Number Systems

Lecture 2 – Intro to Info. & Comm. Technologies

Number Systems

Number System Name	**Number System and Description**
Decimal Number System	Base 10 Digits used: 0 to 9
Binary Number System	Base 2 Digits used : 0, 1
Octal Number System	Base 8. Digits used : 0 to 7
Hexadecimal Number System	Base 16. Digits used: 0 to 9, Letters used : A- F

Binary Number System

- Each digit of a binary number is called a bit. For example the number $(1010)_2$ has 4 bits.
- The more the number of bits, the more the total numbers that can be represented. For example if we use 8 bits, then total numbers that can be represented by 8 bits is $2^8=256$.

Binary Number System (contd)

- Suppose we have 8 bits. We know that we can store 256 numbers in these 8 bits.
- If we store only positive numbers, then the smallest number that we can store is 0, and the largest number that we can store is 2^8-1= 255.
- 0 in binary (using 8 bits) is $(00000000)_2$ and 255 in binary is $(11111111)_2$

How to compute the decimal value of a binary number

- Technique
  - Multiply each bit by 2^n, where n is the position of the digit. So, 2^n is the value of the bit.
  - The weight is the position of the bit, starting from 0 on the right
  - Add the results

How to compute the decimal value of a binary number

- **Example**
- Binary Number: 10101
- Calculating Decimal Equivalent:

Step	Binary Number	Decimal Number
Step 1	10101	$(1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)$
Step 2	10101	16 + 0 + 4 + 0 + 1
Step 3	10101	21

How to compute the decimal value of a binary number (shortcut)

Suppose we want to find the decimal value of binary number 10101011.

128	64	32	16	8	4	2	1
1	0	1	0	1	0	1	1

Simply add the decimal values where the corresponding bit is 1.

Hexadecimal Number System

The **hexadecimal number system** has 16 digits.
0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

Examples of hexadecimal numbers:

1. 1F
2. 1AB
3. 100
4. 54

How to compute the decimal value of a hexadecimal number

- Technique
  - Multiply each digit by 16^n, where n is the position of the digit. So, 16^n is the value of the digit.
  - positions of the bit start from 0 on the right
  - Add the results

How to compute the decimal value of a hexadecimal number

Step	Hex Number	Decimal Number
Step 1	19FDE	$(1 \times 16^4) + (9 \times 16^3) + (F \times 16^2) + (D \times 16^1) + (E \times 16^0)$
Step 2	19FDE	$(1 \times 16^4) + (9 \times 16^3) + (15 \times 16^2) +(13 \times 16^1) + (14 \times 16^0)$
Step 3	19FDE	65536+ 36864 + 3840 + 208 + 14
Step 4	19FDE	106462

Decimal Number Systems

- One of the most common number systems.
- Has 10 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Decimal to Binary

- Technique
  - Divide by two, keep track of the remainder

$125_{10} = ?_2$

```
2 | 125
  -----
2 | 62    1
  -----
2 | 31    0
  -----
2 | 15    1
  -----
2 | 7     1
  -----
2 | 3     1
  -----
    1     1
```

$125_{10} = 1111101_2$

Decimal to Hexadecimal

$1234_{10} = ?_{16}$

16	1234	
16	77	2
16	4	13 = D
	0	4

$1234_{10} = 4D2_{16}$

Converting Hexadecimal to Binary and Vice Versa

Hexadecimal to Binary

- We know that the largest digit in hexadecimal is F, and this digit requires 4 bits for binary representation.

- So, find 4-digit binary value of each hexadecimal digit.

- The binary equivalent of the hexadecimal number is simply the concatenation of 4-digit binary values of all digits in order.

Conversion Chart

Decimal	Binary	Hex
00	0000	0
01	0001	1
02	0010	2
03	0011	3
04	0100	4
05	0101	5
06	0110	6
07	0111	7
08	1000	8
09	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Hexadecimal to Binary

Example

$10AF_{16} = ?_2$

1 0 A F

0001 0000 1010 1111

$10AF_{16} = 0001000010101111_2$

Binary to Hexadecimal

- Make Quadruplets of bits from right to left.
- Find the hexadecimal value of each quadruplet.
- The final hexadecimal number is the concatenation of hex digits in order.

