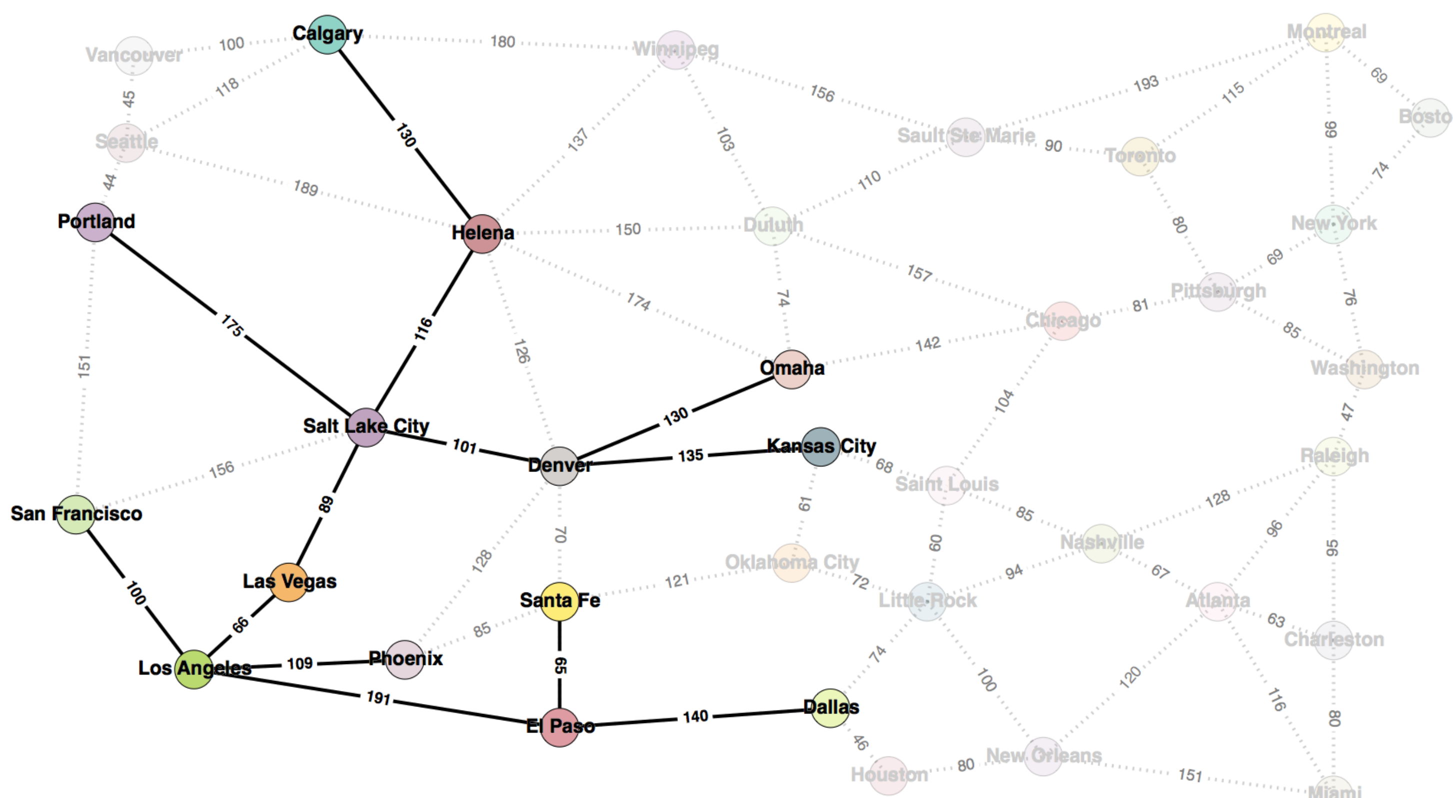
Artificial Intelligence

Search Agents

Uninformed search



Uninformed search

Use no domain knowledge!

Strategies:

1. Breadth-first search (BFS): Expand shallowest node

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3. Depth-limited search (DLS): Depth first with depth limit

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4. Iterative-deepening search (IDS): DLS with increasing limit

Uninformed search

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Strategies:

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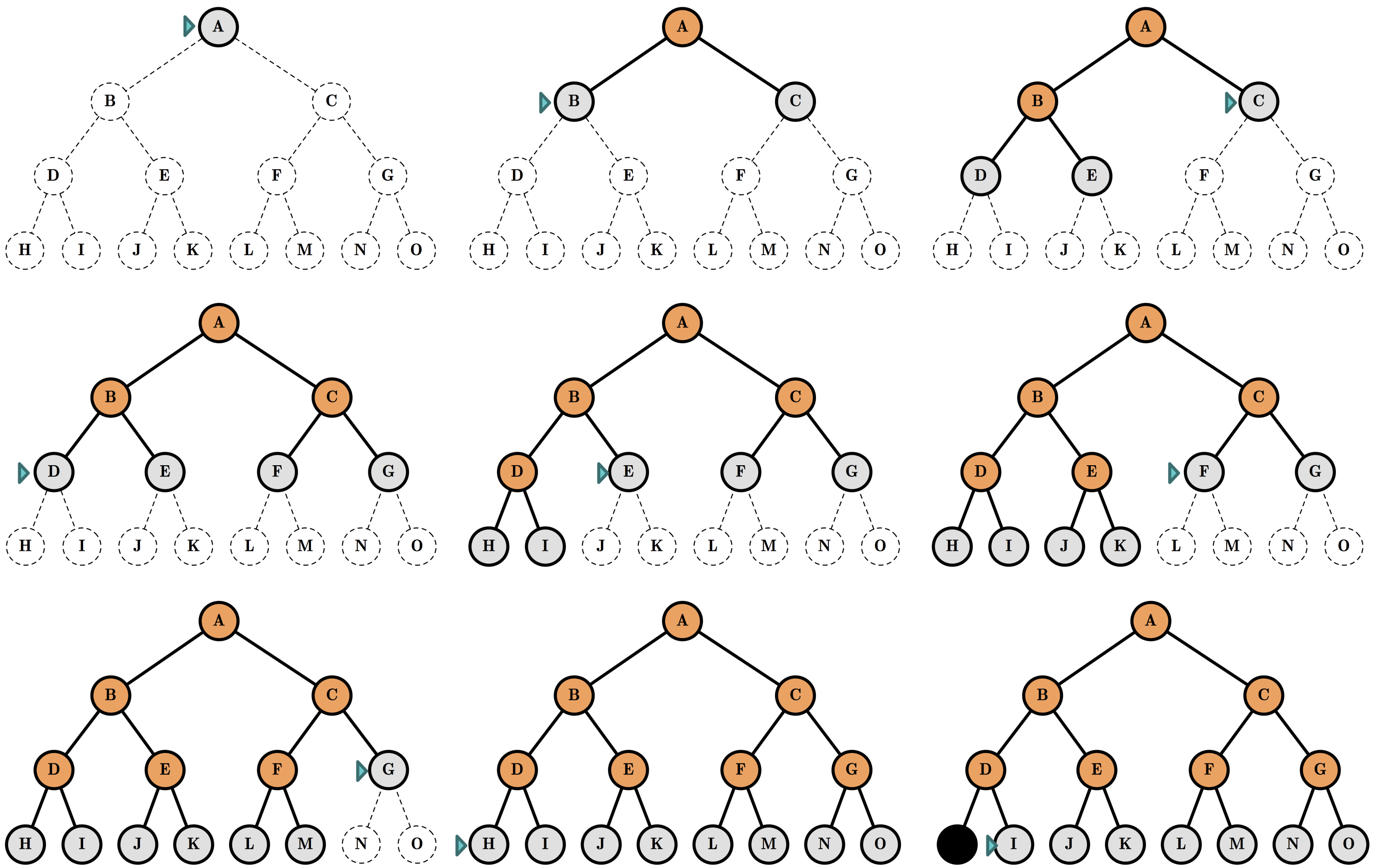
3. Depth-limited search (DLS): Depth first with depth limit
4. Iterative-deepening search (IDS): DLS with increasing limit
5. Uniform-cost search (UCS): Expand least cost node

Breadth-first search (BFS)

BFS: Expand shallowest first.



BFS search

```
function BREADTH-FIRST-SEARCH(initialState, goalTest)
    returns SUCCESS or FAILURE :

    frontier = Queue.new(initialState)
    explored = Set.new()

    while not frontier.isEmpty():
        state = frontier.dequeue()
        explored.add(state)

        if goalTest(state):
            return SUCCESS(state)

        for neighbor in state.neighbors():
            if neighbor not in frontier ∪ explored:
                frontier.enqueue(neighbor)

    return FAILURE
```

BFS Criteria

BFS criteria?

BFS

- **Complete** Yes (if b is finite)
- **Time** $1 + b + b^2 + b^3 + \ldots + b^d = O(b^d)$
- **Space** $O(b^d)$
  Note: If the *goal test* is applied at expansion rather than generation then $O(b^{d+1})$
- **Optimal** Yes (if cost = 1 per step).
- **implementation**: fringe: FIFO (Queue)

Question: If time and space complexities are exponential, why use BFS?

BFS

How bad is BFS?

BFS

How bad is BFS?

Depth	Nodes	Time	Memory
2	110	.11 milliseconds	107 kilobytes
4	11,110	11 milliseconds	10.6 megabytes
6	10^6	1.1 seconds	1 gigabyte
8	10^8	2 minutes	103 gigabytes
10	10^{10}	3 hours	10 terabytes
12	10^{12}	13 days	1 petabyte
14	10^{14}	3.5 years	99 petabytes
16	10^{16}	350 years	10 exabytes

Time and Memory requirements for breadth-first search for a branching factor b=10; 1 million nodes per second; 1,000 bytes per node.

BFS

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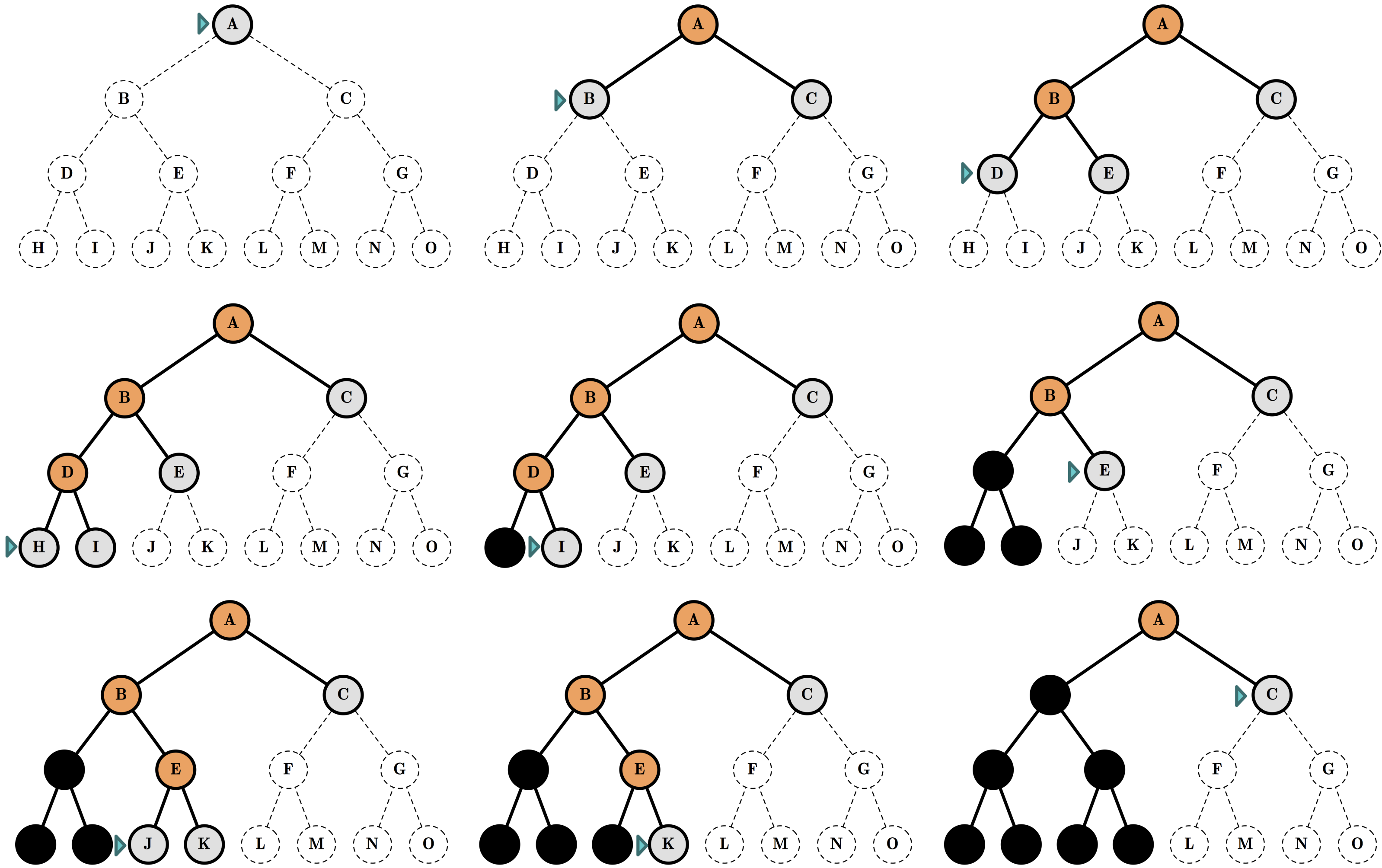
Time and Memory requirements for breadth-first search for a branching factor b=10; 1 million nodes per second; 1,000 bytes per node.

