}, \frac{-1}{3}$

$\frac{-1}{3}, \frac{1}{3}, \frac{1}{3}$

4(a)[10]: Use the Gram – Schmidt process to find the orthogonal set of vectors $\{v_1, v_2, v_3\}$ and orthonormal set of vectors $\{q_1, q_2, q_3\}$ by considering standard inner product between the vectors

4(b)[10]. Also find a matrix R and verify $A = QR$ where, $Q = [\ q_1 \mid q_2 \mid q_3\ ]$ and R is given below

$$R = \begin{bmatrix} \langle u_1, q_1\rangle & \langle u_2, q_1\rangle & \langle u_3, q_1\rangle \\ 0 & \langle u_2, q_2\rangle & \langle u_3, q_2\rangle \\ 0 & 0 & \langle u_3, q_3\rangle \end{bmatrix}.$$

General Linear Transformations

Question#5 [2+2+2+2+2][CLO-5]: Let $T: R^2 \to R^3$ be the linear transformation defined by the formula

$$T(x_1, x_2) = (x_1 + 3x_2\ , x_1 - x_2,\ x_1)$$

a. Find the Standard Basis Matrix (A) for the above transformation.
b. Find the rank of A i.e. rank(A).
c. Find the nullity of A i.e. null(A).
d. Find the rank of the A^t i.e. rank(A^t).
e. Find the nullity of the A^t i.e. null(A^t).

Question#6 [10][CLO-5]: Let $T: R^2 \to R^3$ be defined as $\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 2x_2 \\ -x_1 \\ 0 \end{bmatrix}$. Find the matrix $[T]_{B',B} = [\ [T(u_1)]_{B'} \mid [T(u_2)]_{B'}]$ relative to the basis $B = \{u_1, u_2\}$ and $B' = \{v_1, v_2, v_3\}$, where

$$u_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, u_2 = \begin{bmatrix} -2 \\ 4 \end{bmatrix}, v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}, v_3 = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}.$$

Question # 7[2+2+2+2+2][CLO-5]: Let $T: R^3 \to R^3$ be a linear operator defined as

$$T(x_1, x_2, x_3) = (0x_1 + x_2 - x_3, x_1 + 0x_2 + 2x_3, -1x_1 + x_2 + 0x_3)$$ defined by $T(X) = AX$ as, where

$$A = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & 2 \\ -1 & 1 & 0 \end{bmatrix}$$

a) Check whether T is One to One.
b) Check whether T is Onto.
c) Find Kernel of T and Basis for Kernel of T.
d) Find Range of T and Basis for Range of T.
e) Find Null space of T and Row space of T.

$$\begin{bmatrix} 1/\sqrt{2} & 0 & \sqrt{6}/6 \\ -1/\sqrt{3} & 1/\sqrt{3} & \sqrt{6}/3 \\ \sqrt{6}/6 & \sqrt{6}/3 & -\sqrt{6}/6 \end{bmatrix} \begin{bmatrix} \sqrt{2} & \sqrt{2} & \sqrt{2} \\ 0 & \sqrt{3} & -\sqrt{3}/3 \\ 0 & 0 & 2\sqrt{6}/3 \end{bmatrix} = \quad 1$$

Equivalence Theorem

6 **Question#8[10]: STATE ONLY** the **Equivalent Statements** (as much as you remember) for the **n x n** Matrix, if it's given that:

a) A is invertible.
b) ....

Note: For each equivalent statement one point will be given. Maximum points are 10.

Similarity of Operators (Bonus)

Question#9[05+05][CLO-3,5]: If $C = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$ and $= \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$, then

a. Find a matrix P (consisting of the Eigen vectors of the matrix C) using Eigenvalues of C & show that $P^{-1}CP = D$. Also, find the dimension of Eigen Spaces associated with each Eigen value.

b. Show that C and D represents the same linear operator $T: R^2 \to R^2$ by showing $P^{-1}CP = D$, where $P = P_{B' \to B} = [[u_1']_B \;\; [u_2']_B]$ and $P^{-1} = P_{B \to B'}$, $B' = \{u_1', u_2'\}$, $B = \{e_1, e_2\}$, $u_1' = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ & $u_2' = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. Here P & P^{-1} represents the transition matrices.

Good Luck

$2 + 2 \qquad 2 - 4$

$-1 - 2 \qquad -1 + 4$

$$\begin{bmatrix} 4 & -2 \\ -3 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

$4 + -2 \qquad 4 - 4$

$-3 + 3 \qquad -3 + 6$