Binary to Hexadecimal

Example:

$1010111011_2 = ?_{16}$

10 1011 1011

2 B B

$1010111011_2 = 2BB_{16}$

Representing Signed Binary Numbers in Computer Memory

Signed Magnitude Representation

- Signed binary numbers can be represented in computer memory by using the most significant bit as a signed bit. This representation is also called sign & magnitude representation.

- If the signed bit is on, the number is negative, and if the signed bit is off, the number is positive.

- **What is the total number of values that can be stored using this signed representation?**

Signed Magnitude Representation

Examples:

- $(01101101)_2 = +(109)_{10}$
- $(11101101)_2 = -(109)_{10}$
- $(00101011)_2 = +(43)_{10}$
- $(10101011)_2 = -(43)_{10}$

Signed Magnitude Representation

- Range of values for positive number:
  1 to 127

- Range of values for negative numbers:
  -127 to -1.

Problem:

- Positive zero and negative zero.

- So, The range for positive numbers becomes +0 to +127, and for negative numbers the range becomes -127 to -0.

Converting a Binary Number Represented by Signed Magnitude into Decimal.

- Suppose there are N bits in the given binary number represented by signed magnitude form. The most significant bit will be used to represent the sign.

- Simply find the decimal equivalent of remaining N-1 bits, and put – sign if the sign bit is 1 else put +.

- **Example:** (11010001)

Find the decimal of (1010001) which is 81. Since the sign bit is 1, so the number is -81.

1's Complement

- To represent a negative number in 1's complement, first find its positive binary counterpart.
- Then inverse the bits.
- **For example:** Represent -2 using 1 byte in 1's complement.
  - 2= 00000010
  - -2= 11111101

Note: Applying 1's complement again on the binary of a negative number (represented by 1's complement) will convert it into the positive number.

Binary value	Ones' complement interpretation	Unsigned interpretation
00000000	+0	0
00000001	1	1
⋮	⋮	⋮
01111101	125	125
01111110	126	126
01111111	127	127
10000000	−127	128
10000001	−126	129
10000010	−125	130
⋮	⋮	⋮
11111101	−2	253
11111110	−1	254
11111111	−0	255

Range of 1's Complement Representation

- The range of signed numbers using one's complement is $-(2^{N-1} - 1)$ to $(2^{N-1} - 1)$ with ± 0.

- **For example:** if we use 8 bits, then range is -127_{10} to $+127_{10}$ with zero being either 00000000 (+0) or 11111111 (–0).

Converting a Binary Number Represented by 1's Complement into Decimal.

- Suppose there are N bits in the given binary number represented by 1's complement.
- If the most significant bit is 0, then the number is positive, and simply find its decimal equivalent.
- If the most significant bit is 1, then the number is negative. So, first apply 1's complement (to convert the negative binary number into positive binary number), then find the decimal equivalent and put – sign.
- **Example:** (11010011)

1. Finding 1's complement since msb is 1: 00101100.
2. Now find the decimal equivalent: 44
3. Put negative sign. (-44)

2’s complement

- First find 1’s complement.
- Now add 1 to the 1’s complement.
- It does not have positive and negative zero problem.
- **For example:** Represent -2 using 1 byte in 2’s complement.
  - 2= 00000010
  - 1’s complement = 11111101
  - 2’s complement= 11111101+1= 11111110
  - -2= 11111110

Note:

Applying 2’s complement again on the binary of a negative number (represented by 2’s complement) will convert it into the binary of the same positive number. For example, apply 2’s complement on the binary of -2 (calculated above) and see the result.

Binary value	Two's complement interpretation	Unsigned interpretation
00000000	0	0
00000001	1	1
⋮	⋮	⋮
01111110	126	126
01111111	127	127
10000000	−128	128
10000001	−127	129
10000010	−126	130
⋮	⋮	⋮
11111110	−2	254
11111111	−1	255

Range of 2’s Complement Representation

- The range of signed numbers using two's complement is $-(2^{N-1})$ to $(2^{N-1} - 1)$. There is only one representation of 0 in this representation.

- **For example:** if we use 8 bits, then range is -128_{10} to $+127_{10}$ with zero being 00000000.

