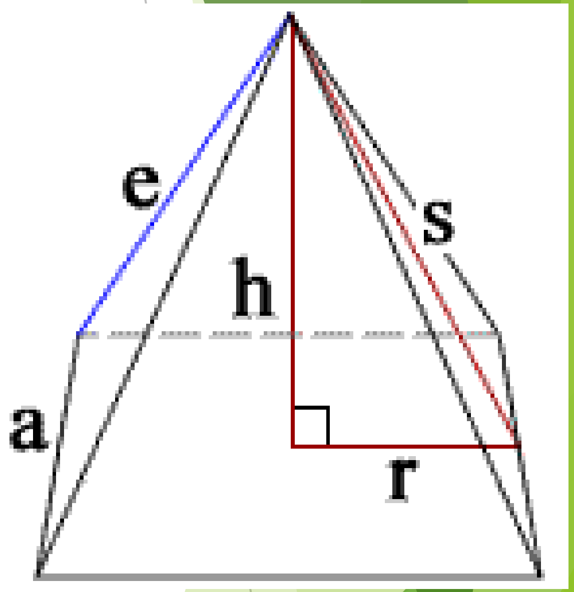
Lecture 04

Mathematics in Computer Science

What is mathematics?

- Mathematics is defined as the science which deals with logic of shape, quantity and arrangement.
- During ancient times in Egypt, the Egyptians used math's and complex mathematic equations like geometry and algebra. That is how they managed to build the pyramids.
- Mathematics is basically related to understanding structure. It is used to do logical analysis, make relevant calculations and eventually to deduce conclusions and pattern.



Role of Math in CS

- What is the importance of the ground in building?
- If you don't have an understanding of the foundation on which you place a structure, that structure may be fairly wobbly and will fail over time.
- Same is the case here with Computer Science

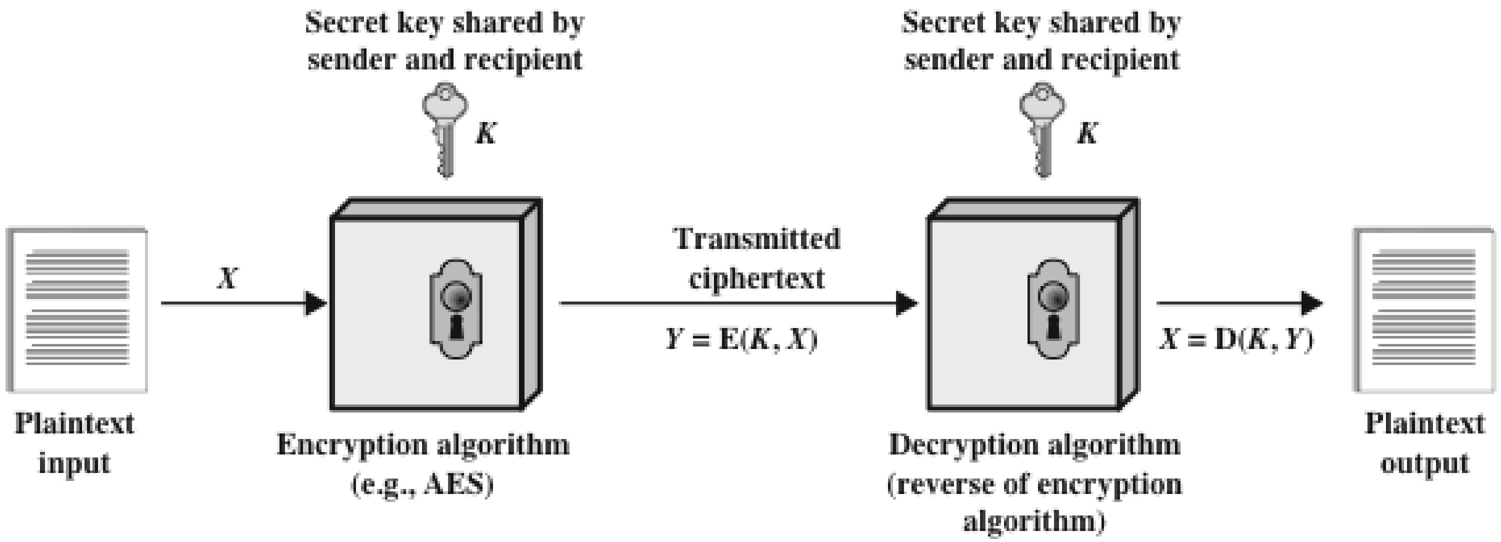
Mathematics is prominent at every layer of Computing:

- The lowest layer with ALU performing the Arithmetic and logical operations.
- The middle layer with Operating system making use of mathematics and algorithms for process management, memory management, disk management, networking operations and more.
- The upper layer with all sort of application and system software using all sort of algorithms to create all sort of magic.
- Right now, when you are reading *this* text, there is mathematics going on behind the scenes to render it on your screen taking into account specific font face/size/color/monitor's resolution/aspect ratio/X,Y coordinates etc. with different algorithms contributing to the final result.

Mathematics applications

Mathematics in Cryptography

- Cryptography is a method of storing and transmitting data in a particular form so that only those for whom it is intended can read and process it.



Plain Text: meet me after the toga party

Cipher: PHHW PH DIWHU WKH WRJD SDUWB

Each element is shifted 3 times to the right

Activity-01

- Can you decipher (Decode) the following messages?

1. Zujge oy ixoiqkz sgzin hkzckkt Vgqoyzgt gtj Otjog

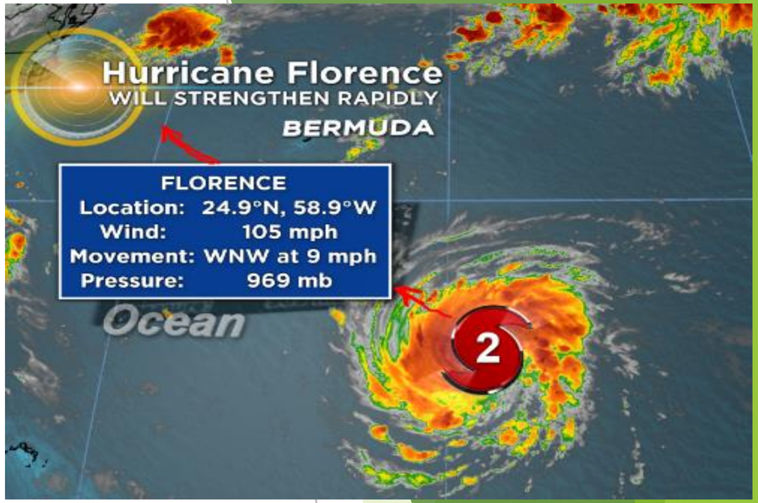
   (Key is 6: Shifted Rightward)

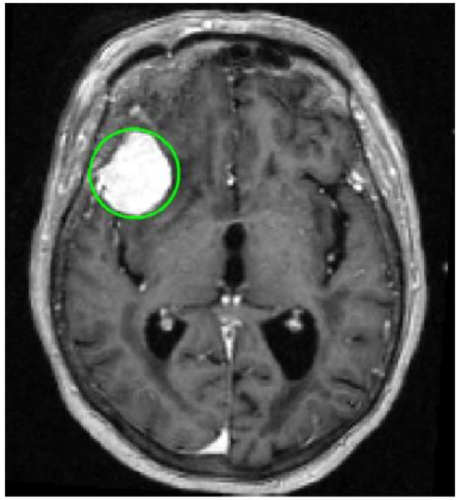
2. tnemeveihca ot sdael taht htiaf eht si msimitpO

