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Sistema de control por realimentación de estados

$$G(s) = \frac{20(s+5)}{s(s+1)(s+4)} \quad \left\{ \begin{array}{l} OS\% \quad 9{,}5\% \\ ts \quad = 0{,}74 \text{ sg} \end{array} \right.$$

$$U(s) \longrightarrow \boxed{\frac{1}{s^3+5s^2+4s}} \xrightarrow{X_1(s)} \boxed{0s^2+20s+100} \longrightarrow Y(s)$$

$$\frac{X_1(s)}{U(s)} = \frac{1}{s^3+5s^2+4s} \longrightarrow (s^3+5s^2+4s)X_1(s) = U(s)$$

$$\dddot{x}_1 + 5\ddot{x}_1 + 4\dot{x}_1 = u$$

Variables de estado

$x_1 = x_1$

$x_2 = \dot{x}_1$

$x_3 = \dot{x}_2 = \ddot{x}_1$

$\dot{x}_3 = \dddot{x}_1$

$$\dot{x}_3 = -5x_3 - 4x_2 + u \quad (1)$$

$$Y(s) = (b_2 s^2 + b_1 s + b_0)\, X_1(s)$$

$$= (0s^2 + 20s + 100)\, X_1(s) \longrightarrow (20s + 100)\, X_1(s)$$

$$= 20\dot{x}_1 + 100x_1$$

$$y = 20x_2 + 100x_1 \quad (2)$$

Representación en espacio de estados

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -4 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$y = \begin{bmatrix} 100 & 20 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\%OS = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \cdot 100$$

$$0{,}095 = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \cdot 100 \longrightarrow \ln(0{,}095) \cdot \ln(e^{-(\zeta\pi/\sqrt{1-\zeta^2})})$$

$$-2{,}3534 = \frac{-\zeta\pi}{\sqrt{1-\zeta^2}} \longrightarrow \cancel{-}2{,}3534(\sqrt{1-\zeta^2}) = \cancel{-}\zeta\pi$$

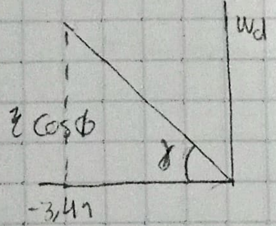
Elevando al cuadrado ambos lados

$$5,5407(1-\xi^2) = \xi^2\pi^2 \rightarrow 5,5407 - 5,5407\xi^2 = \xi^2\pi^2 \rightarrow 5,5407 = \xi^2\pi^2 + 5,5407\xi^2$$

$$5,5407 = \xi^2(\pi^2 + 5,5407) \rightarrow \xi^2 = \frac{5,5407}{\pi^2 + 5,5407} \rightarrow \xi = \sqrt{\frac{5,5407}{\pi^2 + 5,5407}}$$

$$\xi = 0,5996$$

En el plano S

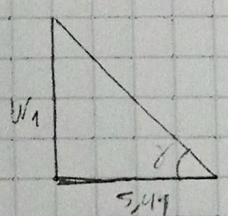


$$S = \sigma + j\omega_d \rightarrow \sigma = \xi\omega_n$$

$$ts = \frac{4}{\sigma} \rightarrow \sigma = \frac{4}{0,74} = 5,405$$

$$\phi = \cos^{-1}(0,5996) = \boxed{53,16°}$$

$$\sigma = \xi\omega_n \rightarrow 5,405 = 0,5996\,\omega_n \rightarrow \boxed{\omega_n = 9,02 \text{ rad/s}}$$

