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Prob & Stats

Discrete Random Variable

When there are jumps → Discrete

SINGLE CASE

Three coins → How many possibilities

$2^3 = n(S) = 8$

$S = \{HHH, HTH, THH, TTH, HHT, HTT, THT, TTT\}$

Let 'x' be the no. of Heads.

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Prob. Dist of X

x / $\cancel{X}$	P(x) or $f(x)$	$x f(x)$	$x^2 f(x)$	$(x-\mu)^2 f(x)$
0	1/8	0	0	$(0-1.5)^2 \frac{1}{8} = 0.28125$
1	3/8	3/8	3/8	0.09375 / 0.09375
2	3/8	6/8	12/8	~~0.375~~ 0.28125
3	1/8	3/8	9/8	~~2.53125~~
	1	1.5	3	0.75

$\mu = E(x) = \sum x f(x)$

$\sigma^2 = E(x^2) - [E(x)]^2$

$= \sum x^2 f(x) - (\mu)^2$

OR

$\sigma^2 = \sum (x-\mu)^2 f(x)$

$E(x) = \sum x f(x) = 1.5$

$\sigma^2 = E(x^2) - [E(x)]^2$

$\sigma^2 = \sum x^2 f(x) - (\mu^2)$

$\sigma^2 = 3 - (1.5)^2$

$\sigma^2 = 0.75$

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~~Q~~ 7, 14, 24

Q-7

a) $\Sigma f(x) = 0.20 + 0.15 + 0.25 + 0.40$

$= 1$

Probability Distribution is valid.

b) $f(30) = 0.25$

c) ~~P~~ $f(20)$ OR $(f(25))$

$f(20) + f(25) = 0.20 + 0.15$

$= 0.35$

d) $f(35) = 0.40$

$P(x > 30)$

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(14) a) $f(200) = 1 - 0.10 - 0.20 - 0.30 - 0.25 - 0.10$

$=$ ~~$\$$~~ 0.05

b) Profitable means that $X > 0$

$P(x > 0) = f(50) + f(100) + f(150) + f(200)$

$= 0.30 + 0.25 + 0.10 + 0.05$

$= 0.70$

c) $P(x \geq 100) = f(100) + f(150) + f(200)$

$= 0.25 + 0.10 + 0.05$

$= 0.40$

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Q24

Expected Profit = $E(X)$

$= \sum x \, f(x)$

Medium-Scale Expansion Profit

$= (50 \times 0.20) + (150 \times 0.5) (200 \times 0.30)$

$= 145$

large Scale Expansion Profit

$= (0 \times 0.20) + (100 \times 0.5) + (300 \times 0.30)$

$= 140$

Medium-Scale is preferred.

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$\sigma^2 = E(X^2) - [E(X)]^2$

$\sigma^2 = \sum x^2 f(x^2) - \mu^2$

Medium $x^2 f(x)$	large $x^2 f(x)$
500	0
11250	5000
12000	27000
23750	32000

$\sigma^2 = 23750 - (145)^2$ $\quad$ $\sigma^2 = 32000 - (140)^2$

$= 2725$ $\quad$ $= 12400$

$\sqrt{\sigma^2} = 52.2$ $\quad$ $\sqrt{\sigma^2} = 111.355$

$C.V = \frac{52.2 \times 100}{145}$ $\quad$ $C.V = \frac{111.355 \times 100}{140}$

$C.V = 36.2\%$ $\quad$ $C.V = 79.53\%$

Prob & Stats

$$\mu_R = E(X^R) = \begin{cases} \sum x^R f(x) & \text{for discrete} \\ \int_{-\infty}^{\infty} x^R f(x)\,dx & \text{for cont.} \end{cases}$$

x	$f(x)$	$F(x)$
0	$f(0)$	$F(0) = f(0)$
1	$f(1)$	$F(1) = f(0) + f(1)$
2	$f(2)$	$F(2) = f(0) + f(1) + f(2)$
⋮	⋮	⋮

① A shipment of 8 similar micro computers to a retail outlet contains 3 that are defective and 5 that are non-defective. If a school makes a random purchase

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of 2 of these computers, find the expected no. of defective computers purchased.

$X \sim$ defective computers

x	$P(X=x)$
0	10/28
1	15/28
2	3/28

$$f(x) = P(X=x) = \begin{cases} \dfrac{\binom{3}{x}\binom{5}{2-x}}{\binom{8}{2}} & x: \text{discrete}, \ x \leq 2, \ x = 0,1,2 \\ 0 & \text{otherwise} \end{cases}$$

$$E(X) = \sum_{i=0}^{2} x f(x) = 0.75 \approx 1$$

② Let X be a discrete random variable with the following probability distribution.

x	$f(x)$	$x f(x)$	$x^2 f(x)$
0	10/28	0	0
1	15/28	15/28	15/28
2	3/28	6/28	12/28

Find $E(g(x))$
where $g(x) = (x-1)^2$

$$\begin{aligned} E[g(x)] &= E[(x-1)^2] \\ &= E(x^2 - 2x + 1) \\ &= E(x^2) - 2E(x) + E(1) \\ &= \sum x^2 f(x) - 2\sum x f(x) + 1 \\ &= 0.96 - 2(0.75) + 1 \\ &= 0.46 \end{aligned}$$

