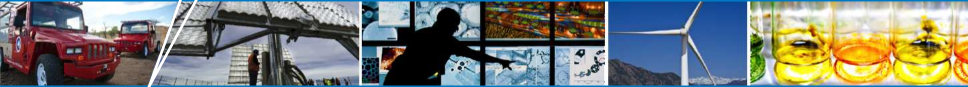
Nonlinear Legendre Spectral Finite Elements for Wind Turbine Blade Dynamics



Qi Wang[1], Michael A. Sprague[1]
Jason Jonkman[1], Nick Johnson[2]

[1]National Renewable Energy Laboratory
[2]Colorado School of Mines

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NREL is a national laboratory of the U.S. Department of Energy, Office of Energy Efficiency and Renewable Energy, operated by the Alliance for Sustainable Energy, LLC.

Motivation

- Beam model currently used in FAST
  - Euler-Bernoulli beam model with shortening effect
  - Two degree-of-freedoms
  - Assumed-mode method
- Beam models used in other wind turbine tools
  - Multibody-formulation
  - Linear beam models
  - Constraints introduced between linear beams to describe large deflections and rotations
  - Finite element method

Objective

- Objective: create efficient high-fidelity beam models for wind turbine blade analysis that can
  - Capture geometrical nonlinearity systematically
  - Capture anisotropic and heterogeneous behavior of composite materials rigorously
  - Modeling moving beams (translation and rotation)
  - Achieve the speed of computational design without significant loss of accuracy comparing to the ultimate accuracy obtained by 3D nonlinear FEA

NWTC static and dynamic blade tests showing typical large, elastic deflections.

Approach

- Implementation
  - Geometrically Exact Beam Theory (GEBT)

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    - Mixed implementation (Yu and Blair, 2012)
  - Legendre Spectral Finite Element (LSFE) (Patera, 1984)

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    - A *p*-version high-order finite element
    - Successfully applied to simulation of fluid dynamics, geophysics, elastodynamics
    - Limited usage in structural dynamics
  - FAST Modularization Framework (Jonkman, 2013)
- Result: BeamDyn, which can be used as a structural module of FAST

GEBT

- Governing Equation

$$\dot{\underline{h}} - \underline{F}' = \underline{f}$$

$$\dot{\underline{g}} + \dot{\tilde{u}}\underline{h} - \underline{M}' - (\tilde{x}_0' + \tilde{u}')\underline{F} = \underline{m}$$

- Constitutive Equation

$$\begin{Bmatrix} \underline{h} \\ \underline{g} \end{Bmatrix} = \underline{\underline{\mathcal{M}}} \begin{Bmatrix} \dot{\underline{u}} \\ \underline{\omega} \end{Bmatrix}$$

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- Timoshenko-like beam model

- Strain Measures

$$\begin{Bmatrix} \underline{\epsilon} \\ \underline{\kappa} \end{Bmatrix} = \begin{Bmatrix} \underline{x}_0' + \underline{u}' - (\underline{\underline{R}}\,\underline{\underline{R}}_0)\bar{\imath}_1 \\ \underline{k} + \underline{\underline{R}}\,\underline{k}_i \end{Bmatrix}$$

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- Geometrically exact: deformed beam geometry is represented exactly

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- Geometrically exact: deformed beam geometry is represented exactly
- Small strains

GEBT (continued)

- Wiener-Milenković Rotation Parameters

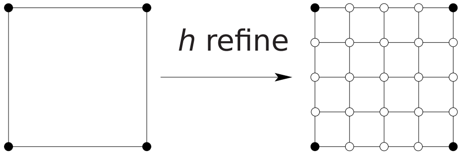
$$\underline{p} = 4\tan\left(\frac{\phi}{4}\right)\bar{n}$$

- Interpolation of 3D Finite Rotation Field (Jelenić and Crisfield, 1999)
  - Rotations do not form a linear space so that they must be “composed” rather than added
  - A rescaling operation is needed to eliminate the singularity existing in the vectorial rotation parameters
  - Rotation field lacks "objectivity"
- Generalized-α Time Integration Scheme
  - Unconditionally stable
  - Second-order accurate
  - High frequency numerical dissipation can be achieved by choosing proper parameters

h-type Finite Elements

FEs in commercial codes (*e.g.*, ABAQUS, ANSYS) typically employ Lagrangian-interpolant shape functions of fixed polynomial order N

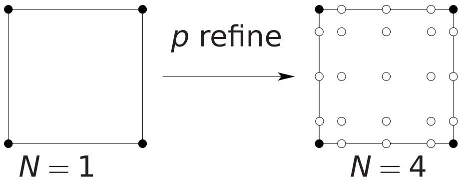
- Typically low order; linear ($N = 1$) or quadratic ($N = 2$)
- Solution improved through h-refinement; increasing number of elements in the domain
- Exhibit *algebraic* convergence for sufficiently smooth problems; error $\sim O(h^{N+1})$



Legendre Spectral Finite Elements

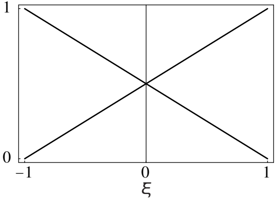
LSFE methods combine the geometric flexibility of the FE method with the accuracy of global spectral methods.

