Constraint Satisfaction

Student:

Student:	Taking classes:		
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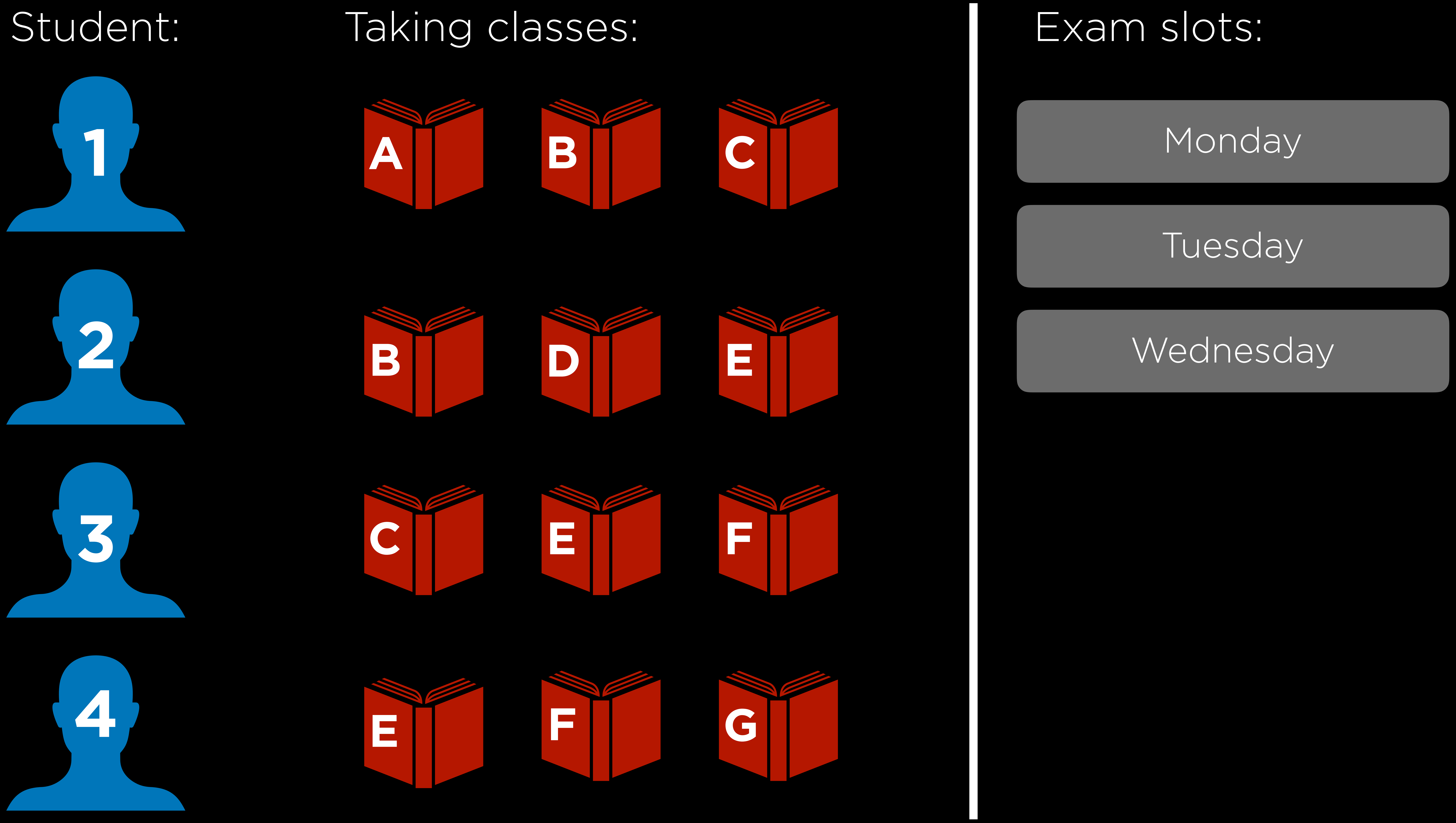
3	C	E	F
4	E	F	G


Student:
Taking classes:
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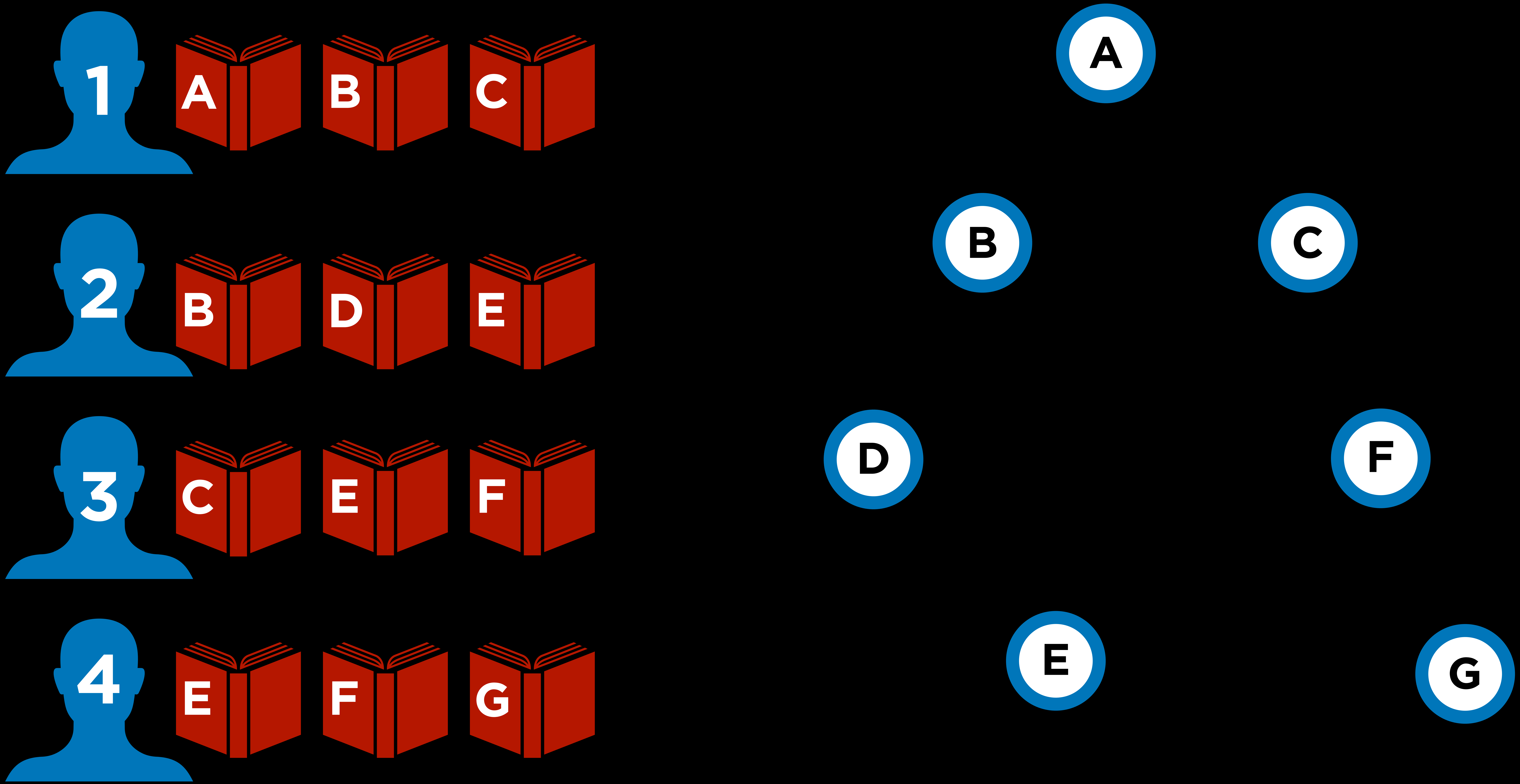

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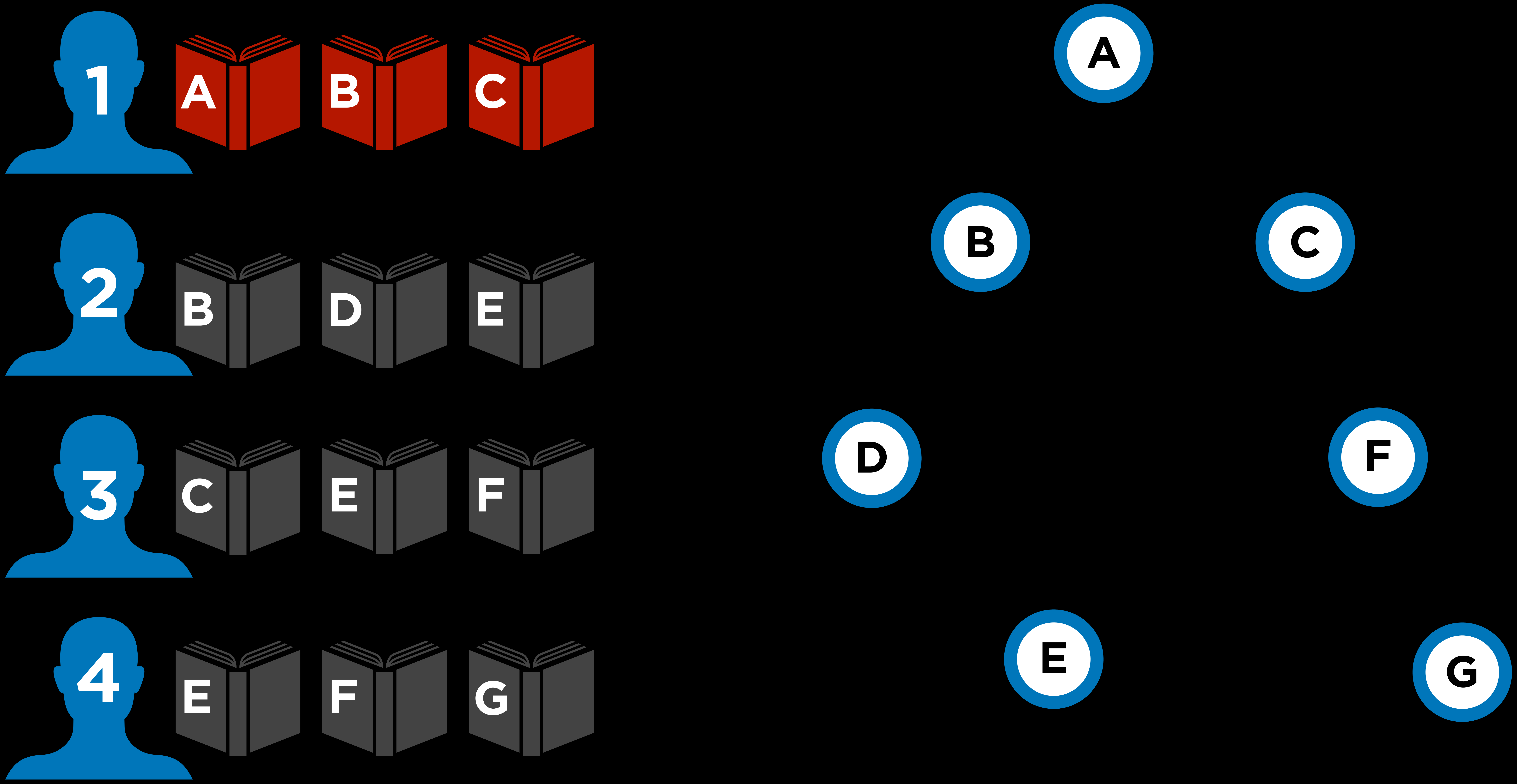
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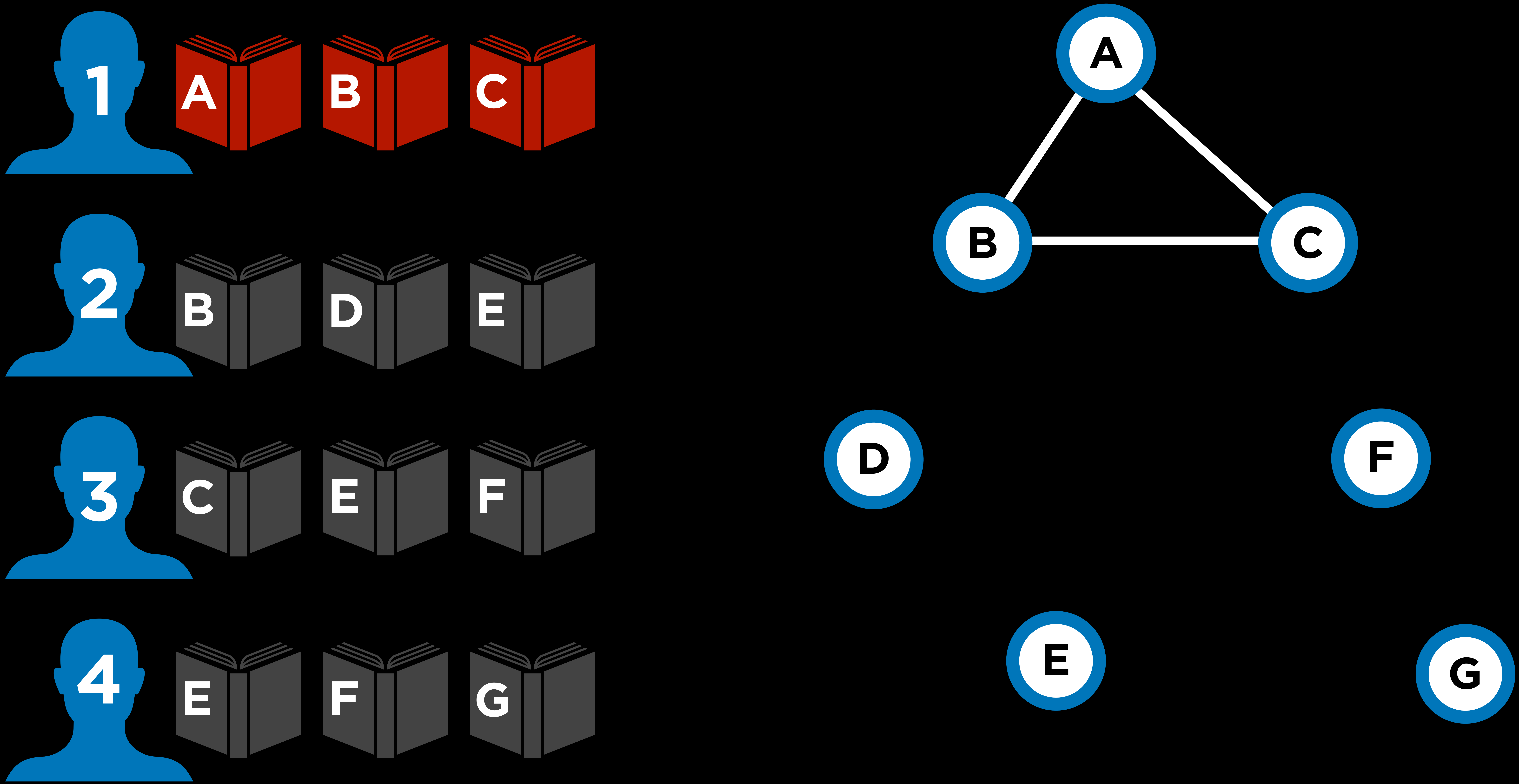

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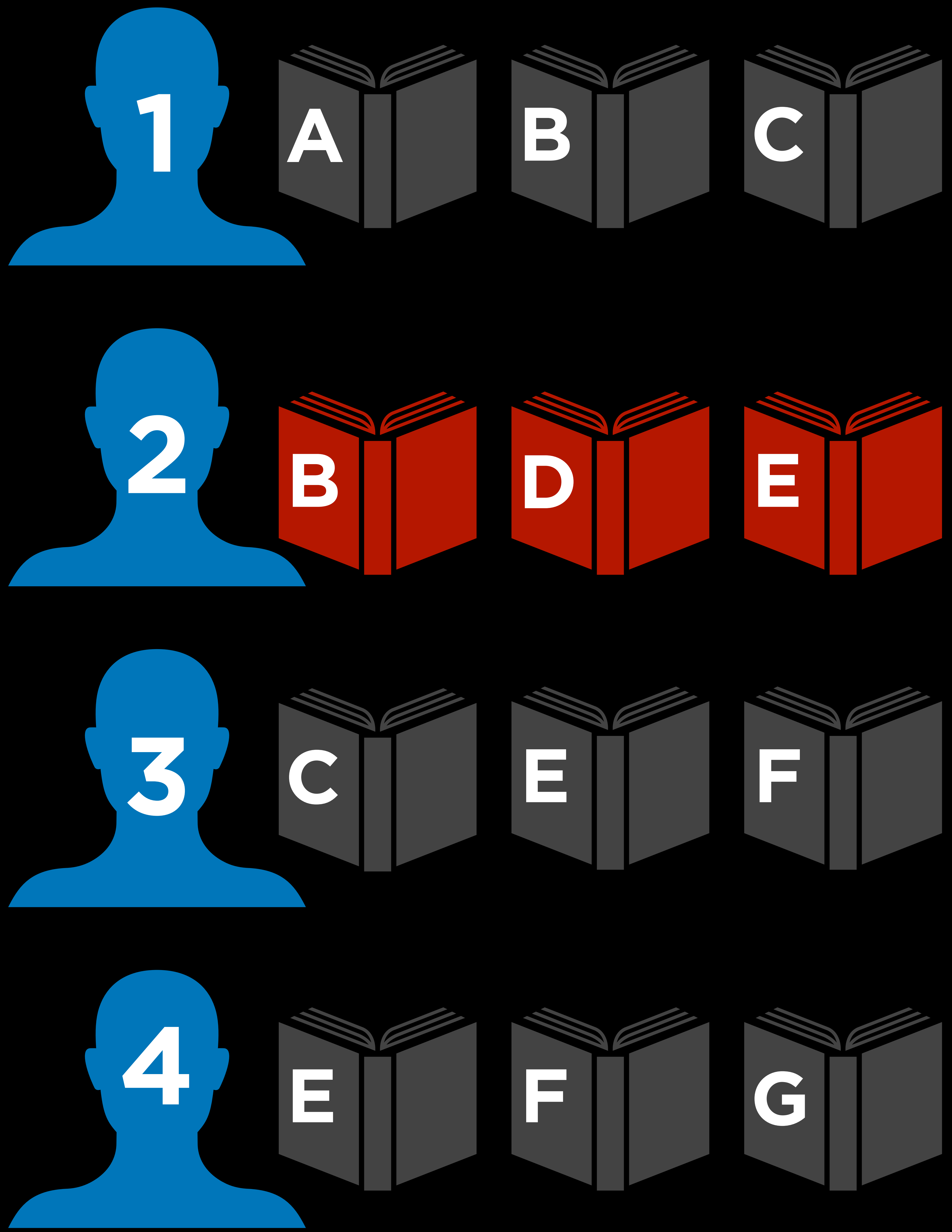
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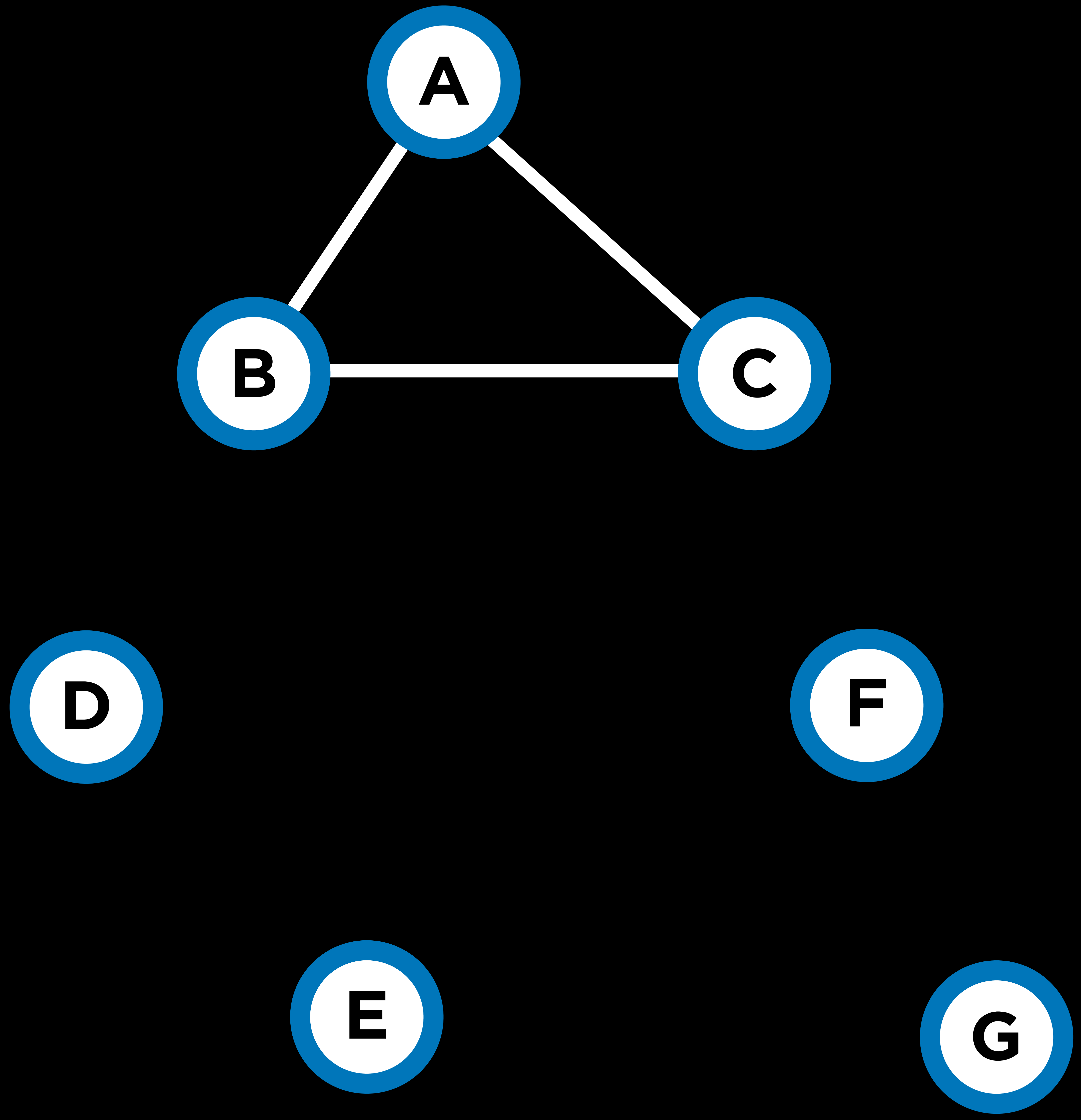
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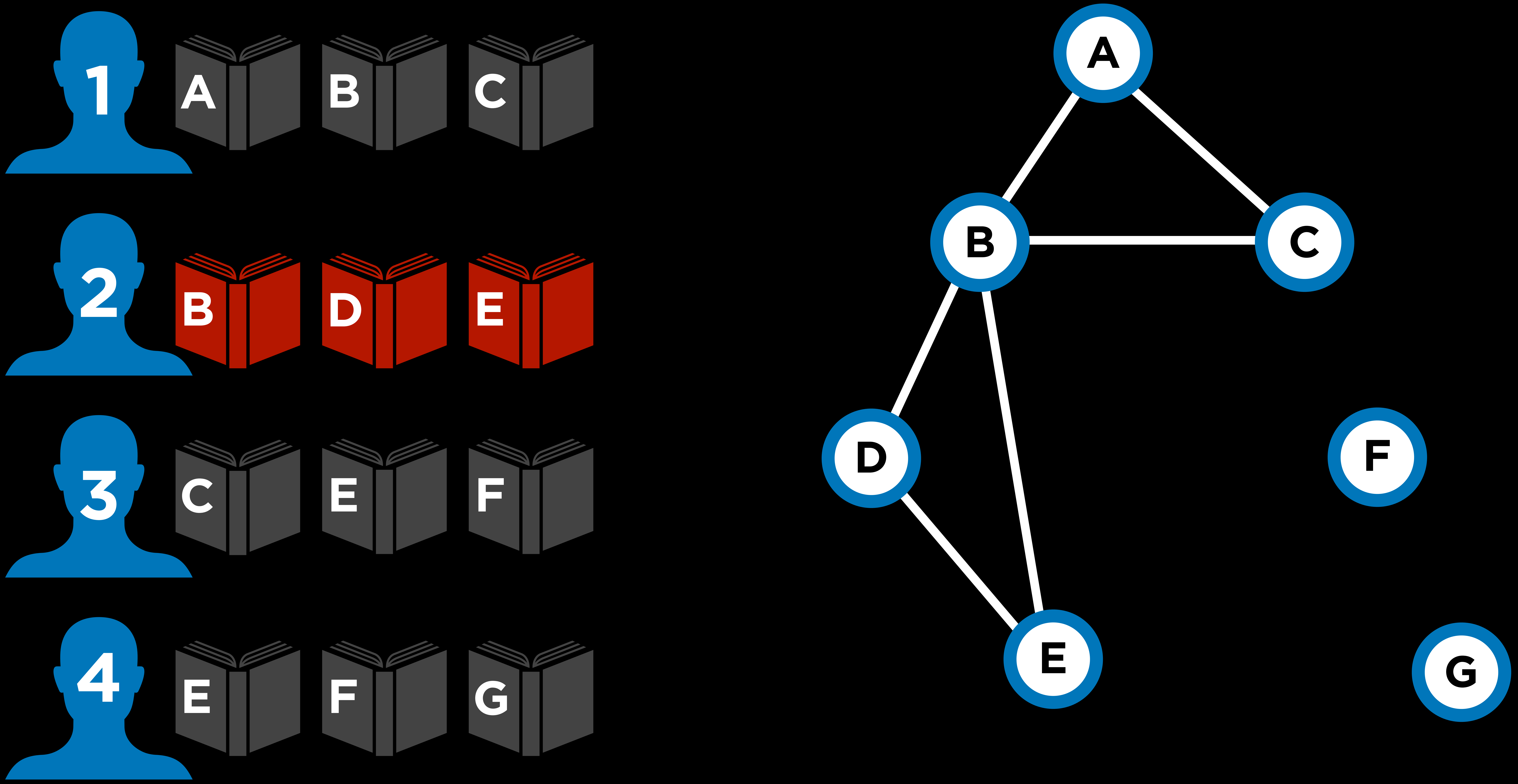
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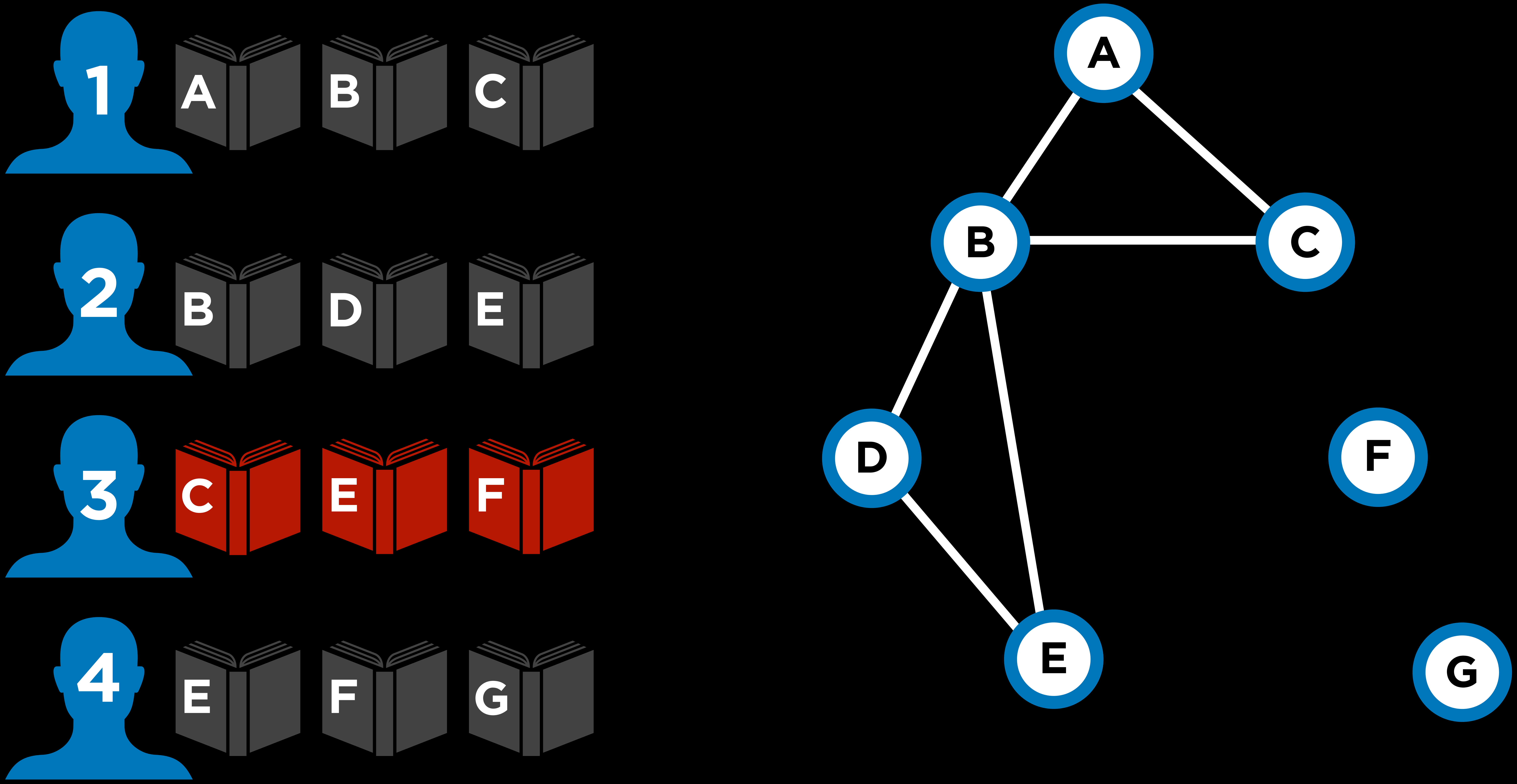

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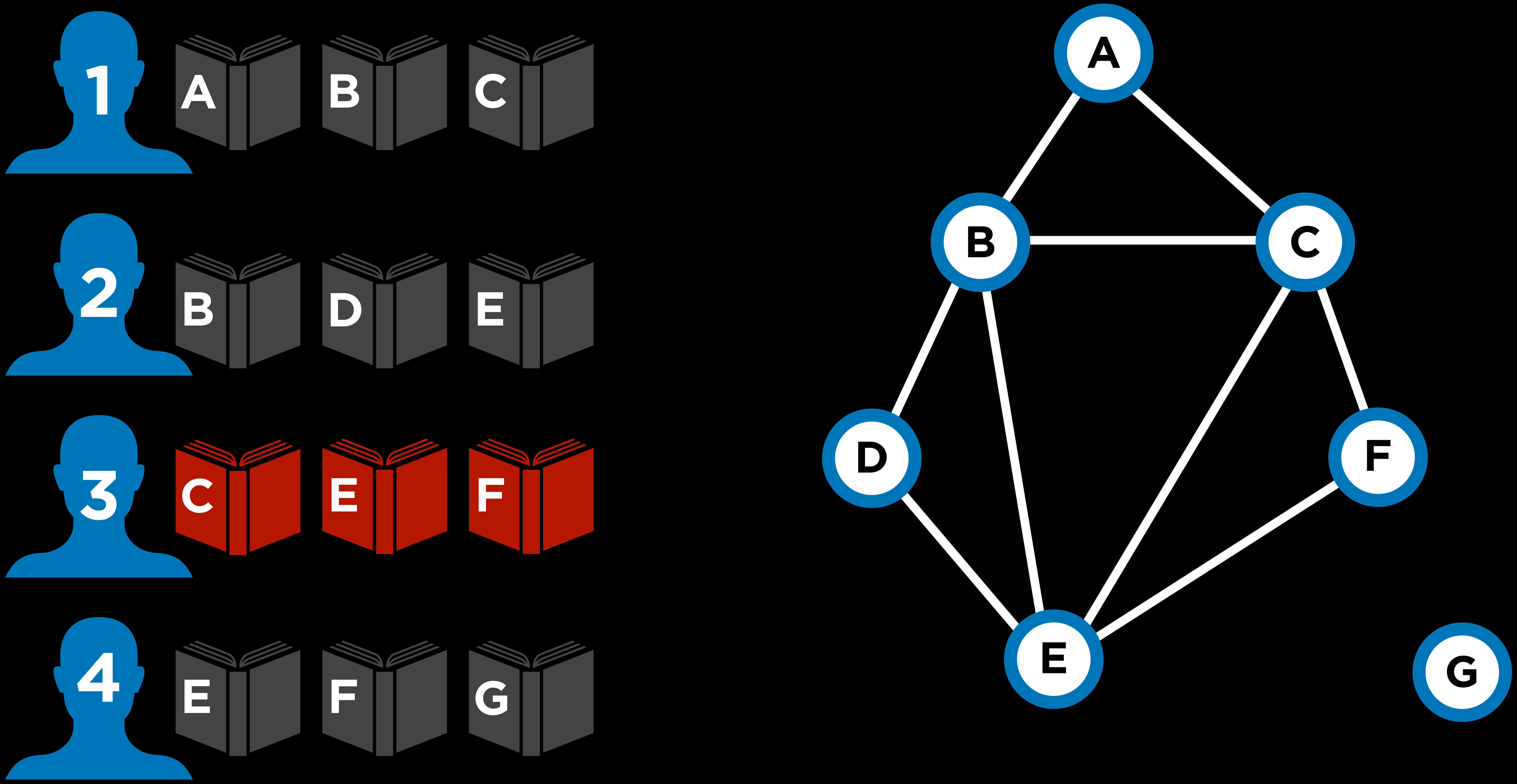