OR

$$E(g(x)) = E[(x-1)^2]$$
$$= \sum_{i=0}^{2} (x-1)^2 f(x)$$
$$= \left(1 \times \frac{10}{28}\right) + \left(0 \times \frac{15}{28}\right)$$
$$+ \left(\frac{1 \times 3}{28}\right) = 0.46$$

③ Let X and Y be the random Variables with joint probability distribution. Find Expected value of $g(x,y) = xy$

$f(x,y)$	x: 0	1	2	
y: 0	3/28	9/28	3/28	15/28
1	3/14	3/14	0	3/7
2	1/28	0	0	1/28
Total:	5/14	15/28	3/28	

$$E(XY) \neq E(X)E(Y)$$
not Independant

$$E[g(x,y)] = E(XY)$$
$$= \sum XY\, f(x,y)$$
$$= 0+0+0+0+3/14$$
$$+0+0+0+0$$
$$= 3/14$$

12/10/23

Prob & Stats

From Previous Question.

$$E(XY) = 3/14 = \sum XY\, f(x,y)$$

$$E(XY) = E(X)E(Y)$$
$$= \sum x f(x) \sum y (f(y))$$

$$3/14 = \left[(0\times 5/14) + (1\times 15/28) + (2\times 3/28)\right]$$
$$\left[(0\times 15/28) + (1\times 3/7) + (2\times 1/28)\right]$$

$$3/14 = \left(0 + \frac{15}{28} + \frac{6}{28}\right)\left(0 + \frac{3}{7} + \frac{2}{28}\right)$$

$$3/14 = \left(\frac{21}{28}\right)\left(\frac{\cancel{14}}{2\cancel{28}}\right) = \frac{21}{28} \times \frac{1}{2}$$

$$3/14 = \frac{21}{56} = \frac{3}{8}$$

$3/14 \neq 3/8 \Rightarrow E(XY) \neq E(X)E(Y)$

NOT INDEPEDENT

Covariance : Joint Variance

Cov (X, Y)

ρ rho – population

r – sample

−1 0 +1

– ——— No Corr. ———

Inverse Direct

↑↓ ↓↑ ↑↑ ↓↓

$$\rho = \frac{Cov(X,Y)}{S.D(x)\, S.D(Y)}$$

↓

Correlation

$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$$

$$\sigma_{XY} = E(XY) - E(X)E(Y)$$
$$= E(XY) - \mu_X \mu_Y$$

$$\sigma_X^2 = E(X^2) - [E(X)]^2$$
$$\sigma_X = \sqrt{E(X^2) - [E(X)]^2}$$

$$\sigma_x = \sqrt{E(x^2) - (\mu_x)^2}$$
$$\sigma_x = \sqrt{\Sigma x^2 f(x) - [\Sigma x f(x)]^2}$$

$$\sigma_y = \sqrt{E(Y)^2 - [E(Y)]^2}$$
$$\sigma_y = \sqrt{E(Y)^2 - (\mu_y)^2}$$
$$\sigma_y = \sqrt{\Sigma y^2 f(y) - [\Sigma y f(y)]^2}$$

Q) Suppose that no. of cars x that pass through a car wash between 4:00 PM - 5:00PM on any sunny friday has the following prob. dist:

x	4	5	6	7	8	9	
$P(X=x)$	$1/12$	$1/12$	$1/4$	$1/4$	$1/6$	$1/6$	$41/6$
$(2x-1)f(x)$	$7/12$	$9/12$	$11/4$	$13/4$	$15/6$	$17/6$	$38/3$

let $g(x) = 2x-1$ represent the amount of money in dollars paid to the attendant by the manager. Find the attendant's expected earning for the particular time period.

$$E(g(x)) = E(2x-1)$$
$$= \Sigma(2x-1) f(x)$$
$$= 38/3 = 12.67$$

OR

$$E(g(x)) = E(2x-1)$$
$$= 2E(x) - E(1)$$
$$= 2\Sigma x f(x) - 1$$
$$= 2(41/6) - 1$$
$$= 41/3 - 1$$
$$= \cancel{3/8}\ 38/3$$
$$= 12.67$$

(WALPOLE CH3+4)

Prob & Stats

Q Two ball point pens are selected at random from a box that contains 3 blue pens, 2 red pens & 3 green pens. If x is the no. of blue pens selected and y is the no. of red pen selected.

Find a) prob function $f(x,y)$

b) $P[(x,y) \in A]$

where A is $[(x,y) \mid x+y \le 1]$

$x \sim$ no. of blue

$y \sim$ no. of red

$$f(x,y) = \frac{\binom{3}{x}\binom{2}{y}\binom{3}{2-x-y}}{\binom{8}{2}}$$

a)

	$f(x,y)$	x: 0	1	2	$f(y)$
y	0	3/28	9/28	3/28	15/28
	1	6/28	6/28	0	12/28
	2	1/28	0	0	1/28
$f(x)$		10/28	15/28	3/28	28/28 = 1

b) $P(x,y) \in A \Rightarrow P(x+y \le 1)$

$$f(0,0) + f(0,1) + f(1,0) = 3/28 + 6/28 + 9/28 = 18/28$$

c) Find the conditional dist. of x given $y=1$ and $P(x=0 \mid y=1)$

$$P(x \mid y=1) = P(x=0 \mid y=1) \quad P(x=1 \mid y=1) \quad P(x=2 \mid y=1)$$

$P(B \mid A) = \dfrac{P(A \cap B)}{P(B)}$

$$P(x=0 \mid y=1) = \frac{P(x=0 \cap y=1)}{P(y=1)}$$

Calculate all like this.

