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Assignment No 2:

MTH105

Multivariable Calculas

Submitted by:

Haider Ali

FA21-BEE-053

Submitted to:

Syed Qazmi Usman

Q#1 Evaluate each of the following limits

a) $$\lim_{(x,y)\to(2,1)} \frac{x^2-2xy}{x^2-4y^2}$$

b) $$\lim_{(x-y)\to(0,0)} \frac{x-4y}{6y+7x}$$

c) $$\lim_{(x,y)\to(0,0)} \frac{x^2-y^6}{xy3}$$

d) $$\lim_{(x,y,z)\to(-1,0,4)} \frac{x^3-ze^{zy}}{6x+2y-3z}$$

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QUESTION 01

Q#1 Evaluate Limit

a) $\lim_{(x,y)\to(2,1)} \frac{x^2-2xy}{x^2-4y^2}$

$= \lim_{(x,y)\to(2,1)} \frac{x(x-2y)}{x^2-(2y)^2}$

$= \lim_{(x,y)\to(2,1)} \frac{x(x-2y)}{(x-2y)(x+2y)}$

$= \lim_{(x,y\to 2,1)} \frac{x}{x+2y}$

$= \frac{2}{2+2(1)}$

$= \frac{2}{4}$

$= \frac{1}{2}$

b) $\lim_{(x,y)\to(0,0)} \frac{x-4y}{6y+7x}$

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put $x=0$

$= \frac{0-4y}{6y} = \frac{-2}{3}$

put $y=0$

$= \frac{x}{7x}$

$= \frac{1}{7}$

$= \left(\frac{-2}{3}, \frac{1}{7}\right)$

c) $\lim_{(x,y)\to(0,0)} \frac{x^2-y^6}{xy^3}$

$x=0$

$= \frac{-y^6}{0(y^3)}$

$= -\infty$

$y=0$

$= \frac{x^2}{x(0)}$

$= \infty$

$= (-\infty, \infty)$

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(4)

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d) $$\lim_{(x,y,z)\to(-1,0,4)} \frac{x^3 - 2e^{zy}}{6x + 2y - 3z}$$

$$= \frac{(-1)^3 - (4)\, e^{2(0)}}{6(-1) + 2(0) - 3(4)}$$

$$= \frac{-1 - 4(1)}{-6 - 12}$$

$$= \frac{-5}{-18}$$

$$= \frac{5}{18}$$

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QUESTION 03

Q# 3 Determine directional derivative of $f(x,y,z) = 4x - y^2 e^{3xz}$ at $(3,-1,0)$ in the direction of $\vec{V} = (-1,4,2)$

Solution:-

$f(x,y,z) = 4x - y^2 e^{3xz}$

$$\frac{\partial}{\partial x} = 4 - zy^2 e^{3xz}$$

at $(3,-1,0)$

$$\frac{\partial}{\partial x} = 4 - 0$$
$$= 4$$

$$\frac{\partial}{\partial y} = -2y e^{3xz}$$

at $(3,-1,0)$

$$= -2(-1) e^{3(0)}$$
$$= 2$$

$$\frac{\partial}{\partial z} = -3y^2 x e^{3xz}$$

at $(3,-1,0)$

$$= -3(-1)^2(3)$$
$$= -9$$

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$f(x,y,z) = \langle 4,2,-9 \rangle$

$$\vec{V} = \frac{(-1,4,2)}{\sqrt{(-1)^2+(4)^2+(2)^2}}$$

$$= \sqrt{21}$$

$$\hat{u} = \frac{-1}{\sqrt{21}} + \frac{4}{\sqrt{21}} j + \frac{2}{\sqrt{21}} k$$

$$= \frac{-4}{\sqrt{21}} i + \frac{8}{\sqrt{21}} j - \frac{18}{\sqrt{21}} k$$

$$D_i f(x,y,z) = \frac{-4}{\sqrt{21}} \hat{i} + \frac{8}{\sqrt{21}} \frac{-18}{\sqrt{21}} \hat{k}$$

QUESTION 04

Find the maximum rate of change of the function at the indicated point and the direction in which this rate of change Occurs.

