Lecture

Graph

Shortest Path

- In graph theory the shortest path between two vertices in a graph is a path between those two vertices in such way that the sum of weights of edges in that path is the smallest than the sum of weights of edges in any path between those two vertices provided the graph is a weighted graph.

- So the shortest path problem is to find such a path between the given vertices.

Single-Source Shortest Path Problem

- **Single-Source Shortest Path Problem** - The problem of finding shortest paths from a source vertex *v* to all other vertices in the graph.
  - Dijkstra's Algorithm
  - Bellman Ford Algorithm

Dijkstra's algorithm

Dijkstra's algorithm - is a solution to the single-source shortest path problem in graph theory.

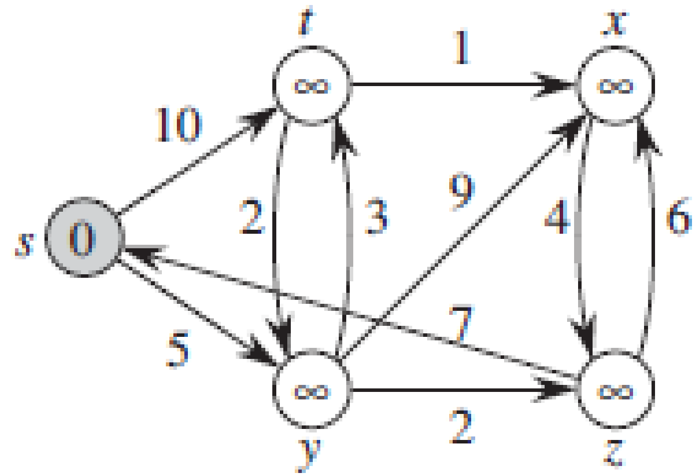
Works on both directed and undirected graphs.

Approach: Greedy

Input: Weighted graph G={E,V} and source vertex $s \in V$,

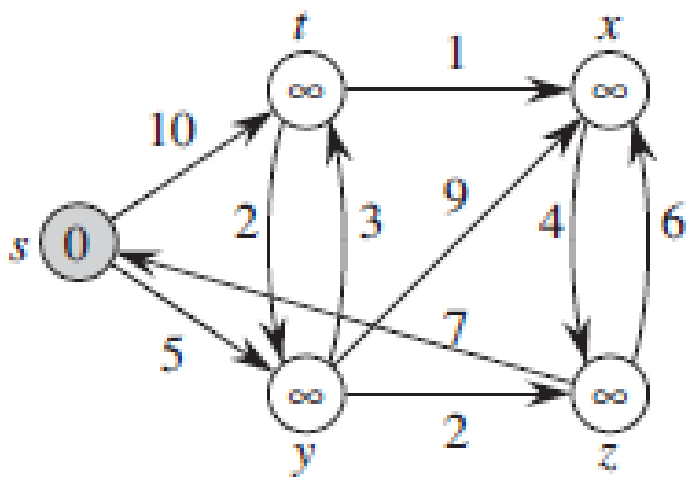
Output: the shortest path from a given source vertex $s \in V$ to all other vertices