$P(x=0 \mid y=1)$	$P(x=1 \mid y=1)$	$P(x=2 \mid y=1)$
$\frac{6/28}{12/28}$	$\frac{6/28}{12/28}$	0
$= 1/2$	$= 1/2$	$= 0$

CONTINUOUS RANDOM VAR.

In discrete

$$\cancel{E(x) =} \sum f(x) = 1$$

In continuous

$$\int_{-\infty}^{x} f(x)\,dx = 1$$

Q let x be a continuous random variable that represent the life in hrs of a certain electronic device. R.D.F is given by

$$f(x) = \begin{cases} \frac{20000}{x^3} & x > 100 \\ 0 & \text{elsewhere} \end{cases}$$

Find expected life of this device.

$$E(X) = \int_{100}^{\infty} x\, f(x)\, dx$$

$$= \int_{100}^{\infty} \cancel{x} \frac{20000}{x^{\cancel{3}2}}\, dx$$

$$= -20000 \int_{100}^{\infty} \frac{-1}{x^2}\, dx$$

$$= -20000\left[\frac{1}{x}\right]_{100}^{\infty}$$

$$= -20000\left[\frac{1}{\infty} - \frac{1}{100}\right]$$

$$= \frac{-20000}{100} = 200$$

$$E(X) = 200$$

$$g(x) = \frac{1}{x}$$

$$E(g(x)) = E\left(\frac{1}{x}\right) = \int_{100}^{\infty} \frac{1}{x} f(x)\,dx$$

$$= \int_{100}^{\infty} \frac{1}{x} \frac{20000}{x^3}\,dx$$

$$= 20000\left[\frac{-1}{3x^3}\right]_{100}^{\infty}$$

$$= 20000\left[\frac{-1}{3(\infty)^3} - \frac{-1}{3(100)^3}\right]$$

$$= 20{,}000\left[0 + \frac{1}{3(100)^3}\right]$$

$$= 0.0066$$

Q $f(x) = \begin{cases} 2(x-1) & 1<x<2 \\ 0 & \text{otherwise} \end{cases}$

$$E(X) = \int_{1}^{2} x\, f(x)\,dx$$

$$= \int_{1}^{2} x\,(2x-2)\,dx$$

$$= \int_{1}^{2} 2x^2 - 2x\,dx$$

$$= 2\int_{1}^{2} x^2 dx - 2\int_{1}^{2} x\,dx$$

$$= 2\left[\frac{x^3}{3}\right]_1^2 - 2\left[\frac{x^2}{2}\right]_1^2$$

$$= 2\left[\frac{8}{3} - \frac{1}{3}\right] - 2\left(\frac{3}{2}\right)$$

$$= 2\left(\frac{7}{3}\right) - 3$$

$$= \frac{14}{3} - 3 = \frac{5}{3}$$

$$V(X) = E(x^2) - [E(x)]^2$$

$$= \int_1^2 x^2\, 2(x-1)\, dx - \left(\frac{5}{3}\right)^2$$

$$= \int_1^2 2x^3 - 2x^2\, dx - \left(\frac{5}{3}\right)^2$$

$$= \frac{17}{6} - \frac{25}{9} = \frac{1}{18} = 0.055\overline{5}$$

Q $$f(x) = \begin{cases} \frac{1}{3}x^2 & -1 < x < 2 \\ 0 & \text{elsewhere} \end{cases}$$

Find $E(4x+3)$

$$= 4(E(X) + 3)$$

$$E(X) = \int_{-1}^{2} x \frac{x^2}{3}\, dx$$

$$= \int_{-1}^{2} \frac{x^3}{3}\, dx = \frac{1}{3}\left[\frac{x^4}{4}\right]_{-1}^{2}$$

$$= \frac{5}{4}$$

$$E(4x+3) = 4\left(\frac{5}{4}\right) + 3 = 8$$

Q The total no. of hrs measured in unit of 100 hrs that a family runs a vaccum cleaner over a period of one year is a continuous random variable X that has the density function.

$$f(x) = \begin{cases} x & 0 < x < 1 \\ 2-x & 1 \leq x < 2 \\ 0 & \text{elsewhere} \end{cases}$$

Find Prob. of one year, a family runs their vaccum cleaner

a) less than 120 hrs

b) b/w 50 & 100 hrs.

a) $P(X) = \int_0^1 x\,dx + \int_1^{1.2} (2-x)\,dx$

$= \left[\frac{x^2}{2}\right]_0^1 + \left[2x\right]_1^{1.2} - \left[\frac{x^2}{2}\right]_1^{1.2}$

$= \left[\frac{1}{2} - 0\right] + \left[2(1.2) - 2\right] - \left[\frac{1}{2} - \frac{(1.2)^2}{2}\right] = \left(\frac{1}{2}\right) + 0.4 + 0.22$

