Cache Memories:
Why Programmers Need to Know!

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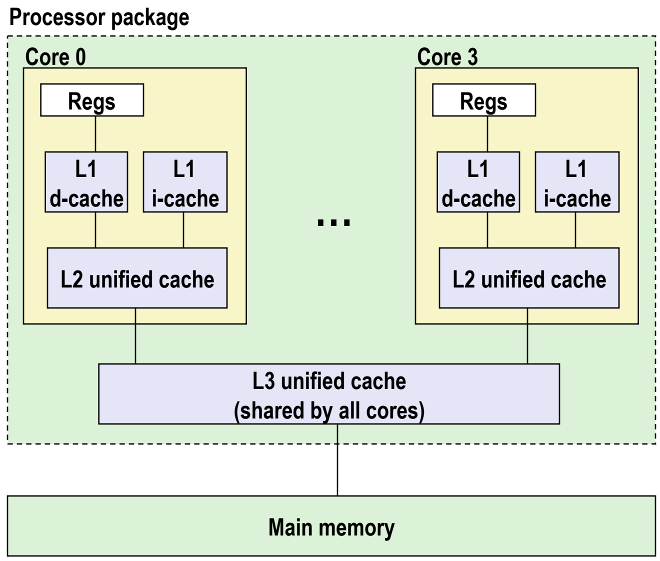
Administrative

- **Homework #6 Due April 11**
- **Y86 Simulator- groups of two (email me group members)**
  - Start this ASAP, get questions out of the way.

Today

- **Performance impact of caches**
  - Rearranging loops to improve spatial locality
  - Using blocking to improve temporal locality
  - Data layout changes to improve locality

Intel Core i7 Cache Hierarchy



L1 i-cache and d-cache:
32 KB, 8-way,
Access: 4 cycles

L2 unified cache:
256 KB, 8-way,
Access: 11 cycles

L3 unified cache:
8 MB, 16-way,
Access: 30-40 cycles

Block size: 64 bytes for all caches.

Cache Performance Metrics

- **Miss Rate**
  - Fraction of memory references not found in cache (misses / accesses) = 1 – hit rate
  - Typical numbers (in percentages):
    - 3-10% for L1
    - can be quite small (e.g., < 1%) for L2, depending on size, etc.
- **Hit Time**
  - Time to deliver a line in the cache to the processor
    - includes time to determine whether the line is in the cache
  - Typical numbers:
    - 1-2 clock cycle for L1
    - 5-20 clock cycles for L2
- **Miss Penalty**
  - Additional time required because of a miss
    - typically 50-200 cycles for main memory (Trend: increasing!)

Cache Performance

CPU time = (CPU execution clock cycles + Memory stall clock cycles) x clock cycle time

Memory stall clock cycles = Memory accesses x Miss rate x Miss penalty

<u>Example</u>

- Assume every instruction takes 1 cycle
- Miss penalty = 20 cycles
- Miss rate = 10%
- 1000 total instructions, 300 memory accesses
- Memory stall cycles? CPU clocks?

Cache Performance

- Memory Stall cycles = 300 * 0.10 * 20 = 600
- CPUclocks = 1000 + 600 = 1600

- 60% slower because of cache misses!

- Change miss penalty to 100 cycles
- CPUclocks = 1000 + 3000 = 4000 cycles

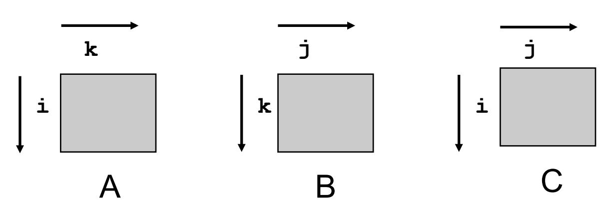
Writing Cache Friendly Code

- **Make the common case go fast**
  - Focus on the inner loops of the core functions
- **Minimize the misses in the inner loops**
  - Repeated references to variables are good (temporal locality)
  - Stride-1 reference patterns are good (spatial locality)

Key idea: Our qualitative notion of locality is quantified through our understanding of cache memories.