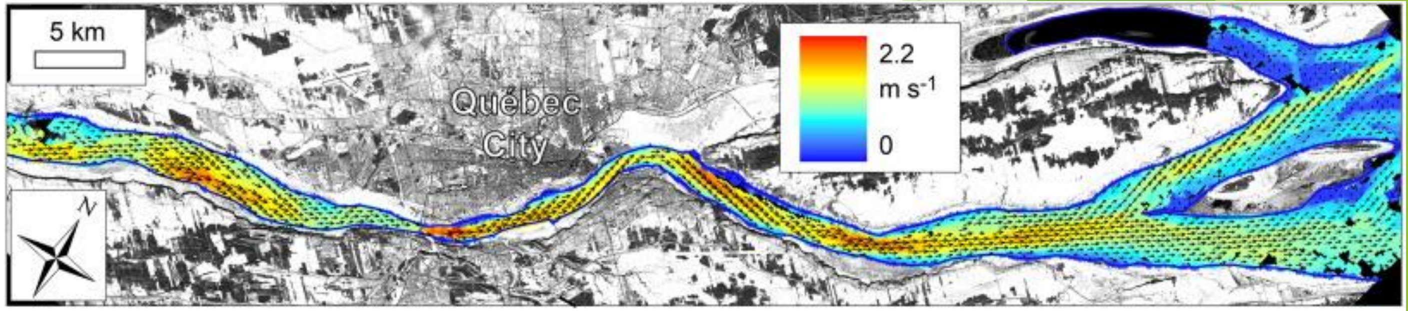
Image Processing & Computer Vision

- Detecting/tracking Hurricanes
- Cancer/Tumor Detection
- Anomaly Detection
- Movement detection from satellite images
- …and many more

All these applications required heavy mathematical operations



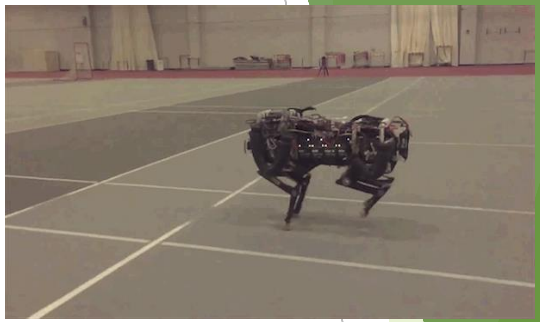


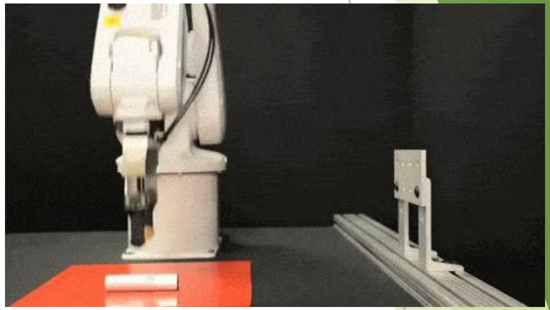


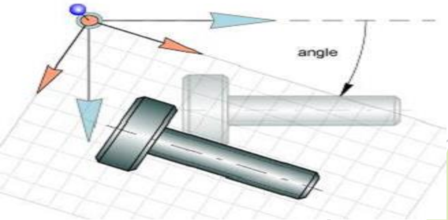
Robotics

- In robotics 3D linear and rotational motion are involved
- Understanding of different coordinate systems are necessary to describe position
  - Coordinates to describe position.
    - rectangular (x,y,z)
    - cylindrical (r,θ,z)
    - spherical (r,θ,φ)
- Similarly pose estimation and object holding

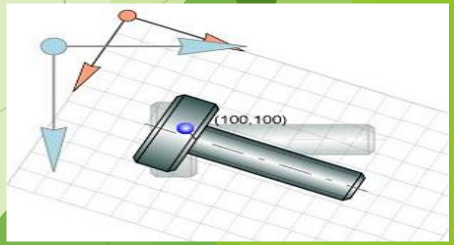
required complex maths







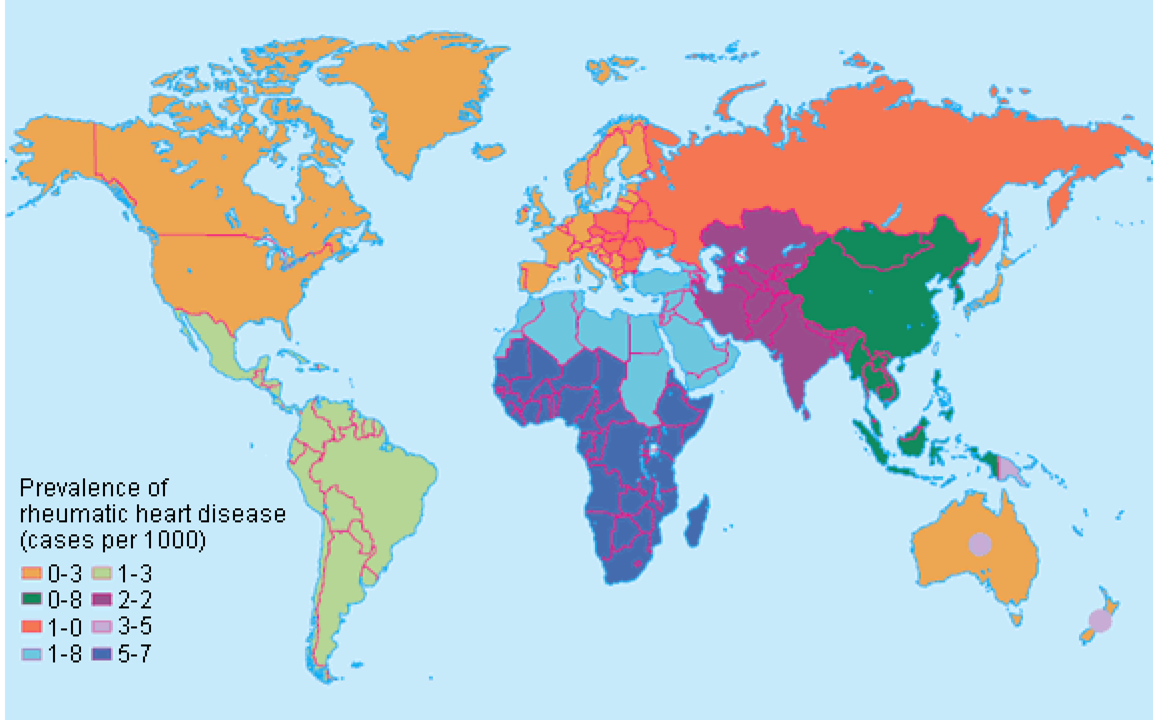
Simple rotation



Rotation & translation

Epidemiology

- Epidemiology is the branch of medicine which deals with the incidence, distribution, and possible control of diseases and other factors relating to health.
- What is your hearth rate now? How you measure it? 100 beats/min
- How smart devices measure blood pressure, heart rate and sugar?
- How the computerized eye-sight checking machine (Auto-Refractometer) works?
- All these techniques required math at backend.



