BeamDyn User's Guide and Theory Manual

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Preface

This document offers a quick reference guide for the BeamDyn software program. It is intended to be used by the general user in combination with the FAST manual. The manual will be updated as new releases are issued and as needed to provide further information on advancements or modifications to the software.

Acknowledgments

The authors are grateful to the U.S. Department of Energy Wind and Water Power Program for supporting the development of this software.

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1 Introduction

BeamDyn is a time-domain structural-dynamics module for slender structures created by the National Renewable Energy Laboratory (NREL) through U.S. Department of Energy Wind and Water Power Program support. The module has been coupled into the FAST aero-hydro-servo-elastic wind turbine multi-physics engineering tool where it used to model blade structural dynamics. BeamDyn is designed to analyze beams that are made of composite materials, initially curved and twisted, and subject to large displacement and rotation deformations. BeamDyn can also be used for static analysis of beams.

The new BeamDyn module follows the requirements of the FAST modularization framework, couples to FAST version 8, and provides new capabilities for modeling initially curved and twisted composite wind turbine blades undergoing large deformation. BeamDyn can also be driven as a stand-alone code to compute the static and dynamic responses of slender structures (blades or otherwise) under prescribed boundary conditions uncoupled from FAST.

BeamDyn is based on the geometrically exact beam theory (GEBT) and is implemented using Legendre spectral finite elements (LSFEs). GEBT supports full geometric nonlinearity and large deflection, with bending, torsion, shear, and extensional degree-of-freedom (DOFs); anisotropic composite material couplings (using full 6×6 mass and stiffness matrices, including bend-twist coupling); and a reference axis that permits blades that are not straight (supporting built-in curve, sweep, and sectional offsets). LFSEs are p-type elements that combine the accuracy of global spectral methods with the geometric modeling flexibility of the h-type finite elements (FEs). For smooth solutions, LSFEs have exponential convergence rates compared to low-order elements that have algebraic convergence. Two spatial numerical integration schemes, including the reduced Gauss quadrature and a trapezoidal over-integration, have been implemented for FE analysis. The trapezoidal scheme is specifically designed for wind turbine blade analysis, where the cross-sectional beam properties may vary dramatically such that a large number of cross-sectional stations along the blade axis is needed. In this scheme, information at all specified stations, including stiffness and mass constants, will be captured such that the common practice of interpolating those to the quadrature points, which is usually lack of sound theoretical foundations, can be avoided. Time integration of the BeamDyn equations of motion is achieved through the implicit generalized-α solver, with user-specified numerical damping. Given the theoretical foundation, powerful numerical tools introduced above, BeamDyn can solve the complicate nonlinear composite beam problem in an efficient manner, for example, finishing the coupled transient dynamic analysis of composite wind turbine blade by a single 5^{th}-order LSFE with dozens of cross-sectional stations.

When coupled with FAST, loads and responses are transferred between BeamDyn, ElastoDyn, ServoDyn, and AeroDyn via the FAST driver program (glue code) to enable aero-elasto-servo interaction at each coupling time step. There is a separate instance of BeamDyn for each blade. At the root node, the six DOFs displacements (three translations and three rotations), velocities, and accelerations are inputs to BeamDyn from ElastoDyn; and the six reaction loads (three force and three moment resultants) at the root of the wind turbine blade are outputs from BeamDyn to ElastoDyn, including the blade-pitch command through ServoDyn. BeamDyn also outputs the local blade displacements, velocities, and accelerations to AeroDyn in order to calculate the local aerodynamic applied loads that become inputs for BeamDyn. In addition, BeamDyn can calculate member internal reaction loads, as requested by the user. Please refers to Figure 1 for the coupled interactions between BeamDyn and other modules in FAST. When uncoupled from FAST, the root motion and applied loads are specified via a stand-alone BeamDyn driver code.

The input file defines the blade geometry, cross-sectional material mass,stiffness, and damping properties, FE resolutions, and other simulation and output control parameters. The blade geometry is defined through a curvilinear blade reference axis by a series of key points in three-dimensional (3D) space along with the initial twist angles at these points. Each member contains at least three key points for the cubic spline fit implemented in BeamDyn; and is considered a LSFE with a parameter defining the order of the element. FE nodes, which are usually not evenly spaced along the element for LSFEs, will be generated by the module based on the mesh information. Blade properties are specified in a non-dimensional coordinate ranging from 0.0 to 1.0 along beam axis and are linear interpolated between two stations if needed by the spatial integration method.

The applied loads to BeamDyn can be either distributed loads at quadrature points or concentrated load at FE nodes

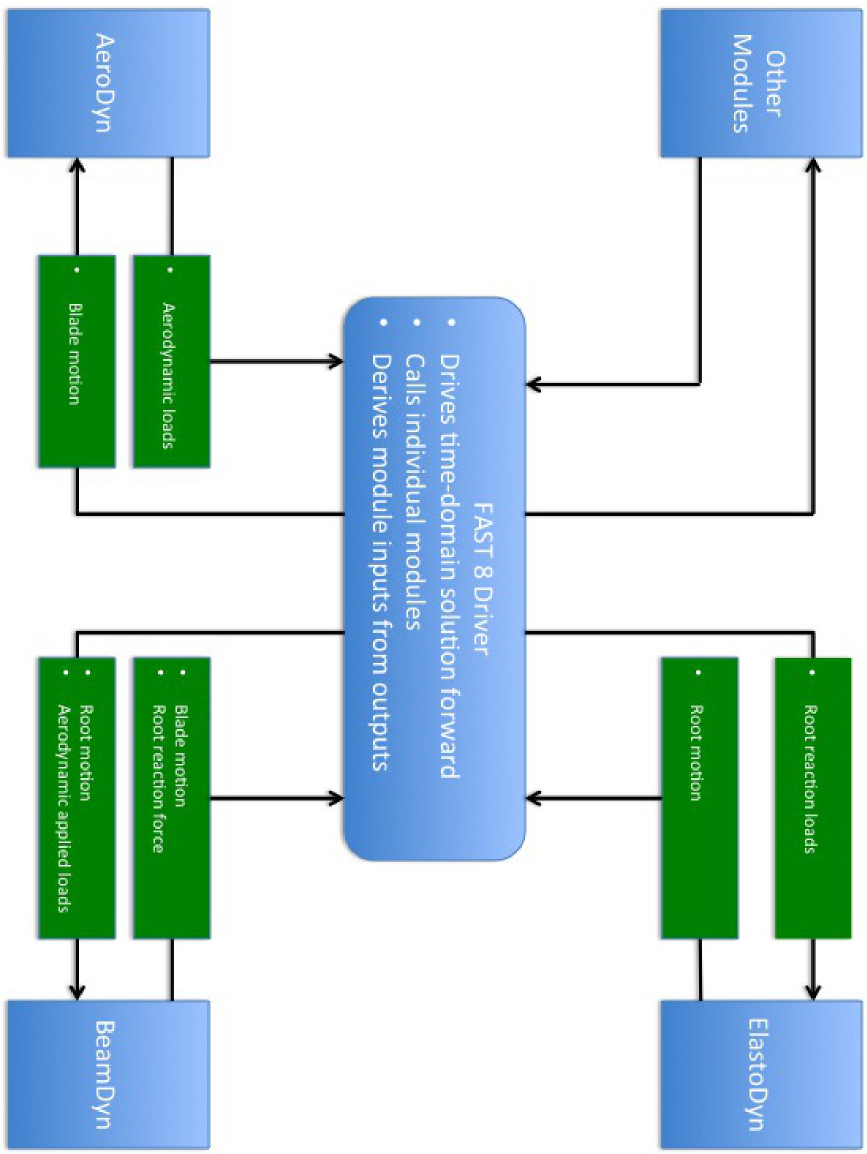


Figure 1. Coupled interaction between BeamDyn and FAST

or a combination of the the two types of loads. When coupled to FAST, the aerodynamic loads computed by AeroDyn are transferred by the glue code at the nodes that are used for quadrature in BeamDyn.

This document is organized as follows. Section 2 details how to obtain the BeamDyn and FAST software archives and run either the stand-alone version of BeamDyn or BeamDyn coupled to FAST. Section 3 describes the BeamDyn input files. Section 4 discusses the output files generated by BeamDyn. Example input files are shown in Appendix A, B, and C. A summary of available output channels is found in Appendix D

2 Running BeamDyn

This section discusses how to obtain and execute BeamDyn from a personal computer. Both the stand-alone version and the FAST-coupled version of the software are considered.

2.1 Downloading the BeamDyn Software

There are two forms of the BeamDyn software to choose from: stand-alone and coupled to the FAST simulator. Alghough the user may not necessarily need both forms, he/she would likely need to be familiar with and run the stand-alone model if building a model of the blade from scratch. The stand-alone version is also helpful for model troubleshooting and may benefit users who are interested in conducting aero-hydro-servo-elastic simulations of onshore/offshore wind turbines. For this reason, BeamDyn can be obtained from two different repositories: one for the stand-alone BeamDyn and one for the coupled solution through FAST.

2.1.1 Stand-Alone BeamDyn Archive

Users can download the stand-alone BeamDyn archive from our Web server at `https://nwtc.nrel.gov/BeamDyn`. The file has a name similar to *BD_v1.00.00a.exe*, but may have a different version number. The user can then download the self-extracting archive (*.exe*) to expand the archive into a folder he/she specifies.

The archive contains the ***bin***, ***CertTest***, ***Compiling***, ***Documentation***, and ***Source*** folders. The ***bin*** folder includes the main executable file, *BeamDyn_win32.exe*, which is used to execute the stand-alone BeamDyn program. The ***CertTest*** folder contains a collection of sample BeamDyn input files and driver input files that can be used as templates for the user's own models. This manila may be found in the ***Documentation*** folder. The ***Compling*** folder contains files for compiling the stand-alone *BeamDyn_v1.00.00.exe* file with either Visual Studio or gFortran. The Fortran source code is located in the ***Source*** folder.

2.1.2 FAST Archive

Download the FAST archive, which includes a coupling to BeamDyn, from our Web server at `https://nwtc.nrel.gov/FAST8`. The file has a name similar to *FAST_v8.12.00.exe*, but may have a different version number. Run the downloaded self-extracting archive (*.exe*) to expand the archive into a user-specified folder. The FAST executable file is located in the archive's *bin* folder. Example models using the NREL 5-MW reference turbine are located in the *CertTest* folder.

2.2 Running BeamDyn

2.2.1 Running the Stand-Alone BeamDyn Program

The stand-alone BeamDyn program, *BeamDyn_v1.00.00.exe*, simulates static and dynamic responses of the user's input model, without coupling to FAST. Unlike the coupled version, the stand-alone software requires the use of a driver file in addition to the primary and blade BeamDyn input files. This driver file specifies inputs normally provided to BeamDyn by FAST, including motions of the blade root and externally applied loads. Both the BeamDyn summary file and the results output file are available when using the stand-alone BeamDyn (see Section 4 for more information regarding the BeamDyn output files).

Run the stand-alone BeamDyn software from a DOS command prompt by typing, for example:

```
>BeamDyn_v1.00.00.exe Dvr_5MW_Dynamic.inp
```

where, *Dvr_5MW_Dynamic.inp* is the name of the BeamDyn driver input file, as described in Section 3.2.

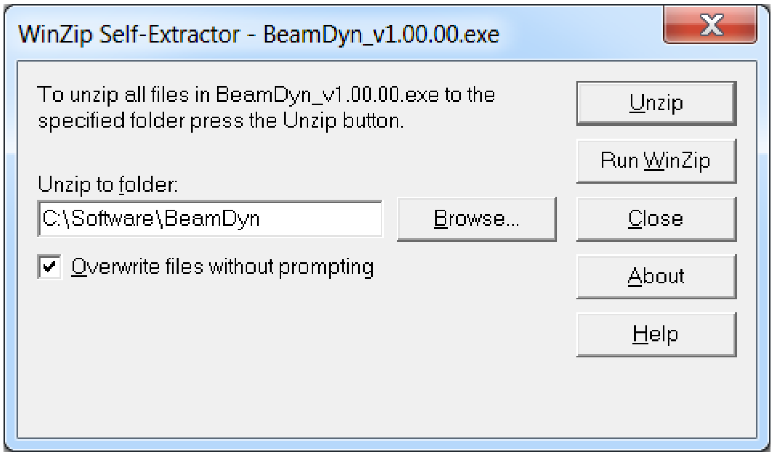


Figure 2. WinZip self-extractor main window

2.2.2 Running BeamDyn Coupled to FAST

Run the coupled FAST software from a DOS command prompt by typing, for example:

```
>FAST_v.8.12.00.exe Test26.fst
```

where *Test26.fst* is the name of the primary FAST input file. This input file has a feature switch to enable or disable the BeamDyn capabilities within FAST, and a corresponding reference to the BeamDyn input file. See the documentation supplied with FAST for further information.