Miss Rate Analysis for Matrix Multiply

- **Assume:**
  - Line size = 32B (big enough for four 64-bit words)
  - Matrix dimension (N) is very large
    - Approximate 1/N as 0.0
  - Cache is not even big enough to hold multiple rows
- **Analysis Method:**
  - Look at access pattern of inner loop



Matrix Multiplication Example

- **Description:**
  - Multiply N x N matrices
  - O(N^3) total operations
  - N reads per source element
  - N values summed per destination
    - but may be able to hold in register

Variable sum held in register

```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

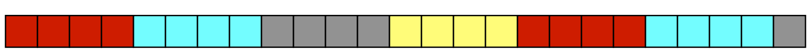

Mapping Arrays to Memory

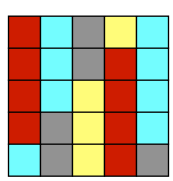
Row-major

0	1	2	3	4
5	6	7	8	9
10	11	12	13	14
15	16	17	18	19
20	21	22	23	24

Column major

0	5	10	15	20
1	6	11	16	21
2	7	12	17	22
3	8	13	18	23
4	9	14	19	24





Part of the Row maps into cache

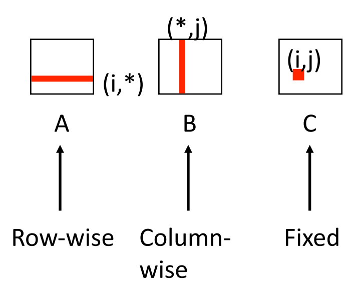
Layout of C Arrays in Memory (review)

- **C arrays allocated in row-major order**
  - each row in contiguous memory locations
- **Stepping through columns in one row:**
  - ```
    for (i = 0; i < N; i++)
      sum += a[0][i];
    ```
  - accesses successive elements
  - if block size (B) > 4 bytes, exploit spatial locality
    - compulsory miss rate = 4 bytes / B
- **Stepping through rows in one column:**
  - ```
    for (i = 0; i < n; i++)
      sum += a[i][0];
    ```
  - accesses distant elements
  - no spatial locality!
    - compulsory miss rate = 1 (i.e. 100%)

Matrix Multiplication (ijk)

```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

Inner loop:



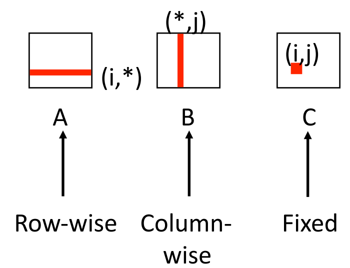
Misses per inner loop iteration:

A	B	C
0.25	1.0	0.0

Matrix Multiplication (jik)

```
/* jik */
for (j=0; j<n; j++) {
  for (i=0; i<n; i++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum
  }
}
```

Inner loop:



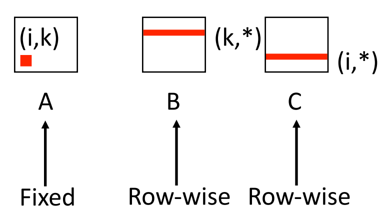
Misses per inner loop iteration:

A	B	C
0.25	1.0	0.0

Matrix Multiplication (kij)

```
/* kij */
for (k=0; k<n; k++) {
  for (i=0; i<n; i++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
```

Inner loop:

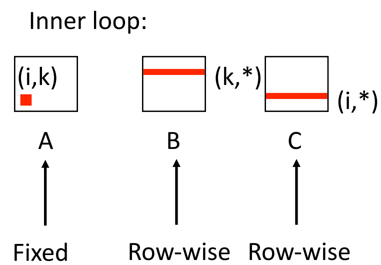


Misses per inner loop iteration:

A	B	C
0.0	0.25	0.25

Matrix Multiplication (ikj)

```
/* ikj */
for (i=0; i<n; i++) {
  for (k=0; k<n; k++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
```

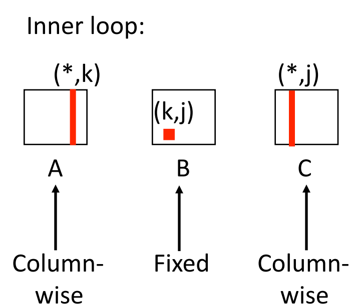


Misses per inner loop iteration:

A	B	C
0.0	0.25	0.25

Matrix Multiplication (jki)

```
/* jki */
for (j=0; j<n; j++) {
  for (k=0; k<n; k++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}
```



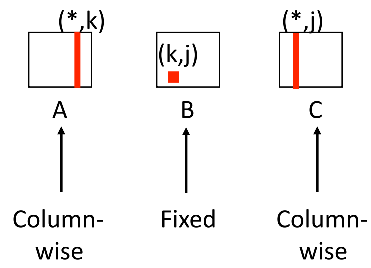
Misses per inner loop iteration:

A	B	C
1.0	0.0	1.0

Matrix Multiplication (kji)

```
/* kji */
for (k=0; k<n; k++) {
  for (j=0; j<n; j++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}
```

Inner loop:



Misses per inner loop iteration:

A	B	C
1.0	0.0	1.0

Summary of Matrix Multiplication

```
for (i=0; i<n; i++) {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

ijk (& jik):
- 2 loads, 0 stores
- misses/iter = **1.25**

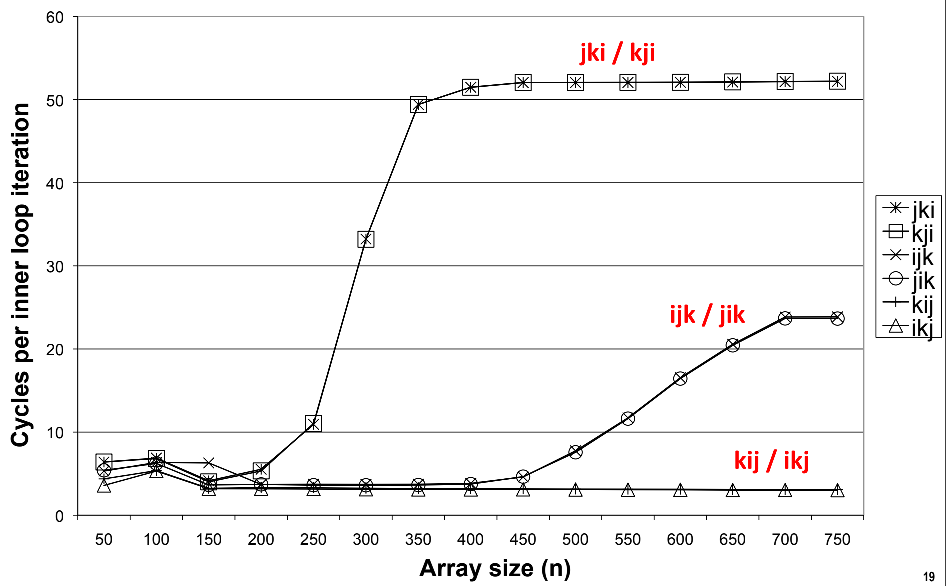
```
for (k=0; k<n; k++) {
 for (i=0; i<n; i++) {
  r = a[i][k];
  for (j=0; j<n; j++)
   c[i][j] += r * b[k][j];
 }
}
```

kij (& ikj):
- 2 loads, 1 store
- misses/iter = **0.5**

```
for (j=0; j<n; j++) {
 for (k=0; k<n; k++) {
  r = b[k][j];
  for (i=0; i<n; i++)
   c[i][j] += a[i][k] * r;
 }
}
```

jki (& kji):
- 2 loads, 1 store
- misses/iter = **2.0**

Core i7 Matrix Multiply Performance



Improving Data Cache Performance

- **Instruction Sequencing**
  - *Loop Interchange*: change nesting of loops to access data in order stored in memory (ijk vs kij, etc.)
  - *Loop Fusion*: Combine 2 independent loops that have same looping and some variables overlap
  - *Blocking*: Improve temporal locality by accessing “blocks” of data repeatedly vs. going down entire columns or rows
- **Data Layout**
  - *Merging Arrays*: Improve spatial locality by single array of compound elements vs. 2 separate arrays
  - *Nonlinear Array Layout*: Mapping 2 dimensional arrays to the linear address space
  - *Pointer-based Data Structures*: node-allocation

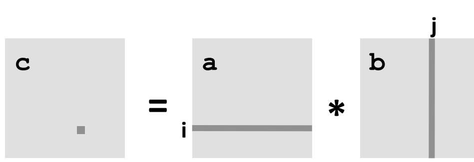
Loop Fusion Example