Memory requirement + exponential time complexity are the biggest handicaps of BFS!

DFS

DFS: Expand **deepest** first.



DFS search

```
function DEPTH-FIRST-SEARCH(initialState, goalTest)
    returns SUCCESS or FAILURE :

    frontier = Stack.new(initialState)
    explored = Set.new()

    while not frontier.isEmpty():
        state = frontier.pop()
        explored.add(state)

        if goalTest(state):
            return SUCCESS(state)

        for neighbor in state.neighbors():
            if neighbor not in frontier ∪ explored:
                frontier.push(neighbor)

    return FAILURE
```

DFS

DFS criteria?

DFS

- **Complete** No: fails in infinite-depth spaces, spaces with loops
  Modify to avoid repeated states along path.
  $\Rightarrow$ complete in finite spaces
- **Time** $O(b^m)$: $1 + b + b^2 + b^3 + \ldots + b^m = O(b^m)$
  bad if m is much larger than d
  but if solutions are dense, may be much faster than BFS.
- **Space** $O(bm)$ linear space complexity! (needs to store only a single path from the root to a leaf node, **along with the remaining unexpanded sibling nodes for each node on the path, hence the m factor**.)
- **Optimal** No
- **Implementation:** fringe: LIFO (Stack)

DFS

How bad is DFS?

Recall for BFS...

Depth	Nodes	Time	Memory
2	110	.11 milliseconds	107 kilobytes
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Depth =16.

We go down from 10 exabytes in BFS to . . . in DFS?

DFS

How bad is DFS?

Recall for BFS...

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2	110	.11 milliseconds	107 kilobytes
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Depth =16.

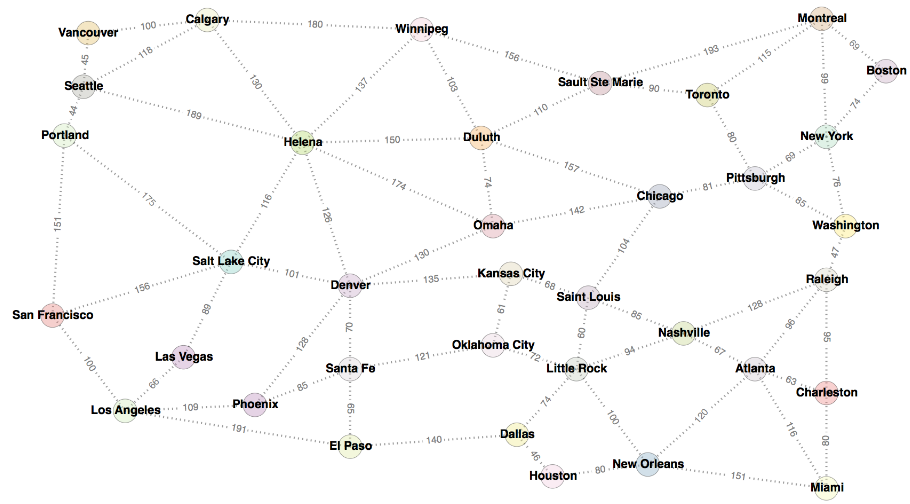
We go down from 10 exabytes in BFS to 156 kilobytes in DFS!

Depth-limited search

- DFS with depth limit l (nodes at level l has no successors).
- Select some limit L in depth to explore with DFS
- Iterative deepening: increasing the limit l

Depth-limited search

- If we know some knowledge about the problem, may be we don't need to go to a full depth.



Idea: any city can be reached from another city in at most L steps with $L < 36$.

Iterative Deepening

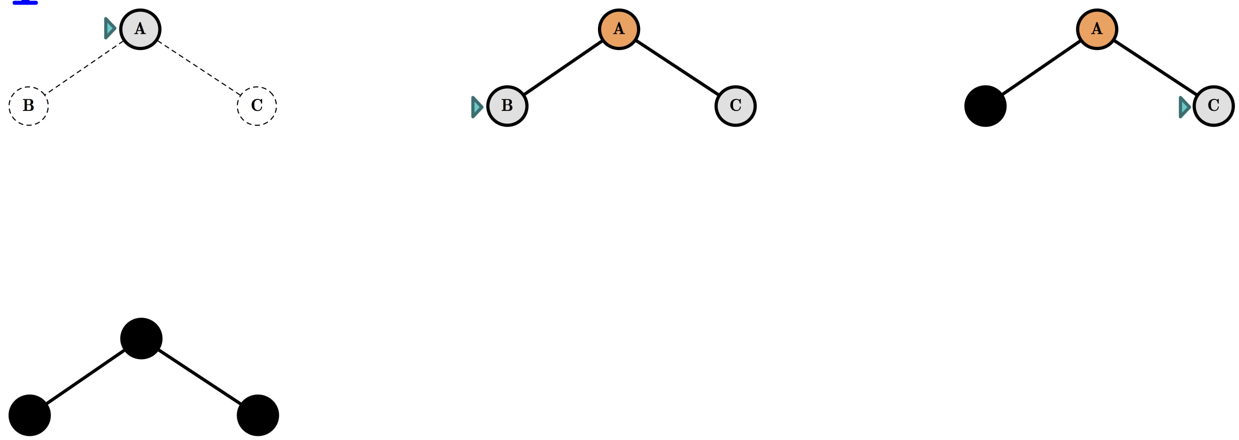
- Combines the benefits of BFS and DFS.
- Idea: Iteratively increase the search limit until the depth of the shallowest solution d is reached.
- Applies DLS with increasing limits.
- The algorithm will stop if a solution is found or if DLS returns a failure (no solution).
- Because most of the nodes are on the bottom of the search tree, it not a big waste to iteratively re-generate the top
- Let's take an example with a depth limit between 0 and 3.