3 Input Files

Users specify the blade model parameters; including its geometry, properties, and FE and output control parameters; via a primary BeamDyn input file and a blade property input file. When used in stand-alone mode, an additional driver input file is required. This driver file specifies inputs normally provided to BeamDyn by FAST, including simulation range, root motions (initial conditions), and externally applied loads.

No lines should be added or removed from the input files, except in tables where the number of rows is specified.

3.1 Units

BeamDyn uses the SI system (kg, m, s, N). Angles are assumed to be in radians unless otherwise specified.

3.2 BeamDyn Driver Input File

The driver input file is only needed for the stand-alone version of BeamDyn and contains inputs that are normally set by FAST, and that are necessary to control the simulation for uncoupled models.

The driver input file begins with two lines of header information, which is for the user but is not used by the software. If BeamDyn run in the stand-alone mode, the results output file will be prefixed with the same name of this driver input file.

A sample BeamDyn driver input file is given in Appendix A

3.2.1 Simulation Control Parameters

t_initial and ***t_final*** specify the starting time of the simulation and ending time of the simulation, respectively. ***dt*** specifies the time step size.

3.2.2 Gravity Parameters

Gx , ***Gy*** , and ***Gz*** specify the components of gravity vector along X, Y, and Z directions in the global coordinate system, respectively. In FAST, this is normally 0, 0, and -9.80665.

3.2.3 Inertial Frame Parameters

This section defines the relation between two inertial frames, the global coordinate system and initial blade reference coordinate system. ***GlbPos(1)*** , ***GlbPos(2)*** , ***GlbPos(3)*** specifies three components of the initial global position vector along X, Y, and Z directions resolved in the global coordinate system, see Figure 3. And the following 3×3 direction cosine matrix (DCM) relates the rotations from global coordinate system to blade coordinate system.

3.2.4 Floating Blade Reference Frame Parameters

This section specifies the parameters that defines the floating blade reference frame, which is a body-attached floating frame and the blade root is cantilevered at the origin of this frame. The floating blade reference fame is assumed to be in a constant rigid-body rotation mode about the origin of the global coordinate system, that is,

$$v_r = \omega_r \times r_t \tag{3.1}$$

where v_r is the root (origin of the floating blade reference frame) translational velocity vector; ω_r is the constant root (origin of the floating blade reference frame) angular velocity vector; and r_t is the global position vector introduced

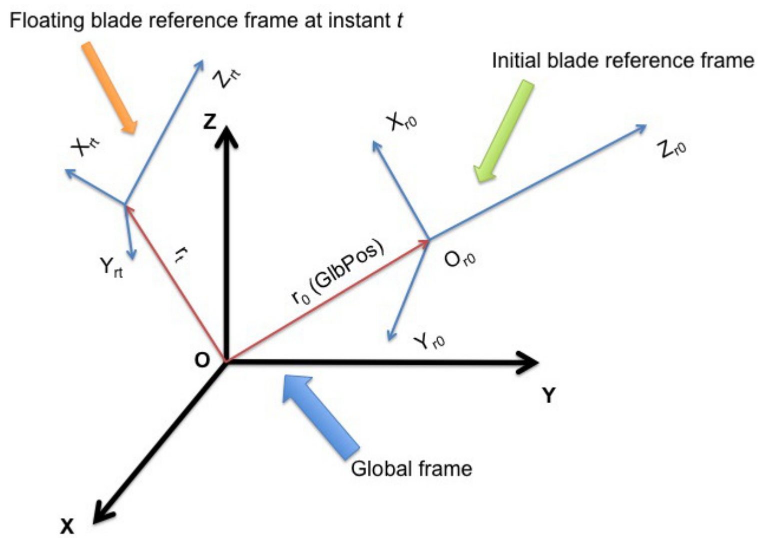


Figure 3. Global and blade coordinate systems in BeamDyn

in the previous section at instant t, see Figure 3. It is pointed out that the floating blade reference frame coincides with the initial floating blade reference frame at the beginning $t = 0$. ***RootVel(4)*** , ***RootVel(5)*** , and ***RootVel(6)*** specify the three components of the root angular velocity vector about X, Y, and Z axises in global coordinate system, respectively. ***RootVel(1)*** , ***RootVel(2)*** , and ***RootVel(3)*** , which are the three components of the root translational velocity vector along X, Y, and Z directions in global coordinate system, respectively, are calculated based on Eq. 3.1. BeamDyn can handle more complicate root motions by change the part, for example, in *BD_InputSolve* subroutine in the Drvier_Beam.f90:

```
u%RootMotion%RotationVel(:,:) = 0.0D0
u%RootMotion%RotationVel(1,1) = IniVelo(5)
u%RootMotion%RotationVel(2,1) = IniVelo(6)
u%RootMotion%RotationVel(3,1) = IniVelo(4)
u%RootMotion%TranslationVel(:,:) = 0.0D0
u%RootMotion%TranslationVel(:,1) = &
MATMUL(BD_Tilde(real(u%RootMotion%RotationVel(:,1),BDKi)),temp_rr)
```

where $IniVelo(5)$, $IniVelo(6)$, and $IniVelo(4)$ are the three components of the root angular velocity vector about X, Y, and Z axises in global coordinate system, respectively; $temp\_rr$ is the global position vector at instant t.

The blade is initialized in the rigid-body motion mode, i.e., based on the root velocity information defined in this section and the position information defined in the previous section, the motion of other points along the blade are initialized as

$$a_{r0} = \omega_r \times (\omega_r \times r_0) \tag{3.2}$$

$$v_0 = v_{r0} + \omega \times P \tag{3.3}$$

$$\omega_0 = \omega_r \tag{3.4}$$

where a_{r0} is the initial root translational acceleration vector; v_0 and ω_0 the initial translational and angular velocity vectors along blade other than the root, respectively; and P is the position vector along the blade relative to the root.

3.2.5 Applied Load

This section defines the applied loads, including distributed and tip concentrated loads, for the analysis. The first six entries ***DistrLoad(i)*** , $i \in [1,6]$, specify three components of uniformly distributed force vector and three components of uniformly distributed moment vector in the global coordinate systems, respectively. The following six entries ***TipLoad(i)*** , $i \in [1,6]$, specify three components of concentrated tip force vector and three components of concentrated tip moment vector in global coordinate system, respectively. The distributed load defined in this section is assumed to be uniform along the blade and constant throughout the simulation; the tip load is a constant concentrated load applied at the tip of a blade.

BeamDyn is capable of handling more complex loading cases, for example, the time-dependent loads, through customizing of the source code (requiring a recompile of stand-alone BeamDyn). The user can define such loads in the *BD_InputSolve* solve in the Driver_Beam.f90 file, which is called every time step. The following section can be modified to define the concentrated load at each FE node:

```
! Define concentrated force vector
u%PointLoad%Force(:,:)   = 0.0D0
! Define concentrated moment vector
u%PointLoad%Moment(:,:) = 0.0D0
```

where the first index in each array ranges from 1 to 3 for load vector components along three directions and the second index of each array ranges from 1 to ***node_total*** , where the latter is the total number of FE nodes. For example, a sinusoidal force along the X direction applied at the 2^{nd} FE node can be defined as

```
! Define concentrated force vector
u%PointLoad%Force(:,:) = 0.0D0
u%PointLoad%Force(1,2)   = 1.0D+03*SIN((2.0*pi)*t/6.0  )
! Define concentrated moment vector
u%PointLoad%Moment(:,:) = 0.0D0
```

with $1.0D+03$ the amplitude and 6.0 the period.

Similar to the concentrated load, the distributed loads can be defined in the same subroutine

```
IF(p%quadrature .EQ. 1) THEN
    DO i=1,p%ngp*p%elem_total+2
        u%DistrLoad%Force(1:3,i) = InitInput%DistrLoad(1:3)
        u%DistrLoad%Moment(1:3,i)= InitInput%DistrLoad(4:6)
    ENDDO
ELSEIF(p%quadrature .EQ. 2) THEN
    DO i=1,p%ngp
        u%DistrLoad%Force(1:3,i) = InitInput%DistrLoad(1:3)
        u%DistrLoad%Moment(1:3,i)= InitInput%DistrLoad(4:6)
    ENDDO
ENDIF
```

where p%ngp is the number of quadrature points; InitInput%DistrLoad(:) is the constant uniformly distributed loads BeamDyn reads from the driver input file, and p%elem_total is the total number of elements. The user can modify "InitInput%DistrLoad(:)" to define the loads based on need.

It is pointed out that the distributed loads are defined at the quadrature points for numerical integrations. For example, if Gauss quadrature is chosen (p%quadratrure .EQ. 1), then the distributed loads are defined at Gauss points plus the two end points of the beam (root and tip).

3.2.6 Primary Input File

InputFile is the file name of the primary BeamDyn input file. This name should be in quotations and can contain an absolute path or a relative path.

3.3 BeamDyn Primary Input File

The BeamDyn primary input file defines the blade geometry, FE and simulation options, output channels, and name of the blade input file. The geometry of the blade is defined by key point coordinates and initial twist angles (in units of degree) in the local blade coordinate system (IEC standard blade system where Z_r is along blade axis from root to tip, X_r directs normally toward the suction side, and Y_r directs normally toward the trailing edge).

The file is organized into several functional sections. Each section corresponds to an aspect of the BeamDyn model.

A sample BeamDyn primary input file is given in Appendix B

The primary input file begins with two lines of header information, which is for the user but is not used by the software.

3.3.1 Simulation Controls

User can set the ***Echo*** flag to "TURE" to have BeamDyn echo the contents of the BeamDyn input file (useful for debugging errors in the input file).

Analysis_Type specifies the type of an analysis. In the current version, there are two options: 1) static analysis, and 2) dynamic analysis. If BeamDyn is run in coupled FAST mode, this entry can be only set to 2, i.e., for dynamic analysis.

rhoinf specifies the numerical damping parameter (spectral radius of the amplification matrix) in the range of $[0.0, 1.0]$ used in the generalized-α time integrator implemented in BeamDyn for dynamic analysis. For ***rhoinf*** $=$ 1.0, no numerical damping is introduced and the generalized-α scheme is identical to the Newmark scheme; for ***rhoinf*** $= 0.0$, maximum numerical damping is introduced to help with convergence.

Quadrature specifies the spatial numerical integration scheme. There are two options: 1) Gauss quadrature; and 2) Trapezoidal quadrature. It is pointed out that in the current version, the Gauss quadrature is implemented in reduced form to improve efficiency and avoid shear locking. In the trapezoidal quadrature, only one member (FE element) can be defined in the following GEOMETRY section of the primary input file.

Refine specifies a refinement parameter used in trapezoidal quadrature. A value greater than unity will split the space between two input stations into Refine number of segments.The keyword "DEFAULT" may be used to set it to 1, i.e., no refinement is needed. This entry is not used in Gauss quadrature.

N_Fact specifies a parameter used in the modified Newton-Ralphson scheme. If ***N_Fact*** $= 1$ a full Newton iteration scheme is used, i.e., the global stiffness matrix is computed and factorized at each iteration; if ***N_Fact*** > 1 a modified Newton iteration scheme is used, i.e., the global stiffness matrix is computed and factorized every ***N_Fact*** iterations within each time step. The keyword "DEFAULT" set ***N_Fact*** $= 5$.

DTBeam specifies the fixed time step of the time-integration in seconds. The keyword "DEFAULT" may be used to indicate that the module should employ the time step prescribed by the driver code (FAST/stand-alone driver program).

NRMax specifies the maximum number of iterations in the Newton-Ralphson scheme. If convergence is not reached within this number of iterations, BeamDyn returns an error message and terminates the simulation. The keyword "DEFAULT" sets ***NRMax*** $= 10$.

Stop_Tol specifies a tolerance parameter used in convergence criteria of a nonlinear solution that is used for the termination of the iteration. The keyword "DEFAULT" sets ***Stop_Tol*** $= 1.0E-05$.

3.3.2 Geometry Parameter

The blade beam model is composed of several *members* in series and each member is defined by at least three key points in BeamDyn. A cubic-spline-fit pre-processor implemented in BeamDyn automatically generates the member based on the key points and then interconnects the members into a blade. There is always a shared key points at adjacent members; therefore the total number of key points is related to number of members and key points in each member.

Member_Total specifies the number of beam members used in the structure.

KP_Total specifies the total number of key points used to define the beam members.

The following section contains ***Member_Total*** lines. Each line has two integers providing the member number (must be 1, 2, 3, etc., sequentially) and the number of key points in this member, respectively. It is noted that the number of key points in each member is not independent of the total number of key points and they should satisfy the following equality:

$$KP_t = \sum_{i=1}^{M_t} n_i - M_t + 1 \tag{3.5}$$

where KP_t is the total number of key points, M_t is the total number of members, and n_i is the number of key points in the i^{th} member. Because cubic splines are implemented in BeamDyn, n_i must be greater than or equal to three. Figure 4 shows two cases for member and key point definition.

The next section defines the key points information. Each key point is defined by three physical coordinates in IEC standard blade coordinate system along with a structural twist angle in the unit of degrees. The structural twist angle is also following the IEC standard which is defined as the twist about the negative Z axis. The key points are entered sequentially and there should be a total of ***KP_Total*** lines for BeamDyn to read in the information, after two header lines.