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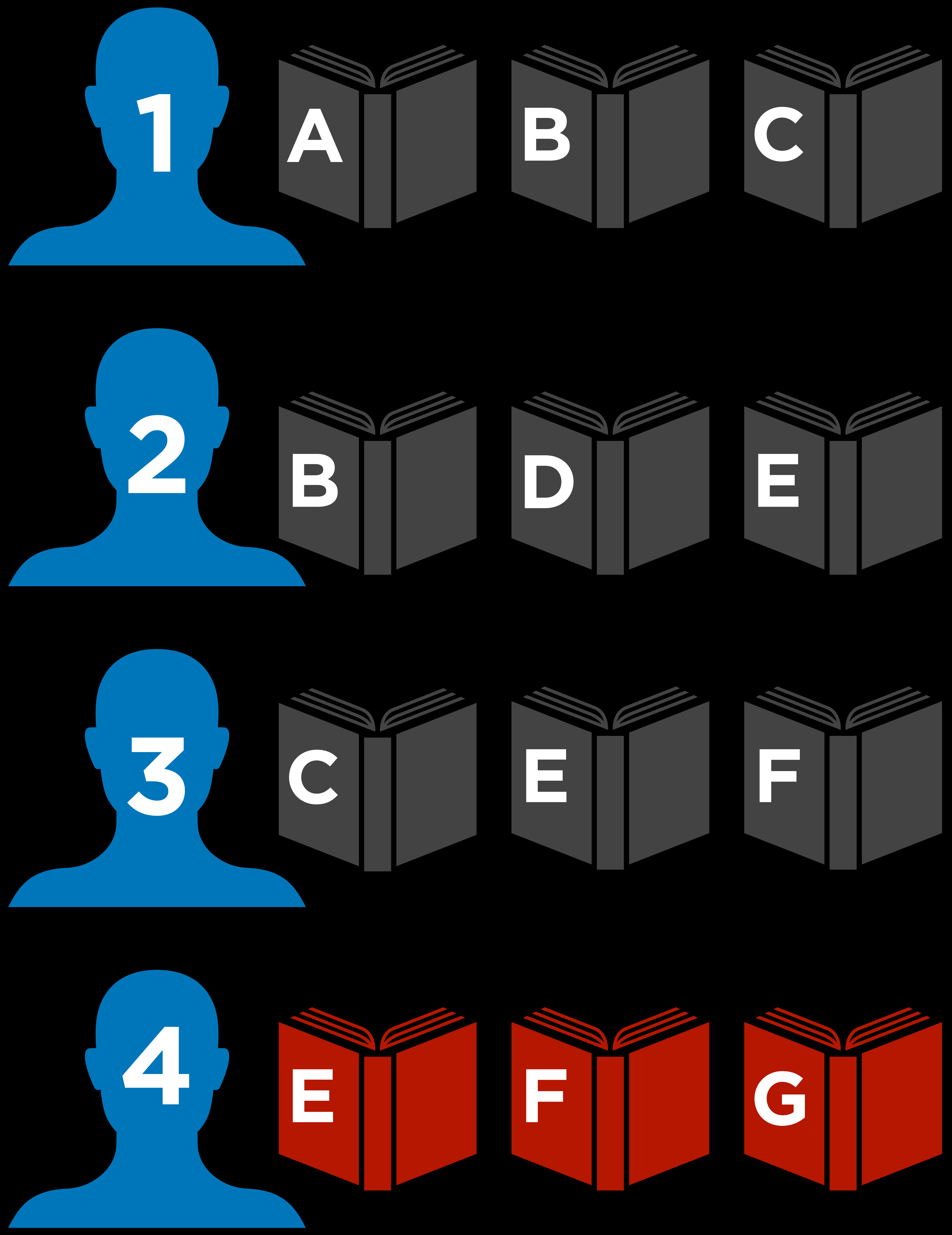
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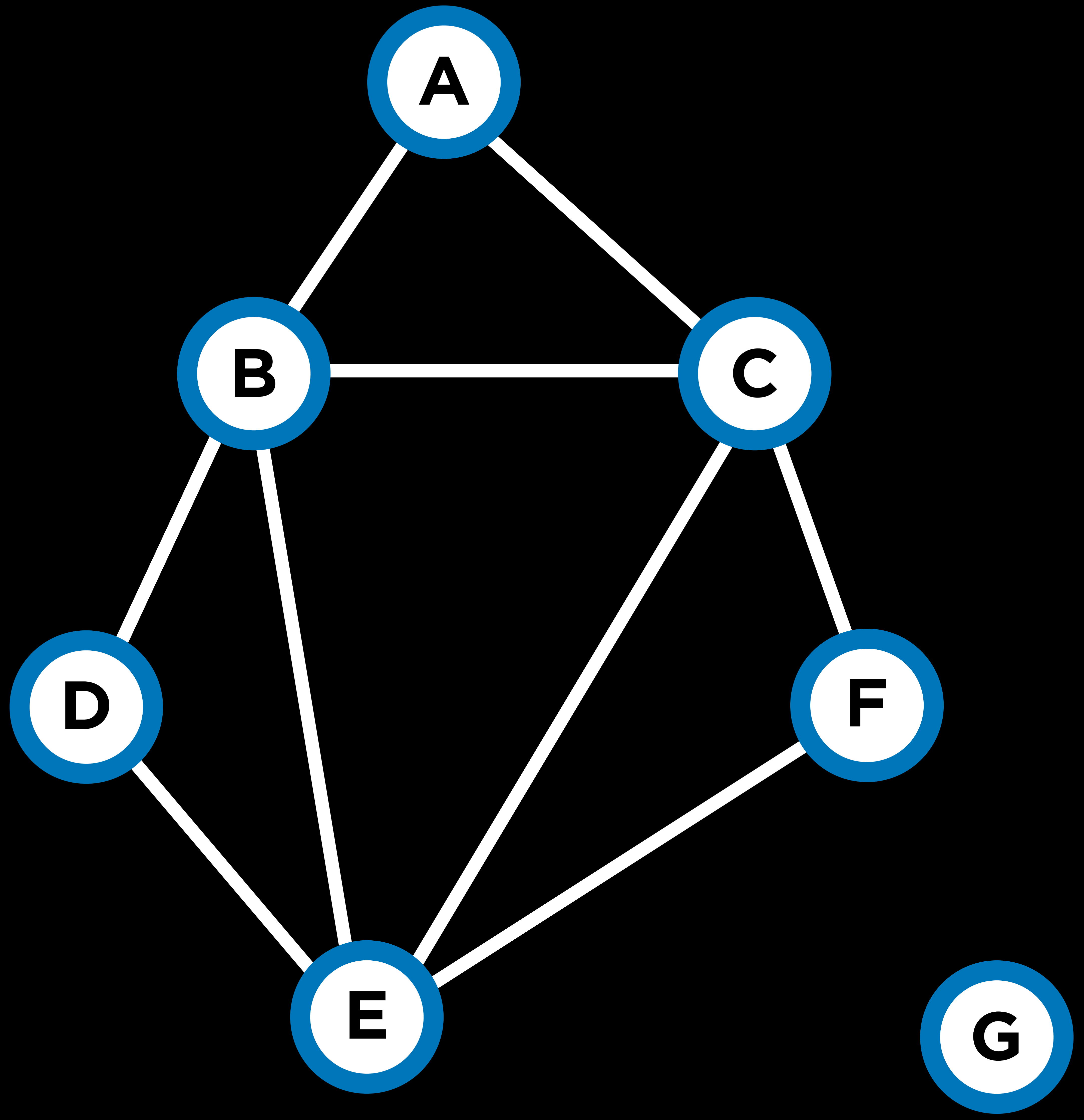
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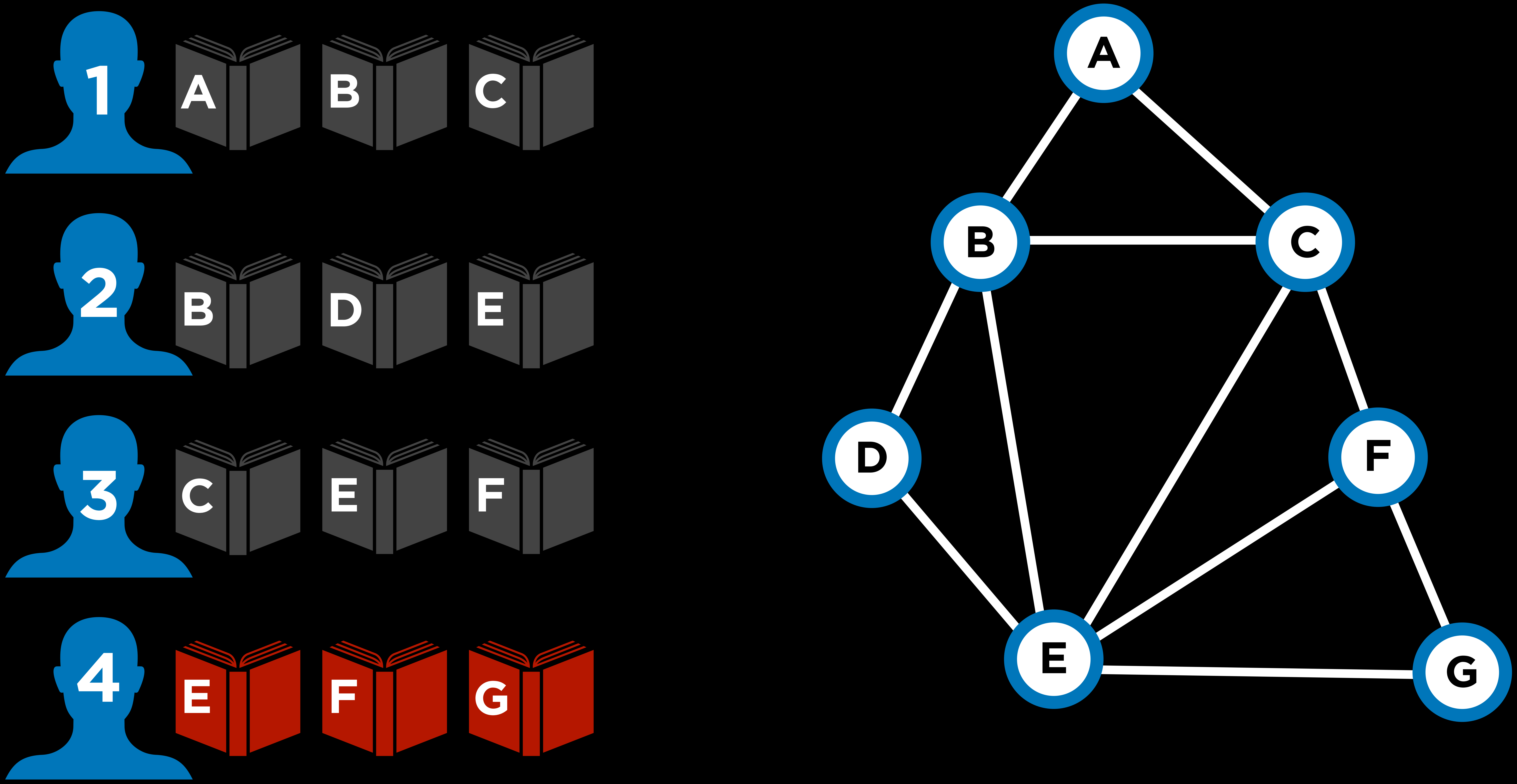
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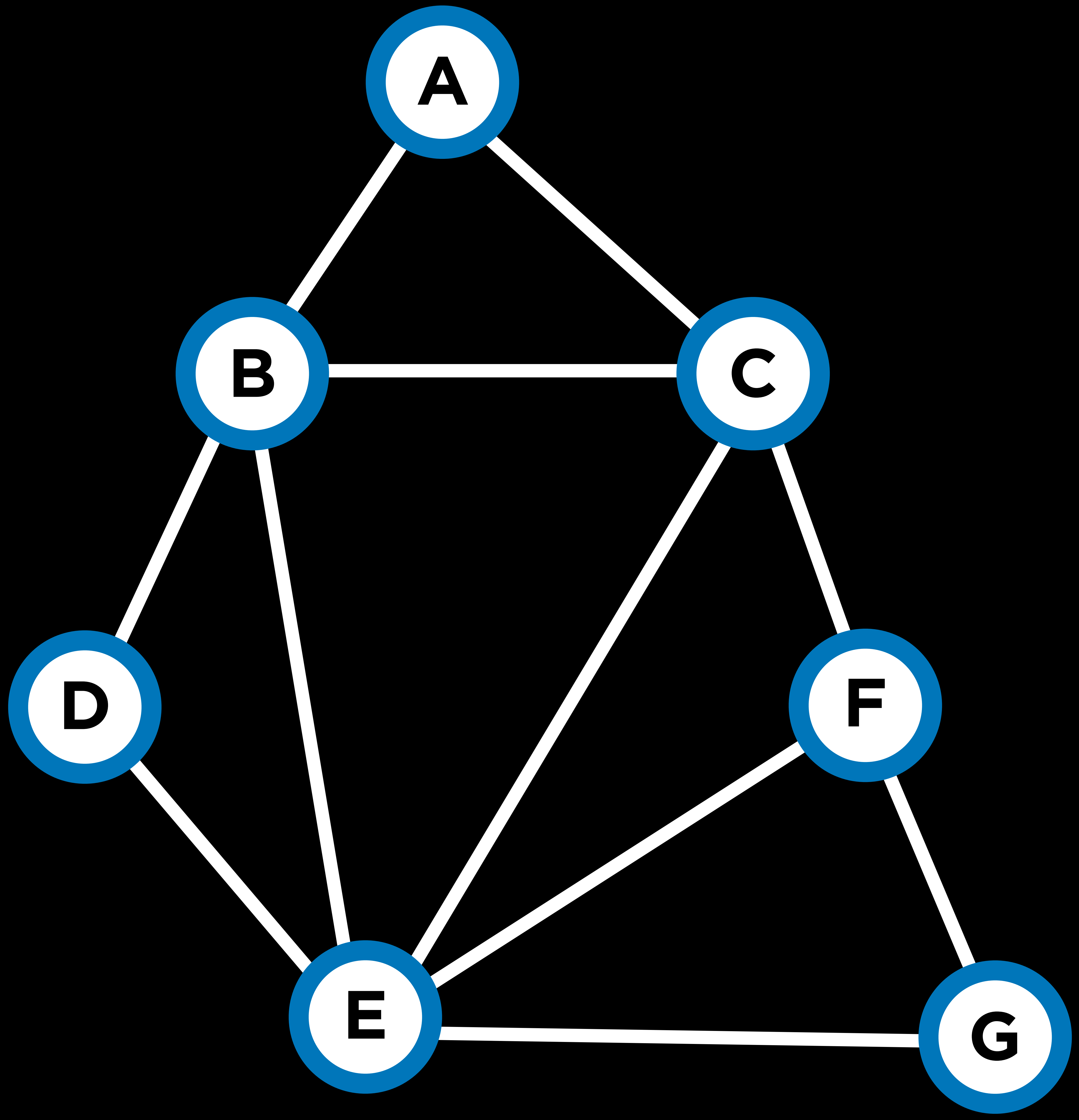
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Constraint Satisfaction Problem

- Set of variables $\{X_1, X_2, ..., X_n\}$
- Set of domains for each variable $\{D_1, D_2, ..., D_n\}$
- Set of constraints C

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

Variables

$$\{(0, 2), (1, 1), (1, 2), (2, 0), \ldots\}$$

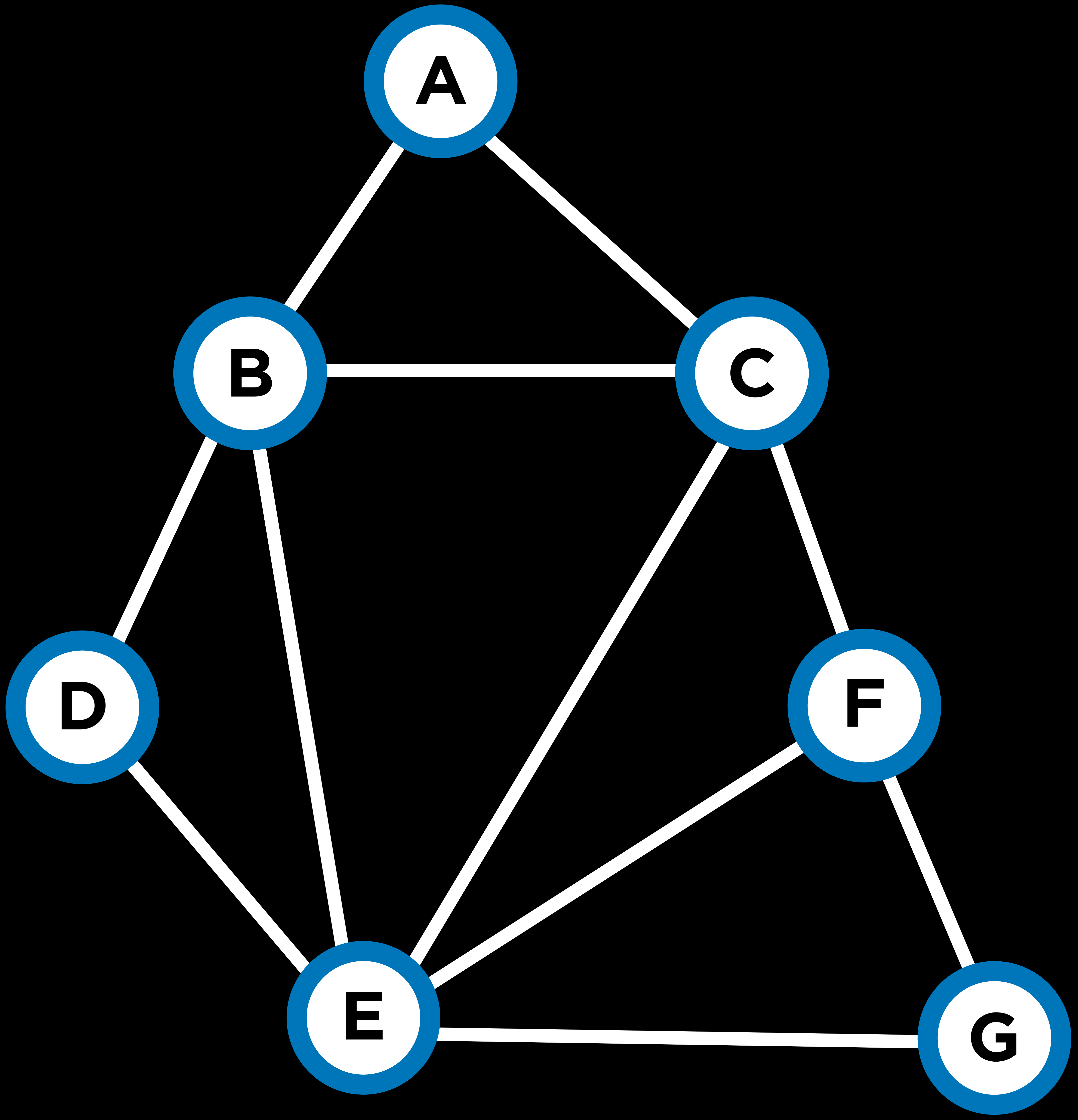
Domains

$$\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

for each variable

Constraints

$$\{(0, 2) \neq (1, 1) \neq (1, 2) \neq (2, 0), \ldots\}$$



Variables

$\{A, B, C, D, E, F, G\}$

Domains

$\{Monday, Tuesday, Wednesday\}$

for each variable

Constraints

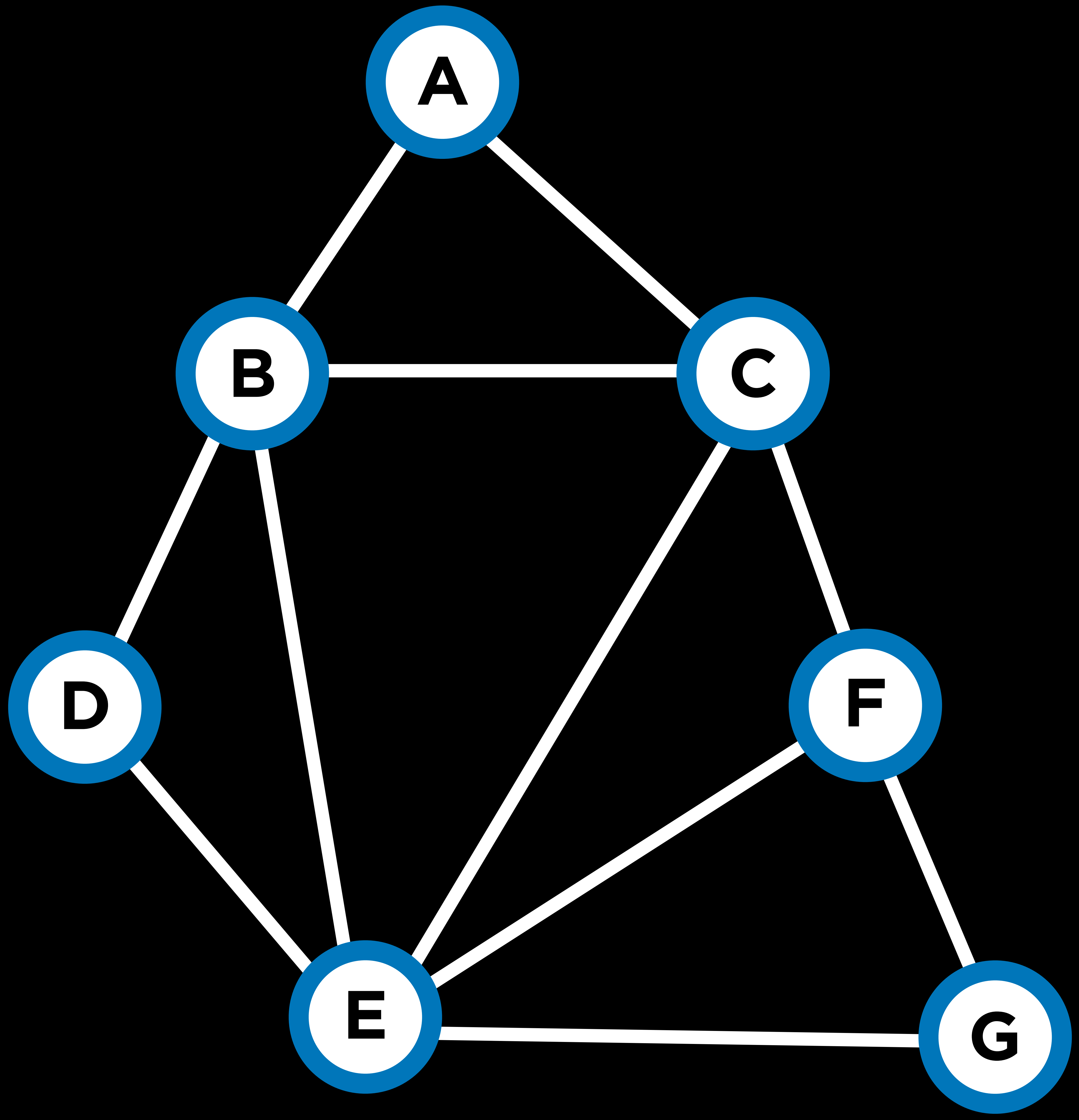
$\{A{\neq}B, A{\neq}C, B{\neq}C, B{\neq}D, B{\neq}E, C{\neq}E, C{\neq}F, D{\neq}E, E{\neq}F, E{\neq}G, F{\neq}G\}$

hard constraints

constraints that must be satisfied in a correct solution

soft constraints

constraints that express some notion of which solutions are preferred over others


A
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unary constraint

constraint involving only one variable

unary constraint

$$\{A \neq Monday\}$$

binary constraint

constraint involving two variables

binary constraint

$\{A \neq B\}$

node consistency

when all the values in a variable's domain satisfy the variable's unary constraints

$\{Mon, Tue, Wed\}$ $\{Mon, Tue, Wed\}$

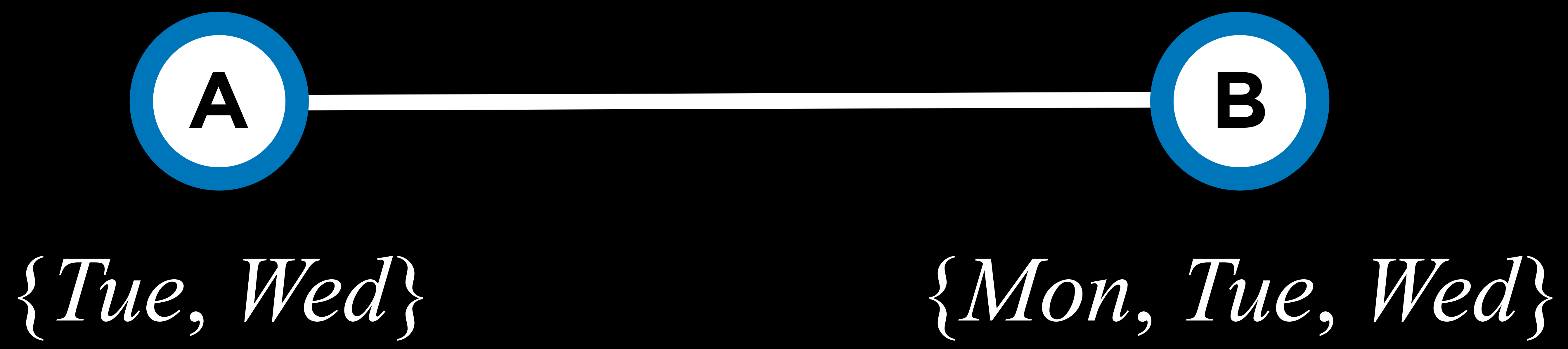
$$\{A \neq Mon, B \neq Tue, B \neq Mon, A \neq B\}$$

$\{Mon, Tue, Wed\}$ $\{Mon, Tue, Wed\}$

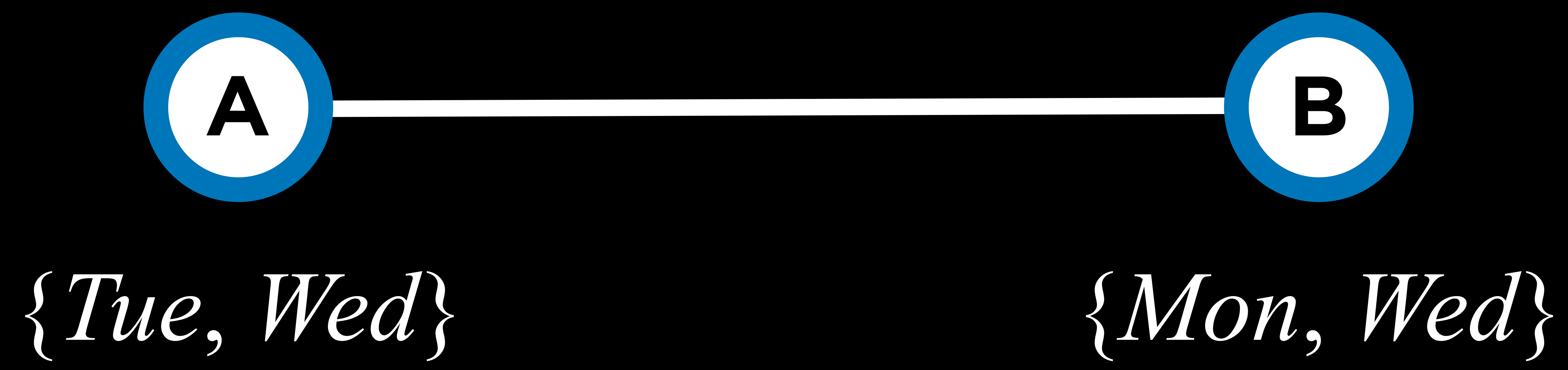
$$\{A \neq Mon, B \neq Tue, B \neq Mon, A \neq B\}$$

$\{Tue, Wed\}$ $\{Mon, Tue, Wed\}$

$$\{A \neq Mon, B \neq Tue, B \neq Mon, A \neq B\}$$



$$\{A \neq Mon, B \neq Tue, B \neq Mon, A \neq B\}$$



$$\{A \neq Mon, B \neq Tue, B \neq Mon, A \neq B\}$$

$\{Tue, Wed\}$ $\{Mon, Wed\}$

$$\{A \neq Mon, B \neq Tue, B \neq Mon, A \neq B\}$$

$\{Tue, Wed\}$ $\{Wed\}$

$$\{A \neq Mon, B \neq Tue, B \neq Mon, A \neq B\}$$

$\{Tue, Wed\}$ $\{Wed\}$

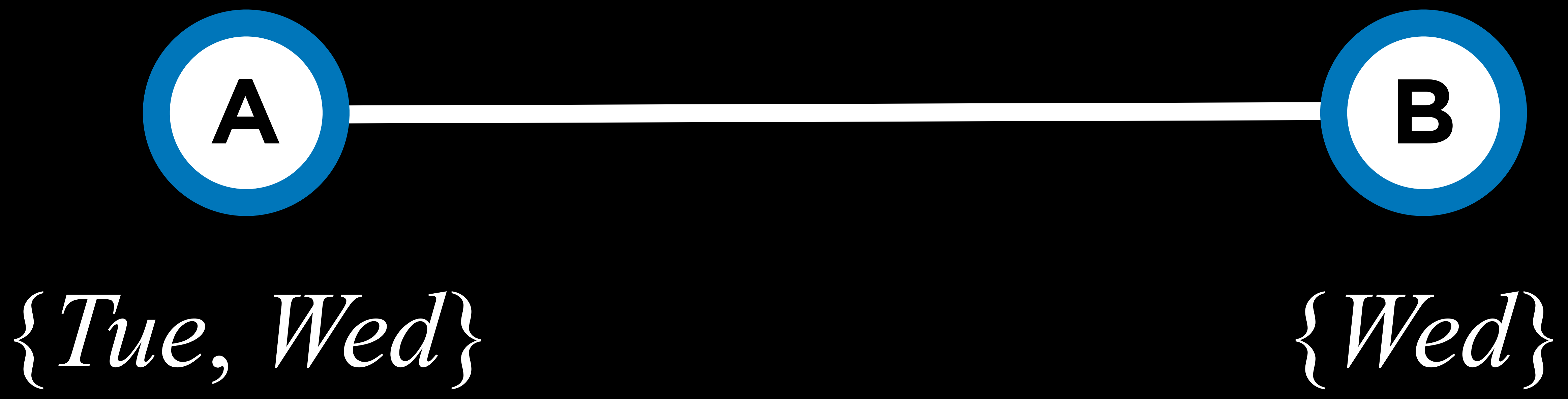
$$\{A \neq Mon, B \neq Tue, B \neq Mon, A \neq B\}$$

arc consistency

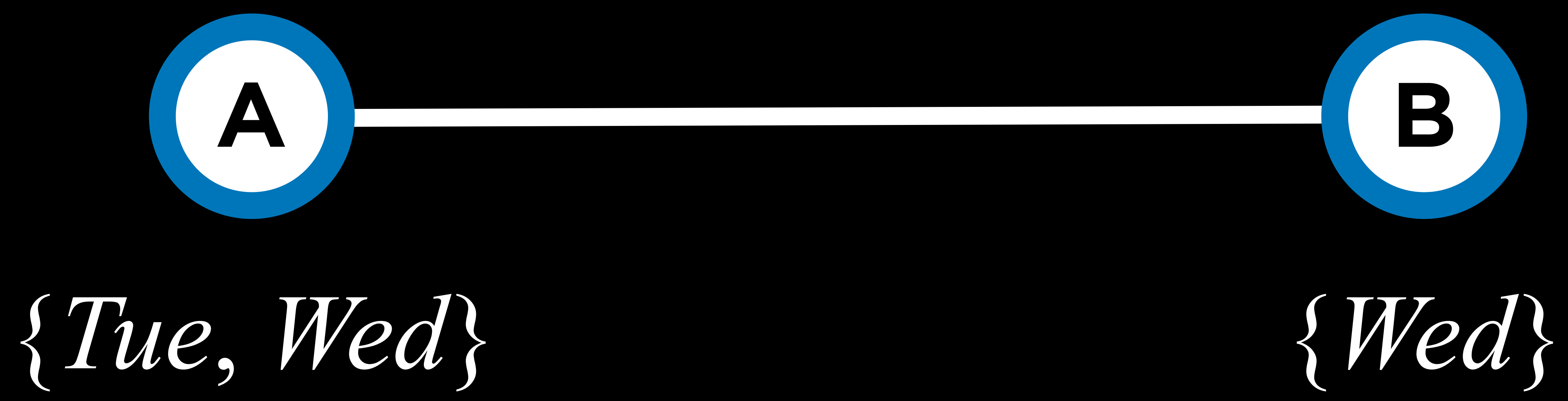
when all the values in a variable's domain satisfy the variable's binary constraints

arc consistency

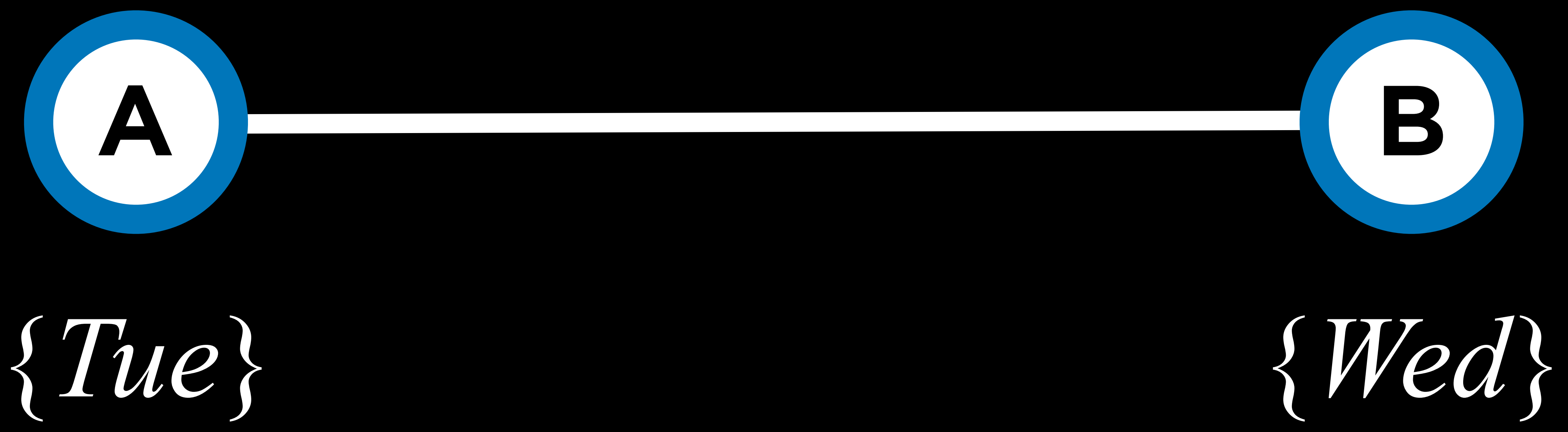
To make X arc-consistent with respect to Y, remove elements from X's domain until every choice for X has a possible choice for Y



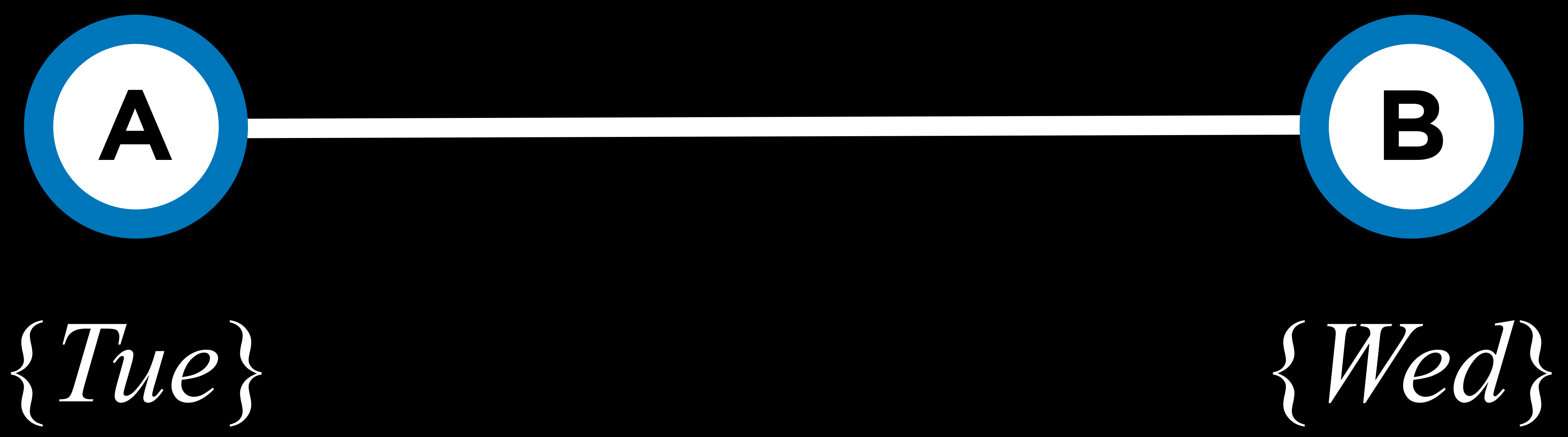
$$\{A \neq Mon, B \neq Tue, B \neq Mon, A \neq B\}$$



$$\{A \neq Mon, B \neq Tue, B \neq Mon, A \neq B\}$$



$$\{A \neq Mon,\ B \neq Tue,\ B \neq Mon,\ A \neq B\}$$



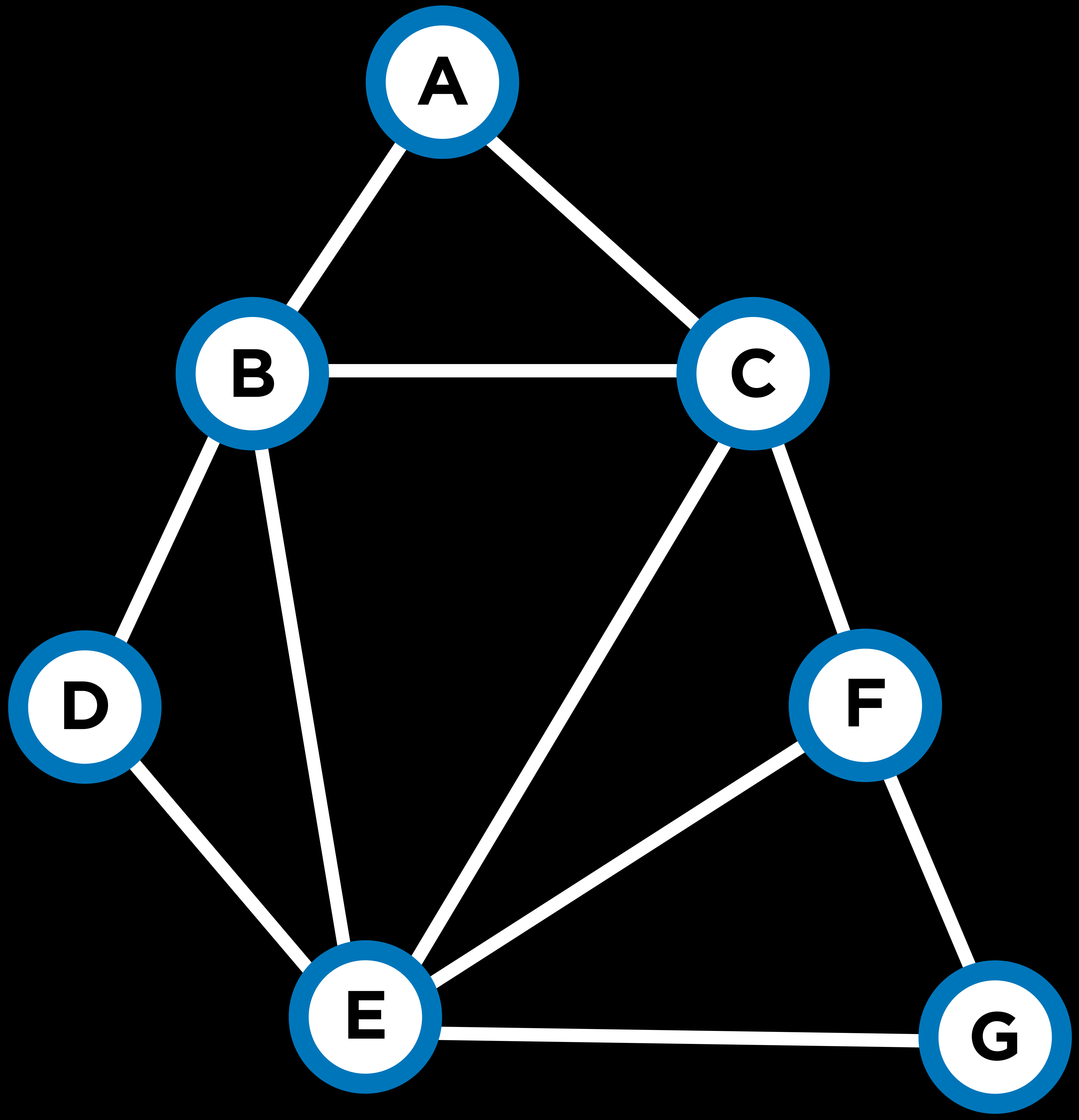
$$\{A \neq Mon, B \neq Tue, B \neq Mon, A \neq B\}$$

