Artificial Intelligence
Fuzzy Systems

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Fuzzy logic

What does fuzzy means?
According to oxford dictionary:

- confused or not explained clearly
- the things which are not certain.

Examples:

- the boy is intelligent
- the building is very tall
- the garden is beautiful
- the place is far from here

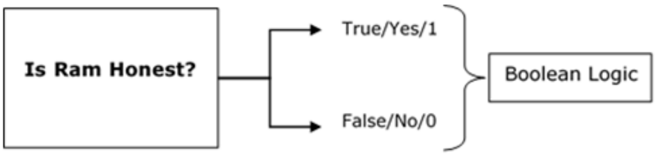
In the given sentences the terms *intelligent, tall, beautiful*, and *far* are not clearly defined. The meaning of the given words are vary person to person.
Consider another example:
Suppose you want to buy a gift of about Rs. 200. So, the about may be $[190, 210]$ or $[180, 225]$. Different people can interpret the *about* in different ways.

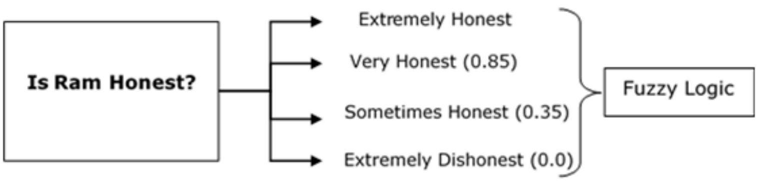
Fuzzy Logic resembles the human decision-making methodology. It deals with vague and imprecise information. This is gross oversimplification of the real-world problems and based on degrees of truth rather than usual true/false or 1/0 like Boolean logic.

Take a look at the following diagram. It shows that in fuzzy systems, the values are indicated by a number in the range from 0 to 1. Here 1.0 represents absolute truth and 0.0 represents absolute falseness. The number which indicates the value in fuzzy systems is called the truth value.

n other words, we can say that fuzzy logic is not logic that is fuzzy, but logic that is used to describe fuzziness. There can be numerous other examples like this with the help of which we can understand the concept of fuzzy logic.

Fuzzy Logic was introduced in 1965 by Lofti A. Zadeh in his research paper “Fuzzy Sets”. He is considered as the father of Fuzzy Logic.





Fuzzy sets:

The set theory of classical is the subset of Fuzzy set theory. Fuzzy logic is based on this theory, which is a generalisation of the classical theory of set (i.e., crisp set) introduced by Zadeh in 1965.

A fuzzy set is a collection of values which exist between 0 and 1. Fuzzy sets are denoted or represented by the tilde ( ) character. The sets of Fuzzy theory were introduced in 1965 by Lofti A. Zadeh and Dieter Klaua. In the fuzzy set, the partial membership also exists. This theory released as an extension of classical set theory.

The fuzzy set can be defined as an order pair such as:

$$A = \{(x, \mu_A(x))\}$$

where $\mu_A(x) \in [0, 1]$ define the membership of the x that belong to set A

Operations on Fuzzy Set

1. Union Operation: The union operation of a fuzzy set is defined by:

$$\mu_{AUB}(x) = max(\mu_A(x), \mu_B(x))$$

Example: Let's suppose A is a set which contains following elements:

$$A = \{(X_1, 0.6), (X_2, 0.2), (X_3, 1), (X_4, 0.4)\}$$

And, B is a set which contains following elements:

$$B = \{(X_1, 0.1), (X_2, 0.8), (X_3, 0), (X_4, 0.9)\}$$

then,

$$AUB = \{(X_1, 0.6), (X_2, 0.8), (X_3, 1), (X_4, 0.9)\}$$

2. Intersection Operation: The intersection operation of fuzzy set is defined by:

$$\mu_{A \cup B}(x) = min(\mu_A(x), \mu_B(x))$$

Example:
Let's suppose A is a set which contains following elements:

$$A = \{(X_1, 0.3), (X_2, 0.7), (X_3, 0.5), (X_4, 0.1)\}$$

And, B is a set which contains following elements:

$$B = \{(X_1, 0.8), (X_2, 0.2), (X_3, 0.4), (X_4, 0.9)\}$$

then,

$$A \cup B = \{(X_1, 0.3), (X_2, 0.2), (X_3, 0.4), (X_4, 0.1)\}$$

3. Complement Operation: The complement operation of fuzzy set is defined by:

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x)$$

Example:
Let's suppose A is a set which contains following elements:

$$A = \{(X_1, 0.3), (X_2, 0.8), (X_3, 0.5), (X_4, 0.1)\}$$

then,

$$\bar{A} = \{(X_1, 0.7), (X_2, 0.2), (X_3, 0.5), (X_4, 0.9)\}$$

Fuzzy Logic Systems Architecture

Basically, there are four parts in the architecture of the fuzzy logic system-

1. Linguistic Variables: Linguistic variables are basically the input and output variables of the system. The values of these variables are mostly words and sentences from the natural languages and no numeric value. Linguistic variables can be decomposed into a set of linguistic terms.

2. Rule Base: It contains all the rules and "if-then" conditions offered by experts to control decision-making. The most recent update in fuzzy logic provides a number of methods for the design and tuning. Moreover, the update has significantly reduced the number of sets of rules.

3. Fuzzification: Fuzzification is the second in this series and it helps to convert inputs. It helps in converting crisp numbers to fuzzy sets. Crisp inputs are measured by sensors and passed into a control system for processing. The module is used to transform inputs into fuzzy input. This unit uses the membership functions to convert the the crisp input into the fuzzy form.