```
/* Before */
for (i = 0; i < N; i = i+1)
   for (j = 0; j < N; j = j+1)
       a[i][j] = 1/b[i][j] * c[i][j];
for (i = 0; i < N; i = i+1)
   for (j = 0; j < N; j = j+1)
       d[i][j] = a[i][j] + c[i][j];
/* After */
for (i = 0; i < N; i = i+1)
   for (j = 0; j < N; j = j+1)
   {   a[i][j] = 1/b[i][j] * c[i][j];
       d[i][j] = a[i][j] + c[i][j];}
```

2 misses per access to a & c vs. one miss per access

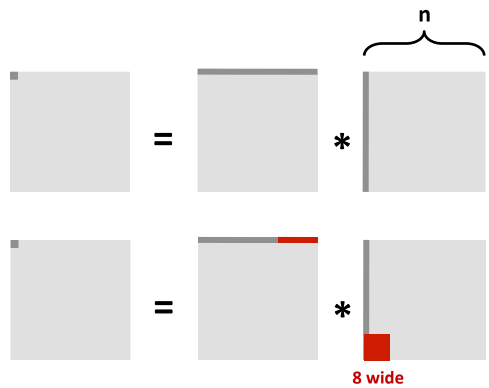
Example: Matrix Multiplication

```
c = (double *) calloc(sizeof(double), n*n);

/* Multiply n x n matrices a and b  */
void mmm(double *a, double *b, double *c, int n) {
    int i, j, k;
    for (i = 0; i < n; i++)
        for (j = 0; j < n; j++)
            for (k = 0; k < n; k++)
            c[i*n+j] += a[i*n + k]*b[k*n + j];
}
```



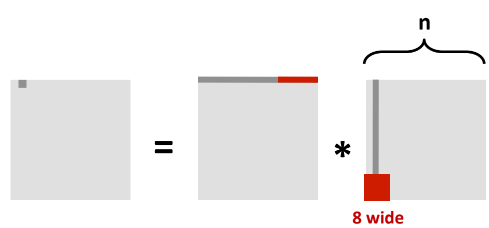
Cache Miss Analysis

- **Assume:**
  - Matrix elements are doubles
  - Cache block = 8 doubles
  - Cache size C << n (much smaller than n)
- **First iteration:**
  - n/8 + n = 9n/8 misses
  - Afterwards in cache:
    (schematic)



Cache Miss Analysis

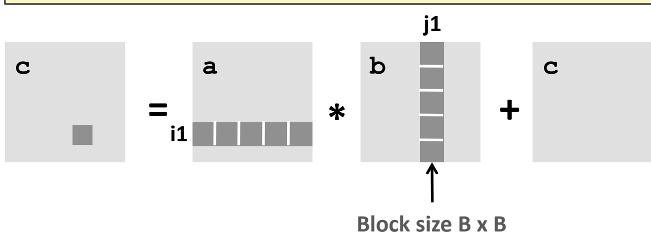
- **Assume:**
  - Matrix elements are doubles
  - Cache block = 8 doubles
  - Cache size C << n (much smaller than n)
- **Second iteration:**
  - Again:
    n/8 + n = 9n/8 misses



- **Total misses:**
  - $9n/8 * n^2 = (9/8) * n^3$

Blocked (Tiled) Matrix Multiplication

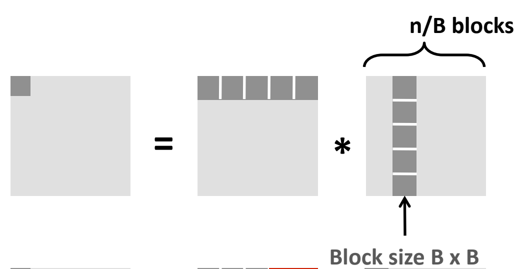
```
c = (double *) calloc(sizeof(double), n*n);