Math in CS
Civil Engineering Apps

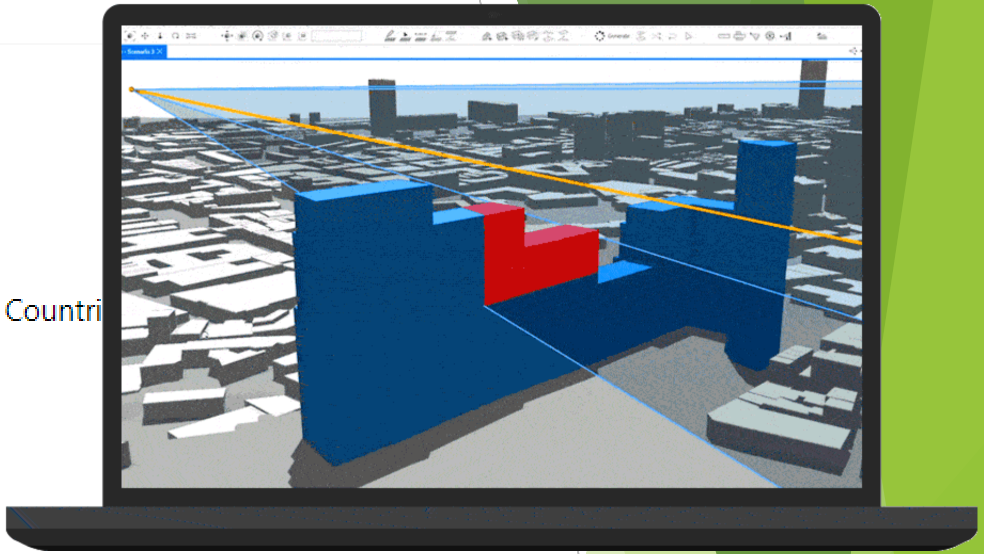
Civil Engineering

WAGmob • Books & reference > Reference

★ ★ ★ ★ ★ WAGmob: Over One million Paying Customers from 175+ Countri
★

More

PEGI 3

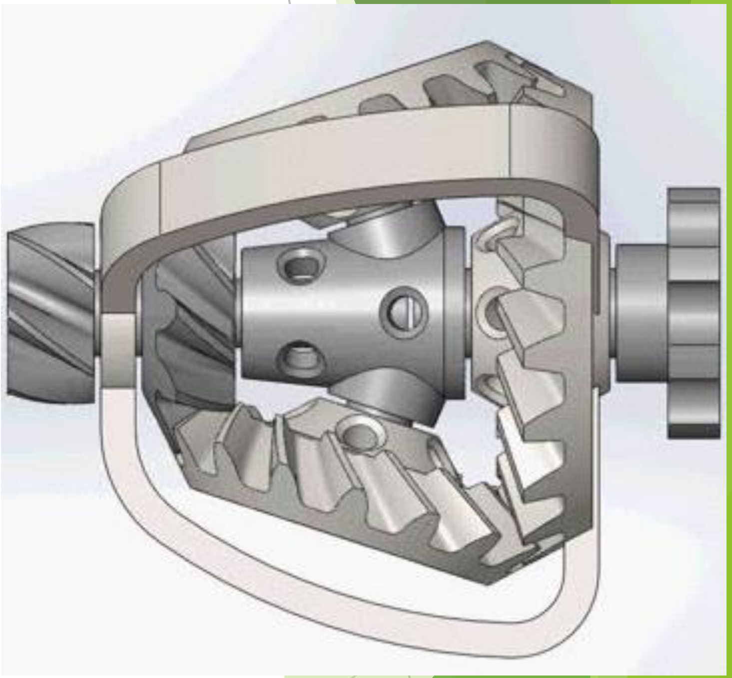


- WAGmob brings you, Simple and Easy, on-the-go learning app for "Civil Engineering". The byte sized app helps you to understand the basics of "Civil Engineering".

How to develop such applications
without understanding of mathematics?

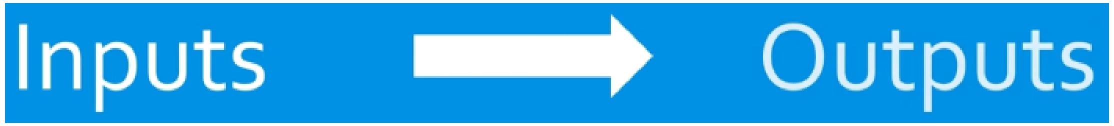
Math in CS
Mechanical Engineering apps

- Some applications for 3D designing in the field of Mechanical Engineering
  - AUTODESK
  - INVENTOR
  - Etc.
- All these application needs heavy math at back end.
- As in given figure there are Force, Rotation, Momentum and Velocity involved.
- By changing one quantity effects other quantities and the changes can be measured/observed through computer



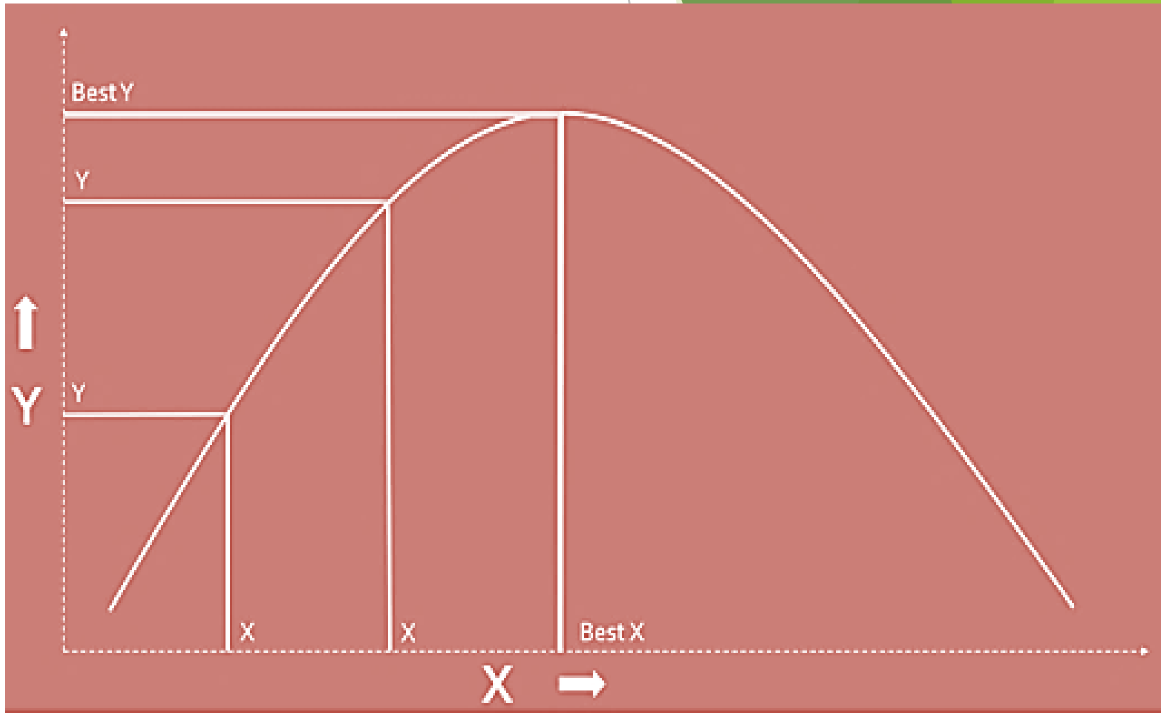
Concept of Optimization

- In mathematical terms, an ***optimization problem*** is the problem of finding the *best* solution from among the set of all *feasible* solutions.
- In simple terms optimization is choosing INPUTS that will result in the best possible OUTPUTS



Optimization

- Let’s take a look at a very simple example of an optimization problem:
- Given a parabola, chose x to get the **largest y**. We can try different x values to see the resulting y value.
- Eventually we can find the maximum y value by choosing Best X.
- You may also have solved this type of problem in calculus class by taking the derivative of the parabola and setting it equal to zero.



