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Assignment #1

Exercise 1.1

Q#9: (b) $(3,-1,1)$

$$2x_1 - 4x_2 - x_3 = 1$$
$$x_1 - 3x_2 + x_3 = 1 \quad \text{—} \ (1)$$
$$3x_1 - 5x_2 - 3x_3 = 1$$

put in eq (1)

$$x_1 - 3x_2 + x_3 = 1$$
$$(3) - 3(-1) + (1) = 1$$
$$3 + 3 + 1 = 1$$
$$7 \neq 1$$

Hence, the given 3-tuple is not a solution of linear system.

(e) $(17,7,5)$ put in eq (1)

$$17 - 3(7) + 5 = 1$$
$$17 - 21 + 5 = 1$$
$$1 = 1$$

Hence, the given 3-tuple is a solution of linear system.

Q#11: (a) $3x - 2y = 4$ — (1)
$6x - 4y = 9$ — (2)

multiple 2 by eq (1)

$$6x - 4y = 8$$
$$\ominus \ \cancel{6x} \oplus \cancel{4y} \ominus 9$$
$$0 = 1$$

The linear system has zero point of intersection and no solution.

These lines are parallel and distinct.

(b) $2x - 4y = 1 \rightarrow 2(2x - 4y) = 2(1)$
$4x - 8y = 2$

$$\begin{array}{r} \cancel{4x} - \cancel{8y} = \cancel{2} \\ \ominus\ \cancel{4x} - \cancel{8y} = \cancel{2} \\ \hline 0 = 0 \end{array}$$

This linear equation has infinetly many points of intersection

$2x - 4y = 1$
$2x = 1 + 4y$
$x = \frac{1}{2} + \frac{4}{2} y$

$x = \frac{1}{2} + 2y$, $y = t$

$\boxed{x = \frac{1}{2} + 2t} \longrightarrow$ parametric equation.

(C)
$$\begin{array}{r} \cancel{x} - 2y = 0 \\ \ominus\ \cancel{x} \oplus 4y = 8 \\ \hline 2y = -8 \\ y = -4 \end{array}$$

$x - 2(-4) = 0$
$x + 8 = 0$
$x = -8$

$(x, y) = (-8, -4)$

This linear equation has a unique solution. And one point of intersection.

CamScanner

Q#13: (a) $7x - 5y = 3$

$7x = 3 + 5y$

$x = \frac{3}{7} + \frac{5}{7}y$ $t = y$

$\boxed{x = \frac{3}{7} + \frac{5}{7}t}$ Parametric equation with 't' as a parameter.

(c) $-8x_1 + 2x_2 - 5x_3 + 6x_4 = 1$

$-8x_1 = 1 - 2x_2 + 5x_3 - 6x_4$

$x_1 = \frac{-1}{8} + \frac{1}{4}x_2 - \frac{5}{8}x_3 + \frac{3}{4}x_4$

$x_2 = r$

$x_3 = s$

$x_4 = t$

$\boxed{x_1 = \frac{-1}{8} + \frac{1}{4}r - \frac{5}{8}s + \frac{3}{4}t}$ parametric equation with 'r', 's' and 't' as parameters.

Q#17: (b) $\begin{bmatrix} 0 & -1 & -5 & 0 \\ 2 & -9 & 3 & 2 \\ 1 & 4 & -3 & 3 \end{bmatrix}$ Add $R_1 + R_3$

$$\begin{bmatrix} 1 & 3 & -8 & 3 \\ 2 & -9 & 3 & 2 \\ 1 & 4 & -3 & 3 \end{bmatrix}$$

Q#21: $y = ax^2 + bx + c$ —— ①

(x_1, y_1)
(x_2, y_2)
(x_3, y_3)

$$\begin{bmatrix} x_1^2 & x_1 & 1 & y_1 \\ x_2^2 & x_2 & 1 & y_2 \\ x_3^2 & x_3 & 1 & y_3 \end{bmatrix}$$

subtitute the coordinates of points in eq ①

$$y_1 = ax_1^2 + x_1 b + c$$
$$y_2 = ax_2^2 + x_2 b + c$$
$$y_3 = ax_3^2 + x_3 b + c$$

This is a linear system in unknown variables a, b, c.

It's augmented matrix is

$$\begin{bmatrix} x_1^2 & x_1 & 1 & y_1 \\ x_2^2 & x_2 & 1 & y_2 \\ x_3^2 & x_3 & 1 & y_3 \end{bmatrix}$$

Exercise 1.2

Q#3 (c)

$$\begin{bmatrix} 1 & 7 & -2 & 0 & -8 & -3 \\ 0 & 0 & 1 & 1 & 6 & 5 \\ 0 & 0 & 0 & 1 & 3 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The linear system,

$$x_1 + 7x_2 - 2x_3 - 8x_5 = -3$$
$$x_3 + x_4 + 6x_5 = 5$$
$$x_4 + 3x_5 = 9$$
$$0 = 0$$

these can be written as,

$$x_4 = 9 - 3x_5 \qquad \text{let } \boxed{x_5 = t}$$
$$x_3 = 5 - x_4 - 6x_5$$
$$x_1 = -3 - 7x_2 + 2x_3 + 8x_5$$

$$\boxed{x_4 = 9 - 3t}$$
$$x_3 = 5 - (9 - 3t) - 6t$$
$$= 5 - 9 + 3t - 6t$$
$$\boxed{x_3 = -4 - 3t}$$

$$x_1 = -3 - 7s + 2(-4 - 3t) + 8t \qquad \text{let } \boxed{x_2 = s}$$
$$= -3 - 7s + -8 - 6t + 8t$$
$$\boxed{x_1 = -11 - 7s + 2t}$$

where s and t are arbitrary values.

Q#4: (c)

$$\begin{bmatrix} 1 & -6 & 0 & 0 & 3 & -2 \\ 0 & 0 & 1 & 0 & 4 & 7 \\ 0 & 0 & 0 & 1 & 5 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x_1 - 6x_2 + 3x_5 = -2$$
$$x_3 + 4x_5 = 7$$
$$x_4 + 5x_5 = 8$$
$$0 = 0$$

$$x_4 = 8 - 5x_5$$ let $\boxed{x_5 = t}$
$$x_3 = 7 - 4x_5$$
$$x_1 = -2 + 6x_2 - 3x_5.$$

$$x_4 = 8 - 5t$$ let $\boxed{x_2 = s}$
$$x_3 = 7 - 4t$$
$$x_1 = -2 + 6s - 3t$$

where s and t are arbitrary values

Q#7: by Guassian elimination (row-echelon form)

$$\begin{aligned} x - y + 2z - w &= -1 \\ 2x + y - 2z - 2w &= -2 \\ -x + 2y - 4z + w &= 1 \\ 3x \qquad\qquad - 3w &= -3 \end{aligned}$$

$$\begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 2 & 1 & -2 & -2 & -2 \\ -1 & 2 & -4 & 1 & 1 \\ 3 & 0 & 0 & -3 & -3 \end{bmatrix}$$

$R_2 - 2R_1$
$R_3 + R_1$
$R_4 - 3R_1$

$$\begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & +3 & 6 & 0 & 0 \end{bmatrix}$$

R_{23}

$$\begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 3 & 6 & 0 & 0 \end{bmatrix}$$

$R_3 - 3R_2$
$R_4 - 3R_2$

$$\begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$x = -1 + y - 2z + w$
$y = 2z$

$$\begin{aligned} x - y + 2z - w &= -1 \\ y - 2z &= 0 \\ 0 &= 0 \\ 0 &= 0 \end{aligned}$$

$\Rightarrow \boxed{y = 2s}$ $\boxed{\begin{matrix} z = s \\ w = t \end{matrix}}$

$\Rightarrow x = -1 + y - 2s + w$
$x = -1 + \cancel{2s} - \cancel{2s} + t$

where s and t have arbitrary values and are ...

$\boxed{x = -1 + t}$

Q#9: Guass- Jordan (reduced row echelon form)

$$x_1 + x_2 + 2x_3 = 8$$
$$-x_1 - 2x_2 + 3x_3 = 1$$
$$3x_1 - 7x_2 + 4x_3 = 10$$

$$\begin{bmatrix} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{bmatrix} \quad \begin{matrix} R_2 + R_1 \\ R_3 - 3R_1 \end{matrix}$$

$$\begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & 3 & 5 & 9 \\ 0 & -10 & -2 & -14 \end{bmatrix} \quad R_2/3$$

$$\begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & 1 & 5/3 & 9 \\ 0 & -10 & -2 & -14 \end{bmatrix} \quad \begin{matrix} R_1 - R_2 \\ R_3 + 10R_2 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 1/3 & 5 \\ 0 & 1 & 5/3 & 3 \\ 0 & 0 & 44/3 & 16 \end{bmatrix} \quad R_3 \times \frac{3}{44}$$

$$\begin{bmatrix} 1 & 0 & 1/3 & 5 \\ 0 & 1 & 5/3 & 3 \\ 0 & 0 & 1 & 12/11 \end{bmatrix}$$

$$\left[\begin{array}{cccc} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{array}\right]$$

$$\left[\begin{array}{cccc} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & -10 & -2 & -14 \end{array}\right] \quad \begin{array}{l} R_2+R_1 \\ R_3-3R_1 \end{array}$$

$R_2/-1$

$$\left[\begin{array}{cccc} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & -10 & -2 & -14 \end{array}\right] \quad \begin{array}{l} R_1-R_2 \\ R_3+10R_2 \end{array}$$

$$\left[\begin{array}{cccc} 1 & 0 & -7 & -17 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & -52 & -104 \end{array}\right] \quad R_3/-52$$

$$\left[\begin{array}{cccc} 1 & 0 & -7 & -17 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & 1 & 2 \end{array}\right] \quad \begin{array}{l} R_1+7R_3 \\ R_2+5R_3 \end{array}$$

$$\left[\begin{array}{cccc} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right] \quad \begin{array}{l} x_1=-3 \\ x_2=1 \\ x_3=2 \end{array}$$