$= 1.12$

b) $\int_{0.5}^{1} x\,dx = \left[\frac{x^2}{2}\right]_{0.5}^{1}$

$= \left[\frac{1}{2} - \frac{(0.5)^2}{2}\right] = 0.375$

Prob & Stats

(Q1) $F(x) = \begin{cases} 0 & x<0 \\ \dfrac{2x^2}{5} & 0<x<1 \\ -\dfrac{3}{5} + \dfrac{2}{5}\left(3x - \dfrac{x^2}{2}\right) & 1<x\leq 2 \\ 1 & x>2 \end{cases}$

P.D.F $P(|x|<1.5)$

↳ Derivative of $F(X)$

$f(x) = \begin{cases} \dfrac{4x}{5} & 0<x<1 \\ \dfrac{2}{5}(3-x) & 1<x\leq 2 \\ 0 & \text{otherwise} \end{cases}$

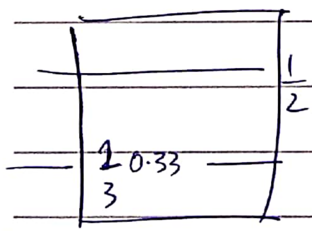
~~2~~ $P(|x|<1.5) = -1.5<x<1.5$

$= \int_{-1.5}^{0} + \int_{0}^{1} + \int_{1}^{1.5}$

calculate.

(2) $f(x) = \begin{cases} 2x & 0<x<1 \\ 0 & \text{otherwise} \end{cases}$

$$\frac{P\left[\left(x\leq \frac{1}{2}\right) \cap \left(\frac{1}{3}<x<\frac{2}{3}\right)\right]}{P\left(\frac{1}{3}<x<\frac{2}{3}\right)}$$



$= 0.5$

$$= \frac{P\left(\frac{1}{3} < x < \frac{1}{2}\right)}{P\left(\frac{1}{3} < x < \frac{2}{3}\right)}$$

$$= \frac{2\left(\frac{x^2}{2}\right)\Big|_{1/3}^{1/2}}{2\left(\frac{x^2}{2}\right)\Big|_{1/3}^{2/3}}$$

DISCRETE

Binomial	Hypergeometric	Poisson
fix trials n	fix trials n	inf. seq. of process
ind.	dep.	ind.
with replacement	without replacement	$\mu = np$
P(S) constant, remains same	P(S) changes on trial to trial	$n > 20$ & $p < 0.05$
n, p	n, N, R	
$\frac{\text{disc. Occur.}}{\text{disc. Trials}}$	$\frac{\text{disc. Occur.}}{\text{disc. Trials}}$	$\frac{\text{disc. Occur}}{\text{Cont. Trials}}$

Experiment can be categorized into S & F

$P(S) + P(F) = 1$

$p + q = 1$

$b(x; n, p)$

$$p(x) = \binom{n}{x} p^x q^{n-x}$$

where $x = 0, 1, 2 \ldots\ldots n$

$p + q = 1$ and $q = 1 - p$

p = prob. of success

q = prob of failure

$$\mu = E(x) = np$$

$$\sigma^2 = V(x) = npq$$

$$\sigma = SD(x) = \sqrt{npq}$$

$h(x; n, N, k)$

$$p(x) = \frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}$$

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Such that

$0 \leq x \leq n$

$0 \leq x \leq K$

$p = \frac{K}{N}$, $q = \frac{N-K}{N}$

Mean $= \mu = E(X) = n \frac{K}{N} = np$

Variance $= \sigma^2 = V(X) = (n)\left(\frac{K}{N}\right)\left(1-\frac{K}{N}\right)\left(\frac{N-n}{N-1}\right) = npq\,fpc$

Standard Deviation : $\sigma = \sqrt{npq\left(\frac{N-n}{N-1}\right)}$

N = no. of units in population.

K = no. of sucesses in population

n = no. of units in sample.

fpc → finite pop. correction factor.

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$P(x; \mu)$

$$p(x) = \frac{e^{-\mu} \mu^x}{x!}$$

where $x = 0, 1, 2, \ldots$

Mean = Var (by property)

$\mu = \sigma^2$

If $\mu = np$ and

n is larger with smaller p

~~n is~~ such as $n > 20$ and

$p < 0.05$. use poisson approximation.

---

b ~ binomial

h ~ hypergeometric

p ~ poisson

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b, p, h

$\boxed{\begin{array}{l}\text{disc. occurences} \\ \text{disc. Trials}\end{array}}$

$\boxed{\begin{array}{l}\text{disc. Occurences} \\ \text{Cont. Trials}\end{array}}$

only p

Q26 $\quad n = 10 \quad p = 0.10$

a) $f(0) = \binom{10}{0}(0.10)^{10}(0.90)^{10-0}$

$= 3.487 \times 10^{-11}$

b) $f(2) = \binom{10}{2}(0.10)^{10}(0.90)^{10-2}$

$= 1.94 \times 10^{-9}$

c) $P(X \leq 2) = P(x=0) + P(x=1) + P(x=2)$

$= \sum_{x=0}^{2} \binom{10}{x}(0.10)^{10}(0.90)^{10-x}$

$= 3.49 \times 10^{-11} + 3.9 \times 10^{-10} + 1.94 \times 10^{-9}$

$= 2.36 \times 10^{-9}$

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d) $P(x \geq 1) = 1 - P(x < 1)$