3.3.3 Mesh Parameter

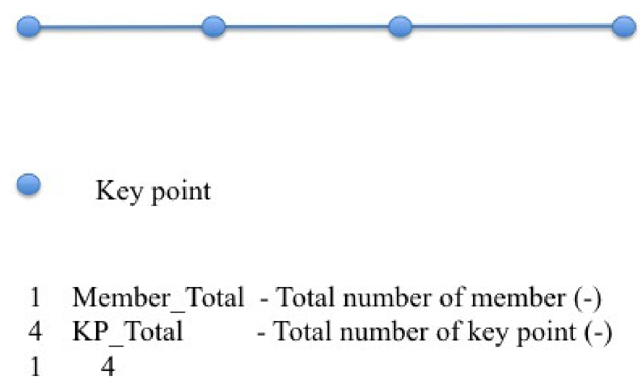
Order_Elem specifies the order of shape functions for each finite element. Because LSFEs are adopted in BeamDyn, it is recommended to refine the mesh by increasing the order of elements (p-type) instead of increasing the number of elements. For Gauss quadrature, ***Order_Elem*** should be greater than one.

3.3.4 Material Parameter

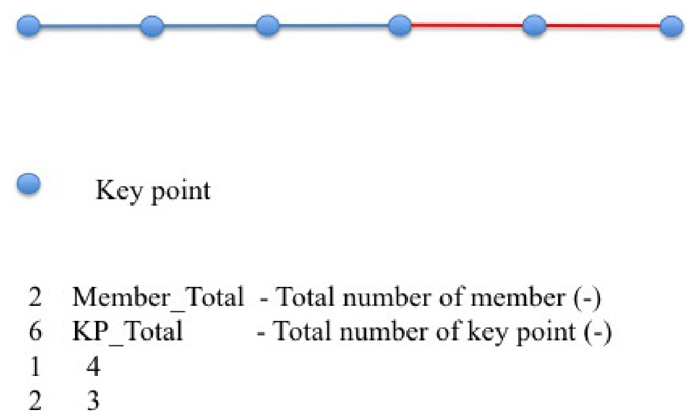
BldFile is the file name of the blade input file. This name should be in quotations and can contain an absolute path or a relative path.

3.3.5 Pitch Actuator Parameter

In this release, the pitch actuator implemented in BeamDyn is not available. The ***UsePitchAct*** should be set to "FALSE" in this version, whereby the input blade-pitch angle prescribed by the driver code is used to orient the blade directly. ***PitchJ***, ***PitchK***, and ***PitchC*** specify the pitch actuator inertial, stiffness, and damping coefficient, respectively. In future releases, specifying ***UsePitchAct*** = TRUE will enable a second-order pitch actuator, whereby the pitch angular orientation, velocity, and acceleration are determined by the actuator based on the input blade-pitch angle prescribed by the driver code.



(a) Case 1



(b) Case 2

Figure 4. Member and key point definition. Case 1: one member defined by four key points; and Case 2: two members defined by six key points.

3.3.6 Outputs

In this section of the primary input file, the user sets flags and switches for the desired output behavior.

Specifying ***SumPrint*** = TRUE causes BeamDyn to generate a summary file with name ***PrimaryInputFile***.*sum*. See Section 4.2 for summary file details.

OutFmt parameter controls the formatting of the results within the stand-alone BeamDyn's output file. It needs to be a valid Fortran format string, but BeamDyn currently does not check the validity. This input is unused when BeamDyn is used coupled to FAST.

NNodeOuts specifies the number of nodes to output to file. Currently, BeamDyn can output quantities at a maximum of nine nodes.

OutNd is a list ***NNodeOuts*** long of node numbers between 1 and $node_total$ (total number of FE nodes), separated by any combination of commas, semicolons, spaces, and/or tabs. The nodal positions are given in the summary file, if output.

The ***OutList*** block contains a list of output parameters. Enter one or more lines containing quoted strings that in turn contain one or more output parameter names. Separate output parameter names by any combination of commas, semicolons, spaces, and/or tabs. If you prefix a parameter name with a minus sign, "-", underscore, "_", or the characters "m" or "M", BeamDyn will multiply the value for that channel by Ð1 before writing the data. The parameters are written in the order they are listed in the input file. BeamDyn allows you to use multiple lines so that you can break your list into meaningful groups and so the lines can be shorter. You may enter comments after the closing quote on any of the lines. Entering a line with the string "END" at the beginning of the line or at the beginning of a quoted string found at the beginning of the line will cause BeamDyn to quit scanning for more lines of channel names. Node-related quantities are generated for the requested nodes identified through the OutNd list above. If BeamDyn encounters an unknown/invalid channel name, it warns the users but will remove the suspect channel from the output file. Please refer to Appendix D for a complete list of possible output parameters and their names.

3.4 Blade Input File

The blade input file defines the cross-sectional properties at various stations along a blade and six damping coefficient for the whole blade. A sample BeamDyn blade input file is given in Appendix C. The blade input file begins with two lines of header information, which is for the user but is not used by the software.

3.4.1 Blade Parameters

Station_Total specifies the number cross-sectional stations along the blade axis used in the analysis.

Damp_Type specifies if structural damping is considered in the analysis. If ***Damp_Type*** = 0, then no damping is considered in the analysis and the six damping coefficient in the next section will be ignored. If ***Damp_Type*** = 1, structural damping will be included in the analysis.

3.4.2 Damping Coefficient

This section specifies six damping coefficients, μ_{ii} with $i \in [1,6]$, for six DOFs (three translations and three rotations). Viscous damping is implemented in BeamDyn where the damping forces are proportional to the strain rate. These are stiffness-proportional damping coefficients, whereby the 6x6 damping matrix at each cross section is scaled from the 6x6 stiffness matrix by these diagonal entries of a 6x6 scaling matrix:

$$\underline{\mathscr{F}}^{Damp} = \underline{\underline{\mu}}\,\underline{\underline{S}}\,\underline{\dot{\epsilon}} \tag{3.6}$$

where $\mathscr{F}^{Damp}$ is the damping force, $\underline{\underline{S}}$ is the 6×6 cross-sectional stiffness matrix, $\dot{\underline{\epsilon}}$ is the strain rate, and $\underline{\underline{\mu}}$ is the damping coefficient matrix defined as

$$\underline{\underline{\mu}} = \begin{bmatrix} \mu_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu_{66} \end{bmatrix} \tag{3.7}$$

3.4.3 Distributed Properties

This section specifies the cross-sectional properties at each of the ***Station_Total*** stations. For each station, a non-dimensional parameter η specifies the station location along the blade axis ranging from $[0.0, 1.0]$. The first and last station parameters must be set to 0.0 and 1.0, respectively.

Following the station location parameter η, there are two 6×6 matrices providing the structural and inertial properties for this cross-section. First is the stiffness matrix and then the mass matrix. It is noted that these matrices are defined in a local coordinate system along the blade axis following the IEC blade coordinate convention with Z_{rl} directing toward the unit tangent vector of the axis. For a cross-section without coupling effects, for example, the stiffness matrix is given as follows:

$$\begin{bmatrix} K_{ShrFlp} & 0 & 0 & 0 & 0 & 0 \\ 0 & K_{ShrEdg} & 0 & 0 & 0 & 0 \\ 0 & 0 & EA & 0 & 0 & 0 \\ 0 & 0 & 0 & EI_{Edg} & 0 & 0 \\ 0 & 0 & 0 & 0 & EI_{Flp} & 0 \\ 0 & 0 & 0 & 0 & 0 & GJ \end{bmatrix} \tag{3.8}$$

where K_{ShrEdg} and K_{ShrFlp} are the edge and flap shear stiffnesses, respectively; EA is the extension stiffness; EI_{Edg} and EI_{Flp} are the edge and flap stiffnesses, respectively; and GJ is the torsional stiffness.

A generalized sectional mass matrix is given by:

$$\begin{bmatrix} m & 0 & 0 & 0 & 0 & -mY_{cm} \\ 0 & m & 0 & 0 & 0 & mX_{cm} \\ 0 & 0 & m & mY_{cm} & -mX_{cm} & 0 \\ 0 & 0 & mY_{cm} & i_{Edg} & -i_{cp} & 0 \\ 0 & 0 & -mX_{cm} & -i_{cp} & i_{Flp} & 0 \\ -mY_{cm} & mX_{cm} & 0 & 0 & 0 & i_{plr} \end{bmatrix} \tag{3.9}$$

where m is the mass density per unit span; X_{cm} and Y_{cm} are the local coordinates of the sectional center of mass, respectively; i_{Edg} and i_{Flp} are the edge and flap mass moments of inertia per unit span, respectively; i_{plr} is the polar moment of inertia per unit span; and i_{cp} is the sectional cross-product of inertia per unit span. It is pointed out that for beam structure, the i_{plr} is given as (although this relationship is not checked by BeamDyn)

$$i_{plr} = i_{Edg} + i_{Flp} \tag{3.10}$$

4 Output Files

BeamDyn produces three types of output files, depending on the options selected: an echo file, a summary file, and a time-series results file. The following sections detail the purpose and contents of these files.

4.1 Echo File

If the user sets the ***Echo*** flag to TRUE in the BeamDyn primary input file, the contents of this file will be echoed to a file with the naming convention ***PrimaryInputFile.ech*** . The echo file is helpful for debugging the input files. The contents of an echo file will be truncated if BeamDyn encounters an error while parsing an input file. The error usually corresponds to the line after the last successfully echoed line.

4.2 Summary File

BeamDyn generates a summary file with the naming convention, ***PrimaryInputFile.sum*** if the ***SumPrint*** parameter is set to TRUE. This file summarizes key information about the simulation, including:

- Blade mass.
- Blade length.
- Blade center of mass.
- Initial global position vector in BD coordinate system.
- Initial global rotation tensor in BD coordinate system.
- Analysis type.
- Numerical damping coefficient.
- Time step size.
- Maximum number of iterations in the Newton-Ralphson solution.
- Convergence parameter in the stopping criterion.
- Factorization frequency in the Newton-Ralphson solution.
- Numerical integration (quadrature) method.
- FE mesh refinement factor used in trapezoidal quadrature.
- Number of elements.
- Number of FE nodes.
- Initial position vectors of FE nodes in BD coordinate system.
- Initial rotation vectors of FE nodes in BD coordinate system.
- Quadrature point position vectors in BD coordinate system. For Gauss quadrature, the physical coordinates of Gauss points are listed. For trapezoidal quadrature, the physical coordinates of the quadrature points are listed.
- Sectional stiffness and mass matrices at quadrature points in local blade reference coordinate system. These are the data being used in calculations at quadrature points and they can be different from the section in Blade Input File since BeamDyn linearly interpolates the sectional properties into quadrature points based on need.
- Initial displacement vectors in BD coordinate system.

Table 1. Transformation between blade coordinate system and BD coordinate system.

Blade Frame	X_{r0}	Y_{r0}	Z_{r0}
BD Frame	x_2	x_3	x_1

- Initial rotation displacement vectors in BD coordinate system.
- Initial translational velocity vectors in BD coordinate system.
- Initial angular velocity vectors in BD coordinate system.
- Requested output information.

All the quantities are outputs in this file in the BD coordinate system, the one being used internally in BeamDyn calculations. The initial blade coordinate system, denoted by a subscript $r0$ that follows the IEC standard, is related to the internal BD coordinate system by Table 1.

4.3 Results File

The BeamDyn time-series results are written to a text-based file with the naming convention ***DriverInputFile.out*** where ***DriverInputFile*** is the name of the driver input file when BeamDyn is run in the stand-alone mode. If BeamDyn is coupled to FAST, then FAST will generate a master results file that includes the BeamDyn results. The results in ***DriverInputFile.out*** are in table format, where each column is a data channel (the first column always being the simulation time), and each row corresponds to a simulation time step. The data channel are specified in the OUTPUT section of the primary input file. The column format of the BeamDyn-generated file is specified using the ***OutFmt*** parameters of the primary input file.

5 BeamDyn Theory

This section focuses on the theory behind the BeamDyn module. The theoretical foundation, numerical tools, and some special handling in the implementation will be introduced.

In this chapter, matrix notation is used to denote vectorial or vectorial-like quantities. For example, an underline denotes a vector $\underline{u}$, a bar denotes unit vector $\bar{n}$, and a double underline denotes a tensor $\underline{\underline{\Delta}}$. Note that sometimes the underlines only denote the dimension of the corresponding matrix.

5.1 Coordinate Systems

Figure 5 shows the coordinate system used in BeamDyn.

5.1.1 Global Coordinate System

The global coordinate system is denoted as X, Y, and Z in Figure 5. This is an inertial frame and in FAST its origin is usually placed at the bottom of the tower as shown.

5.1.2 BD Coordinate System

The BD coordinate system is denoted as x_1, x_2, and x_3 respectively in Figure 5. This is an inertial frame used internally in BeamDyn and its origin is placed at the initial position of the blade root point.