Solution Set $= \{(x_1, x_2, x_3) = (-3, 1, 2)\}$

Q#13:

$$2x_1 - 3x_2 + 4x_3 - x_4 = 0 \quad \text{—①}$$
$$7x_1 + x_2 + 8x_3 + 9x_4 = 0 \quad \text{—②}$$
$$2x_1 + 8x_2 + x_3 - x_4 = 0 \quad \text{—③}$$

no of equations = 3
no of unknowns = 4

4>3, thus, this homogeneous linear system has infinitely many solutions

Q#21:
$$2I_1 - I_2 + 3I_3 + 4I_4 = 9$$
$$I_1 - 2I_3 + 7I_4 = 11$$
$$3I_1 - 3I_2 + I_3 + 5I_4 = 8$$
$$2I_1 + I_2 + 4I_3 + 4I_4 = 10$$

$$\begin{bmatrix} 2 & -1 & 3 & 4 & 9 \\ 1 & 0 & -2 & 7 & 11 \\ 3 & -3 & 1 & 5 & 8 \\ 2 & 1 & 4 & 4 & 10 \end{bmatrix}$$

R_{12}

$$\begin{bmatrix} 1 & 0 & -2 & 7 & 11 \\ 2 & -1 & 3 & 4 & 9 \\ 3 & -3 & 1 & 5 & 8 \\ 2 & 1 & 4 & 4 & 10 \end{bmatrix} \quad \begin{matrix} \\ R_2 - 2R_1 \\ R_3 - 3R_1 \\ R_4 - 2R_1 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & -2 & 7 & 11 \\ 0 & -1 & 7 & -10 & -13 \\ 0 & -3 & 7 & -16 & -25 \\ 0 & 1 & 8 & -10 & -12 \end{bmatrix} \quad \frac{R_2}{-1}$$

$$\begin{bmatrix} 1 & 0 & -2 & 7 & 11 \\ 0 & 1 & -7 & 10 & 13 \\ 0 & -3 & 7 & -16 & -25 \\ 0 & 1 & 8 & -10 & -12 \end{bmatrix} \quad \begin{matrix} R_3 + 3R_2 \\ R_4 - R_2 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & -2 & 7 & 11 \\ 0 & 1 & -7 & 10 & 13 \\ 0 & 0 & -14 & 14 & 14 \\ 0 & 0 & 15 & -20 & -25 \end{bmatrix} \quad R_3/-14$$

$$\begin{bmatrix} 1 & 0 & -2 & 7 & 11 \\ 0 & 1 & -7 & 10 & 13 \\ 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & 15 & -20 & -25 \end{bmatrix} \quad \begin{matrix} R_1 + 2R_3 \\ R_2 + 7R_3 \\ R_4 - 15R_3 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 5 & 9 \\ 0 & 1 & 0 & 3 & 6 \\ 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & -5 & -10 \end{bmatrix} \quad R_4/-5$$

$$\begin{bmatrix} 1 & 0 & 0 & 5 & 9 \\ 0 & 1 & 0 & 3 & 6 \\ 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} \quad \begin{matrix} R_1 - 5R_4 \\ R_2 - 3R_4 \\ R_3 + R_4 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} \quad \boxed{\begin{matrix} I_1 = -1 \\ I_2 = 0 \\ I_3 = 1 \\ I_4 = 2 \end{matrix}}$$

Q#25:

$$\begin{aligned} x + 2y - 3z &= 4 \\ 3x - y + 5z &= 2 \\ 4x + y + (a^2-14)z &= a+2 \end{aligned}$$

$$\left[\begin{array}{cccc} 1 & 2 & -3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & a^2-14 & a+2 \end{array}\right] \quad \begin{array}{l} R_2 - 3R_1 \\ R_3 - 4R_1 \end{array}$$

$$\left[\begin{array}{cccc} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & -7 & a^2-2 & a-14 \end{array}\right] \quad \frac{R_2}{-7}$$

$$\left[\begin{array}{cccc} 1 & 2 & -3 & 4 \\ 0 & 1 & -2 & 10/7 \\ 0 & -7 & a^2-2 & a-14 \end{array}\right] \quad \begin{array}{l} R_1 - 2R_2 \\ R_3 + 7R_2 \end{array}$$

$$\left[\begin{array}{cccc} 1 & 0 & 1/7 & 8/7 \\ 0 & 1 & 2 & 10/7 \\ 0 & 0 & a^2-16 & a-4 \end{array}\right]$$

when $\boxed{a=-4}$ then, third row becomes.

$0=-8$ no solution.

Thus, when $\boxed{a=4}$, third row becomes $0=0$, infinitely many solutions.

for all remaining values of a (i.e $a \neq 4$, $a \neq -4$) the system has one unique solution.

Exercise 1.3

Q#5: (g) $(DA)^T$

$$D = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} \qquad A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}$$

DA

$3 \times \boxed{3 = 3} \times 2$

$$= \begin{bmatrix} 3+(-5)+2 & 0+10+2 \\ -3+0+1 & 0+0+1 \\ 9+(-2)+4 & 0+4+4 \end{bmatrix}^T$$

$$= \begin{bmatrix} 0 & 12 \\ -2 & 1 \\ 11 & 8 \end{bmatrix}^T$$

$$= \begin{bmatrix} 0 & -2 & 11 \\ 12 & 1 & 8 \end{bmatrix} \text{ Ans.}$$

(i) $tr(DD^T)$ → trace.

$$= tr \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}^T$$

$$= tr \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & -1 & 3 \\ 5 & 0 & 2 \\ 2 & 1 & 4 \end{bmatrix}$$

$$= tr\begin{bmatrix} 1+25+4 & -1+0+2 & 3+10+8 \\ -1+0+2 & 1+0+1 & -3+0+4 \\ 3+10+8 & -3+0+4 & 9+4+16 \end{bmatrix}$$

$$= tr\begin{bmatrix} 30 & 1 & 21 \\ 1 & 2 & 1 \\ 21 & 1 & 29 \end{bmatrix}$$

$$= 30+2+29$$

$$= \boxed{61} \text{ Ans.}$$

Q#7:

$$A = \begin{bmatrix} 3 & -2 & 7 \\ 6 & 5 & 4 \\ 0 & 4 & 9 \end{bmatrix}$$

$$B = \begin{bmatrix} 6 & -2 & 4 \\ 0 & 1 & 3 \\ 7 & 7 & 5 \end{bmatrix}$$

(a) first row of $AB = [R_1A]B$

$$= \begin{bmatrix} 3 & -2 & 7 \end{bmatrix}\begin{bmatrix} 6 & -2 & 4 \\ 0 & 1 & 3 \\ 7 & 7 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 67 & 41 & 41 \end{bmatrix}$$

(b) third row of $AB = [R_3A][B]$

$$= \begin{bmatrix} 0 & 4 & 9 \end{bmatrix}\begin{bmatrix} 6 & -2 & 4 \\ 0 & 1 & 3 \\ 7 & 7 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 63 & 67 & 57 \end{bmatrix}$$

(c) Second column of $AB = A[C_2B]$

$$= \begin{bmatrix} 3 & -2 & 7 \\ 6 & 5 & 4 \\ 0 & 4 & 9 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} 41 \\ 21 \\ 67 \end{bmatrix}$$

(d) first column of $BA = B[C_1A]$

$$= \begin{bmatrix} 6 & -2 & 4 \\ 0 & 1 & 3 \\ 7 & 7 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \\ 63 \end{bmatrix}$$

(e) third row of $AA = [R_3A]A$

$$= [0 \; 4 \; 9] \begin{bmatrix} 3 & -2 & 7 \\ 6 & 5 & 4 \\ 0 & 4 & 9 \end{bmatrix}$$

$$= [24 \quad 56 \quad 97]$$

(f) third column of $AA = A[C_3A]$

$$= \begin{bmatrix} 3 & -2 & 7 \\ 6 & 5 & 4 \\ 0 & 4 & 9 \end{bmatrix} \begin{bmatrix} 7 \\ 4 \\ 9 \end{bmatrix} = \begin{bmatrix} 76 \\ 98 \\ 97 \end{bmatrix}$$

Q#8:

(a) first column of AB = A [$C_1 B$]

$$= \begin{bmatrix} 3 & -2 & 7 \\ 6 & 5 & 4 \\ 0 & 4 & 9 \end{bmatrix} \begin{bmatrix} 6 \\ 0 \\ 7 \end{bmatrix} = \begin{bmatrix} 67 \\ 64 \\ 63 \end{bmatrix}$$

(b) C_3 BB = B[$C_3 B$]

$$= \begin{bmatrix} 6 & -2 & 4 \\ 0 & 1 & 3 \\ 7 & 7 & 5 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 38 \\ 18 \\ 74 \end{bmatrix}$$

(c) R_2 of BB = [$R_2 B$] B

$$= \begin{bmatrix} 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} 6 & -2 & 4 \\ 0 & 1 & 3 \\ 7 & 7 & 5 \end{bmatrix} = \begin{bmatrix} 21 & 22 & 18 \end{bmatrix}$$

(d) C_1 of AA = A[$C_1 A$]

$$= \begin{bmatrix} 3 & -2 & 7 \\ 6 & 5 & 4 \\ 0 & 4 & 9 \end{bmatrix} \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix} = \begin{bmatrix} -3 \\ 48 \\ 24 \end{bmatrix}$$

(e) C_3 of AB = A [$C_3 B$]

$$= \begin{bmatrix} 3 & -2 & 7 \\ 6 & 5 & 4 \\ 0 & 4 & 9 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 41 \\ 49 \\ 57 \end{bmatrix}$$

(f) R_1 of BA = [$R_1 B$] A

$$= \begin{bmatrix} 6 & -2 & 4 \end{bmatrix} \begin{bmatrix} 3 & -2 & 7 \\ 6 & 5 & 4 \\ 0 & 4 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -6 & 70 \end{bmatrix}$$

Q#9: **(a)** C_1 of AA = linear combination.