Dijkstra's Algorithm



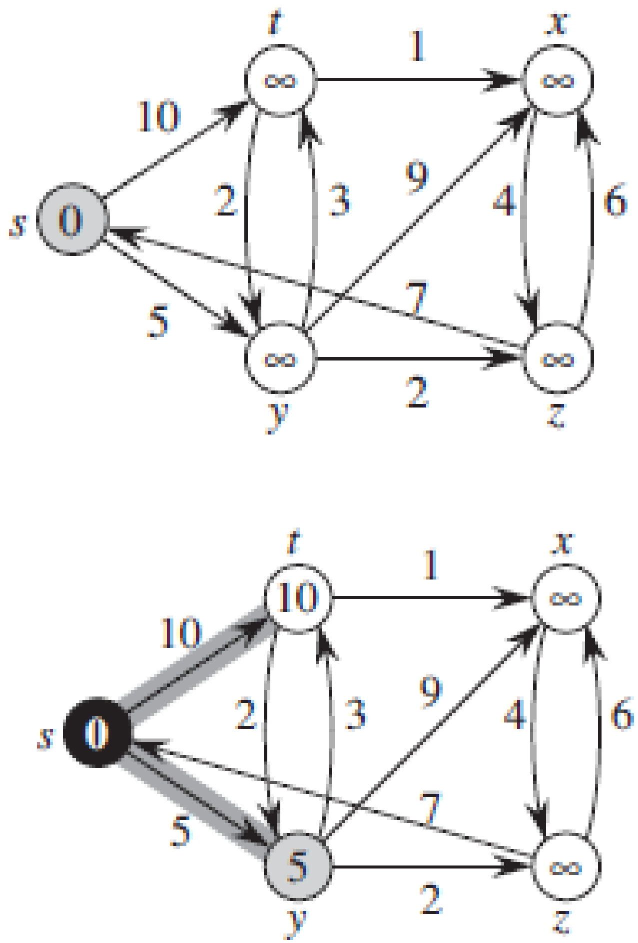
d[s] -> 0
for each vertex $v \in \{V[G] - S\}$
    do $d[v] = \infty$
        $\pi[v] = NIL$
$S = \emptyset$
$Q = G.V$
While $Q \neq \emptyset$
do $u = EXTRACT\min(Q)$
    $S = SU\{u\}$
    for each vertex $v \in Adj[u]$
        do $if\ (v \in Q\ and\ d[v] > d[u] + w(u,v))$
            $d[v] = d[u] + w(u,v))$
            $\pi[v] = u$

Example



Example

while Q = ϕ		*u* *(Extract-min(Q))*	*S*	*Adj* [u]	Relax (u,v,w) if	d[V]	Π [V]
1	T	s	{s}	t	T	10	s
				y	T	5	s
2	T	y	{s,y}	t	T	8	y
				x	T	14	y
				z	T	7	y
3	T	z	{s.y.z}	x	T	13	z
4	T	t	{s,y,z,t}	x	T	9	t
5	T	x	{s, y, z, t.x}	-	-	-	-
End of while loop							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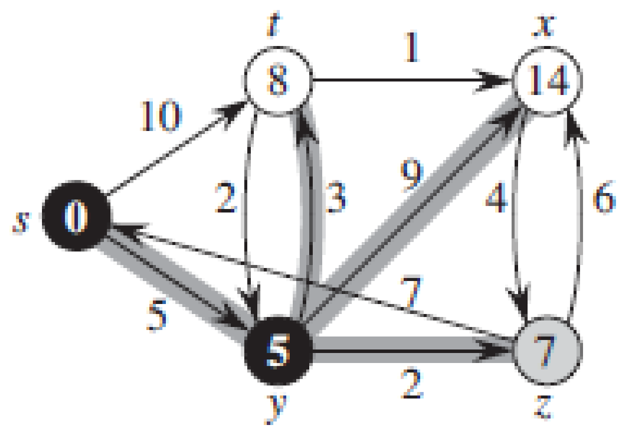
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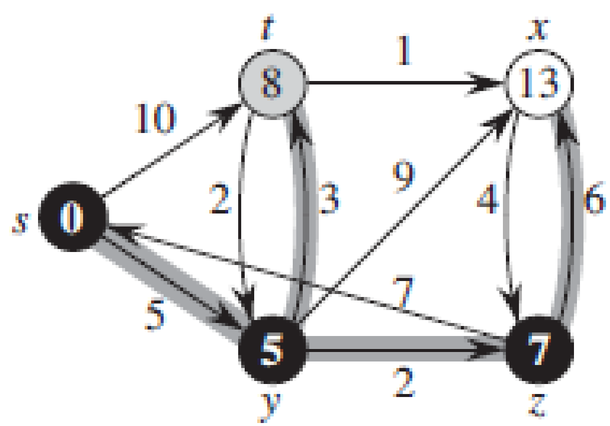
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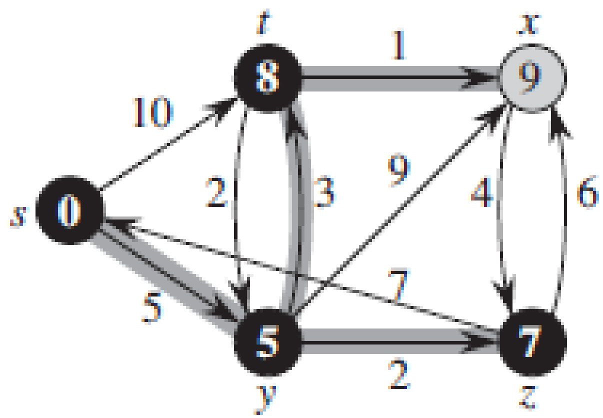
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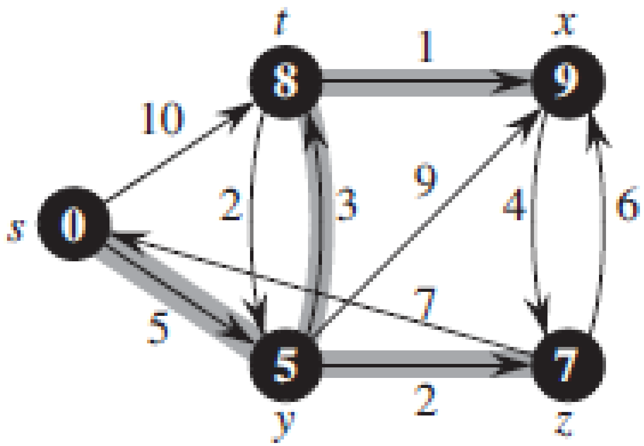
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Analysis: O(E log V)