5.1.3 Blade Reference Coordinate System

The blade reference coordinate system is denoted as X_r, Y_r, and Z_r in Figure 5 at instant t. This is a floating frame that attaches at the blade root and is rotating with the blade. Its origin is at the blade root and the directions of axes following the IEC standard, i.e., Z_r is pointing along the blade axis from root to tip; Y_r pointing towards the trailing edge of the blade and parallel with the chord line at the zero-twist blade station; and X_r is orthogonal with the Y_r and Z_r axes, such that they form a right-handed coordinate system. It is noted that the initial blade reference coordinate system, denoted by subscript $r0$, coincides with the BD coordinate system, which is used internally in BeamDyn and introduced in the previous section. The axis convention relations between the initial blade reference coordinate system and the BD coordinate system can be found in Table 1.

5.2 Geometrically Exact Beam Theory

The theoretical foundation of BeamDyn is the geometrically exact beam theory. This theory features the capability of beams that are initially curved and twisted and subjected to large displacement and rotations. Along with a proper two-dimensional (2D) cross-sectional analysis, the coupling effects between all six DOFs, including extension, bending, shear, and torque, can be captured by GEBT as well . The term, “geometrically exact" refer to the fact that there is no approximation made on the geometries, including both initial and deformed geometries, in formulating the equations Hodges (2006).

The governing equations of motion for geometrically exact beam theory can be written as Bauchau (2010)

$$\dot{\underline{h}} - \underline{F}' = \underline{f} \tag{5.1}$$

$$\dot{\underline{g}} + \dot{\tilde{u}}\underline{h} - \underline{M}' + (\tilde{x}_0' + \tilde{u}')^T \underline{F} = \underline{m} \tag{5.2}$$

where $\underline{h}$ and $\underline{g}$ are the linear and angular momenta resolved in the inertial coordinate system, respectively; $\underline{F}$ and $\underline{M}$ are the beam’s sectional force and moment resultants, respectively; $\underline{u}$ is the one-dimensional (1D) displacement of a

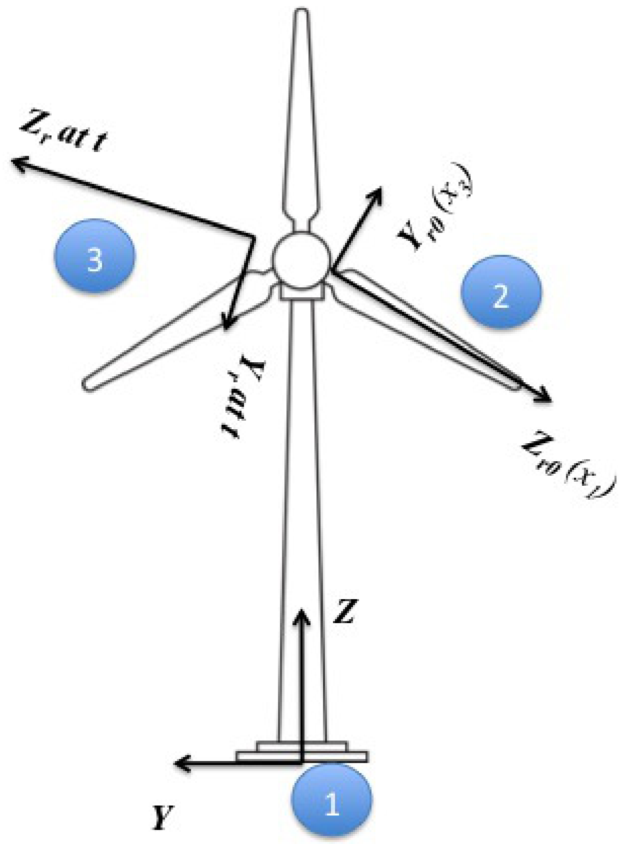


Figure 5. Global, blade, and internal coordinate systems in BeamDyn. Illustration by Al Hicks, NREL

point on the reference line; $\underline{x}_0$ is the position vector of a point along the beam's reference line; and $\underline{f}$ and $\underline{m}$ are the distributed force and moment applied to the beam structure. The notation $(\bullet)'$ indicates a derivative with respect to beam axis x_1 and $(\dot{\bullet})$ indicates a derivative with respect to time. The tilde operator $(\widetilde{\bullet})$ defines a skew-symmetric tensor corresponding to the given vector. In the literature, it is also termed as "cross-product matrix". For example,

$$\widetilde{n} = \begin{bmatrix} 0 & -n_3 & n_2 \\ n_3 & 0 & -n_1 \\ -n_2 & n_1 & 0 \end{bmatrix}$$

The constitutive equations relate the velocities to the momenta and the 1D strain measures to the sectional resultants as

$$\begin{Bmatrix} \underline{h} \\ \underline{g} \end{Bmatrix} = \underline{\underline{\mathscr{M}}} \begin{Bmatrix} \underline{\dot{u}} \\ \underline{\omega} \end{Bmatrix} \tag{5.3}$$

$$\begin{Bmatrix} \underline{F} \\ \underline{M} \end{Bmatrix} = \underline{\underline{\mathscr{C}}} \begin{Bmatrix} \underline{\varepsilon} \\ \underline{\kappa} \end{Bmatrix} \tag{5.4}$$

where $\underline{\underline{\mathscr{M}}}$ and $\underline{\underline{\mathscr{C}}}$ are the 6×6 sectional mass and stiffness matrices, respectively (note that they are not really tensors); $\underline{\varepsilon}$ and $\underline{\kappa}$ are the 1D strains and curvatures, respectively; and, $\underline{\omega}$ is the angular velocity vector that is defined by the rotation tensor $\underline{\underline{R}}$ as $\underline{\omega} = axial(\underline{\underline{\dot{R}}}\,\underline{\underline{R}}^T)$. The axial vector $\underline{a}$ associated with a second-order tensor $\underline{\underline{A}}$ is denoted $\underline{a} = axial(\underline{\underline{A}})$ and its components are defined as

$$\underline{a} = axial(\underline{\underline{A}}) = \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix} = \frac{1}{2} \begin{Bmatrix} A_{32} - A_{23} \\ A_{13} - A_{31} \\ A_{21} - A_{12} \end{Bmatrix} \tag{5.5}$$

The 1D strain measures are defined as

$$\begin{Bmatrix} \underline{\varepsilon} \\ \underline{\kappa} \end{Bmatrix} = \begin{Bmatrix} \underline{x}_0' + \underline{u}' - (\underline{\underline{R}}\,\underline{\underline{R}}_0)\bar{\imath}_1 \\ \underline{k} \end{Bmatrix} \tag{5.6}$$

where $\underline{k} = axial[(\underline{\underline{RR_0}})'(\underline{\underline{RR_0}})^T]$ is the sectional curvature vector resolved in the inertial basis; $\underline{\underline{R}}_0$ is the initial rotation tensor; and $\bar{\imath}_1$ is the unit vector along x_1 direction in the inertial basis. These three sets of equations, including equations of motion Eq. (5.1) and (5.2), constitutive equations Eq. (5.3) and (5.4), and kinematical equations Eq. (5.6), provide a full mathematical description of elasticity problems.

5.3 Numerical Implementation with Legendre Spectral Finite Elements

For a displacement-based finite element implementation, there are six degree-of-freedoms at each node: three displacement components and three rotation components. Here we use $\underline{q}$ to denote the elemental displacement array as $\underline{q} = \begin{bmatrix} \underline{u}^T & \underline{c}^T \end{bmatrix}$ where $\underline{u}$ is the displacement and $\underline{c}$ is the rotation-parameter vector. The acceleration array can thus be defined as $\underline{a} = \begin{bmatrix} \underline{\ddot{u}}^T & \underline{\dot{\omega}}^T \end{bmatrix}$. For nonlinear finite-element analysis, the discretized and incremental forms of displacement, velocity, and acceleration are written as

$$\underline{q}(x_1) = \underline{\underline{N}}\,\underline{\hat{q}} \quad \Delta\underline{q}^T = \begin{bmatrix} \Delta\underline{u}^T & \Delta\underline{c}^T \end{bmatrix} \tag{5.7}$$

$$\underline{v}(x_1) = \underline{\underline{N}}\,\underline{\hat{v}} \quad \Delta\underline{v}^T = \begin{bmatrix} \Delta\underline{\dot{u}}^T & \Delta\underline{\omega}^T \end{bmatrix} \tag{5.8}$$

$$\underline{a}(x_1) = \underline{\underline{N}}\,\underline{\hat{a}} \quad \Delta\underline{a}^T = \begin{bmatrix} \Delta\underline{\ddot{u}}^T & \Delta\underline{\dot{\omega}}^T \end{bmatrix} \tag{5.9}$$

where $\underline{\underline{N}}$ is the shape function matrix and $(\hat{\cdot})$ denotes a column matrix of nodal values.

The displacement fields in an element are approximated as

$$\underline{u}(\xi) = \sum_{k=1}^{p+1} h^k(\xi)\underline{\hat{u}}^k \tag{5.10}$$

$$\underline{u}'(\xi) = \sum_{k=1}^{p+1} h^{k\prime}(\xi)\underline{\hat{u}}^k \tag{5.11}$$

where $h^k(\xi)$, the component of shape function matrix $\underline{\underline{N}}$, is the p^{th}-order polynomial Lagrangian-interpolant shape function of node k, $k = \{1,2,...,p+1\}$, $\underline{\hat{u}}^k$ is the k^{th} nodal value, and $\xi \in [-1,1]$ is the element natural coordinate. However, as discussed in Bauchau et al. (2008), the 3D rotation field cannot simply be interpolated as the displacement field in the form of

$$\underline{c}(\xi) = \sum_{k=1}^{p+1} h^k(\xi)\underline{\hat{c}}^k \tag{5.12}$$

$$\underline{c}'(\xi) = \sum_{k=1}^{p+1} h^{k\prime}(\xi)\underline{\hat{c}}^k \tag{5.13}$$

where $\underline{c}$ is the rotation field in an element and $\underline{\hat{c}}^k$ is the nodal value at the k^{th} node, for three reasons: 1) rotations do not form a linear space so that they must be "composed" rather than added; 2) a rescaling operation is needed to eliminate the singularity existing in the vectorial rotation parameters; 3) the rotation field lacks objectivity, which, as defined by Jelenić and Crisfield (1999), refers to the invariance of strain measures computed through interpolation to the addition of a rigid-body motion. Therefore, we adopt the more robust interpolation approach proposed by Jelenić and Crisfield (1999) to deal with the finite rotations. Our approach is described as follows

Step 1: Compute the nodal relative rotations, $\underline{\hat{r}}^k$, by removing the reference rotation, $\underline{\hat{c}}^1$, from the finite rotation at each node, $\underline{\hat{r}}^k = (\underline{\hat{c}}^{1-}) \oplus \underline{\hat{c}}^k$. It is noted that the minus sign on $\underline{\hat{c}}^1$ denotes that the relative rotation is calculated by removing the reference rotation from each node. The composition in that equation is an equivalent of $\underline{\underline{R}}(\underline{\hat{r}}^k) = \underline{\underline{R}}^T(\underline{\hat{c}}^1)\ \underline{\underline{R}}(\underline{c}^k)$.

Step 2: Interpolate the relative-rotation field: $\underline{r}(\xi) = h^k(\xi)\underline{\hat{r}}^k$ and $\underline{r}'(\xi) = h^{k\prime}(\xi)\underline{\hat{r}}^k$. Find the curvature field $\underline{\kappa}(\xi) = \underline{\underline{R}}(\underline{\hat{c}}^1)\underline{\underline{H}}(\underline{r})\underline{r}'$, where $\underline{\underline{H}}$ is the tangent tensor that relates the curvature vector $\underline{k}$ and rotation vector $\underline{c}$ as

$$\underline{k} = \underline{\underline{H}}\ \underline{c}' \tag{5.14}$$

Step 3: Restore the rigid-body rotation removed in Step 1: $\underline{c}(\xi) = \underline{\hat{c}}^1 \oplus \underline{r}(\xi)$.

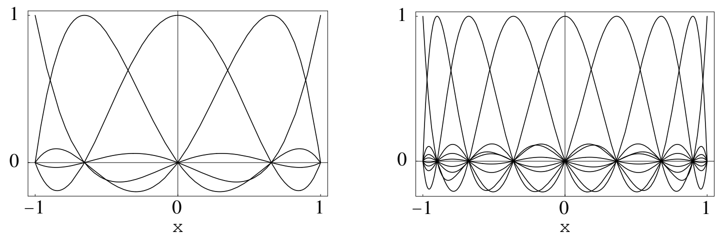
Note that the relative-rotation field can be computed with respect to any of the nodes of the element; we choose node 1 as the reference node for convenience. In the LSFE approach, shape functions (i.e., those composing $\underline{\underline{N}}$) are p^{th}-order Lagrangian interpolants, where nodes are located at the $p+1$ Gauss-Lobatto-Legendre (GLL) points in the $[-1,1]$ element natural-coordinate domain. Figure 6 shows representative LSFE basis functions for fourth- and eighth-order elements. Note that nodes are clustered near element endpoints.

