MSE609_Assignment 2

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Q1:

TechSavvy Inc. acquired a malware detector last year from ByteGuard Solutions to promptly identify and quarantine malicious software that might threaten their systems. Malicious software accounts for 15% of the software pieces they encounter. ByteGuard's malware detector successfully identifies 990 out of 1000 malware pieces but mistakenly flags 2 out of 1000 legitimate software pieces on average per year. Given that the system quarantined a software, what is the probability it is legitimate? Define two variables: A software being malware (M) and the detector quarantining a software (Q).

Solution:

① Definitions
- M : A software being malware.
- L : The software is legitimate, $L = \bar{M}$.
- Q : The detector quarantining a software.

② Use Bayes' theorem to calculate:
The probability of the system flagging the software when ~~it~~ is actually legitimate.
⇒ which is : $P(L|Q)$

$$P(L|Q) = \frac{P(Q|L)\,P(L)}{P(Q|L)\,P(L) + P(Q|M)\,P(M)}$$

③ Given conditions:
- $P(M) = 15\% = 0.15$
- $P(L) = 1 - 15\% = 0.85$
- $P(Q|L) = \frac{2}{1000} = 0.002$
- $P(Q|M) = \frac{990}{1000} = 0.99$

④ Computation:

$$P(L|Q) = \frac{0.002 \times 0.85}{0.002 \times 0.85 + 0.99 \times 0.15} = \frac{0.0017}{0.1502} \approx 0.0113$$

⑤ Answer : $P(L|Q) \approx 1.13\%$

Q2:

Taylor has the option to study at two different libraries for the final exams: Library A and Library B. Based on past experience, Taylor knows the following:

- The noise level at Library A is uniformly distributed between 0 and 15 decibels.
- The noise level at Library B is uniformly distributed between 0 and 20 decibels.
- Taylor chooses Library A 70% of the time and Library B 30% of the time.

One day, Taylor finds the noise level to be exactly 10 decibels. Using Bayes' theorem, calculate the probability that Taylor was at Library A and the probability that Taylor was at Library B, given this noise level.

Solution:

① Definitions.

A: Taylor was at Library A
B: Taylor was at Library B
L: The noise level

② Use Bayes' theorem to calculate:

The probability that Taylor was at Library A and B

$\Rightarrow$ which is $P(A|L)$, $P(B|L)$

$$P(A|L) = \frac{P(L|A)P(A)}{P(L|A)P(A) + P(L|B)P(B)}$$

$$P(B|L) = 1 - P(A|L)$$

③ Given conditions.

- $P(A) = 0.7$
- $P(B) = 0.3$
- $L=10$. This is a uniform continuous distribution, so we use the probability density function (PDF) value at that point as the likelihood in Bayes' theorem.

$P(0 \le L \le 15) = f_A(10) = \frac{1}{15}$, $\Rightarrow P(L|A) = \frac{1}{15}$

$P(0 \le L \le 20) = f_B(10) = \frac{1}{20}$. $\Rightarrow P(L|B) = \frac{1}{20}$

④ Computation:

$$P(A|L=10) = \frac{\frac{1}{15} \times 0.7}{\frac{1}{15} \times 0.7 + \frac{1}{20} \times 0.3} = \frac{28}{37} \approx 0.7568$$

$$P(B|L=10) = 1 - P(A|L=10) = 1 - \frac{28}{37} \approx 0.2432$$

⑤ Answer:

$P(A|L=10) \approx 0.7568$

$P(B|L=10) \approx 0.2432$

Q3:

Suppose we have a bag of five balls with two different colors: red and blue. After drawing four balls without replacement, we observe three red balls and one blue ball. What is the most likely original composition of red balls in the bag?

Solution:

① Likelihood function:

Using the Binomial PMF form, the likelihood of observing exactly 3 red balls is:

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}, \quad k=3,\ n=4.$$

$$P(X=3) = \binom{4}{3} p^3 (1-p)^{4-3} = 4p^3(1-p).$$

$$p = \frac{\text{number of red balls}}{5}$$

② The possible values of p:

- If the bag has 0 red balls ($p=0$):
  $P(X=3) = 4 \times 0^3 \times 1 = 0$
- If the bag has 1 red ball ($p = \frac{1}{5} = 0.2$):
  $P(X=3) = 4 \times 0.2^3 \times (1-0.2) = 0.0256$
- If the bag has 2 red balls ($p = \frac{2}{5} = 0.4$):
  $P(X=3) = 4 \times 0.4^3 \times (1-0.4) = 0.1536$
- If the bag has 3 red balls ($p = \frac{3}{5} = 0.6$):
  $P(X=3) = 4 \times 0.6^3 \times (1-0.6) = 0.3456$
- If the bag has 4 red balls ($p = \frac{4}{5} = 0.8$):
  $P(X=\cancel{4}3) = 4 \times 0.8^3 \times (1-0.8) = 0.4096$
- If the bag has 5 red balls ($p = \frac{5}{5} = 1$):
  $P(X=3) = 4 \times 1^3 \times (1-1) = 0$

③ Compare likelihoods:

The maximum probability is $p=0.8 \Rightarrow$ 4 red balls, 1 blue ball

④ Answer:

The most likely composition is 4 red balls and 1 blue ball.