Converting a Binary Number Represented by 2’s Complement into Decimal.

- Suppose there are N bits in the given binary number represented by 2’s complement.

- If the most significant bit is 0, then number is positive and simply find its decimal equivalent.

- If the most significant bit is 1, then number is negative. So, first apply 2’s complement (to convert it into positive binary number), then find the decimal equivalent of positive binary number and put – sign.

- **Example:** (11010011)

1. Finding 2’s complement since msb is 1: 00101101.
2. Now find the decimal equivalent: 45
3. Put negative sign. (-45)

Data Representation in Computers

What is Data

Data is simply

any **numbers**, **letters** or **symbols** that can be entered into a computer system.

Data values **don’t have any meaning** unless we put them into **context**

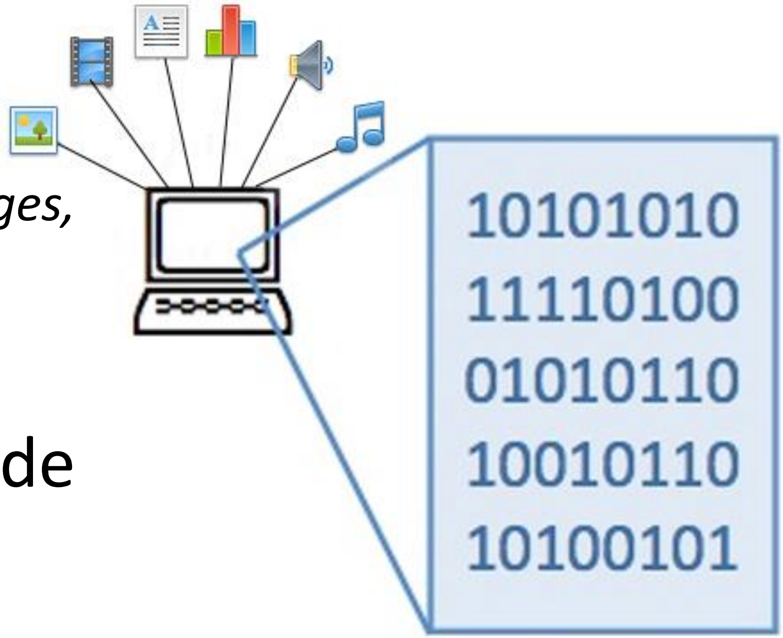
Information = Data + Context

Computers Process Data

- Computers are used to process all types of information in a broad spectrum fields.
  - **Numeric data** consisting of Integers and real numbers are used in programs calculating payroll. We typically perform **arithmetic operations** on numeric data.
  - **Strings** of **alphabets** and numbers (**Alphanumeric Data**) are processed in customer record keeping systems.
  - **Multimedia** content including images, sound and text are frequently used in a large collection of application areas.
  - **Signals** representing various types of information like temperature, pressure, presence or absence of objects etc. are processed by computers in Robotics, IoT, monitoring and control applications.

How is Data Actually Stored in Computer

Everything that is stored and processed inside a computer *(all data, information, instructions, files, images, etc.)* is stored as **Binary Numbers**

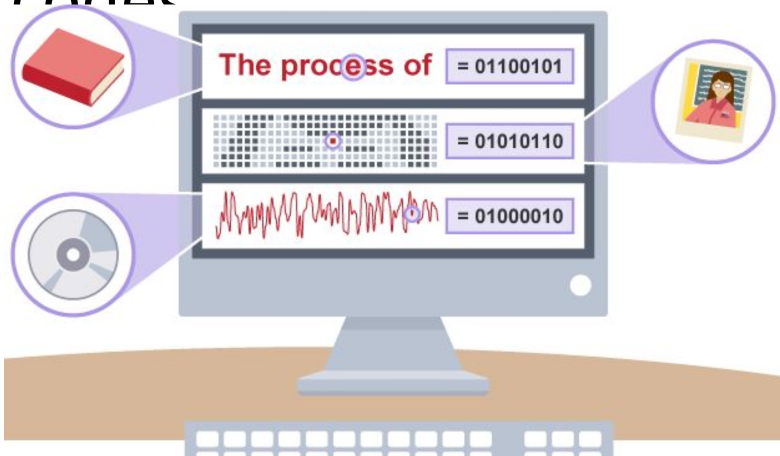


Digital computers have been made such that all data and instructions(program) for processing must be stored in computers memory before processing.