Arc Consistency

```
function REVISE(csp, X, Y):
    revised = false
    for x in X.domain:
        if no y in Y.domain satisfies constraint for (X, Y):
            delete x from X.domain
            revised = true
    return revised
```

Arc Consistency

```
function AC-3(csp):
    queue = all arcs in csp
    while queue non-empty:
        (X, Y) = DEQUEUE(queue)
        if REVISE(csp, X, Y):
            if size of X.domain == 0:
                return false
            for each Z in X.neighbors - {Y}:
                ENQUEUE(queue, (Z, X))
    return true
```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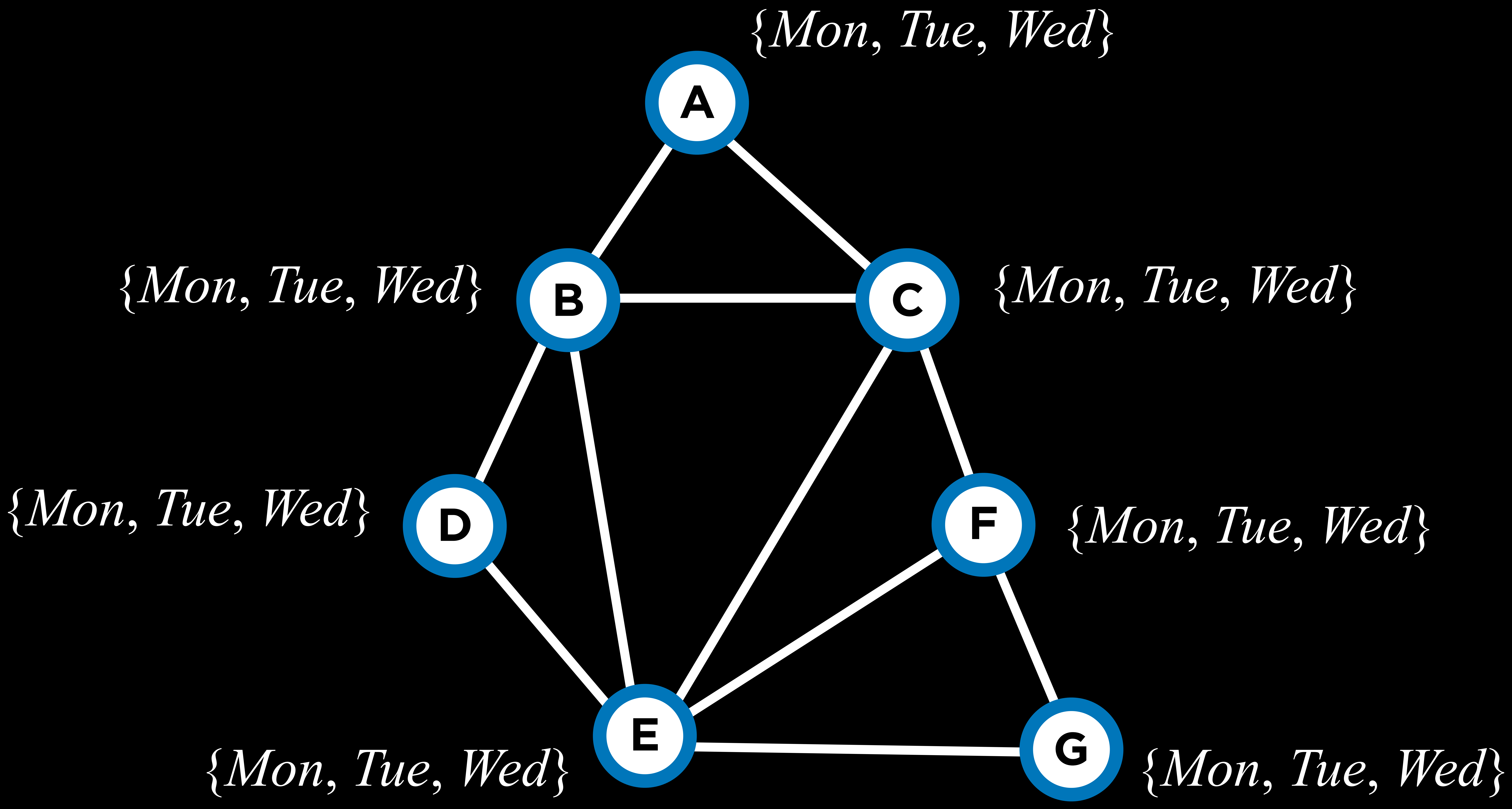
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Search Problems

- initial state
- actions
- transition model
- goal test
- path cost function

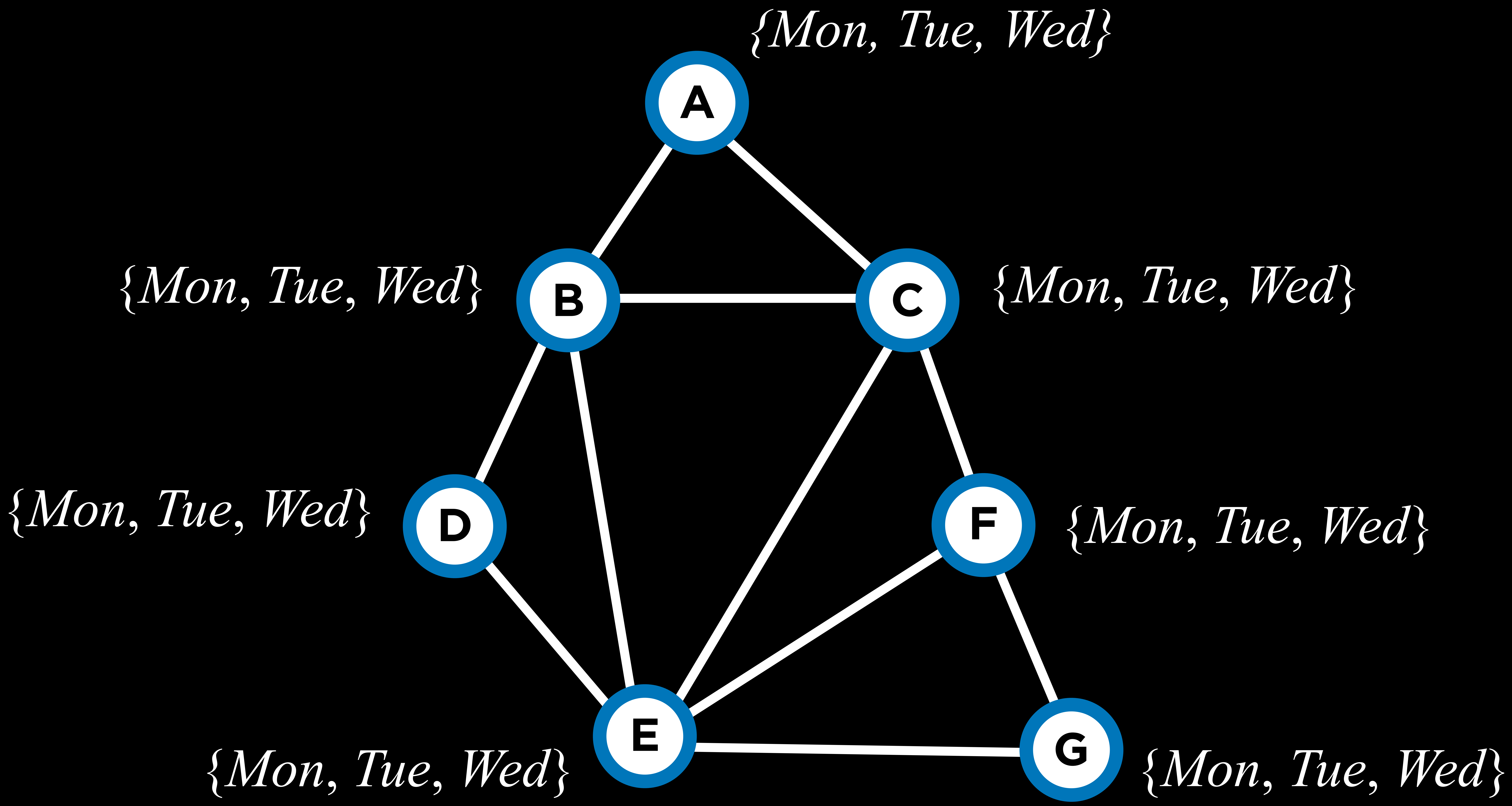
CSPs as Search Problems

- initial state: empty assignment (no variables)
- actions: add a $\{variable = value\}$ to assignment
- transition model: shows how adding an assignment changes the assignment
- goal test: check if all variables assigned and constraints all satisfied
- path cost function: all paths have same cost

Backtracking Search

Backtracking Search

```
function BACKTRACK(assignment, csp):
    if assignment complete: return assignment
    var = SELECT-UNASSIGNED-VAR(assignment, csp)
    for value in DOMAIN-VALUES(var, assignment, csp):
        if value consistent with assignment:
            add {var = value} to assignment
            result = BACKTRACK(assignment, csp)
            if result ≠ failure: return result
        remove {var = value} from assignment
    return failure
```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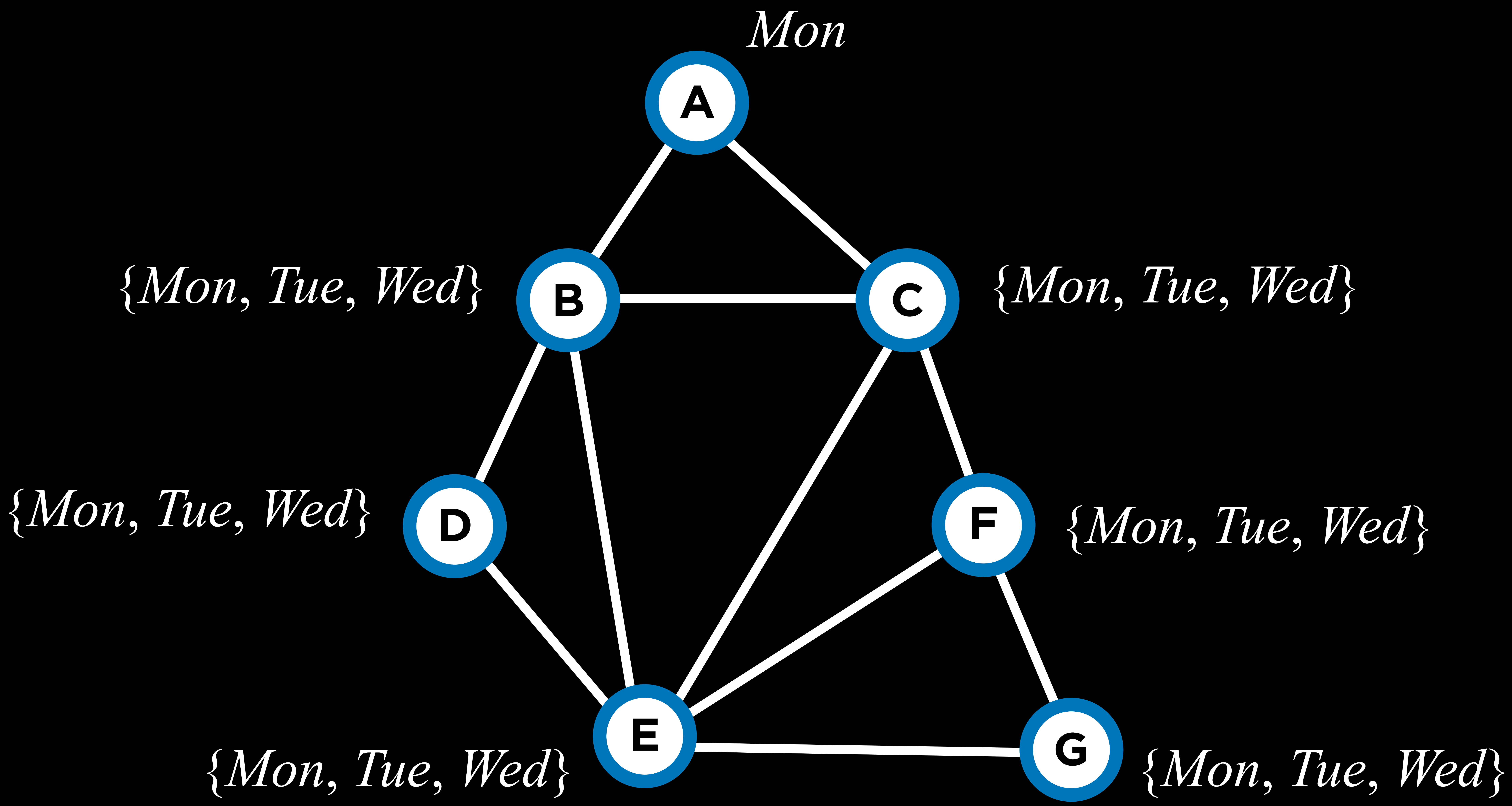
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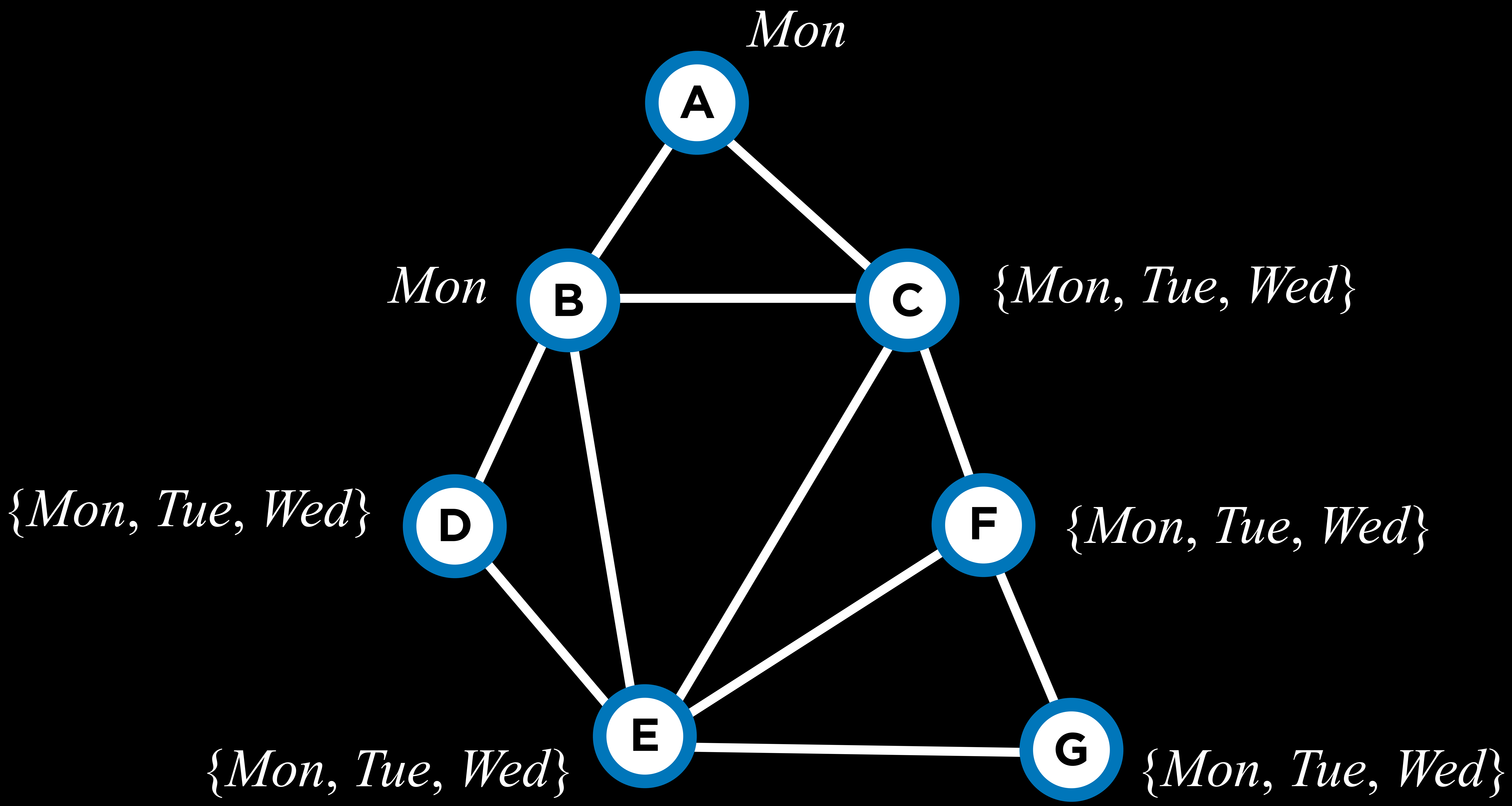
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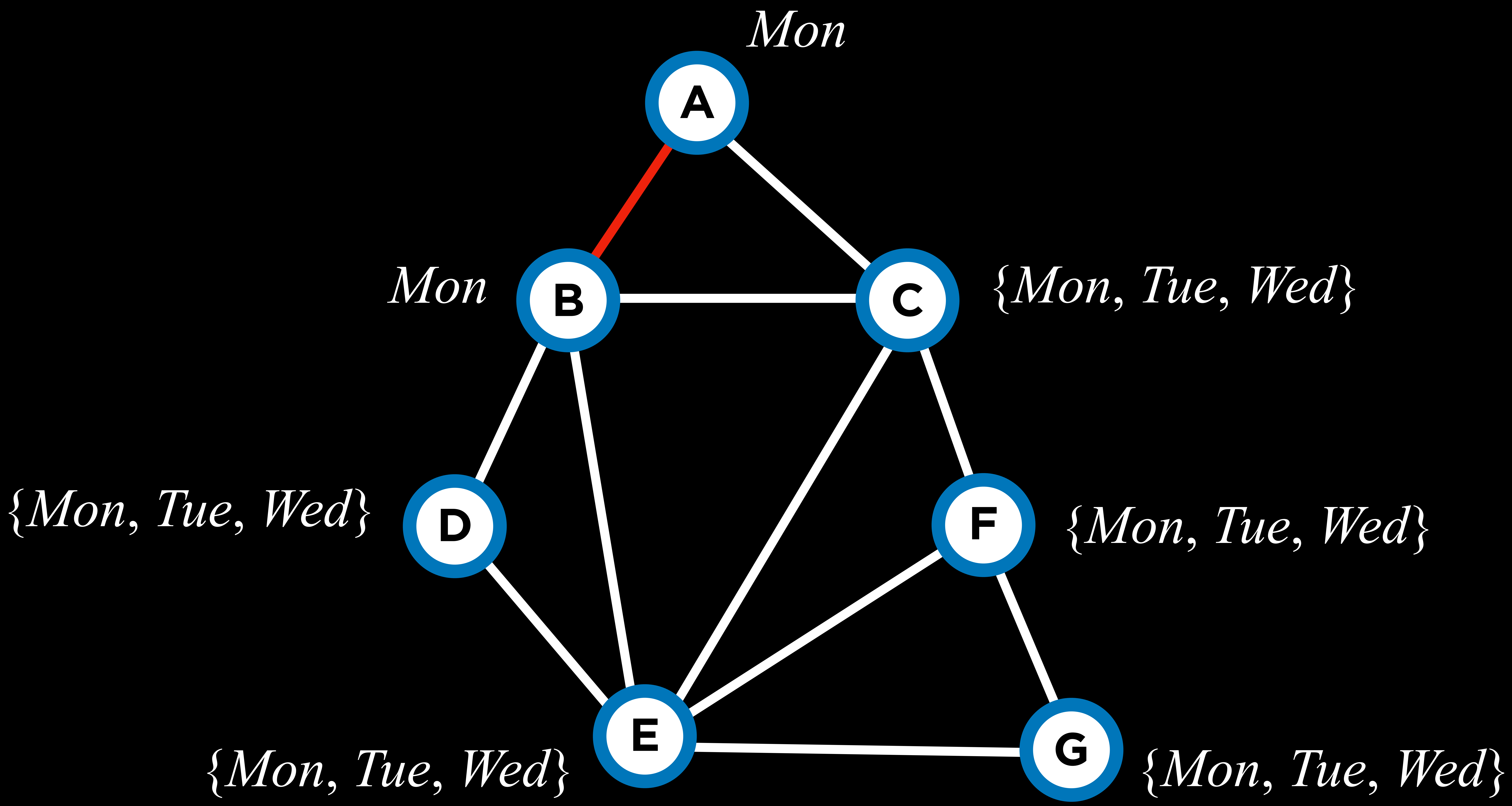
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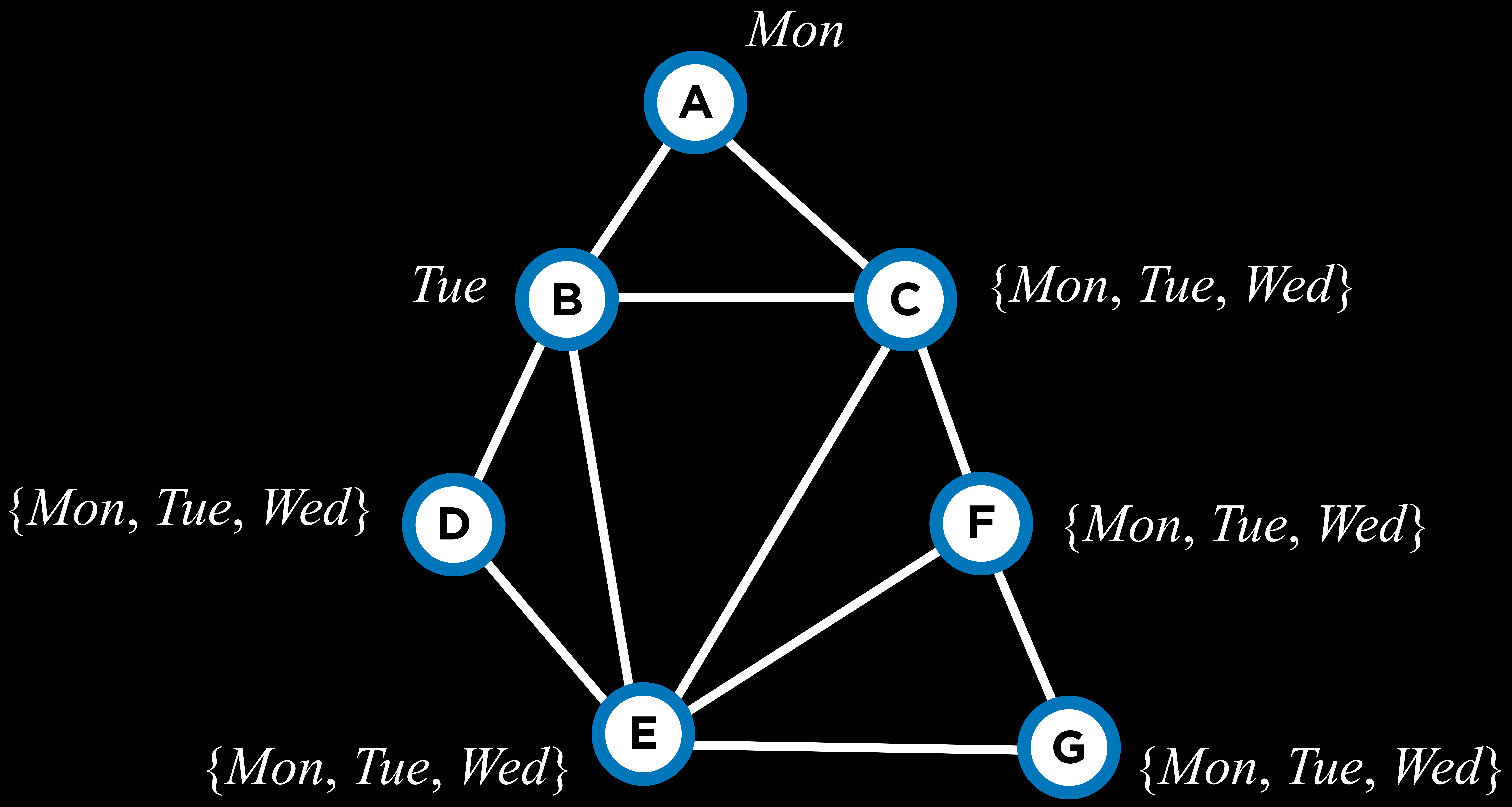
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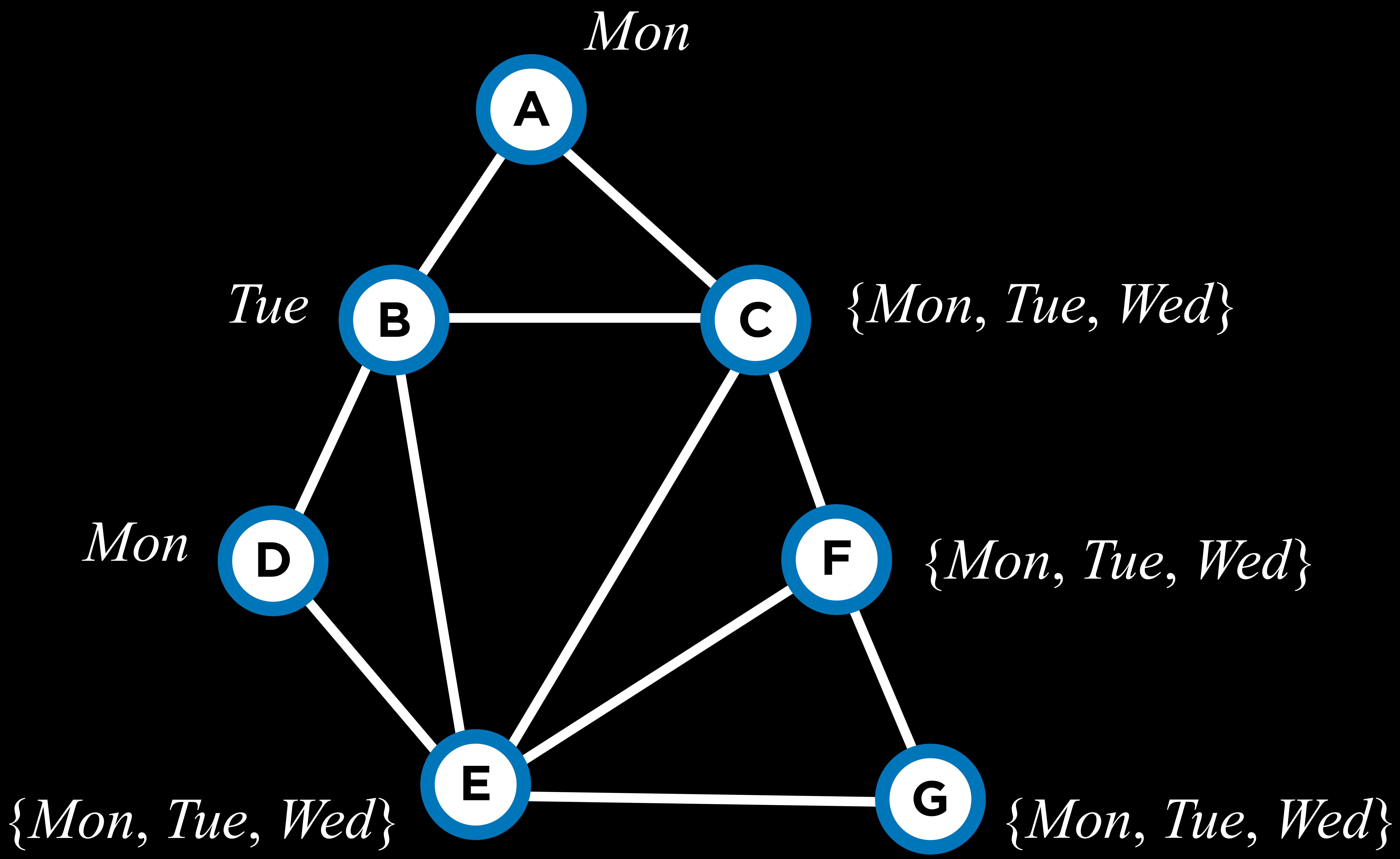
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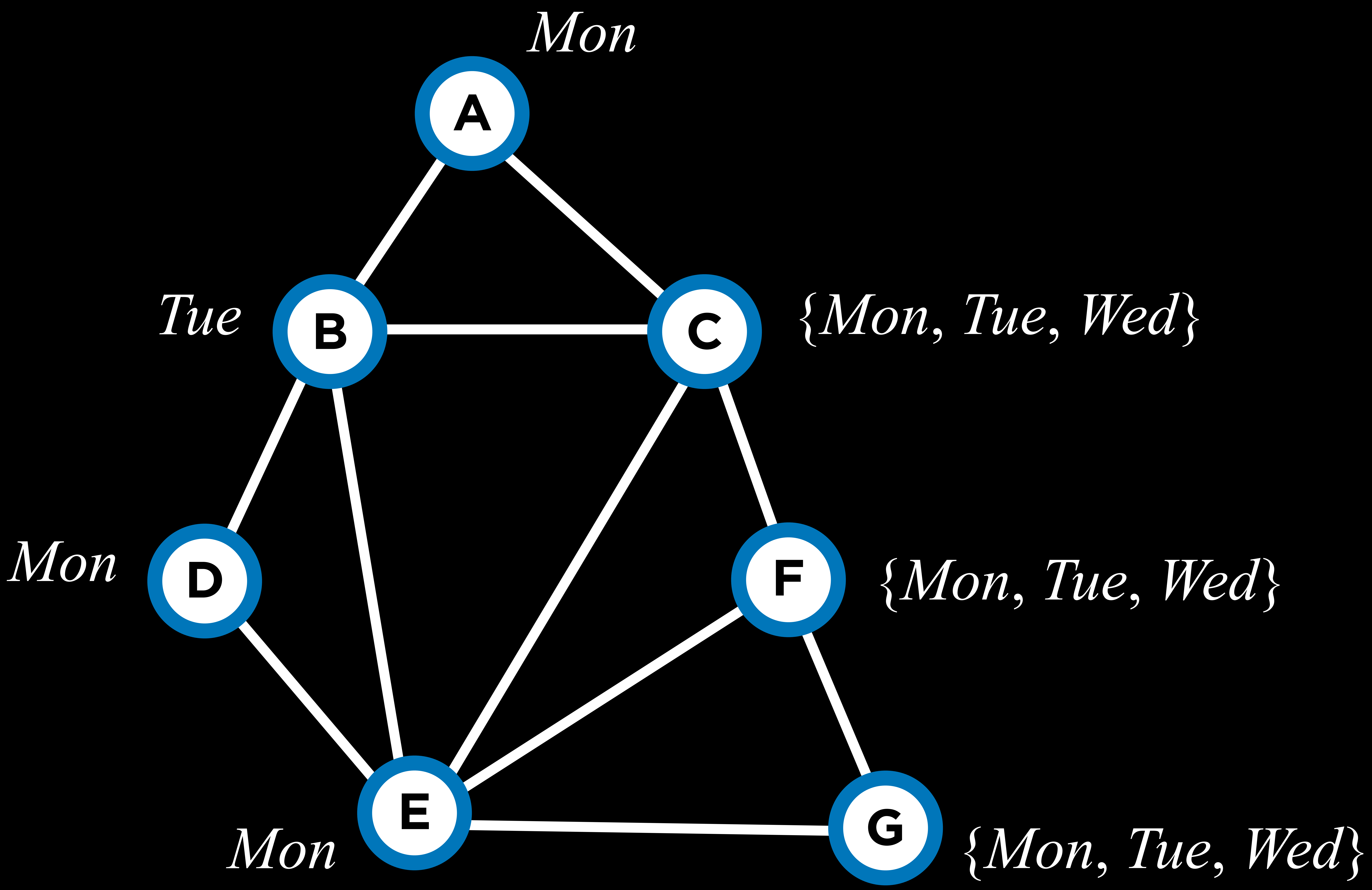
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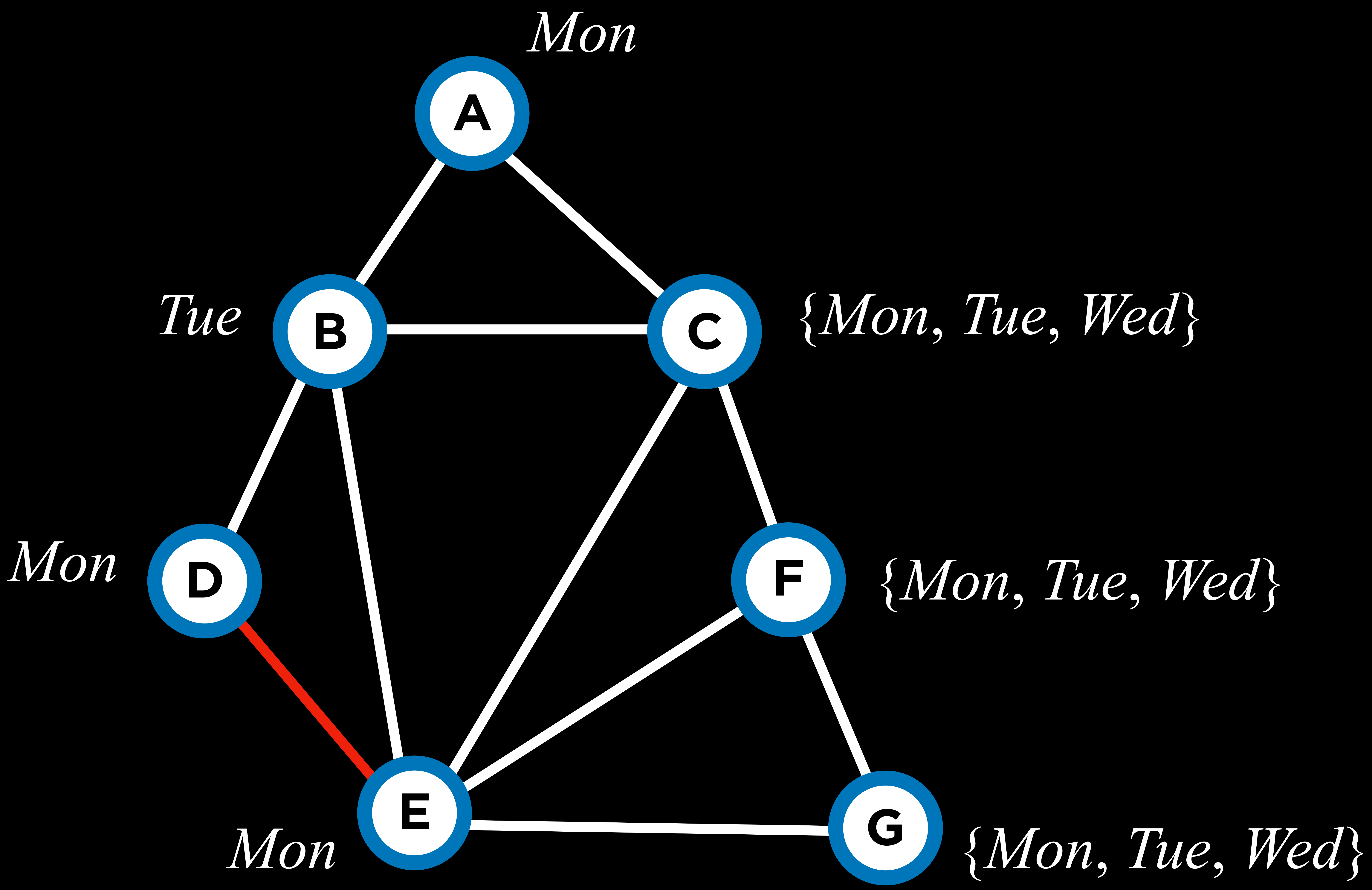

