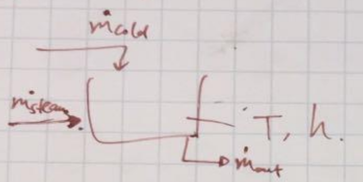
In this part of experiment, A MPC controller was designed with certain constraints and optimization criteria for one-tank system AND controlled response of the discretized model for certain setpoint changes was simulated.

The **linearized discrete time state space representation** is derived as shown below.

1 tank system:



$$u = \begin{bmatrix} \dot{m}_{cold} \\ \dot{m}_{steam} \end{bmatrix}.$$

$$\underbrace{\begin{bmatrix} \dot{h} \\ \dot{T} \end{bmatrix}}_{2\times1} = \underbrace{A}_{2\times2} \begin{bmatrix} h \\ T \end{bmatrix} + \underbrace{B}_{2\times2} \begin{bmatrix} \dot{m}_{cold} \\ \dot{m}_{steam} \end{bmatrix}.$$

MB: known: S.S.: $h_{ss} = 0.22$, $\dot{m}_{c,\,s.s.} = 0.12\ \frac{kg}{s}$ (from data).

$\boxed{\text{Derive}}$:

$$\frac{dh}{dt} = -\frac{R}{\rho A_c} \cdot h^{\frac{1}{2}} + \frac{1}{\rho A_c} \cdot \dot{m}_c \quad ①$$

$$0 = -\frac{R}{\rho A_c} \cdot h^{\frac{1}{2}} + \frac{1}{\rho A_c} \cdot \dot{m}_c$$

$$\therefore \boxed{R = \frac{\dot{m}_c}{h_{ss}^{\frac{1}{2}}} = 0.2558}$$

(from data)

EB: known s.s: $T_{css} = 22.8°C$, $\dot{m}_{steam,\,ss} = \frac{18.7}{3600}\ kg/s$. $\boxed{T_{ss} = 25.7\ °C}$

$$\frac{dT}{dt} = \frac{1}{\rho A_c} \cdot \dot{m}_c T_c \cdot h^{-1} + \frac{\Delta H \cdot F}{\rho A_c \cdot C_p} \cdot \dot{m}_{steam} \cdot h^{-1} - \frac{1}{\rho A_c} \cdot (\dot{m}_c \cdot T \cdot h^{-1}). \quad ②$$

S.S. $0 = (\ -\ -\ \ )$

$$\boxed{F = \frac{m_{css}(T_{ss} - T_{css}) \cdot C_p}{m_{steam_{ss}} \cdot \Delta H} = 0.1247.}$$

Linearization: $h' = h - h_{ss}$; $T' = T - T_{ss}$.

$$\frac{dh'}{dt} = \Bigg[\underbrace{\left(-\frac{k}{\rho A_c}\right) \times 0.5 \times h_{ss}^{-0.5}}_{a_{11}}\Bigg] \times h' + \underbrace{\left[\frac{1}{\rho A_c}\right]}_{b_{11}} \times \dot{m}_c'$$

$$\frac{dT'}{dt} = \underbrace{\Bigg[\left(\frac{1}{\rho A_c} \times \dot{m}_{css} \times T_{css}\right) \cdot (-1) \times h_{ss}^{-2} + \left(\frac{\Delta H \cdot F}{\rho A_c \cdot C_p} \times \dot{m}_{steam\,ss}\right) \times (-1) \times h_{ss}^{-2} + \left(-\frac{1}{\rho A_c} \times \dot{m}_{c\,ss} \cdot T_{ss}\right) \times (-1) \times h_{ss}^{-2}\Bigg]}_{a_{21}} \cdot h'$$

$$+ \underbrace{\left[\left(-\frac{1}{\rho A_c} \cdot \dot{m}_{css} \cdot h_{ss}^{-1}\right)\right]}_{a_{22}} T'$$

$$+ \underbrace{\left[\left(\frac{1}{\rho A_c} \times T_{css} \times h_{ss}^{-1}\right) + \left(-\frac{1}{\rho A_c} \times T_{ss} \times h_{ss}^{-1}\right)\right]}_{b_{21}} \times \dot{m}_c'$$

$$+ \underbrace{\left[\left(\frac{\Delta H \cdot F}{\rho A_c \cdot C_p} \times h_{ss}^{-1}\right)\right]}_{b_{22}} \times \dot{m}_{steam}'$$

The response of discretized model for the set point changes (level +0.05 m, temperature + 5 C) is shown as the following figures. From the figure, it is illustrated that the response of the system to the set point change finally stabilizes to steady state value. Although for steam flow rate, deviation does not go back to 0, it goes back and stabilizes at 10^-5 magnitude which is close enough to 0. Consequently, the conclusion can be made that the MPC is well-designed for this one tank system.