How to store *text* and *pictures* as numbers?

- The solution is to use **numeric codes**:
  - Different **letters** in a text document are given different numeric codes
  - Different **pixels** (colored dots) in an image are given different numeric codes
  - Different **sounds** in a music file are given different numeric codes

Everything is numbers!



Memory Measuring Units

(As viewed by computer scientists)

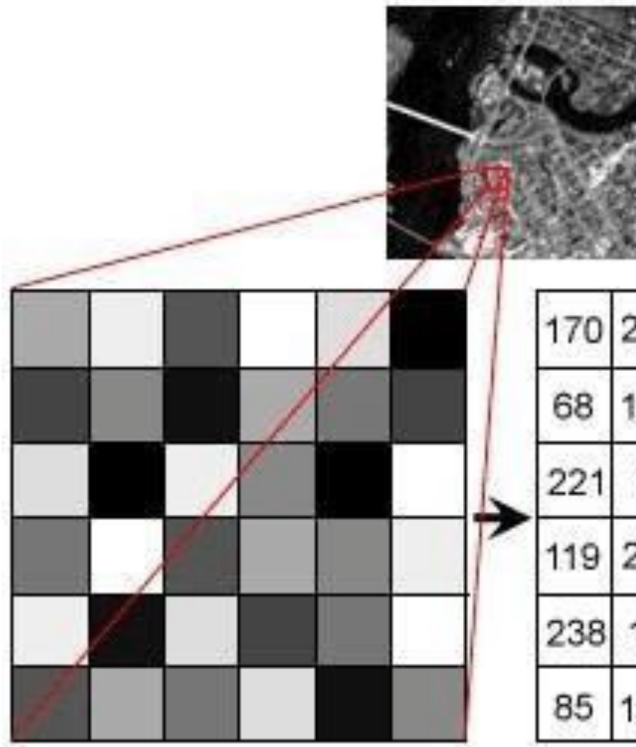
- Bits can be grouped together to make them easier to work with. A group of 8 bits is called a byte.

UNIT	ABBREVIATION	STORAGE
Bit	B	Binary Digit, Single 1 or 0
Nibble	-	4 bits
Byte/Octet	B	8 bits
Kilobyte	KB	1024 bytes
Megabyte	MB	1024 KB
Gigabyte	GB	1024 MB
Terabyte	TB	1024 GB
Petabyte	PB	1024 TB
Exabyte	EB	1024 PB
Zettabyte	ZB	1024 EB
Yottabyte	YB	1024 ZB

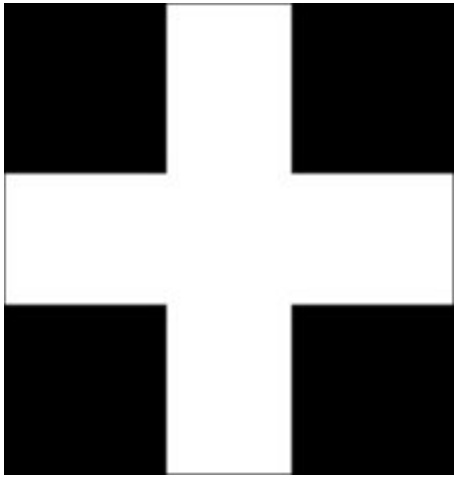
Storage units (www.byte-notes.com

Most computers can process millions of bits every second. A hard drive's storage capacity is measured in gigabytes or terabytes. RAM is often measured in megabytes or gigabytes.