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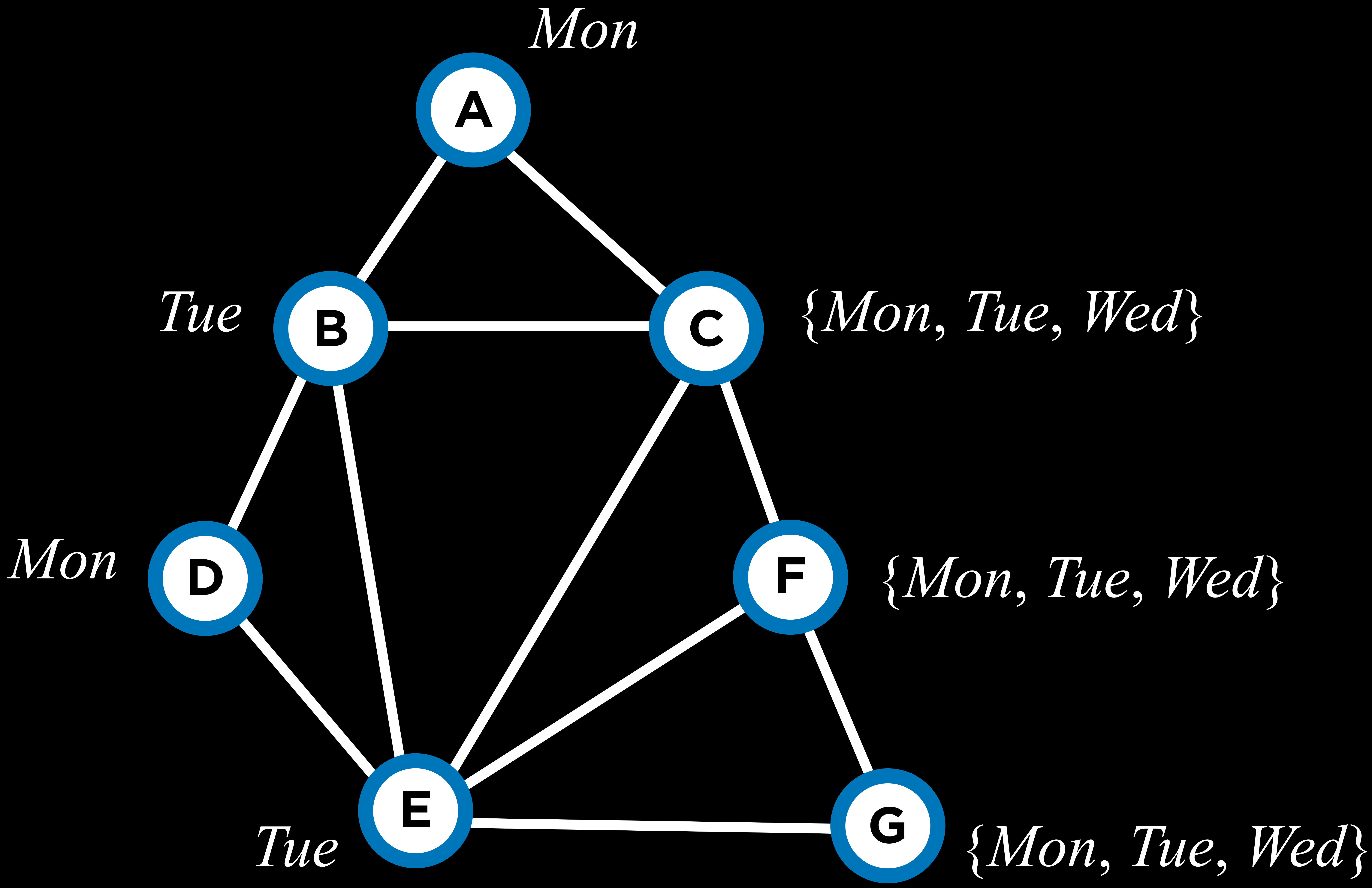
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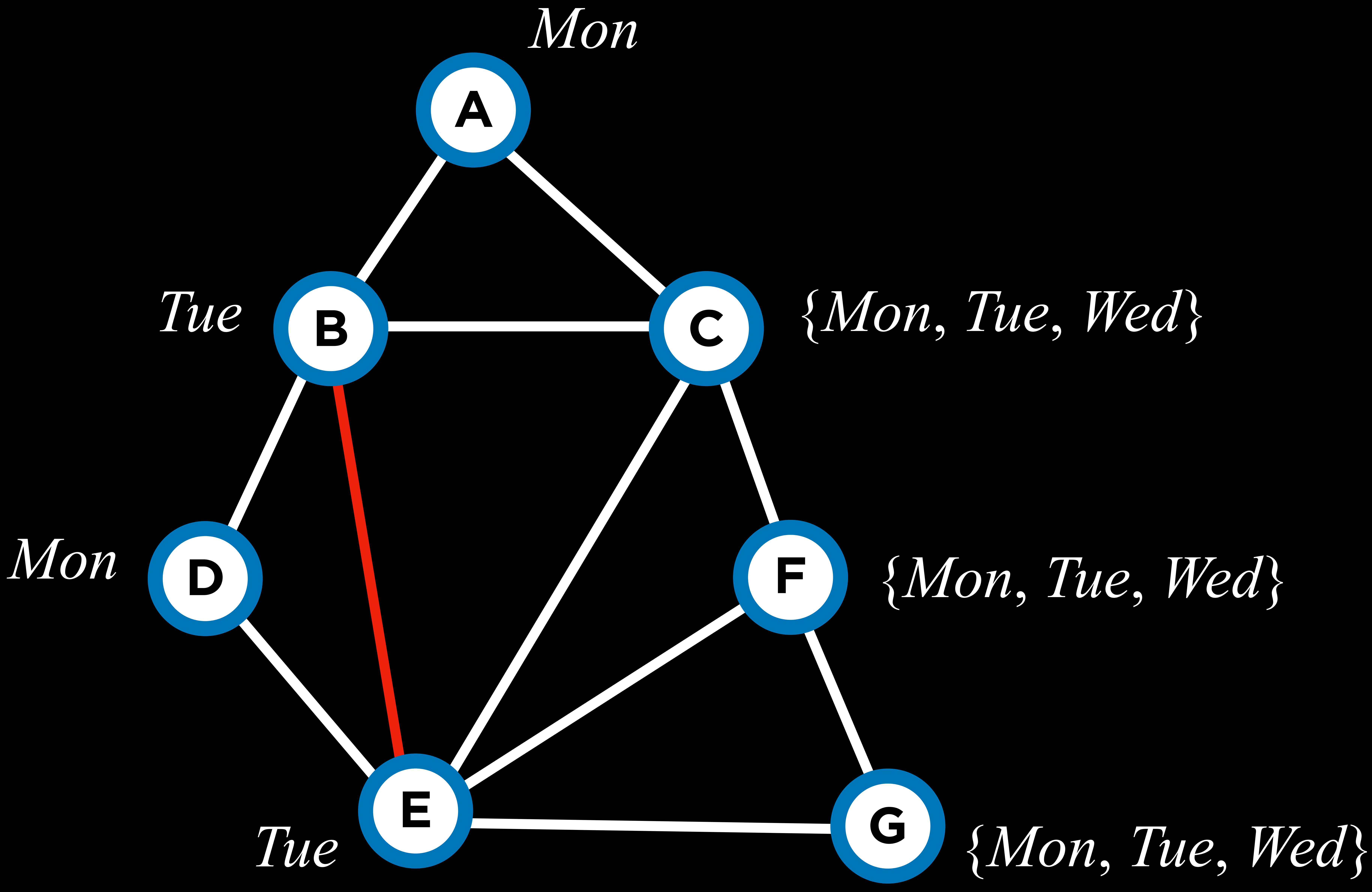

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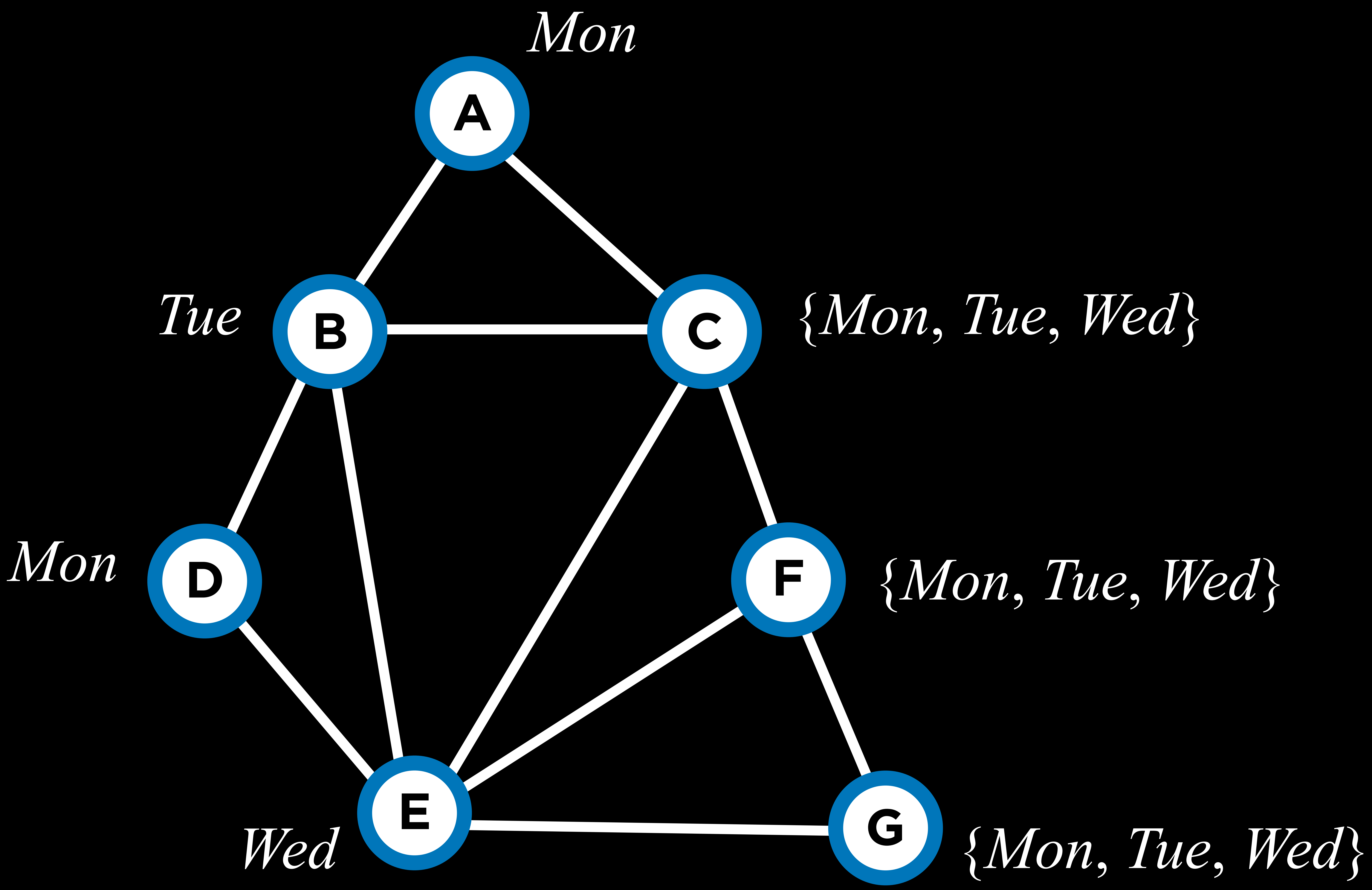
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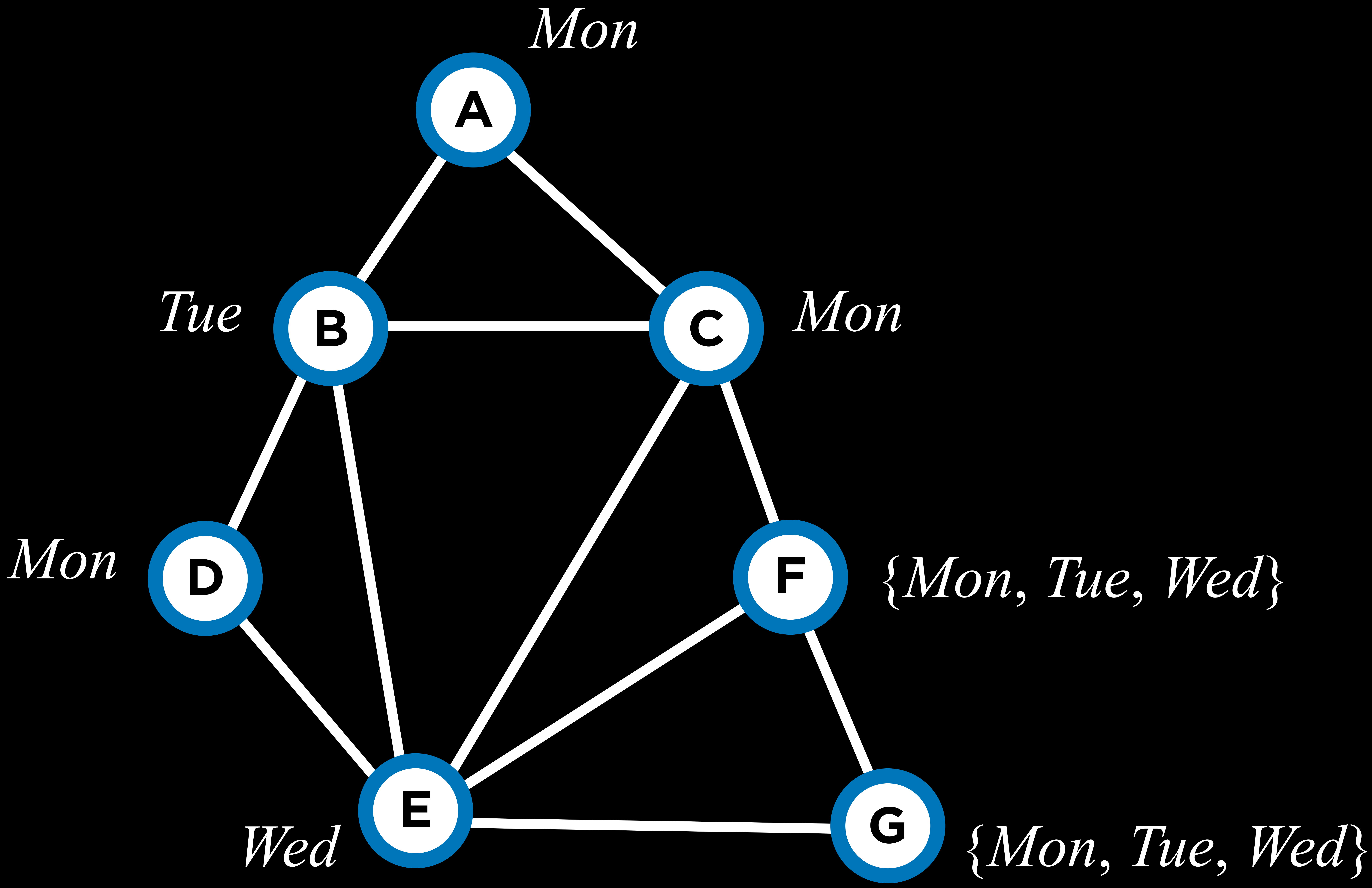
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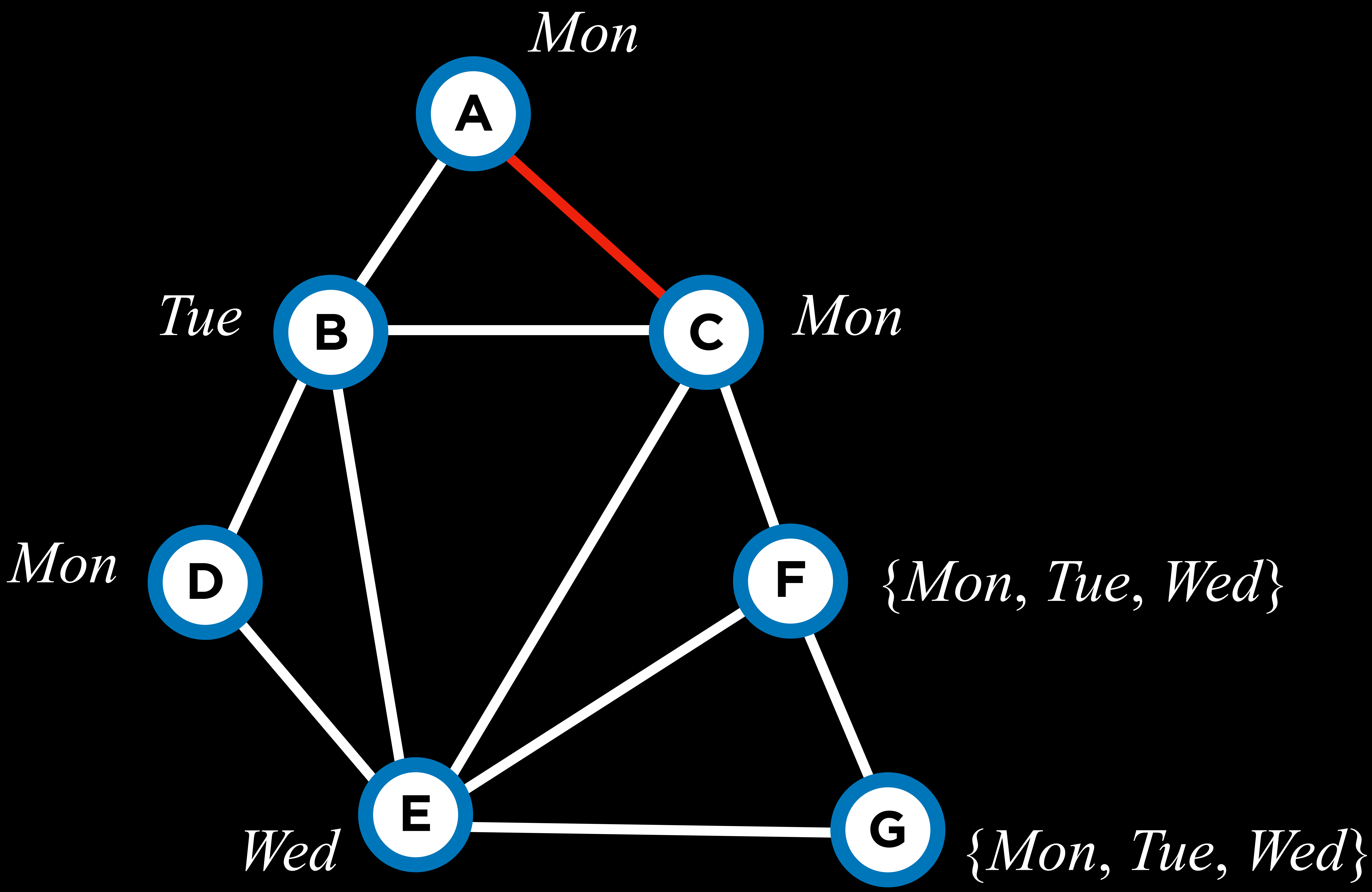
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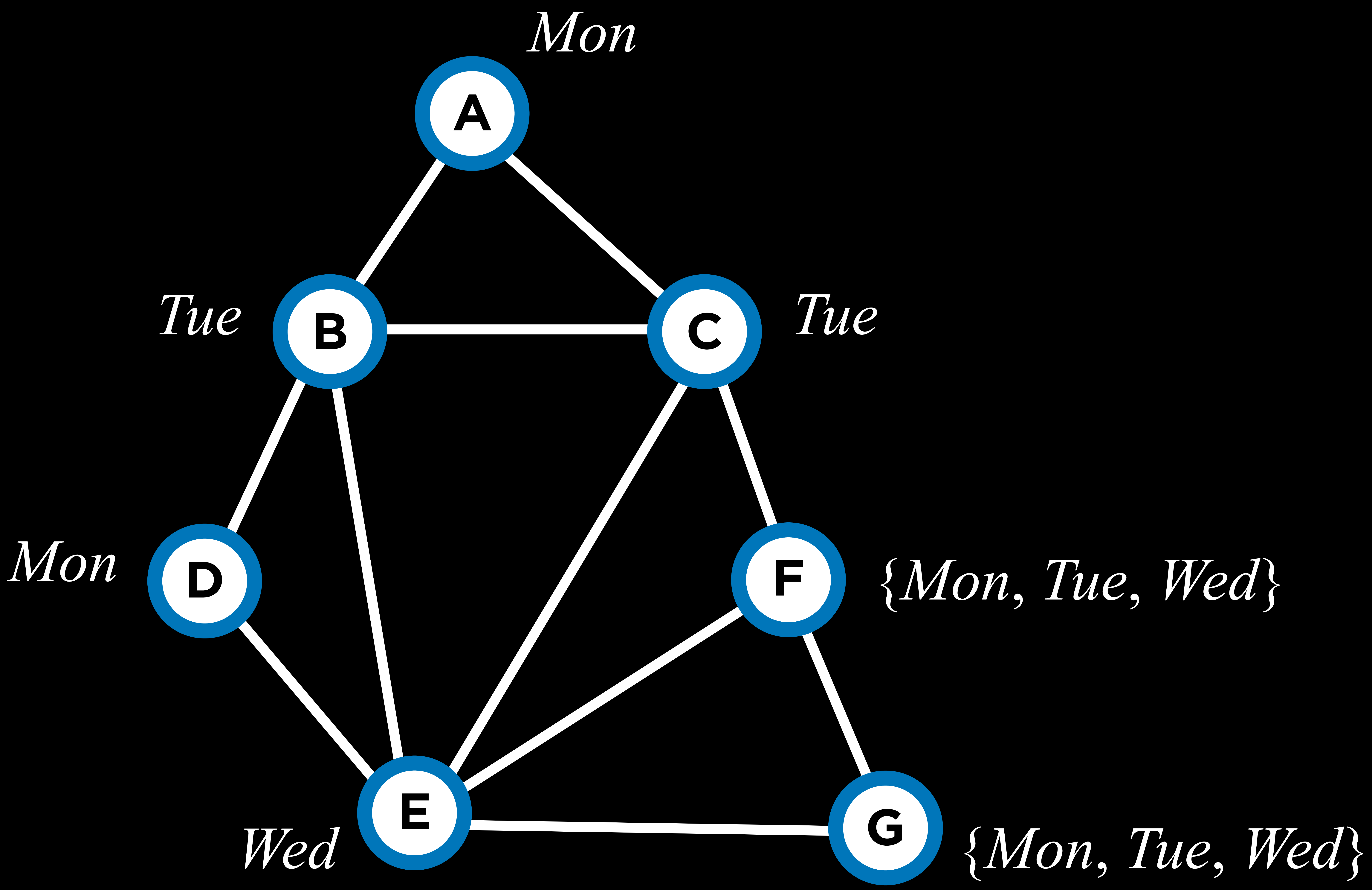
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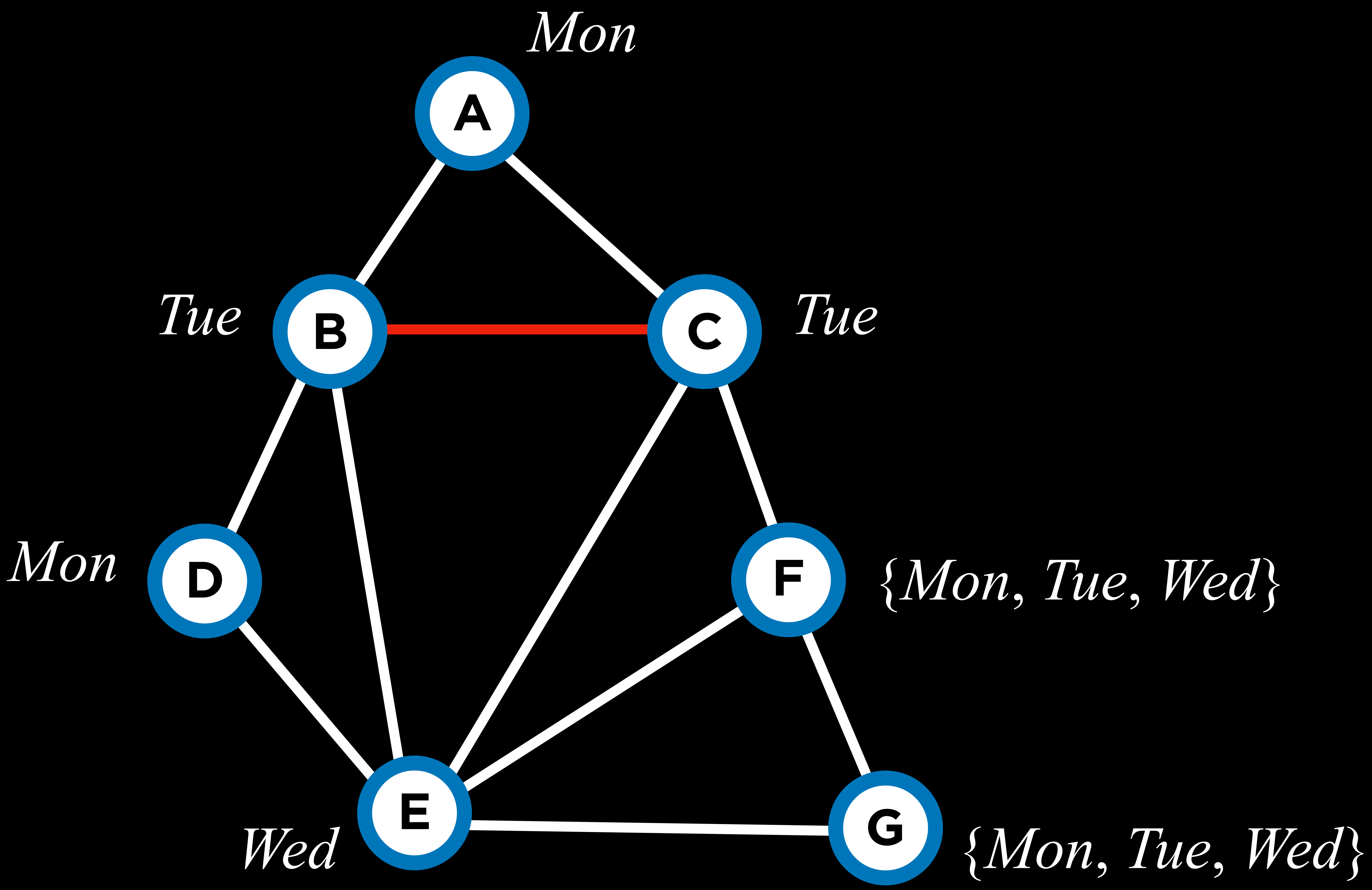