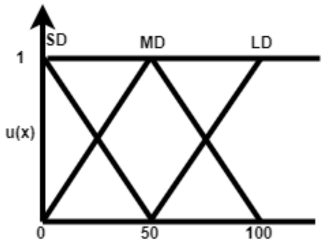
4. Inference Engine: The third one helps in determining the degree of match between fuzzy inputs and fuzzy rules. Based on that percentage it decides which rule is needed to be implemented. After it, to develop the control actions, applied rules are combined. Generally, the process helps in simulating the human reasoning process and that's by making fuzzy inference on the inputs and "if-then" rules.

5. Defuzzification: In this module, the transformation of a fuzzy set into crisp value takes place. There are a number of techniques available to do so, and it's up to the programmer to select the best from the available ones.

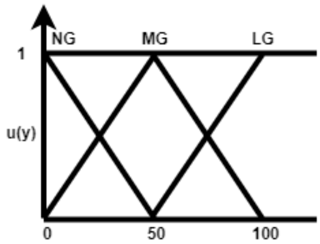
Classical Crisp Set VS Fuzzy Sets

Classical Set Theory	Fuzzy Set Theory
1. This theory is a class of those sets having sharp boundaries.	1. This theory is a class of those sets having unsharp boundaries.
2. This set theory is defined by exact boundaries only 0 and 1.	2. This set theory is defined by ambiguous boundaries.
3. In this theory, there is no uncertainty about the boundary's location of a set.	3. In this theory, there always exists uncertainty about the boundary's location of a set.
4. This theory is widely used in the design of digital systems.	4. It is mainly used for fuzzy controllers.

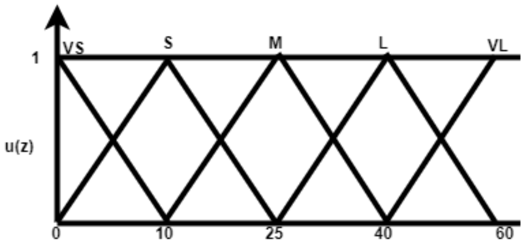
Designing a controller for automatic washing machine (Example): Consider the fuzzy controller that controls the domestic washing machine. Assume that the input is the dirt (in percent) and grease (in percent) on the clothes. Each of the inputs use three descriptors such as $Dirt = \{small-dirt(SD), medium{-}dirt(MD), large{-}dirt(LD)\}$, $Grease = \{no{-}grease(NG), medium{-}grease(MD), large{-}grease(LG)\}$. The output variable is the washing time (in minutes) to wash the clothes. The output variable contains five descriptors, $Washing{-}time = \{very{-}short(VS), short(S), medium(M), large(L), very{-}large(VL)\}$. Note that, the input dirt is represented by x, the grease is represented by y, and the output washing time is represented by z. For each of the input and output variables the membership functions are given below:



(a)



(b)



(c)

The fuzzy system contains a collection of rules called the knowledge base, which is used for inferencing. Here, the knowledge is given by:

$$(1)\ SD\ and\ NG\ \rightarrow VS$$
$$(2)\ MD\ and\ NG\ \rightarrow S$$
$$(3)\ LD\ and\ NG\ \rightarrow M$$
$$(4)\ SD\ and\ MG\ \rightarrow M$$
$$(5)\ MD\ and\ MG\ \rightarrow M$$
$$(6)\ LD\ and\ MG\ \rightarrow L$$
$$(7)\ SD\ and\ LG\ \rightarrow L$$
$$(8)\ MD\ and\ LG\ \rightarrow L$$
$$(9)\ LD\ and\ LG\ \rightarrow VL$$

Assume that the dirt is 60% and the grease is 70% determine the washing time in minutes.

SOLUTION:

The 60% 0f dirt maps to the membership functions medium dirt and large dirt. Which can be given by:

$$\mu_{MD}(x) = \frac{100 - x}{50}$$
$$\mu_{LD}(x) = \frac{x - 50}{50}$$

Similarly the 70% of grease maps to:

$$\mu_{MG}(y) = \frac{100 - y}{50}$$
$$\mu_{LG}(x) = \frac{y - 50}{50}$$

When $x = 60$

$$\mu_{MD}(60) = \frac{100 - 60}{50} = \frac{4}{5}$$
$$\mu_{LD}(60) = \frac{60 - 50}{50} = \frac{1}{5}$$

when $y = 70$

$$\mu_{MG}(70) = \frac{100 - 70}{50} = \frac{3}{5}$$

$$\mu_{LG}(x) = \frac{70-50}{50} = \frac{2}{5}$$

Thus the following 4 rules are fired:

$$MD \ and \ MG \rightarrow M$$
$$MD \ and \ LG \rightarrow L$$
$$LD \ and \ MG \rightarrow L$$
$$LD \ and \ LG \rightarrow VL$$

Compute the strength of each rule:

$$s_1 = min(4/5, 3/5) = 3/5$$
$$s_2 = min(4/5, 2/5) = 2/5$$
$$s_3 = min(1/5, 3/5) = 1/5$$
$$s_4 = min(1/5, 2/5) = 1/5$$

Defuzzification: Select the rule which has the highest strength

$$S_1 = max(s_1, s_2, s_3, s_4) = 3/5$$

, which corresponds to following rule.

$$MD \ and \ MG \rightarrow M$$

The medium washing-time, $\mu_M(z)$, have the the following two membership functions:

$$\mu_M(z) = \frac{z-10}{15}$$

and

$$\mu_M(z) = \frac{40-z}{15}$$

Thus

$$\frac{3}{5} = \frac{z-10}{15} ------(1)$$
$$\frac{3}{5} = \frac{40-z}{15} ------(2)$$

Solving eq(1) and eq(2) we get the following values of z's

$$z = 19 \ and \ z = 31$$

The average time is:

$$z^* = \frac{19+31}{2} = 25 \ minutes$$