/* Multiply n x n matrices a and b  */
void mmm(double *a, double *b, double *c, int n) {
    int i, j, k;
    for (i = 0; i < n; i+=B)
        for (j = 0; j < n; j+=B)
            for (k = 0; k < n; k+=B)
                /* B x B mini matrix multiplications */
                for (i1 = i; i1 < i+B; i++)
                    for (j1 = j; j1 < j+B; j++)
                        for (k1 = k; k1 < k+B; k++)
                            c[i1*n+j1] += a[i1*n + k1]*b[k1*n + j1];
}
```



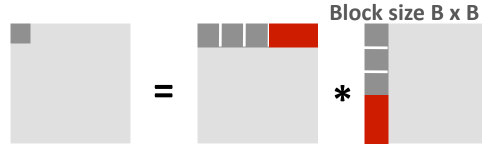
Cache Miss Analysis

- **Assume:**
  - Cache block = 8 doubles
  - Cache size C << n (much smaller than n)
  - Three blocks fit into cache: $3B^2 < C$

- **First (block) iteration:**
  - $B^2/8$ misses for each block
  - $2n/B * B^2/8 = nB/4$ (omitting matrix c)

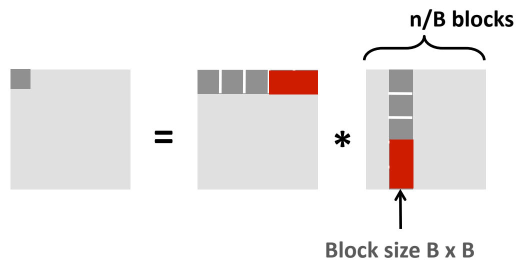


  - Afterwards in cache (schematic)



Cache Miss Analysis

- **Assume:**
  - Cache block = 8 doubles
  - Cache size C << n (much smaller than n)
  - Three blocks fit into cache: $3B^2 < C$
- **Second (block) iteration:**
  - Same as first iteration
  - $2n/B * B^2/8 = nB/4$

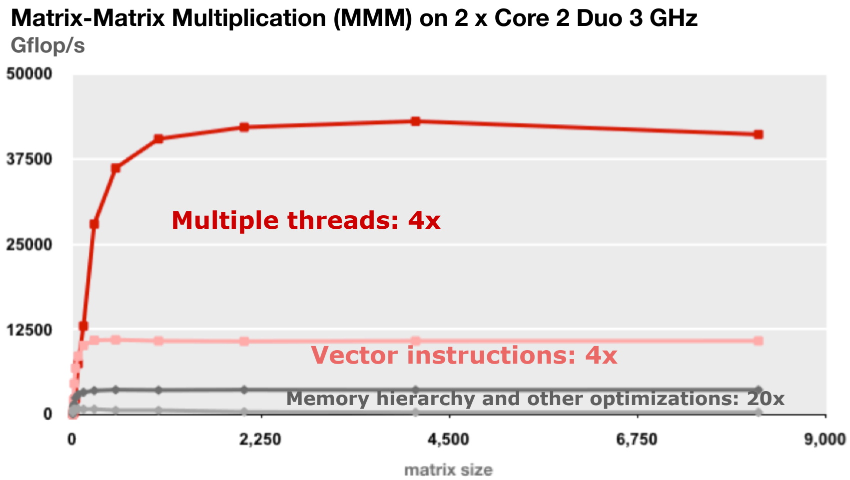


- **Total misses:**
  - $nB/4 * (n/B)^2 = n^3/(4B)$

Summary

- **No blocking: $(9/8) * n^3$**
- **Blocking: $1/(4B) * n^3$**
- **Suggest largest possible block size B, but limit $3B^2 < C$!**
- **Reason for dramatic difference:**
  - Matrix multiplication has inherent temporal locality:
    - Input data: $3n^2$, computation $2n^3$
    - Every array elements used O(n) times!
  - But program has to be written properly

MMM Plot: Analysis



- Reason for 20x: Blocking or tiling, loop unrolling, array scalarization, instruction scheduling, search to find best choice
- Effect: fewer register spills, L1/L2 cache misses, and TLB misses

Data Layout Optimizations

- So far program control
- Changes the order in which memory is accessed

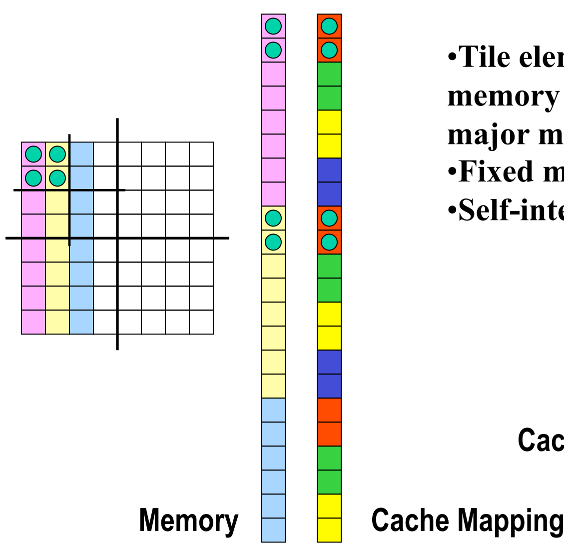
- We can also change the way our data structures map to memory
- 2-dimensional array
- Pointer-based data structures

Merging Arrays Example

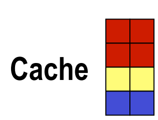
```
/* Before */
int val[SIZE];
int key[SIZE];