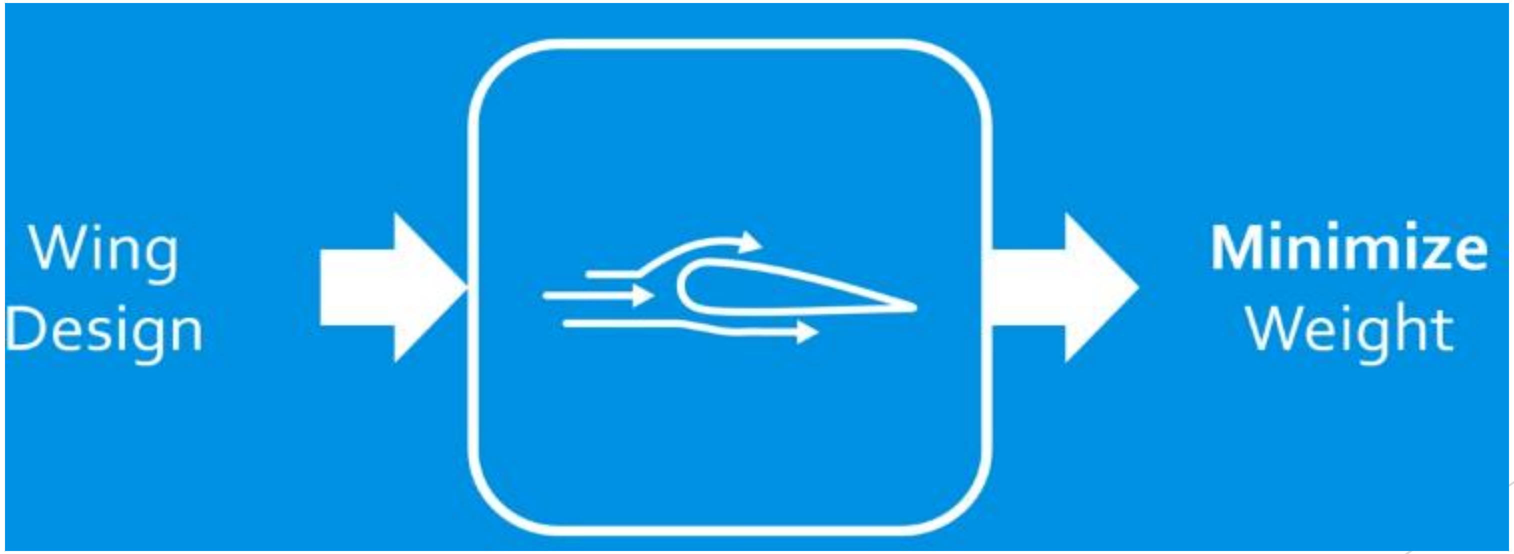
Where can we use OPTIMIZATION?

Bridge Construction

- Designing a bridge that can carry the maximum load possible for a given cost.
- Input is Design and output is to Maximize Load Bearing

Airplane Wing Design

- Design an airplane wing to minimize weight while maintaining strength.
- Input is Wing Design and output is to Minimize Weight



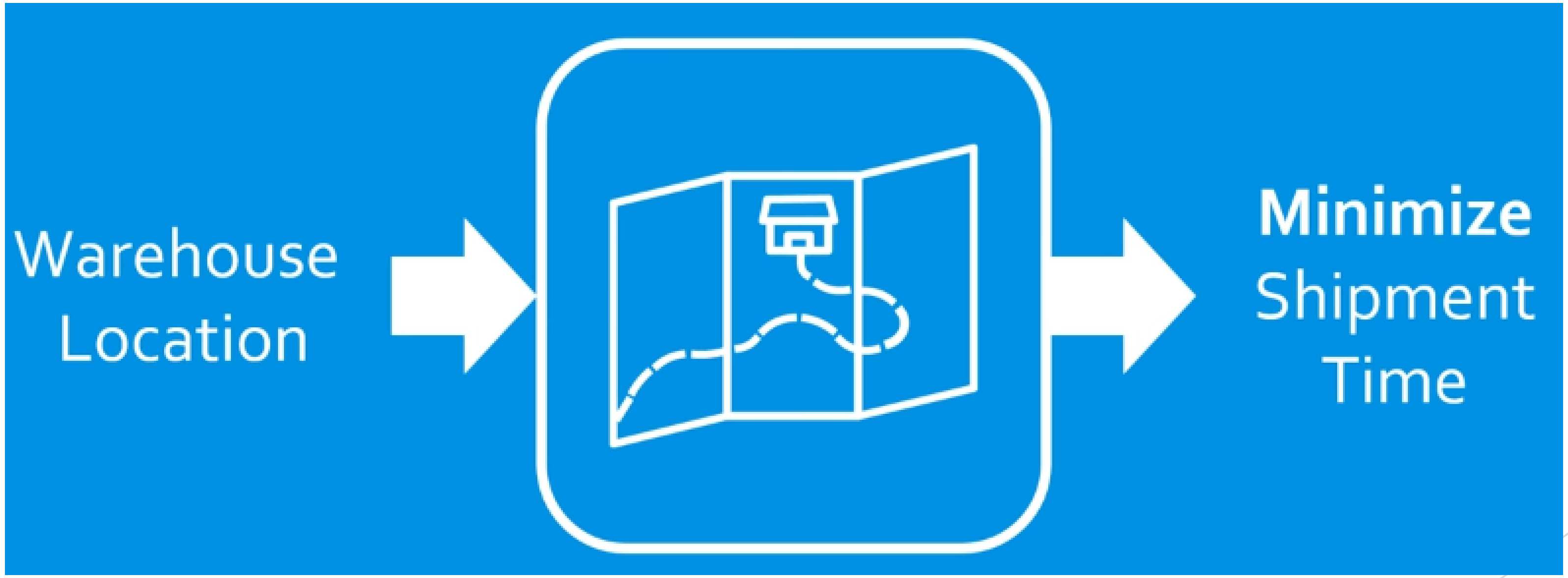
Stock Market

- Selecting the best set of stocks to invest in to maximize returns based on predicted performance.
- Input is Stock Portfolio and Output is to maximize returns on investment

