

COS10003 COMPUTER-LOGIC ESSENTIALS

ASSIGNMENT 2 LOGIC AND SETS

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Question 1:

Considering:

We have a total of 465 students is defined as

Set U (Universe)

We have a total of 101 students did not join any activities

>> The total of students did join at least one activity is:

$$465 - 101 = 364 \text{ (students)}$$

$$(A \cup B \cup C)$$

In the set of total students, there are:

- Set of students joined a student club is Set A
- Set of students ate at a cafe is Set B
- Set of students went to a gym is Set C

Set of students joined club and ate at coffee is $A \cap B$

Set of students joined club and went to gym is $A \cap C$

Set of students joined club ate at coffee and went to gym is $A \cap B \cap C$

- a) According to question we didn't know the total of students ate at coffee and went to the gym.

>> So the total of students ate at coffee and went to gym is the missing value.

Considering that set is $B \cap C$ as x

According to the Fundamental laws and Inclusion/Exclusion techniques, we have the formula:

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

$$\begin{aligned} \leftrightarrow & |A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - x + |A \cap B \cap C| \\ \leftrightarrow & x = |A| + |B| + |C| - |A \cap B| - |A \cap C| + |A \cap B \cap C| - |A \cup B \cup C| \\ \leftrightarrow & x = 220 + 159 + 208 - 68 - 126 + 32 - 364 \\ \leftrightarrow & x = 61 \text{ (students)} \end{aligned}$$

Answer: There are 61 students ate at the coffee and went to gym.

b)

- Students joined school club and ate at coffee but not went to gym:

$$x + 32 = 68$$

$$x = 68 - 32$$

$$x = 36 \text{ (students)}$$

- Students joined school club and went to gym but not ate at coffee:

$$y + 32 = 126$$

$$y = 126 - 32$$

$$y = 94 \text{ (students)}$$

- Students ate at coffee and went to gym but not joined school club:

$$y + 32 = 61$$

$$y = 61 - 32$$

$$y = 61 - 32 = 29 \text{ (students)}$$

- Students only joined school club:

$$a + 32 + 36 + 94 = 220$$

$$a = 220 - 32 - 36 - 94$$

$$a = 58 \text{ (students)}$$

- Students only ate at coffee:

$$b + 32 + 36 + 29 = 159$$

$$b = 159 - 36 - 32 - 29$$

$$b = 62 \text{ (students)}$$

- Students only went to gym:

$$c + 94 + 32 + 29 = 208$$

$$c = 208 - 94 - 32 - 29$$

c = 53 (students)

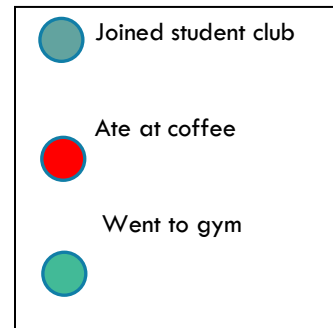
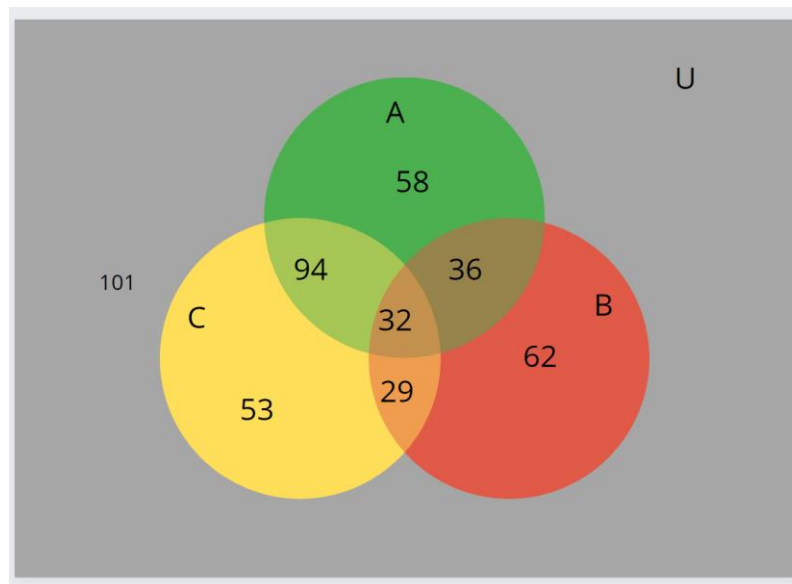