$$= 3\begin{bmatrix}3\\6\\0\end{bmatrix} + 6\begin{bmatrix}-2\\5\\4\end{bmatrix} + 0\begin{bmatrix}7\\4\\9\end{bmatrix} = \begin{bmatrix}-3\\48\\24\end{bmatrix}$$

$$C_2 \text{ of } AA = -2\begin{bmatrix}3\\6\\0\end{bmatrix} + 5\begin{bmatrix}-2\\5\\4\end{bmatrix} + 4\begin{bmatrix}7\\4\\9\end{bmatrix} = \begin{bmatrix}12\\29\\56\end{bmatrix}$$

$$C_3 \text{ of } AA = 7\begin{bmatrix}3\\6\\0\end{bmatrix} + 4\begin{bmatrix}-2\\5\\4\end{bmatrix} + 9\begin{bmatrix}7\\4\\9\end{bmatrix} = \begin{bmatrix}76\\98\\97\end{bmatrix}$$

(b)

$$C_1 \text{ of } BB = 6\begin{bmatrix}6\\0\\7\end{bmatrix} + 0\begin{bmatrix}-2\\1\\7\end{bmatrix} + 7\begin{bmatrix}4\\3\\5\end{bmatrix} = \begin{bmatrix}64\\21\\77\end{bmatrix}$$

$$C_2 \text{ of } BB = -2\begin{bmatrix}6\\0\\7\end{bmatrix} + 1\begin{bmatrix}-2\\1\\7\end{bmatrix} + 7\begin{bmatrix}4\\3\\5\end{bmatrix} = \begin{bmatrix}14\\22\\28\end{bmatrix}$$

$$C_3 \text{ of } BB = 4\begin{bmatrix}6\\0\\7\end{bmatrix} + 3\begin{bmatrix}2\\-1\\7\end{bmatrix} + 5\begin{bmatrix}4\\3\\5\end{bmatrix} = \begin{bmatrix}38\\18\\74\end{bmatrix}$$

Q#11 (b):

$$\begin{aligned}
4x_1 \qquad\quad - 3x_3 + x_4 &= 1\\
5x_1 + x_2 \qquad\quad - 8x_4 &= 3\\
2x_1 - 5x_2 + 9x_3 - x_4 &= 0\\
3x_2 - x_3 + 7x_4 &= 2
\end{aligned}$$

$$\underset{A}{\begin{bmatrix}4 & 0 & -3 & 1\\5 & 1 & 0 & -8\\2 & -5 & 9 & -1\\0 & 3 & -1 & +7\end{bmatrix}}\underset{x}{\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}} = \underset{b}{\begin{bmatrix}1\\3\\0\\2\end{bmatrix}}$$

Q#15:

$$\overset{1\times3}{\begin{bmatrix} k & 1 & 1 \end{bmatrix}} \overset{3\times3}{\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & -3 \end{bmatrix}} \overset{3\times1}{\begin{bmatrix} k \\ 1 \\ 1 \end{bmatrix}} = 0$$

$$= \begin{bmatrix} k & 1 & 1 \end{bmatrix} \begin{bmatrix} k+1 \\ k+2 \\ -1 \end{bmatrix}$$

$$= k(k+1) + k+2 - 1$$

$$= k^2 + k + k + 2 - 1$$

$$= k^2 + 2k + 1$$

$$= (k+1)^2 = 0$$

$$k+1 = 0$$

$$\boxed{k = -1} \text{ Ans.}$$

Q#17: $A = \overset{2\times2}{\begin{bmatrix} 4 & -3 \\ 2 & -1 \end{bmatrix}}$ $B = \overset{2\times3}{\begin{bmatrix} 0 & 1 & 2 \\ -2 & 3 & 1 \end{bmatrix}}$ row-col expansion

$$= \begin{bmatrix} 4 \\ 2 \end{bmatrix} \times \begin{bmatrix} 0 & 1 & 2 \end{bmatrix} + \begin{bmatrix} -3 \\ -1 \end{bmatrix} \times \begin{bmatrix} -2 & 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 4 & 8 \\ 0 & 2 & 4 \end{bmatrix} + \begin{bmatrix} 6 & -9 & -3 \\ 2 & -3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -5 & 5 \\ 2 & -1 & 3 \end{bmatrix}$$

Q#24:

$$\begin{bmatrix} a-b & b+a \\ 3d+c & 2d-c \end{bmatrix} = \begin{bmatrix} 8 & 1 \\ 7 & 6 \end{bmatrix}$$

$a - b = 8$

$b + a = 1$

$3d + \cancel{c} = 7$

$\oplus \; 2d \cancel{-c} = 6$

$5d = 13$

$\boxed{d = \frac{13}{5}}$

$a - \cancel{b} = 8$

$\oplus \; a + \cancel{b} = 1$

$2a = 9$

$\boxed{a = \frac{9}{2}}$

$b = 1 - \frac{9}{2}$

$\boxed{b = -\frac{7}{2}}$

$2\left(\frac{13}{5}\right) - 6 = c$

$\boxed{c = -\frac{4}{5}}$

Exercise 1.4

Q#13

$$(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$$

$$A = \begin{bmatrix} 2 & -3 \\ 4 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

$$ABC = \begin{bmatrix} 2 & -3 \\ 4 & 4 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 \\ 4 & 4 \end{bmatrix}\begin{bmatrix} 6 & 3 \\ 10 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} -18 & -12 \\ 64 & 36 \end{bmatrix} \qquad |ABC| = 120$$

$$\text{adj } ABC = \begin{bmatrix} 36 & 12 \\ -64 & -18 \end{bmatrix}$$

$$(ABC)^{-1} = \begin{bmatrix} 36/120 & 12/120 \\ -64/120 & -18/120 \end{bmatrix}$$

$$= \begin{bmatrix} 3/10 & 1/10 \\ -8/15 & -3/20 \end{bmatrix} \qquad |A| = 20 \quad |C| = 6$$

$$|B| = 1$$

$$C^{-1}.B^{-1}.A^{-1} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/3 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 1/5 & 3/20 \\ -1/5 & 1/10 \end{bmatrix}$$

$$= \begin{bmatrix} 3/10 & 1/10 \\ -8/15 & -3/20 \end{bmatrix}$$

L.H.S = R.H.H Hence proved.

Q#17: $(I + 2A)^{-1} = \begin{bmatrix} -1 & 2 \\ 4 & 5 \end{bmatrix}$

$[(I+2A)^{-1}]^{-1}$

$= I + 2A = \begin{bmatrix} -5/13 & 2/13 \\ 4/13 & 1/13 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + 2A = \begin{bmatrix} -5/13 & 2/13 \\ 4/13 & 1/13 \end{bmatrix}$

$2A = \begin{bmatrix} -18/13 & 2/13 \\ 4/13 & -12/13 \end{bmatrix}$

$A = \begin{bmatrix} -9/13 & 1/13 \\ 4/13 & -6/13 \end{bmatrix}$ Ans

Q#19. $A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$

(a) $= A^3$ $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 11 & 4 \\ 8 & 3 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 41 & 15 \\ 30 & 11 \end{bmatrix}$

(b) $(A^3)^{-1}$ $|A^3| = 1$

$A^{-3} = \begin{bmatrix} 11 & -15 \\ -30 & 41 \end{bmatrix}$

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Q#21 (b) $p(x) = 2x^2 - x + 1$

$$A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$$

$$= 2A^2 - A + I$$

$$= 2\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= 2\begin{bmatrix} 11 & 4 \\ 8 & 3 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & 8 \\ 16 & 6 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 19 & 7 \\ 14 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 20 & 7 \\ 14 & 6 \end{bmatrix} \text{ Ans.}$$

Q#25: $3x_1 - 2x_2 = -1$

$4x_1 + 5x_2 = 3$

$$\underset{A}{\begin{bmatrix} 3 & -2 \\ 4 & 5 \end{bmatrix}} \underset{x}{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}} = \underset{B}{\begin{bmatrix} -1 \\ 3 \end{bmatrix}}$$

$$x = A^{-1}B$$

$$= \begin{bmatrix} 5/23 & 2/23 \\ -4/23 & 3/23 \end{bmatrix}\begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1/23 \\ 13/23 \end{bmatrix}$$

Q#46: (a) idempotent $A^2 = A$
$I - A$ is indempotent

$= (I-A)^2$
$= (I-A)(I-A)$
$= II - AI - AI + AA$ as $\boxed{A^2 = A}$
$= I - A - A + A^2$
$= I - A - \underline{A + A}$

$= I - A$ Hence proved.

(b) $2A - I$ invertible and it's own inverse

$= (2A - I)(2A - I)$
$= (2A)(2A) - 2AI - I2A + II$
$= 4A^2 - 2A - 2A + I$
$= 4A^2 - 4A + I$ $\boxed{A^2 = A}$
$= \cancel{4A} - \cancel{4A} + I$
$= I$ (inverse of identity matrix is identity itself)

Exercise 1.5

Q#2 (d) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

elementry matrix that diff from identity matrix from one single row operation

Not an elementary matrix. (2 row operations required)

Q#4(d) $\begin{bmatrix} 1 & 0 & -1/7 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

row operation = $R1 + \frac{1}{7}R3$

Corresponding E·Matrix = $\begin{bmatrix} 1 & 0 & 1/7 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Q#6 (c) $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $A = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$

verify product EA results from applying rowop to A.

row op corresponding to E?

row operation corresponding to E : $R_2 \times 5$ of indentity matri.

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} \qquad E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$EA = \begin{bmatrix} 1 & 4 \\ 10 & 25 \\ 3 & 6 \end{bmatrix} \text{ — } ①$$

row operation on $A = \begin{bmatrix} 1 & 4 \\ 10 & 25 \\ 3 & 6 \end{bmatrix}$ — ②

$R_2 \times 5$

eq ①=② Hence verified.