5.4 Wiener-Milenković Rotation Parameter

In BeamDyn, the 3D rotations are represented as Wiener-Milenković parameters defined in the following equation:

$$\underline{c} = 4\tan\left(\frac{\phi}{4}\right)\bar{n} \tag{5.15}$$

where ϕ is the rotation angle and $\bar{n}$ is the unit vector of the rotation axis. It can be observed that the valid range for this parameter is $|\phi| < 2\pi$. The singularities existing at integer multiples of $\pm 2\pi$ can be removed by a rescaling



(a) $p = 4$ (b) $p = 8$

Figure 6. Representative $p+1$ Lagrangian-interpolant shape functions in the element natural coordinates for (a) fourth- and (b) eighth-order LSFEs, where nodes are located at the Gauss-Lobatto-Legendre points.

operation at π as:

$$\underline{r} = \begin{cases} 4(q_0\underline{p} + p_0\underline{q} + \tilde{p}\underline{q})/(\Delta_1 + \Delta_2), & \text{if } \Delta_2 \geq 0 \\ -4(q_0\underline{p} + p_0\underline{q} + \tilde{p}\underline{q})/(\Delta_1 - \Delta_2), & \text{if } \Delta_2 < 0 \end{cases} \tag{5.16}$$

where $\underline{p}$, $\underline{q}$, and $\underline{r}$ are the vectorial parameterization of three finite rotations such that $\underline{\underline{R}}(\underline{r}) = \underline{\underline{R}}(\underline{p})\underline{\underline{R}}(\underline{q})$, $p_0 = 2 - \underline{p}^T\,\underline{p}/8$, $q_0 = 2 - \underline{q}^T\,\underline{q}/8$, $\Delta_1 = (4-p_0)(4-q_0)$, and $\Delta_2 = p_0 q_0 - \underline{p}^T\,\underline{q}$. It is noted that the rescaling operation could cause a discontinuity of the interpolated rotation field; therefore a more robust interpolation algorithm will be introduced in the next section where the rescaling-independent relative-rotation field is interpolated.

The rotation tensor expressed in terms of Wiener-Milenković parameters is

$$\underline{\underline{R}}(\underline{c}) = \frac{1}{(4-c_0)^2} \begin{bmatrix} c_0^2 + c_1^2 - c_2^2 - c_3^2 & 2(c_1c_2 - c_0c_3) & 2(c_1c_3 + c_0c_2) \\ 2(c_1c_2 + c_0c_3) & c_0^2 - c_1^2 + c_2^2 - c_3^2 & 2(c_2c_3 - c_0c_1) \\ 2(c_1c_3 - c_0c_2) & 2(c_2c_3 + c_0c_1) & c_0^2 - c_1^2 - c_2^2 + c_3^2 \end{bmatrix} \tag{5.17}$$

where $\underline{c} = [c_1 \;\; c_2 \;\; c_3]^T$ is the Wiener-Milenković parameter and $c_0 = 2 - \frac{1}{8}\underline{c}^T\,\underline{c}$. The relation between rotation tensor and direction cosine matrix (DCM) is

$$\underline{\underline{R}} = (\underline{\underline{DCM}})^T \tag{5.18}$$

Interested users are referred to Bauchau et al. (2008) and Wang et al. (2013) for more details on the rotation parameter and its implementation in GEBT.

5.5 Linearization Process

The nonlinear governing equations introduced in the previous section are solved by Newton-Ralphson method, where a linearization process is needed. The linearization of each term in the governing equations are presented in this section.

According to Bauchau (2010), the linearized governing equations in Eq. (5.1) and (5.2) are in the form of

$$\underline{\underline{\hat{M}}}\Delta\underline{\hat{a}} + \underline{\underline{\hat{G}}}\Delta\underline{\hat{v}} + \underline{\underline{\hat{K}}}\Delta\underline{\hat{q}} = \underline{\hat{F}}^{ext} - \underline{\hat{F}} \tag{5.19}$$

where the $\underline{\underline{\hat{M}}}$, $\underline{\underline{\hat{G}}}$, and $\underline{\underline{\hat{K}}}$ are the elemental mass, gyroscopic, and stiffness matrices, respectively; $\underline{\hat{F}}$ and $\underline{\hat{F}}^{ext}$ are the elemental forces and externally applied loads, respectively. They are defined for an element of length l along x_1 as follows

$$\underline{\underline{\hat{M}}} = \int_0^l \underline{\underline{N}}^T \underline{\underline{\mathscr{M}}}\, \underline{\underline{N}} dx_1 \tag{5.20}$$

$$\underline{\underline{\hat{G}}} = \int_0^l \underline{\underline{N}}^T \underline{\underline{\mathscr{G}}}^I\, \underline{\underline{N}} dx_1 \tag{5.21}$$

$$\underline{\underline{\hat{K}}} = \int_0^l \left[\underline{\underline{N}}^T (\underline{\underline{\mathscr{K}}}^I + \underline{\underline{\mathscr{Q}}})\, \underline{\underline{N}} + \underline{\underline{N}}^T \underline{\underline{\mathscr{P}}}\, \underline{\underline{N}}' + \underline{\underline{N}}'^T \underline{\underline{\mathscr{C}}}\, \underline{\underline{N}}' + \underline{\underline{N}}'^T \underline{\underline{\mathscr{O}}}\, \underline{\underline{N}}\right] dx_1 \tag{5.22}$$

$$\underline{\hat{F}} = \int_0^l (\underline{\underline{N}}^T \underline{\mathscr{F}}^I + \underline{\underline{N}}^T \underline{\mathscr{F}}^D + \underline{\underline{N}}'^T \underline{\mathscr{F}}^C) dx_1 \tag{5.23}$$

$$\underline{\hat{F}}^{ext} = \int_0^l \underline{\underline{N}}^T \underline{\mathscr{F}}^{ext} dx_1 \tag{5.24}$$

where $\underline{\mathscr{F}}^{ext}$ is the applied load vector. The new matrix notations in Eqs. (5.20) to (5.24) are briefly introduced here. $\underline{\mathscr{F}}^C$ and $\underline{\mathscr{F}}^D$ are elastic forces obtained from Eq. (5.1) and (5.2) as

$$\underline{\mathscr{F}}^C = \begin{Bmatrix} \underline{F} \\ \underline{M} \end{Bmatrix} = \underline{\underline{\mathscr{C}}} \begin{Bmatrix} \underline{\varepsilon} \\ \underline{\kappa} \end{Bmatrix} \tag{5.25}$$

$$\underline{\mathscr{F}}^D = \begin{bmatrix} \underline{\underline{0}} & \underline{\underline{0}} \\ (\tilde{x}_0' + \tilde{u}')^T & \underline{\underline{0}} \end{bmatrix} \underline{\mathscr{F}}^C \equiv \underline{\underline{\Upsilon}}\, \underline{\mathscr{F}}^C \tag{5.26}$$

where $\underline{\underline{0}}$ denotes a 3×3 null matrix. The $\underline{\underline{\mathscr{G}}}^I$, $\underline{\underline{\mathscr{K}}}^I$, $\underline{\underline{\mathscr{O}}}$, $\underline{\underline{\mathscr{P}}}$, $\underline{\underline{\mathscr{Q}}}$, and $\underline{\mathscr{F}}^I$ in Eq. (5.21), Eq. (5.22), and Eq. (5.23) are defined as

$$\underline{\underline{\mathscr{G}}}^I = \begin{bmatrix} \underline{\underline{0}} & (\widetilde{\tilde{\omega} m \underline{\eta}})^T + \tilde{\omega} m \tilde{\eta}^T \\ \underline{\underline{0}} & \tilde{\omega} \underline{\underline{\rho}} - \widetilde{\underline{\underline{\rho}} \underline{\omega}} \end{bmatrix} \tag{5.27}$$

$$\underline{\underline{\mathscr{K}}}^I = \begin{bmatrix} \underline{\underline{0}} & \dot{\tilde{\omega}} m \tilde{\eta}^T + \tilde{\omega}\tilde{\omega} m \tilde{\eta}^T \\ \underline{\underline{0}} & \ddot{\tilde{u}} m \tilde{\eta} + \underline{\underline{\rho}} \dot{\tilde{\omega}} - \widetilde{\underline{\underline{\rho}} \dot{\underline{\omega}}} + \tilde{\omega} \underline{\underline{\rho}} \tilde{\omega} - \tilde{\omega} \widetilde{\underline{\underline{\rho}} \underline{\omega}} \end{bmatrix} \tag{5.28}$$

$$\underline{\underline{\mathscr{O}}} = \begin{bmatrix} \underline{\underline{0}} & \underline{\underline{C}}_{11} \tilde{E}_1 - \tilde{F} \\ \underline{\underline{0}} & \underline{\underline{C}}_{21} \tilde{E}_1 - \tilde{M} \end{bmatrix} \tag{5.29}$$

$$\underline{\underline{\mathscr{P}}} = \begin{bmatrix} \underline{\underline{0}} & \underline{\underline{0}} \\ \tilde{F} + (\underline{\underline{C}}_{11} \tilde{E}_1)^T & (\underline{\underline{C}}_{21} \tilde{E}_1)^T \end{bmatrix} \tag{5.30}$$

$$\underline{\underline{\mathscr{Q}}} = \underline{\underline{\Upsilon}}\, \underline{\underline{\mathscr{O}}} \tag{5.31}$$

$$\underline{\mathscr{F}}^I = \begin{Bmatrix} m \ddot{\underline{u}} + (\dot{\tilde{\omega}} + \tilde{\omega}\tilde{\omega}) m \underline{\eta} \\ m \tilde{\eta} \ddot{\underline{u}} + \underline{\underline{\rho}} \dot{\underline{\omega}} + \tilde{\omega} \underline{\underline{\rho}} \underline{\omega} \end{Bmatrix} \tag{5.32}$$

where m is the mass density per unit length, $\underline{\eta}$ is the location of the sectional center of mass, $\underline{\underline{\rho}}$ is the moment of inertia tensor, and the following notations were introduced to simplify the above expressions

$$\underline{E}_1 = \underline{x}_0' + \underline{u}' \tag{5.33}$$

$$\underline{\underline{\mathscr{C}}} = \begin{bmatrix} \underline{\underline{C}}_{11} & \underline{\underline{C}}_{12} \\ \underline{\underline{C}}_{21} & \underline{\underline{C}}_{22} \end{bmatrix} \tag{5.34}$$

5.6 Damping Forces and Linearization

A viscous damping model has been implemented into BeamDyn to account for the structural damping effect. The damping force is defined as

$$\underline{f}_d = \underline{\underline{\mu}}\, \underline{\underline{\mathscr{C}}} \begin{Bmatrix} \dot{\underline{\varepsilon}} \\ \dot{\underline{\kappa}} \end{Bmatrix} \tag{5.35}$$

where $\underline{\underline{\mu}}$ is a user-defined damping-coefficient diagonal matrix. The damping force can be recast in two separate parts, like $\underline{\mathscr{F}}^C$ and $\underline{\mathscr{F}}^D$ in the elastic force, as

$$\underline{\mathscr{F}}^C_d = \begin{Bmatrix} \underline{F}_d \\ \underline{M}_d \end{Bmatrix} \tag{5.36}$$

$$\underline{\mathscr{F}}^D_d = \begin{Bmatrix} \underline{0} \\ (\tilde{x}_0' + \tilde{u}')^T \underline{F}_d \end{Bmatrix} \tag{5.37}$$

The linearization of the structural damping forces are as follows:

$$\Delta \underline{\mathscr{F}}^C_d = \underline{\underline{\mathscr{S}}}_d \begin{Bmatrix} \Delta \underline{u}' \\ \Delta \underline{c}' \end{Bmatrix} + \underline{\underline{\mathscr{O}}}_d \begin{Bmatrix} \Delta \underline{u} \\ \Delta \underline{c} \end{Bmatrix} + \underline{\underline{\mathscr{G}}}_d \begin{Bmatrix} \Delta \dot{\underline{u}} \\ \Delta \underline{\omega} \end{Bmatrix} + \underline{\underline{\mu}}\, \underline{\underline{C}} \begin{Bmatrix} \Delta \dot{\underline{u}}' \\ \Delta \underline{\omega}' \end{Bmatrix} \tag{5.38}$$

$$\Delta \underline{\mathscr{F}}^D_d = \underline{\underline{\mathscr{P}}}_d \begin{Bmatrix} \Delta \underline{u}' \\ \Delta \underline{c}' \end{Bmatrix} + \underline{\underline{\mathscr{Q}}}_d \begin{Bmatrix} \Delta \underline{u} \\ \Delta \underline{c} \end{Bmatrix} + \underline{\underline{\mathscr{X}}}_d \begin{Bmatrix} \Delta \dot{\underline{u}} \\ \Delta \underline{\omega} \end{Bmatrix} + \underline{\underline{\mathscr{Y}}}_d \begin{Bmatrix} \Delta \dot{\underline{u}}' \\ \Delta \underline{\omega}' \end{Bmatrix} \tag{5.39}$$

5.7 Convergence Criterion and Generalized-α Time Integrator

The system of nonlinear equations in Eqs. (5.1) and (5.2) are solved using the Newton-Raphson method with the linearized form in Eq. (5.19). In the present implementation, an energy-like stopping criterion has been chosen, which is calculated as

$$\|\Delta \mathbf{U}^{(i)T} \left({}^{t+\Delta t}\mathbf{R} - {}^{t+\Delta t}\mathbf{F}^{(i-1)} \right) \| \leq \| \varepsilon_E \left(\Delta \mathbf{U}^{(1)T} \left({}^{t+\Delta t}\mathbf{R} - {}^{t}\mathbf{F} \right) \right) \| \tag{5.40}$$

where $\|\cdot\|$ denotes the Euclidean norm, $\Delta\mathbf{U}$ is the incremental displacement vector, $\mathbf{R}$ is the vector of externally applied nodal point loads, $\mathbf{F}$ is the vector of nodal point forces corresponding to the internal element stresses, and ε_E is the user-defined energy tolerance. The superscript on the left side of a variable denotes the time-step number (in a dynamic analysis), while the one on the right side denotes the Newton-Raphson iteration number. As pointed out by Bathe and Cimento (1980), this criterion provides a measure of when both the displacements and the forces are near their equilibrium values.