$= 1 - P(x = 0)$

$= 1 - 1.348 \times 10^{-11}$

$E(X) = np = 10 \times 0.10$

$V(x) = npq = 10 \times 0.10 \times 0.90$

$\sigma = \sqrt{npq} =$

Q = 28

$p = 0.23$

$n = 6$

a) $p(2) = \binom{6}{2} (0.23)^{2} (0.77)^{6-2}$

b) $n = 6$

$P(x \geq 2) = 1 - P(x < 2)$

$\cancel{= 1 - P(P}$

$= 1 - [P(x=0) + P(x=1)]$

c) $n = 10$

$$P(x=0)\ \binom{10}{0}(0.23)^{10}(0.77)^{10-0}$$

Q42 Poisson.

$\mu = 7/\text{min}$

a) $p(0) = \dfrac{e^{-7}\, 7^{0}}{0!}$

b) $p(x \geq 2) = 1 - p(x < 2)$

$$= 1 - \left[\left(\frac{e^{-7}\, 7^{0}}{0!}\right) + \left(\frac{e^{-7}\, 7^{1}}{1!}\right)\right]$$

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c) $P(x \geq 1) = 1 - P(x < 1)$

$= 1 - P(x = 0)$

$= 1 - \frac{e^{-3.5}\, 3.5^0}{0!}$ $\quad \mu = \frac{7}{6} \times 30$

d) $P(x \geq 5)$

$= 1 - \displaystyle\sum_{x=0}^{4} \frac{e^{-7}\, 7^x}{x!}$

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Prob & Stats

Q62

$$P(x \geq 3) = 1 - P(x < 3)$$

$$= 1 - \sum_{x=0}^{3} \frac{e^{-1.5}\, 1.5^{x}}{x!}$$

$$= 1 - [(0.223) + (0.335) + (0.251)]$$

$$= 0.19$$

Q $\frac{0.5}{100} = 0.005$

$\mu = np = 100 \times 0.005 = 0.5$

$$f(x) = \frac{e^{-\mu}\, \mu^{x}}{x!}$$

$$= \frac{e^{-0.5}\, 0.5^{0}}{0!}$$

$$= 0.6065$$

61% of the boxes

Q50

Texas	Hawaii	Total	
40	20	60	N

X_H = Hawaii

$$p(x) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$$

a) $P(x_H = 0)$

$$p(x) = \frac{\binom{20}{0}\binom{60-20}{10-0}}{\binom{60}{10}}$$

$= 0.0112$

b) $P(x_H = 1)$

$$p(x) = \frac{\binom{20}{1}\binom{60-20}{10-1}}{\binom{60}{10}}$$

$= 0.0725$

c) $P(x \geq 2) = 1 - P(x < 2)$

$= 1 - (0.0112 + 0.725)$

$= 0.9163$

d) $P(x_T = 9)$

$$= \frac{\binom{40}{9}\binom{60-40}{10-9}}{\binom{60}{10}}$$

$= 0.0725$

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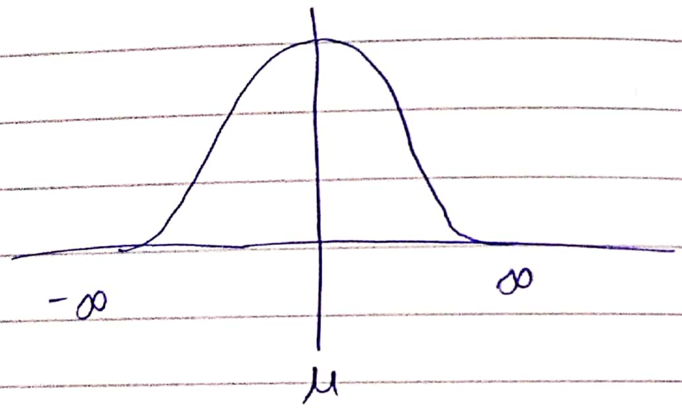
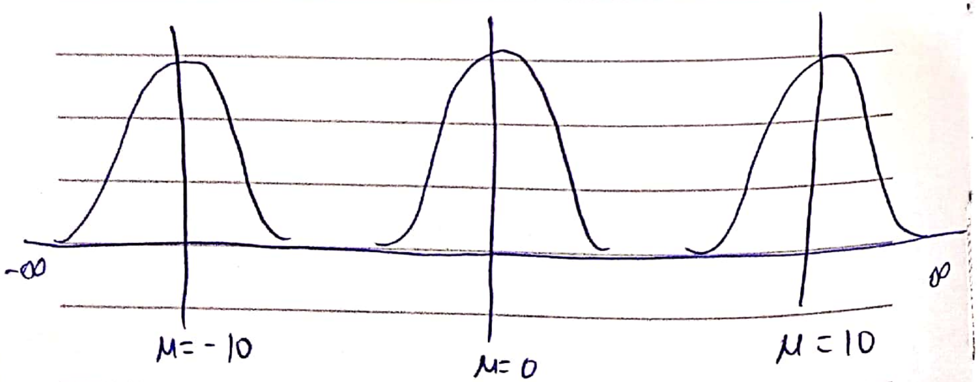
NORMAL DISTRIBUTION:

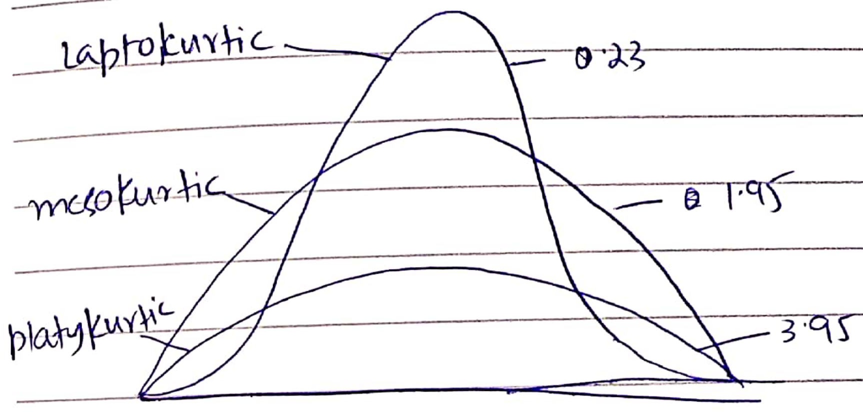


Mean = med = mode

-- If mean is changed in normal distribution, graph moves on the x-axis.



-- If σ is changed, graph is moved on the y-axis.



Data more spread → σ is high

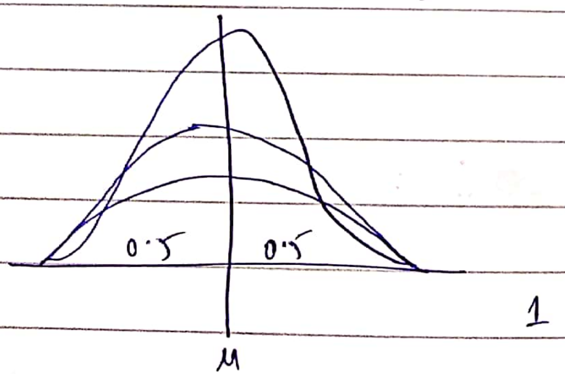
Properties:

1) μ = location parameter
   σ = shape parameter
2) Highest point on normal dist. is at mean = med = mode.
3) Mean of the distribution can

be any numerical value. +ve, −ve or zero.

4) Normal distribution is symmetric and bell shape, skewness measure is zero. Left of the mean is a mirror image of the right of the mean.

5) S. Dev determines how flat & wider the curve is.

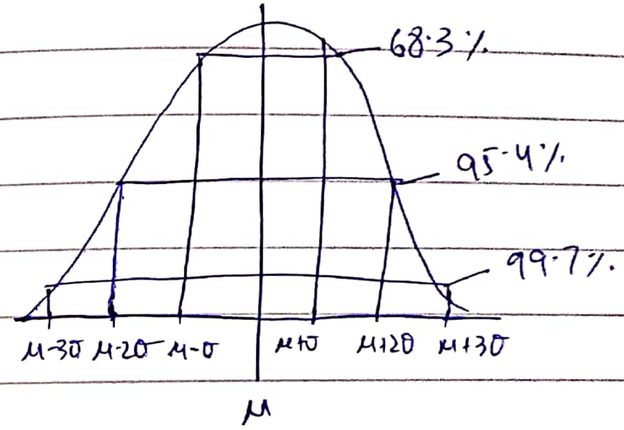


6) The total area under the curve is ONE.

7) $\mu \pm \sigma = 68.3\%$

$\mu \pm 2\sigma = 95.4\%$

$\mu \pm 3\sigma = 99.7\%$



Prob & Stats

Normally distribution

↓

Standard Normal distribution

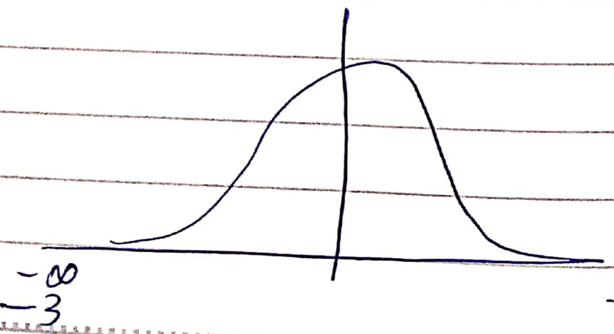
3.3 NUST 4.0

3.3 QAU 5.0

Ratio %

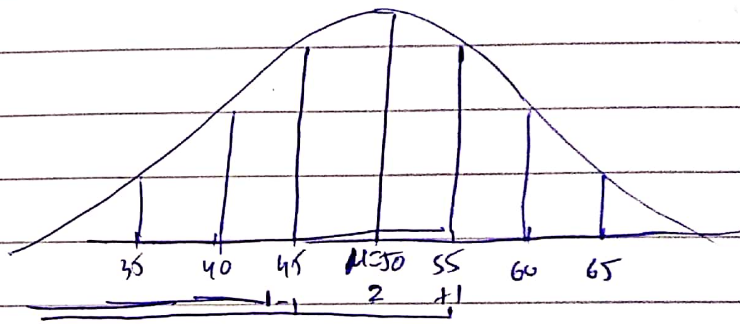
* Bring on scale to see which is better

$Z = \frac{x-\mu}{\sigma}$ $\mu_z = 0$, $\sigma_z = 1$



Q9 $\mu = 50$, $\sigma = 5$

a)