- Solution improved through increased basis polynomial order (*p*-refinement)
- LSFEs employ Lagrangian interpolant shape functions with nodes at Gauss-Lobatto-Legendre (GLL) points
- *Exponential* convergence rates for sufficiently smooth solutions

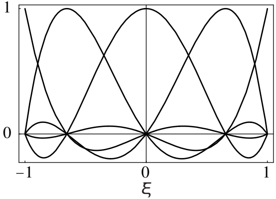


LSFE: Example 1D Basis Functions

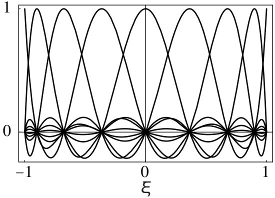
$(N+1)$ LSFE nodes located in $[-1,1]$ at zeros of $(1-\xi^2)P'_N(\xi)$, where $P_N(\xi)$ is the N-degree Legendre polynomial



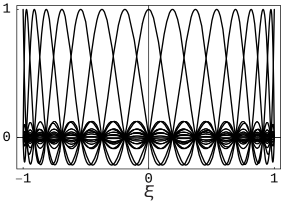
(a) $N=1$



(b) $N=4$

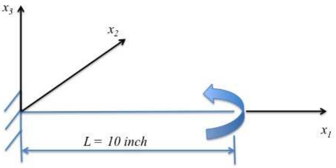


(c) $N=8$



(d) $N=16$

Example 1: A Cantilevered Beam



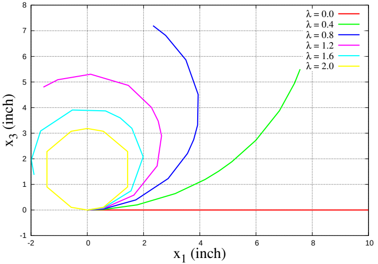
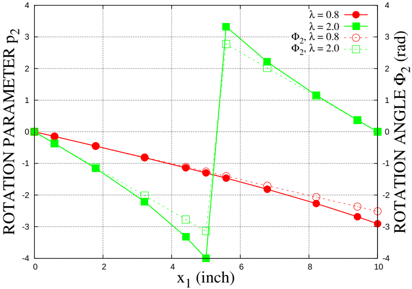


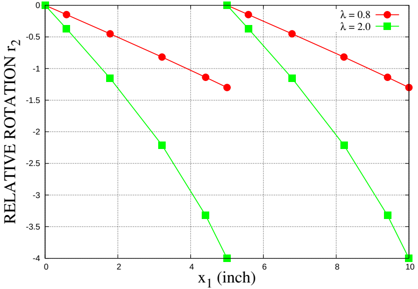
Table: Comparison of analytical and BeamDyn-calculated tip displacements u_1 and u_3 (in inches) of a cantilever beam subjected to a constant moment; the BeamDyn model was composed of two 5^{th}-order LSFEs.

λ	Analytical (u_1)	BeamDyn (u_1)	Analytical (u_3)	BeamDyn (u_3)
0.4	-2.4317	-2.4317	5.4987	5.4987
0.8	-7.6613	-7.6613	7.1978	7.1979
1.2	-11.5591	-11.5591	4.7986	4.7986
1.6	-11.8921	-11.8921	1.3747	1.3747
2.0	-10.0000	-10.0000	0.0000	0.0000

Example 1: 3D Rotations



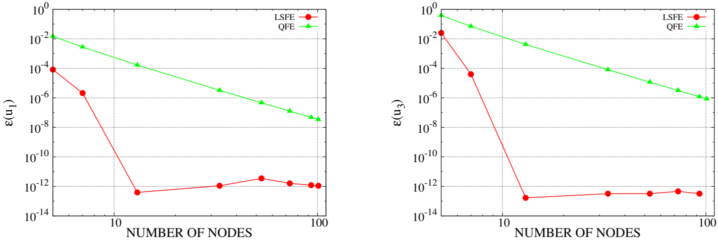
(a) Rotation parameter p_2 and rotation angle ϕ_2



(b) Relative rotation r_2

Figure: (a) Wiener-Milenković rotation parameters and rotation angles along the beam axis x_1 as calculated by BeamDyn for two tip moments; (b) relative rotations in the two elements.