Warehouse placement

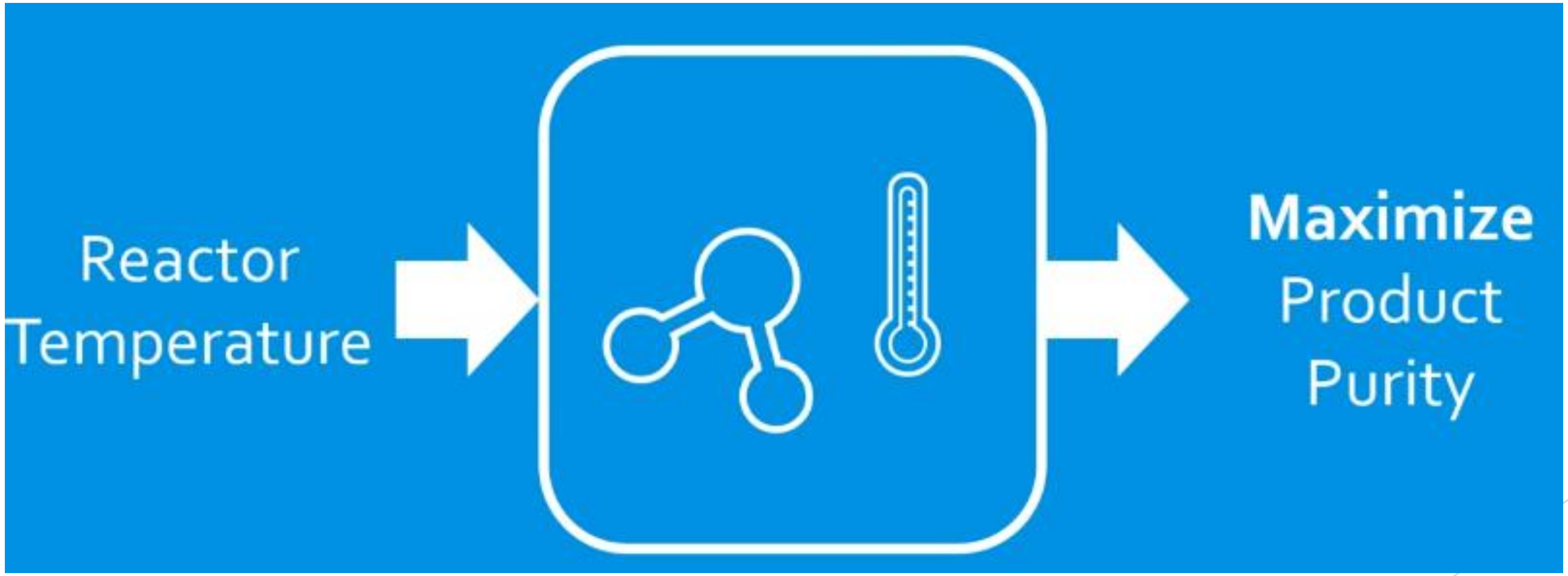
Inputs → Outputs

- Choosing the optimal location for a warehouse to minimize shipment times to potential customers.



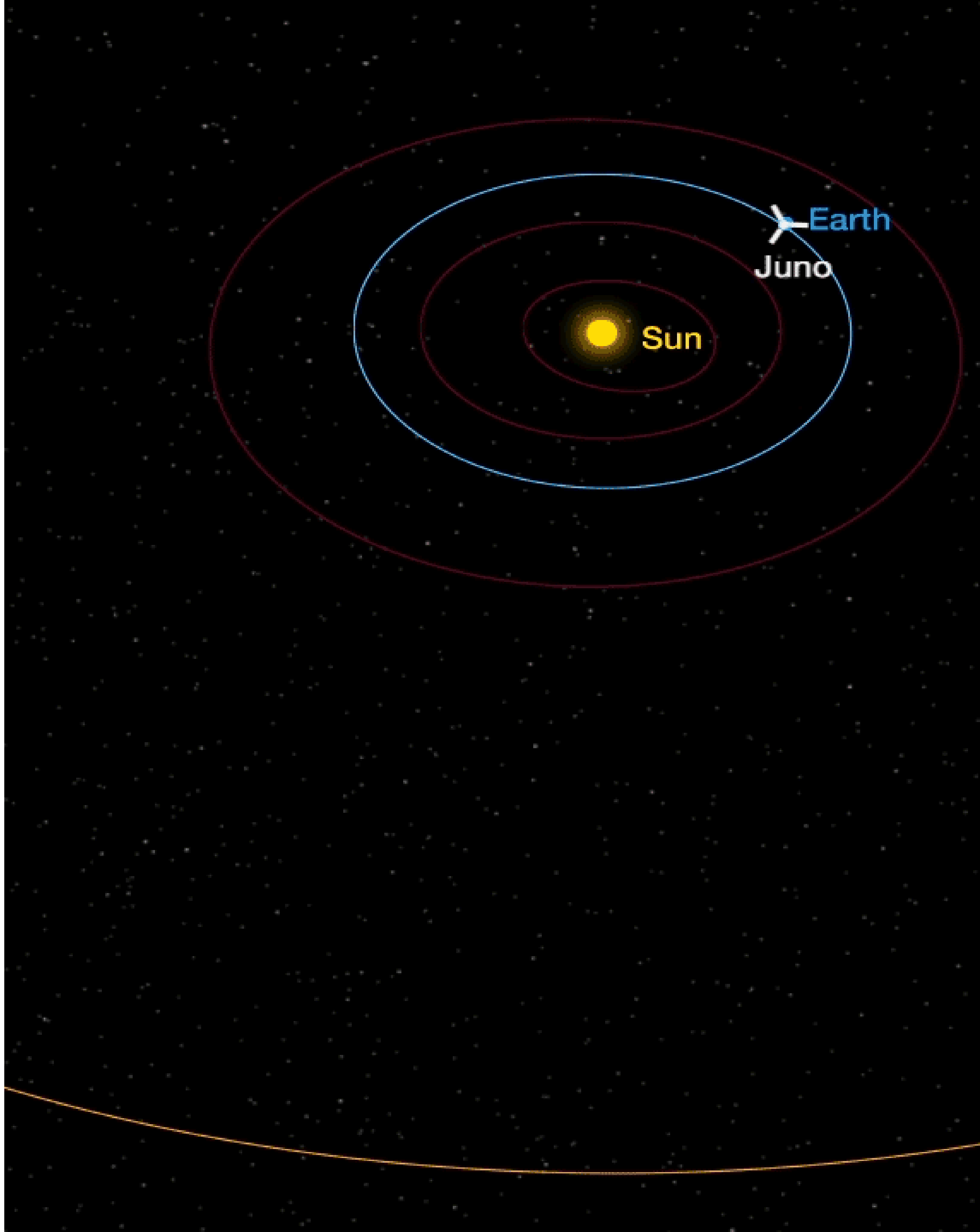
Temperature control of a chemical reaction

- Controlling the temperature of a chemical reaction throughout a process to maximize the purity of a desired product.



Space Shuttle / Suborbital Vehicle trajectory optimization

- Even optimization is involved in life critical missions
- Trajectory of space shuttle is calculated and optimized to its target position by minimizing fuel cost and time.



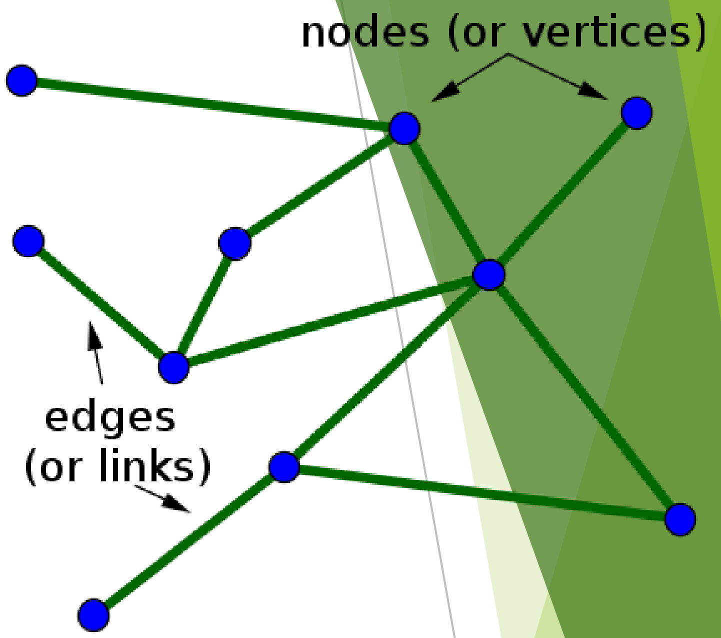
Optimization

Continue…

- As you can see, optimization is a powerful tool in many applications.
- This is just a small sampling of the many fields that make use of optimization techniques to improve the quality of their solutions.
- If something can be modeled mathematically, it can usually be optimized.

