Ejemplo 1 Fracciones parciales

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$$X(s) = \frac{2s^3 + 8s^2 + 4s + 8}{s(s+1)(s^2+4s+8)}$$

En fracciones parciales:

$$= \frac{K_1}{s} + \frac{K_2}{s+1} + \frac{A}{s+2+j2} + \frac{A^*}{s+2-j2}$$

$$K_1 = sX(s)\Big|_{s=0}$$

$$K_1 = \cancel{s}\left(\frac{2s^3 + 8s^2 + 4s + 8}{\cancel{s}(s+1)(s^2+4s+8)}\right)\Big|_{s=0}$$

$$\boxed{K_1 = 1}$$

$$K_2 = (s+1)Xs\Big|_{s=-1}$$

$$K_2 = (s+1)\left(\frac{2s^3 + 8s^2 + 4s + 8}{s(s+1)(s^2+4s+8)}\right)\Big|_{s=-1}$$

$$K_2 = \frac{-2 + 8 - 4 + 8}{-1(1 - 4 + 8)} = -\frac{10}{5} = -2$$

$$\boxed{K_2 = -2}$$

$$A = (s+2+j2)\,X(s)\Big|_{s=-2-j2}$$

$$A = (s+2+j2)\left(\frac{2s^3 + 8s^2 + 4s + 8}{s(s+1)(s^2+4s+8)}\right)\Big|_{s=-2-j2}$$

$$A = (s+2+j2)\left(\frac{2s^3 + 8s^2 + 4s + 8}{s(s+1)(s+2+j2)(s+2-j2)}\right)\Big|_{s=-2-j2}$$

$$A = \frac{2(-2-j2)^3 + 8(-2-j2)^2 + 4(-2-j2) + 8}{(-2-j2)\left[(-2-j2)+1\right]\left[(-2-j2)^2 + 4(-2-j2) + 8\right]}$$

Resolviendo parte por parte

$2s^3 = 2[-2 - j2]^3$ $\quad -j24 \quad 24$

$= 2[(-2)^3 + 3(-2)^2(-j2) + 3(-2)(-j2)^2 + (-j2)^3]$

$(-j2)^3 = (-1)^3 j^3 2^3$

$= -1 \cdot j^2 j8$

$= -8(-1) j$

$= j8$

$2s^3 = 2[-8 - j24 + 24 + j8]$

$2s^3 = 2[16 - j16]$

$\boxed{2s^3 = 32 - j32}$

$8s^2 = 8[-2 - j2]^2$

$8s^2 = 8[4 + 2(4j) - 4]$

$8s^2 = 8[8j]$

$\boxed{8s^2 = j64}$

Numerador: $32 - j32 + j64 + 4(-2 - j2) + 8$

$+ \cancel{-8} - j8 + 8$

$\boxed{32 + j24}$

Denominador: $(-2 - j2)(-2 - j2 + 1)(\cancel{-2} - j2 + 2 - j2)$

$(-2 - j2)(-2 - j2 + 1)(-j4)$

$(j8 - 8)(-1 - j2)$

$-j8 + 16 + 8 + j16$

$\boxed{24 + j8}$

$A = \frac{32 + j24}{24 + j8} = \frac{8(4 + j3)}{8(3 + j)} = \frac{4 + j3}{3 + j} \cdot \frac{3 - j}{3 - j} = \frac{12 - 4j + 9j + 3}{9 - \cancel{j8} + j8 + 1}$

$A = \frac{15 + 5j}{10} = \frac{3 + j}{2} = 1{,}5 + j0{,}5$

$\boxed{A = 1{,}5 + j0{,}5}$

Entonces tenemos:

$X(s) = \frac{1}{s} - \frac{2}{s+1} + \frac{1{,}5 + j0{,}5}{s + 2 + j2} + \frac{1{,}5 - j0{,}5}{s + 2 - j2}$