Time integration is performed using the generalized-α scheme in BeamDyn, which is an unconditionally stable (for linear systems), second-order accurate algorithm. The scheme allows for users to choose integration parameters that introduce high-frequency numerical dissipation. More details regarding the generalized-α method can be found in Bauchau (2010); Chung and Hulbert (1993).

5.8 Calculation of Reaction Force

A BeamDyn Driver Input File

```
------- BEAMDYN V1.00.00 Driver INPUT FILE ------------------------------------
Dynamic analysis of rotating NREL 5MW blade under gravity force
---------------------- SIMULATION CONTROL -------------------------------------
0          t_initial     - Starting time of simulation (s)
30.0       t_final       - Ending time of simulation   (s)
2E-03      dt            - Time increment size         (s)
---------------------- GRAVITY PARAMETER --------------------------------------
0.0        Gx            - Component of gravity vector along X direction (m/s^2)
-9.8       Gy            - Component of gravity vector along Y direction (m/s^2)
0.0        Gz            - Component of gravity vector along Z direction (m/s^2)
---------------------- FRAME PARAMETER --------------------------------------
0.0        GlbPos(1)     - Component of position vector of the reference blade frame along X direction (m)
0.0        GlbPos(2)     - Component of position vector of the reference blade frame along Y direction (m)
1.0        GlbPos(3)     - Component of position vector of the reference blade frame along Z direction (m)
---The following 3 by 3 matrix is the direction cosine matirx ,GlbDCM(3,3),
---relates global frame to reference blade frame
1.000E+00    0.000E+00   0.000E+00
0.000E+00    1.000E+00   0.000E+00
0.000E+00    0.000E+00   1.000E+00
---------------------- ROOT VELOCITY PARAMETER ----------------------------------
1.0006     RootVel(4)     - Component of angular velocity vector of the beam root about X axis (rad/s)
0.0        RootVel(5)     - Component of angular velocity vector of the beam root about Y axis (rad/s)
0.0        RootVel(6)     - Component of angular velocity vector of the beam root about Z axis (rad/s)
---------------------- APPLIED FORCE ----------------------------------
0.0        DistrLoad(1)  - Component of distributed force vector along X direction (N/m)
0.0        DistrLoad(2)  - Component of distributed force vector along Y direction (N/m)
0.0        DistrLoad(3)  - Component of distributed force vector along Z direction (N/m)
0.0        DistrLoad(4)  - Component of distributed moment vector along X direction (N-m/m)
0.0        DistrLoad(5)  - Component of distributed moment vector along Y direction (N-m/m)
0.0        DistrLoad(6)  - Component of distributed moment vector along Z direction (N-m/m)
0.0        TipLoad(1)    - Component of concentrated force vector at blade tip along X direction (N)
0.0        TipLoad(2)    - Component of concentrated force vector at blade tip along Y direction (N)
0.0        TipLoad(3)    - Component of concentrated force vector at blade tip along Z direction (N)
0.0        TipLoad(4)    - Component of concentrated moment vector at blade tip along X direction (N-m)
0.0        TipLoad(5)    - Component of concentrated moment vector at blade tip along Y direction (N-m)
0.0        TipLoad(6)    - Component of concentrated moment vector at blade tip along Z direction (N-m)
---------------------- PRIMARY INPUT FILE --------------------------------------
"BeamDyn_Input_5MW.inp"    InputFile - Name of the primary input file
```

B BeamDyn Primary Input File: NREL 5-MW Reference Wind Turbine

```
--------- BEAMDYN V1.01.* INPUT FILE -------------------------------------------
NREL 5MW blade primary input file
---------------------- SIMULATION CONTROL --------------------------------------
TRUE     Echo            - Echo input data to "<RootName>.ech" (flag)
2         analysis_type  - 1: Static analysis; 2: Dynamic analysis (switch)
0.0       rhoinf         - Numerical Damping Parameter for Generalized-alpha integrator
2         quadrature     - 1: Gauss; 2: Trapezoidal (switch)
DEFAULT   refine         - Refinement factor for quadrature 2 (-). DEFAULT = 1
DEFAULT   n_fact         - Factorization frequency (-). DEFAULT = 5
DEFAULT   DTBeam         - Time step size (s).
DEFAULT   NRMax          - Max number of iterations in Newton-Ralphson algorithm (-). DEFAULT = 10
DEFAULT   stop_tol       - Tolerance for stopping criterion (-)
---------------------- GEOMETRY PARAMETER --------------------------------------
1     member_total       - Total number of members (-)
49    kp_total           - Total number of key points (-)
1    49                  - Member number; Number of key points in this member
kp_xb kp_yb   kp_zb    initial_twist
(m)    (m)    (m)     (deg)
  0.000000   0.000000   0.000000  13.308000
  0.000000   0.000000   0.199875  13.308000
  0.000000   0.000000   1.199865  13.308000
  0.000000   0.000000   2.199855  13.308000
  0.000000   0.000000   3.199845  13.308000
  0.000000   0.000000   4.199835  13.308000
  0.000000   0.000000   5.199825  13.308000
  0.000000   0.000000   6.199815  13.308000
  0.000000   0.000000   7.199805  13.308000
  0.000000   0.000000   8.201025  13.308000
  0.000000   0.000000   9.199785  13.308000
  0.000000   0.000000  10.199775  13.308000
  0.000000   0.000000  11.199765  13.181000
  0.000000   0.000000  12.199755  12.848000
  0.000000   0.000000  13.200975  12.192000
  0.000000   0.000000  14.199735  11.561000
  0.000000   0.000000  15.199725  11.072000
  0.000000   0.000000  16.199715  10.792000
  0.000000   0.000000  18.200925  10.232000
  0.000000   0.000000  20.200290   9.672000
```

```
  0.000000   0.000000  22.200270   9.110000
  0.000000   0.000000  24.200250   8.534000
  0.000000   0.000000  26.200230   7.932000
  0.000000   0.000000  28.200825   7.321000
  0.000000   0.000000  30.200190   6.711000
  0.000000   0.000000  32.200170   6.122000
  0.000000   0.000000  34.200150   5.546000
  0.000000   0.000000  36.200130   4.971000
  0.000000   0.000000  38.200725   4.401000
  0.000000   0.000000  40.200090   3.834000
  0.000000   0.000000  42.200070   3.332000
  0.000000   0.000000  44.200050   2.890000
  0.000000   0.000000  46.200030   2.503000
  0.000000   0.000000  48.201240   2.116000
  0.000000   0.000000  50.199990   1.730000
  0.000000   0.000000  52.199970   1.342000
  0.000000   0.000000  54.199950   0.954000
  0.000000   0.000000  55.199940   0.760000
  0.000000   0.000000  56.199930   0.574000
  0.000000   0.000000  57.199920   0.404000
  0.000000   0.000000  57.699915   0.319000
  0.000000   0.000000  58.201140   0.253000
  0.000000   0.000000  58.699905   0.216000
  0.000000   0.000000  59.199900   0.178000
  0.000000   0.000000  59.699895   0.140000
  0.000000   0.000000  60.199890   0.101000
  0.000000   0.000000  60.699885   0.062000
  0.000000   0.000000  61.199880   0.023000
  0.000000   0.000000  61.500000   0.000000
---------------------- MESH PARAMETER -----------------------------------------
5         order_elem     - Order of interpolation (basis) function (-)
---------------------- MATERIAL PARAMETER -------------------------------------
"5MW_Blade_IEC.inp"  BldFile - Name of file containing properties for blade (quoted string)
---------------------- PITCH ACTUATOR PARAMETERS -------------------------------
False     UsePitchAct - Whether a pitch actuator should be used (flag)
200       PitchJ      - Pitch actuator inertia (kg-m^2) [used only when UsePitchAct is true]
2.0E+7    PitchK      - Pitch actuator stiffness (kg-m^2/s^2) [used only when UsePitchAct is true]
5.0E+5    PitchC      - Pitch actuator damping (kg-m^2/s) [used only when UsePitchAct is true]
```

```
---------------------- OUTPUTS --------------------------------------------------
True      SumPrint       - Print summary data to "<RootName>.sum" (flag)
"ES10.3E2"    OutFmt          - Format used for text tabular output, excluding the time channel.
      2   NNodeOuts      - Number of nodes to output to file [0 - 9] (-)
 1, 3     OutNd          - Nodes whose values will be output  (-)
          OutList        - The next line(s) contains a list of output parameters.  See
OutListParameters.xlsx.
"RootFxr, RootFyr, RootFzr"
"RootMxr, RootMyr, RootMzr"
"N1Fxl,N1Fyl,N1Fzl"
"N1Mxl,N1Myl,N1Mzl"
"TipTDxr, TipTDyr, TipTDzr"
"TipRDxr, TipRDyr, TipRDzr"
END of input file (the word "END" must appear in the first 3 columns of this last OutList line)
---------------------------------------------------------------------------------
```