σ $\mu \pm 5$

$\mu \pm 10$

$\mu \pm 15$

b) $P(45 < x < 55)$

$P\left(\frac{45-50}{5} < \frac{x-\mu}{\sigma} < \frac{55-50}{5}\right)$

$P(-1 < Z < +1)$

$= P(z<1) - P(z<-1)$
$= 0.8413 - 0.1587$
$= 0.6826$

Q12

a) $P(0 \le z \le 0.83)$

$= P(z \le 0.83) - P(z < 0)$
$= 0.7967 - 0.5$
$= 0.2967$

b) $P(-1.57 \le z \le 0)$

$= P(z \le 0) - P(z < -1.57)$
$= 0.5 - 0.0582$
$= 0.4418$

c) $P(z > 0.44)$

$= 1 - (z < 0.44)$
$= 1 - 0.6700$
$= 0.33$

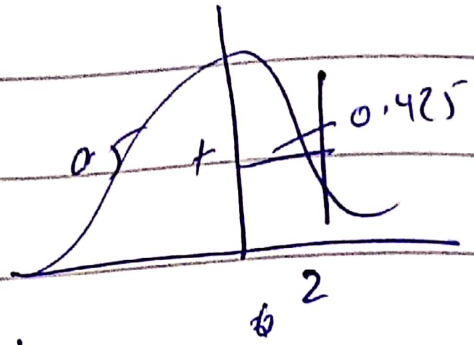
d) $P(z \ge -0.23)$

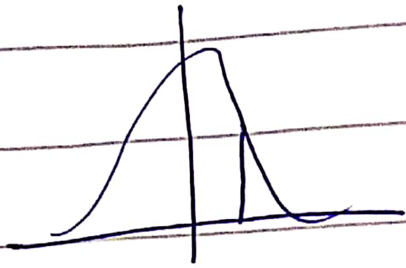
$= 1 - (z < -0.23)$
$= 1 - 0.4880$
$=$

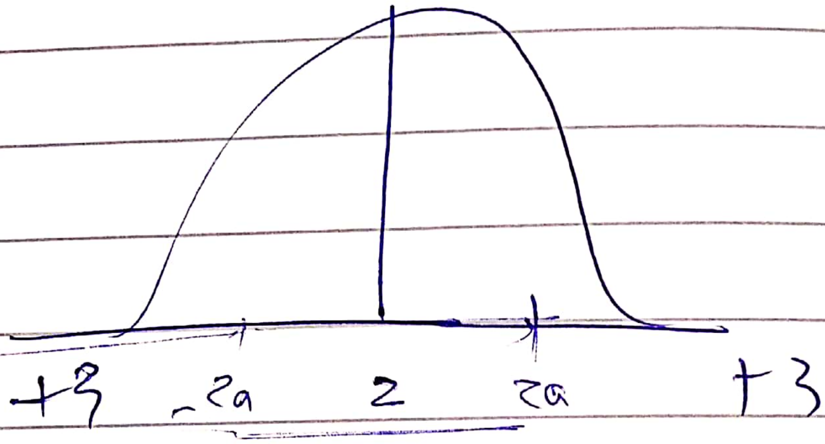
e) $P(z < 1.20)$
$= 0.8849$

Q14

a) $P(z \le z_a) = 0.9750$
$z_a = 1.96$
$P(z \le 1.96) = 0.9750$

b) z  $= 0.9750$

d) = 



~~$-z <$~~

$-z_a < z < z_a$

$P(z < z_a) - P(z < -z_a) = 0.9030$

$\downarrow$

1.403

0.9236

z ~~$\;\;$~~ $+0.9030$

0.5

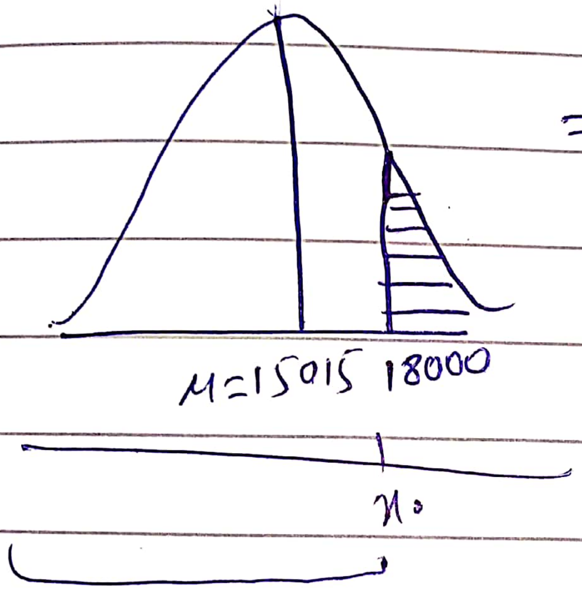
Prob & Stats

Q-17 $\mu = 15015$

$\sigma = 3540$

debt $(x) \sim$ Normally dist

a) $P(x > 18000) = 1 - P(x < 18000)$



$= 1 - P\left(\frac{x-\mu}{\sigma} < \frac{18000-15015}{3540}\right)$

$= 1 - P(z < 0.84)$

$= 1 - 0.7995$

$= 0.2005$

~~$= 1 - P(z < 0.84)$~~

b) $P(x < 10000)$

$= P\left[\frac{x-\mu}{\sigma} < \frac{10000 - 15015}{3540}\right)$

$= P(z < -1.416)$

$= P(z < 1.42)$

$= 0.778$

c) $P(12000 < x < 18000)$

$= P\left(\frac{12000 - \cancel{105}\ 15015}{3540} < \frac{x - \cancel{\mu}}{\sigma} < \frac{18000 - 15015}{3540}\right)$

$= P(-0.85 < \cancel{z}\ z < 0.84)$

