Corrección del parcial 1

Haider Santiago Calderón Rodríguez - 20211005075

① Representar en espacio de estados y hallar la función de transferencia.

$$\dddot{x} + \ddot{x} + 2\dot{x} + x = 2f(t) \longrightarrow \dddot{x} = 2f - \ddot{x} - 2\dot{x} - x$$

$q_1 = x$

$q_2 = \dot{q}_1 = \dot{x}$

$q_3 = \dot{q}_2 = \ddot{x}$

$\dot{q}_3 = \dddot{x}$

$$\dot{q}_3 = 2f - q_3 - 2q_2 - q_1$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -1 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} f$$

$$\begin{bmatrix} x \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

Para hallar la función de transferencia:

$$\dddot{x} + \ddot{x} + 2\dot{x} + x = 2f(t)$$

$$\downarrow \mathcal{L}$$

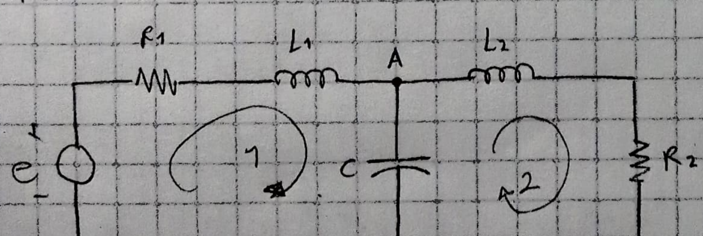
$$s^3 \mathcal{L}[x] + s^2 \mathcal{L}[x] + 2s\mathcal{L}[x] + \mathcal{L}[x] = 2\mathcal{L}[f(t)]$$

$$\mathcal{L}[x](s^3 + s^2 + 2s + 1) = 2\mathcal{L}[f(t)]$$

$$X(s^3 + s^2 + 2s + 1) = 2F$$

$$\frac{X}{F} = \frac{2}{s^3 + s^2 + 2s + 1}$$

② Encontrar una expresión en espacio de estados valida para el sistema. output V_{R2}



$V_L = L\dot{i}_L$

$i_C = C\dot{V}_C$

Malla 1: $-e(t) + V_{R1} + V_{L1} + V_C = 0$

$-e(t) + i_{L1}R_1 + V_{L1} + V_C = 0$ ①

Malla 2:

$V_{L2} + V_{R2} - V_C = 0$ ②

Nodo A: $i_{L1} - i_C - i_{L2} = 0$ ③

De ①: $V_{L1} = e(t) - i_{L1}R_1 - V_c \rightarrow \dot{i}_{L1} = \frac{e(t)}{L_1} - \frac{i_{L1}R_1}{L_1} - \frac{V_c}{L_1}$

De ②: $V_{L2} = V_c - V_{R2}$

De ③: $i_c = i_{L2} - i_{L1}$

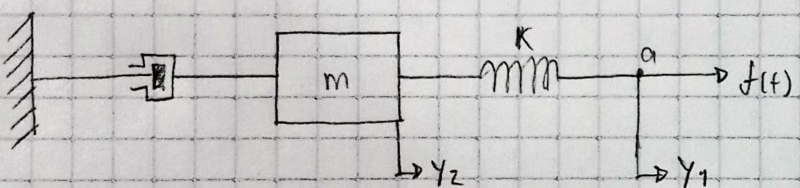
$$\dot{i}_{L2} = \frac{V_c}{L_2} - \frac{V_R}{L_2}$$

$$\dot{V}_c = \frac{i_{L2}}{c} - \frac{i_{L1}}{c}$$

$$\begin{bmatrix} \dot{i}_{L1} \\ \dot{i}_{L2} \\ \dot{V}_c \end{bmatrix} = \begin{bmatrix} R_1/L_1 & 0 & -1/L_1 \\ 0 & 0 & 1/L_2 \\ -1/c & 1/c & 0 \end{bmatrix} \begin{bmatrix} i_{L1} \\ i_{L2} \\ V_c \end{bmatrix} + \begin{bmatrix} 1/L_1 \\ 0 \\ 0 \end{bmatrix} [e(t)]$$

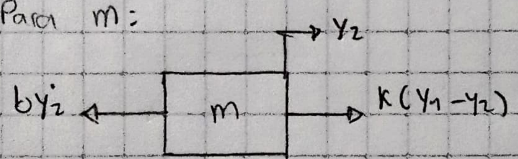
$$[V_{R2}] = \begin{bmatrix} 0 & R_2 & 0 \end{bmatrix} \begin{bmatrix} i_{L1} \\ i_{L2} \\ V_c \end{bmatrix}$$

③ Hallar una expresión en espacio de estados valida para el sistema.
outputs: y_1, y_2

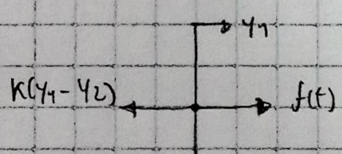


Diagramas de cuerpo libre:

Para m:



Para a:



$$-b\dot{y}_2 + Ky_1 - Ky_2 = m\ddot{y}_2$$

$$\ddot{y}_2 = -\frac{b}{m}\dot{y}_2 + \frac{K}{m}y_1 - \frac{K}{m}y_2 \quad ①$$

$$-Ky_1 + Ky_2 + f = 0 \quad ②$$

$q_1 = y_1$

$q_2 = y_2$

$q_3 = \dot{q}_2 = \dot{y}_2$

$\dot{q}_3 = \ddot{y}_2$

Remplazando en ①:

$$\dot{q}_3 = \frac{K}{m}q_1 - \frac{K}{m}q_2 - \frac{b}{m}q_3 \quad ③$$

Remplazando en ②:

$$-Kq_1 + Kq_2 + f = 0$$

$$q_1 = q_2 + \frac{f}{K} \quad ④$$

Remplazando ④ en ③:

$$\dot{q}_3 = \frac{K}{m}\left(q_2 + \frac{f}{K}\right) - \frac{K}{m}q_2 - \frac{b}{m}q_3$$

$$\dot{q}_2 = \frac{k}{m} q_2 + \frac{f}{m} - \frac{k}{m} q_1 - \frac{b}{m} q_3$$

$$\dot{q}_3 = \frac{f}{m} - \frac{b}{m} q_3$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -b/m \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/m \end{bmatrix} f$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$