C BeamDyn Blade Input File: NREL 5-MW Reference Wind Turbine Blade

```
 ------- BEAMDYN V1.00.* INDIVIDUAL BLADE INPUT FILE --------------------------
NREL 5MW Blade
---------------------- BLADE PARAMETERS --------------------------------------
49   station_total    - Number of blade input stations (-)
 1   damp_flag        - Damping flag: 0: no damping; 1: damped
 ---------------------- DAMPING COEFFICIENT------------------------------------
  mu1        mu2        mu3        mu4        mu5        mu6
  (-)        (-)        (-)        (-)        (-)        (-)
1.0E-03    1.0E-03    1.0E-03    1.0E-03    1.0E-03    1.0E-03
---------------------- DISTRIBUTED PROPERTIES---------------------------------
 0.000000
  9.729480E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
  0.000000E+00    9.729480E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
  0.000000E+00    0.000000E+00    9.729480E+09    0.000000E+00    0.000000E+00    0.000000E+00
  0.000000E+00    0.000000E+00    0.000000E+00    1.811360E+10    0.000000E+00    0.000000E+00
  0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.811000E+10    0.000000E+00
  0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    5.564400E+09

  6.789350E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -1.154189E-01
  0.000000E+00    6.789350E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
  0.000000E+00    0.000000E+00    6.789350E+02    1.154189E-01    0.000000E+00    0.000000E+00
  0.000000E+00    0.000000E+00    1.154189E-01    9.730400E+02    0.000000E+00    0.000000E+00
  0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    9.728600E+02    0.000000E+00
 -1.154189E-01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.945900E+03

 0.003250
  9.729480E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
  0.000000E+00    9.729480E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
  0.000000E+00    0.000000E+00    9.729480E+09    0.000000E+00    0.000000E+00    0.000000E+00
  0.000000E+00    0.000000E+00    0.000000E+00    1.811360E+10    0.000000E+00    0.000000E+00
  0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.811000E+10    0.000000E+00
  0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    5.564400E+09

  6.789350E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -1.154189E-01
  0.000000E+00    6.789350E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
  0.000000E+00    0.000000E+00    6.789350E+02    1.154189E-01    0.000000E+00    0.000000E+00
  0.000000E+00    0.000000E+00    1.154189E-01    9.730400E+02    0.000000E+00    0.000000E+00
  0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    9.728600E+02    0.000000E+00
```

```
-1.154189E-01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.945900E+03

0.019510
 1.078950E+09    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    1.078950E+09    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    1.078950E+10    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    1.955860E+10    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.942490E+10    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    5.431590E+09

 7.733630E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.785695E+01
 0.000000E+00    7.733630E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    7.733630E+02   -1.785695E+01    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00   -1.785695E+01    1.066380E+03    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.091520E+03    0.000000E+00
 1.785695E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    2.157900E+03

0.035770
 1.006723E+09    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    1.006723E+09    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    1.006723E+10    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    1.949780E+10    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.745590E+10    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    4.993980E+09

 7.405500E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -2.547492E+00
 0.000000E+00    7.405500E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    7.405500E+02    2.547492E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    2.547492E+00    1.047360E+03    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    9.660900E+02    0.000000E+00
-2.547492E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    2.013450E+03

0.052030
 9.867780E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    9.867780E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    9.867780E+09    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    1.978880E+10    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.528740E+10    0.000000E+00
```

```
 0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00     4.666590E+09

 7.400420E+02     0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00    -3.215482E+01
 0.000000E+00     7.400420E+02     0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00
 0.000000E+00     0.000000E+00     7.400420E+02     3.215482E+01     0.000000E+00     0.000000E+00
 0.000000E+00     0.000000E+00     3.215482E+01     1.099750E+03     0.000000E+00     0.000000E+00
 0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00     8.738100E+02     0.000000E+00
-3.215482E+01     0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00     1.973560E+03

0.068290
 7.607860E+08     0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00
 0.000000E+00     7.607860E+08     0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00
 0.000000E+00     0.000000E+00     7.607860E+09     0.000000E+00     0.000000E+00     0.000000E+00
 0.000000E+00     0.000000E+00     0.000000E+00     1.485850E+10     0.000000E+00     0.000000E+00
 0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00     1.078240E+10     0.000000E+00
 0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00     3.474710E+09

 5.924960E+02     0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00    -3.491579E+01
 0.000000E+00     5.924960E+02     0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00
 0.000000E+00     0.000000E+00     5.924960E+02     3.491579E+01     0.000000E+00     0.000000E+00
 0.000000E+00     0.000000E+00     3.491579E+01     8.730200E+02     0.000000E+00     0.000000E+00
 0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00     6.485500E+02     0.000000E+00
-3.491579E+01     0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00     1.521570E+03

0.084550
 5.491260E+08     0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00
 0.000000E+00     5.491260E+08     0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00
 0.000000E+00     0.000000E+00     5.491260E+09     0.000000E+00     0.000000E+00     0.000000E+00
 0.000000E+00     0.000000E+00     0.000000E+00     1.022060E+10     0.000000E+00     0.000000E+00
 0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00     7.229720E+09     0.000000E+00
 0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00     2.323540E+09

 4.502750E+02     0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00    -2.924086E+01
 0.000000E+00     4.502750E+02     0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00
 0.000000E+00     0.000000E+00     4.502750E+02     2.924086E+01     0.000000E+00     0.000000E+00
 0.000000E+00     0.000000E+00     2.924086E+01     6.414900E+02     0.000000E+00     0.000000E+00
 0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00     4.567600E+02     0.000000E+00
-2.924086E+01     0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00     1.098250E+03
```

```
0.100810
 4.971300E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    4.971300E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    4.971300E+09    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    9.144700E+09    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    6.309540E+09    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.907870E+09

 4.240540E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -3.272849E+01
 0.000000E+00    4.240540E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    4.240540E+02    3.272849E+01    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    3.272849E+01    5.937300E+02    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    4.005300E+02    0.000000E+00
-3.272849E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    9.942600E+02

0.117070
 4.493950E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    4.493950E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    4.493950E+09    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    8.063160E+09    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    5.528360E+09    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.570360E+09

 4.006380E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -3.362955E+01
 0.000000E+00    4.006380E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    4.006380E+02    3.362955E+01    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    3.362955E+01    5.471800E+02    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    3.516100E+02    0.000000E+00
-3.362955E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    8.987900E+02

0.133350
 4.034800E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    4.034800E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    4.034800E+09    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    6.884440E+09    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    4.980060E+09    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.158260E+09
```

```
 3.820620E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -3.887099E+01
 0.000000E+00    3.820620E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    3.820620E+02    3.887099E+01    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    3.887099E+01    4.908400E+02    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    3.161200E+02    0.000000E+00
-3.887099E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    8.069600E+02

0.149590
 4.037290E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    4.037290E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    4.037290E+09    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    7.009180E+09    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    4.936840E+09    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.002120E+09

 3.996550E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -4.299488E+01
 0.000000E+00    3.996550E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    3.996550E+02    4.299488E+01    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    4.299488E+01    5.038600E+02    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    3.036000E+02    0.000000E+00
-4.299488E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    8.074600E+02

0.165850
 4.169720E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    4.169720E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    4.169720E+09    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    7.167680E+09    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    4.691660E+09    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    8.559000E+08

 4.263210E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -6.748235E+01
 0.000000E+00    4.263210E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    4.263210E+02    6.748235E+01    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    6.748235E+01    5.447000E+02    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    2.892400E+02    0.000000E+00
-6.748235E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    8.339400E+02
```

```
0.182110
 4.082350E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    4.082350E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    4.082350E+09    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    7.271660E+09    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    3.949460E+09    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    6.722700E+08

 4.168200E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -9.267993E+01
 0.000000E+00    4.168200E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    4.168200E+02    9.267993E+01    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    9.267993E+01    5.699000E+02    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    2.465700E+02    0.000000E+00
-9.267993E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    8.164700E+02

0.198370
 4.085970E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    4.085970E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    4.085970E+09    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    7.081700E+09    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    3.386520E+09    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    5.474900E+08

 4.061860E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -1.249266E+02
 0.000000E+00    4.061860E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    4.061860E+02    1.249266E+02    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    1.249266E+02    6.012800E+02    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    2.159100E+02    0.000000E+00
-1.249266E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    8.171900E+02

0.214650
 3.668340E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    3.668340E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    3.668340E+09    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    6.244530E+09    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    2.933740E+09    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    4.488400E+08
```

```
 3.814200E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -1.158983E+02
 0.000000E+00    3.814200E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    3.814200E+02    1.158983E+02    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    1.158983E+02    5.465600E+02    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.871100E+02    0.000000E+00
-1.158983E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    7.336700E+02

0.230890
 3.147760E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    3.147760E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    3.147760E+09    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    5.048960E+09    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    2.568960E+09    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    3.359200E+08

 3.528220E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -9.356487E+01
 0.000000E+00    3.528220E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    3.528220E+02    9.356487E+01    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    9.356487E+01    4.687100E+02    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.608400E+02    0.000000E+00
-9.356487E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    6.295500E+02

0.247150
 3.011580E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    3.011580E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    3.011580E+09    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    4.948490E+09    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    2.388650E+09    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    3.113500E+08

 3.494770E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -9.065783E+01
 0.000000E+00    3.494770E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    3.494770E+02    9.065783E+01    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    9.065783E+01    4.537600E+02    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.485600E+02    0.000000E+00
-9.065783E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    6.023200E+02

0.263410
```

```
 2.882620E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    2.882620E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    2.882620E+09    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    4.808020E+09    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    2.271990E+09    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    2.919400E+08

 3.465380E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -8.665876E+01
 0.000000E+00    3.465380E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    3.465380E+02    8.665876E+01    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    8.665876E+01    4.362200E+02    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.403000E+02    0.000000E+00
-8.665876E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    5.765200E+02

0.295950
 2.613970E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    2.613970E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    2.613970E+09    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    4.501400E+09    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    2.050050E+09    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    2.610000E+08

 3.393330E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -7.857256E+01
 0.000000E+00    3.393330E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    3.393330E+02    7.857256E+01    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    7.857256E+01    3.981800E+02    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.246100E+02    0.000000E+00
-7.857256E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    5.227900E+02

0.328460
 2.357480E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    2.357480E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    2.357480E+09    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    4.244070E+09    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.828250E+09    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    2.288200E+08

 3.300040E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -6.726142E+01
```

```
 0.000000E+00    3.300040E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    3.300040E+02    6.726142E+01    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    6.726142E+01    3.620800E+02    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.094200E+02    0.000000E+00
-6.726142E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    4.715000E+02

0.360980
 2.146860E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    2.146860E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    2.146860E+09    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    3.995280E+09    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.588710E+09    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    2.007500E+08

 3.219900E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -6.418549E+01
 0.000000E+00    3.219900E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    3.219900E+02    6.418549E+01    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    6.418549E+01    3.350100E+02    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    9.436000E+01    0.000000E+00
-6.418549E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    4.293700E+02

0.393500
 1.944090E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    1.944090E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    1.944090E+09    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    3.750760E+09    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.361930E+09    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.743800E+08

 3.138200E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -6.063944E+01
 0.000000E+00    3.138200E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    3.138200E+02    6.063944E+01    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    6.063944E+01    3.085700E+02    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    8.024000E+01    0.000000E+00
-6.063944E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    3.888100E+02

0.426020
 1.632700E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
```

```
 0.000000E+00    1.632700E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    1.632700E+09    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    3.447140E+09    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.102380E+09    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.444700E+08

 2.947340E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -4.419242E+01
 0.000000E+00    2.947340E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    2.947340E+02    4.419242E+01    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    4.419242E+01    2.638700E+02    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    6.267000E+01    0.000000E+00
-4.419242E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    3.265400E+02

0.458550
 1.432400E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    1.432400E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    1.432400E+09    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    3.139070E+09    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    8.758000E+08    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.199800E+08

 2.871200E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -4.427678E+01
 0.000000E+00    2.871200E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    2.871200E+02    4.427678E+01    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    4.427678E+01    2.370600E+02    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    4.942000E+01    0.000000E+00
-4.427678E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    2.864800E+02

0.491060
 1.168760E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    1.168760E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    1.168760E+09    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    2.734240E+09    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    6.813000E+08    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    8.119000E+07

 2.633430E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -3.489821E+01
 0.000000E+00    2.633430E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
```

```
 0.000000E+00    0.000000E+00    2.633430E+02    3.489821E+01    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    3.489821E+01    1.964100E+02    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    3.734000E+01    0.000000E+00
-3.489821E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    2.337500E+02

0.523580
 1.047430E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    1.047430E+08    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    1.047430E+09    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    2.554870E+09    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    5.347200E+08    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    6.909000E+07

 2.532070E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -3.370945E+01
 0.000000E+00    2.532070E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    2.532070E+02    3.370945E+01    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    3.370945E+01    1.803400E+02    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    2.914000E+01    0.000000E+00
-3.370945E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    2.094800E+02

0.556100
 9.229500E+07    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    9.229500E+07    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    9.229500E+08    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    2.334030E+09    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    4.089000E+08    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    5.745000E+07

 2.416660E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -3.391782E+01
 0.000000E+00    2.416660E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    2.416660E+02    3.391782E+01    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    3.391782E+01    1.624300E+02    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    2.216000E+01    0.000000E+00
-3.391782E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.845900E+02

0.588620
 7.608200E+07    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    7.608200E+07    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
```

```
 0.000000E+00    0.000000E+00    7.608200E+08    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    1.828730E+09    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    3.145400E+08    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    4.592000E+07

 2.206380E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -3.077900E+01
 0.000000E+00    2.206380E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    2.206380E+02    3.077900E+01    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    3.077900E+01    1.348300E+02    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.733000E+01    0.000000E+00
-3.077900E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.521600E+02

0.621150
 6.480300E+07    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    6.480300E+07    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    6.480300E+08    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    1.584100E+09    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    2.386300E+08    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    3.598000E+07

 2.002930E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -3.031234E+01
 0.000000E+00    2.002930E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    2.002930E+02    3.031234E+01    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    3.031234E+01    1.163000E+02    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.330000E+01    0.000000E+00
-3.031234E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.296000E+02

0.653660
 5.397000E+07    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    5.397000E+07    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    5.397000E+08    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    1.323360E+09    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.758800E+08    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    2.744000E+07

 1.794040E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -3.124859E+01
 0.000000E+00    1.794040E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    1.794040E+02    3.124859E+01    0.000000E+00    0.000000E+00
```

```
 0.000000E+00   0.000000E+00   3.124859E+01   9.798000E+01   0.000000E+00   0.000000E+00
 0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00   9.960000E+00   0.000000E+00
-3.124859E+01   0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00   1.079400E+02

0.686180
 5.311500E+07   0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00
 0.000000E+00   5.311500E+07   0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00
 0.000000E+00   0.000000E+00   5.311500E+08   0.000000E+00   0.000000E+00   0.000000E+00
 0.000000E+00   0.000000E+00   0.000000E+00   1.183680E+09   0.000000E+00   0.000000E+00
 0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00   1.260100E+08   0.000000E+00
 0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00   2.090000E+07

 1.650940E+02   0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00  -4.114473E+01
 0.000000E+00   1.650940E+02   0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00
 0.000000E+00   0.000000E+00   1.650940E+02   4.114473E+01   0.000000E+00   0.000000E+00
 0.000000E+00   0.000000E+00   4.114473E+01   9.893000E+01   0.000000E+00   0.000000E+00
 0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00   7.300000E+00   0.000000E+00
-4.114473E+01   0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00   1.062300E+02

0.718700
 4.600100E+07   0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00
 0.000000E+00   4.600100E+07   0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00
 0.000000E+00   0.000000E+00   4.600100E+08   0.000000E+00   0.000000E+00   0.000000E+00
 0.000000E+00   0.000000E+00   0.000000E+00   1.020160E+09   0.000000E+00   0.000000E+00
 0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00   1.072600E+08   0.000000E+00
 0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00   1.854000E+07

 1.544110E+02   0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00  -4.018083E+01
 0.000000E+00   1.544110E+02   0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00
 0.000000E+00   0.000000E+00   1.544110E+02   4.018083E+01   0.000000E+00   0.000000E+00
 0.000000E+00   0.000000E+00   4.018083E+01   8.578000E+01   0.000000E+00   0.000000E+00
 0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00   6.220000E+00   0.000000E+00
-4.018083E+01   0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00   9.200000E+01

0.751220
 3.757500E+07   0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00
 0.000000E+00   3.757500E+07   0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00
 0.000000E+00   0.000000E+00   3.757500E+08   0.000000E+00   0.000000E+00   0.000000E+00
```

```
 0.000000E+00    0.000000E+00    0.000000E+00    7.978100E+08    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    9.088000E+07    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.628000E+07

 1.389350E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -3.133540E+01
 0.000000E+00    1.389350E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    1.389350E+02    3.133540E+01    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    3.133540E+01    6.996000E+01    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    5.190000E+00    0.000000E+00
-3.133540E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    7.515000E+01

0.783760
 3.288900E+07    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    3.288900E+07    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    3.288900E+08    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    7.096100E+08    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    7.631000E+07    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.453000E+07

 1.295550E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -2.953206E+01
 0.000000E+00    1.295550E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    1.295550E+02    2.953206E+01    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    2.953206E+01    6.141000E+01    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    4.360000E+00    0.000000E+00
-2.953206E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    6.577000E+01

0.816260
 2.440400E+07    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    2.440400E+07    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    2.440400E+08    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    5.181900E+08    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    6.105000E+07    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    9.070000E+06

 1.072640E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -2.209638E+01
 0.000000E+00    1.072640E+02    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    1.072640E+02    2.209638E+01    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    2.209638E+01    4.544000E+01    0.000000E+00    0.000000E+00
```

```
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    3.360000E+00    0.000000E+00
-2.209638E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    4.880000E+01

0.848780
 2.116000E+07    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    2.116000E+07    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    2.116000E+08    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    4.548700E+08    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    4.948000E+07    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    8.060000E+06

 9.877600E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -2.139686E+01
 0.000000E+00    9.877600E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    9.877600E+01    2.139686E+01    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    2.139686E+01    3.957000E+01    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    2.750000E+00    0.000000E+00
-2.139686E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    4.232000E+01

0.881300
 1.815200E+07    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    1.815200E+07    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    1.815200E+08    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    3.951200E+08    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    3.936000E+07    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    7.080000E+06

 9.024800E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -2.056210E+01
 0.000000E+00    9.024800E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
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 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    2.210000E+00    0.000000E+00
-2.056210E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    3.630000E+01

0.897560
 1.602500E+07    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    1.602500E+07    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    1.602500E+08    0.000000E+00    0.000000E+00    0.000000E+00
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```