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(a)

$f(x,y) = \sqrt{x^2+y^2}$ at $(-2,3)$

Solution:-

$$\frac{\partial}{\partial x} = \frac{1}{2\sqrt{x^2+y^2}} \quad 2x$$

at $(-2,3)$

$$= \frac{1}{\cancel{2}\sqrt{4+9}} \cdot \cancel{2}(-2)$$

$$= \frac{-2}{\sqrt{13}}$$

$$\frac{\partial}{\partial y} = \frac{1}{\cancel{2}\sqrt{x^2+y^2}} \quad \cancel{2}y$$

at $(-2,3)$

$$= \frac{1}{\sqrt{2^2+3^2}} \quad (3)$$

$$= \frac{1}{\sqrt{13}} \quad 3$$

$$= \frac{3}{\sqrt{13}}$$

$$\hat{u} = \frac{-2}{\sqrt{13}}\,i + \frac{3}{\sqrt{13}}\,j$$

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(b)

$f(x,y,z) = e^{2y}\cos(y-2z)$ at $(4,-2,0)$

Solution:-

$$\frac{\partial}{\partial x} = 2e \quad \cos(y-2z)$$

at $(4,-2,0)$

$$= 2e^{8}\cos(-2;0)$$

$$= 2e^{8}\cos(-2)$$

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial y}\left[e^{2x}\cos(y-2z)\right]$$

$$= -e^{2x}\sin(y-2z)$$

at $(4,-2,0)$

$$= -e^{8}\left[\sin(-2,-0)\right]$$

$$= -e^{8}\sin(-2)$$

$$\frac{\partial}{\partial z} = \frac{\partial}{\partial z} e^{2}\cos(y-2z)$$

$$= 2e^{2x}\sin(y-2z)$$

at $(4,-2,0)$

$$= 2e^{8}\sin(-2,0)$$

$$= 2e^{8}\sin(-2$$

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$f(x,y,z) = \langle 2e^{8}(0.999), -e^{8}(-0,034), 2e^{8}(-0,034)$

$\hat{u} = 2e^{8}(0.999)\hat{i} + e^{8}(0.034)\hat{j} - 2e^{8}(0.034)\hat{k}$

QUESTION 02

Q#2 Determine $D_{\vec{u}} f$ for the given function in the indicated direction

(a)

$f(x,y) = \cos\left(\frac{x}{y}\right)$ in the direction of $\vec{v} = (3,-4)$

$\vec{v} = (3,-4)$

for $\frac{\partial}{\partial x} = \frac{\partial}{\partial x} \cos(x/y)$

Solving

$\frac{\partial}{\partial x} = \frac{-\sin\frac{x}{y}}{y}$

for $\frac{\partial}{\partial y} = \frac{\partial}{\partial y} \cos(x/xy)$

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$$\frac{\delta}{\delta y} = \frac{x \sin\left(\frac{x}{y}\right)}{y^2}$$

So

$$f_x = \frac{-(\sin \frac{x}{y})}{y}$$

and $$f_y = \frac{x\,(\sin(\frac{x}{y}))}{y^2}$$

Now for

$$D_u f(x,y) = \frac{-(\sin \frac{x}{y})}{y}(3) + \frac{x(\sin \frac{x}{y})}{y^2}(-4)$$

$$= \frac{-3(\sin \frac{x}{y})}{y} - \frac{4(\sin \frac{x}{y})}{y^2}$$

(b)

$f(x,y,z) = x^2y^3 - 4xz$ in direction of $\vec{v} = (-1,2,0)$

$\bar{v} = (-1,2,0)$

finding $f_x = \frac{\delta}{\delta x}(x^2y^3 - 4xz)$

$$= 2xy^3 - 4z$$

For $f_y = \frac{\delta}{\delta y}(x^2y^3 - 4xz]$

$$= 3y^2x^2$$

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For $f = \frac{6}{6z}(x^2y^3 - 4xz)$

$= -4x$

Now for $D_{\vec{u}} f$

$D_{\vec{u}} f = (2xy^3 - 4z)(-1) + (3x^2y^2)(2)(4x)(0)$

$= -2xy^3 + 4z + 6x^2y^2$

$D_{\vec{u}} f = -2xy^3 + 4z + 6x^2y^2$

QUESTION 05

Compute div $\vec{F}$ and Curl $\vec{f}$

(a)

$\vec{F} = x^2y\hat{i} - (z^3 - 3x)\hat{j} + 4y^2\hat{k}$

$\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$

$\nabla \cdot F$ for divergence

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$$\nabla \cdot F = \frac{\partial}{\partial x}(x^2 y) - \frac{\partial}{\partial y}(z^3 - 3x) + \frac{\partial}{\partial}(4y^2)$$

$$= 2xy - 0 + 0$$

~~$\nabla \cdot F = 2xy$~~

~~Now for curl $\nabla \times F$~~

~~$\nabla \times F \quad \partial / \partial x$~~

$$\nabla \cdot F = 2xy$$

Now for curl $\nabla \times F$

$$\nabla \times F = \begin{vmatrix} i & j & k \\ \partial/\partial x & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 y & -z^3 + 3x & 4y^2 \end{vmatrix}$$