Example 1: Convergence study



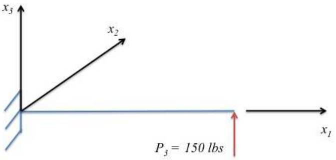
(a) u_1 (b) u_3

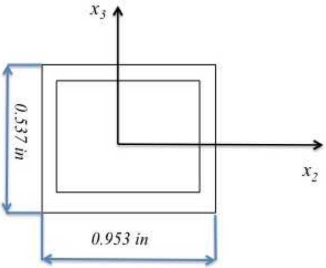
Figure: Normalized error of the (a) u_1 and (b) u_3 tip displacements of a cantilever beam under constant tip moment ($\lambda = 1.0$) as a function of the total number of nodes. Results were calculated with BeamDyn (LSFE) and Dymore (QFE).

- Dymore is a well-known FE based multibody dynamics code for the comprehensive modeling of flexible multibody systems
- Conventional quadratic elements are limited to algebraic convergence.
- LSFEs (with *p*-refinement) exhibit highly desirable exponential convergence to machine-precision error.
- The total numbers of iterations in Newton-Raphson method are the same.

Example 2: Composite beam

- Sketch of Example 2



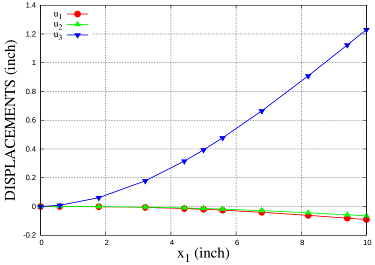


- Sectional stiffness matrix

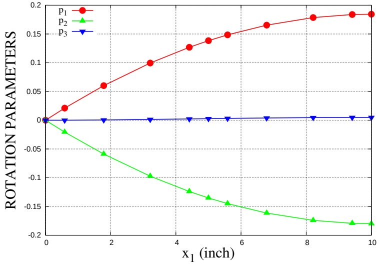
$$C^* = 10^3 \times \begin{bmatrix} 1368.17 & 0 & 0 & 0 & 0 & 0 \\ 0 & 88.56 & 0 & 0 & 0 & 0 \\ 0 & 0 & 38.78 & 0 & 0 & 0 \\ 0 & 0 & 0 & 16.96 & 17.61 & -0.351 \\ 0 & 0 & 0 & 17.61 & 59.12 & -0.370 \\ 0 & 0 & 0 & -0.351 & -0.370 & 141.47 \end{bmatrix}$$

- A composite boxed-beam.
- Units of C^*_{ij} (lb), $C^*_{i,j+3}$ (lb.in), and $C^*_{i+3,j+3}$ (lb.in^2) for $i, j = 1, 2, 3$; these units are adopted for consistency with those used in (Yu et al., 2002).

Example 2: Results



(a) *Displacements*



(b) *Rotations*

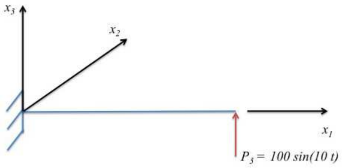
Figure: Displacements and rotation parameters along beam axis for Example 2.

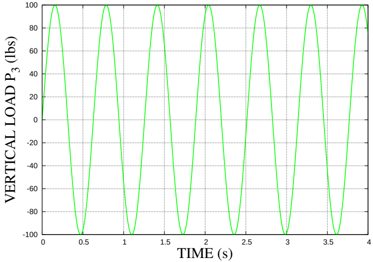
Table: Numerically determined tip displacements and rotation parameters of a composite beam in Example 2 as calculated by BeamDyn (LSFE) and Dymore (QFE)

	u_1 (inch)	u_2(inch)	u_3(inch)	p_1	p_2	p_3
BeamDyn	-0.09064	-0.06484	1.22998	0.18445	-0.17985	0.00488
Dymore	-0.09064	-0.06483	1.22999	0.18443	-0.17985	0.00488

Example 3: A composite beam - Dynamics

- Sketch of Example 3





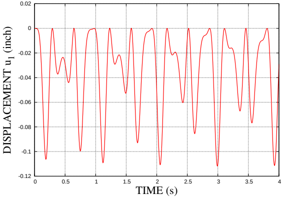
- Sectional mass matrix

$$M^* = 10^{-2} \times \begin{bmatrix} 8.538 & 0 & 0 & 0 & 0 & 0 \\ 0 & 8.538 & 0 & 0 & 0 & 0 \\ 0 & 0 & 8.538 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.4433 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.40972 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0336 \end{bmatrix}$$

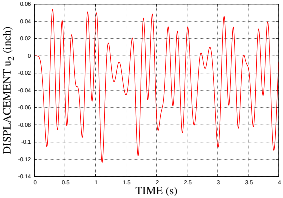
- A composite boxed-beam under applied sinusoidal force
- The units associated with the mass matrix values are M^*_{ii} (lb.s^2/in^2) and $M^*_{i+3,i+3}$ (lb.s^2) for $i = 1, 2, 3$.