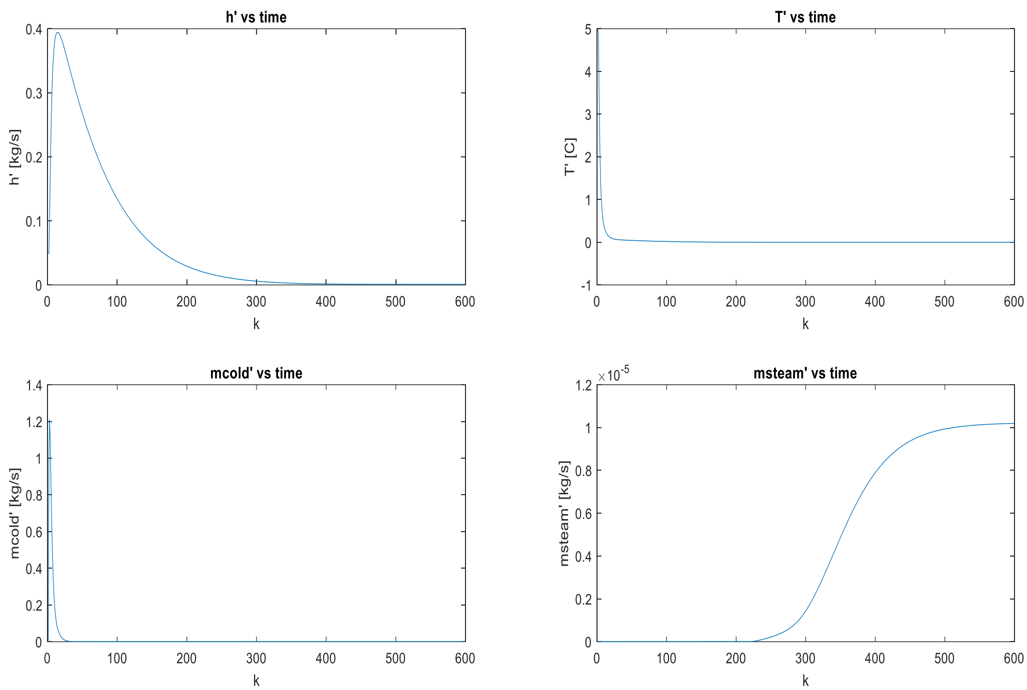


Figure 1. The response of discretized model for the set point change of states (level +0.05 m, temperature + 5 C)

Matlab code for MPC:

See ‘PartI_Q6.m’ matlab file in package in the given link: https://github.com/Haihan-W/UALabDataAnalysis/tree/master/LAB_MPC_Controller