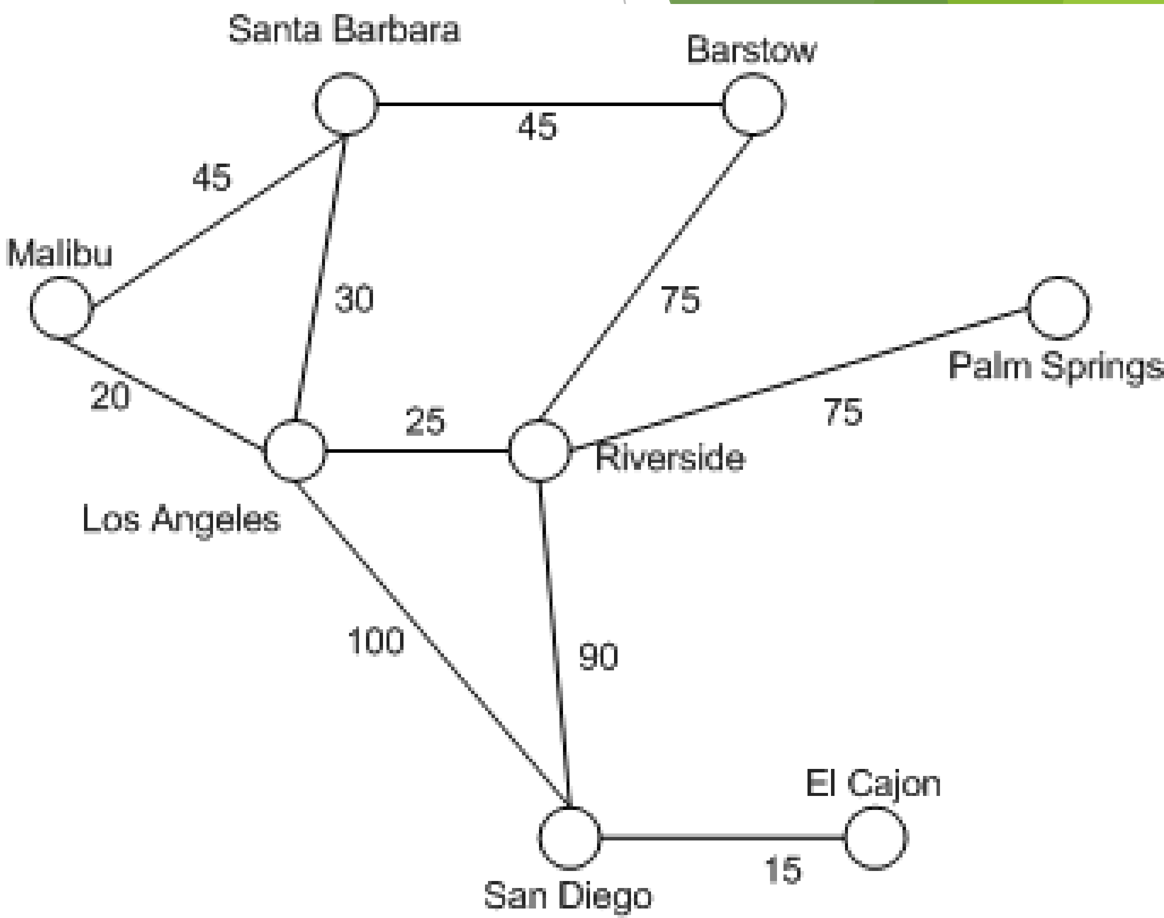
Introduction to Graph

- In Computer Science, graph is a collection of connected NODES (vertices) with the help of EDGES (links)
- A node is a thing or an entity and we can assign some value to it, so a person, a car and a city are examples of a node.
- An edge can be define as a relation of some sort between two or more nodes
- A perfect layman's example might be **Facebook**.
- The network of you, your friends, and their friends etc. are collectively referred to as the **social graph**
- Here people are nodes connected through friendship edges



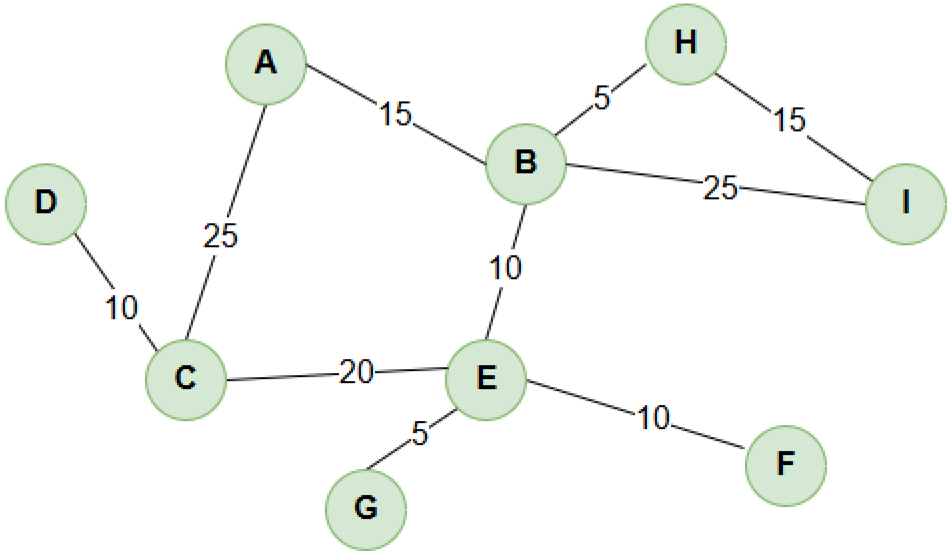
Graph

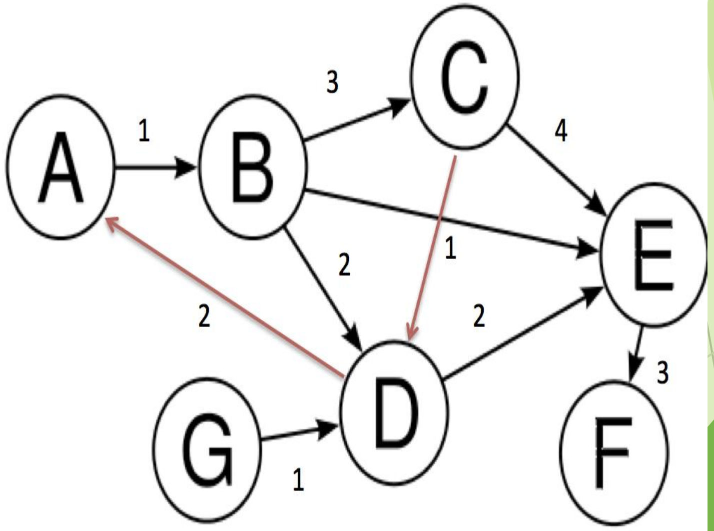
- Similarly we can shows connected cities using graph as in given figure
- Cities are represented as nodes (vertices) and the road's distance is represented on edges.



Types of Graphs

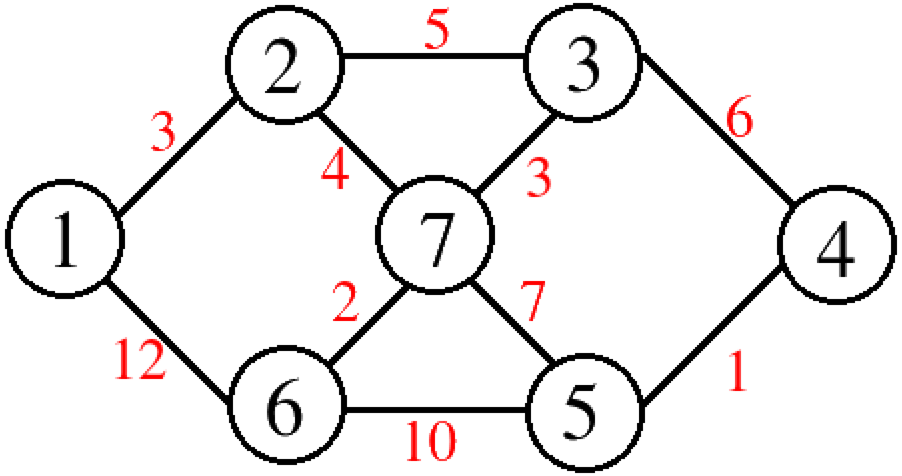
- There are two types of graphs
  - Undirected
  - Directed