/* After */
struct merge {
   int val;
   int key;
};
struct merge merged_array[SIZE];
```

Reducing conflicts between val & key

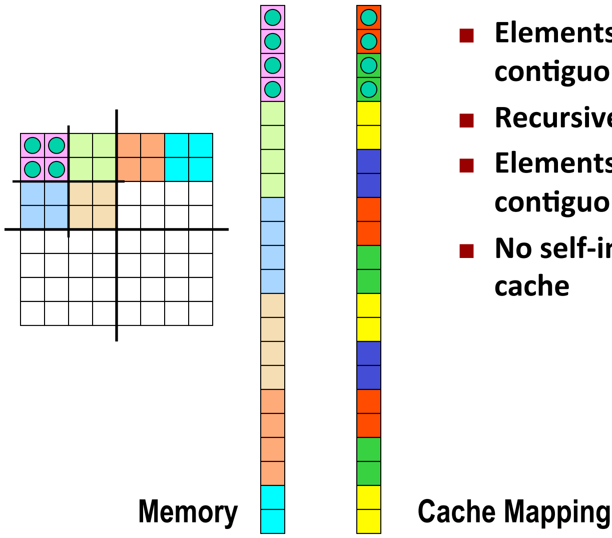
Layout and Cache Behavior



- Tile elements spread out in memory because of column-major mapping
- Fixed mapping into cache
- Self-interference in cache



Making Tiles Contiguous



- Elements of a quadrant are contiguous
- Recursive layout
- Elements of a tile are contiguous
- No self-interference in cache

Pointer-based Data Structures

- Linked List, Binary Tree
- Basic idea is to group linked elements close together in memory
- Need relatively static traversal pattern
- Or could do it during garbage collection/compaction

Reducing I-Cache Misses by Compiler Optimizations

- **Instructions**
  - Reorder procedures in memory to reduce misses
  - Profiling to look at conflicts

Concluding Observations

- **Programmer can optimize for cache performance**
  - How data structures are organized
  - How data are accessed
    - Nested loop structure
    - Blocking (tiling) is a general technique
- **All systems favor "cache friendly code"**
  - Getting absolute optimum performance is very platform specific
    - Cache sizes, line sizes, associativities, etc.
  - Can get most of the advantage with generic code
    - Keep working set reasonably small (temporal locality)
    - Use small strides (spatial locality)

Cache Memory Summary

- **Cost Effective Memory Hierarchy**
- **Work by exploiting locality (temporal & spatial)**
- **Associativity, Blocksize, Capacity (ABCs of caches)**
- **Know how a cache works**
  - Break address into tag, index, block offset
- **Know how to draw a block diagram of a cache**
- **CPU cycles/time, Memory Stall Cycles**
- **Your programs and cache performance**

Next Time

- **Exceptions and Interrupts**