Sistema rotacional y traslacional

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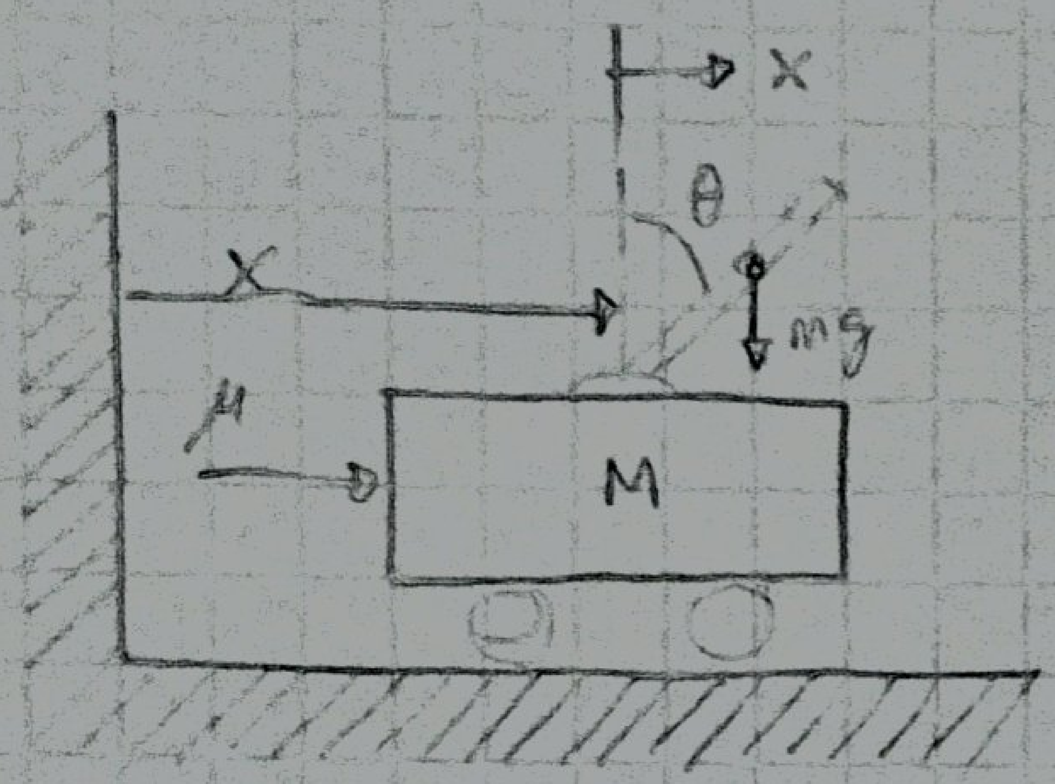

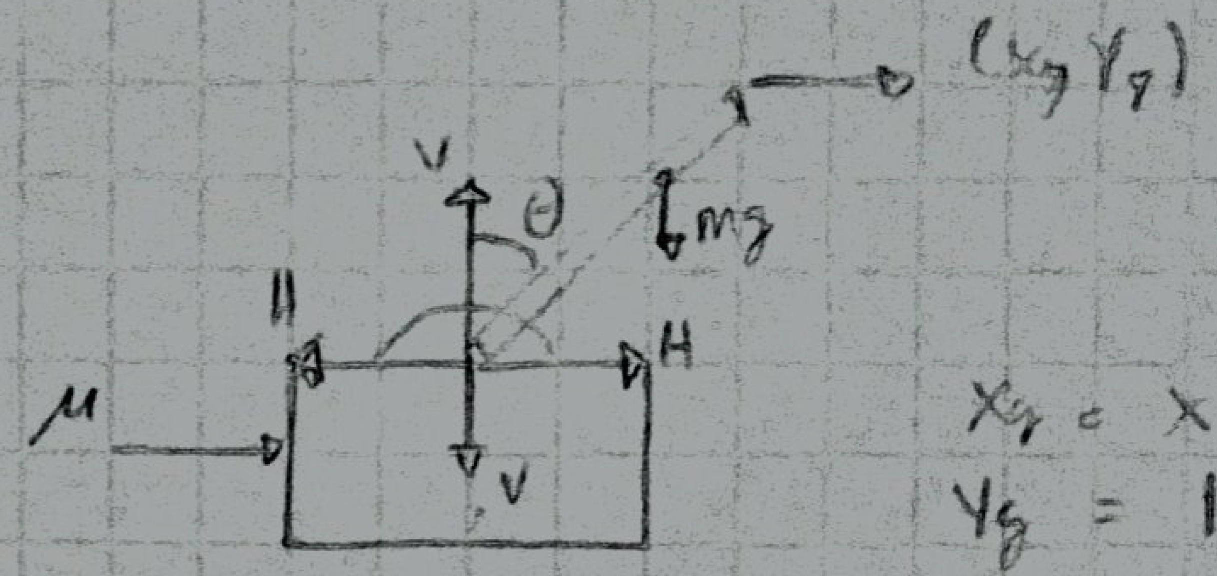

$X_g = x + l \operatorname{sen}\theta$

$Y_g = l\cos\theta$

movimiento rotacional : $I\ddot{\theta} = Vl\operatorname{sen}\theta + Hl\cos\theta$ (3-11)

movimiento horizontal

$$\frac{m\,d^2(x + l\operatorname{sen}\theta)}{dt^2} = H$$

$$= m\ddot{x} + \frac{m\,d^2(l\operatorname{sen}\theta)}{dt^2}$$

$$= m\ddot{x} + \frac{m\,d(l\cos(\theta)\dot{\theta})}{dt}$$

$$= m\ddot{x} + \frac{m\,l\,d\cos(\theta)\dot{\theta}}{dt}$$

$$= m\ddot{x} + ml[-\operatorname{sen}(\theta)\dot{\theta}\dot{\theta} + \cos(\theta)\ddot{\theta}]$$

$$= m\ddot{x} - ml\operatorname{sen}(\theta)\dot{\theta}^2 + ml\cos\theta \quad (3\text{-}10)$$