Representation of Graph in Computer

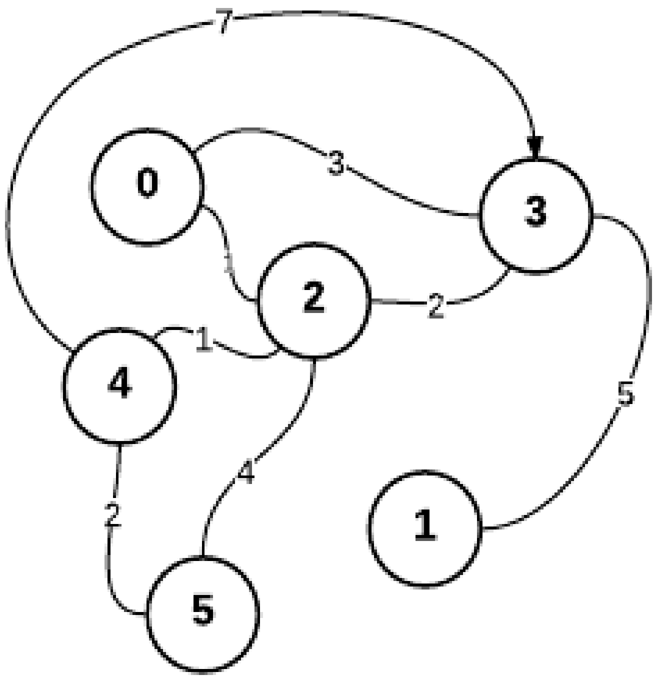
- Adjacency Matrix

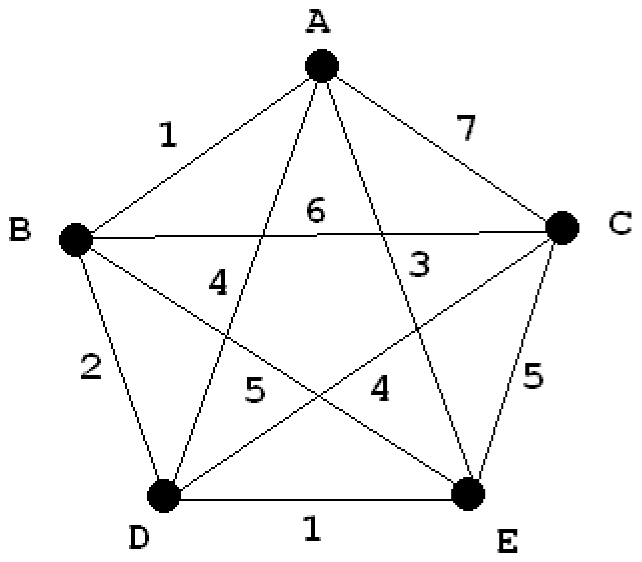


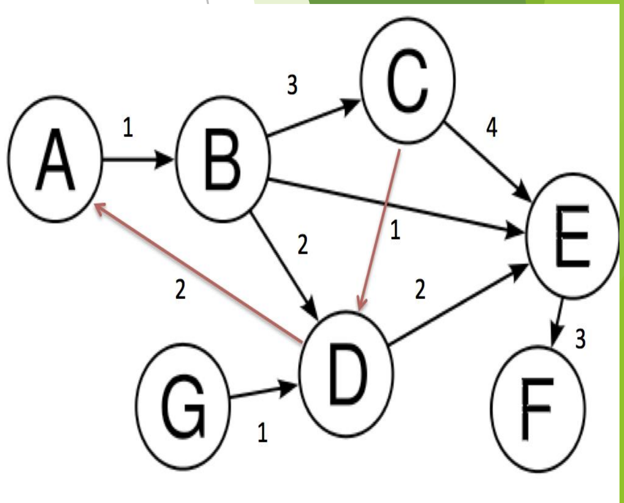
Vertices	1	2	3	4	5	6	7
1	0	3	0	0	0	12	0
2	3	0	5	0	0	0	4
3	0	5	0	6	0	0	3
4	0	0	6	0	1	0	0
5	0	0	0	1	0	10	7
6	12	0	0	0	10	0	2
7	0	4	3	0	7	2	0

Activity-02

Represent the following graph in form of adjacency matrix







Activity-03

Draw a graph from each of the following adjacency matrices

$$
\begin{array}{c}
\begin{array}{cccc} \quad a & b & c & d \end{array} \\
\begin{array}{c} a \\ b \\ c \\ d \end{array}
\begin{pmatrix}
0 & 3 & 0 & 2 \\
3 & 0 & 0 & 1 \\
0 & 0 & 1 & 2 \\
2 & 1 & 2 & 0
\end{pmatrix}
\end{array}
$$

	1	2	3	4	5	6
1	0	1	1	0	0	0
2	-1	0	0	1	0	0
3	-1	0	0	1	0	0
4	0	-1	-1	0	1	0
5	0	0	0	-1	0	1
6	0	0	0	0	-1	0

	A	B	C	D	E	F	G
A	0	0	1	1	0	1	0
B	0	0	0	1	1	0	0
C	1	0	0	0	0	1	0
D	1	1	0	0	0	1	0
E	0	1	0	1	0	0	0
F	1	0	1	1	0	0	0
G	0	0	0	0	0	0	0

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7

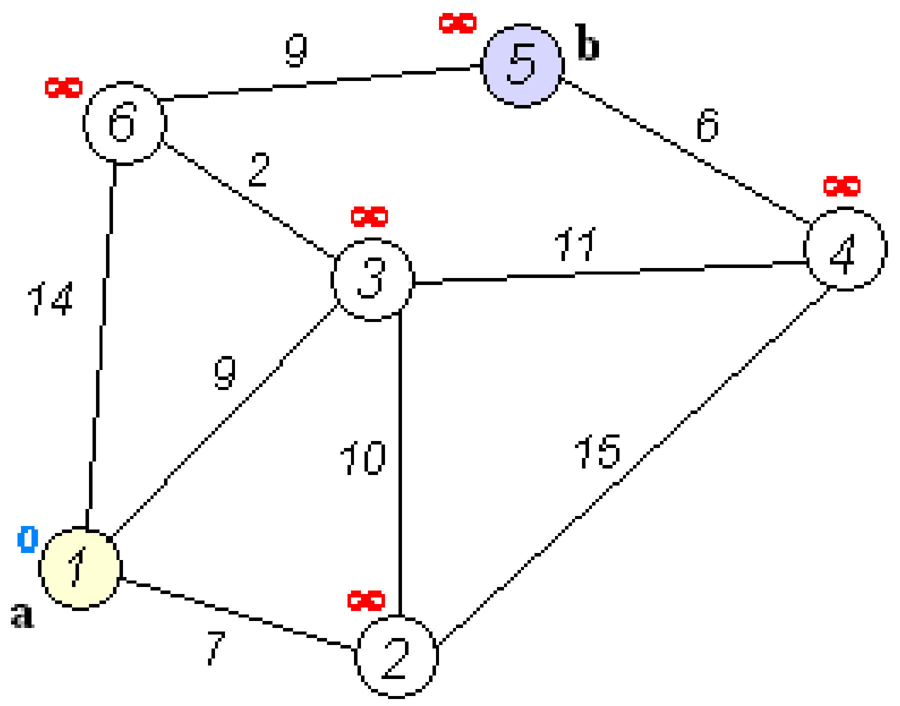
Working with Graphs

Point to Ponder

- Do you know:
  - How does facebook suggest friends?