Diagram about student activities

c)

i. Did not go a gym (could both join student club and ate at a coffee)

$$>> x = 58 + 36 + 62 + 101$$

$$>> x = 257 \text{ (students)}$$

Answer: There were 156 students do not go to gym.

ii. Joined student club but did not eat at a coffee

$$>> x = 58 + 94$$

$$>> x = 152 \text{ (students)}$$

Answer: There were 152 students join student club but did not eat at a coffee.

iii. Did one only of joined a student club, eat at a cafe or went to a gym.

$$>> x = 58 + 53 + 62$$

$$>> x = 173 \text{ (students)}$$

Answer: There were 173 students did one join student club, eat at a coffee or went to a gym.

d)

i. Did not go to a gym nor eat at a cafe.

Answer: $(B \cup C)'$

ii. Joined a student club and ate at a cafe but did not go to a gym.

Answer: $C' \cup (A \cup B)$

Question 2:

a: Adita plays esports

d: David plays esports

h: Huyen plays cricket

a)

i. $h \wedge (d \vee a)$

Answer: Huyen plays cricket and Adita or David plays esports

ii. $d \rightarrow \neg a \vee h$

Answer: If David plays esports, then Adita could not play esports or Huyen could play cricket

iii. $\neg (h \vee d)$

Answer: Neither Huyen play cricket nor David play esports.

b)

i. If David plays esports, then Adita plays esports.

Answer: $d \rightarrow a$

ii. Neither Adita nor David play esports.

Answer: $\neg(a \vee d)$

iii. Adita plays esports if and only if Huyen plays cricket and David plays esports.

Answer: $a \leftrightarrow (h \wedge d)$

Question 3:

Considering that:

Height ≤ 100 is

a

Weight > 10 is

b

Statement: $(a \vee b) \wedge (\neg a \vee b) \wedge a$

Associative law: $(a \vee b) \wedge a \wedge (\neg a \vee b)$

Absorption law: $a \wedge (\neg a \vee b)$

Domination law: $a \wedge b$ (what need to prove)

Question 4:

We have: relation S on $Z \times Z$ where $Z = \{a; b; c; d; e\}$

$S = \{(a; a); (b; b); (a; b); (b; a); (c; c); (d; d); (e; e); (c; e); (d; e); (e; c); (e; d)\}$

A equivalence relation depends on 3 properties: reflective, symmetric and transitive.

Answer:

Reflectivity: $A = \{(a, a); (b, b); (c, c); (d, d); (e, e)\}$

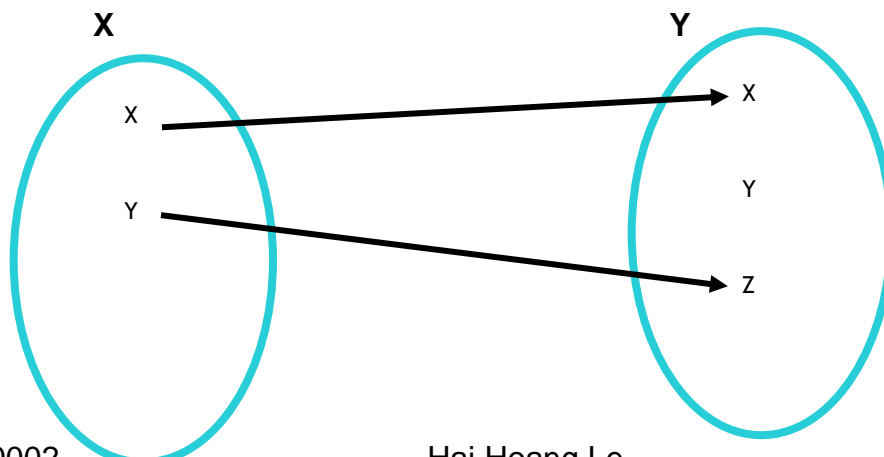
Symmetric: $B = \{(a, b); (b, a); (c, e); (e, c); (e, d); (d, e)\}$