$$= \hat{i}\left[\left(\frac{\partial}{\partial y}(4y^2) + \frac{\partial}{\partial z}(-z^3 + 3x)\right] - \hat{j}\left[\left(\frac{\partial}{\partial x}(4y^2) - \frac{\partial}{\partial z}(x^2 y)\right]$$

$$+ \hat{x}\left[\frac{\partial}{\partial x}(-z^3 + 3x) - \frac{\partial}{\partial y}(x^2 y)\right.$$

$$= (8y - 3z^2)\hat{i} - \hat{j}(0 - 0) + \hat{k}(3 - x^2)$$

$$\nabla \times F = 8y - 3z^2 + (3 - x^2)\hat{k}$$

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(b)

$$\vec{F} = (8x + 2z^2)\hat{i} + \frac{x^3y^2}{z}\hat{j} - (z - 7x)\hat{k}$$

As $\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$

For Div $\vec{F}$

$$\nabla \cdot F = (8x + 2z^2)\frac{\partial}{\partial x} + \left(\frac{x^3y^2}{z}\right)\frac{\partial}{\partial y} + (-z + 7x)\frac{\partial}{\partial z}$$

$$= 8 + \frac{2x^3y}{z} - 1$$

$$\nabla \cdot F = \frac{2x^3y}{z} + 7$$

For Curl $\vec{F}$

$$\nabla \times F = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ (8x + 2z^2) & \frac{x^3y^2}{z} & -z + 7x \end{vmatrix}$$

$$= i\left[\left(\frac{\partial}{\partial y}(-z + 7x) - \left(\frac{\partial}{\partial z}\right)\left(\frac{x^3y^2}{z}\right)\right) - j\left[\left(\frac{\partial}{\partial x}(-z + 7x)\right.\right.\right.$$

$$- \frac{\partial}{\partial z}(8x + 2z^2)\Big] + k\left[\left(\frac{\partial}{\partial x}\left(\frac{x^3y^2}{z}\right) - \frac{\partial}{\partial y}(8x + 2z^2)\right)\right.$$

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$$= \left(-\frac{2x^3y^2}{z^2}\right)i - (7 - 4z)j + \left(\frac{3x^2y^2}{z} - 0\right)k$$

$$\nabla \times F = \left(-\frac{2x^3y^2}{z^2}\right)\hat{i} - (7-4z)\hat{j} + \frac{3x^2y^2}{z}\hat{k}$$

QUESTION 07

Q#7 Given the following information use the chain rule to determine $\frac{dz}{dt}$.

where $z = \cos(yx^2)$ $x = t^4 - 2t$, $y = 1 - t^6$

(a)

Find $\frac{dz}{dt}$

$z = \frac{x^2 - w}{y^4}$, $x = t^3 + 7$, $y = \cos(2t)$, $w = 4t$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt} \quad (1)$$

So

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$$\frac{dx}{dt} = \frac{d}{dt}(t^3+7)$$

$$= 3t^2$$

$$\frac{\delta z}{\delta x} = \frac{1}{y^4}(2x)$$

$$\frac{dy}{dt} = \frac{d}{dt}(\cos(2t))$$

$$= -\sin 2t \quad (2)$$

$$\frac{\delta z}{\delta y} = \frac{d}{dy}\left(\frac{x^2-4t}{y^4}\right)$$

$$= \frac{-}{y^5}(x^2-4t)$$

putting values in eq (i)

$$\frac{dz}{dt} = \frac{2x}{y^4}\cdot 3t^2 + \left(\frac{-5}{y^5}(x^2-4t)\right)\cdot$$

$$(-2\sin 2t)$$

$$\frac{dz}{dt} = \frac{2}{y^4}\left(3xt^2 + \frac{5(x^2-4t)\sin 2t}{y}\right)$$

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(b)

Find $\frac{dz}{dx}$, when

$z = x^2y^4 - 2y$, $y = \sin x^2$

put y value in z.