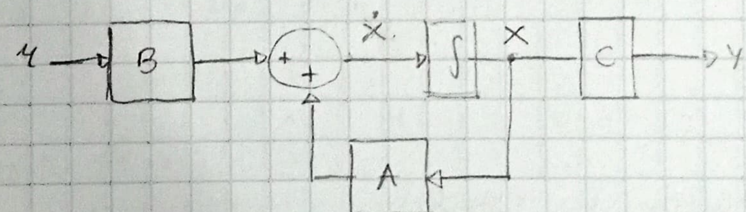


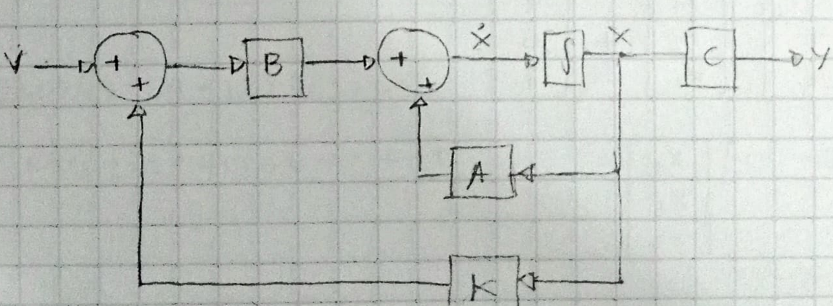
$$\tan\phi = \frac{\omega_d}{5,41}$$

$$\tan^{1}(53,16)(5,41) = \boxed{\omega_d = 7,2146}$$

Realimentación en espacio de estados

$$\dot{x} = Ax + Bx$$
$$y = Cx$$



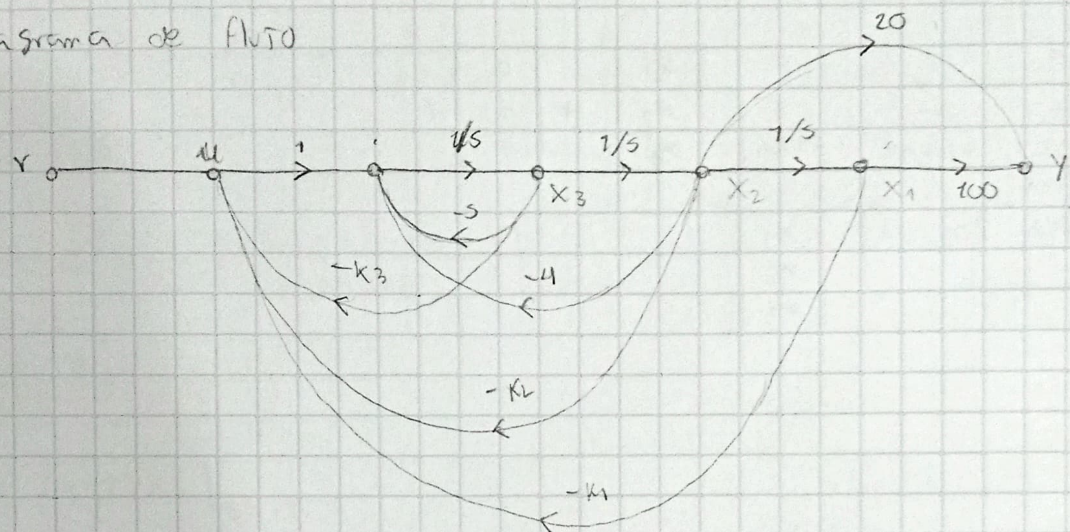


$$\dot{x} = Ax + Bu$$
$$= Ax + B(-ux + v)$$
$$= Ax - BKx + Bv \rightarrow \dot{x} = (A - B)x + v$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}0&1&0\\0&0&1\\0&-4&-5\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}+\begin{bmatrix}0\\0\\1\end{bmatrix}u$$

$$y=\begin{bmatrix}100&20&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$$

Diagrama de fluJO



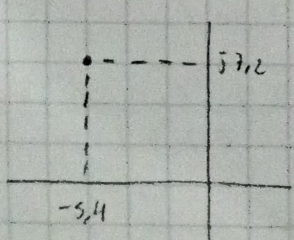
$$\begin{aligned}\dot{x}_3&=-4x_2-5x_3+u\\&=-4x_2-5x_3+[-k_3x_3-k_2x_2-k_1x_1]+r\\&=-4x_2-5x_3-k_3x_3-k_2x_2-k_1x_1+r\\&=-k_1x_1-(4+k_2)x_2-(5+k_3)x_3+r\end{aligned}$$

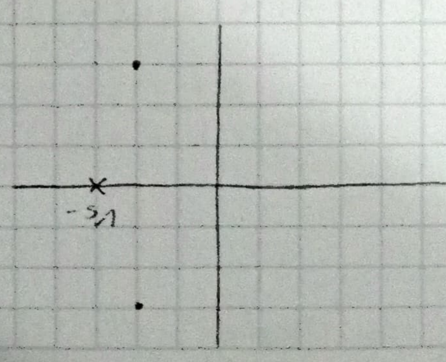
$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}0&1&0\\0&0&1\\-k_1&-(4+k_2)&-(5+k_3)\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}+\begin{bmatrix}0\\0\\1\end{bmatrix}r$$

$$\det(sI-A(BM))=\boxed{s^3+(5+k_3)s^2+(4+k_2)s+k_1=0}$$

$$\nabla=(s+5{,}4-j7{,}2)(s+5{,}4+j7{,}2)(s+5{,}1)$$

$$s^3+15{,}95s^2+136{,}22s+413{,}83=0\leftarrow T$$





$$s^3 + (5 + k_3)s^2 + (4 + k_2)s_2 + k_1 = s^3 + 15{,}4s^2 + 136{,}22s + 413{,}83$$

$$(5 + k_3)s^2 = 15{,}9s^2 \qquad (4 + k_2)s = 136{,}22s$$

$$5 + k_3 = 15{,}9 \qquad 4 + k_2 = 136{,}21 \qquad \boxed{k_1 = 413{,}83}$$

$$\boxed{k_3 = 10{,}9} \qquad \boxed{k_2 = 132{,}22}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -413{,}8 & -136{,}12 & -15{,}4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v$$

$$y = \begin{bmatrix} 100 & 20 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$