9 Important Properties of Mean and Variance of Random Variables

Property 1: $E(X + Y) = E(X) + E(Y)$. (X and Y are random variables)

Property 2: $E(X_1 + X_2 + \dots + X_n) = E(X_1) + E(X_2) + \dots + E(X_n) = \Sigma_i E(X_i)$.

Property 3: $E(XY) = E(X) E(Y)$. Here, X and Y must be independent.

Property 4: $E[aX] = a E[X]$ and $E[X + a] = E[X] + a$, where a is a constant

Property 5: For any random variable, $X > 0$, $E(X) > 0$.

Property 6: $E(Y) \geq E(X)$ if the random variables X and Y are such that $Y \geq X$.

Property 7: The variance of a constant is 0.

Property 8: $V[aX + b] = a^2 V(X)$, where a and b are constants, X is random variable.

Property 9: $V(a_1X_1 + a_2 X_2 + \dots + a_nX_n) = a_1^2 V(X_1) + a_2^2 V(X_2) + \dots + a_n^2 V(X_n)$.

If $X \geq 0$, then $E(X) \geq 0$

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5. Suppose we have two independent random variable one with parameters $E[X] = 4$ and $\text{Var}(X) = 3$, and the other with parameters $E[Y] = 9$ and $\text{Var}(Y) = 6$.

- a. What is $E[X + Y + 2]$?
- b. What is $E[3X + 2Y - 5]$?
- c. What is $\text{Var}(3X + 2)$?
- d. What is $\text{Var}(2(X + Y + 1))$?

$E[X]+E[Y]+2=4+9+2=15$

$3E[X]+2E[Y]-5=3(4)+2(9)-5=12+18-5=25$

Where: $Var(X)=3$,therefore: :$9Var(X)=27$

$4Var(X+Y+1) =4(Var(X)+Var(Y))=4(3+6)=4(9)=36$

Let X be a random variable and $Y = 2X + 1$. What is the variance of Y if variance of X is 5 ?

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Example 6.14

Determine the mean and variance of a discrete random variable, given its distribution as follows:

$X = x$	1	2	3	4	5	6
$F_x(x)$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	1

Solution

From the given data, you first calculate the probability distribution of the random variable. Then using it you calculate mean and variance.

X	$p(x)$
1	$F(1) = \frac{1}{6}$
2	$F(2) - F(1) = \frac{2}{6} - \frac{1}{6} = \frac{1}{6}$
3	$F(3) - F(2) = \frac{3}{6} - \frac{2}{6} = \frac{1}{6}$
4	$F(4) - F(3) = \frac{4}{6} - \frac{3}{6} = \frac{1}{6}$
5	$F(5) - F(4) = \frac{5}{6} - \frac{4}{6} = \frac{1}{6}$
6	$F(6) - F(5) = 1 - \frac{5}{6} = \frac{1}{6}$

The probability mass function is

$X = x$	1	2	3	4	5	6
$P(x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Mean of the random variable $X = E(X) = \sum_x x\, P_X(x)$

$$= \left(1\times\frac{1}{6}\right)+\left(2\times\frac{1}{6}\right)+\left(3\times\frac{1}{6}\right)+\left(4\times\frac{1}{6}\right)+\left(5\times\frac{1}{6}\right)+\left(6\times\frac{1}{6}\right)$$

$$= \frac{1}{6}(1+2+3+4+5+6)$$

$$= \frac{7}{2}$$

$$E\left(X^2\right) = \sum_x x^2\, P_X(x)$$

$$= \left(1^2\times\frac{1}{6}\right)+\left(2^2\times\frac{1}{6}\right)+\left(3^2\times\frac{1}{6}\right)+\left(4^2\times\frac{1}{6}\right)+\left(5^2\times\frac{1}{6}\right)+\left(6^2\times\frac{1}{6}\right)$$

$$= \frac{1}{6}\left(1^2+2^2+3^2+4^2+5^2+6^2\right)$$

$$= \frac{91}{6}$$

Variance of the Random Variable $X = V(X) = E\left(X^2\right) - [E(X)]^2$

$$= \frac{91}{6} - \left(\frac{7}{2}\right)^2$$

$$= \frac{35}{12}$$

Example 6.18

Suppose the probability mass function of the discrete random variable is

$X = x$	0	1	2	3
$p(x)$	0.2	0.1	0.4	0.3

What is the value of $E(3X + 2X^2)$?