How Images are Stored in Memory



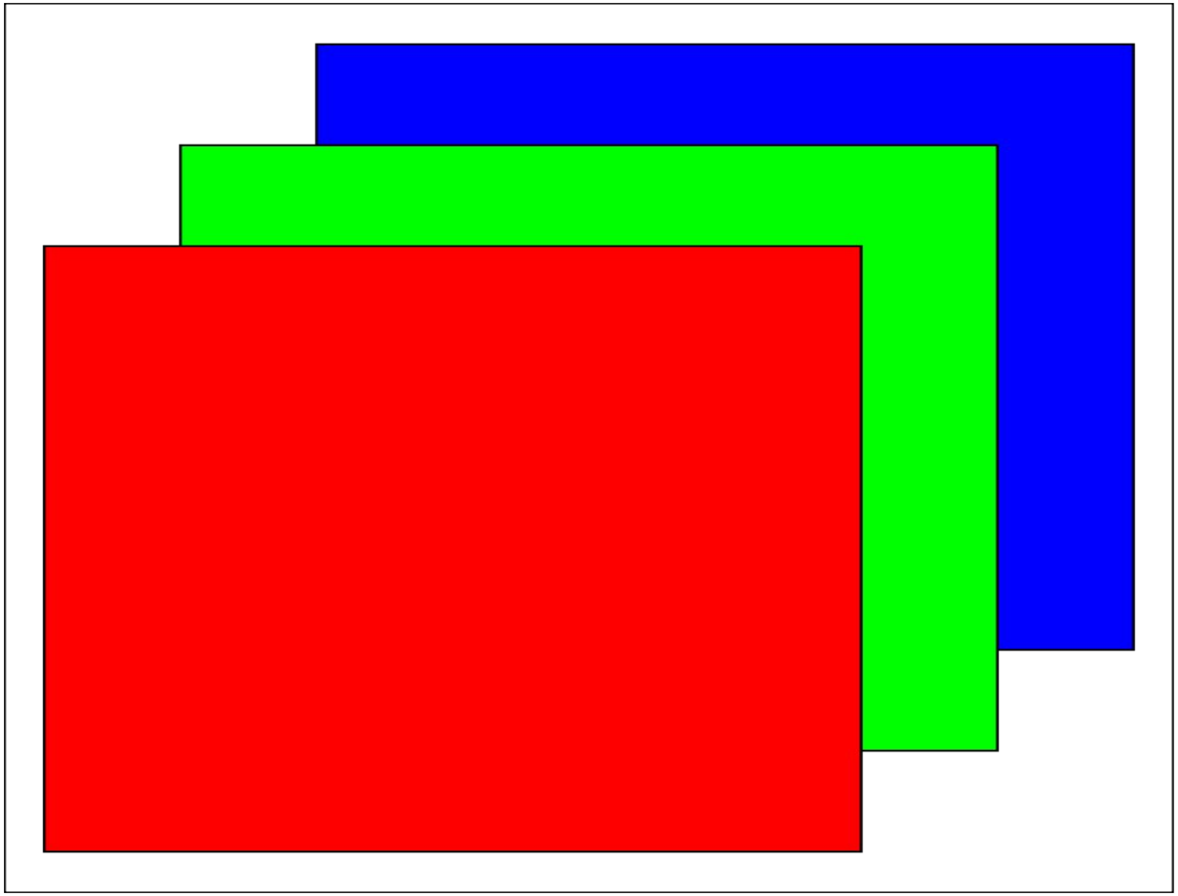
170	238	85	255	221	0
68	136	17	170	119	68
221	0	238	136	0	255
119	255	85	170	136	238
238	17	221	68	119	255
85	170	119	221	17	136



0	0	255	0	0
0	0	255	0	0
0	0	255	0	0
255	255	255	255	255
0	0	255	0	0
0	0	255	0	0
0	0	255	0	0