- 2-6 -> O(V)
- 8 -> O(V log V)
- 10-13 -> O(E log (V))

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3. do $d[v] = \infty$
4. $\pi[v] = NIL$
5. $S = \emptyset$
6. $Q = G.V$
7. While $Q \neq \emptyset$
8. do $u = EXTRACT \min(Q)$
9. $S = SU\{u\}$
10. for each vertex $v \in Adj[u]$
11. do $if\ (v \in Q\ and\ d[v] > d[u] + w(u,v)\ )$
12. $d[v] = d[u] + w(u,v))$
13 $\pi[v] = u$

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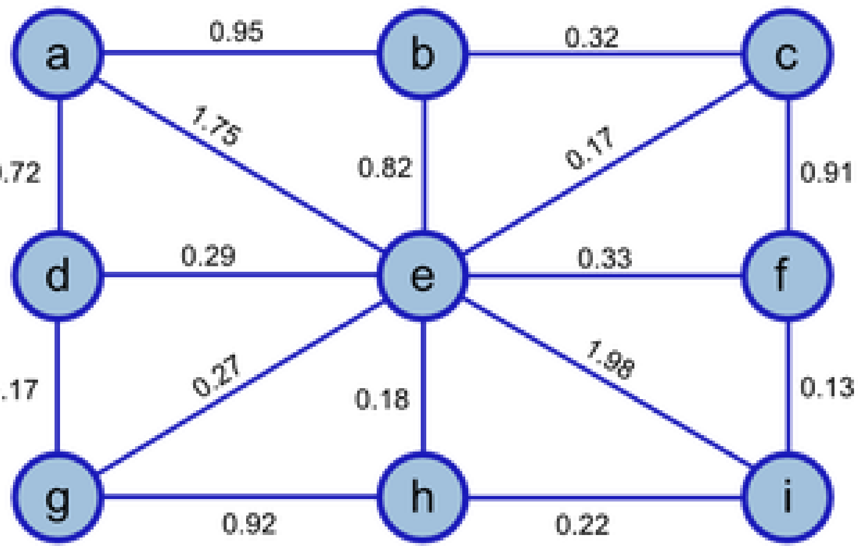
Task
Apply Djikstra: Start Node a.



Bellman Ford Algorithm Homework