NDCG

A way to measure effectiveness of web search engine algorithms

$$ext{CG}_{ ext{p}} = \sum_{i=1}^p rel_i$$

$$egin{aligned} ext{DCG}_{ ext{p}} &= \sum_{i=1}^p rac{rel_i}{\log_2(i+1)} = rel_1 + \sum_{i=2}^p rac{rel_i}{\log_2(i+1)} \end{aligned}$$

$$ext{DCG}_{ ext{p}} = \sum_{i=1}^p rac{2^{rel_i}-1}{\log_2(i+1)}$$

NDCG: Normalized DCG

DCG: Discounted Cumulative Gain

CG: Cumulative Gain

CG:
$$3 + 2 + 3 + 0 + 1 + 2 = 11$$

DCG:

i	$reli_i$	$\log_2(i+1)$	$\frac{rel_i}{\log_2(i+1)}$
1	3	1	3
2	2	1.585	1.262
3	3	2	1.5
4	0	2.322	0
5	1	2.585	0.387
6	2	2.807	0.712

= 6.861

NDCG: Normalized DCG

$$ext{nDCG}_{ ext{p}} = rac{DCG_p}{IDCG_p}$$
 ,

Ideal ordering?

3, 3, 2, 2, 1, 0

$$\text{nDCG}_6 = \frac{DCG_6}{IDCG_6} = \frac{6.861}{7.141} = 0.961$$



How is related with Airbnb evaluation?

For each new user, you are to make a maximum of 5 predictions on the country of the first booking. The ground truth country is marked with relevance = 1, while the rest have relevance = 0.

[FR] gives a
$$NDCG = \frac{2^1 - 1}{log_2(1+1)} = 1.0$$
[US, FR] gives a $DCG = \frac{2^0 - 1}{log_2(1+1)} + \frac{2^1 - 1}{log_2(2+1)} = \frac{1}{1.58496} = 0.6309$