Q#13:

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \quad R_3 - R_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 0 & 1 \end{array}\right] \quad R_3 - R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & -2 & -1 & -1 & 1 \end{array}\right] \quad R_3/-2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1/2 & 1/2 & -1/2 \end{array}\right] \quad \begin{matrix} R_1 - R_3 \\ R_2 - R_3 \end{matrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & -1/2 & 1/2 \\ 0 & 1 & 0 & -1/2 & 1/2 & 1/2 \\ 0 & 0 & 1 & 1/2 & 1/2 & -1/2 \end{array}\right]$$

Then inverse is $\begin{bmatrix} 1/2 & -1/2 & 1/2 \\ -1/2 & 1/2 & 1/2 \\ 1/2 & 1/2 & -1/2 \end{bmatrix}$

Q#21: $\begin{bmatrix} c & c & c \\ 1 & c & c \\ 1 & 1 & c \end{bmatrix}$ invertible matrix

$= \begin{bmatrix} 1 & 1 & c \\ 1 & c & c \\ c & c & c \end{bmatrix}$ R_{13}

$R_2 - R_1$
$R_3 - cR_1$

$= \begin{bmatrix} 1 & 1 & c \\ 0 & c-1 & 0 \\ 0 & 0 & c-c^2 \end{bmatrix}$

invertible matrix $\Rightarrow$ reduced row echelon form is identity

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$c - 1 = 1$
$c - c^2 = 0$
$c(1-c) = 0$

if, $c=0$ $c=1$, the last matrix contains atleast one row of zeroes, therefore it cannot be reduced to identity.

Otherwise, if $(c \neq 0, c \neq 1)$,

$R_2 \times \left(\frac{1}{c-1}\right)$ and $R_3 \left(\frac{1}{c-c^2}\right)$ would result in row echelon form with 1's on main diagonal.

Subsequent elementary row operations can lead them to identity matrix so,

value of c other than 0 and 1, matrix is invertible.

Q#25: $\begin{bmatrix} 1 & 0 & -2 \\ 0 & 4 & 3 \\ 0 & 0 & 1 \end{bmatrix}$ $R_2/4$ $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$R_2 - \frac{3}{4}R_3$ $\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 3/4 \\ 0 & 0 & 1 \end{bmatrix}$ $E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$R_1 + 2R_3$ $\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\frac{3}{4} \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $E_3 = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$(E_3 E_2 E_1)A = I$

$$A = (E_3 E_2 E_1)^{-1} I = \boxed{E_1^{-1} E_2^{-1} E_3^{-1} = A}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/4 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -3/4 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and

$$\boxed{A^{-1} = E_3 E_2 E_1} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -3/4 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Exercise 1.6

Q#1 :

$$x_1 + x_2 = 2$$
$$5x_1 + 6x_2 = 9$$

inverting the coefficient matrix and find $x = A^{-1}b$

$$A = \begin{bmatrix} 1 & 1 \\ 5 & 6 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad b = \begin{bmatrix} 2 \\ 9 \end{bmatrix}$$

$$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 5 & 6 & 0 & 1 \end{array}\right] \quad R_2 - 5R_1$$

$$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 1 & -5 & 1 \end{array}\right] \quad R_1 - R_2$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 6 & 0 \\ 0 & 1 & -5 & 1 \end{array}\right] \quad A^{-1} = \begin{bmatrix} 6 & 0 \\ -5 & 1 \end{bmatrix}$$

$$x = A^{-1}b$$
$$= \begin{bmatrix} 6 & 0 \\ -5 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix} \qquad \boxed{\begin{array}{l} x_1 = 3 \\ x_2 = -1 \end{array}}$$

Q#3:

$$x_1 + 3x_2 + x_3 = 4$$
$$2x_1 + 2x_2 + x_3 = -1$$
$$2x_1 + 3x_2 + x_3 = 3$$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 2 & 2 & 1 & 0 & 1 & 0 \\ 2 & 3 & 1 & 0 & 0 & 1 \end{array}\right] \quad \begin{array}{l} R_2 - 2R_1 \\ R_3 - 2R_1 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & -4 & -1 & -2 & 1 & 0 \\ 0 & -3 & -1 & -2 & 0 & 1 \end{array}\right] \quad R_2/-4$$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1/4 & 1/2 & -1/4 & 0 \\ 0 & -3 & -1 & -2 & 0 & 1 \end{array}\right] \quad \begin{array}{l} R_1 - 3R_2 \\ R_3 + 3R_2 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1/4 & -1/2 & 3/4 & 0 \\ 0 & 1 & 1/4 & 1/2 & -1/4 & 0 \\ 0 & 0 & -1/4 & -1/2 & -3/4 & 1 \end{array}\right] \quad R_3 \times -4$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1/4 & -1/2 & 3/4 & 0 \\ 0 & 1 & 1/4 & 1/2 & -1/4 & 0 \\ 0 & 0 & 1 & 2 & 3 & -4 \end{array}\right] \quad \begin{array}{l} R_1 - 1/4R_3 \\ R_2 - 1/4R_3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 2 & 3 & -4 \end{array}\right] \quad A^{-1} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 2 & 3 & -4 \end{bmatrix}$$

$$x = A^{-1}b = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 2 & 3 & -4 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ -7 \end{bmatrix}$$

$$\boxed{\begin{array}{l} x_1 = -1 \\ x_2 = +4 \\ x_3 = -7 \end{array}}$$

*linear system is consistent.

↓

if no row is zero.

Q#15:

$$x_1 - 2x_2 + 5x_3 = b_1$$
$$4x_1 - 5x_2 + 8x_3 = b_2$$
$$-3x_1 + 3x_2 - 3x_3 = b_3$$

$$\begin{bmatrix} 1 & -2 & 5 & b_1 \\ 4 & -5 & 8 & b_2 \\ -3 & 3 & -3 & b_3 \end{bmatrix} \quad \begin{matrix} R_2 - 4R_1 \\ R_3 + 3R_1 \end{matrix}$$

$$\begin{bmatrix} 1 & -2 & 5 & b_1 \\ 0 & 3 & -12 & b_2 - 4b_1 \\ 0 & -3 & 12 & b_3 + 3b_1 \end{bmatrix} \quad R_3 + R_2$$

$$\begin{bmatrix} 1 & -2 & 5 & b_1 \\ 0 & 3 & -12 & b_2 - 4b_1 \\ 0 & 0 & 0 & b_3 + 3b_1 + b_2 - 4b_1 \end{bmatrix}$$

$$-b_1 + b_2 + b_3 = 0 \quad \text{i.e.} \quad \boxed{b_1 = b_1 + b_3}$$

Exercise 1.7

Q#1:
(a) upper triangular $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$
(b) lowertriangular $\begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}$
(c) diagonal matrix, both upper and lower triangular
(d) upper triangular

Q#5:

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} -3 & 2 & 0 & 4 & -4 \\ 1 & -5 & 3 & 0 & 3 \\ -6 & 2 & 2 & 2 & 2 \end{bmatrix}$$

find product by inspection.

$$= \begin{bmatrix} 5(-3) & 5(2) & 5(0) & 5(4) & 5(-4) \\ 2(1) & 2(-5) & 2(3) & 2(0) & 2(3) \\ -3(-6) & -3(2) & -3(2) & -3(2) & -3(2) \end{bmatrix}$$

$$= \begin{bmatrix} -15 & 10 & 0 & 20 & -20 \\ 2 & -10 & 6 & 0 & 6 \\ 18 & -6 & -6 & -6 & -6 \end{bmatrix}$$

Q#19:

$$\begin{bmatrix} 0 & 6 & -1 \\ 0 & 7 & -4 \\ 0 & 0 & -2 \end{bmatrix}$$

by inspection.

upper triangular with 0 in its main diagonal
thus, not invertible.

** invertible only if diagonal entries are <u>all non-zero</u>

Q#21:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & -5 & 0 & 0 \\ 4 & -3 & 4 & 0 \\ 1 & -2 & 1 & 3 \end{bmatrix}$$ lower triangular matrix.

has main diagonal with all non-zero entries thus, its invertible.

Q#26:

$$A = \begin{bmatrix} 2 & a-2b+2c & 2a+b+c \\ 3 & 5 & a+c \\ 0 & -2 & 7 \end{bmatrix}$$

$A = A^T$

$$A^T = \begin{bmatrix} 2 & 3 & 0 \\ a-2b+2c & 5 & -2 \\ 2a+b+c & a+c & 7 \end{bmatrix}$$

$a-2b+2c = 3$
$2a+b+c = 0$ by guass-jordan
$a+c = -2$

$$\begin{bmatrix} 1 & -2 & 2 & 3 \\ 2 & 1 & 1 & 0 \\ 1 & 0 & 1 & -2 \end{bmatrix}$$ $R_2 - 2R_1$, $R_3 - R_1$

$$\begin{bmatrix} 1 & -2 & 2 & 3 \\ 0 & 5 & -3 & -6 \\ 0 & 2 & -1 & -5 \end{bmatrix}$$ $R_2/5$

$$\begin{bmatrix} 1 & -2 & 2 & 3 \\ 0 & 1 & -3/5 & -6/5 \\ 0 & 2 & -1 & -5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 2 & 3 \\ 0 & 1 & -3/5 & -6/5 \\ 0 & 2 & -1 & -5 \end{bmatrix} \quad \begin{matrix} R_1 + 2R_2 \\ R_3 - 2R_2 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 4/5 & 3/5 \\ 0 & 1 & -3/5 & -6/5 \\ 0 & 0 & 1/5 & -13/5 \end{bmatrix} \quad R_3 \times 5$$

$$\begin{bmatrix} 1 & 0 & 4/5 & 3/5 \\ 0 & 1 & -3/5 & -6/5 \\ 0 & 0 & 1 & -13 \end{bmatrix} \quad \begin{matrix} R_1 - 4/5 R_3 \\ R_2 + 3/5 R_3 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 11 \\ 0 & 1 & 0 & -9 \\ 0 & 0 & 1 & -13 \end{bmatrix} \quad \boxed{\begin{matrix} a = 11 \\ b = -9 \\ c = -13 \end{matrix}}$$

Q#28: $A = \begin{bmatrix} x - \frac{1}{2} & 0 & 0 \\ x & x - 1/3 & 0 \\ x^2 & x^3 & x + 1/4 \end{bmatrix}$

A a triangular matrix, it is invertible if all entries on its main diagonal is non-zero.

i.e.

$$x \neq \frac{1}{2}, \quad x \neq \frac{1}{3}, \quad x \neq -\frac{1}{4}$$

This lower triangular matrix is invertible for any value of x except $\frac{1}{2}, \frac{1}{3}, \frac{-1}{4}$.