Solution

$$E(X) = \sum_x x P_X(x)$$

$$= (0 \times 0.2) + (1 \times 0.1) + (2 \times 0.4) + (3 \times 0.3)$$

$$= 1.8$$

$$E(X^2) = \sum_x x^2 P_X(x)$$

$$= (0^2 \times 0.2) + (1^2 \times 0.1) + (2^2 \times 0.4) + (3^2 \times 0.3)$$

$$= 4.4$$

$$E(3X + 2X^2) = 3E(X) + 2E(X^2)$$

$$= (3 \times 1.8) + (2 \times 4.4)$$

$$= 14.2$$

Example 6.22

A commuter train arrives punctually at a station every 25 minutes. Each morning, a commuter leaves his house and casually walks to the train station. Let X denote the amount of time, in minutes, that commuter waits for the train from the time he reaches the train station. It is known that the probability density function of X is

$$f(x) = \begin{cases} \frac{1}{25}, & for\, 0 < x < 25 \\ 0, & otherwise. \end{cases}$$

Obtain and interpret the expected value of the random variable X.

Solution:

Expected value of the random variable is

$$E(X) = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$= \int_0^{25} x \frac{1}{25} dx$$

$$= \frac{1}{25} \int_0^{25} x\, dx$$

$$= \frac{1}{25} \left[\frac{x^2}{2} \right]_0^{25}$$

$$= 12.5$$

Therefore, the expected waiting time of the commuter is 12.5 minutes.

Consider a random variable X with probability density function

$$f(x)=\begin{cases}4x^3, if\ 0<x<1\\ 0,\ otherwise\end{cases}$$

Find E(X) and V(X).

Solution

We know that,

$$\begin{aligned}E(X)&=\int_{-\infty}^{\infty} x\,f(x)dx\\ &=\int_0^1 x\,4x^3\,dx\\ &=4\left[\frac{x^5}{5}\right]_0^1\\ E(X)&=\frac{4}{5}\end{aligned}$$

$$\begin{aligned}E\left(X^2\right)&=\int_{-\infty}^{\infty} x^2 f(x)dx\\ &=\int_0^1 x^2\,4x^3\,dx\\ &=4\left[\frac{x^6}{6}\right]_0^1\\ &=\frac{4}{6}\\ V(X)&=E\left(X^2\right)-\left[E(X)\right]^2\\ &=\frac{4}{6}-\left[\frac{4}{5}\right]^2\\ &=\frac{2}{75}\end{aligned}$$

Definition 6.11

If X is a random variable, then the r^{th} moment of X, usually denoted by μ_r, is defined as

$$\mu'_r = E\left(X^r\right)=\begin{cases}\sum_x x^r p(x), & \text{for discrete random variable}\\ \int_{-\infty}^{\infty} x^r f(x)dx, & \text{for continuous random variable}\end{cases}$$

provided the expectation exists.

Given $E(X) = 5$ and $E(Y) = -2$, then $E(X - Y)$ is

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A formula or equation used to represent the probability distribution of a continuous random variable is called

probability density function

If c is a constant, then E (c) is

C

$E[X - E(X)]$ is equal to

0

$E[X - E(X)]^2$ is

V (X)

15. If the random variable takes negative values, then the negative values will have

(a) positive probabilities

(b) negative probabilities

(c) constant probabilities

(d) difficult to tell

$\int_{-\infty}^{\infty} f(x)dx$ is always equal to______________________1

The probability function of a random variable is defined as

$X = x$	-1	-2	0	1	2
$P(x)$	k	2k	3k	4k	5k

Then k is equal to

(c) 1/15

E(aX +bY)