(3.1). main code:

```
clear all
%tolerance
options.StepTolerance = 1e-10;
   rho=1000;
   D_T=0.145;
   Ac=0.25*pi()*(D_T)^2;
   delta_H=2100;
   cp=4.1855;

   %SS inputs
   mcss=7.12/60;
   Tcss=23;
   msteamss=18.36/3600;
```

```
%SS states
hss=0.22;
Tss=25.7;
F = (mcss*Tss-mcss*Tcss)/(msteamss*delta_H/cp);
R=mcss/(hss.^0.5);

A=zeros(2);
B=zeros(2);
  % f1=dh1'/dt=a11*h1'+b11*mh'
  A(1,1)=(-R/(rho*Ac))*0.5*hss^-0.5;
  B(1,1)=(1/(rho*Ac));

  %f4=dT1'/dt=a41*h1'+a44*T1'+b41*mh'+b43*msteam
  A(2,1)=((mcss*(Tss-Tcss)-msteamss*F*delta_H/cp)/rho/Ac/hss^2)
  A(2,2)=(-1/(rho*Ac)* mcss*hss^-1);
  B(2,1)=(1/(rho*Ac)*Tcss*hss^-1)+(-1/(rho*Ac)*Tss*hss^-1);
  B(2,2)=(delta_H*F/(rho*Ac*cp)*hss^-1);

  % sampling time = 1 s
  C=eye(2)
  D=0
  sys=ss(A,B,C,D)
  sysd = c2d(sys,1)

  Ad=sysd.a;
  Bd=sysd.b;
  Cd=sysd.c;
  Dd=sysd.d;


  Q=[1,0;0,1];
  S=[16,0;0,5];
  Rm=0;

  Qbar=dlyap(Ad',Cd'*Q*Cd);
  % N=5( horizon)
  N=20
  Fm=zeros(2*N,2)
  Fm(1:2,1:2)=S;
  G=zeros(2*N,2);
  for i=1:2:2*N
    G(i:i+1,1:2)=Bd'*Qbar*Ad^((i+1)/2);
  end

  H=zeros(2*N);
```

```
for i=1:2:2*N
    for j=1:2:2*N
        if i==j
            H(i:i+1,j:j+1)=Bd'*Qbar*Bd+R+2*S;
        elseif i==1 && j==3
            H(i:i+1,j:j+1)=Bd'*Ad'*Qbar*Bd-S;

            elseif j==1 && i==3
                H(i:i+1,j:j+1)=(Bd'*Ad'*Qbar*Bd-S)';
                elseif i< j %upper diagonal
                    power=(j+1)/2-1-((i+1)/2-1)
                    H(i:i+1,j:j+1)=Bd'*(Ad')^(power)*Qbar*Bd;
                    elseif i>j
                        power=(i+1)/2-1-((j+1)/2-1)
                        H(i:i+1,j:j+1)=(Bd'*(Ad')^(power)*Qbar*Bd)';

        end







    end
end

H=(H+H')/2

% constraint
LB=zeros(2*N,1)
UB= zeros(2*N,1)
for i=1:N
    UB(i*2-1,1)=9.2;
    UB(i*2,1)=30;
end
delta_u1max=12;
delta_u2max=2;
w=zeros(2*N)
for i=1:2*N

        if i<= N
            w(i,2*i-1)=1
            if i>1
            w(i,(i-1)*2-1)=-1
            end
        end
```

```
        if i>N
            w(i,2*(i-20))=1
            if i>N+1
            w(i,(i-20-1)*2)=-1
            end
        end




 end

 I=eye(2*N)


 Am=[I;-I;w;-w]

% bm
bm1=zeros(2*N,1)
bm2=zeros(2*N,1)
bm3=zeros(2*N,1)
bm4=zeros(2*N,1)

u1m=9.2
u2m=30

for i=1:2:2*N
    bm1(i,1)= u1m;

    bm1(i+1,1)=u2m;
end
for i=1:2*N



       if i<=N
           bm3(i,1)=delta_u1max;
       else
           bm3(i,1)=delta_u2max;
       end
end
bm4=bm3;
```

```
 x(:,1)=[0.05;5];
 u(:,1)=[0,0];
 kend=600
for k=2:kend


    %x(:,k)=Ad*x(:,k-1)+Bd*u(:,k-1);
     options = odeset('AbsTol',1e-10,'RelTol',1e-10);
     initialu=u(:,k-1);
     initialu(1,1)=u(1,k-1)+mcss;
     initialu(2,1)=u(2,k-1)+msteamss;
     initialx=x(:,k-1);
     initialx(1,1)=x(1,k-1)+hss;
     initialx(2,1)=x(2,k-1)+Tss;
     [t,Tandh]=ode45(@Q6ode, [k-2:1:k+100], initialx ,options,R,F,
initialu);
     x(1,k)=Tandh(2,1)-hss;
     x(2,k)=Tandh(2,2)-Tss;

    bm3(1,1)=delta_u1max+u(1,k-1);
    bm3(N+1,1)=delta_u2max+u(2,k-1);
    bm4(1,1)=delta_u1max-u(1,k-1);
    bm4(N+1,1)=delta_u2max-u(2,k-1);
    bm=[bm1;bm2;bm3;bm4];
    Fquad=G*x(:,k)-Fm*u(:,k-1);
    u_mpc=quadprog(H,Fquad,Am,bm,[],[],LB,UB);
    u(1,k)=u_mpc(1,1);
    u(2,k)=u_mpc(2,1);


end
figure(1)
subplot(2,2,1)

 plot(1:kend,x(1,:))
 xlabel('k')
 ylabel('h'' [kg/s]')
 title('h'' vs time')
subplot(2,2,2)
 plot(1:kend,x(2,:))
 xlabel('k')
 ylabel('T'' [C]')
 title('T'' vs time')
subplot(2,2,3)
  plot(1:kend,u(1,:))
  xlabel('k')
```

```
ylabel('mcold" [kg/s]')
title('mcold" vs time')
subplot(2,2,4)
plot(1:kend,u(2,:))
xlabel('k')
ylabel('msteam" [kg/s]')
title('msteam" vs time')
```

(3.2). odefunction:

```
function dydt=T_ode_dynamic(t,Tandh,R,F,inpvec)



  h=Tandh(1)
  T=Tandh(2)


  %tolerance
options.StepTolerance = 1e-10;
  rho=1000;
  D_T=0.145;
  Ac=0.25*pi()*(D_T)^2;
  delta_H=2100;
  cp=4.1855;



  Tc=23;

  hss=0.22;
  Tss=25.7;
  msteamss=18.36/3600;
  mcss=7.12/60;

  %input vector
  mc=inpvec(1,1)
  msteam=inpvec(2,1)


  dydt(1)=-R/(rho*Ac)*h^0.5+1/(rho*Ac)*mc

  dydt(2)=(mc*Tc/(rho*Ac)+msteam*delta_H*F/(rho*Ac*cp))*h^-1 -
mc/(rho*Ac)*(T*h^-1)
```

```
    dydt=[dydt(1);dydt(2)]
end
```