$z = x^2(\sin^2)^4 - 2\sin x^2$

taking differentiation with respect to x

$$\frac{dz}{dx} = \frac{d}{dx}\left[x^2(\sin x^2)^4 - 2\sin x^2\right]$$

$$= 2x\left[(\sin(x^2))^4\right] + x^2\left[4(\sin^2 y)\right]^3 \cdot \cos(x^2)\cdot 2x - 4x\cos(x^2)$$

$$= 2x\sin^4(x^2) + 8x^3\cos x^2\sin^3 x^2 - 4x\cos(x^2)$$

(c)

Compute $\frac{dy}{dx}$ for the following equation

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$$x^2y^4-3z = \sin xy$$

differentiating w.r.t x on b/s

$$\frac{d}{dx}(x^2y^4-3) = \frac{d}{dx}\sin xy$$

$$\frac{dy}{dx}x^2 3y^3+y^4 2x = \cos xy\cdot\left(y+\frac{dy}{dx}\cdot x\right)$$

$$\frac{dy}{dx}(3x^2y^3)-\cos xy\frac{dy}{dx}\cdot x = \cos xy - y - y^4 2x$$

$$\frac{dy}{dx}\left(3x^2y^3-\cos(xy)\cdot x\right) = \cos xy\cdot y - y^4 2x$$

$$\frac{dy}{dx} = \frac{\cos xy\cdot y - 2y^4 x}{3x^2y^3\cdot\cos xy\cdot x}$$

$$\frac{dy}{dx} = \frac{y}{x}\left(\frac{\cos xy - 2y^3x}{3xy^3-\cos xy}\right)$$

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QUESTION 06

Q# 6 Determine if the vector field is Conservative.

Def^n: A Vector field on two (or three) dimensional space is a function $\vec{F}$ assigns to each point (x, y) or (x, y, z) two (or three dimensinal vector given by $\vec{F}(x, y)$ (or $\vec{F}(x, y, z)$)

(a)

$$\vec{F} = x^2 y \hat{i}$$

$$\vec{F} = \left(4x^2 + \frac{3x^2 y}{z^2}\right)\hat{i} + \left(8xy + \frac{x^3}{z^2}\right)\hat{i} + \left(11 - \frac{2x^3 y}{z^3}\right)\hat{k}$$

Solution:

The vector field is conservate if and only if

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$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad \frac{\partial N}{\partial z} = \frac{\partial P}{\partial y} \quad \frac{\partial M}{\partial z} = \frac{\partial P}{\partial x}$$

$$\vec{F} = \underbrace{\left(4y^2 + \frac{3x^2 y}{z^2}\right)}_{M} \hat{i} + \underbrace{\left(8xy + \frac{x^3}{z^2}\right)}_{N} \hat{i}$$

$$+ \left(11 - \frac{2x^3 y}{z^3}\right) \hat{k} \Rightarrow P$$

$$\frac{\partial M}{\partial y} = 8y + \frac{3x^2}{z}, \quad \frac{\partial N}{\partial x} = 8y - \frac{3x^2}{z^2}$$

$$\frac{\partial N}{\partial z} = x^3 z^{-2} = x^3(-2)z^{-3} = \frac{-2x^3}{z^3}$$

$$\frac{\partial P}{\partial y} = \frac{-2x^3}{z^3}$$

$$\frac{\partial M}{\partial z} = 4x^2 + \frac{3x^2 y}{z^2} = 3x^2 y(-2)z^{-3}$$

$$= \frac{-6x^2 y}{z^3}$$

$$\frac{\partial P}{\partial x} = \frac{\partial}{\partial x}\left(11 - \frac{2x^2 y}{z^3}\right) = \frac{-6x^2 y}{z^3}$$

Hence

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}, \quad \frac{\partial N}{\partial z} = \frac{\partial P}{\partial y} \quad \frac{\partial M}{\partial z} = \frac{\partial P}{\partial x}$$

Since the vector field is conservative.

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(b)

$\vec{F} = 6x\hat{i} + (2x - y^2)\hat{i} + (6z - x^3)\hat{x}$

The vector field is conservative if and only if

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \cdot \frac{\partial N}{\partial z} = \frac{\partial P}{\partial y}, \frac{\partial M}{\partial z} = \frac{\partial P}{\partial x}$

$\vec{F} = \underset{M}{\underline{6x\hat{i}}} + \underset{N}{\underline{(2x - y^2)\hat{i}}} + \underset{P}{\underline{(6z - x^3)\hat{k}}}$

$\frac{\partial M}{\partial y} = \frac{\partial (6x)}{\partial y} = 0, \frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(2x - y^2)$

$= 2$

$\frac{\partial N}{\partial z} = \frac{\partial}{\partial z}(2x - y^2) = 0, \frac{\partial P}{\partial y} = \frac{\partial}{\partial y}(6z - x^3) = 0$

$\frac{\partial M}{\partial z} = \frac{\partial}{\partial z} 6x = 0, \quad \frac{\partial P}{\partial x} = \frac{\partial}{\partial x}(6z - x^3)$

$= -3x^2$

So,

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}, \frac{\partial N}{\partial z} = \frac{\partial P}{\partial y}, \frac{\partial M}{\partial z} \neq \frac{\partial P}{\partial x}$

Since, the vector field is not Conservative.