* Word Problems at Last.

CHAPTER #2 :

Exercise 2.1

Q#3:

$$A = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 4 & -1 & 1 & 6 \\ 0 & 0 & -3 & 3 \\ 4 & 1 & 0 & 14 \\ 4 & 1 & 3 & 2 \end{bmatrix} \end{array}$$

(a) $M_{13} = \begin{vmatrix} 0 & 0 & 3 \\ 4 & 1 & 14 \\ 4 & 1 & 2 \end{vmatrix} = 3\begin{vmatrix} 4 & 1 \\ 4 & 1 \end{vmatrix} = \boxed{0}$

$C_{13} = (-1)^{1+3} M_{13} = 1 \cdot M_{13} = \boxed{0}$

(b) $M_{23} = \begin{vmatrix} 4 & -1 & 6 \\ 4 & 1 & 14 \\ 4 & 1 & 2 \end{vmatrix}$

$= 4\begin{vmatrix} 1 & 14 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 4 & 14 \\ 4 & 2 \end{vmatrix} + 6\begin{vmatrix} 4 & 1 \\ 4 & 1 \end{vmatrix}$

$= 4(2-14) + 1(8-56) + 6(0)$

$= \boxed{-96}$

$C_{23} = (-1)^{2+3} M_{23} = -M_{23} = \boxed{96}$

(C) $M_{22} = \begin{vmatrix} 4 & 1 & 6 \\ 4 & 0 & 14 \\ 4 & 3 & 2 \end{vmatrix} = -4\begin{vmatrix} 1 & 6 \\ 3 & 2 \end{vmatrix} - 14\begin{vmatrix} 4 & 1 \\ 4 & 3 \end{vmatrix}$

$= -4(2-18) - 14(12-4)$

$= \boxed{-48}$

$C_{22} = (-1)^{2+2} M_{22} = \boxed{-48}$

(d) $M_{21} = \begin{vmatrix} -1 & 1 & 6 \\ 1 & 0 & 14 \\ 1 & 3 & 2 \end{vmatrix}$

$= -1\begin{vmatrix} -1 & 6 \\ 1 & 2 \end{vmatrix} - 14\begin{vmatrix} -1 & 1 \\ 1 & 3 \end{vmatrix}$

$= -1(-2-6) - 14(-3-1)$

$= \boxed{72}$

$C_{21} = (-1)^{2+1} M_{21} = \boxed{-72}$

Q#14: Use Arrow technique :-

$\begin{vmatrix} c & -4 & 3 \\ 2 & 1 & c^2 \\ 4 & c-1 & 2 \end{vmatrix} \begin{matrix} c & -4 \\ 2 & 1 \\ 4 & c-1 \end{matrix}$

$= [2c - 16c^2 + 6(c-1)] - [12 + c^3(c-1) - 16]$

$= 2c - 16c^2 + 6c - 6 - 12 - c^4 + c^3 + 16$

$= -16c^2 + c^3 - c^4 + 8c - 2$

$= \boxed{-c^4 + c^3 - 16c^2 + 8c - 2}$ Ans.

Q#25: det(A)

$$A = \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \\ \end{array}$$

$$A = \begin{bmatrix} 3 & 3 & 0 & 5 \\ 2 & 2 & 0 & -2 \\ 4 & 1 & -3 & 0 \\ 2 & 10 & 3 & 2 \end{bmatrix}$$

expansion by third column.

$$= 0 - 0 + (-3)\begin{vmatrix} 3 & 3 & 5 \\ 2 & 2 & -2 \\ 2 & 10 & 2 \end{vmatrix} - 3\begin{vmatrix} 3 & 3 & 5 \\ 2 & 2 & -2 \\ 4 & 1 & 0 \end{vmatrix}$$

$$\begin{vmatrix} 3 & 3 & 5 \\ 2 & 2 & -2 \\ 2 & 10 & 2 \end{vmatrix} = 3\begin{vmatrix} 2 & -2 \\ 10 & 2 \end{vmatrix} - 3\begin{vmatrix} 2 & -2 \\ 2 & 2 \end{vmatrix} + 5\begin{vmatrix} 2 & 2 \\ 2 & 10 \end{vmatrix}$$

$$= 3(24) - 3(8) + 5(16)$$

$$= \boxed{128}$$

$$\begin{vmatrix} 3 & 3 & 5 \\ 2 & 2 & -2 \\ 4 & 1 & 0 \end{vmatrix} = 3\begin{vmatrix} 2 & -2 \\ 1 & 0 \end{vmatrix} - 3\begin{vmatrix} 2 & -2 \\ 4 & 0 \end{vmatrix} + 5\begin{vmatrix} 2 & 2 \\ 4 & 1 \end{vmatrix}$$

$$= 3(2) - 3(8) + 5(-6)$$

$$= \boxed{-48}$$

Therefore $\det(A) = 0 - 0 + (-3)(128) - 3(-48)$

$$= \boxed{-240}$$

Q#29:
$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 4 & 3 & 0 \\ 1 & 2 & 3 & 8 \end{bmatrix}$$

Determinant of a lower triangular matrix is the product of entries on its main diagonal

$\det A = 0 \times 2 \times 3 \times 8$

$= \boxed{0}$

Q#33(b):
$$\begin{vmatrix} \sin\theta & \cos\theta & 0 \\ -\cos\theta & \sin\theta & 0 \\ \sin\theta-\cos\theta & \sin\theta+\cos\theta & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{vmatrix}$$

$= (\sin^2\theta) + (\cos^2\theta)$

$= \boxed{1}$ So $|A|$ is independent of θ.

Exercise 2.2:

Q#9:
$$\begin{bmatrix} 3 & -6 & 9 \\ -2 & 7 & -2 \\ 0 & 1 & 5 \end{bmatrix}$$

take 3 common from R_1

$$= 3\begin{vmatrix} 1 & -2 & 3 \\ -2 & 7 & -2 \\ 0 & 1 & 5 \end{vmatrix} \quad R_2 + 2R_1$$

$$= 3\begin{vmatrix} 1 & -2 & 3 \\ 0 & 3 & 4 \\ 0 & 1 & 5 \end{vmatrix} \quad R_{23} \text{ were interchanged}$$

after interchanging Row 3 and Row 2.

$$= 3(-1)\begin{vmatrix} 1 & -2 & 3 \\ 0 & 1 & 5 \\ 0 & 3 & 4 \end{vmatrix} \quad R_3+(-3)R_2$$

$$= (3)(-1)\begin{vmatrix} 1 & -2 & 3 \\ 0 & 1 & 5 \\ 0 & 0 & -11 \end{vmatrix}$$

$$= 3(-1)(-11)\begin{vmatrix} 1 & -2 & 3 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{vmatrix}$$

lower triangular matrix

$$= 3(-1)(-11)(1)$$

$$\det(A) = \boxed{33}$$

Q#19:

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}\begin{vmatrix} a+g & b+h & c+i \\ d & e & f \\ g & h & i \end{vmatrix}$$

Given: $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -6$

$R_1 - R_3$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = \boxed{-6}$$

Q#27:

$$\begin{array}{ccc} C_1 & C_2 & C_3 \end{array}$$

$$\begin{vmatrix} a_1+b_1 & a_1-b_1 & c_1 \\ a_2+b_2 & a_2-b_2 & c_2 \\ a_3+b_3 & a_3-b_3 & c_3 \end{vmatrix} = -2\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$C_2 - C_1$

$$= \begin{vmatrix} a_1+b_1 & -2b_1 & c_1 \\ a_2+b_2 & -2b_2 & c_2 \\ a_3+b_3 & -2b_3 & c_3 \end{vmatrix}$$

$$= -2\begin{vmatrix} a_1+b_1 & b_1 & c_1 \\ a_2+b_2 & b_2 & c_2 \\ a_3+b_3 & b_3 & c_3 \end{vmatrix}$$

take -2 common from C_2

$$= -2\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \quad \overset{C_1 - C_2}{=} R.H.S$$ Hence proved.

Exercise 2.3

Q#3: $\det(kA) = k^n \det(A)$

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 3 & 2 & 1 \\ 1 & 4 & 5 \end{bmatrix} ; \quad k = -2$$

⇒ arrow technique

$$\det(kA) = \begin{vmatrix} -4 & 2 & -6 \\ -6 & -4 & -2 \\ -2 & -8 & -10 \end{vmatrix} \begin{matrix} -4 & 2 \\ -6 & -4 \\ -2 & -8 \end{matrix}$$

$$= (-160 + 8 - 288) - (-48 - 64 + 120)$$

$$= \boxed{-448}$$

$$k^n(\det A) = (-2)^3 \begin{vmatrix} 2 & -1 & 3 \\ 3 & 2 & 1 \\ 1 & 4 & 5 \end{vmatrix} = (-8)[(20 - 1 + 36) - (6 + 8 - 15)]$$

$$= \boxed{-448}$$

(proved)

Q#13: $A = \begin{bmatrix} 2 & 0 & 0 \\ 8 & 1 & 0 \\ -5 & 5 & 6 \end{bmatrix}$

$= 6\begin{vmatrix} 2 & 0 \\ 8 & 1 \end{vmatrix} = 6(2-0)$ non-singular.

$= \boxed{12} \neq 0$ therefore, A is inVertible.

Q#17: $A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 1 & 6 \\ k & 3 & 2 \end{bmatrix} \begin{matrix} 1 & 2 \\ 3 & 1 \\ k, & 3, \end{matrix}$

$= (2 + 12k + 36) - (4k + 18 + 12)$

$= 8 + 8k$

$= 8(1+k)$ $*$ if $\boxed{k \neq -1}$

$= 8 + 8k$ then A will be inveritble.

