PRACTICAL LESSON

Lec_02

Problem-1 $T(n) = 3T(n/2) + n^2$

Solution: $T(n) = 3T(n/2) + n^2 => T(n) = \Theta(n^2)$ (Master Theorem Case 3.a)

Problem-2 $T(n) = 4T(n/2) + n^2$

Solution: $T(n) = 4T(n/2) + n^2 => T(n) = \Theta(n^2 \log n)$ (Master Theorem Case 2.a)

Problem-3 $T(n) = T(n/2) + n^2$

Solution: $T(n) = T(n/2) + n^2 => \Theta(n^2)$ (Master Theorem Case 3.a)

Problem-4 $T(n) = 2^n T(n/2) + n^n$

Solution: $T(n) = 2^n T(n/2) + n^n =>$ Does not apply (a is not constant)

Problem-5 $T(n) = 16T(n/4) + n$

Solution: $T(n) = 16T(n/4) + n => T(n) = \Theta(n^2)$ (Master Theorem Case 1)

Problem-6 $T(n) = 2T(n/2) + n\log n$

Solution: $T(n) = 2T(n/2) + n\log n => T(n) = \Theta(n\log^2 n)$ (Master Theorem Case 2.a)

Problem-7 $T(n) = 2T(n/2) + n/\log n$

Solution: $T(n) = 2T(n/2) + n/\log n => T(n) = \Theta(n\log\log n)$ (Master Theorem Case 2. b)

Problem-8 $T(n) = 2T(n/4) + n^{051}$

Solution: $T(n) = 2T(n/4) + n^{051} => T(n) = \Theta(n^{0.51})$ (Master Theorem Case 3.b)

Problem-9 $T(n) = 0.5T(n/2) + 1/n$

Solution: $T(n) = 0.5T(n/2) + 1/n =>$ Does not apply (a < 1)

Problem-10 $T(n) = 6T(n/3) + n^2 \log n$

Solution: $T(n) = 6T(n/3) + n^2 \log n => T(n) = \Theta(n^2 \log n)$ (Master Theorem Case 3.a)

Problem-11 $T(n) = 64T(n/8) - n^2 logn$

Solution: $T(n) = 64T(n/8) - n^2 logn$ => Does not apply (function is not positive)

Problem-12 $T(n) = 7T(n/3) + n^2$

Solution: $T(n) = 7T(n/3) + n^2 => T(n) = \Theta(n^2)$ (Master Theorem Case 3.as)

Problem-13 $T(n) = 4T(n/2) + logn$

Solution: $T(n) = 4T(n/2) + logn => T(n) = \Theta(n^2)$ (Master Theorem Case 1)

Problem-14 $T(n) = 16T(n/4) + n!$

Solution: $T(n) = 16T(n/4) + n! => T(n) = \Theta(n!)$ (Master Theorem Case 3.a)

Problem-15 $T(n) = \sqrt{2}\,T(n/2) + logn$

Solution: $T(n) = \sqrt{2}\,T(n/2) + logn => T(n) = \Theta(\sqrt{n})$ (Master Theorem Case 1)

Problem-16 $T(n) = 3T(n/2) + n$

Solution: $T(n) = 3T(n/2) + n => T(n) = \Theta(n^{log3})$ (Master Theorem Case 1)

Problem-17 $T(n) = 3T(n/3) + \sqrt{n}$

Solution: $T(n) = 3T(n/3) + \sqrt{n} => T(n) = \Theta(n)$ (Master Theorem Case 1)

Problem-18 $T(n) = 4T(n/2) + cn$

Solution: $T(n) = 4T(n/2) + cn => T(n) = \Theta(n^2)$ (Master Theorem Case 1)

Problem-19 $T(n) = 3T(n/4) + nlogn$

Solution: $T(n) = 3T(n/4) + nlogn => T(n) = \Theta(nlogn)$ (Master Theorem Case 3.a)

Problem-20 $T(n) = 3T(n/3) + n/2$

Solution: $T(n) = 3T(n/3) + n/2 => T(n) = \Theta(nlogn)$ (Master Theorem Case 2.a)

Problem-21 Find the complexity of the below recurrence:

$$T(n) = \begin{cases} 3T(n-1), & if \ n > 0, \\ 1, & otherwise \end{cases}$$

Solution: Let us try solving this function with substitution.

$$T(n) = 3T(n-1)$$

$$T(n) = 3(3T(n-2)) = 3^2 T(n-2)$$

$$T(n) = 3^2(3T(n-3))$$

Problem-22 Find the complexity of the below recurrence:

$$T(n) = \begin{cases} 2T(n-1) - 1, & if \ n > 0, \\ 1, & otherwise \end{cases}$$

Solution: Let us try solving this function with substitution.

$$T(n) = 2T(n-1) - 1$$

$$T(n) = 2(2T(n-2) - 1) - 1 = 2^2 T(n-2) - 2 - 1$$

$$T(n) = 2^2(2T(n-3) - 2 - 1) - 1 = 2^3 T(n-4) - 2^2 - 2^1 - 2^0$$

$$T(n) = 2^n T(n-n) - 2^{n-1} - 2^{n-2} - 2^{n-3} \ .... \ 2^2 - 2^1 - 2^0$$

$$T(n) = 2^n - 2^{n-1} - 2^{n-2} - 2^{n-3} \ .... \ 2^2 - 2^1 - 2^0$$

$$T(n) = 2^n - (2^n - 1) \ [note: \ 2^{n-1} + 2^{n-2} + \cdots + 2^0 = 2^n]$$

$$T(n) = 1$$

$\therefore$ Time Complexity is $O(1)$. Note that while the recurrence relation looks exponential, the solution to the recurrence relation here gives a different result.