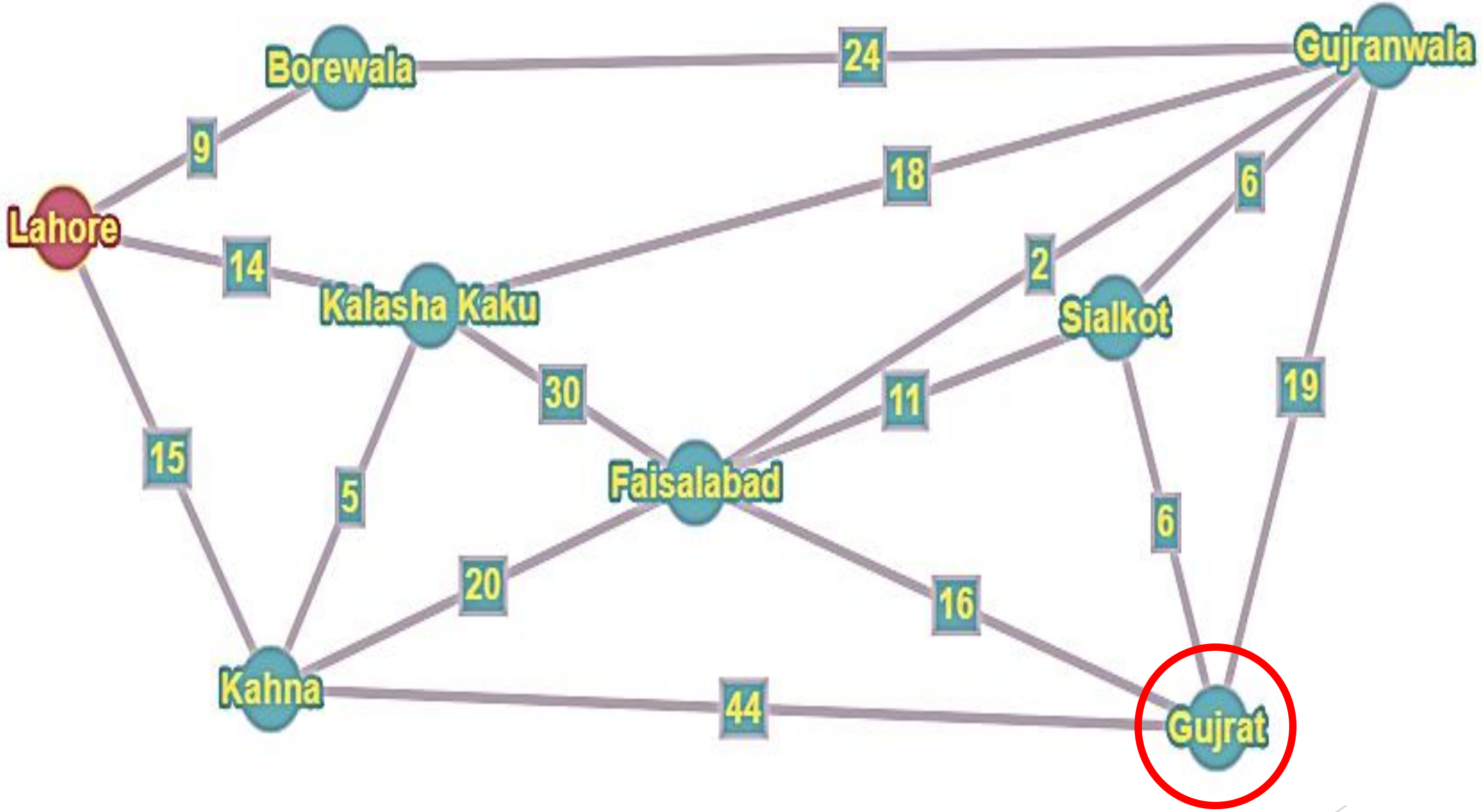
Finding shortest path between two nodes

- Finding shortest path
- Can you find the shortest path between node 1 and 5 in given graph?



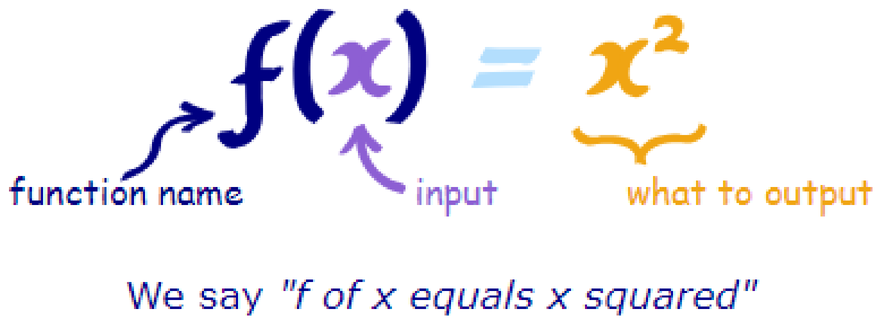
Activity-04

- Can you find the shortest path from Lahore to Gujrat?



Functions

- A function relates an input to an output.
- It is like a machine that has an input and an output. And the output is related somehow to the input
- Representation of a function



https://www.mathsisfun.com/sets/function.html

A function ***relates*** an input to an output. How?

Example: this tree grows 20 cm every year, so the height of the tree is ***related*** to its age using the function ***h***:

$$h(\text{age}) = \text{age} \times 20$$

So, if the age is 10 years, the height is:

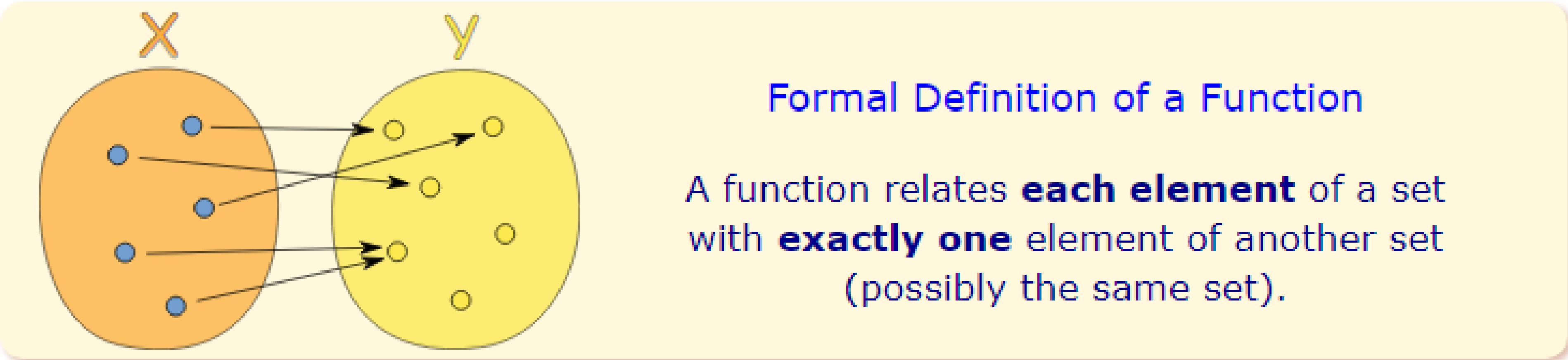
$$h(10) = 10 \times 20 = 200 \text{ cm}$$

Here are some example values:

age	$h(\text{age}) = \text{age} \times 20$
0	0
1	20
3.2	64
15	300
...	...

A function has a special rule

1. It must work for **every** possible input value
2. And it has only **one relationship** for each input value



End of Lecture 04