Transitive: we could have (c, e) and (e, d), but (c, d) $\notin S$ so $C = \emptyset$

Question 5:

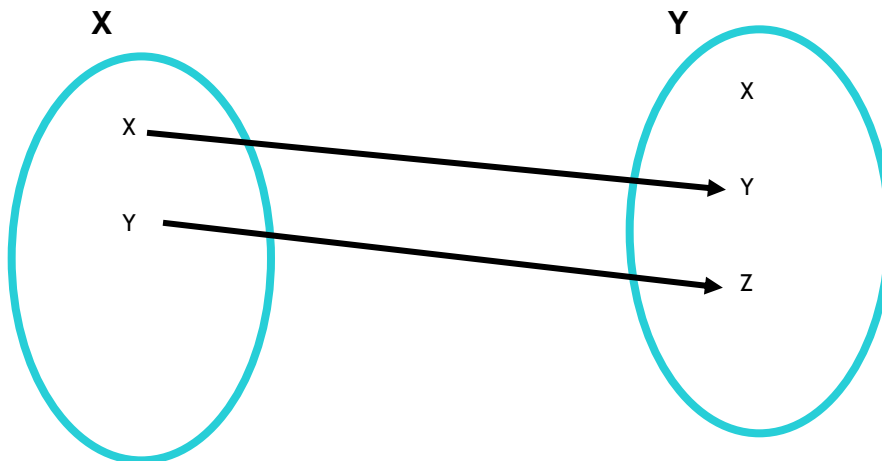
We have the domain $X = \{x, y\}$ and co-domain $Y = \{x, y, z\}$:

a) Possible function $f : X \rightarrow Y$



Answer: There are 2 possible function $f: X \rightarrow Y$

b) Possible injective function



Answer:

There are two possible functions that is injective function

Example of injective function (one to one function): $\{(x, y); (y, z)\}$

Example of not injective function: $\{(x, y); (x, z)\}$

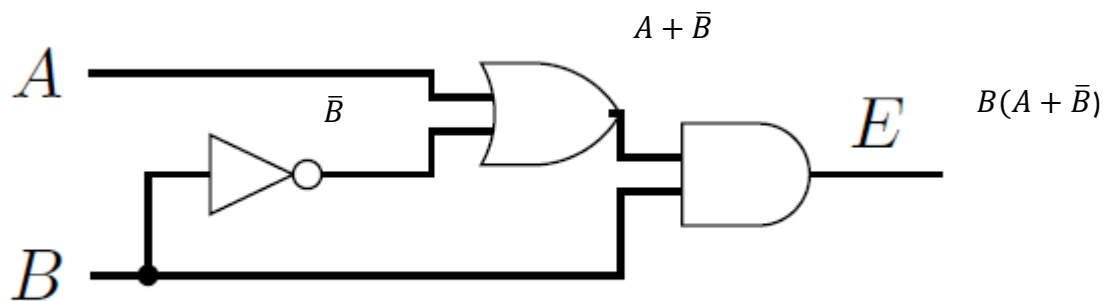
c) Possible bijective function

Answer:

In this case there are any bijective functions because domain X has 2 values and the range Y has 3 values which means that there is one value won't be used.

Question 6:

a) i.