$= P(z < 0.84) - P(z < -0.85)$

$= 0.7995 - 0.1977$

$=$

18 $\mu = 30$

$\sigma = 8.20$

a) $P(x \geq 40)$

$= 1 - P(x < 40)$

$= 1 - P\left(\frac{x - \cancel{\mu}}{\sigma} < \frac{40 - 30}{8.20}\right)$

$= 1 - P(z < 1.2195)$

$= 1 - P(z < 1.22)$

$= 1 - 0.8888$

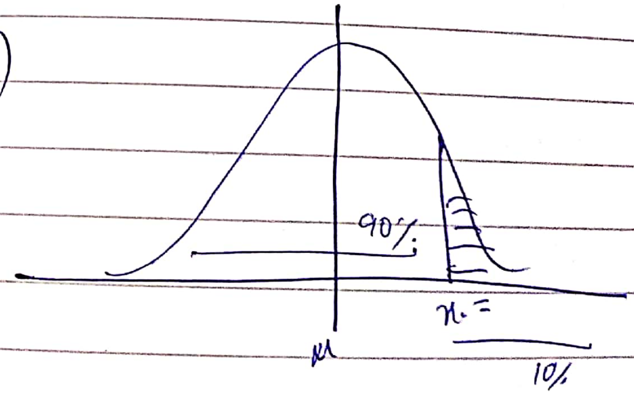
$= 0.1112$

b) $P(x \leq 20)$

$= \cancel{1 - P}\ P\left(\frac{x - \mu}{\sigma} < \frac{20 - 30}{8.2}\right)$

$= P(z < -1.22)$

c)



$z = \frac{x - \mu}{\sigma}$

$x = \mu + \sigma z$

$x = \cancel{8}\ 30 + (8.2)(1.28)$

$P(z < 0.90) = \cancel{1.28}$

$z = 1.28$

~~P~~

$= \cancel{8}\ x = 40.50$

19 $\mu = 328$

$\sigma = 92$

$P(x < \cancel{500}\ 250)$

$P\left(\frac{x-\mu}{\sigma} < \frac{250-328}{92}\right)$

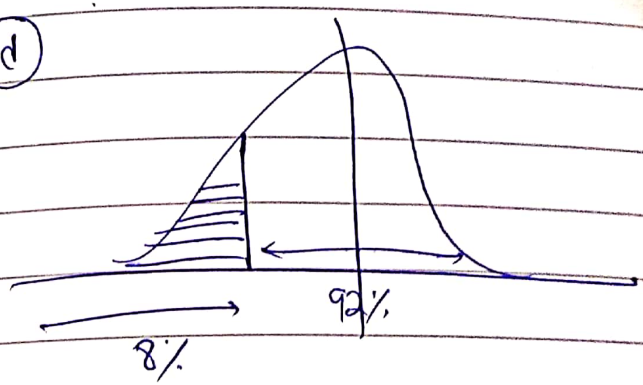
$P(z < -0.84)$

$z = 1.0$

$x = \mu + \sigma z$

$x = 328 + 92(1).$

(d)



$\cancel{P(x < 0.08) = P\left(\frac{x-\mu}{\sigma} < \frac{0.08-328}{92}\right)}$

$\cancel{= P(z < -3.56)}$

$P(z < 0.08) \Rightarrow z = -1.41$

$x = \sigma z + \mu$

$x = (92 \times -1.41) + 328$

$x = 198.28$

Date

Q42 $\mu = 6312$

$z < 0.05 \Rightarrow -1.645$

$\frac{x-\mu}{\sigma} = z$

$\frac{x-\mu}{z} = \sigma$

$\frac{1000-6312}{-1.645} = \cancel{3320}$ 3229

highest possible $6312 + 3320$

$0.03 < \cancel{\mathbf{x}}\, z < \underline{1}$

$P(z<1) - P(z < 0.03)$

$-(-1.88)$

Scanned with CamScanner

Date

$z = 4.88$

$x = \sigma z + \mu$

$x = 3229\,(4.88) + 6312$

$x = 22069$

$1 - P(z < 0.03)$

$1 + 1.88$

$x = 15611$