Q#21: $A = \begin{bmatrix} 2 & -3 & 5 \\ 0 & 1 & -3 \\ 0 & 0 & 2 \end{bmatrix}$

$\det = 2\begin{vmatrix} 1 & -3 \\ 0 & 2 \end{vmatrix} = 2(2-0)$

$= \boxed{4}$ invertible.

cofactors of A are :-

$C_{11} = \begin{vmatrix} 1 & -3 \\ 0 & 2 \end{vmatrix} = 2$ $\quad C_{21} = -\begin{vmatrix} -3 & 5 \\ 0 & 2 \end{vmatrix} = 6$ $\quad C_{31} = \begin{vmatrix} -3 & 5 \\ 1 & -3 \end{vmatrix} = 4$

$C_{12} = -\begin{vmatrix} 0 & -3 \\ 0 & 2 \end{vmatrix} = 0$ $\quad C_{22} = \begin{vmatrix} 2 & 5 \\ 0 & 2 \end{vmatrix} = 4$ $\quad C_{32} = -\begin{vmatrix} 2 & 5 \\ 0 & -3 \end{vmatrix} = 6$

$C_{13} = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0$ $\quad C_{23} = -\begin{vmatrix} 2 & -3 \\ 0 & 0 \end{vmatrix} = 0$ $\quad C_{33} = \begin{vmatrix} 2 & -3 \\ 0 & 1 \end{vmatrix} = 2$

$\therefore \begin{bmatrix} 2 & 0 & 0 \\ 6 & 4 & 0 \\ 4 & 6 & 2 \end{bmatrix}$ = matrix of cofactors.

$$\text{adj}A = \begin{bmatrix} 2 & 6 & 4 \\ 0 & 4 & 6 \\ 0 & 0 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{adj}(A) = \frac{1}{4}\begin{bmatrix} 2 & 6 & 4 \\ 0 & 4 & 6 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 & 3/2 & 1 \\ 0 & 1 & 3/2 \\ 0 & 0 & 1/2 \end{bmatrix} \quad \text{Ans.}$$

Q#27:

$$x_1 - 3x_2 + x_3 = 4$$
$$2x_1 - x_2 = -2$$
$$4x_1 \quad - 3x_3 = 0$$

$$A = \begin{bmatrix} 1 & -3 & 1 \\ 2 & -1 & 0 \\ 4 & 0 & -3 \end{bmatrix} \qquad |A| = (3+0+0) - (-4+0+18) = \boxed{-11}$$

$$\det(Ax_1) = \begin{vmatrix} 4 & -3 & 1 \\ -2 & -1 & 0 \\ 0 & 0 & 3 \end{vmatrix} = |Ax_1| = -3\begin{vmatrix} 4 & -3 \\ -2 & -1 \end{vmatrix}$$

$$= (-3)(-4-6) = \boxed{30}$$

$$|Ax_2| = \begin{vmatrix} 1 & 4 & 1 \\ 2 & -2 & 0 \\ 4 & 0 & 3 \end{vmatrix}$$

$$= (6+0+0) - (-8+0-24) = \boxed{38}$$

$$|Ax_3| = \begin{vmatrix} 1 & -3 & 4 \\ 2 & -1 & -2 \\ 4 & 0 & 0 \end{vmatrix} = 4\begin{vmatrix} -3 & 4 \\ -1 & 2 \end{vmatrix} = 4(6+4) = \boxed{40}$$

$$x_1 = \frac{|Ax_1|}{|A|} = \boxed{\frac{30}{-11}} \qquad x_2 = \frac{|Ax_2|}{|A|} = \boxed{\frac{-38}{11}} \qquad x_3 = \frac{|Ax_3|}{|A|} = \boxed{\frac{-40}{11}}$$

Q#35(d): $\det((2A)^{-1})$ $\quad \det(A) = 7$

$= \dfrac{1}{\det(2A)}$ $\quad \Rightarrow$ matrix is 3×3

$\Rightarrow \det(kA) = k^n \det(A)$

$= \dfrac{1}{2^3 \det(A)}$

$= \dfrac{1}{8 \times 7}$

$= \boxed{\dfrac{1}{56}}$ Ans.

Chapter #3

Exercise 3.1

Q#12: $u = (1, 2, -3, 5, 0)$
$v = (0, 4, -1, 1, 2)$
$w = (7, 1, -4, -2, 3)$

(a) $v + w = (7, 5, -5, -1, 5)$ Ans

(b) $3(2u - v) = 3[(2, 4, -6, 10, 0) - (0, 4, -1, 1, 2)]$
$= 3(2, 0, -5, 9, -2)$
$= (6, 0, -15, 27, -6)$ Ans

(c) $(3u - v) - (2u + 4w)$
$= [(3, 6, -9, 15, 0) - (0, 4, -1, 1, 2)] - [(2, 4, -6, 10, 0) + (28, 4, -16, -8, 12)]$
$= (3, 2, -8, 14, -2) - (30, 8, -22, 2, 12)$
$= (-27, -6, 14, 12, -14)$ Ans.

(d) $\frac{1}{2}(w-5v+2u)+v$

$= \frac{1}{2}[(7,1,-4,-2,3)-(0,20,-5,5,10)+(2,4,-6,10,0)]$
$+ (0,4,-1,1,2)$

$= \frac{1}{2}(9,-15,-5,3,-7)+(0,4,-1,1,2)$

$= (9/2, -7/2, -7/2, 5/2, -3/2)$ Ans

Q#20: $c_1(-1,0,2)+c_2(2,2,-2)+c_3(1,-2,1)$
$= (-6, 12, 4)$

L.H.S =

$= (-c_1, 0, 2c_1)+(2c_2, 2c_2, -2c_2)+(c_3, -2c_3, c_3)$
$= (-6, 12, 4)$

So,

$-c_1+2c_2+c_3=-6$
$2c_2-2c_3=12$
$2c_1-2c_2+c_3=4$

$$\begin{bmatrix} -1 & 2 & 1 & -6 \\ 0 & 2 & -2 & 12 \\ 2 & -2 & 1 & 4 \end{bmatrix}$$

reduced row echelon form. $-1R_1$

$$\begin{bmatrix} 1 & -2 & -1 & 6 \\ 0 & 2 & -2 & 12 \\ 2 & -2 & 1 & 4 \end{bmatrix} \quad R_3+(-2)R_1$$

$$\begin{bmatrix} 1 & -2 & -1 & 6 \\ 0 & 2 & -2 & 12 \\ 0 & 2 & 3 & -8 \end{bmatrix} \quad R_2/2$$

$$\begin{bmatrix} 1 & -2 & -1 & 6 \\ 0 & 1 & -1 & 6 \\ 0 & 2 & 3 & -8 \end{bmatrix} \quad \begin{matrix} R_1+2R_2 \\ R_3-2R_2 \end{matrix}$$

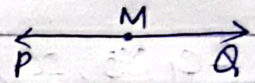
$$\begin{bmatrix} 1 & 0 & -3 & 18 \\ 0 & 1 & -1 & 6 \\ 0 & 0 & 5 & -20 \end{bmatrix} \quad R_3/5$$

$$\begin{bmatrix} 1 & 0 & -3 & 18 \\ 0 & 1 & -1 & 6 \\ 0 & 0 & 1 & -4 \end{bmatrix} \quad \begin{matrix} R_1+3R_3 \\ R_2+R_3 \end{matrix}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & -4 \end{array}\right]$$

$$\boxed{c_1 = 6}, \quad \boxed{c_2 = 2}, \quad \boxed{c_3 = -4}$$

Q#23: $P(2,3,-2)$
$Q(7,-4,1)$



(a) $\overrightarrow{OM} = \overrightarrow{OP} + \frac{1}{2}\overrightarrow{PQ}$

$= (2,3,-2) + \frac{1}{2}\left[(7,-4,1) - (2,3,-2)\right]$

$= (2,3,-2) + \frac{1}{2}(5,-7,3)$

$= \left(9/2, -\frac{1}{2}, -\frac{1}{2}\right)$ Ans.

(b) $\overrightarrow{ON} = \overrightarrow{OP} + \frac{3}{4}\overrightarrow{PQ}$

$= (2,3,-2) + 3/4\,(7-2, -4-3, 1+2)$

$= \left(\frac{23}{4}, -9/4, \frac{1}{4}\right)$ Ans.

Exercise 3.2:

Q# 1 (b): $V = (1,0,2,1,3)$

$$\|V\| = \sqrt{1^2+2^2+1^2+3^2} = \boxed{\sqrt{15}}$$

$$\hat{V} = \frac{\vec{V}}{\|V\|} = \left(\frac{1}{\sqrt{15}}, 0, \frac{2}{\sqrt{15}}, \frac{1}{\sqrt{15}}, \frac{3}{\sqrt{15}}\right);$$

oppositely directed

$$\frac{-\vec{V}}{\|V\|} = \frac{-1}{\sqrt{15}}(1,0,2,1,3)$$

$$-\hat{V} = \left(-\frac{1}{\sqrt{15}}, 0, \frac{-2}{\sqrt{15}}, \frac{-1}{\sqrt{15}}, \frac{-3}{\sqrt{15}}\right) \text{ Ans.}$$

Q# 11(b): $u = (0,-2,-1,1)$, $v = (-3,2,4,4)$

→ Euclidean distance between u and v

→ $\cos\theta$

→ acute?, obtuse?, 90°?

$$d(u,v) = \|u-v\| = \sqrt{(0+3)^2+(-2-2)^2+(-1-4)+(1-4)^2}$$

$$= \boxed{\sqrt{59}}$$

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|u\|\|v\|} = \frac{(0)(-3)+(-2\times2)+(-4)+4}{\sqrt{59}}$$

$$= \boxed{\frac{-4}{\sqrt{6}\sqrt{45}}}$$

Since $\boxed{\vec{u}\cdot\vec{v} < 0}$ so, angle is obtuse

Q#17: (a). $u = (-3, 1, 0)$ Verify, Cauchy-Schwarz inequality holds.
$v = (2, -1, 3)$

$|u \cdot v| = |-6 + (-1) + 0| = \boxed{7}$

$\|u\|\|v\| = \sqrt{-3^2 + 1^2 + 0^2}\sqrt{2^2 + (-1)^2 + 3^2} = \boxed{\sqrt{10}\sqrt{14}}$

Since, $|u \cdot v| = \sqrt{49} \leq \sqrt{140} = \sqrt{10}\sqrt{14} = \|u\|\|v\|$,

the Cauchy-Schwarz inequality holds.