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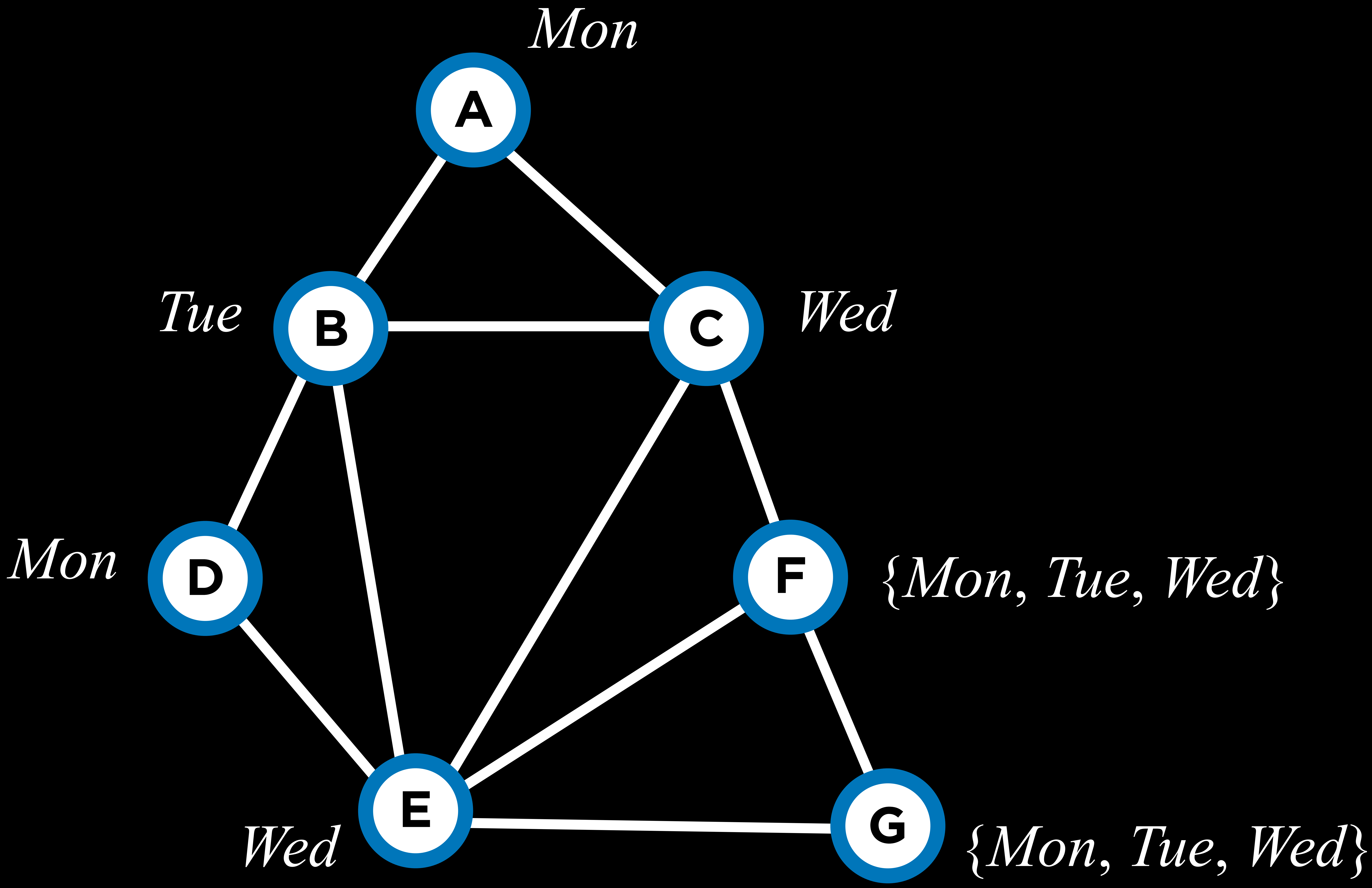
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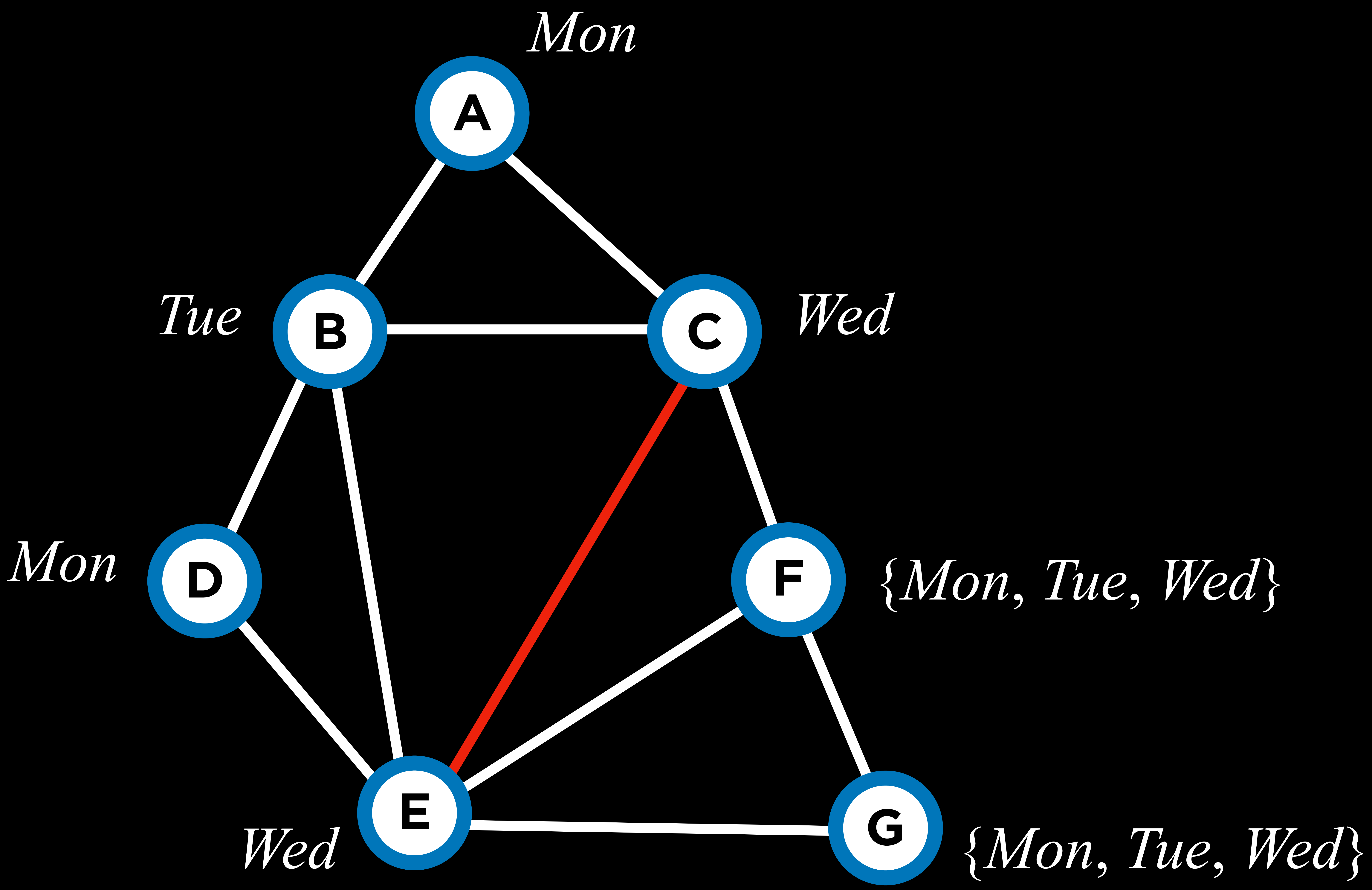
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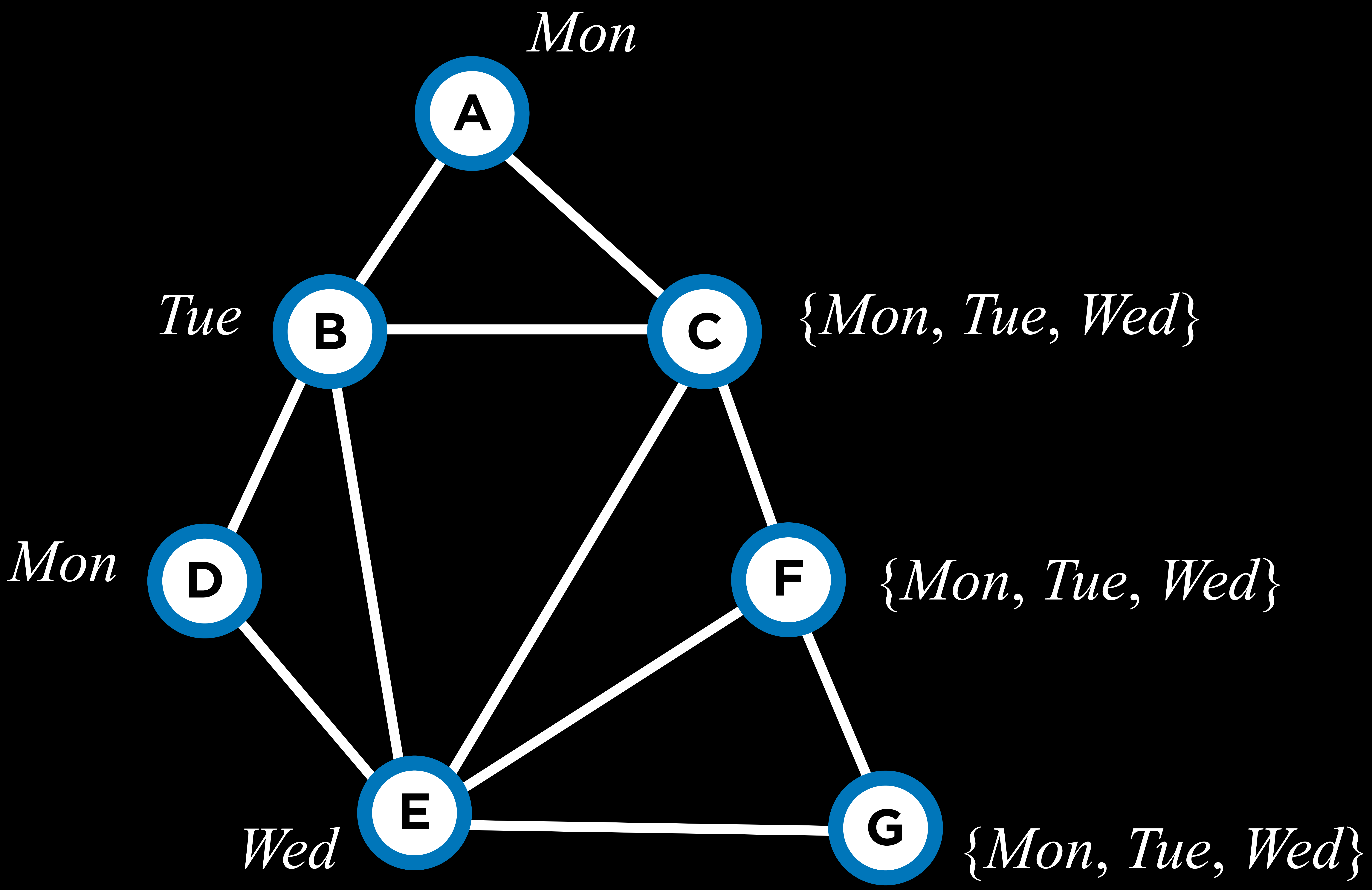
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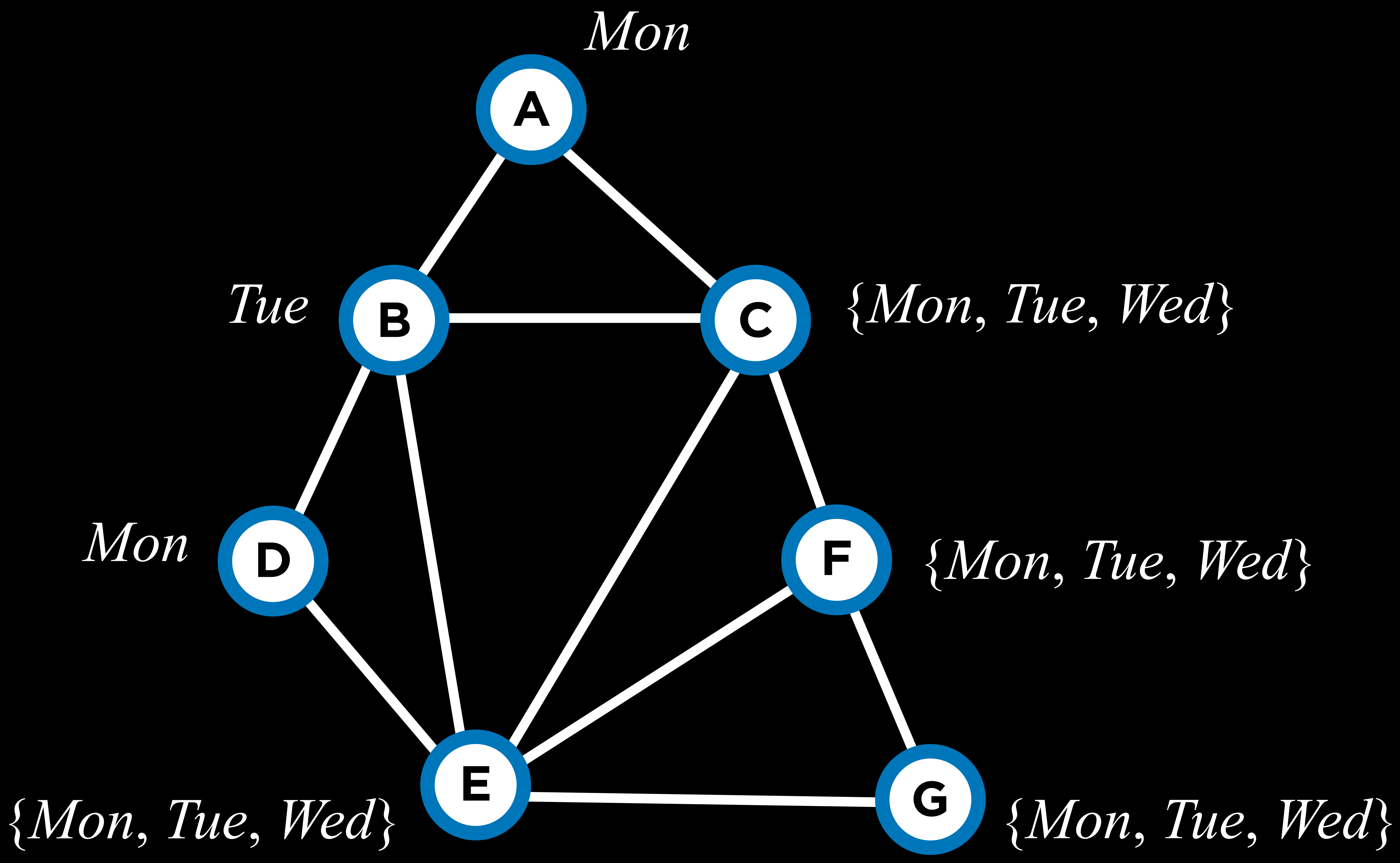
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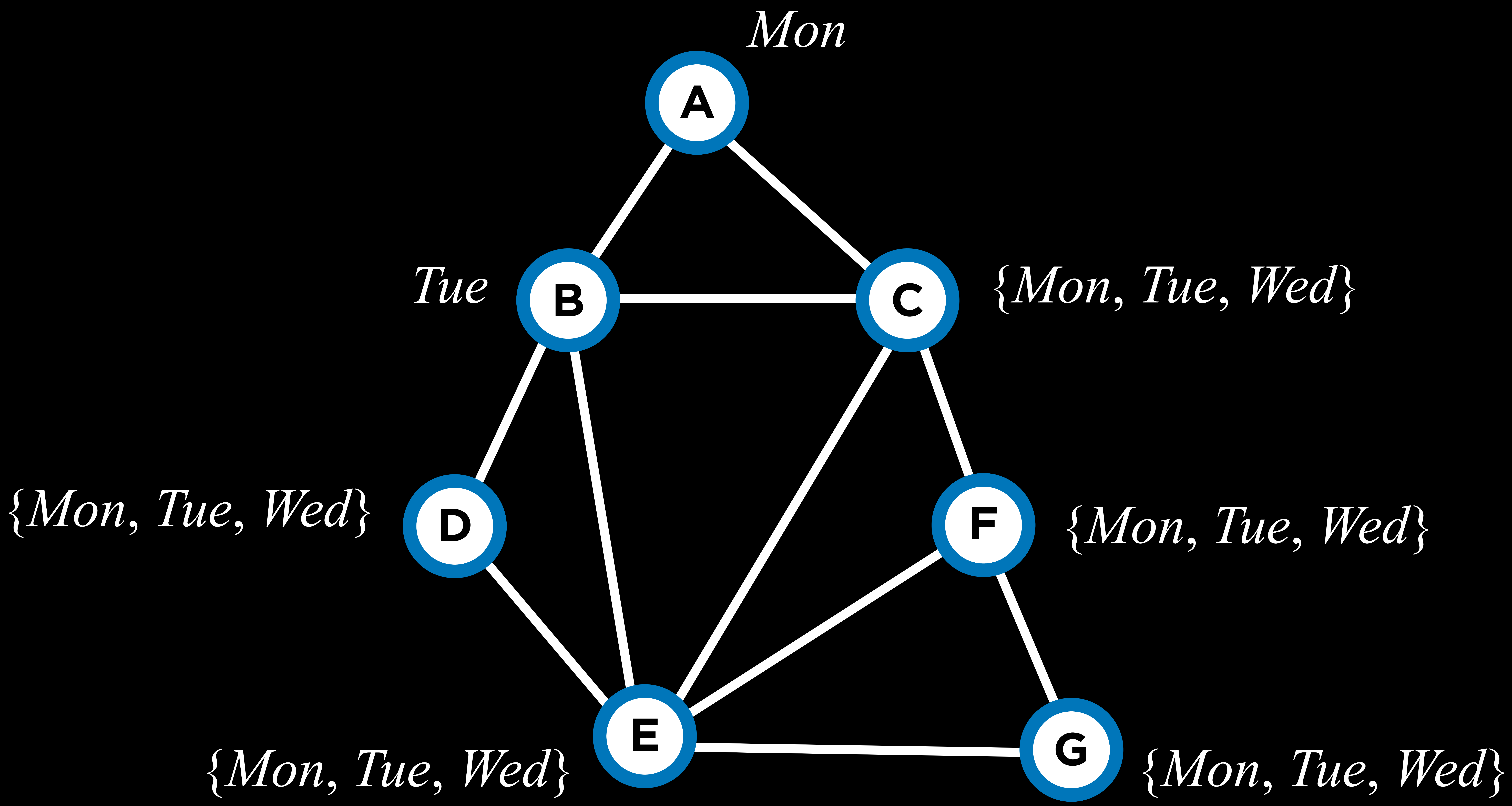
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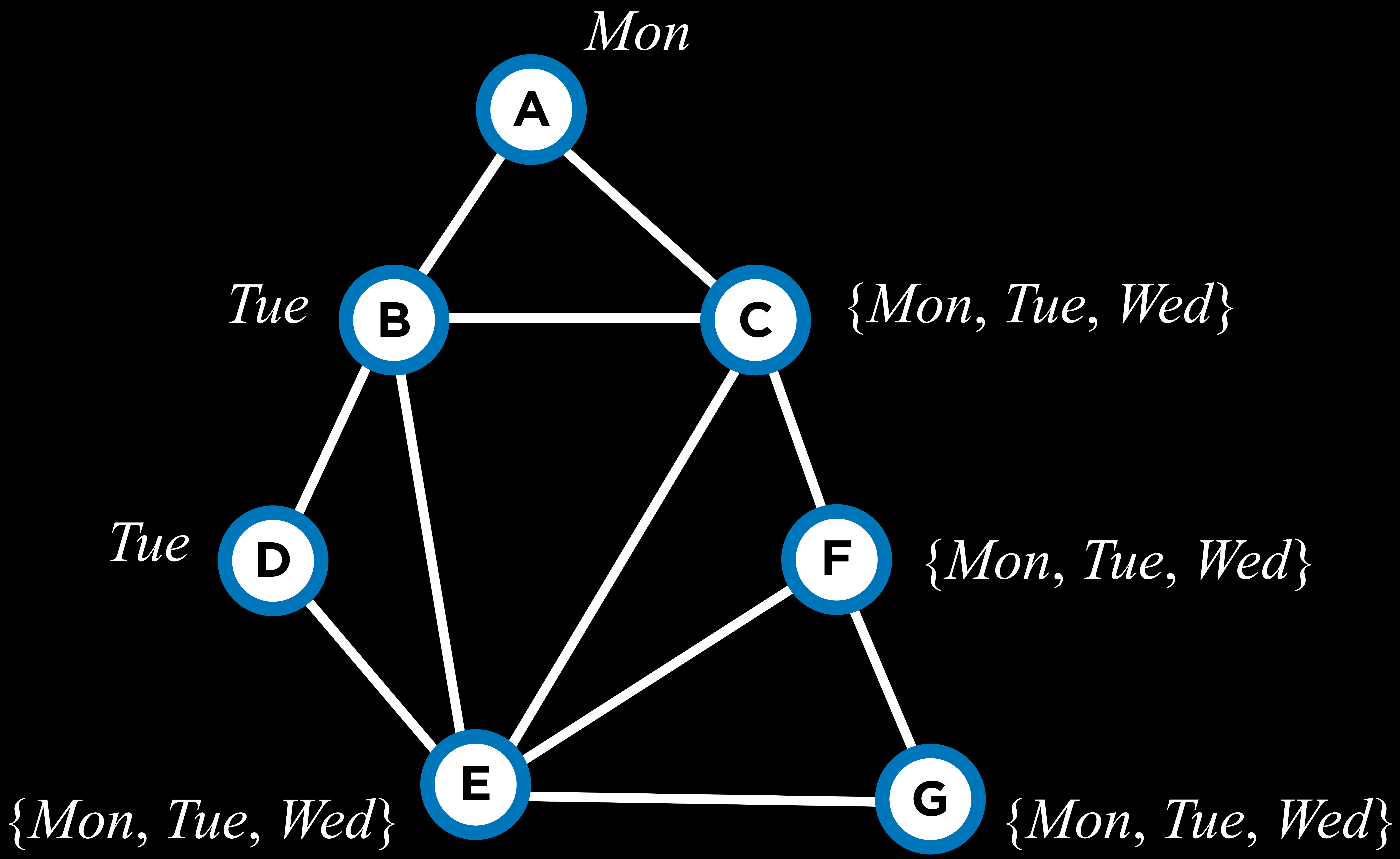

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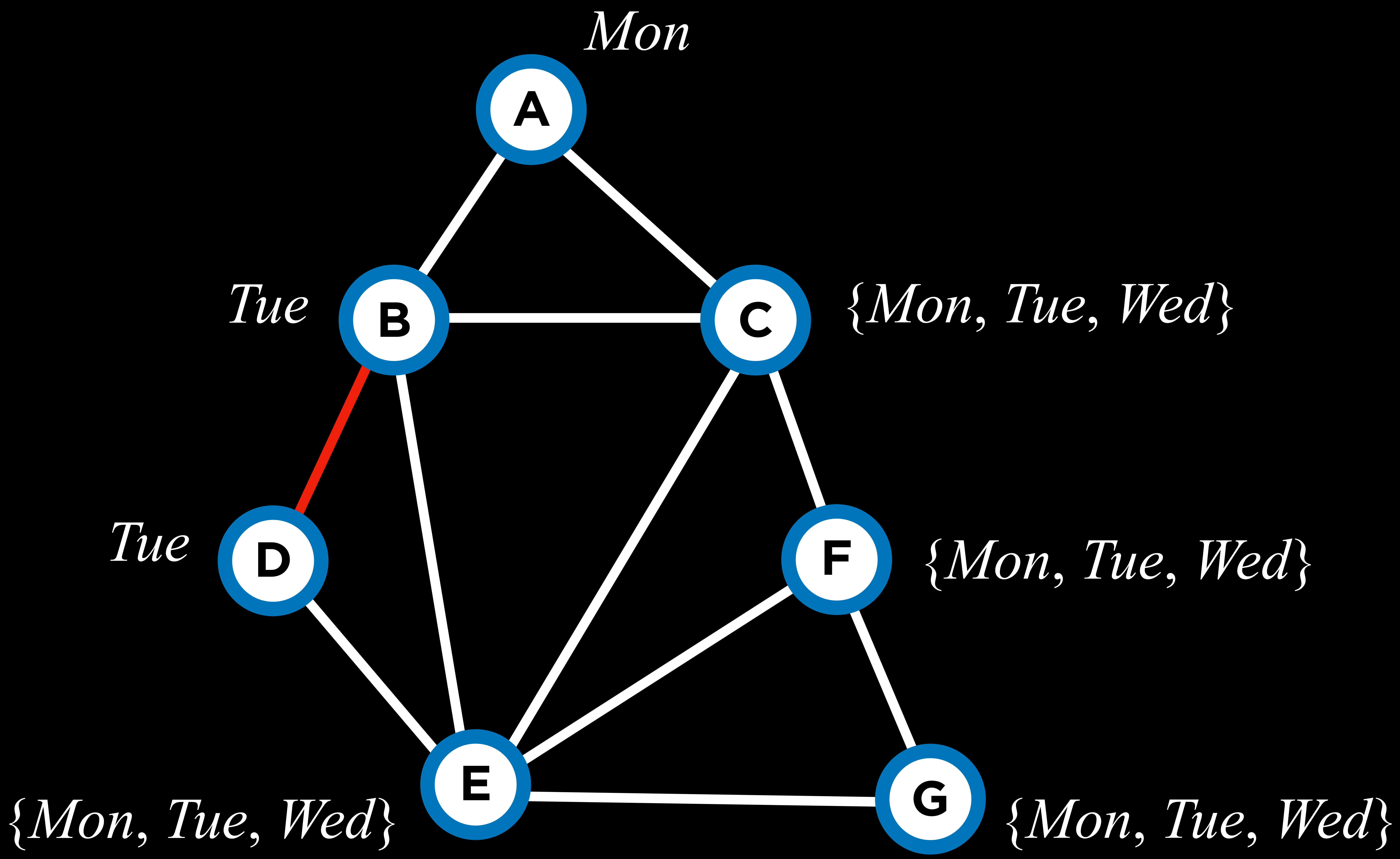
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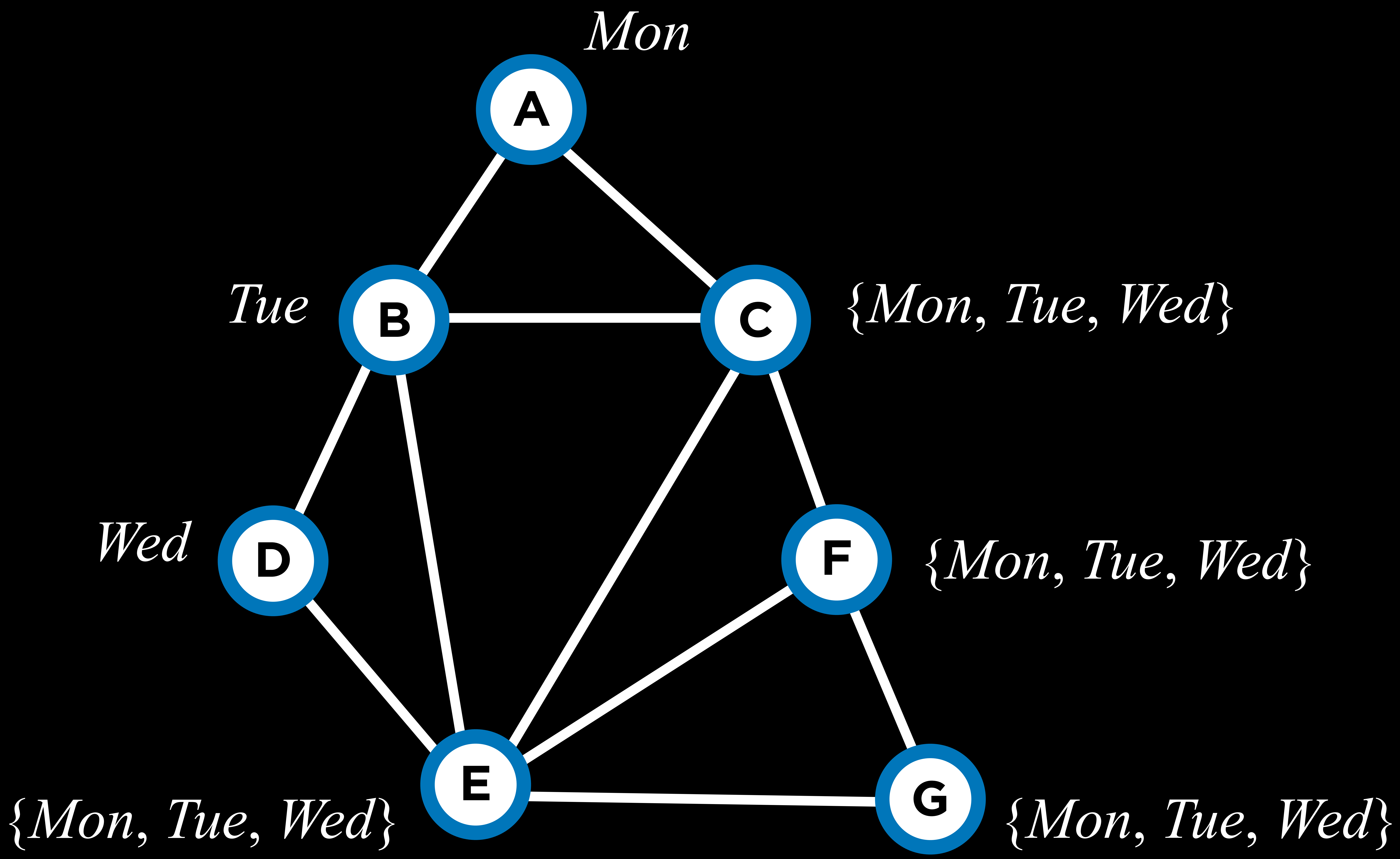

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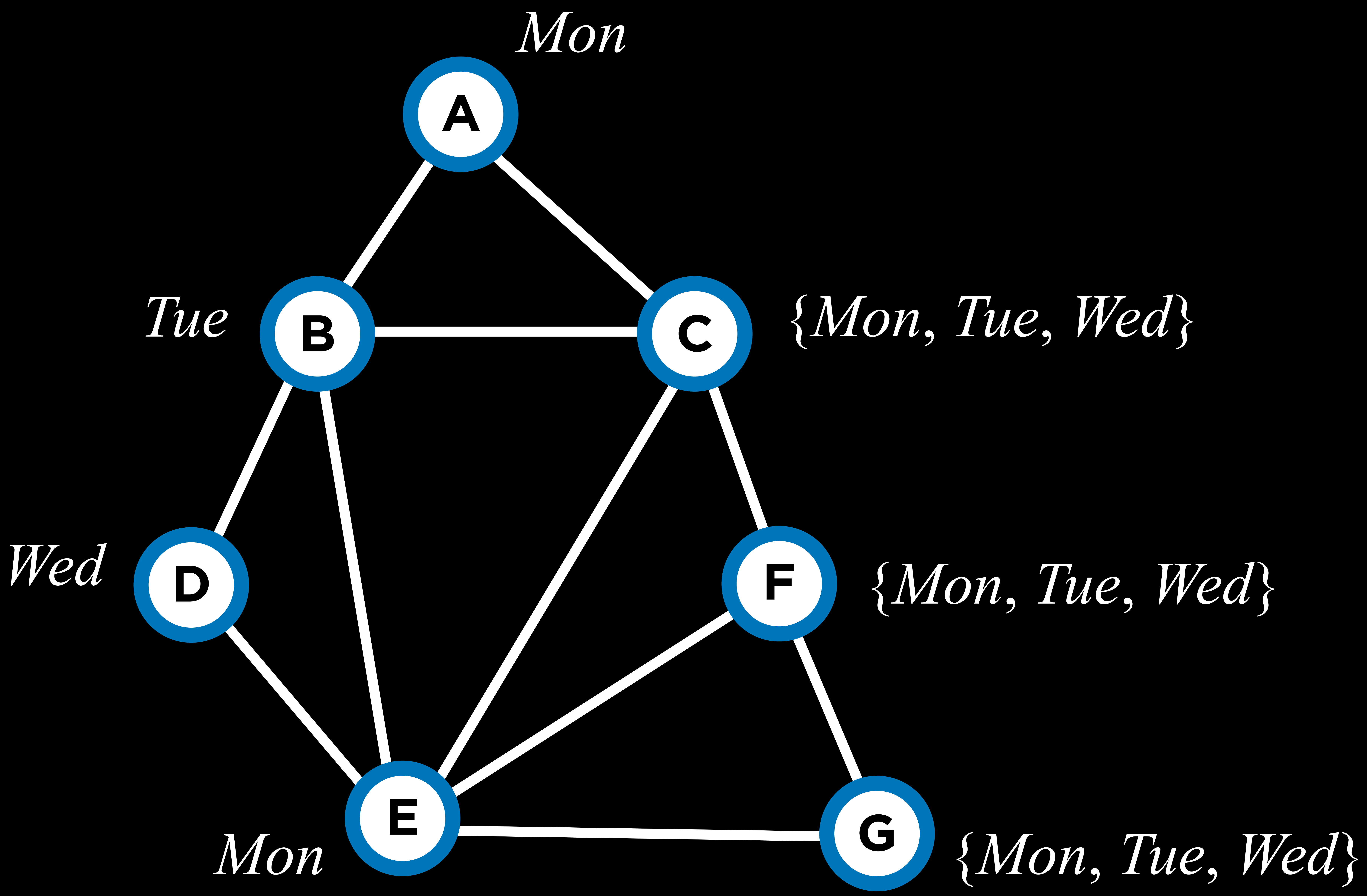
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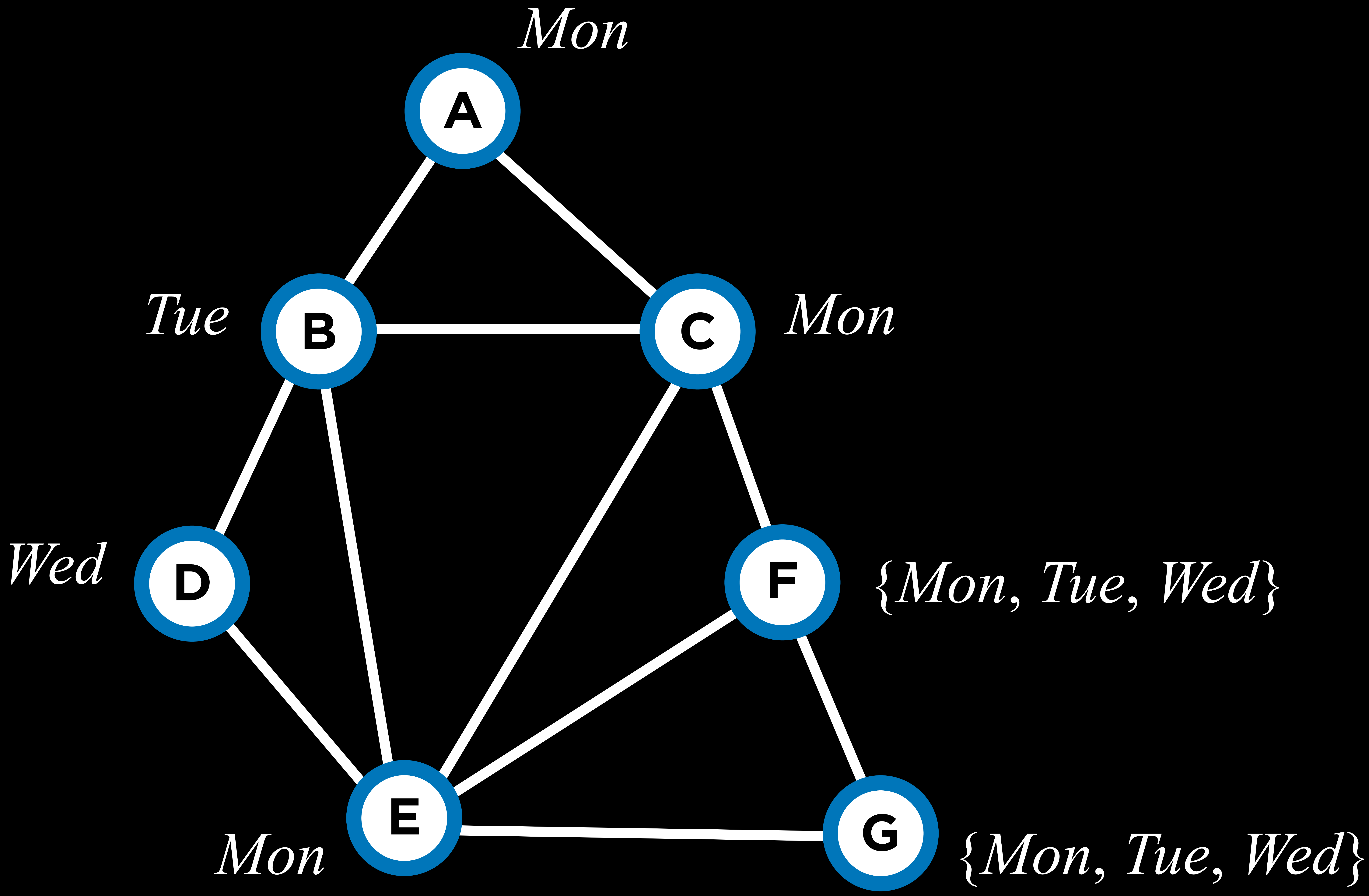