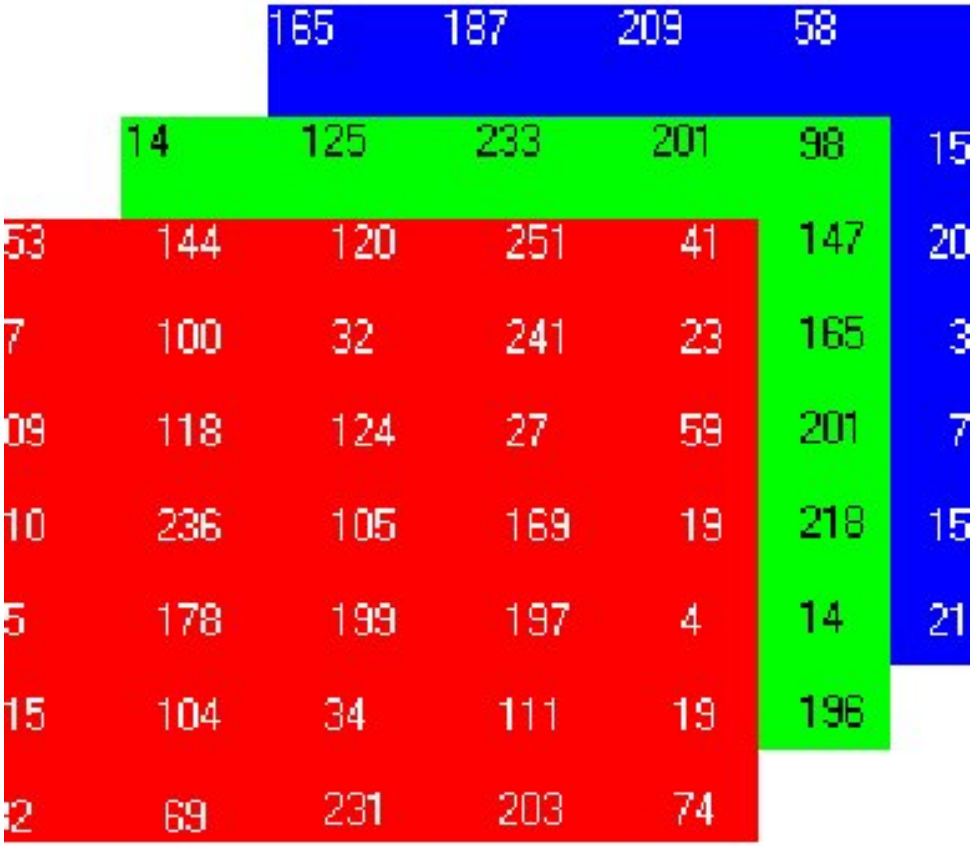
How To Represent Color Images in Memory

RGB



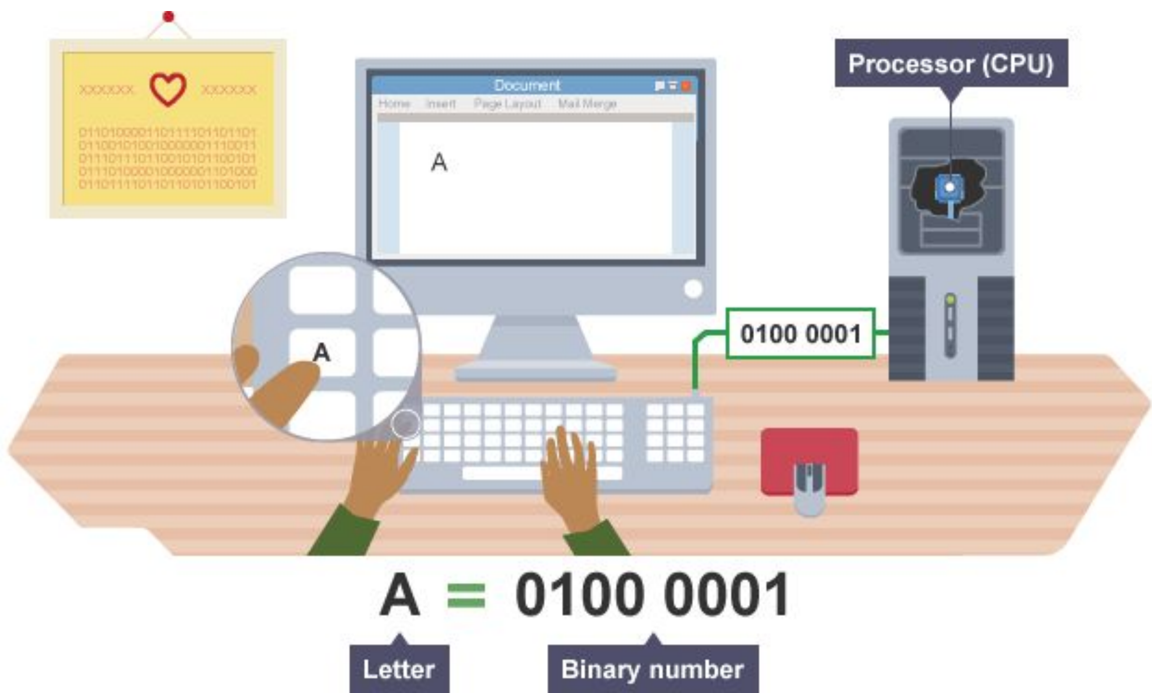
Three Separate Matrices for R, G and B, respectively.