(b). $u = (0, 2, 2, 1)$, $v = (1, 1, 1, 1)$

$|u \cdot v| = 0 + 2 + 2 + 1 = \boxed{5}$

$\|u\|\|v\| = \sqrt{4+4+1}\sqrt{1+1+1+1} = \sqrt{9}\sqrt{4} = \boxed{6}$

Since $5 \leq 6$ thus, it holds verified.

Cauchy-Schwarz inequality

Q#21:

$v = (v_1, v_2, v_3)$ in R^3

$\cos\alpha = \dfrac{v_1}{\|v\|}$, $\cos\beta = \dfrac{v_2}{\|v\|}$, $\cos\gamma = \dfrac{v_3}{\|v\|}$

We have, $\|i\| = \|j\| = \|k\| = 1$ Therefore,

$$\cos\alpha = \frac{v \cdot i}{\|v\|\|i\|} = \frac{(v_1) + v_2(0) + v_3(0)}{\|v\|} = \frac{v_1}{\|v\|}$$

$$\cos\beta = \frac{v \cdot j}{\|v\|\|j\|} = \frac{0 + v_2 + 0}{\|v\|} = \frac{v_2}{\|v\|}$$

$$\cos\gamma = \frac{v \cdot k}{\|v\|\|k\|} = \frac{0 + 0 + v_3}{\|v\|} = \frac{v_3}{\|v\|}$$

Exercize 3.3.

Q#1. (c) $u = (3,-2,1,3)$
$v = (-4,1,-3,7)$

$u \cdot v = -12 + (-2) + (-3) + 21$
$= 4 \neq 0$ therefore u and v are not orthognal.

Q#3: $P(-1,3,-2)$
$n = (-2,1,-1)$

$\Rightarrow -2(x-(-1)) + 1(y-3) - 1(z-(-2)) = 0$
0 can be written as.

$$\boxed{-2(x+1) + (y-3) - (z+2) = 0}$$

Q#9: $2y = 8x - 4z + 5$ and $x = \frac{1}{2}z + \frac{1}{4}y$
$-8x + 2y + 4z = 5$
yields a normal vector $(-8, 2, 4)$

$x - \frac{1}{4}y - \frac{1}{2}z = 0$ yields a normal vector $(1, -\frac{1}{4}, -\frac{1}{2})$

• the two normal vectors are parallel therefore the planes are parallel as well
$(-8,2,4) = -8(1, -\frac{1}{4}, -\frac{1}{2})$

Q#11: $3x - y + z = 4$
$x + 2z = -1$

normal vectors of plane are not orthogonal:
$(3,-1,1) \cdot (1,0,2) = (3\times1) + (-1\times0) + (1\times2)$
$= \boxed{5 \neq 0}$

therefore, the planes are not prependicular.

Q#13 (b): $u = (3,0,4)$
$a = (2,3,3)$

$$\|\text{proj}_a \vec{u}\| = \frac{|\vec{u}\cdot\vec{a}|}{\|a\|}$$
$$= \frac{|3(2) + (0)(3) + 4(3)|}{\sqrt{2^2+3^2+3^2}}$$
$$= \boxed{\frac{18}{\sqrt{22}}}$$

Q#15: $u = (6,2)$, $a = (3,-9)$

$$u\cdot a = 6(3) + 2(-9) = 0$$
$$\|a^2\| = (3)^2 + (-9)^2 = 90$$

the vector component of u along a is $\text{proj}_a u = \frac{\vec{u}\cdot\vec{a}}{\|a\|^2}\vec{a}$

$$= \frac{0}{90}(3,-9) = \boxed{(0,0)}$$

the vector component of u orthogonal to a is
$$u - \text{proj}_a u = (6,2) - (0,0) = \boxed{(6,2)}$$

Q#21: $(-3,1)$; $4x + 3y + 4 = 0$

The distance between the point and the line is

$$D = \frac{|4(-3) + (3)(1) + 4|}{\sqrt{4^2+3^2}} = \frac{5}{\sqrt{25}} = \boxed{1}$$

Q#29:

$u = (1,0,1)$ $v = (0,1,1)$

In order for $\vec{w} = (a,b,c)$ to be orthogonal to both $(1,0,1)$ and $(0,1,1)$, we must have $a+c=0$ and $b+c=0$. These equations form a linear system whose augmented matrix is:

$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$ is already in reduced row echelon form.

For real number t, the solutions are $a = -t$, $b = -t$, $c = t$.
arbitrary

Since $\vec{w}$ is also required to be a unit vector, we must have $\|\vec{w}\| = \sqrt{(-t)^2 + (-t)^2 + t^2} = \sqrt{3t^2} = 1$

This yields $t = \pm \frac{1}{\sqrt{3}}$, consequently there are two possible vectors that satisfy the given conditions:

$\left(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right)$ and $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}} \right)$

Q#30: (a) $\vec{v} = (a, b)$ are orthogonal vectors.
$\vec{w} = (-b, a)$

$\vec{v} \cdot \vec{w} = (a)(-b) + (b)(a) = 0$ therefore $\vec{v}$ and $\vec{w}$ are orthogonal vectors.

(b) $\vec{v} = (2, -3)$

$(3, 2)$ and $(-3, -2)$

(c) $\vec{v} = (-3, 4)$
$(4, 3)$ and $(4, 3)$

Exercise # 3.4

Q#4: Point : $(-9,3,4)$
Vector : $\vec{v} = (-1,6,0)$

$$(x,y,z) = (-9,3,4) + t(-1,6,0)$$

This, yields the parametric equations

$$x = -9-t$$
$$y = 3+6t$$
$$z = 4$$

Q#8: $x = (1-t)(0,-5,1)$

A point on the line : $(0,-5,1)$
a vector parallel to the line : $(0,5,-1)$

Q#12: Point : $(0,5,-4)$
Vectors : $\vec{v_1} = (0,0,-5)$ and $\vec{v_2} = (1,-3,-2)$

Vector equation:

$$(x,y,z) = (0,5,-4) + t_1(0,0,-5) + t_2(1,-3,-2).$$

This yields parametric equations:

$$x = t_2, \quad y = 5-3t_2, \quad z = -4-5t_1-2t_2$$

Q#18:

$$x_1 + 3x_2 - 4x_3 = 0$$
$$2x_1 + 6x_2 - 8x_3 = 0$$

The augmented matrix of linear system $\begin{bmatrix} 1 & 3 & -4 & 0 \\ 2 & 6 & -8 & 0 \end{bmatrix}$ has the reduced row echelon form $\begin{bmatrix} 1 & 3 & -4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. A general solution of the system, $x_1 = -3s + 4t$

$$x_2 = s$$
$$x_3 = t$$

expressed in vector form as $\vec{x} = (-3s+4t, s, t)$ is orthogonal to the rows of the coefficient matrix of the original system

$\vec{r_1} = (1, 3, -4)$

$\vec{r_2} = (2, 6, -8)$ since

$\vec{r_1} \cdot \vec{x} = (1)(-3s+4t) + 3(s) + (-4)(t) = 0$

and

$\vec{r_2} \cdot \vec{x} = (2)(-3s+4t) + 6(s) + (-8)(t) = 0$

Q#23:

(a) $x = x_0 + t\vec{v}$

The image of x under multiplication by A is

$y = Ax = A(x_0 + t\vec{v})$

$y = Ax_0 + t(A\vec{v})$

Since, $x = x_0 + \vec{v}t$ is equation of line in R^n so, $v \neq 0$ and A is invertible so $Av \neq 0$.

So, $Ax_0 + t(A\vec{v})$ is also equation of line in R^n.

This shows image of line under multiplication by A is itself a line.

(b) $A = \begin{bmatrix} 2 & 1 \\ 3 & -4 \end{bmatrix}$ $x = (1,3) + t(2,-1)$

$y = Ax = Ax_0 + t(Av)$

$y = Ax = \begin{bmatrix} 2 & 1 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \end{bmatrix} + t\begin{bmatrix} 2 & 1 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix}$

$y = Ax = \begin{bmatrix} 5 \\ -9 \end{bmatrix} + t\begin{bmatrix} 3 \\ 10 \end{bmatrix}$ $y = (x, y)$

$(x,y) = (5,-9) + t(3,10) = (5+3t, -9+10t)$

$x = 5 + 3t$

$y = -9 + 10t$

<u>Parametric Eq:</u> $x = 5 + 3t$

$y = -9 + 10t$.

Exercise # 3.5:

Q# 5:

$\vec{u} \times (\vec{v} \times \vec{w})$

$$\vec{v} \times \vec{w} = \left(\begin{vmatrix} 2 & -3 \\ 6 & 7 \end{vmatrix}, -\begin{vmatrix} 0 & -3 \\ 2 & 7 \end{vmatrix}, \begin{vmatrix} 0 & 2 \\ 2 & 6 \end{vmatrix} \right)$$

$$= (32, -6, -4)$$

$$\vec{u} \times (\vec{v} \times \vec{w}) = \left(\begin{vmatrix} 2 & -1 \\ -6 & -4 \end{vmatrix}, -\begin{vmatrix} 3 & -1 \\ -32 & -4 \end{vmatrix}, \begin{vmatrix} 3 & 2 \\ 32 & -6 \end{vmatrix} \right)$$

$$= (-14, -20, -82)$$

$$\vec{u} \times (\vec{v} \times \vec{w}) = (\vec{u} \cdot \vec{w})\vec{v} - (\vec{u} \cdot \vec{v})\vec{w}$$

$$= [(3)(2) + 2(6) + (-1)(7)](0, 2, -3) - [(3)(0) + (2)(2) + (-1)(-3)](2, 6, 7)$$

$$= 11(0, 2, -3) - 7(2, 6, 7)$$

$$= (0, 22, -33) - (14, 42, 49)$$

$$= \boxed{(-14, -20, -82)}$$

Q#8: $\vec{u} = (1, 1, -2)$, $\vec{v} = (2, -1, 2)$

$$\vec{u} \times \vec{v} = \left(\begin{vmatrix} 1 & -2 \\ -1 & 2 \end{vmatrix}, -\begin{vmatrix} 1 & -2 \\ 2 & 2 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} \right) = (0, -6, -3) \text{ is}$$

orthogonal to both $\vec{u}$ and $\vec{v}$.