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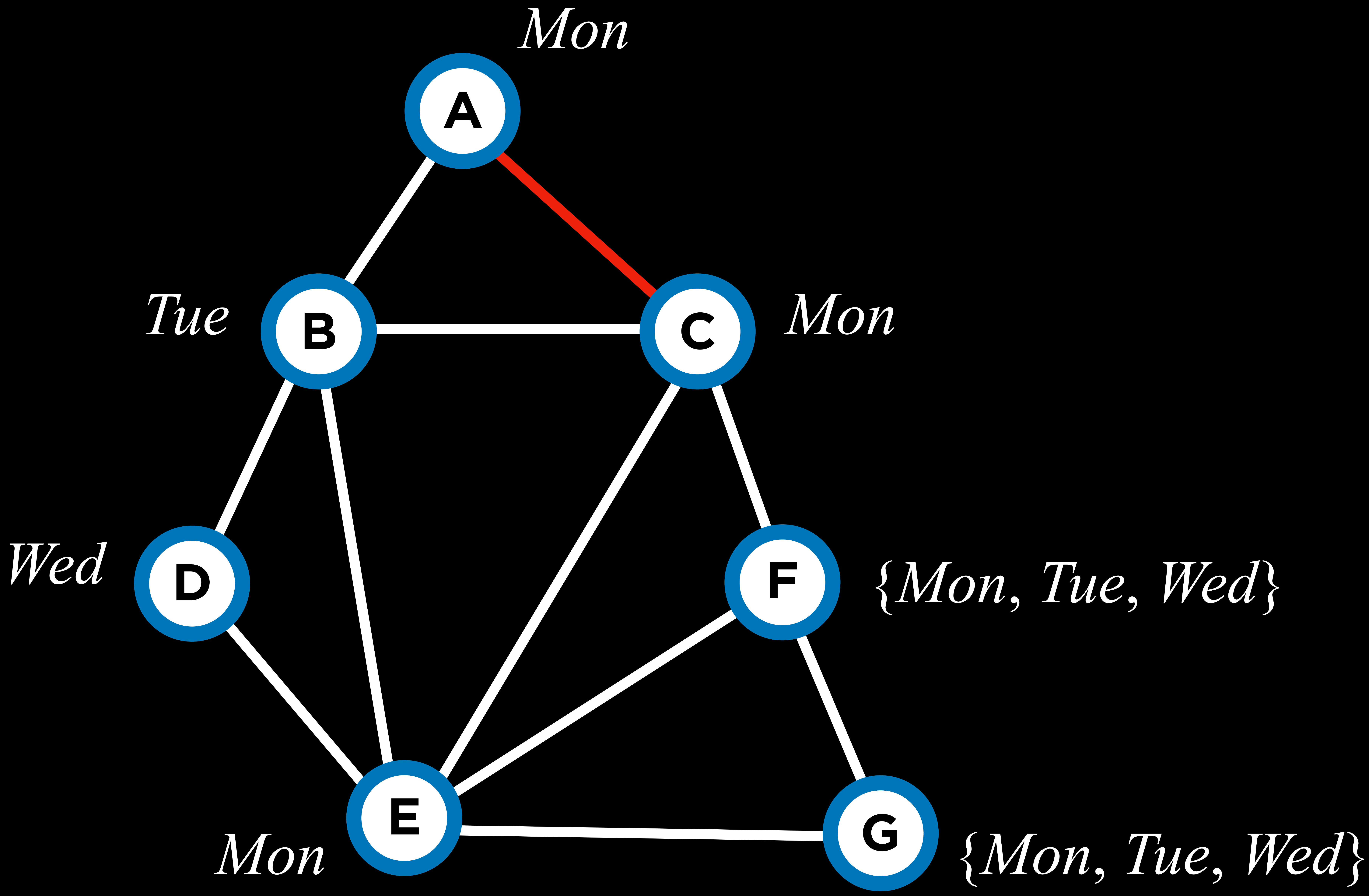
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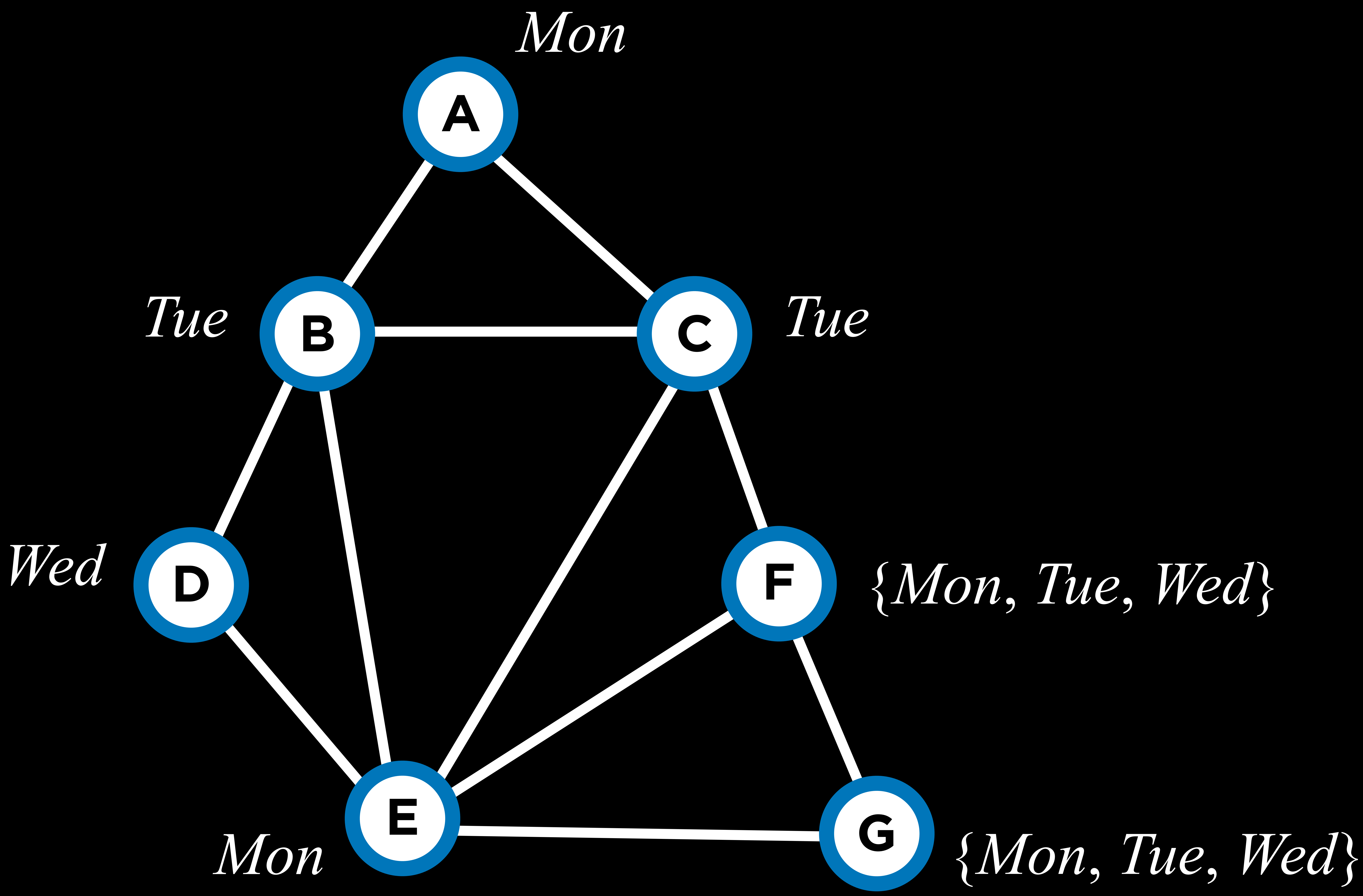
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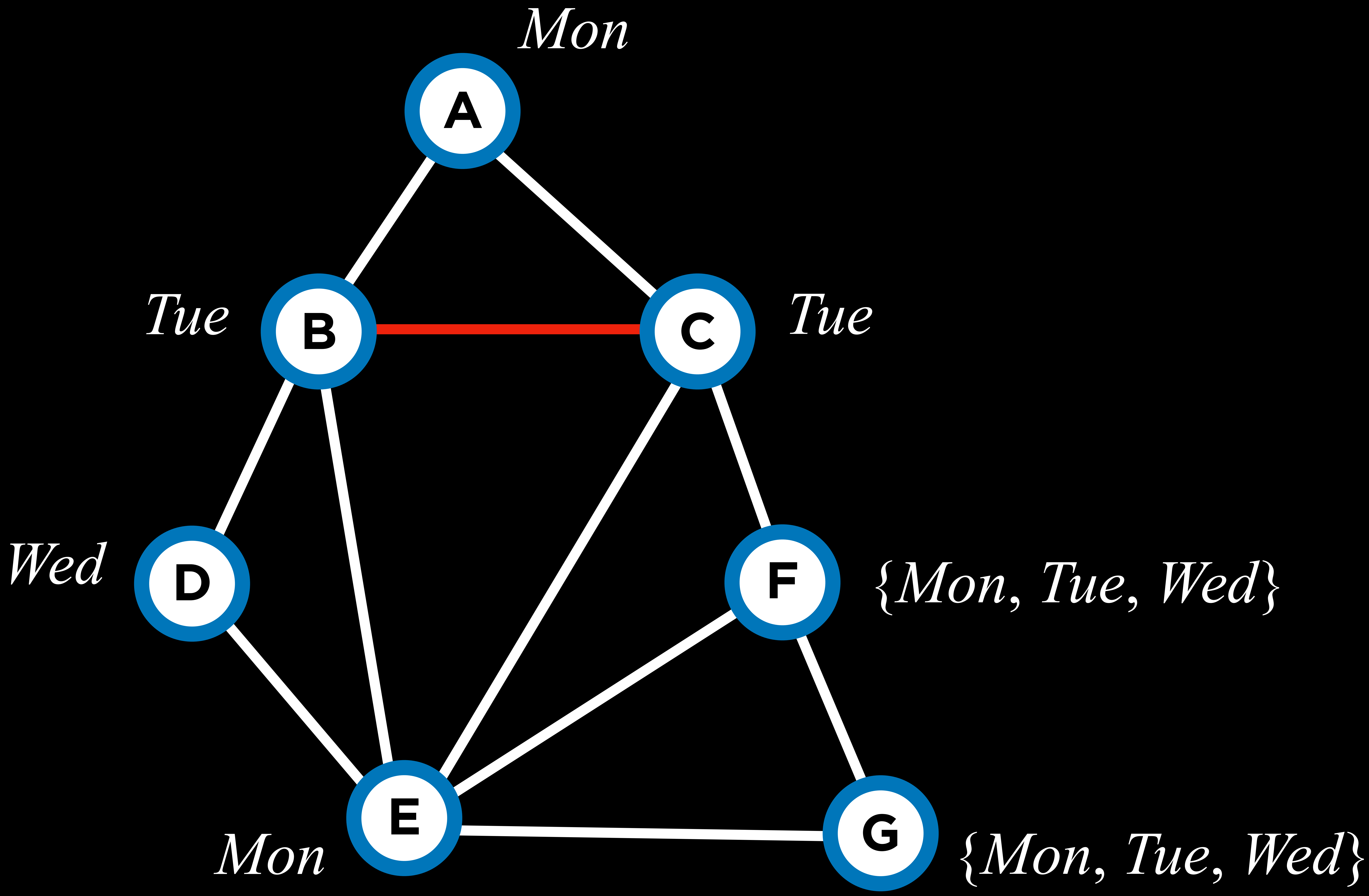
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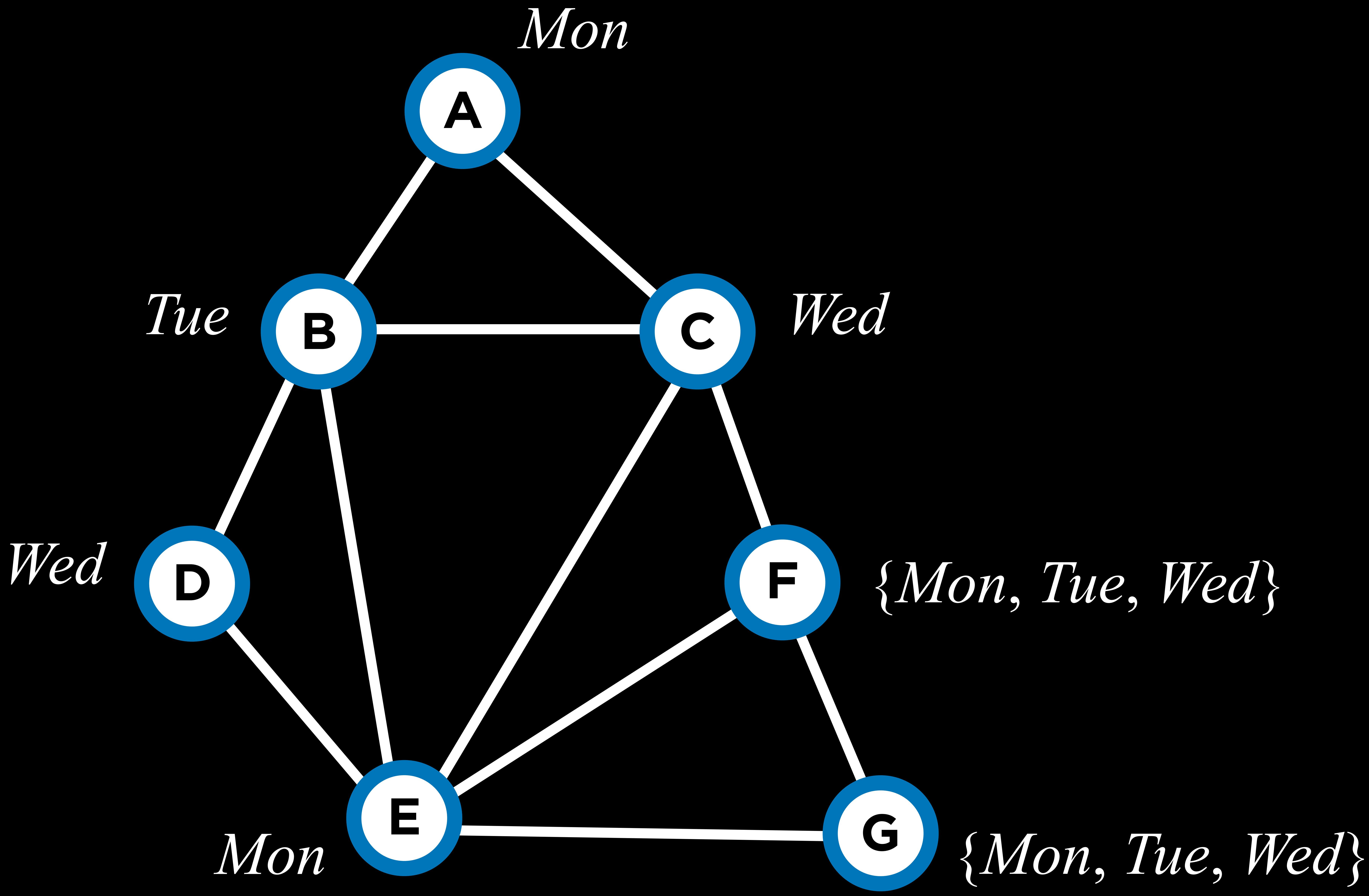
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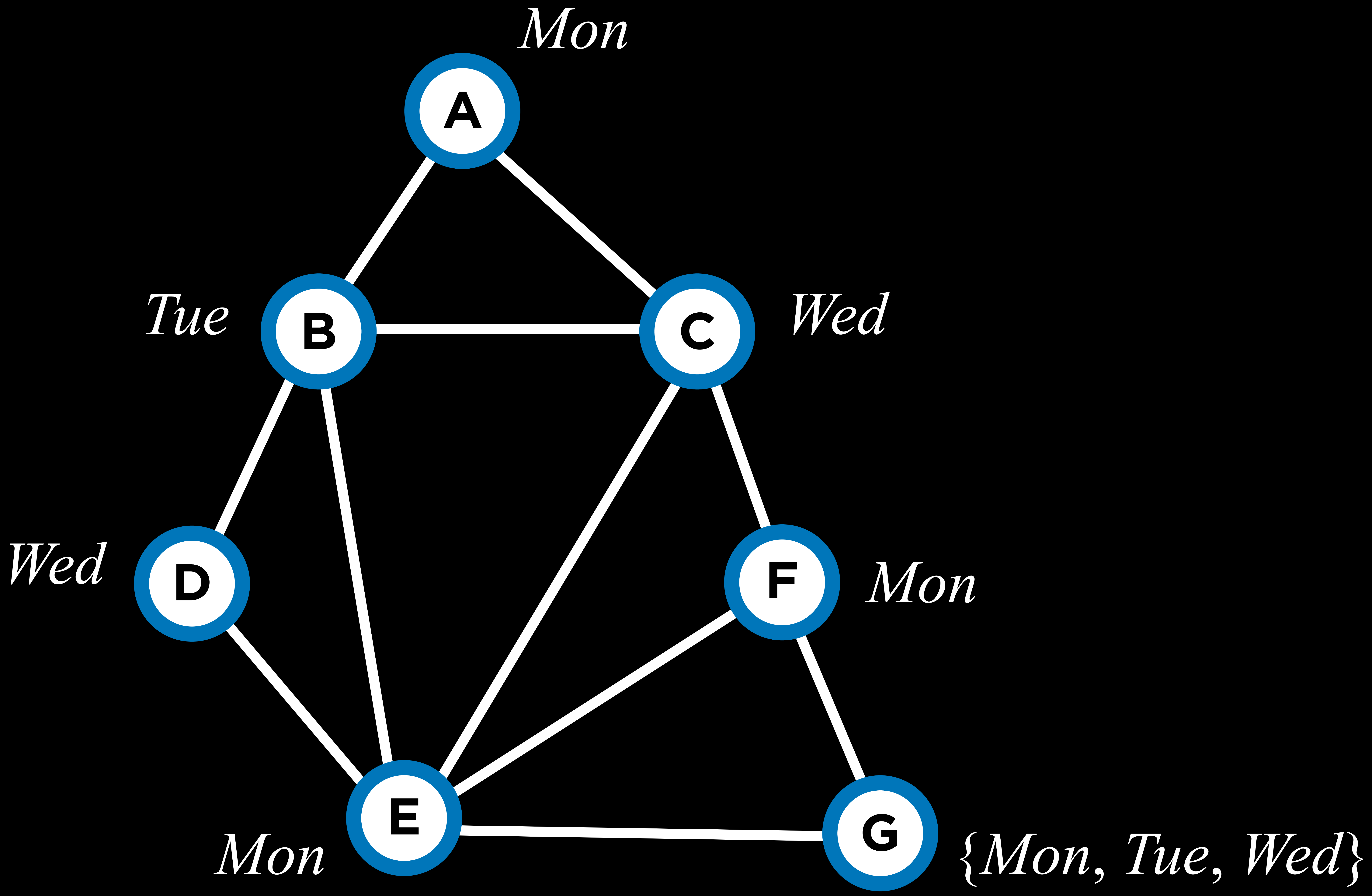
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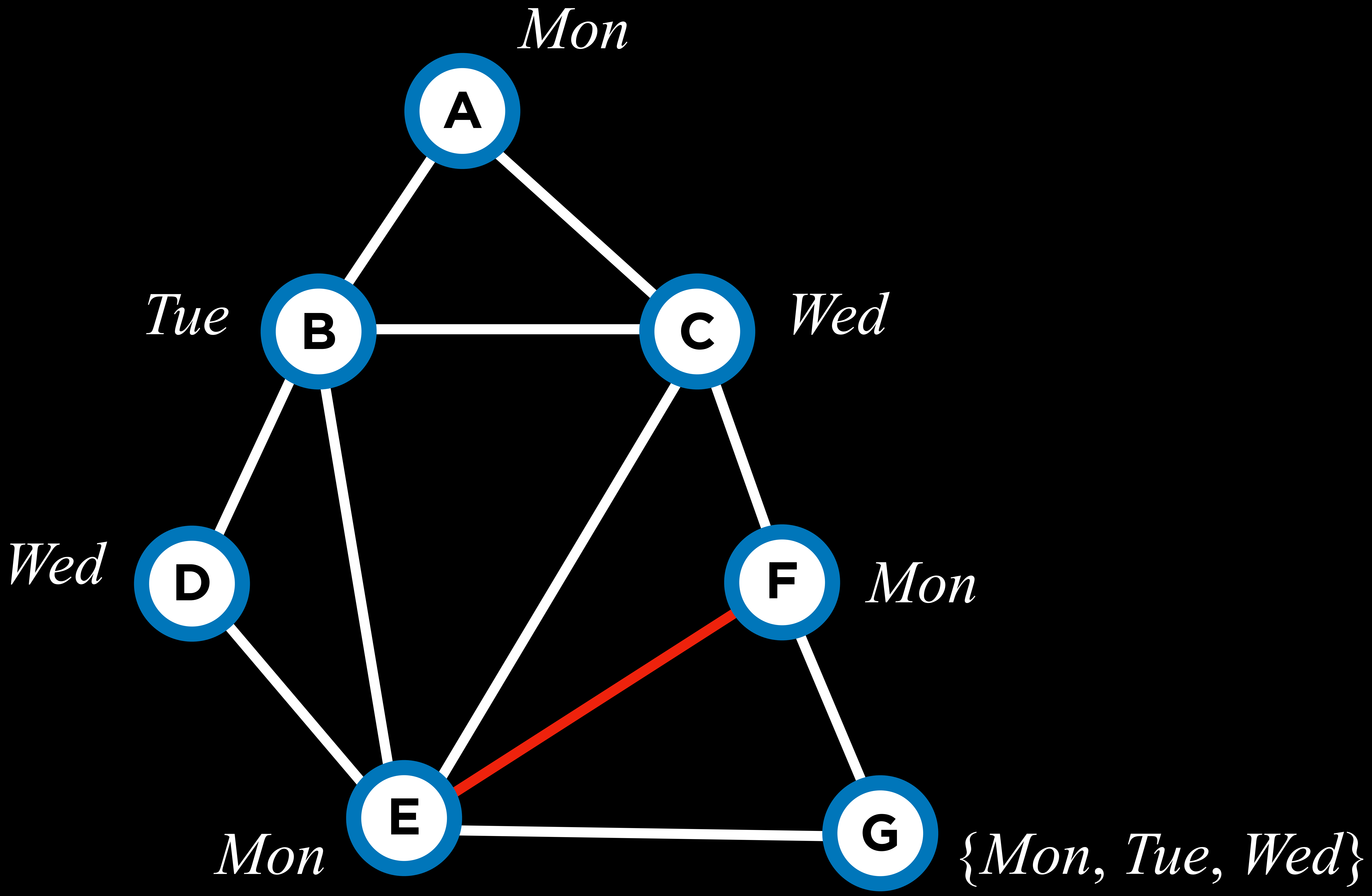
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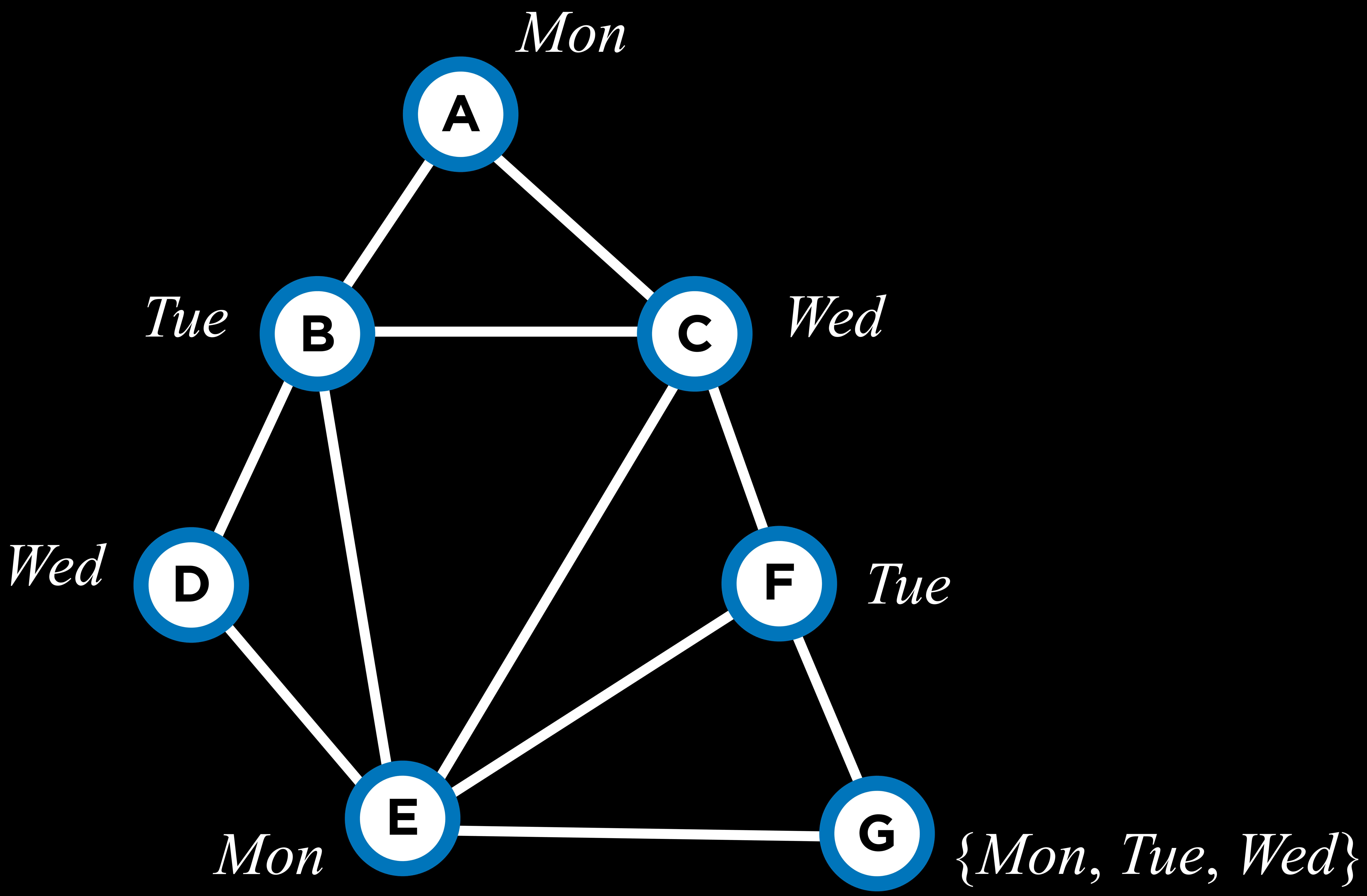
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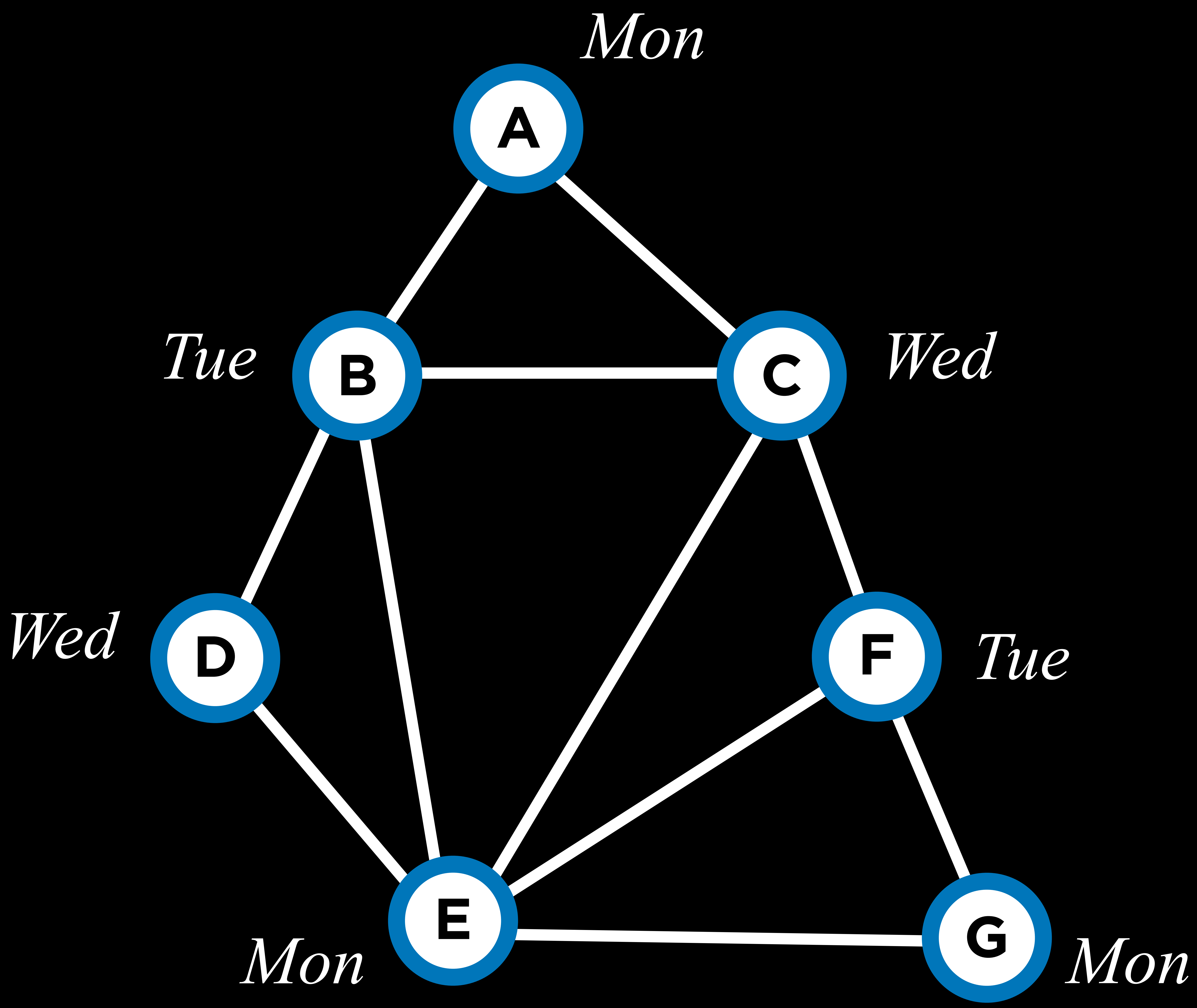
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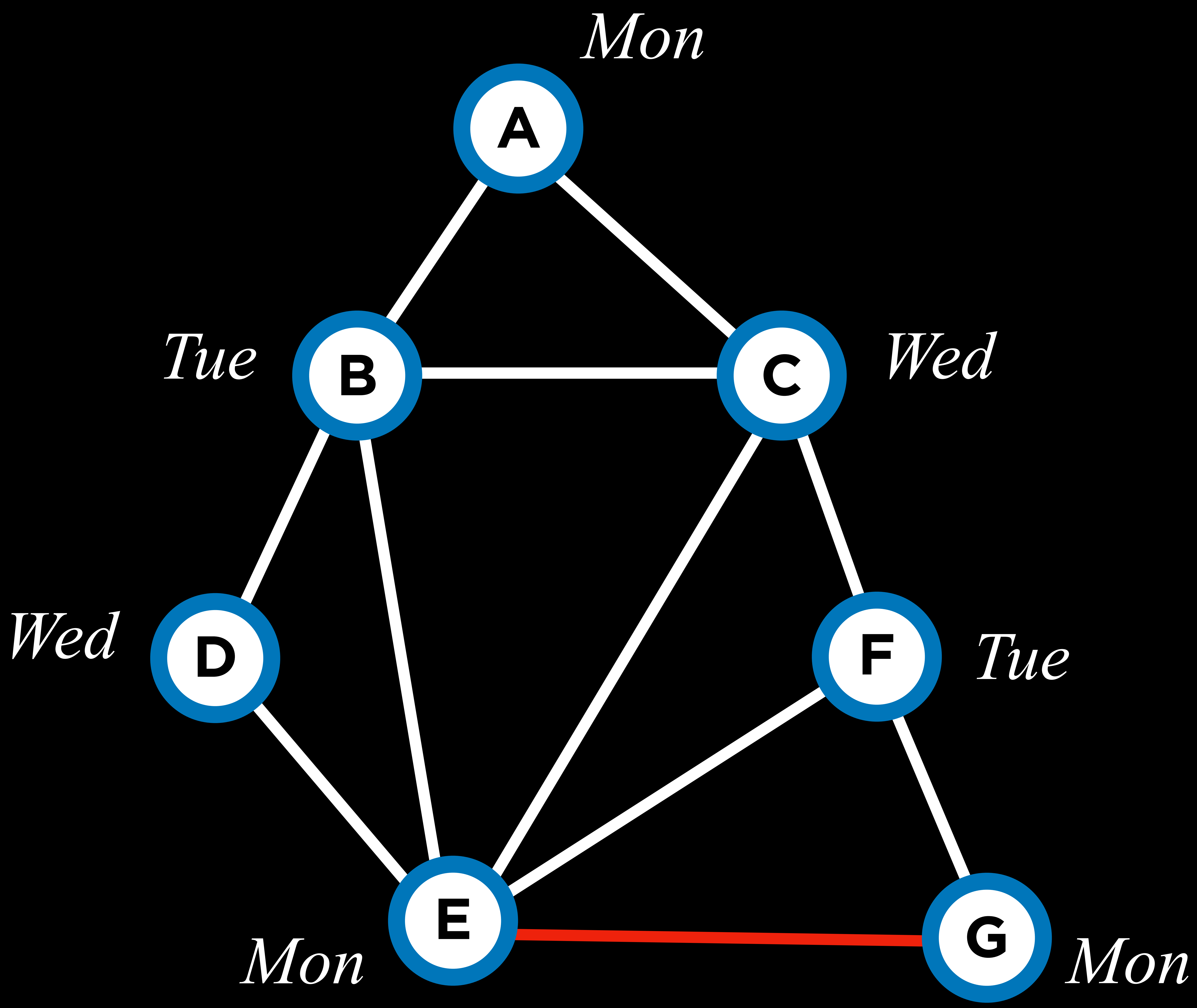
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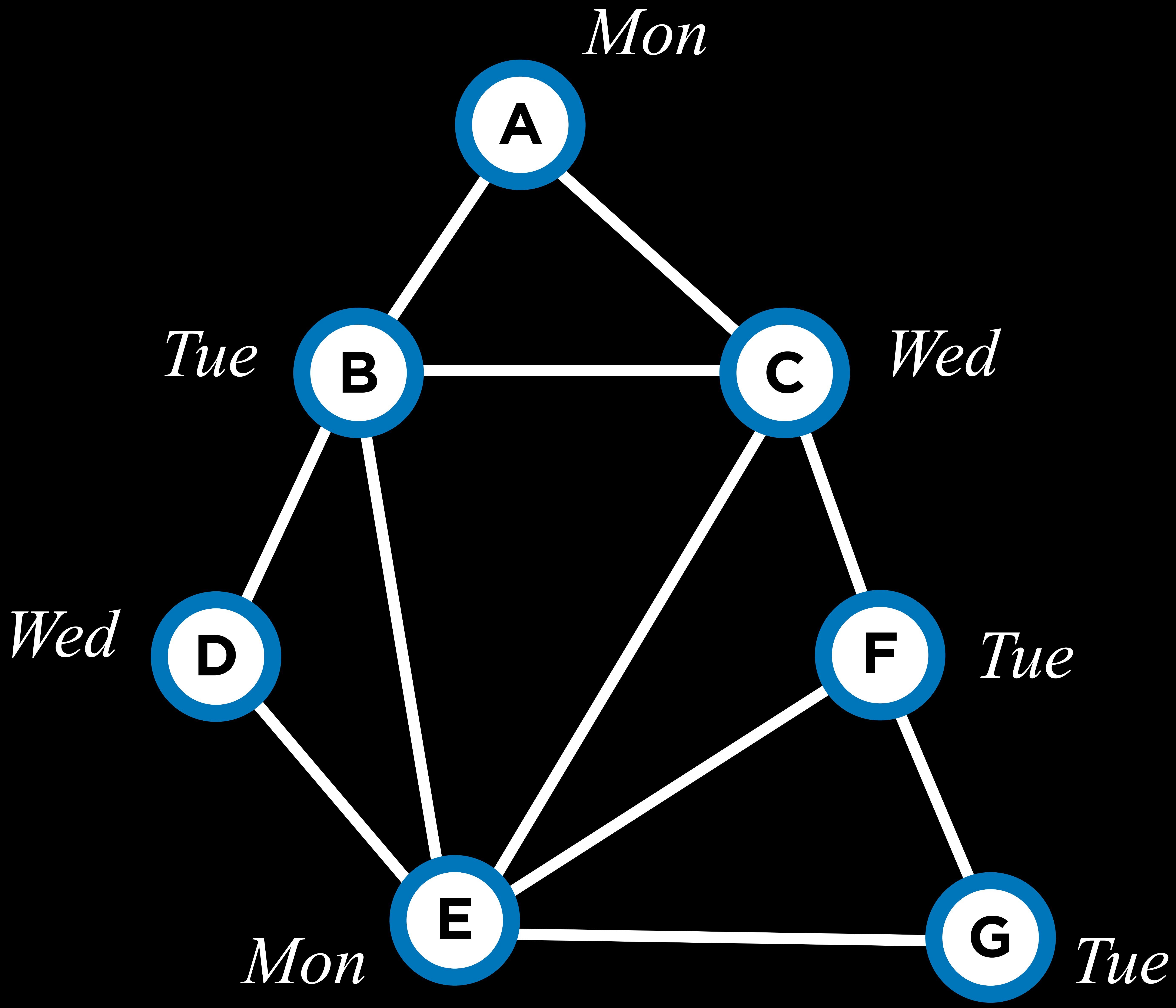
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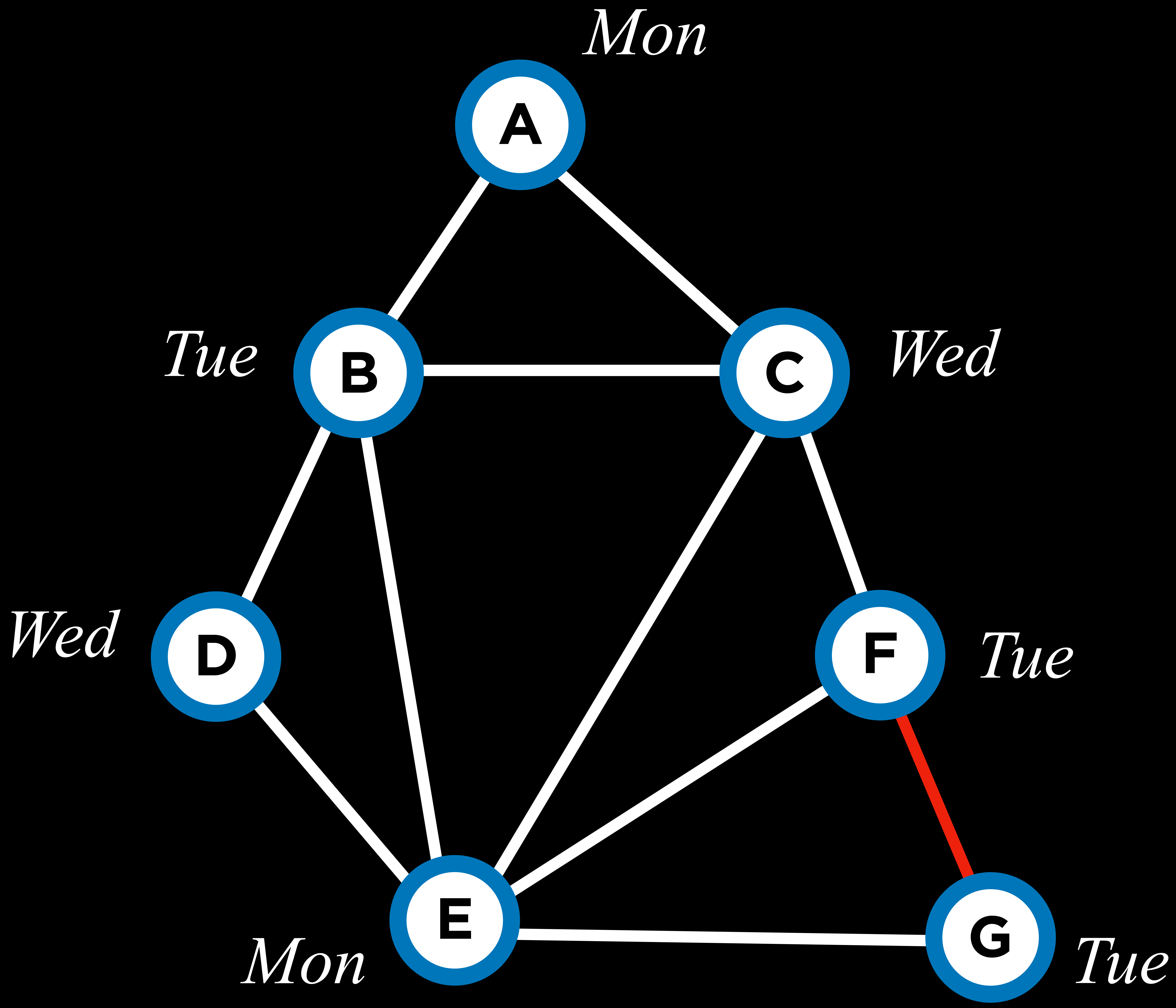
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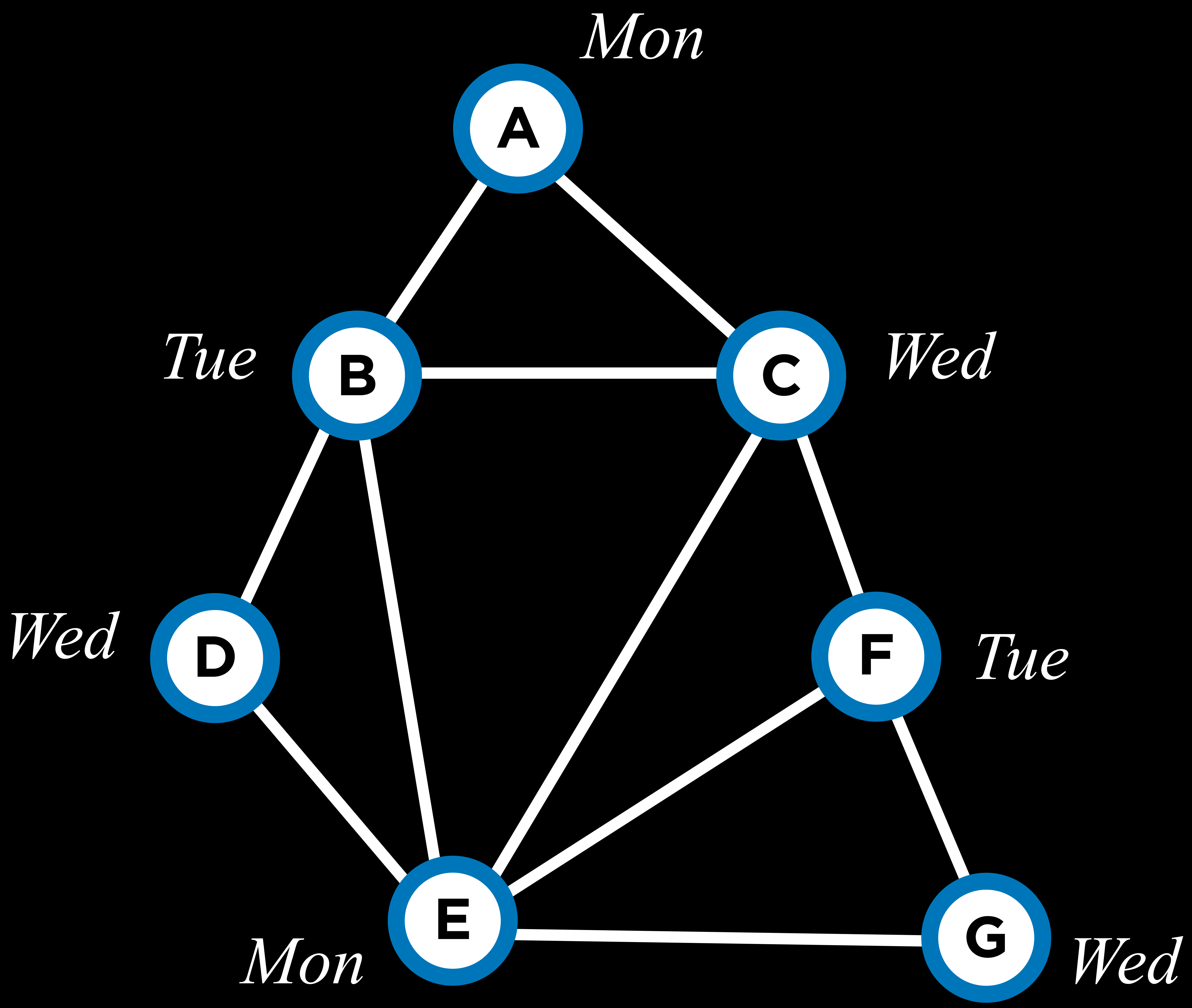
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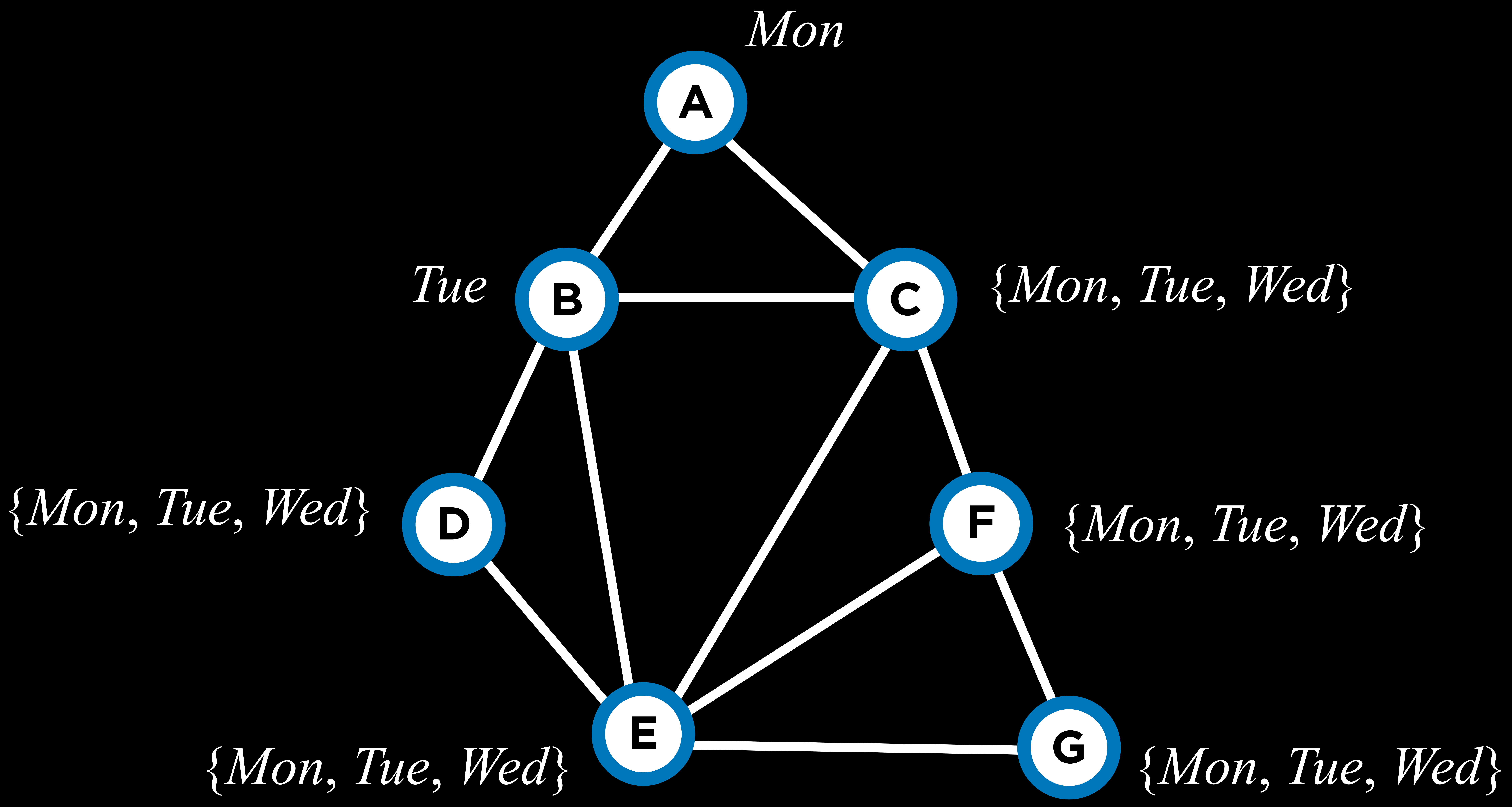