Iterative Deepening

Limit = 0

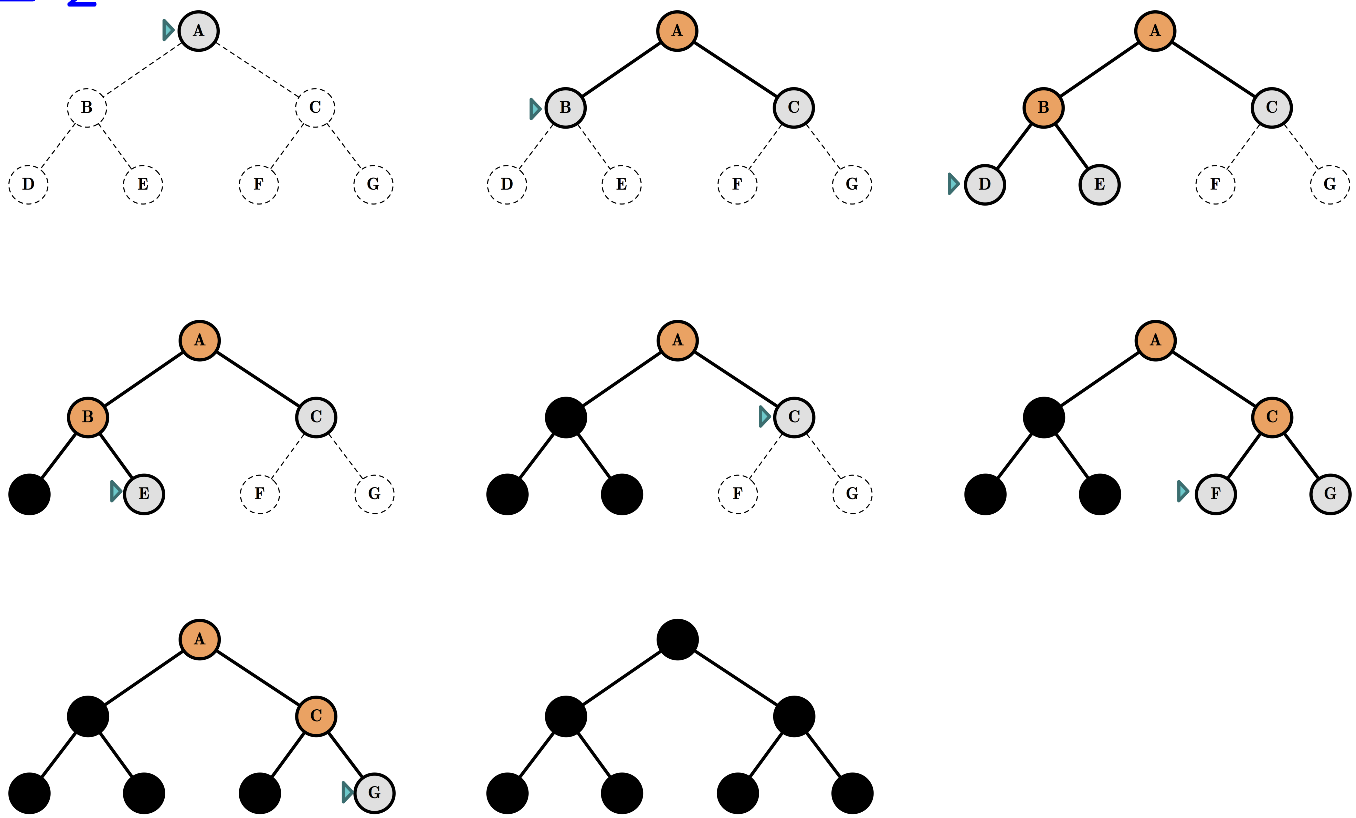
Iterative Deepening

Limit = 1



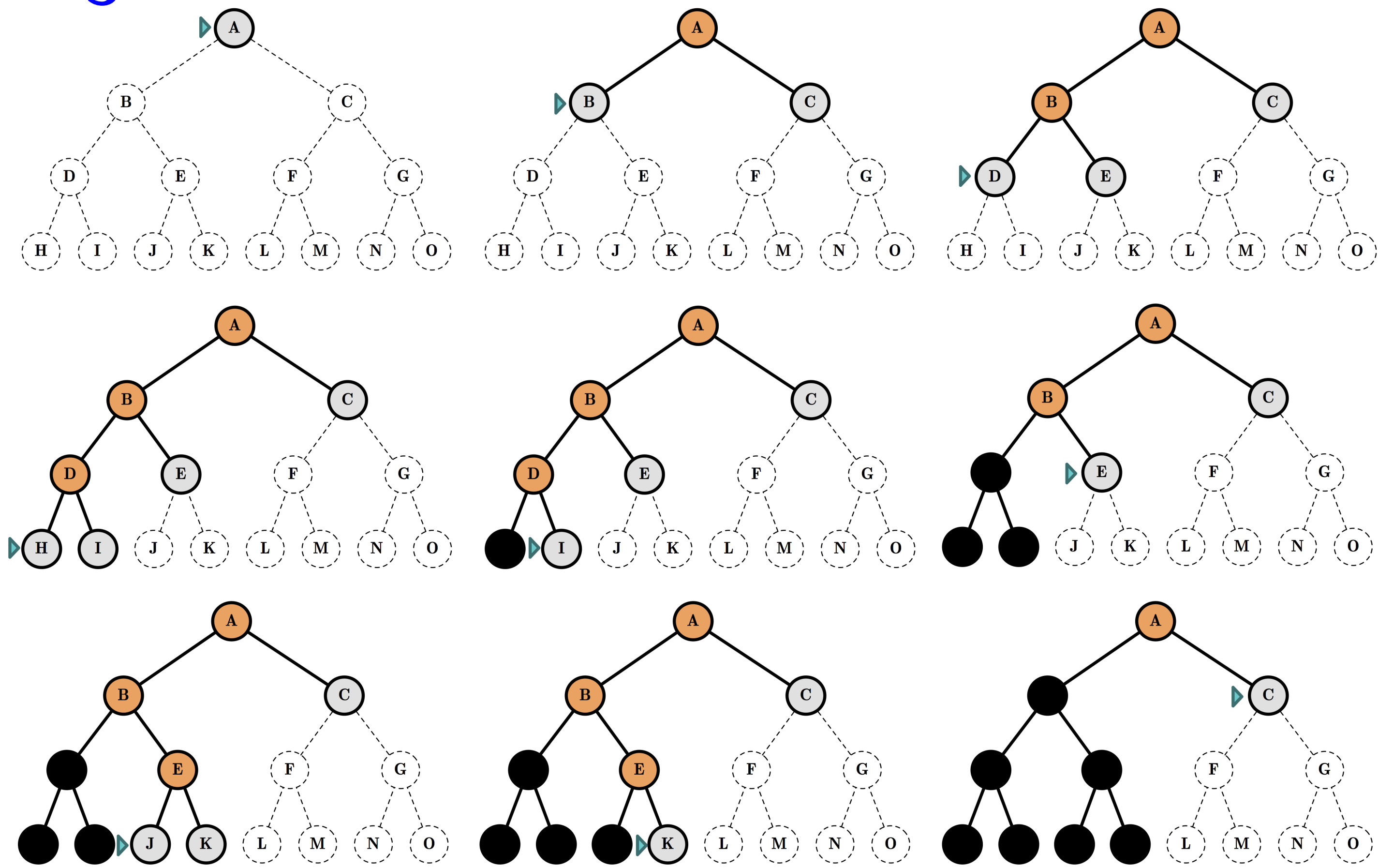
Iterative Deepening

Limit = 2

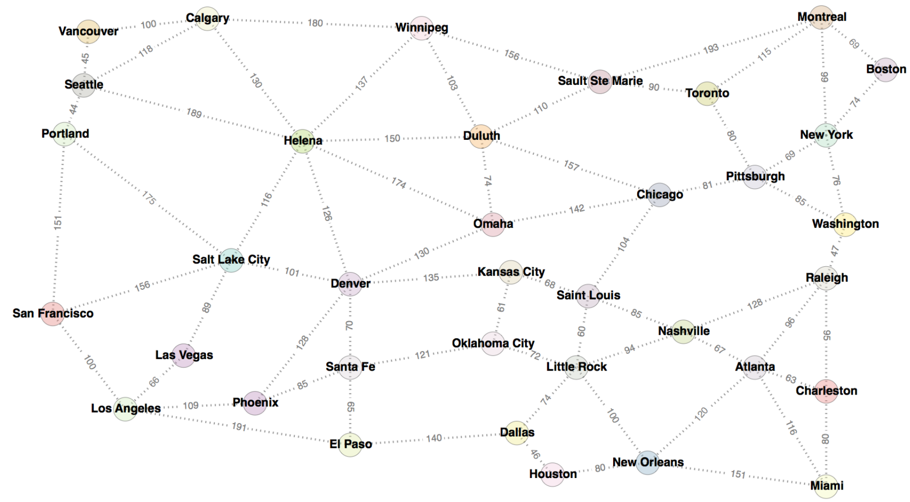


Iterative Deepening

Limit = 3

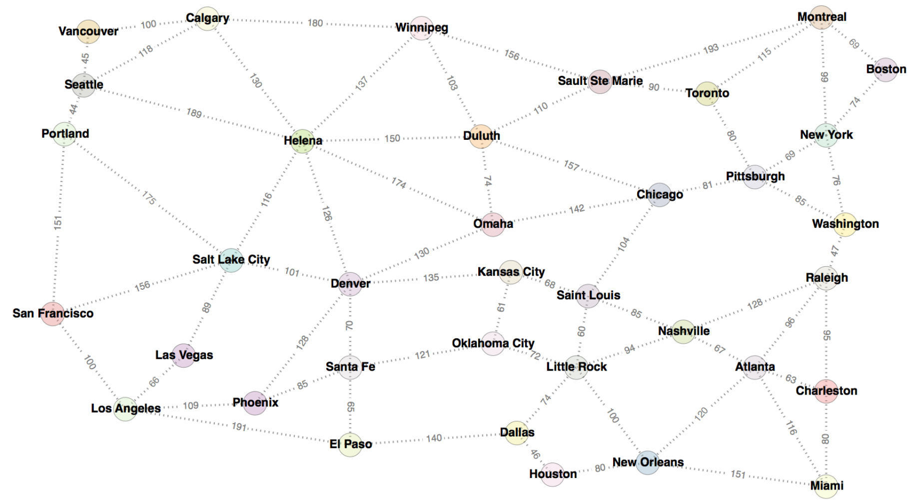


Uniform-cost search



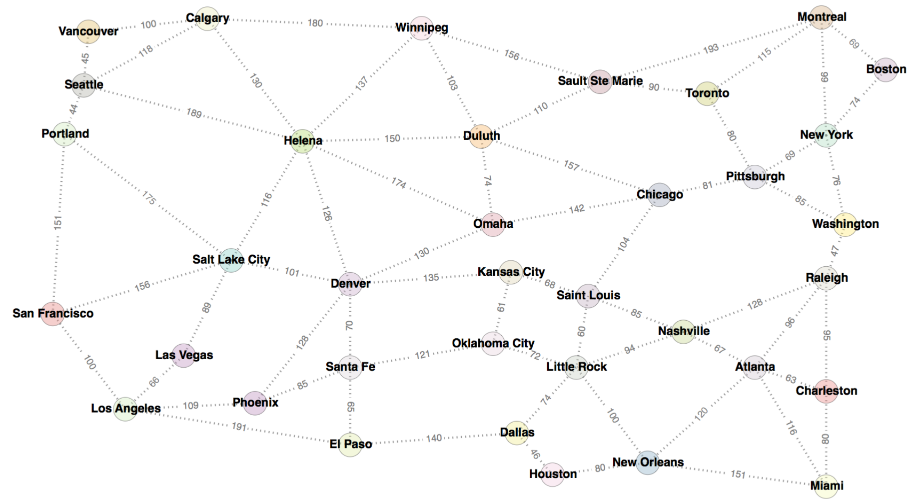
- The arcs in the search graph may have weights (different cost attached). How to leverage this information?

Uniform-cost search



- The arcs in the search graph may have weights (different cost attached). How to leverage this information?
- BFS will find the shortest path which may be costly.
- We want the cheapest not shallowest solution.

Uniform-cost search



- The arcs in the search graph may have weights (different cost attached). How to leverage this information?
- BFS will find the shortest path which may be costly.
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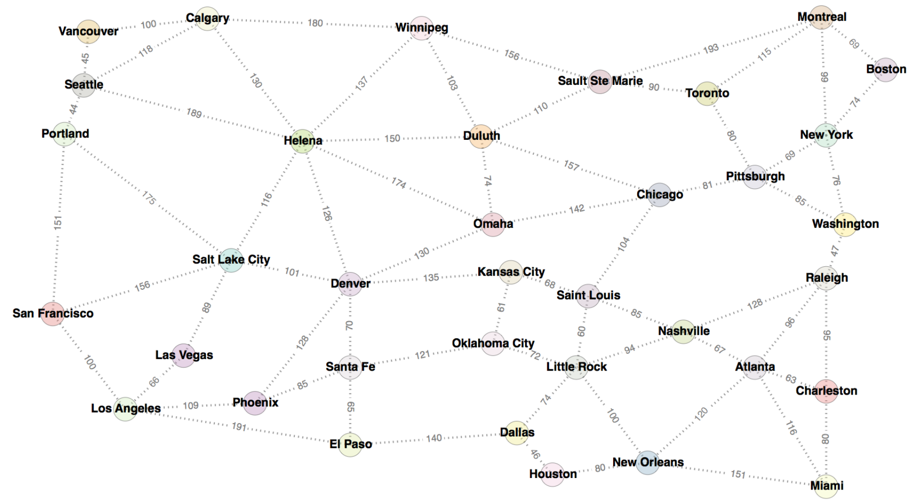
- Modify BFS: Prioritize by cost not depth $\rightarrow$ **Expand node n with the lowest path cost $g(n)$**

Uniform-cost search

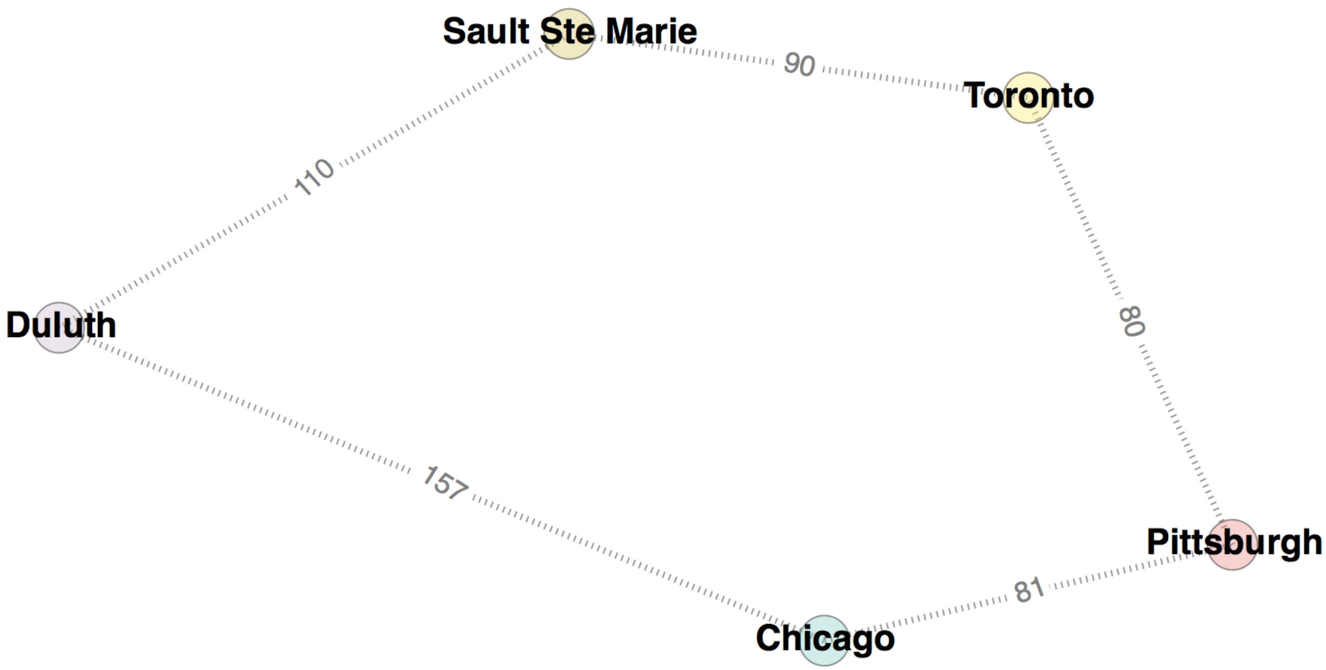


- The arcs in the search graph may have weights (different cost attached). How to leverage this information?
- BFS will find the shortest path which may be costly.
- We want the cheapest not shallowest solution.
- Modify BFS: Prioritize by cost not depth $\rightarrow$ **Expand node n with the lowest path cost $g(n)$**
- Explores increasing costs.

UCS algorithm

```
function UNIFORM-COST-SEARCH(initialState, goalTest)
    returns SUCCESS or FAILURE :  /* Cost f(n) = g(n) */

    frontier = Heap.new(initialState)
    explored = Set.new()

    while not frontier.isEmpty():
        state = frontier.deleteMin()
        explored.add(state)

        if goalTest(state):
            return SUCCESS(state)

        for neighbor in state.neighbors():
            if neighbor not in frontier ∪ explored:
                frontier.insert(neighbor)
            else if neighbor in frontier:
                frontier.decreaseKey(neighbor)

    return FAILURE
```

Uniform-cost search



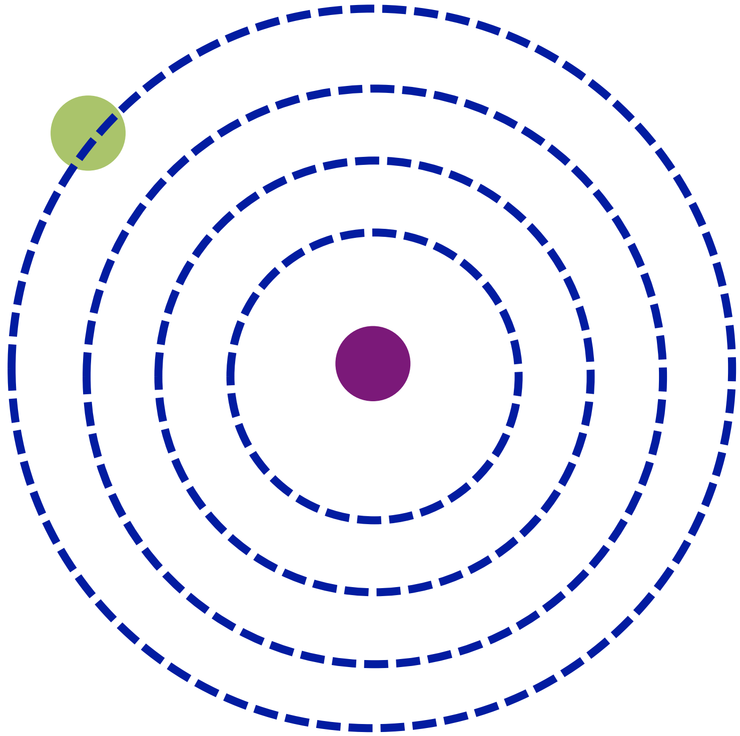
Go from Chicago to Sault Ste Marie. Using BFS, we would find Chicago-Duluth-Sault Ste Marie. However, using UCS, we would find Chicago-Pittsburgh-Toronto-Sault Ste Marie, which is actually the shortest path!