Movimiento horizontal del carro

$M\ddot{x} = \mu - H$ (3-12)

Como θ es cercano a cero

$\operatorname{sen}\theta \approx 0$

$\cos\theta = 1$

$\theta\dot{\theta}^2 = 0$

De (3-11) : $I\ddot{\theta} = Vl\theta - Hl$ (3-13)

De (3-10) : $m\ddot{x} + ml\ddot{\theta} = H$

$m(\ddot{x} + l\ddot{\theta}) = H$ (3-14)

movimiento vertical

$$\frac{m\,d^2(l\cos(\theta))}{dt^2} = V - mg \quad (3\text{-}11)$$

De (3-11) : $0 = V - mg$ (3-15)

Remplazando (3-14) en (3-12)

$M\ddot{x} = \mu - m(\ddot{x} + l\ddot{\theta})$

$M\ddot{x} = \mu - m\ddot{x} - ml\ddot{\theta}$

$M\ddot{x} + m\ddot{x} + ml\ddot{\theta} = \mu$

$\ddot{x}(M+m) + ml\ddot{\theta} = \mu$ (3-16)

De (3-13), (3-14) y (3-15)

$I\ddot{\theta} = Vl\theta - Hl$; $H = m(\ddot{x} + l\ddot{\theta})$; $V - mg = 0$

$I\ddot{\theta} = mgl\theta - ml\ddot{x} - ml^2\ddot{\theta}$

$mgl\theta = I\ddot{\theta} + ml\ddot{x} + ml^2\ddot{\theta}$

$mgl\theta = \ddot{\theta}(I + ml^2) + ml\ddot{x}$ (3-17)

De (3-16) : $\ddot{x} = \dfrac{\mu - ml\ddot{\theta}}{M+m}$

Y remplazando en (3-17) :

$$mgl\theta = \ddot{\theta}(I + ml^2) + ml\left(\frac{\mu - ml\ddot{\theta}}{M+m}\right)$$

$$mgl\theta = I\ddot{\theta} + ml^2\ddot{\theta} + \frac{ml\mu}{M+m} - \frac{m^2l^2\ddot{\theta}}{M+m}$$

$$mgl\theta = \frac{I\ddot{\theta}(M+m) - m^2l^2\ddot{\theta}}{M+m} + \frac{ml^2\ddot{\theta}(M+m) + ml\mu}{M+m}$$

$$mgl\theta = \frac{I\ddot{\theta}M + I\ddot{\theta}m - \cancel{m^2l^2\ddot{\theta}} + ml^2\ddot{\theta}M + \cancel{m^2l^2\ddot{\theta}} + ml\mu}{M+m}$$

$$mgl\theta = \frac{\ddot{\theta}(IM + Im + ml^2M) + ml\mu}{M+m}$$

$$\boxed{\frac{(M+m)mgl\theta - ml\mu}{(IM + Im + ml^2M)} = \ddot{\theta}} \quad (1)$$

De (3-16): $\ddot{\theta} = \dfrac{\mu - \ddot{x}(M+m)}{ml}$

Y remplazando en (3-17):

$$mgl\theta = \left(\frac{\mu - \ddot{x}(M+m)}{ml}\right)(I + ml^2) + ml\ddot{x}$$

$$mgl\theta = \frac{I\mu}{ml} - \frac{I(M+m)\ddot{x}}{ml} + \frac{ml^2\mu}{ml} - \frac{ml^2(M+m)\ddot{x}}{ml} + ml\ddot{x}$$

$$mgl\theta = \frac{I\mu + ml^2\mu}{ml} - I(M+m)\ddot{x} - ml^2(M+m)\ddot{x} + ml\ddot{x}$$

$$mgl\theta = \frac{\mu(I + ml^2)}{ml} - \ddot{x}\left(\frac{I(M+m) - ml^2(M+m)}{ml}\right) + ml\ddot{x}$$

$$mgl\theta = \frac{\mu(I + ml^2)}{ml} - \ddot{x}\left(\frac{(M+m)(I + ml^2)}{ml}\right) + ml\ddot{x}$$

$$\boxed{\ddot{x} = -m^2l^2g\theta\left(\frac{1}{I(M+m) + Mml^2}\right) + \mu\left(\frac{I + ml^2}{I(M+m) + Mml^2}\right)} \quad (2)$$

$q_1 = x$ $\qquad$ $q_3 = \theta$

$q_2 = \dot{q}_1 = \dot{x}$ $\qquad$ $q_4 = \dot{q}_3 = \dot{\theta}$

$\dot{q}_2 = \ddot{x}$ $\qquad$ $\dot{q}_4 = \ddot{\theta}$

Reemplazando en (1) y en (2):

$$\dot{q}_2 = \frac{-m^2 l^2 g}{I(M+m) + Mml^2} q_3 + u\left(\frac{I + ml^2}{I(M+m) + Mml^2}\right)$$

$$\dot{q}_4 = \frac{mgl(M+m)}{I(M+m) + Mml^2} q_3 - u\left(\frac{ml}{I(M+m) + Mml^2}\right)$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{m^2 l^2 g}{I(M+m) + Mml^2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{mgl(M+m)}{I(M+m) + Mml^2} & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{I + ml^2}{I(M+m) + Mml^2} \\ 0 \\ \frac{ml}{I(M+m) + Mml^2} \end{bmatrix} u$$

$$\begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$