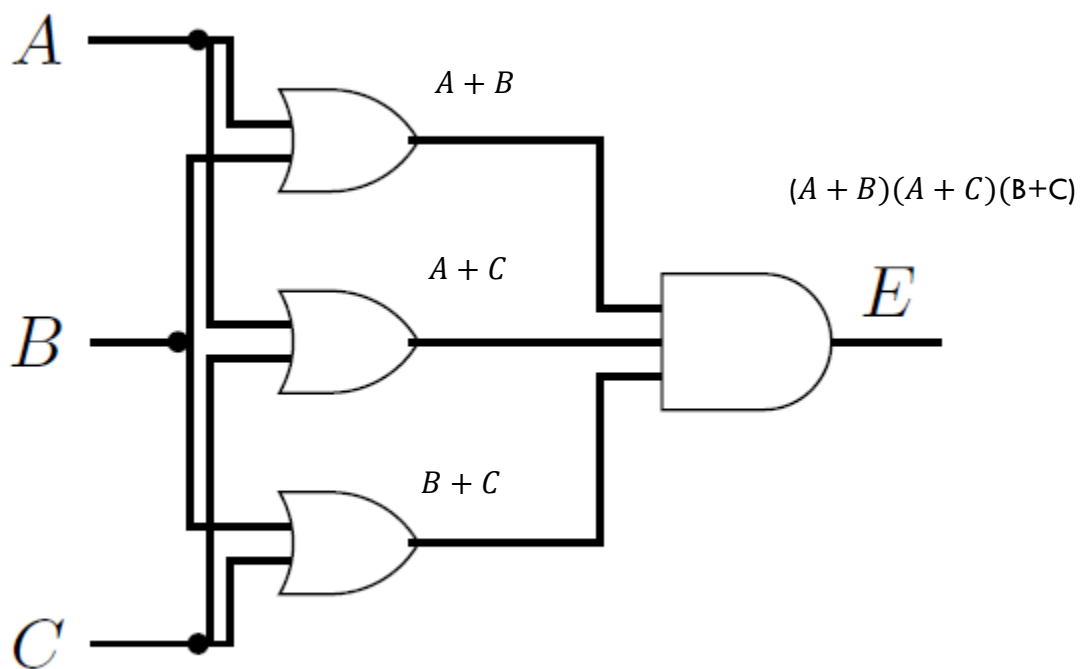


ii.

A	B	\bar{B}	$A + \bar{B}$	$E = B(A + \bar{B})$
1	1	0	1	1

b)

i.

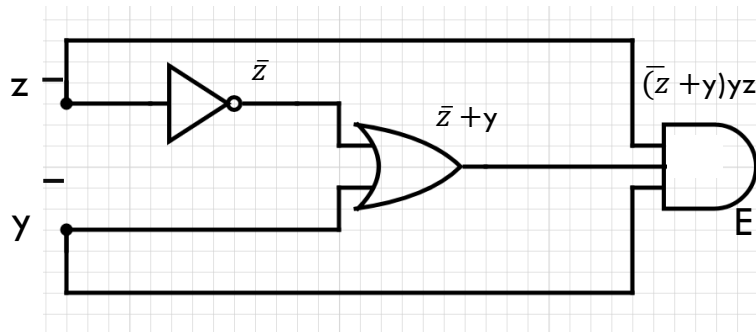


ii.

A	B	C	$A + B$	$A + C$	$B + C$	$E = (A + B)(A + C)(B + C)$
1	1	0	1	1	1	1

Question 7

a) $E = (z' + y) y z$



b)

$E = (z' + y) y z$

Distributive law: $E = (yz' + yy)z$

Idempotent law: $E = (yz')z$

Distributive law: $E = yzz'$

Complement law: $E = yz$

Answer $E = yz$

c) **Answer:**

Original circuit: Size = 3, Depth = 3

Simplified circuit: Size = 1, Depth = 1