Uniform-cost search

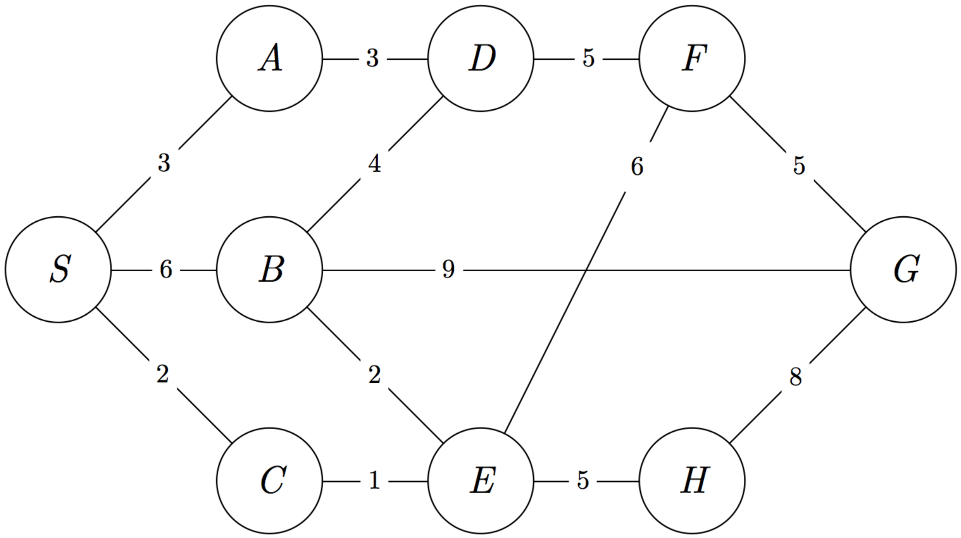
- **Complete** Yes, if solution has a finite cost.
- **Time**
  - Suppose C^*: cost of the optimal solution
  - Every action costs at least ϵ (bound on the cost)
  - The effective depth is roughly C^*/ϵ (how deep the *cheapest* solution could be).
  - $O(b^{C^*/\epsilon})$
- **Space** # of nodes with $g \leq$ cost of optimal solution, $O(b^{C^*/\epsilon})$
- **Optimal** Yes
- **Implementation**: fringe = queue ordered by path cost $g(n)$, lowest first = Heap!

Uniform-cost search

While complete and optimal, UCS explores the space in every direction because no information is provided about the goal!

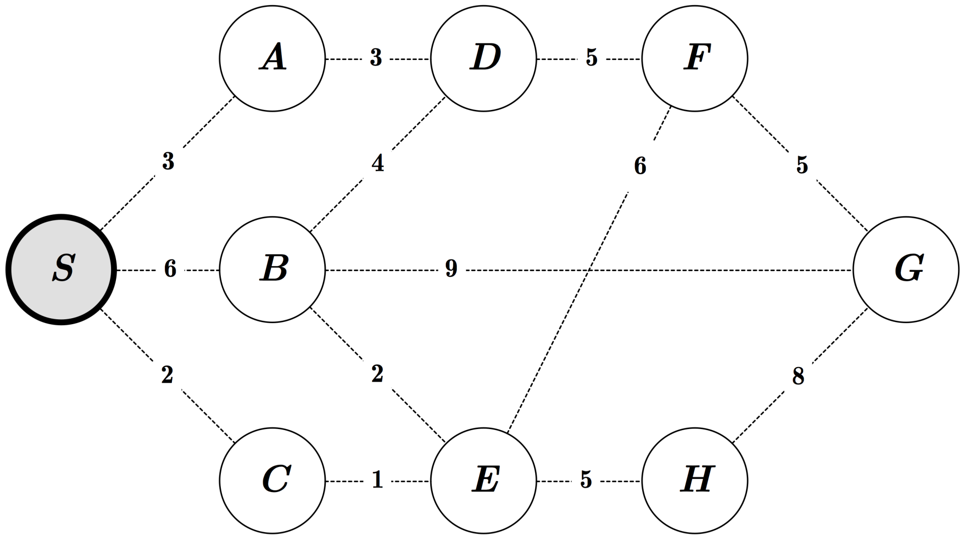


Exercise

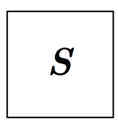


Question: What is the **order of visits of the nodes** and the **path** returned by BFS, DFS and UCS?

Exercise: BFS

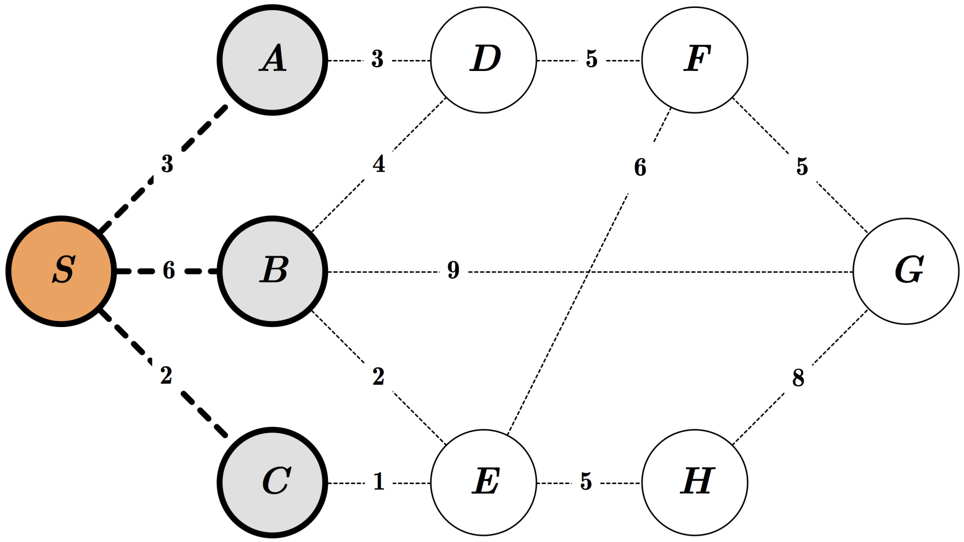


Queue:



Order of Visit:

Exercise: BFS



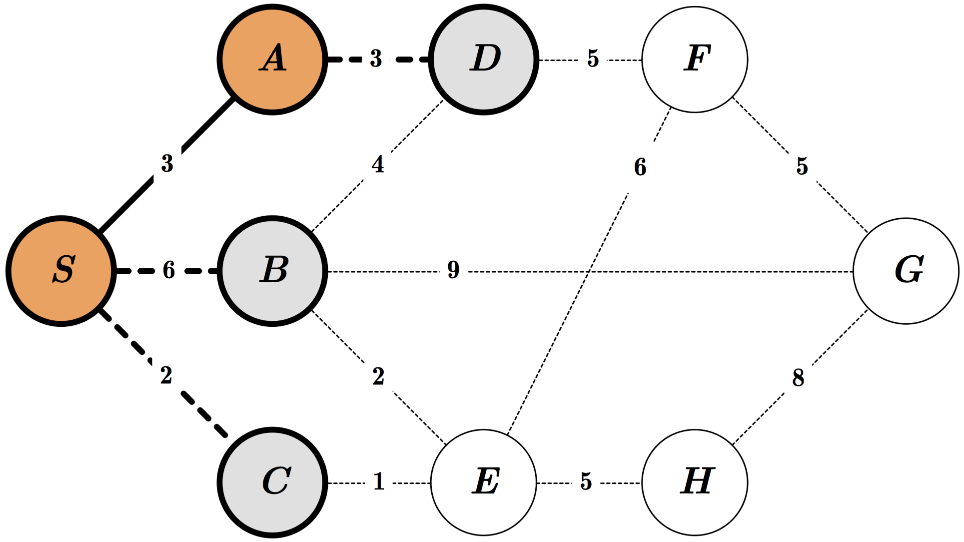
Queue:

S	A	B	C

Order of Visit:

S

Exercise: BFS



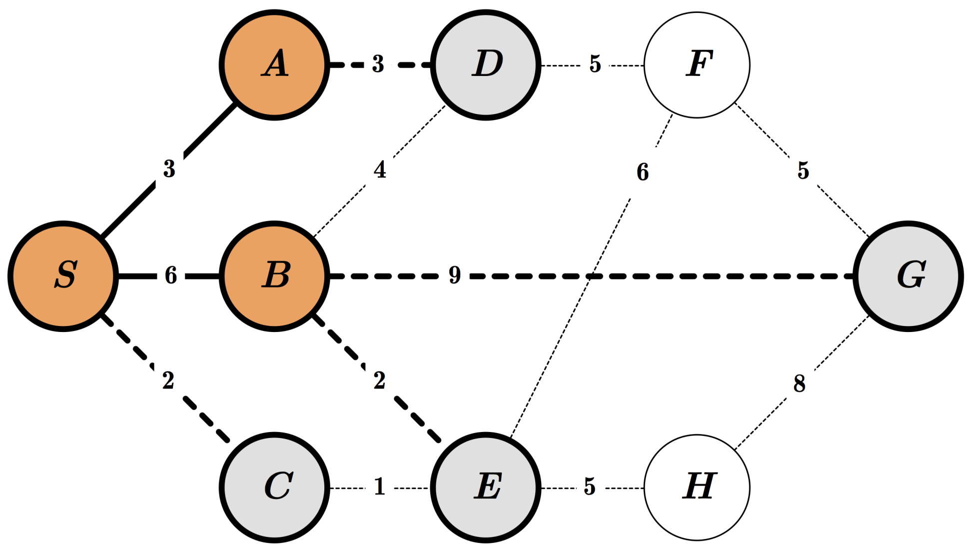
Queue:

S	A	B	C	D

Order of Visit:

S A

Exercise: BFS



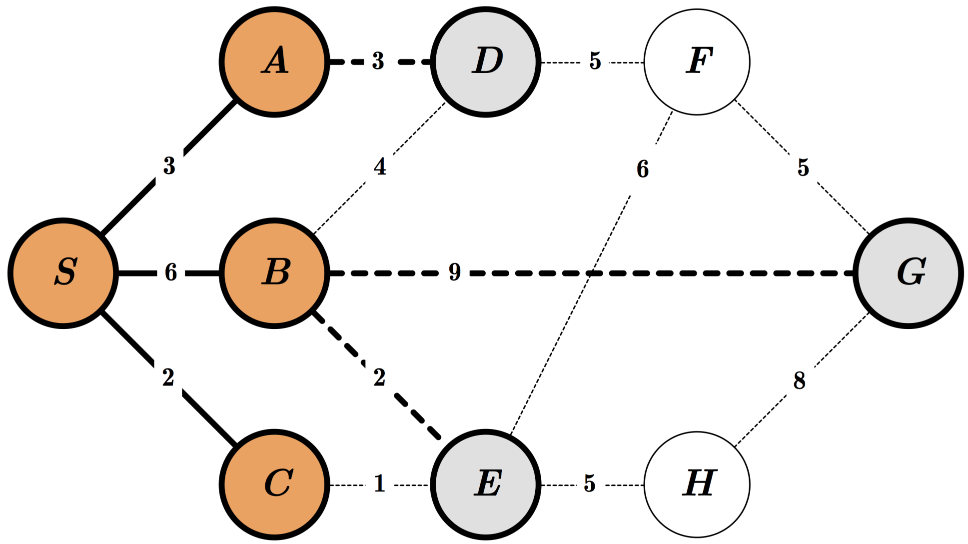
Queue:

S	A	B	C	D	E	G

Order of Visit:

S A B

Exercise: BFS



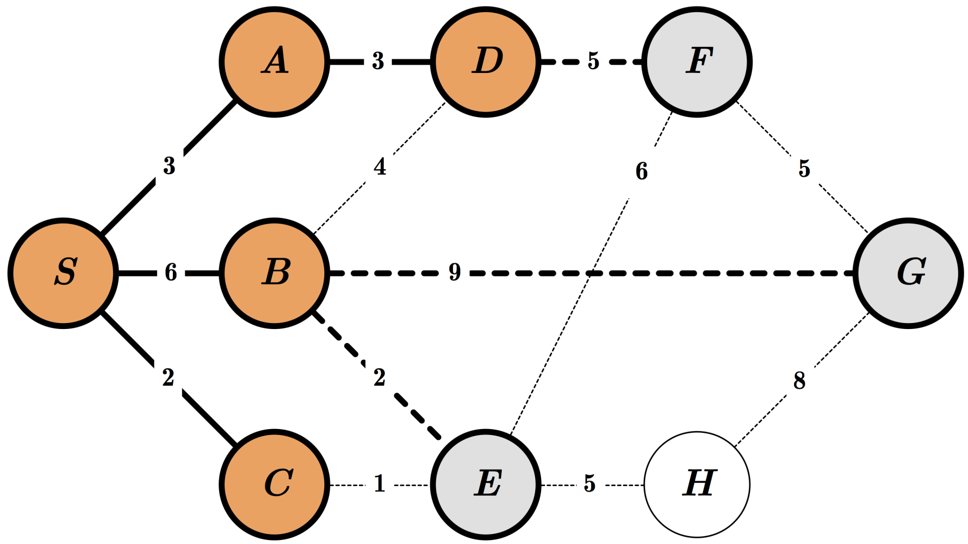
Queue:

S	A	B	C	D	E	G

Order of Visit:

S A B C

Exercise: BFS



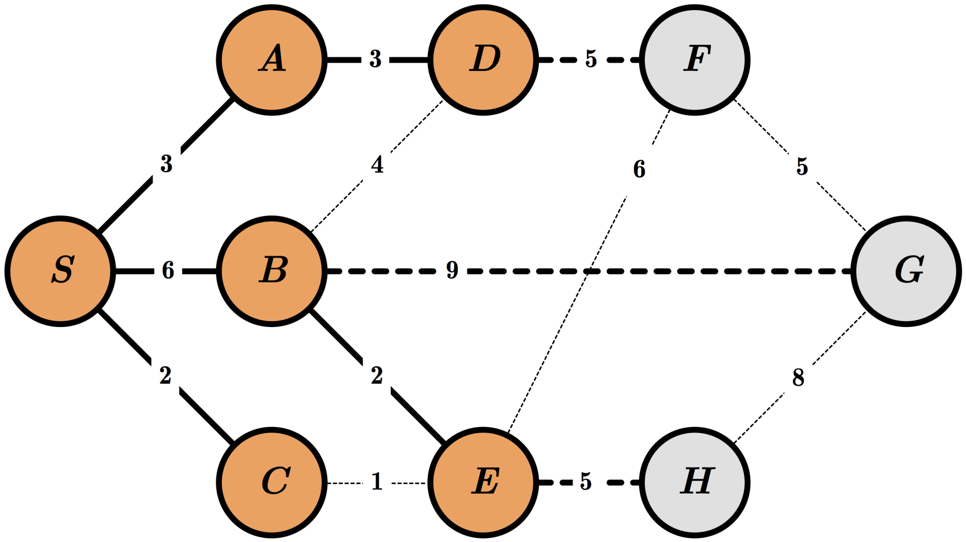
Queue:

S	A	B	C	D	E	G	F

Order of Visit:

S A B C D

Exercise: BFS



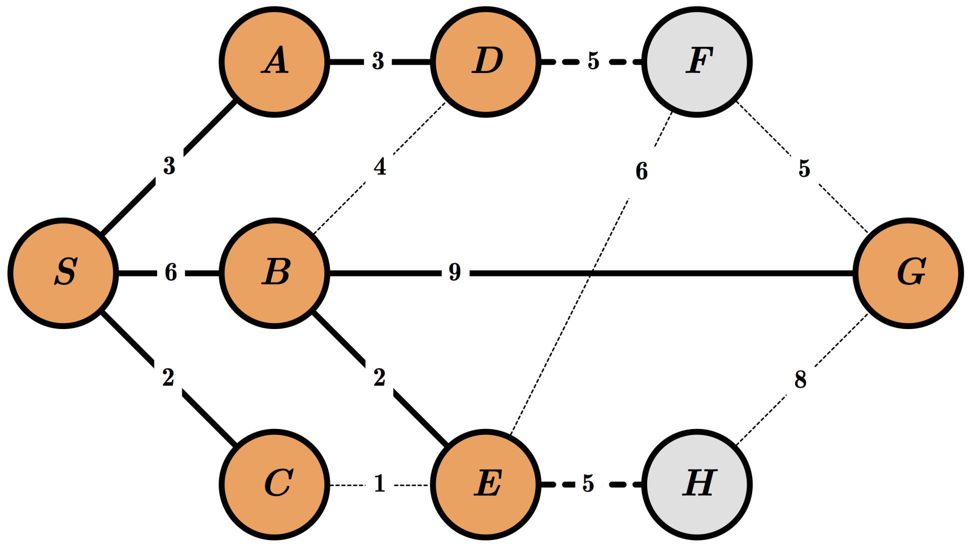
Queue:

S	A	B	C	D	E	G	F	H

Order of Visit:

S A B C D E

Exercise: BFS



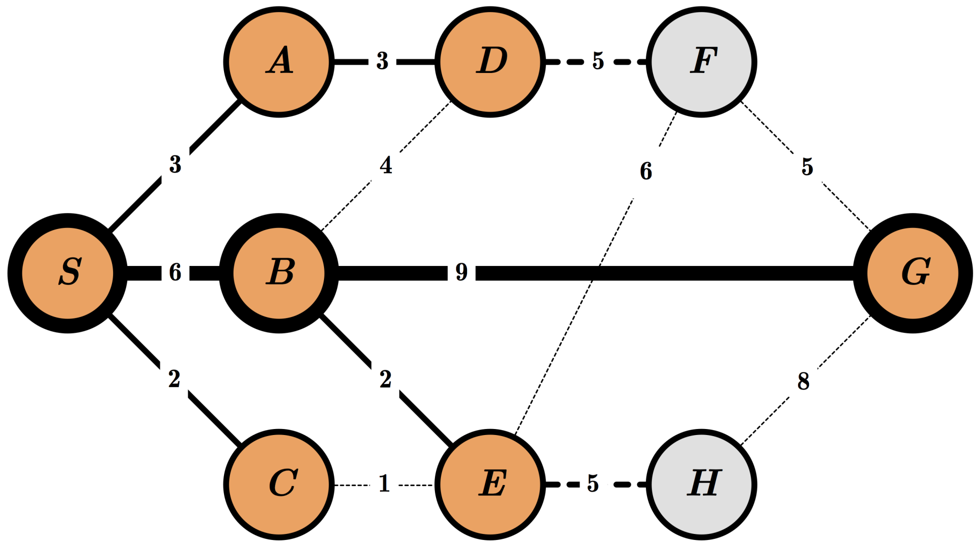
Queue:

S	A	B	C	D	E	G	F	H

Order of Visit:

S A B C D E G

Exercise: BFS



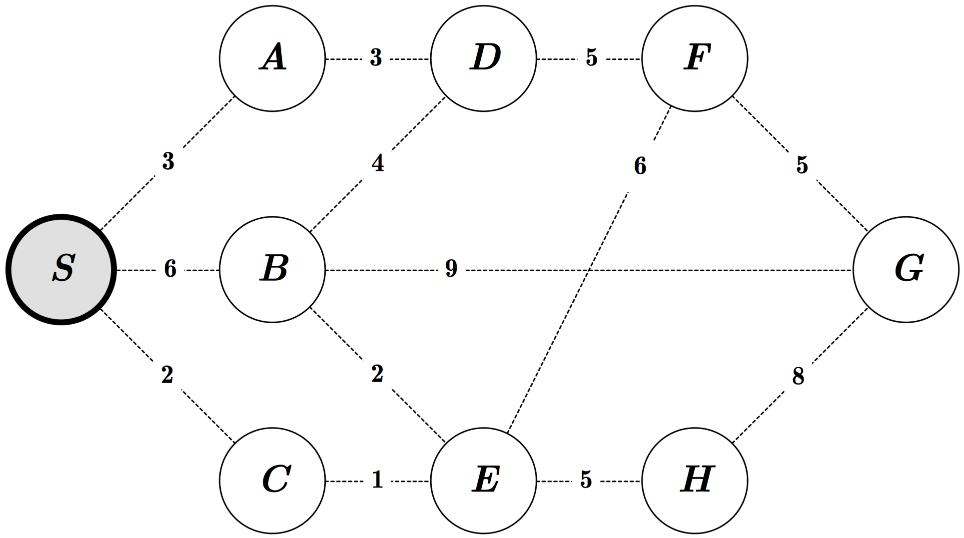
Queue:

S	A	B	C	D	E	G	F	H

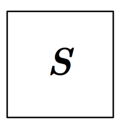
Order of Visit:

S A B C D E G

Exercise: DFS

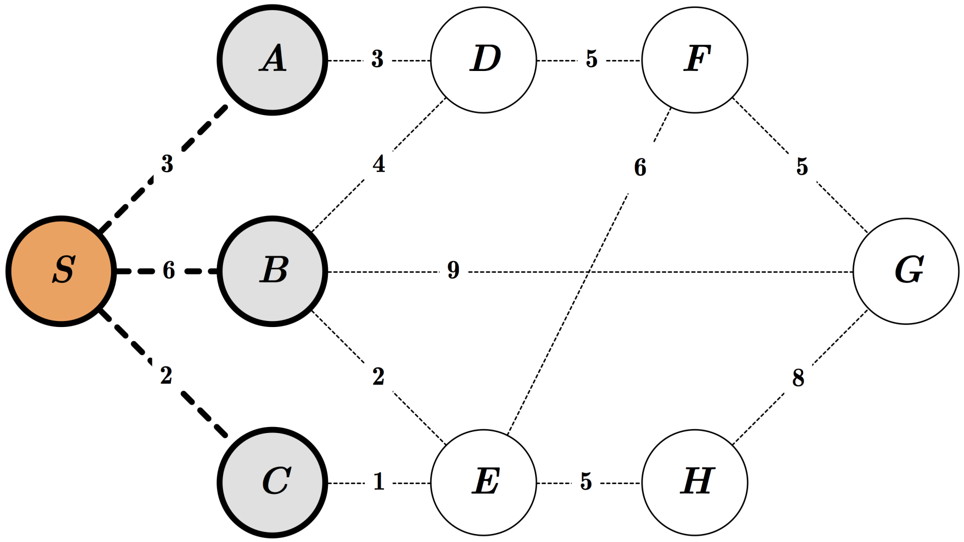


Stack:



Order of Visit:

Exercise: DFS



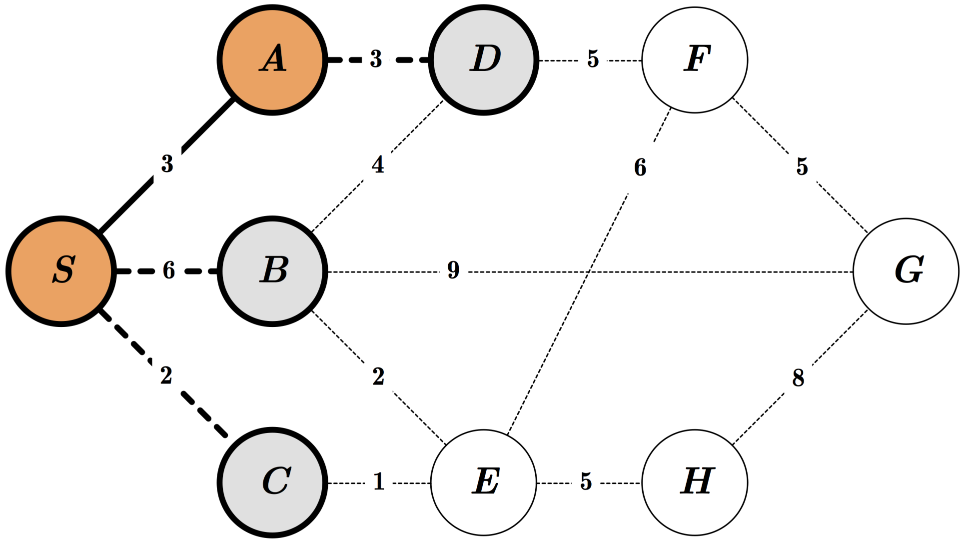
Stack:

S	C	B	A

Order of Visit:

S

Exercise: DFS



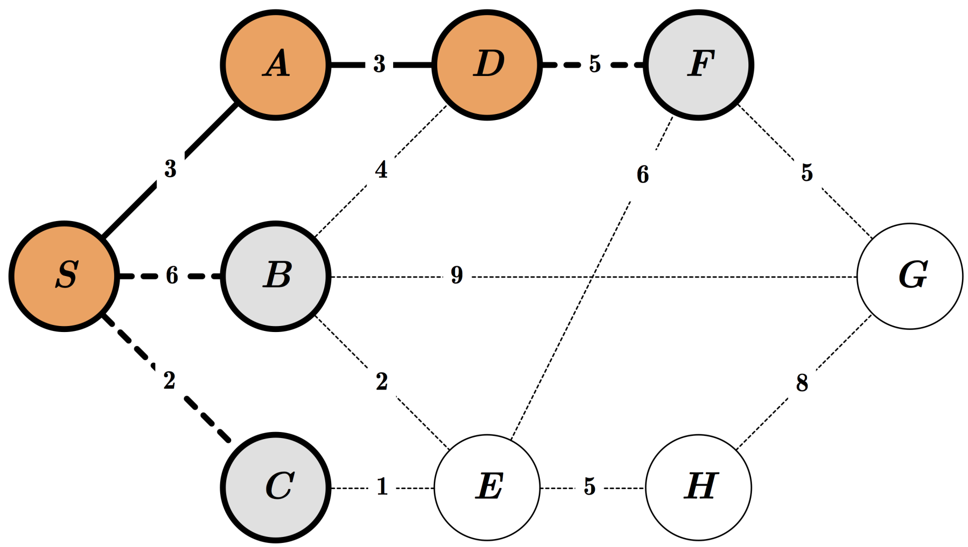
Stack:

S	C	B	A	D

Order of Visit:

S A

Exercise: DFS



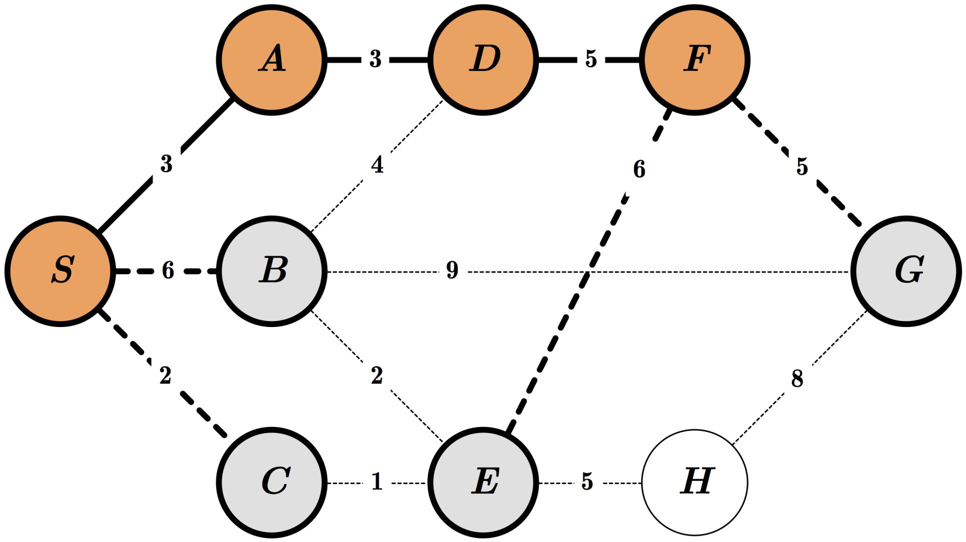
Stack:

S	C	B	A	D	F

Order of Visit:

S A D

Exercise: DFS



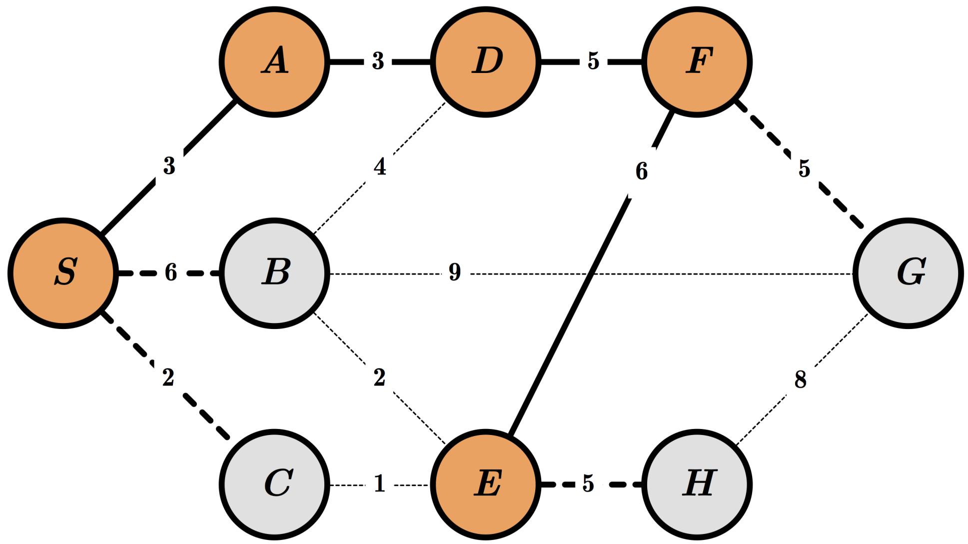
Stack:

S	C	B	A	D	F	G	E

Order of Visit:

$S \quad A \quad D \quad F$

Exercise: DFS



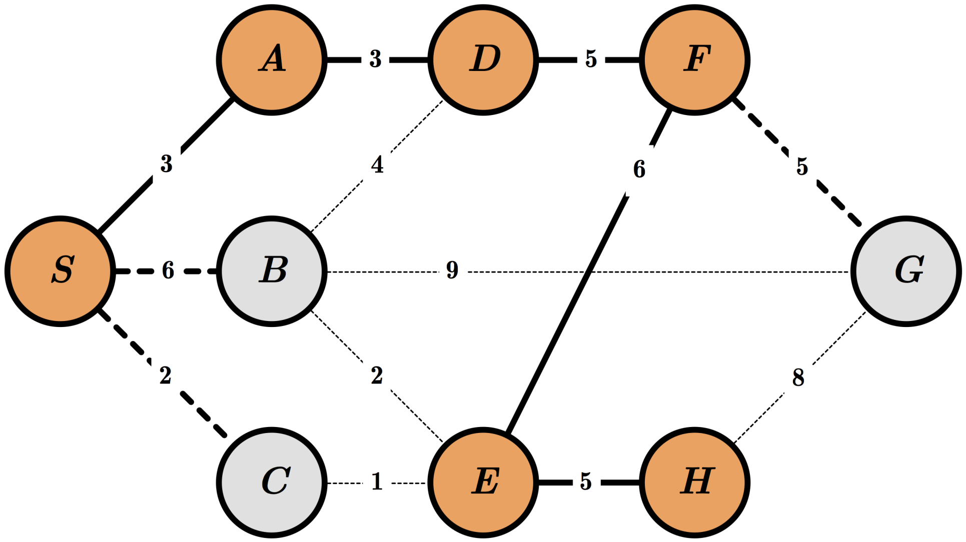
Stack:

S	C	B	A	D	F	G	E	H

Order of Visit:

S A D F E

Exercise: DFS



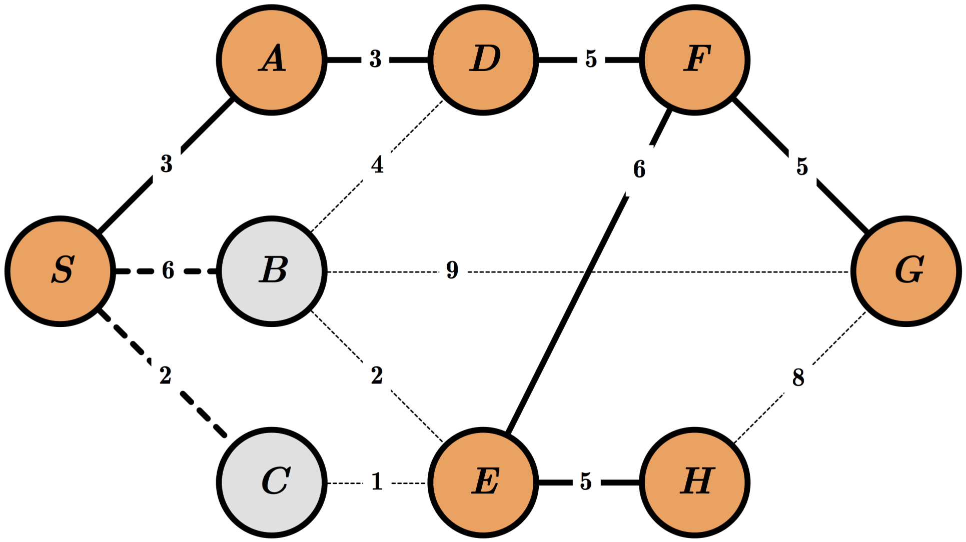
Stack:

S	C	B	A	D	F	G	E	H

Order of Visit:

$S \quad A \quad D \quad F \quad E \quad H$

Exercise: DFS



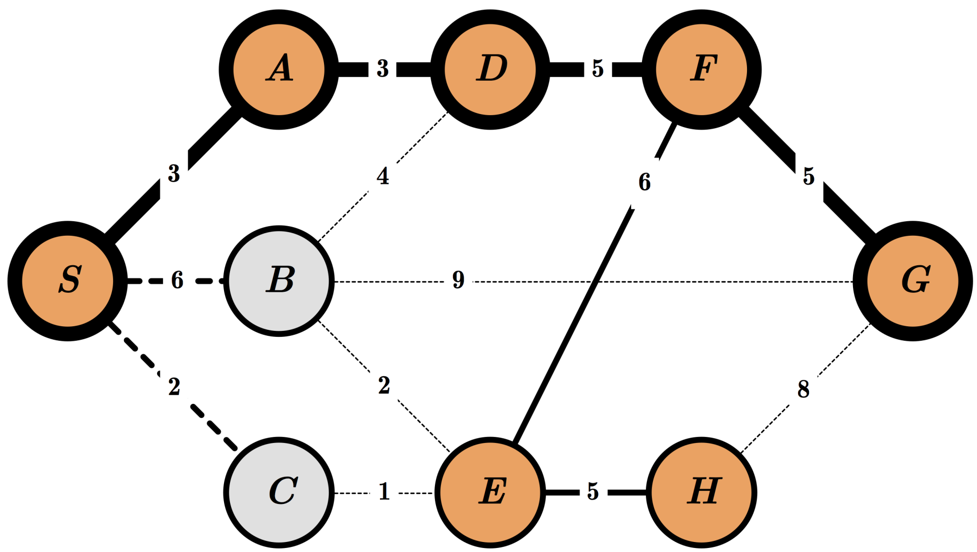
Stack:

S	C	B	A	D	F	G	E	H

Order of Visit:

S A D F E H G

Exercise: DFS



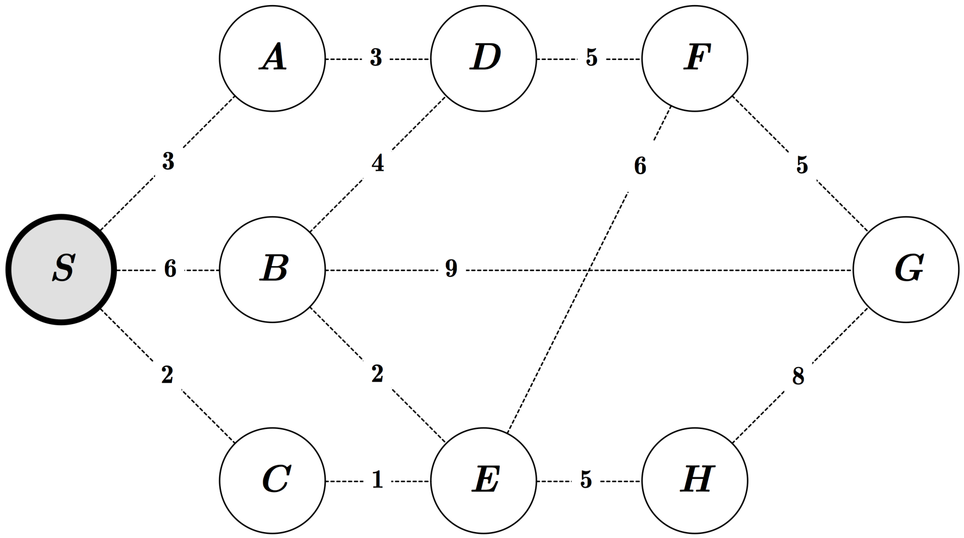
Stack:

S	C	B	A	D	F	G	E	H

Order of Visit:

S A D F E H G

Exercise: UCS

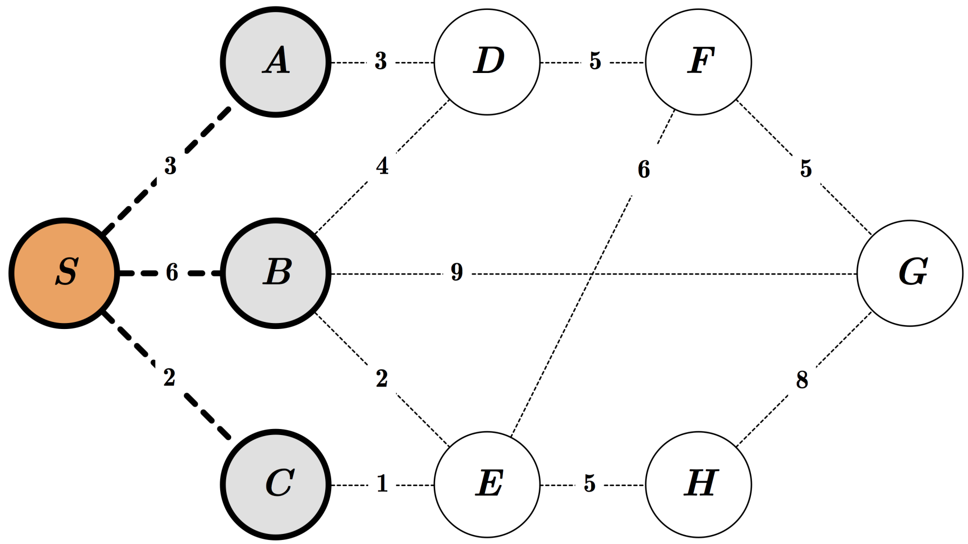


Priority Queue:

S 0

Order of Visit:

Exercise: UCS



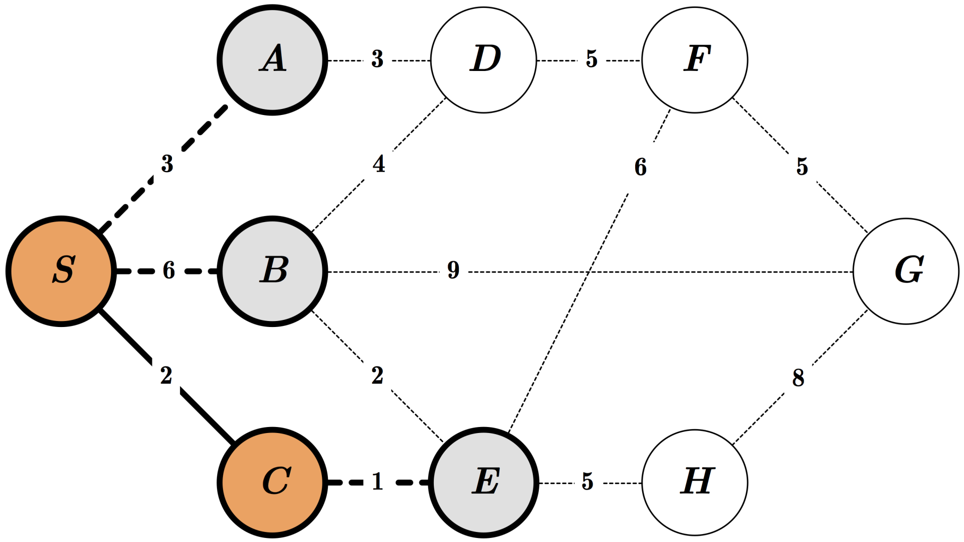
Priority Queue:

S 0	C 2	A 3	B 6

Order of Visit:

S

Exercise: UCS



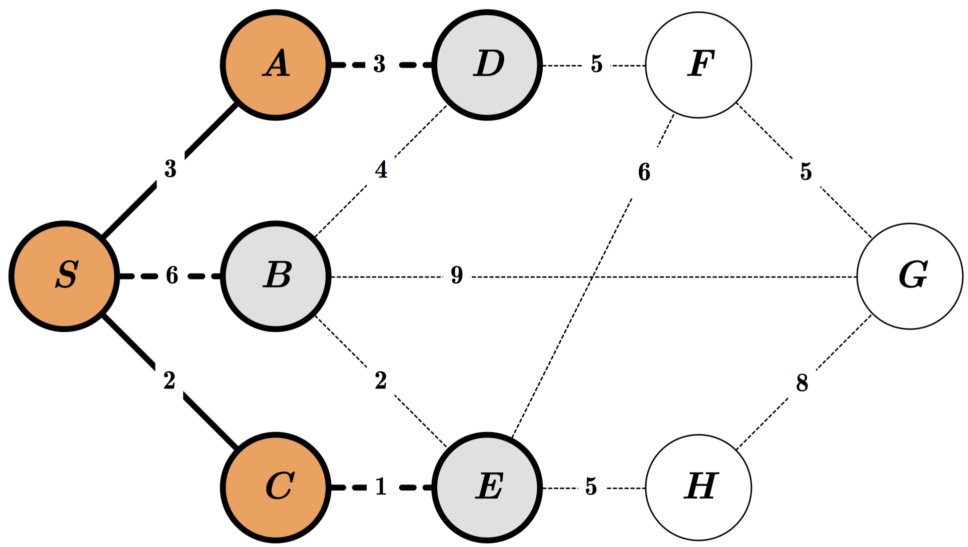
Priority Queue:

S 0	C 2	A 3	E 3	B 6

Order of Visit:

S C

Exercise: UCS



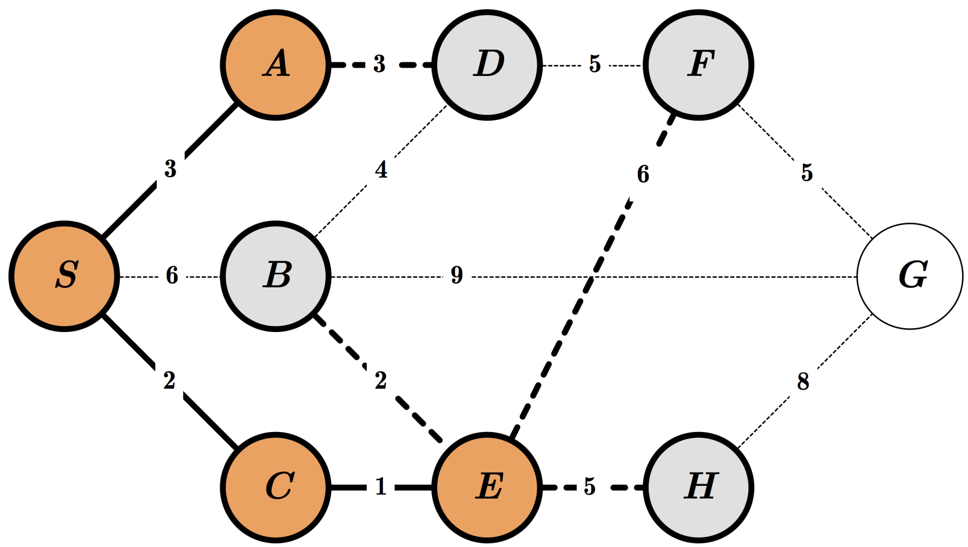
Priority Queue:

S 0	C 2	A 3	E 3	B 6	D 6

Order of Visit:

S C A

Exercise: UCS



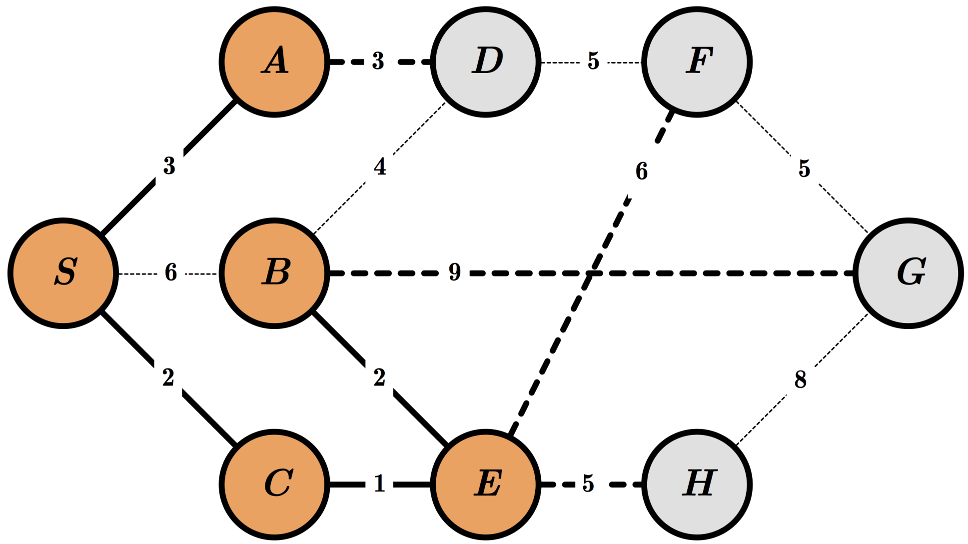
Priority Queue:

S 0	C 2	A 3	E 3	B 5	D 6	H 8	F 9

Order of Visit:

S C A E

Exercise: UCS



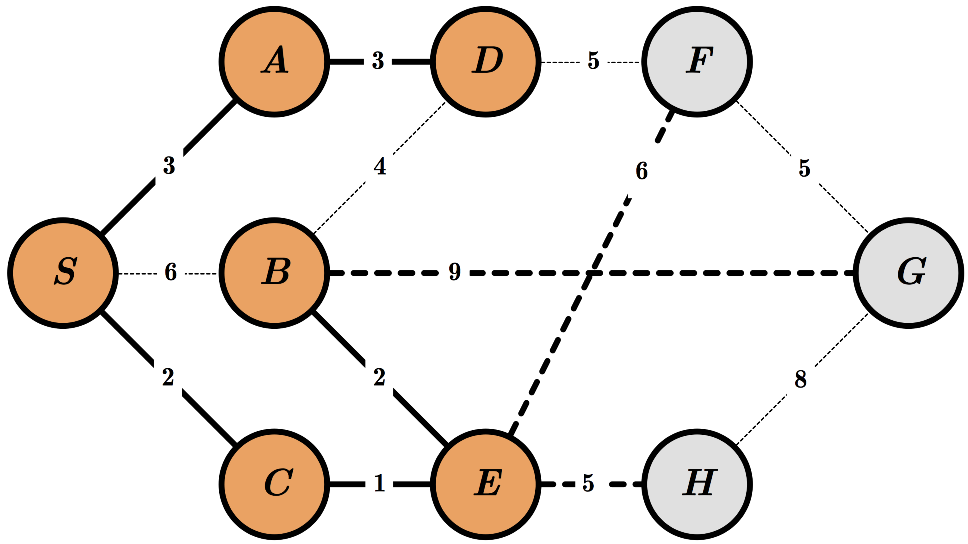
Priority Queue:

S 0	C 2	A 3	E 3	B 5	D 6	H 8	F 9	G 14

Order of Visit:

S C A E B

Exercise: UCS



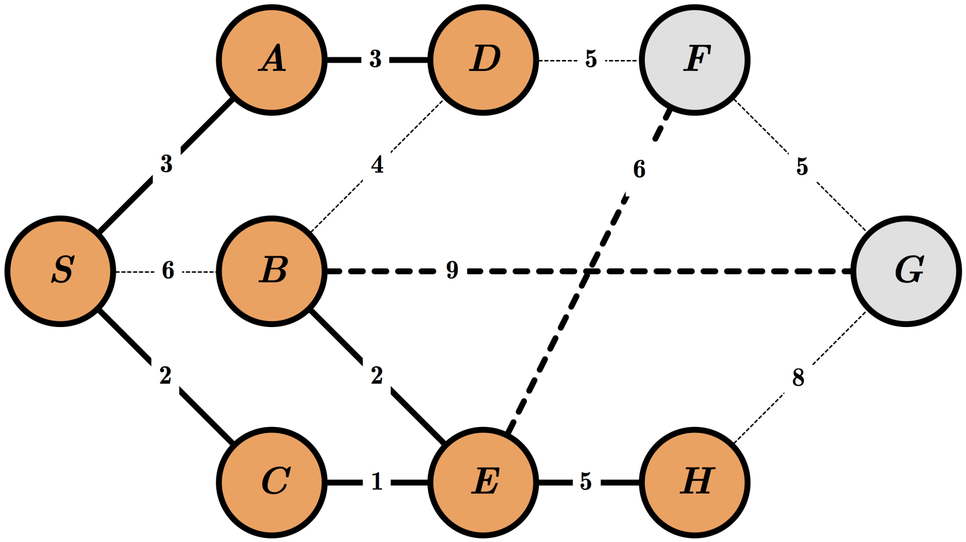
Priority Queue:

S 0	C 2	A 3	E 3	B 5	D 6	H 8	F 9	G 14

Order of Visit:

S C A E B D

Exercise: UCS



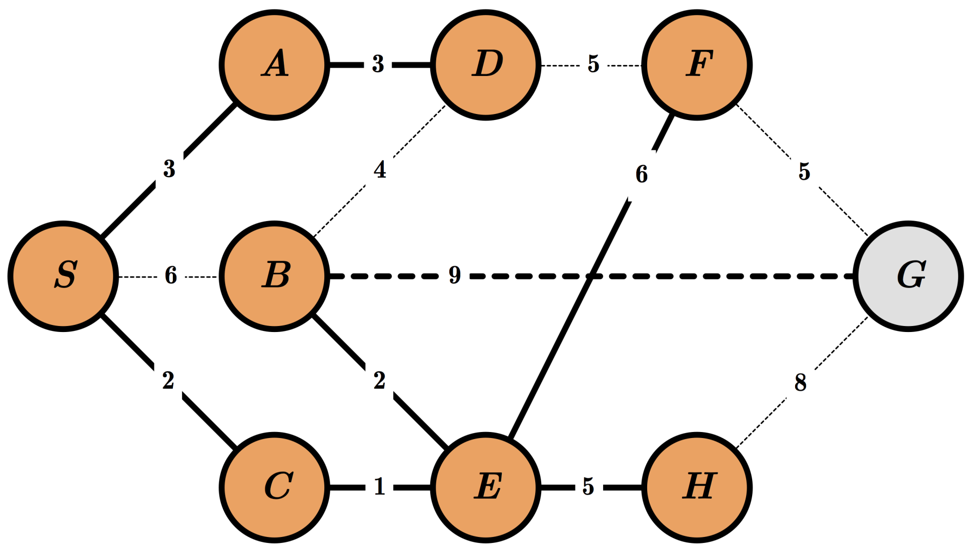
Priority Queue:

S 0	C 2	A 3	E 3	B 5	D 6	H 8	F 9	G 14

Order of Visit:

S C A E B D H

Exercise: UCS



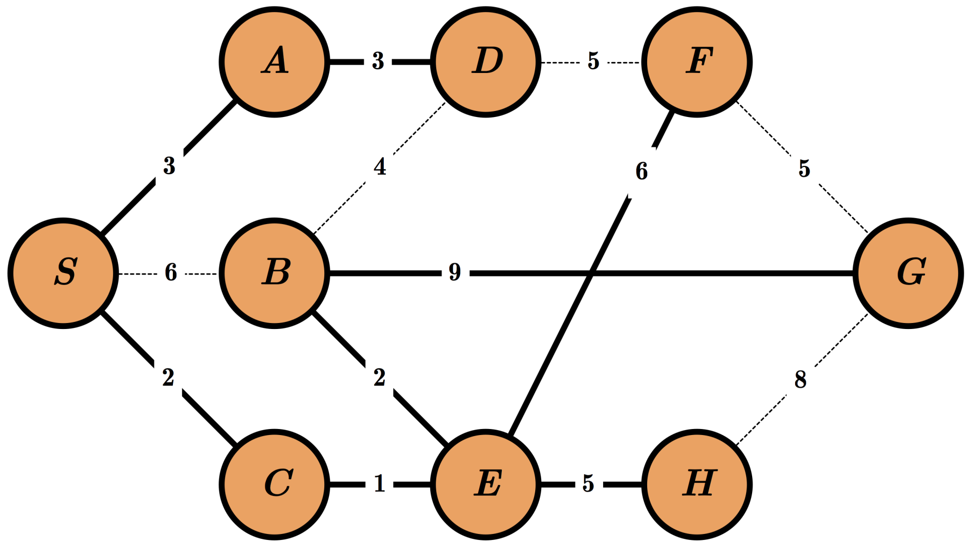
Priority Queue:

S 0	C 2	A 3	E 3	B 5	D 6	H 8	F 9	G 14

Order of Visit:

S C A E B D H F

Exercise: UCS



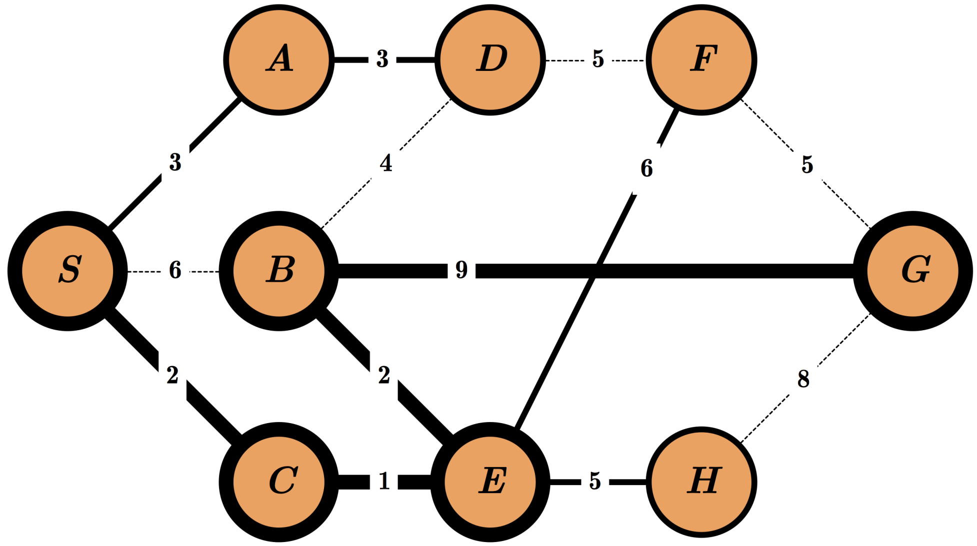
Priority Queue:

S 0	C 2	A 3	E 3	B 5	D 6	H 8	F 9	G 14

Order of Visit:

S C A E B D H F G

Exercise: UCS



Priority Queue:

S 0	*C* 2	*A* 3	*E* 3	*B* 5	*D* 6	*H* 8	*F* 9	*G* 14

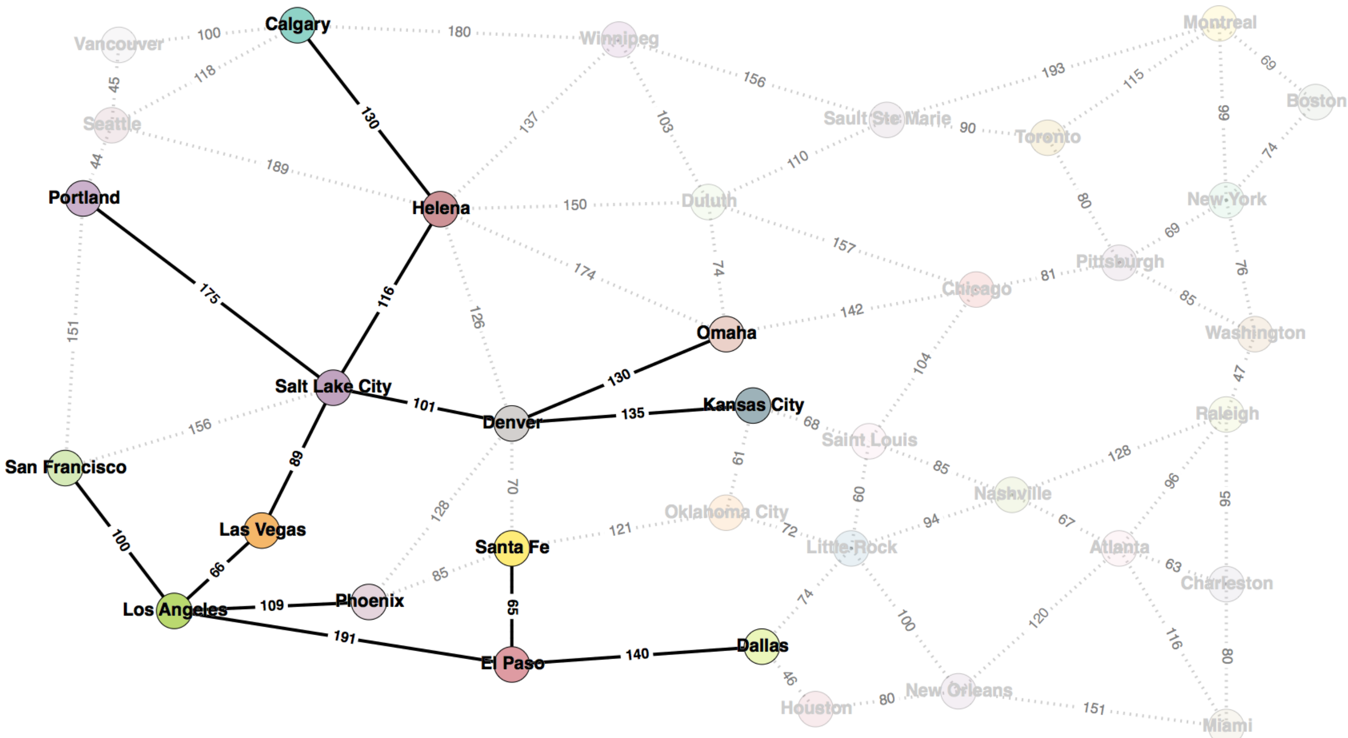
Order of Visit:

S *C* *A* *E* *B* *D* *H* *F* *G*

Examples using the map

Start: Las Vegas

Goal: Calgary



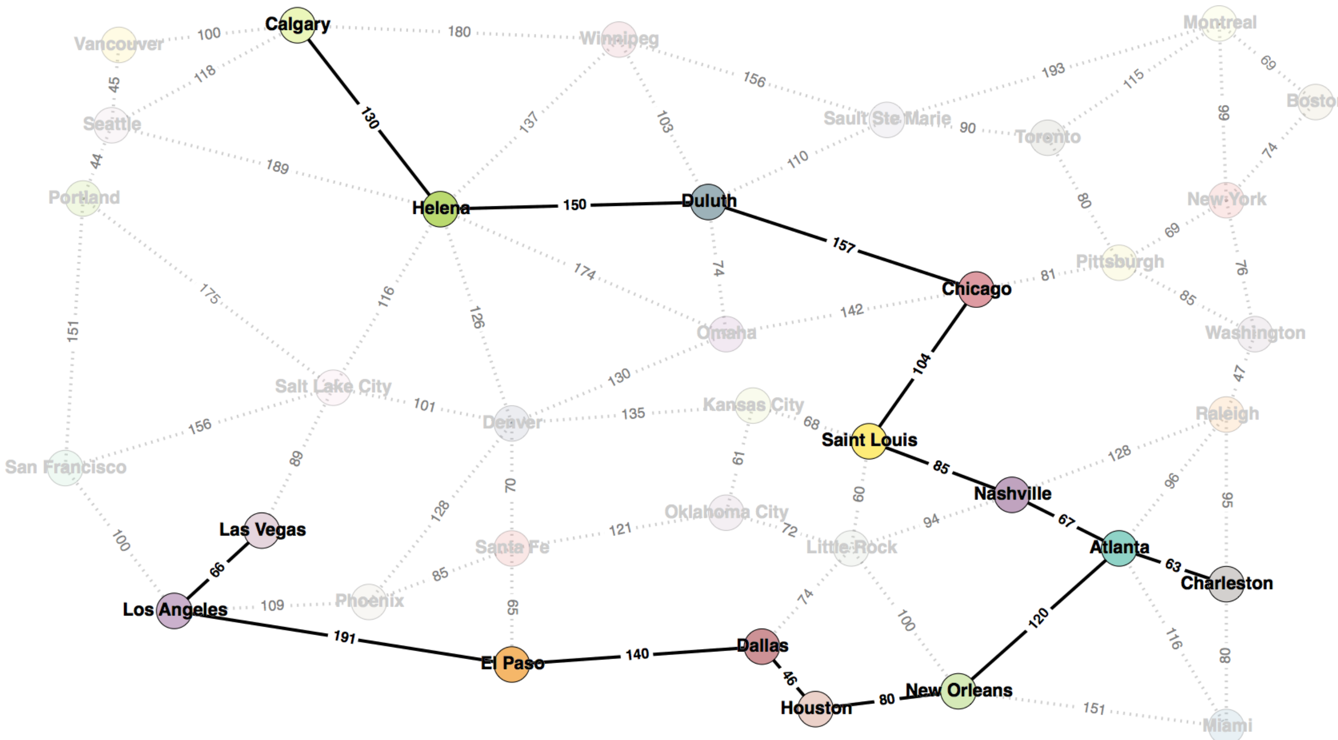
BFS

Order of Visit: Las Vegas, Los Angeles, Salt Lake City, El Paso, Phoenix, San Francisco, Denver, Helena, Portland, Dallas, Santa Fe, Kansas City, Omaha, Calgary.

Examples using the map

Start: Las Vegas

Goal: Calgary



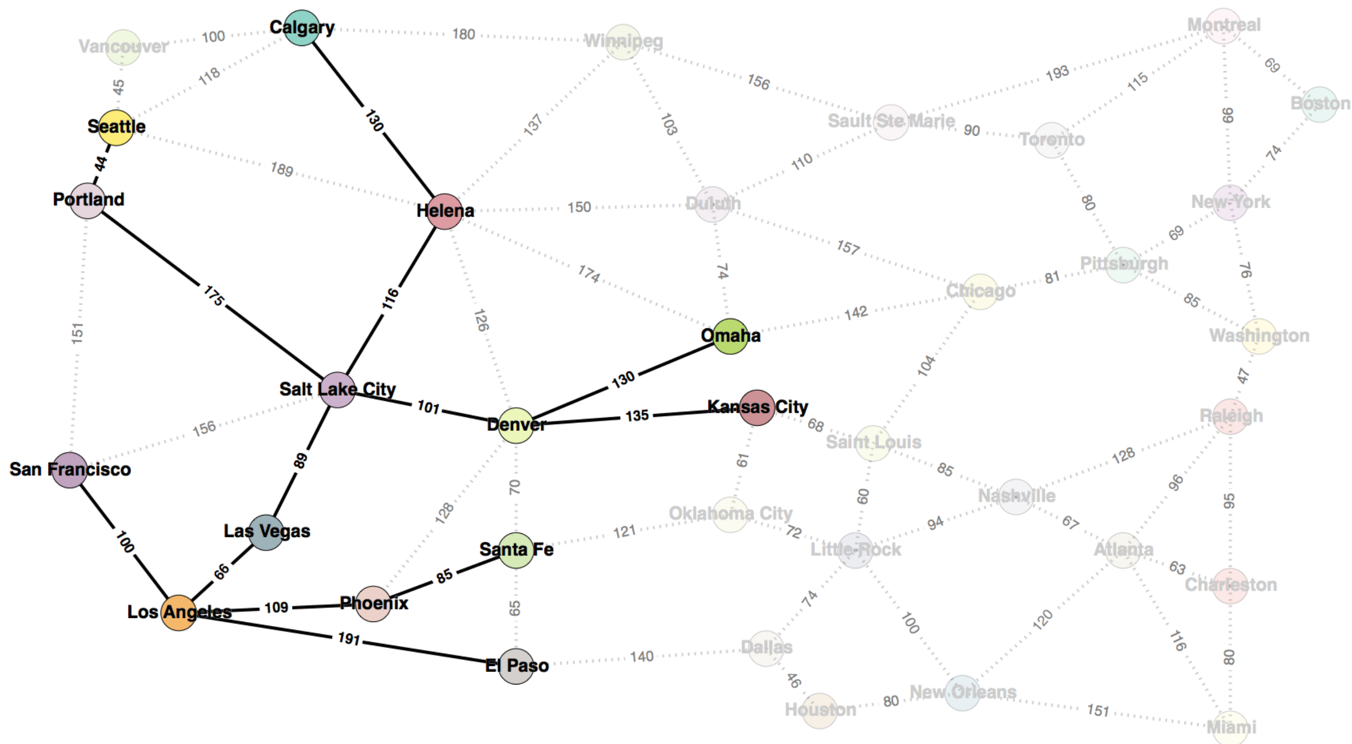
DFS

Order of Visit: Las Vegas, Los Angeles, El Paso, Dallas, Houston, New Orleans, Atlanta, Charleston, Nashville, Saint Louis, Chicago, Duluth, Helena, Calgary.

Examples using the map

Start: Las Vegas

Goal: Calgary



UCS

Order of Visit: Las Vegas, Los Angeles, Salt Lake City, San Francisco, Phoenix, Denver, Helena, El Paso, Santa Fe, Portland, Seattle, Omaha, Kansas City, Calgary.

Credit

- Artificial Intelligence, A Modern Approach. Stuart Russell and Peter Norvig. Third Edition. Pearson Education.

  http://aima.cs.berkeley.edu/