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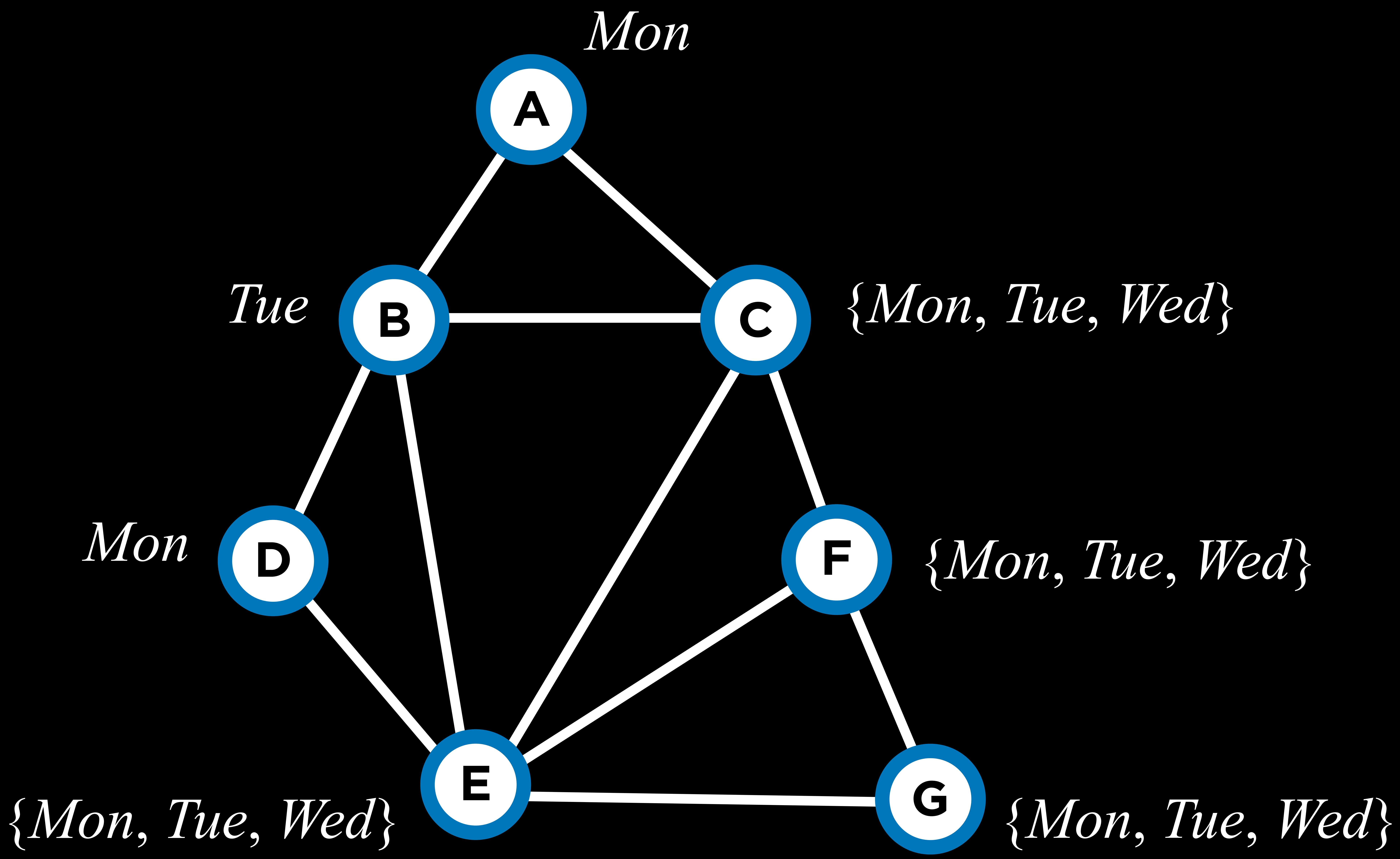

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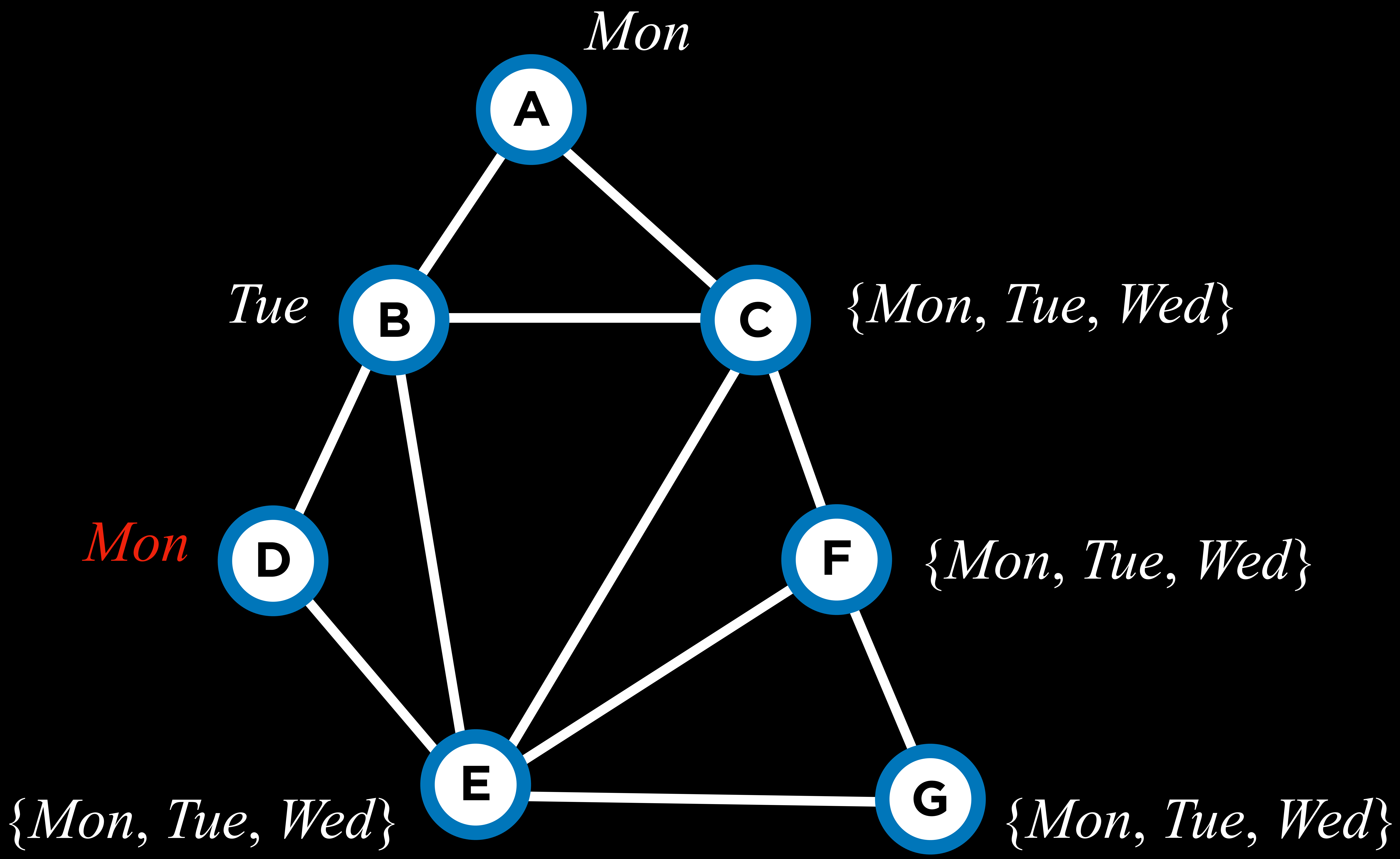
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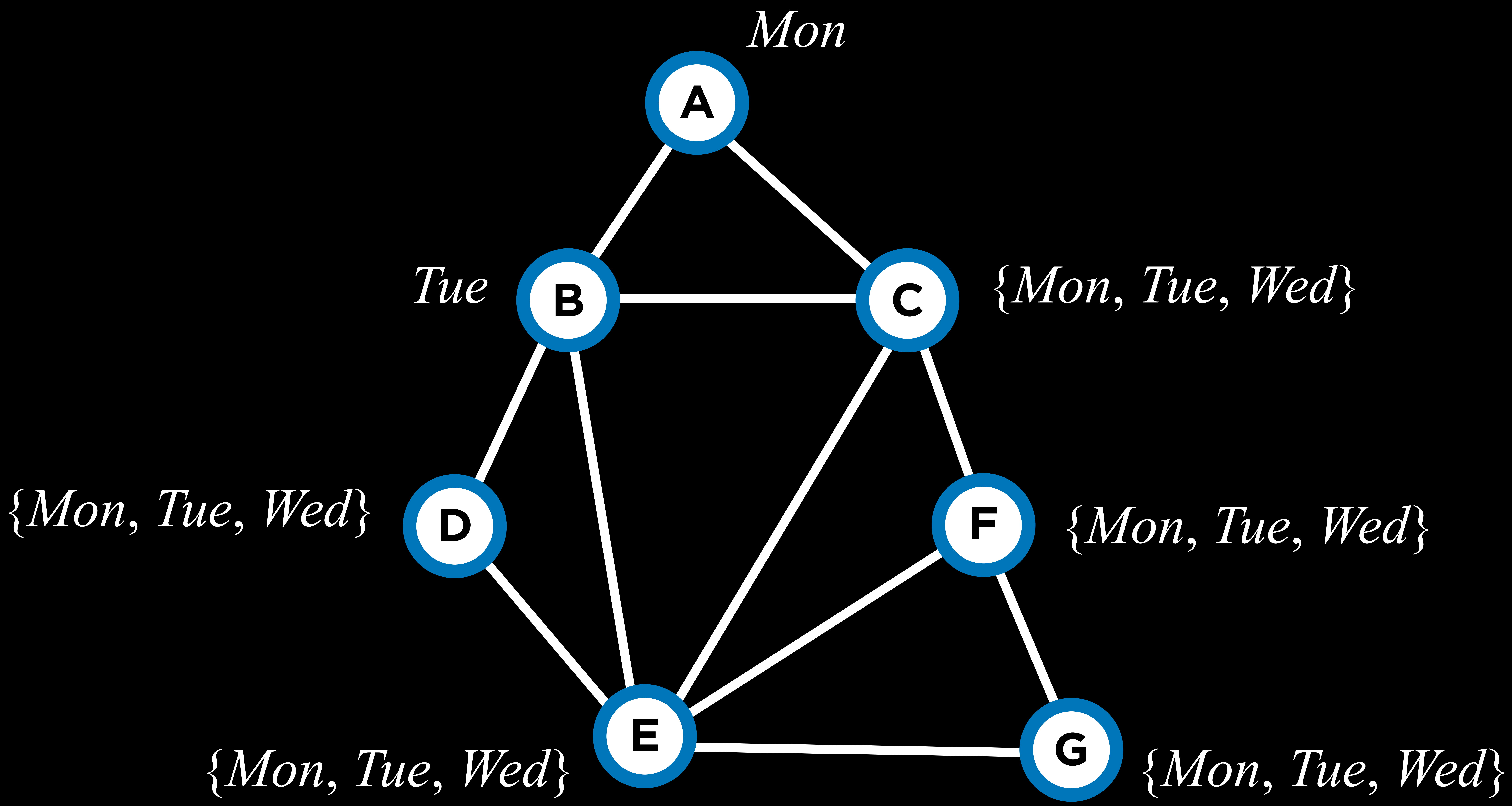
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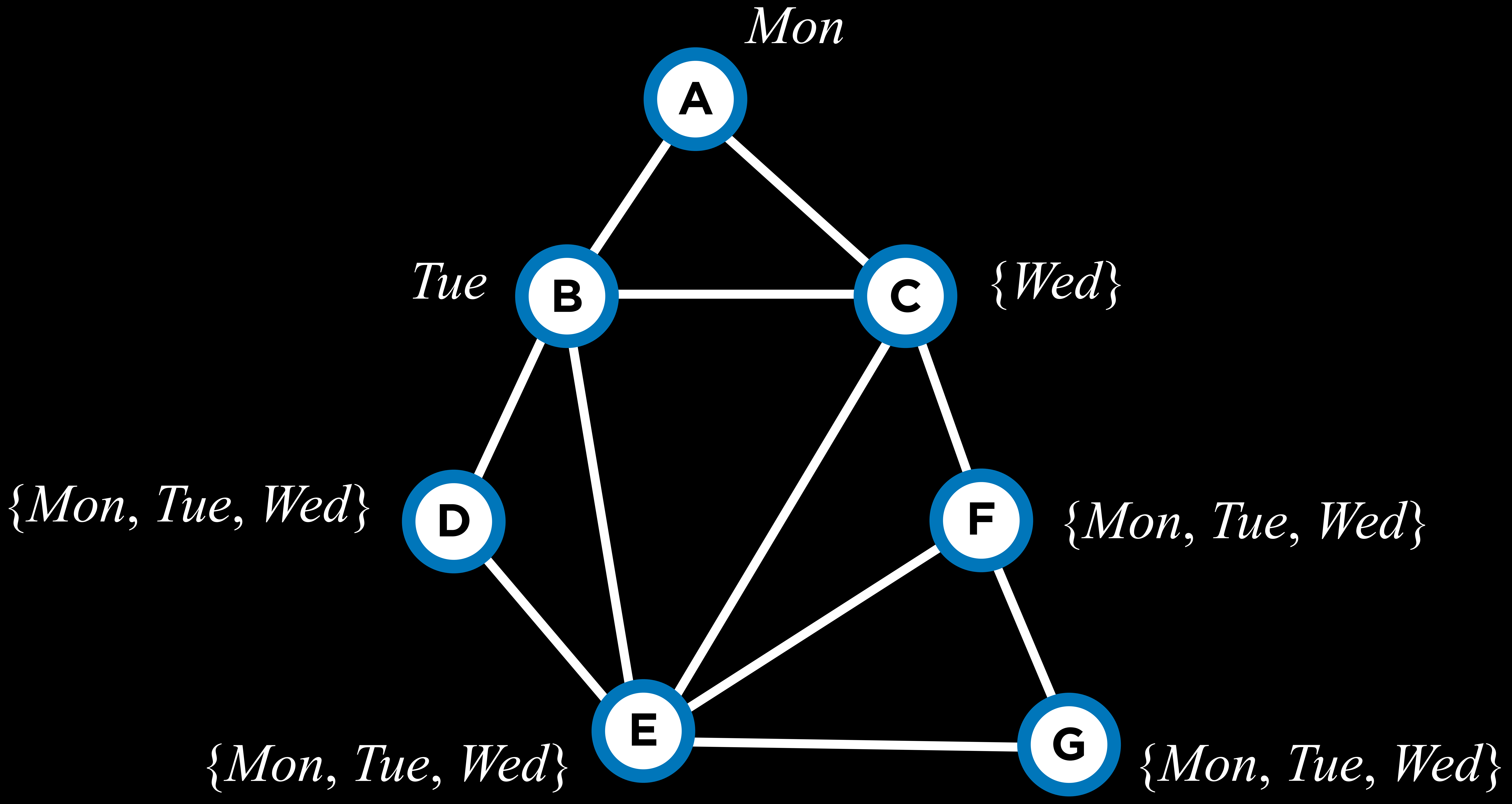
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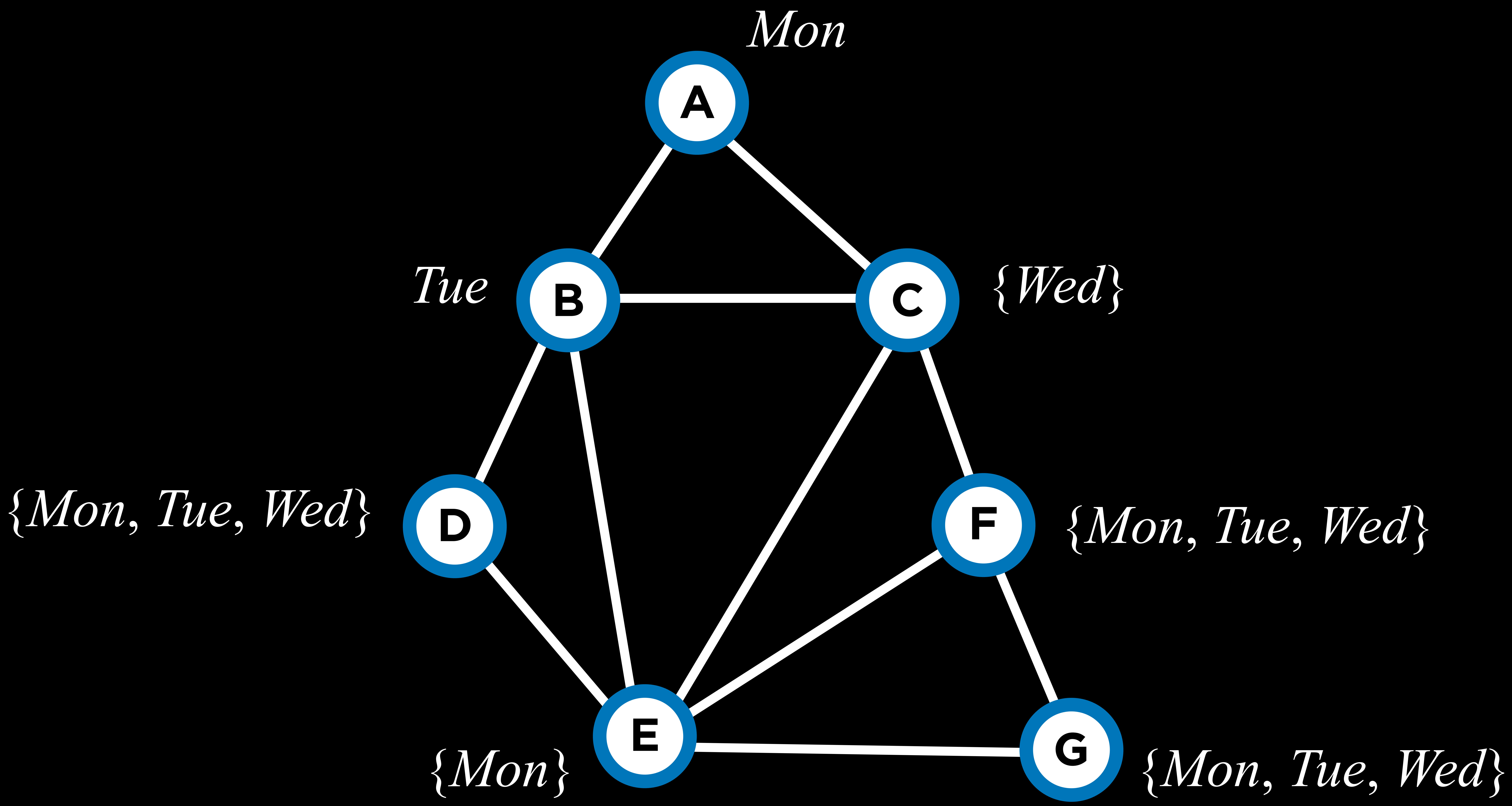
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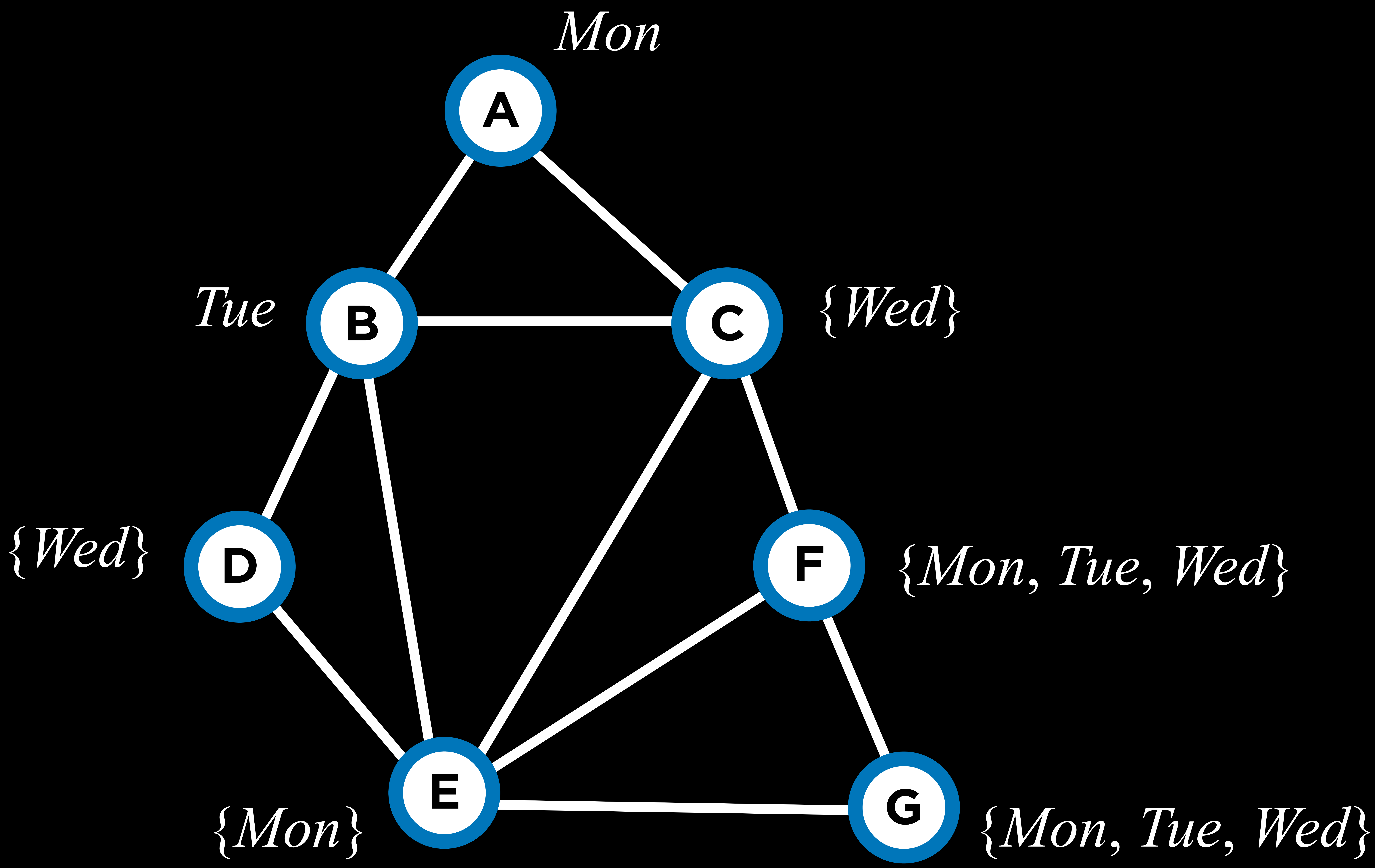
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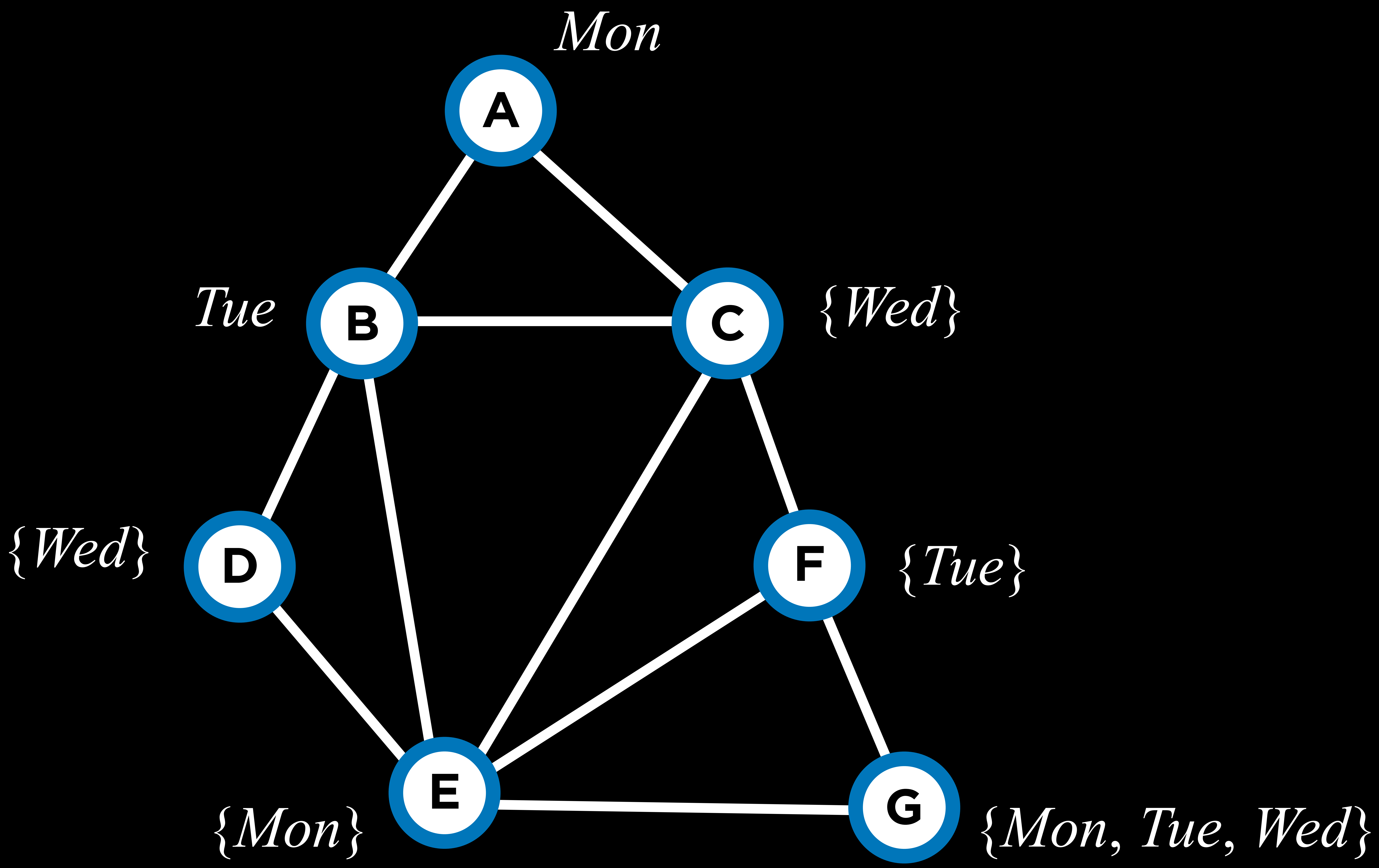
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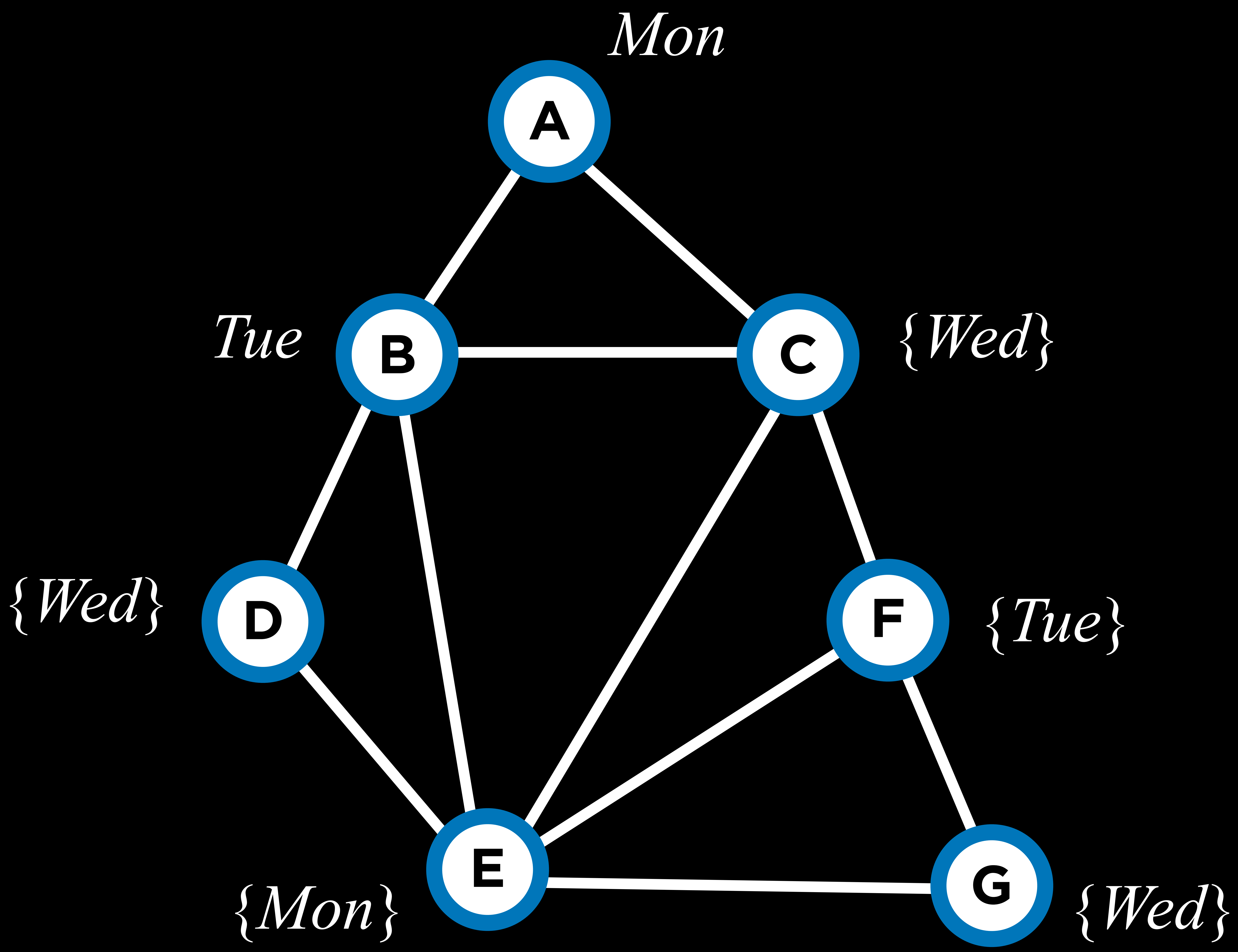