Q#10: $\vec{u} = (3,-1,4)$, $\vec{v} = (6,-2,8)$

$$\vec{u}\times\vec{v} = \left(\begin{vmatrix} -1 & 4 \\ -2 & 8 \end{vmatrix}, -\begin{vmatrix} 3 & 4 \\ 6 & 8 \end{vmatrix}, \begin{vmatrix} 3 & -1 \\ 6 & -2 \end{vmatrix} \right) = (0,0,0)$$

The area of parallelogram determined by both $\vec{u}$ and $\vec{v}$ is $\|\vec{u}\times\vec{v}\| = \sqrt{(0)^2+(0)^2+(0)^2}$

$$= \boxed{0}$$

Q#15: $P_1(2,6,-1)$, $P_2(1,1,1)$, $P_3(4,6,2)$

$$\overrightarrow{P_1P_2} = (-1,-5,2)$$

$$\overrightarrow{P_1P_3} = (2,0,3)$$

$$\overrightarrow{P_1P_2}\times\overrightarrow{P_1P_3} = \left(\begin{vmatrix} -5 & 2 \\ 0 & 3 \end{vmatrix}, -\begin{vmatrix} -1 & 2 \\ 2 & 3 \end{vmatrix}, \begin{vmatrix} -1 & -5 \\ 2 & 0 \end{vmatrix} \right)$$

$$= (-15, 7, 10)$$

The area of the triangle is $\frac{1}{2}\|\overrightarrow{P_1P_2}\times\overrightarrow{P_1P_3}\|$

$$= \frac{1}{2}\sqrt{(-15)^2+7^2+10^2}$$

$$= \boxed{\frac{\sqrt{374}}{2}}$$

Q#21: $\vec{u} = (-2,0,6)$, $\vec{v} = (1,-3,1)$

$\vec{w} = (-5,-1,1$

$$\vec{u}\cdot(\vec{v}\times\vec{w}) = \begin{vmatrix} -2 & 0 & 6 \\ 1 & -3 & 1 \\ -5 & -1 & 1 \end{vmatrix} = \boxed{-92}$$

Q#29: Simplify $(\vec{u}+\vec{v}) \times (\vec{u}-\vec{v})$

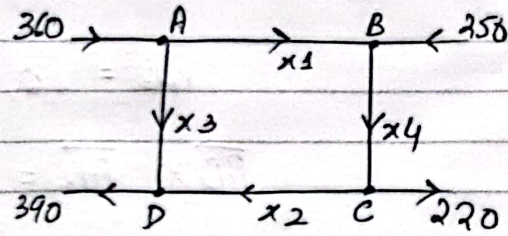
$$(\vec{u}+\vec{v} \times (\vec{u}-\vec{v})$$

$$= (\vec{u}\times\vec{u}) - (\vec{u}\times\vec{v}) + (\vec{v}+\vec{u}) + (\vec{v}\times\vec{v})$$

$$= 0-(-(\vec{v}\times\vec{u})) + (\vec{v}+\vec{u}) + 0$$

$$= \boxed{2(\vec{v}\times\vec{u})}$$ Ans.

Word Problems:



Q#1: (i)

$x_1 + x_3 = 360$ — ①

$x_1 + 250 = x_4$ — ②

$x_2 + x_3 = 390$ — ③

$x_4 = x_2 + 220$ — ④

Subtract ④ from ②

$x_4 = x_1 + 250$

$x_4 = x_2 + 220$

$- \quad - \quad -$

$0 = x_1 - x_2 + 30$

$x_1 - x_2 = -30$

Let $x_3 = a$

from ① $x_1 = 360 - a$

from ② $x_2 = 390 - a$

(ii)

$x_1 = 360 - x_3$

$x_2 = 390 - x_3$

Thus, traffic flow along AB i.e x_1 and CD i.e x_2 can be expressed in terms of traffic along AD i.e x_3.

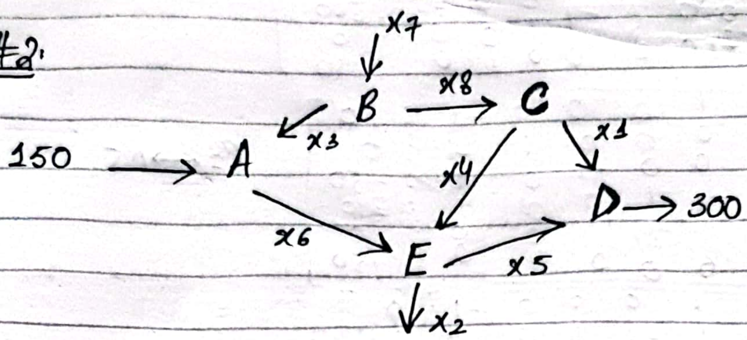
(iii) If AD and CD is closed i.e $x_3 = 0$

Then, $x_1 = 360 - 0$
$= 360$

Similarly, $x_2 = 390$
$x_3 = 0$

which is unique solution to the problem.

Q#2:



A	$x_3 + 150 = x_6$	$x_3 - x_6 = -150$
B	$x_7 = x_3 + x_8$	$x_7 - x_3 - x_8 = 0$
C	$x_8 = x_1 + x_4$	$x_8 - x_1 - x_4 = 0$
D	$x_1 + x_5 = 300$	$x_1 + x_5 = 300$
E	$x_4 + x_6 = x_2 + x_5$	$x_4 + x_6 - x_2 - x_5 = 0$
Total	$x_7 + 150 = x_2 + 300$	$x_7 - x_2 = 150.$

$$\left[\begin{array}{cccccccc:c}
0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & -150 \\
0 & 0 & -1 & 0 & 0 & 0 & 1 & -1 & 0 \\
-1 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 300 \\
0 & -1 & 0 & 1 & -1 & 1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 150
\end{array}\right]$$

$R_1 \leftrightarrow R_3$

$$= \left[\begin{array}{cccccccc|c} -1 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & -150 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 300 \\ 0 & -1 & 0 & 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 150 \end{array}\right]$$

$= (-1)R_1$

$$\left[\begin{array}{cccccccc|c} 1 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & -150 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 300 \\ 0 & 0 & 0 & 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 150 \end{array}\right]$$

$R_4 - R_1$

$$= \left[\begin{array}{cccccccc|c} 1 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & -150 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 1 & 300 \\ 0 & -1 & 0 & 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 150 \end{array}\right]$$

$R_2 \leftrightarrow R_5$

$$= \left[\begin{array}{cccccccc|c} 1 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & -150 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 1 & 300 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 150 \end{array}\right]$$

$(-1)R_2$

$$= \left[\begin{array}{cccccccc|c} 1 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & -150 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 1 & 300 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 1 & 1 & 150 \end{array}\right]$$

$R_3 + R_2$

$$= \left[\begin{array}{cccccccc|c} 1 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & -150 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 1 & 300 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 & -1 & 1 & 0 & 150 \end{array}\right]$$

$R_5 + R_3$

$$= \left[\begin{array}{cccccccc|c} 1 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & -150 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 1 & 300 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & -1 & -150 \\ 0 & 0 & 0 & 1 & 1 & -1 & 1 & 0 & 150 \end{array}\right]$$

$(-1)R_4$

$$
= \left[\begin{array}{cccccccc|c}
1 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\
0 & 1 & 0 & -1 & 1 & -1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & -150 \\
0 & 0 & 0 & 1 & -1 & 0 & 0 & -1 & -300 \\
0 & 0 & 0 & 0 & 0 & -1 & 1 & -1 & -150 \\
0 & 0 & 0 & -1 & 1 & -1 & 1 & 0 & 150
\end{array}\right]
$$

$R_1 - R_4, \quad R_2 + R_4, \quad R_3 + R_4$

$$
= \left[\begin{array}{cccccccc|c}
1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 300 \\
0 & 1 & 0 & 0 & 0 & -1 & 0 & -1 & -300 \\
0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & -150 \\
0 & 0 & 0 & 1 & -1 & 0 & 0 & -1 & -300 \\
0 & 0 & 0 & 0 & 0 & -1 & 1 & -1 & -150 \\
0 & 0 & 0 & 0 & 0 & -1 & 1 & -1 & -150
\end{array}\right]
$$

$(-1)R_5$

$$
= \left[\begin{array}{ccccccccc|c}
1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 300 \\
0 & 1 & 0 & 0 & 0 & -1 & 0 & -1 & -1 & -300 \\
0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & -150 \\
0 & 0 & 0 & 1 & -1 & 0 & 0 & -1 & -1 & -300 \\
0 & 0 & 0 & 0 & 0 & +1 & -1 & 1 & 1 & 150 \\
0 & 0 & 0 & 0 & 0 & -1 & 1 & -1 & -1 & -150
\end{array}\right]
$$

$R_2 + R_5, \quad R_3 + R_5, \quad R_6 + R_5$

$$
= \left[\begin{array}{cccccccc|c}
1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 300 \\
0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & -150 \\
0 & 0 & 1 & 0 & 0 & 0 & -1 & 1 & 0 \\
0 & 0 & 0 & 1 & -1 & 0 & 0 & -1 & -300 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & 150 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
$$

$x_1 + x_5 = 300$ $\quad$ $x_1 = 300 - x_5$

$x_2 - x_7 = -150$ $\quad$ $x_2 = x_7 - 150$

$x_3 - x_7 + x_8 = 0$ $\quad$ $x_3 = x_2 - x_8$

$x_4 - x_5 - x_8 = -300$ $\quad$ $x_4 = x_5 + x_8 - 300$

$x_6 - x_7 + x_8 = 150$ $\quad$ $x_6 = x_7 - x_8 + 150$

Let $x_5 = t$, $x_7 = s$, $x_8 = v$ then,

$x_1 = 300 - t$	$x_2 = s - 150$
$x_3 = s - v$	$x_4 = t + v - 300$
$x_5 = t$	$x_6 = s - v + 150$
$x_7 = s$	$x_8 = v$

$\longleftrightarrow$

CamScanner