```
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    3.467000E+07    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    6.090000E+06

 8.300100E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -1.919315E+01
 0.000000E+00    8.300100E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    8.300100E+01    1.919315E+01    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    1.919315E+01    3.012000E+01    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.930000E+00    0.000000E+00
-1.919315E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    3.205000E+01

0.913820
 1.092300E+07    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    1.092300E+07    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    1.092300E+08    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    3.047300E+08    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    3.041000E+07    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    5.750000E+06

 7.290600E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -1.080904E+01
 0.000000E+00    7.290600E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    7.290600E+01    1.080904E+01    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    1.080904E+01    2.015000E+01    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.690000E+00    0.000000E+00
-1.080904E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    2.184000E+01

0.930080
 1.000800E+07    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    1.000800E+07    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    1.000800E+08    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    2.814200E+08    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    2.652000E+07    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    5.330000E+06

 6.877200E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -1.055375E+01
 0.000000E+00    6.877200E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    6.877200E+01    1.055375E+01    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    1.055375E+01    1.853000E+01    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.490000E+00    0.000000E+00
```

```
-1.055375E+01   0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00   2.002000E+01

0.938210
 9.224000E+06   0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00
 0.000000E+00   9.224000E+06   0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00
 0.000000E+00   0.000000E+00   9.224000E+07   0.000000E+00   0.000000E+00   0.000000E+00
 0.000000E+00   0.000000E+00   0.000000E+00   2.617100E+08   0.000000E+00   0.000000E+00
 0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00   2.384000E+07   0.000000E+00
 0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00   4.940000E+06

 6.626400E+01   0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00  -1.019273E+01
 0.000000E+00   6.626400E+01   0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00
 0.000000E+00   0.000000E+00   6.626400E+01   1.019273E+01   0.000000E+00   0.000000E+00
 0.000000E+00   0.000000E+00   1.019273E+01   1.711000E+01   0.000000E+00   0.000000E+00
 0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00   1.340000E+00   0.000000E+00
-1.019273E+01   0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00   1.845000E+01

0.946360
 6.323000E+06   0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00
 0.000000E+00   6.323000E+06   0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00
 0.000000E+00   0.000000E+00   6.323000E+07   0.000000E+00   0.000000E+00   0.000000E+00
 0.000000E+00   0.000000E+00   0.000000E+00   1.588100E+08   0.000000E+00   0.000000E+00
 0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00   1.963000E+07   0.000000E+00
 0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00   4.240000E+06

 5.934000E+01   0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00  -5.619498E+00
 0.000000E+00   5.934000E+01   0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00
 0.000000E+00   0.000000E+00   5.934000E+01   5.619498E+00   0.000000E+00   0.000000E+00
 0.000000E+00   0.000000E+00   5.619498E+00   1.155000E+01   0.000000E+00   0.000000E+00
 0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00   1.100000E+00   0.000000E+00
-5.619498E+00   0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00   1.265000E+01

0.954470
 5.332000E+06   0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00
 0.000000E+00   5.332000E+06   0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00
 0.000000E+00   0.000000E+00   5.332000E+07   0.000000E+00   0.000000E+00   0.000000E+00
 0.000000E+00   0.000000E+00   0.000000E+00   1.378800E+08   0.000000E+00   0.000000E+00
 0.000000E+00   0.000000E+00   0.000000E+00   0.000000E+00   1.600000E+07   0.000000E+00
```

```
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    3.660000E+06

 5.591400E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -5.042325E+00
 0.000000E+00    5.591400E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    5.591400E+01    5.042325E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    5.042325E+00    9.770000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    8.900000E-01    0.000000E+00
-5.042325E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.066000E+01

0.962600
 4.453000E+06    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    4.453000E+06    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    4.453000E+07    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    1.187900E+08    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.283000E+07    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    3.130000E+06

 5.248400E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -4.493155E+00
 0.000000E+00    5.248400E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    5.248400E+01    4.493155E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    4.493155E+00    8.190000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    7.100000E-01    0.000000E+00
-4.493155E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    8.900000E+00

0.970730
 3.690000E+06    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    3.690000E+06    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    3.690000E+07    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    1.016300E+08    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.008000E+07    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    2.640000E+06

 4.911400E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -3.946310E+00
 0.000000E+00    4.911400E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    4.911400E+01    3.946310E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    3.946310E+00    6.820000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    5.600000E-01    0.000000E+00
-3.946310E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    7.380000E+00
```

```
0.978860
 2.992000E+06    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    2.992000E+06    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    2.992000E+07    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    8.507000E+07    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    7.550000E+06    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    2.170000E+06

 4.581800E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -3.251245E+00
 0.000000E+00    4.581800E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    4.581800E+01    3.251245E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    3.251245E+00    5.570000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    4.200000E-01    0.000000E+00
-3.251245E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    5.990000E+00

0.986990
 2.131000E+06    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    2.131000E+06    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    2.131000E+07    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    6.426000E+07    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    4.600000E+06    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.580000E+06

 4.166900E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -2.260127E+00
 0.000000E+00    4.166900E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    4.166900E+01    2.260127E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    2.260127E+00    4.010000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    2.500000E-01    0.000000E+00
-2.260127E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    4.260000E+00

0.995120
 4.850000E+05    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    4.850000E+05    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    4.850000E+06    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    6.610000E+06    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    2.500000E+05    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    2.500000E+05
```

```
 1.145300E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -6.169731E-01
 0.000000E+00    1.145300E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    1.145300E+01    6.169731E-01    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    6.169731E-01    9.400000E-01    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    4.000000E-02    0.000000E+00
-6.169731E-01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    9.800000E-01

1.000000
 3.530000E+05    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    3.530000E+05    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    3.530000E+06    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    5.010000E+06    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.700000E+05    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    1.900000E+05

 1.031900E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00   -5.346274E-01
 0.000000E+00    1.031900E+01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    1.031900E+01    5.346274E-01    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    5.346274E-01    6.800000E-01    0.000000E+00    0.000000E+00
 0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    2.000000E-02    0.000000E+00
-5.346274E-01    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    7.000000E-01
```

D BeamDyn List of Output Channels

This is a list of all possible output parameters for the BeamDyn module. The names are grouped by meaning, but can be ordered in the OUTPUTS section of the BeamDyn primary input file as the user sees fit. Nβ, refers to output node β, where β is a number in the range [1,9], corresponding to entry β in the ***OutNd*** list. When coupled to FAST, "$B\alpha$ is prefixed to each output name, where α is a number in the range [1,3], corresponding to the blade number. The outputs are expressed in one of the following three coordinate systems:

- r: a floating reference coordinate system fixed to the root of the moving beam; when coupled to FAST for blades, this is equivalent to the IEC blade (b) coordinate system.
- l: a floating coordinate system local to the deflected beam.
- g: the global inertial frame coordinate system; when coupled to FAST, this is equivalent to FAST's global inertial frame (i) coordinate system.

Channel Name(s)	Units	Description
RootFxr, RootFyr, RootFzr	(N), (N), (N)	Root reaction forces expressed in r
RootMxr, RootMyr, RootMzr	(N m), (N m), (N m)	Root reaction moments expressed in r
TipTDxr, TipTDyr, TipTDzr	(m), (m), (m)	Tip translational deflection (relative to the undeflected position) expressed in r
TipRDxr, TipRDyr, TipRDzr	(-), (-), (-)	Tip angular/rotational deflection Wiener-Milenković parameter (relative to the undeflected orientation) expressed in r
TipTVXg, TipTVYg, TipTVZg	(m/s), (m/s), (m/s)	Tip translational velocities (absolute) expressed in g
TipRVXg, TipRVYg, TipRVZg	(deg/s), (deg/s), (deg/s)	Tip angular/rotational velocities (absolute) expressed in g
TipTAXg, TipTAYg, TipTAZg	(m/s^2), (m/s^2), (m/s^2)	Tip translational accelerations (absolute) expressed in g
TipRAXg, TipRAYg, TipRAZg	(deg/s^2), (deg/s^2), (deg/s^2)	Tip angular/rotational accelerations (absolute) expressed in g
NβFxl, NβFyl, NβFzl	(N), (N), (N)	Sectional force resultants at Nβ expressed in l
NβMxl, NβMyl, NβMzl	(N m), (N m), (N m)	Sectional moment resultants at Nβ expressed in l
NβTDxr, NβTDyr, NβTDzr	(m), (m), (m)	Sectional translational deflection (relative to the undeflected position) at Nβ expressed in r
NβRDxr, NβRDyr, NβRDzr	(-), (-), (-)	Sectional angular/rotational deflection Wiener-Milenković parameter (relative to the undeflected orientation) at Nβ expressed in r
NβTVXg, NβTVYg, NβTVZg	(m/s), (m/s), (m/s)	Sectional translational velocities (absolute) at Nβ expressed in g
NβRVXg, NβRVYg, NβRVZg	(deg/s), (deg/s), (deg/s)	Sectional angular/rotational velocities (absolute) at Nβ expressed in g
NβTAXg, NβTAYg, NβTAZg	(m/s^2), (m/s^2), (m/s^2)	Sectional translational accelerations (absolute) at Nβ expressed in g
NβRAXg, NβRAYg, NβRAZg	(deg/s^2), (deg/s^2), (deg/s^2)	Sectional angular/rotational accelerations (absolute) at Nβ expressed in g
NβPFxl, NβPFyl, NβPFzl	(N), (N), (N)	Applied point forces at Nβ expressed in l
NβPMxl, NβPMyl, NβPMzl	(N m), (N m), (N m)	Applied point moments at Nβ expressed in l
NβDFxl, NβDFyl, NβDFzl	(N/m), (N/m), (N/m)	Applied distributed forces at Nβ expressed in l
NβDMxl, NβDMyl, NβDMzl	(N m/m), (N m/m), (N m/m)	Applied distributed moments at Nβ expressed in l

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