Example 3: Tip displacement field

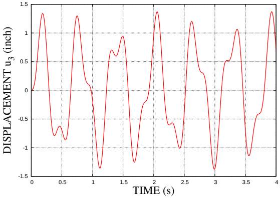
- Displacement u_1



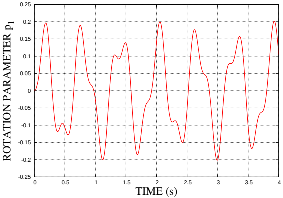
- Displacement u_2



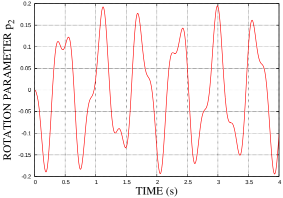
- Displacement u_3



- Rotation p_1



- Rotation p_2



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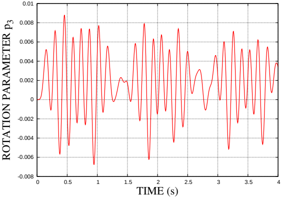
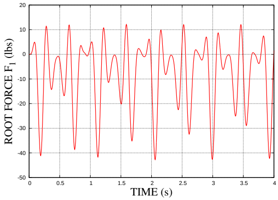
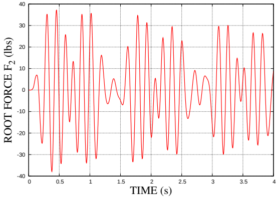


Example 3: Root forces and moments

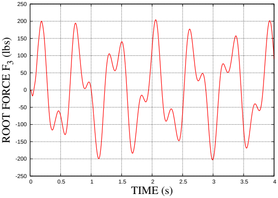
- Force F_1



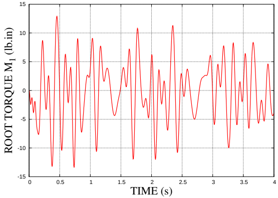
- Force F_2



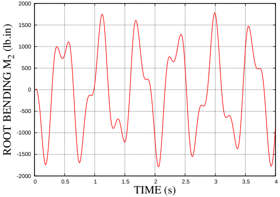
- Force F_3



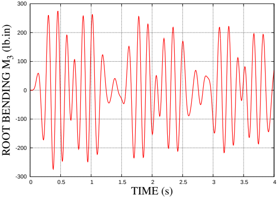
- Torque M_1



- Moment M_2



- Moment M_3



Example 3: Convergence study

- Normalized Root-Mean-Square (RMS) error

$$\varepsilon_{\mathrm{RMS}}(u_1) = \sqrt{\frac{\sum_{k=0}^{n_{max}} \left[u_1^k - u_b(t^k)\right]^2}{\sum_{k=0}^{n_{max}} \left[u_b(t^k)\right]^2}}$$

  - $u_b(t)$ is the benchmark solution: a highly resolved numerical solution obtained by BeamDyn with one 20^{th}-order element
  - Time step size: $\Delta t_b = 1.0 \times 10^{-4}$ s
- Test calculations
  - Case 1: $\Delta t_1 = 5.0 \times 10^{-3}$ s
  - Case 2: $\Delta t_2 = \frac{\Delta t_1}{2} = 2.5 \times 10^{-3}$ s

Example 3: Convergence study (continued)

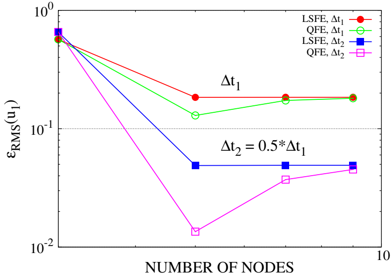


Figure: Normalized RMS error of tip displacement u_1 histories over $0 \leq t \leq 4$ as a function of number of nodes as calculated by BeamDyn (LSFEs) and Dymore (QFEs).

- For a fixed Δt, both Dymore (QFEs) and BeamDyn (LSFEs) converge with spatial refinement to the same error level. BeamDyn is converged with only five nodes, whereas Dymore requires at least nine nodes.
- The converged error levels are due exclusively to time-discretization error. We note that the converged error for $\Delta t_2 = \Delta t_1/2$ is one-fourth that for Δt_1, which is expected for the second-order-accurate time integrator.

Summary

- Conclusion
  - Implemented Legendre spectral finite elements based on geometrically exact beam theory
  - Exponential convergence rates observed
  - Verified against analytical solution, numerical solution for composite beams with elastic couplings
- Future Work
  - Extend to model moving beams: translation and rotation
  - Integrate into FAST for wind turbine analysis
  - Full-blade verification and validation

Questions?

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