How To Represent Color Images in Memory



How can we represent a character?

- IDEA.
  - Assign numeric codes to characters and represent each character in a Byte using it’s numeric code.
  - Can we assign numeric codes of our choice to each character?. What might be a problem with this approach?



How can we represent a character?

- **IDEA**
- Create a Standard coding scheme so that information can be easily shared between devices from different vendors.
- Standard Codes
  - ASCII (American Standard Code for Information Interchange)
  - Unicode
  - Unicode Transformation Format(UTF) UTF-8, UTF-16
  - ANSI Character Set

ASCII Character Encoding

Letter Number Punctuation Symbol Other undefined

ASCII (1977/1986)

	_0	_1	_2	_3	_4	_5	_6	_7	_8	_9	_A	_B	_C	_D	_E	_F
0_	NUL 0000 0	SOH 0001 1	STX 0002 2	ETX 0003 3	EOT 0004 4	ENQ 0005 5	ACK 0006 6	BEL 0007 7	BS 0008 8	HT 0009 9	LF 000A 10	VT 000B 11	FF 000C 12	CR 000D 13	SO 000E 14	SI 000F 15
1_	DLE 0010 16	DC1 0011 17	DC2 0012 18	DC3 0013 19	DC4 0014 20	NAK 0015 21	SYN 0016 22	ETB 0017 23	CAN 0018 24	EM 0019 25	SUB 001A 26	ESC 001B 27	FS 001C 28	GS 001D 29	RS 001E 30	US 001F 31
2_	SP 0020 32	! 0021 33	" 0022 34	# 0023 35	$ 0024 36	% 0025 37	& 0026 38	' 0027 39	(0028 40	) 0029 41	* 002A 42	+ 002B 43	, 002C 44	- 002D 45	. 002E 46	/ 002F 47
3_	0 0030 48	1 0031 49	2 0032 50	3 0033 51	4 0034 52	5 0035 53	6 0036 54	7 0037 55	8 0038 56	9 0039 57	: 003A 58	; 003B 59	< 003C 60	= 003D 61	> 003E 62	? 003F 63
4_	@ 0040 64	A 0041 65	B 0042 66	C 0043 67	D 0044 68	E 0045 69	F 0046 70	G 0047 71	H 0048 72	I 0049 73	J 004A 74	K 004B 75	L 004C 76	M 004D 77	N 004E 78	O 004F 79
5_	P 0050 80	Q 0051 81	R 0052 82	S 0053 83	T 0054 84	U 0055 85	V 0056 86	W 0057 87	X 0058 88	Y 0059 89	Z 005A 90	[005B 91	\ 005C 92	] 005D 93	^ 005E 94	_ 005F 95
6_	` 0060 96	a 0061 97	b 0062 98	c 0063 99	d 0064 100	e 0065 101	f 0066 102	g 0067 103	h 0068 104	i 0069 105	j 006A 106	k 006B 107	l 006C 108	m 006D 109	n 006E 110	o 006F 111
7_	p 0070 112	q 0071 113	r 0072 114	s 0073 115	t 0074 116	u 0075 117	v 0076 118	w 0077 119	x 0078 120	y 0079 121	z 007A 122	{ 007B 123	| 007C 124	} 007D 125	~ 007E 126	DEL 007F 127

ASCII to Binary 0s and 1s

Converting the text "hope" into binary

Characters:	h	o	p	e
ASCII Values:	104	111	112	101
Binary Values:	01101000	01101111	01110000	01100101
Bits:	8	8	8	8

Recommended

- https://www.youtube.com/watch?v=1GSjbWt0c9M

- https://www.khanacademy.org/computing/computer-science/how-computers-work2/v/khan-academy-and-codeorg-introducing-how-computers-work

- https://www.youtube.com/watch?v=ptzGI9VaZmQ

Activity