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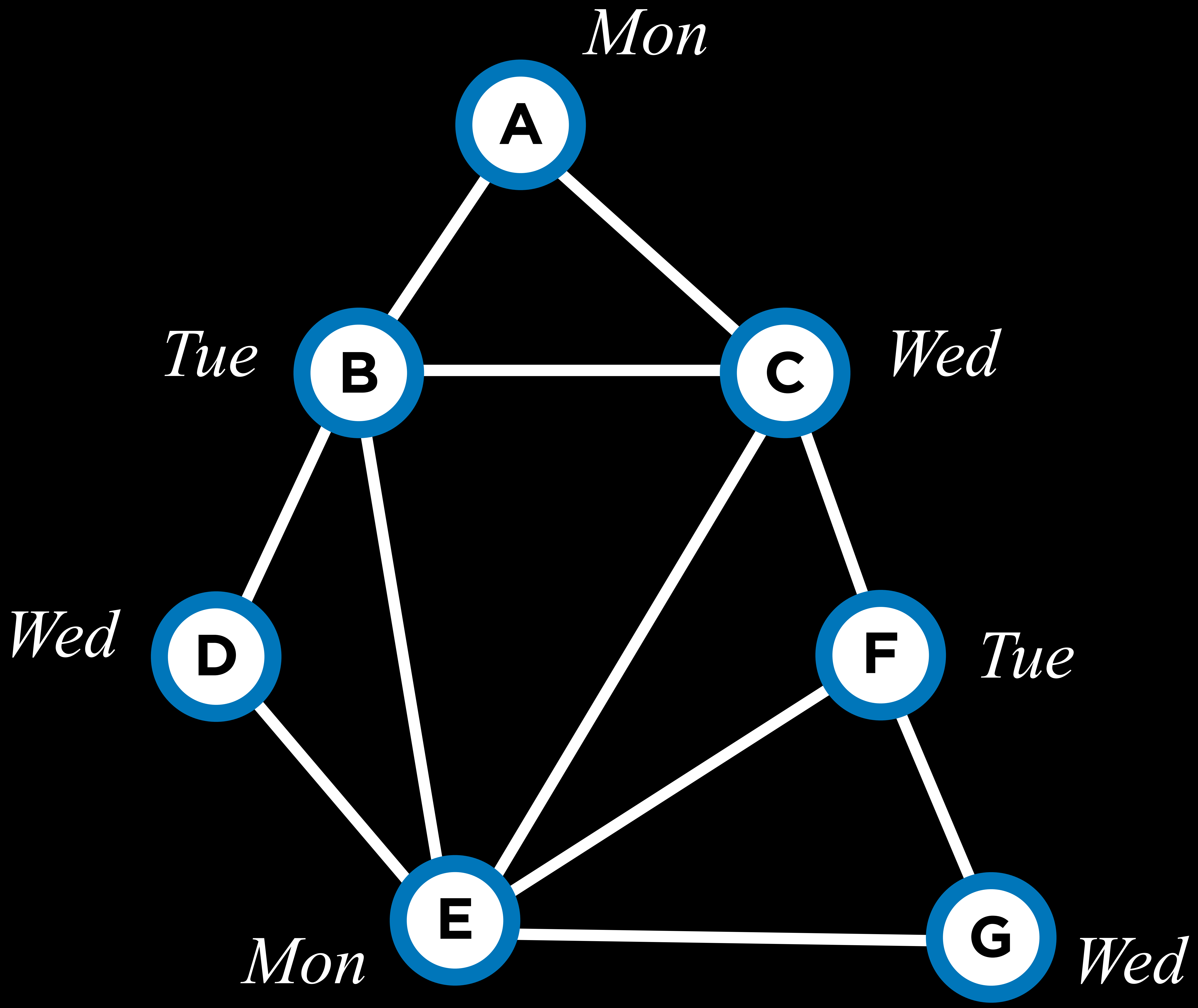
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maintaining arc-consistency

algorithm for enforcing arc-consistency every time we make a new assignment

maintaining arc-consistency

When we make a new assignment to X, calls $\mathrm{AC\text{-}3}$, starting with a queue of all arcs (Y, X) where Y is a neighbor of X

```
function BACKTRACK(assignment, csp):
    if assignment complete: return assignment
    var = SELECT-UNASSIGNED-VAR(assignment, csp)
    for value in DOMAIN-VALUES(var, assignment, csp):
        if value consistent with assignment:
            add {var = value} to assignment
            inferences = INFERENCE(assignment, csp)
            if inferences ≠ failure: add inferences to assignment
            result = BACKTRACK(assignment, csp)
            if result ≠ failure: return result
        remove {var = value} and inferences from assignment
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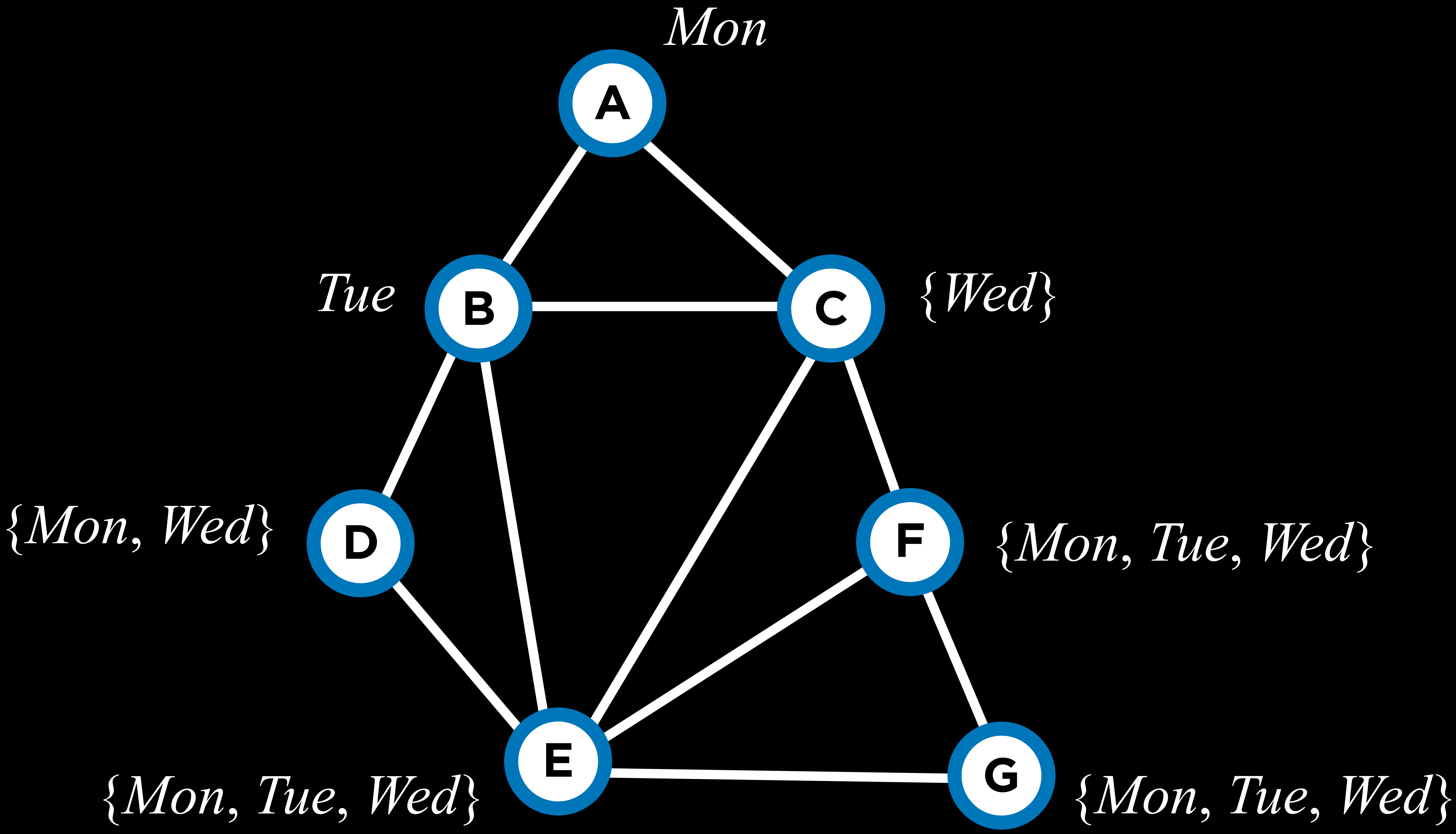
SELECT-UNASSIGNED-VAR

- **minimum remaining values (MRV)** heuristic: select the variable that has the smallest domain
- **degree** heuristic: select the variable that has the highest degree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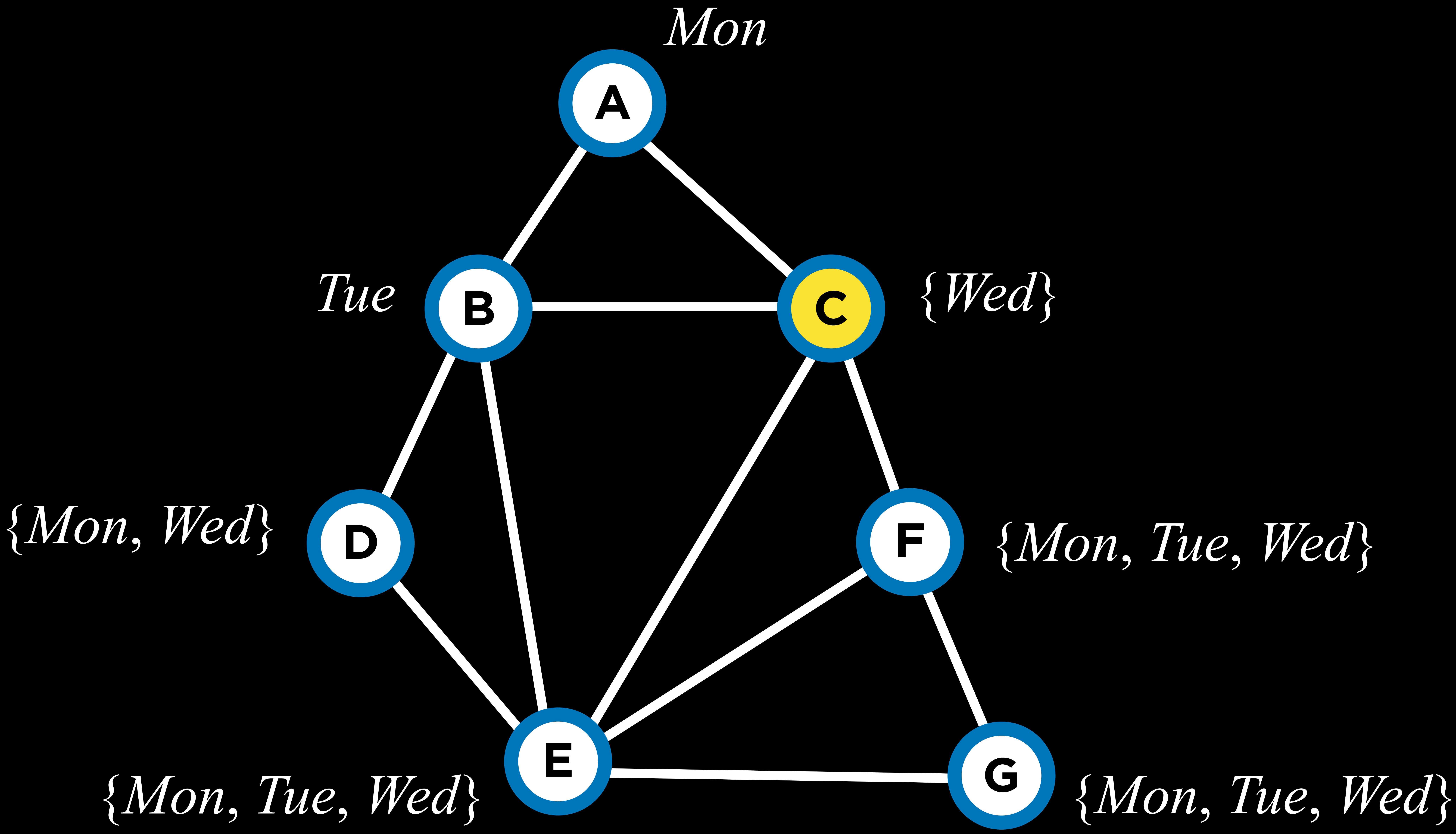
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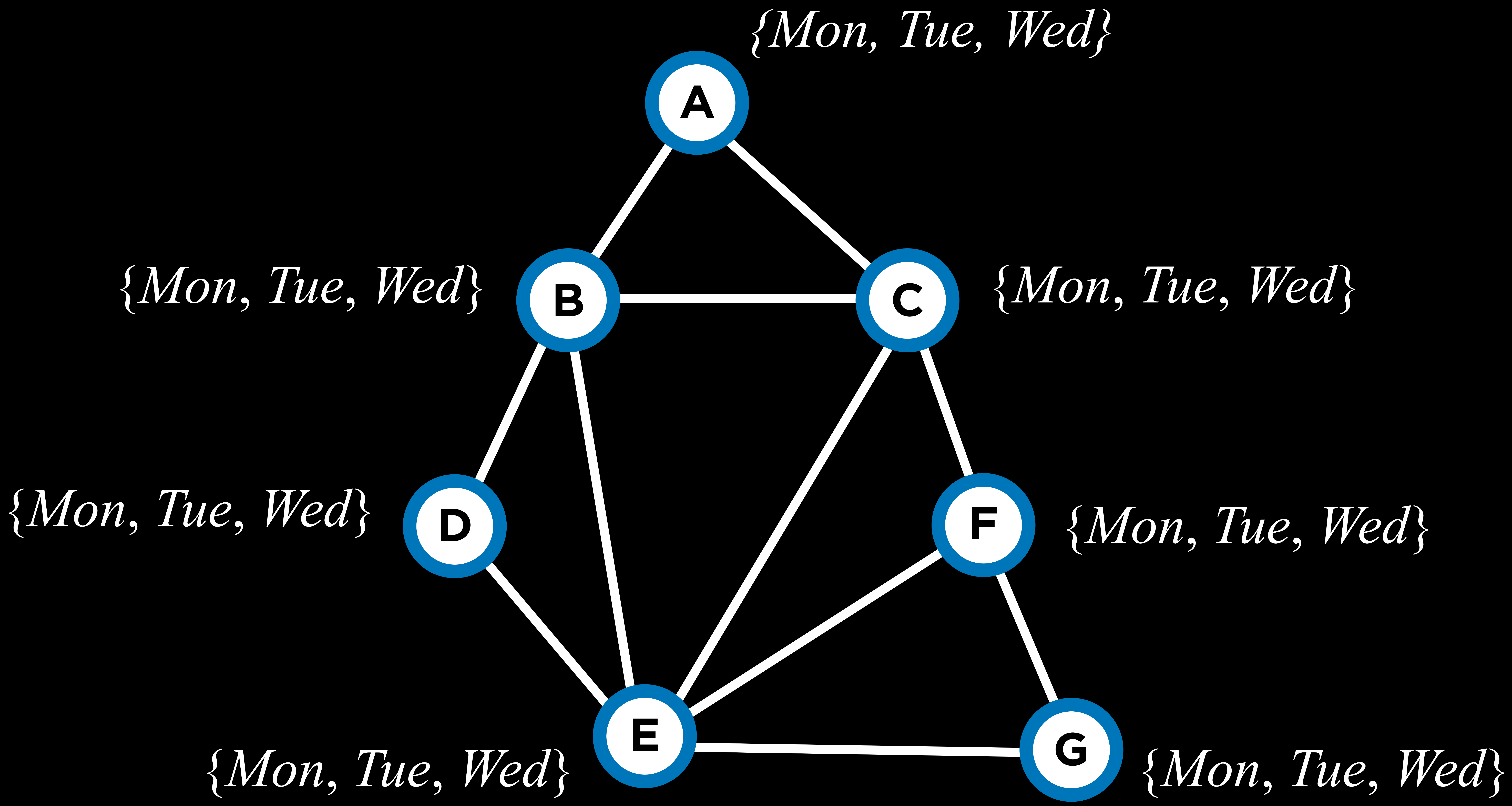
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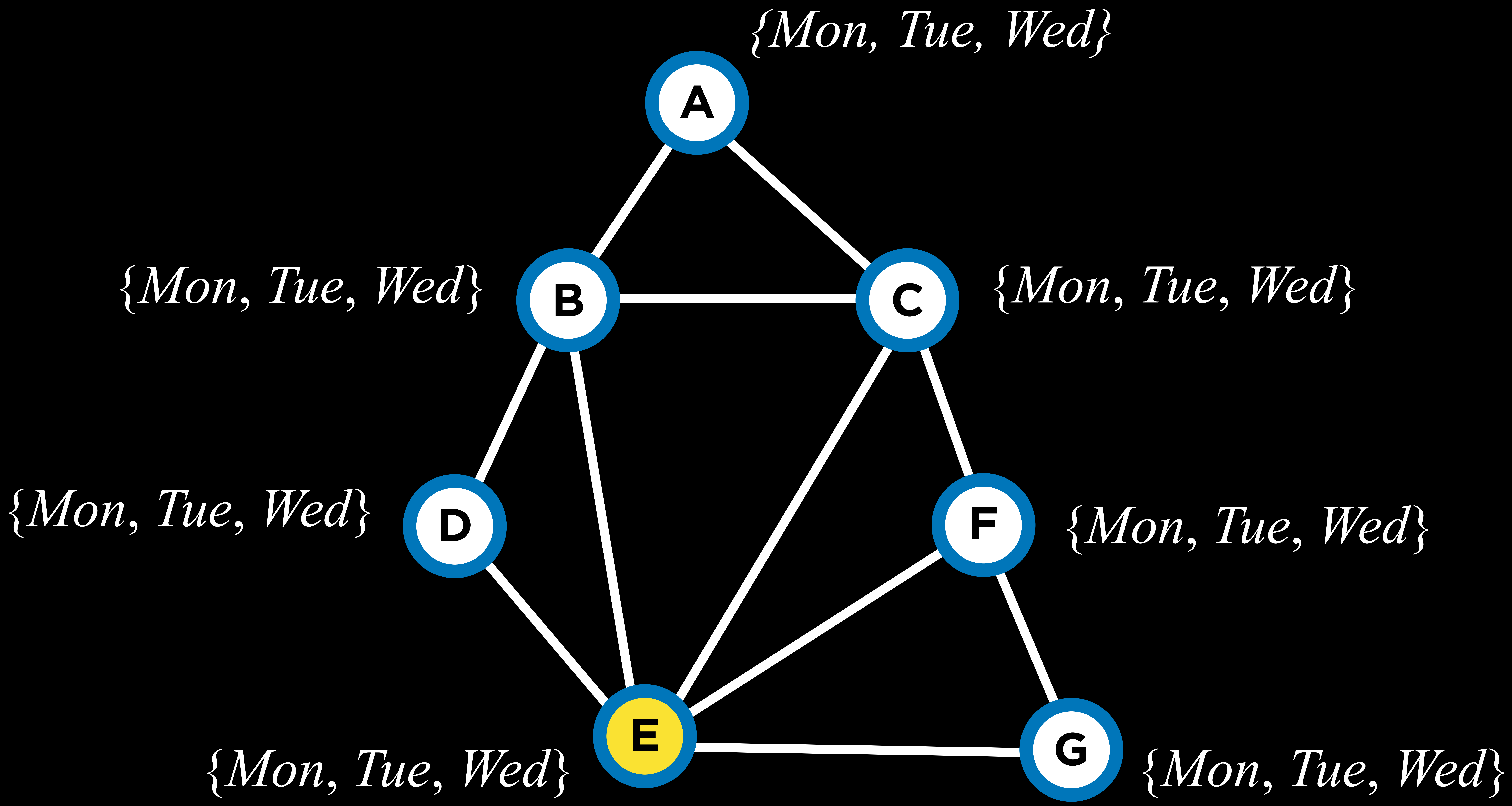

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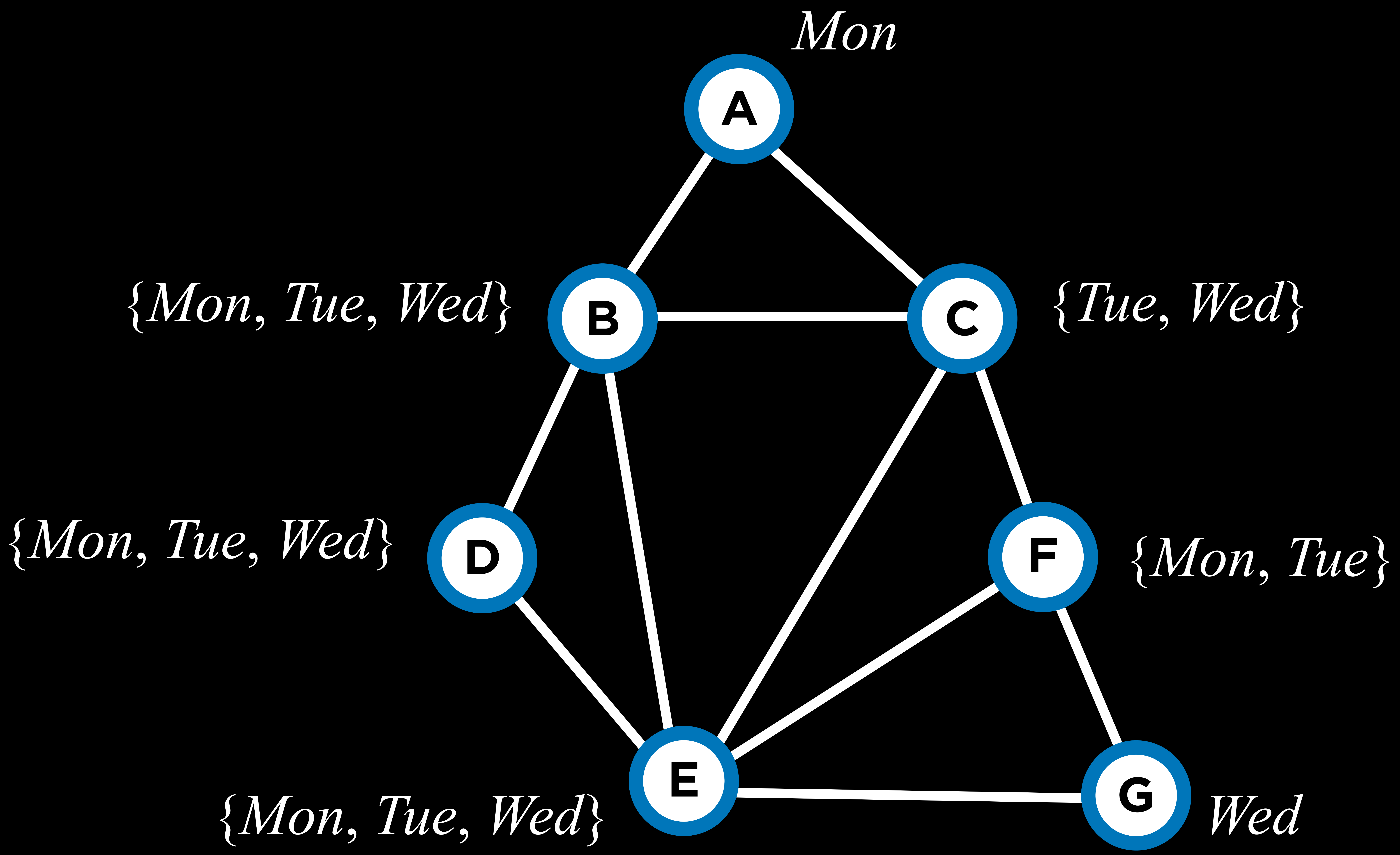
Domain-Values

- **least-constraining values** heuristic: return variables in order by number of choices that are ruled out for neighboring variables
    - try least-constraining values first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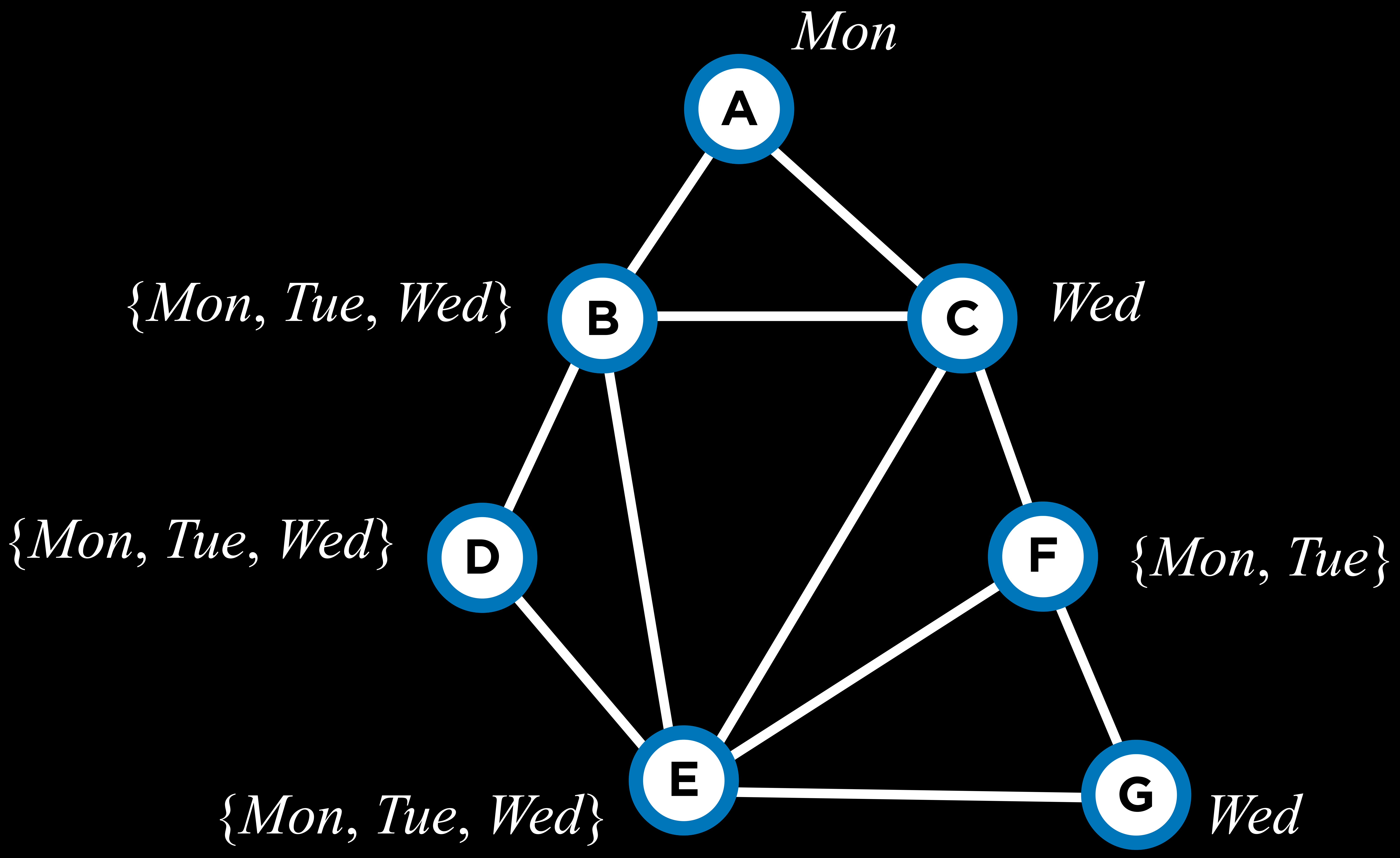
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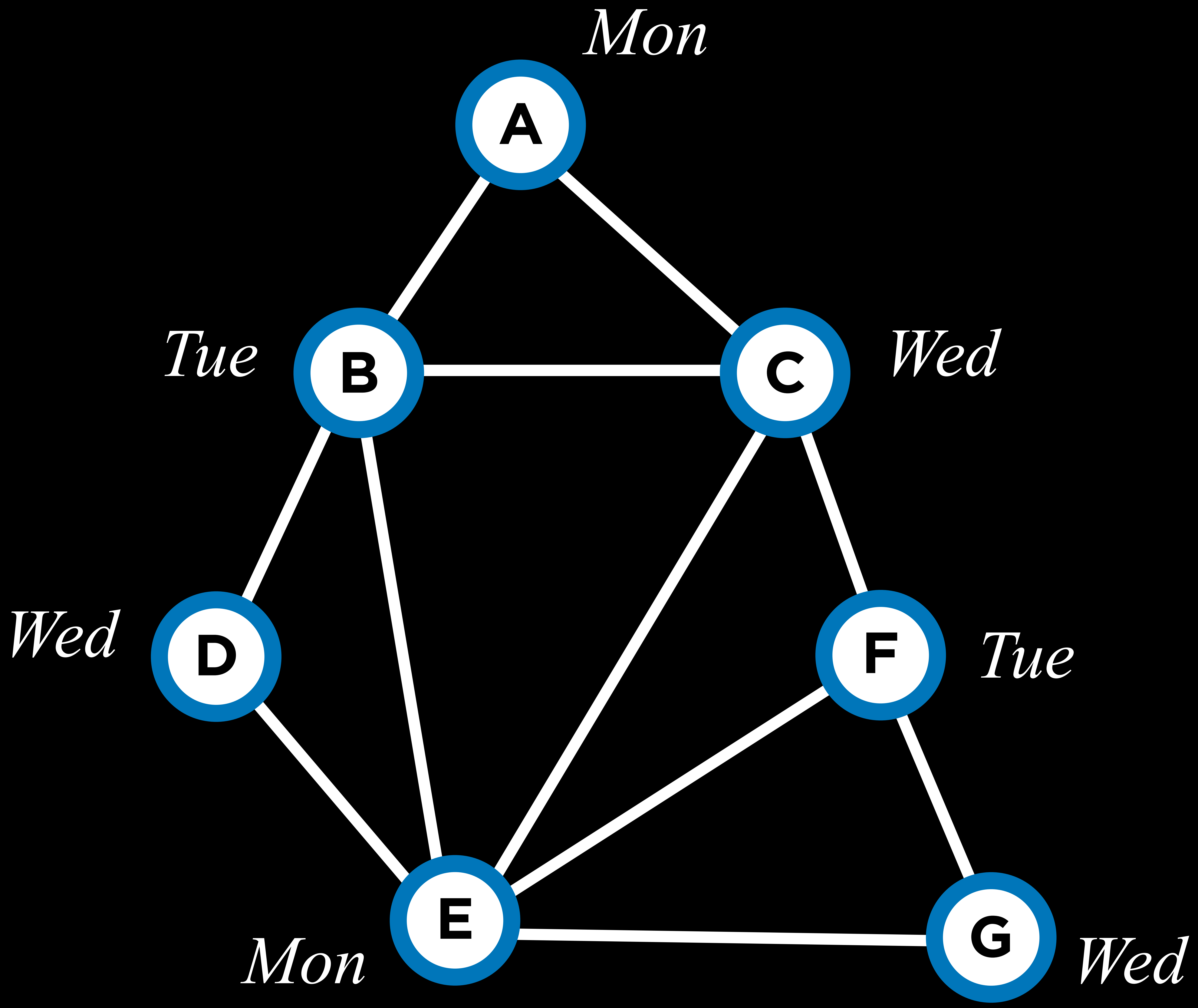
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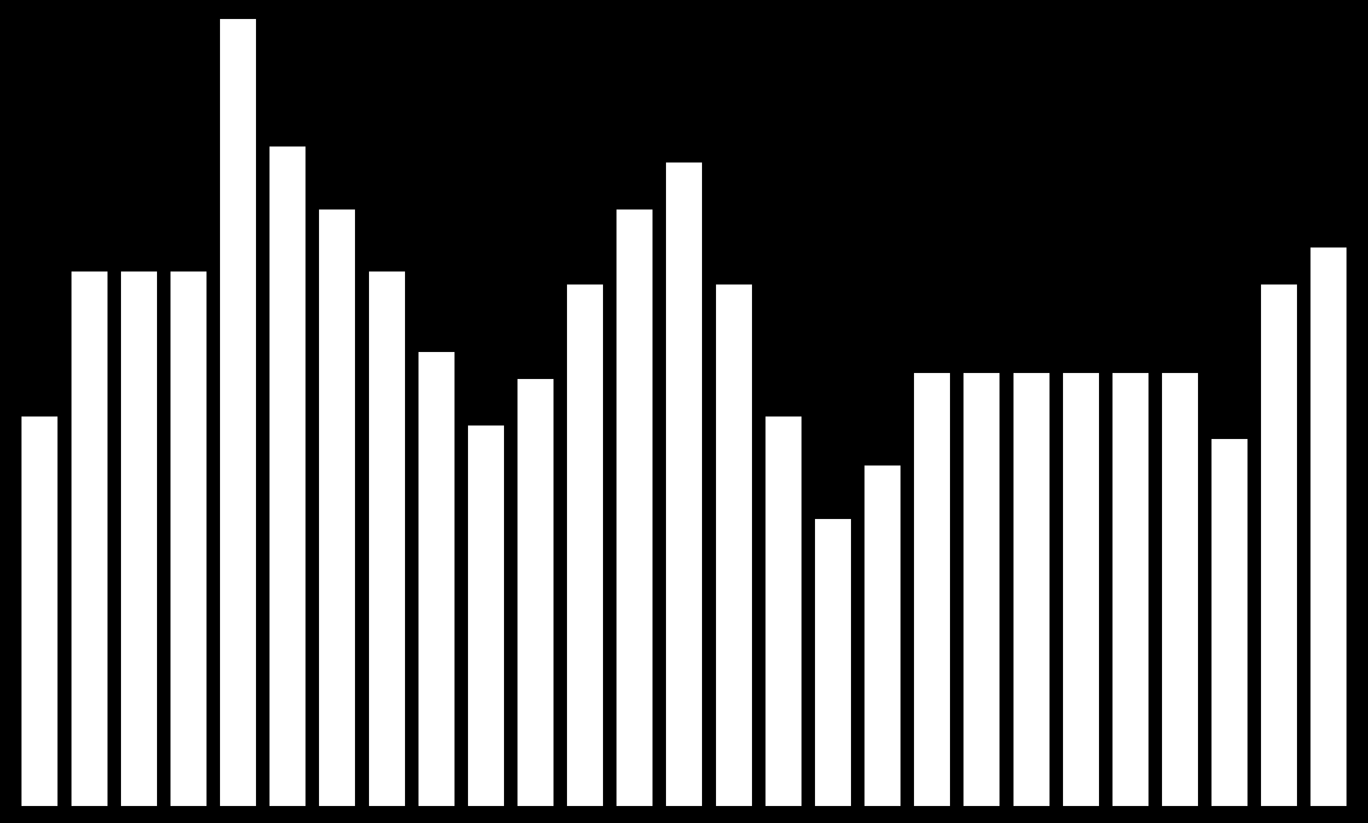
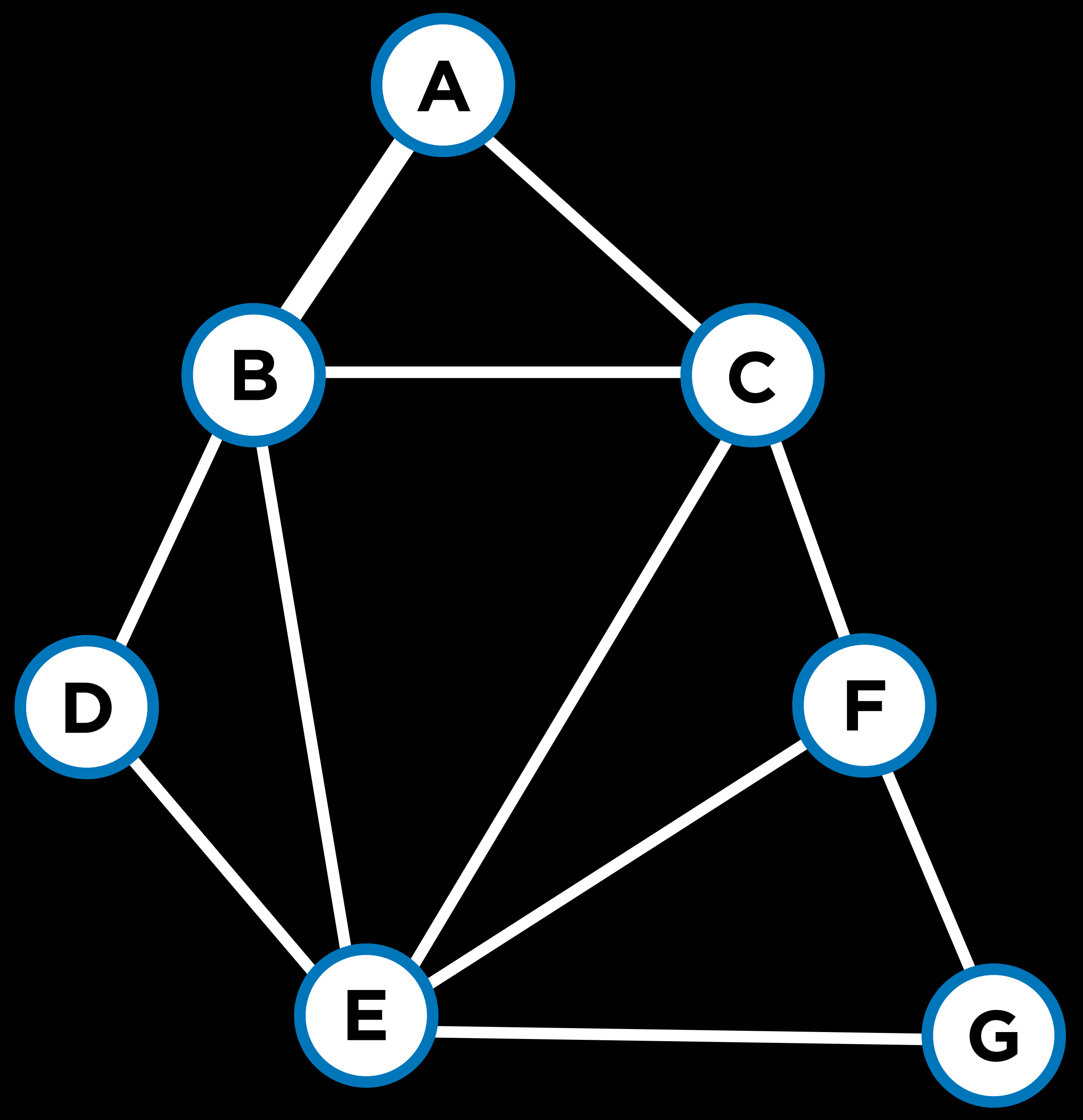
Problem Formulation



$$50x_1 + 80x_2$$
$$5x_1 + 2x_2 \leq 20$$
$$(-10x_1) + (-12x_2) \leq -90$$



Local Search

Linear Programming

Constraint Satisfaction

Optimization

Introduction to

Artificial Intelligence

with Python