Modeling parameters of structured clays as a multivariate normal distribution

Jianye Ching and Kok-Kwang Phoon

Abstract: This study explores the possibility of modeling liquidity index, undrained shear strength, remolded undrained shear strength, preconsolidation stress, and vertical effective stress of structured clays (sensitive or quick clays) as a multivariate normal distribution. The literature is replete with correlation equations between two soil parameters. Consistent synthesis of more than two soil parameters through construction of a multivariate probability distribution function is rare, despite obvious practical usefulness of such an approach. This study compiles a large database of structured clays to construct the multivariate probability distribution among the aforementioned five soil parameters. This multivariate distribution is then used to simulate the correlations between soil parameters of interest and to derive useful equations for Bayesian inference. This constructed multivariate distribution and equations are further validated by another independent database of structured clays as well as by empirical equations proposed in the literature.

Key words: correlation, multivariate normal distribution, site characterization, reliability-based design.

Résumé : Cette étude explore la possibilité de modéliser l’indice de liquidité, la résistance au cisaillement non drainé, la résistance au cisaillement non drainé d’un échantillon intact, la contrainte de préconsolidation, et la contrainte verticale effective d’argiles structurées (argiles sensibles ou rapides) en tant de distribution normale multivariée. La littérature abonde d’équations et de corrélations entre deux paramètres de sol. La synthèse consistante de plus de deux paramètres de sol par la construction d’une fonction de probabilité selon une distribution normale multivariée est rare, malgré les applications évidentes d’une telle approche. Cette étude compile une grande base de données sur les argiles structurées afin de construire la distribution des probabilités multivariées pour les cinq paramètres nommés précédemment. Cette distribution multivariée est ensuite utilisée pour simuler les corrélations entre les paramètres d’intérêt du sol et pour dériver les équations utiles pour l’inférence Bayésienne. Cette distribution multivariée construite et les équations sont par la suite validées à l’aide d’une autre base de données indépendante sur les argiles structurées, ainsi qu’avec des équations empiriques proposées dans la littérature.

Mots-clés : corrélation, distribution normale multivariée, caractérisation de sites, conception basée sur la fiabilité.

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Introduction

Multivariate information is usually available in a typical site investigation. For instance, when undisturbed samples are extracted for oedometer and triaxial tests, test indices, such as the unit weight, plastic limit (PL), liquid limit (LL), and natural water content, are commonly determined in close proximity. A number of these test indices could be correlated to a measure of the undrained shear strength (s_u). It is straightforward to update the first two moments (mean and coefficient of variation) of s_u conditioning on a single test index (e.g., overconsolidation ratio (OCR)). In fact, the geotechnical engineering literature is replete with such pairwise correlations. However, it is not obvious how to conduct the same analysis conditioning on multiple test indices. The rigorous approach is to construct a multivariate probability distribution function from the multivariate information.

This paper examines the feasibility of adopting the multivariate normal distribution to model the correlation structure among liquidity index (LI), undrained shear strength (s_u), remolded undrained shear strength (s_u^{re}), preconsolidation stress (σ'_p), and vertical effective stress (σ'_v). A large database of structured clays is compiled to construct the multivariate probability distribution of {LI, s_u, s_u^{re}, σ'_p, σ'_v} as a multivariate normal distribution. The database contains 345 clay samples with wide ranges of simultaneously known {LI, s_u, s_u^{re}, σ'_p, σ'_v}. This multivariate normal distribution is then used to simulate the correlations among {LI, s_u, s_u^{re}, σ'_p, σ'_v} and to derive useful equations for further Bayesian inference. To ensure the analysis results are useful, the conclusions are further validated by another independent large database of 1792 structured clay samples where {LI, s_u, s_u^{re}, σ'_p, σ'_v} are only partially known (i.e., one or more components are not reported in the original source) and also by empirical equations proposed in literature.

To our knowledge, this paper presents the first attempt to construct a multivariate probability distribution from empiri-

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J. Ching. Department of Civil Engineering, National Taiwan University, Taipei, Taiwan.
K.-K. Phoon. Department of Civil and Environmental Engineering, National University of Singapore, Singapore; National Taiwan University of Science and Technology, Taiwan.

Corresponding author: Jianye Ching (e-mail: jyching@gmail.com).

cal multivariate geotechnical data. When multivariate geotechnical data exist in a sufficient amount, it is of significant practical usefulness to construct a multivariate probability distribution function. The usefulness is as follows: (*i*) it is possible to derive the mean and coefficient of variation (c.o.v.) of any parameter systematically using the constructed multivariate distribution given the information of the subset of other parameters and (*ii*) it may be possible to study this data theoretically using the multivariate distribution if new strong pairwise correlations can be found either among the original components or some derived components.

Existing pairwise correlations among $\{\mathrm{LI}, s_u, s_u^{re}, \sigma'_p, \sigma'_v\}$

Pairwise correlations among $\{\mathrm{LI}, s_u, s_u^{re}, \sigma'_p, \sigma'_v\}$ have been studied for decades. For insensitive clay, Wroth and Wood (1978) derived the relation between s_u^{re} and LI based on the modified Cam Clay model

$$[1] \qquad s_u^{re} = s_{u,L}^{re} R \exp[-\mathrm{LI} \times \log(R)]$$

where $s_{u,L}^{re}$ ($\approx$ 1.7 kN/m^2) is the s_u^{re} value at the liquid limit and R ($\approx$ 100) is the ratio between the s_u^{re} at the plastic and liquid limits. Empirically, Koumoto and Houlsby (2001) proposed

$$[2] \qquad w = a(s_u^{re}/P_a)^{-b}$$

where w is the water content (%), a and b are parameters depending on clay types, and P_a is atmospheric pressure. The close relation between s_u^{re} and LI has been observed empirically by other researchers, such as Mitchell (1976), Leroueil et al. (1983), Locat and Demers (1988), Hong et al. (2003), etc. They all showed that the s_u^{re} versus LI data points lie in a relatively narrow band.

For sensitive and quick clays, the correlation between the undisturbed undrained shear strength, s_u, and water content is more uncertain. This is to be expected given that index properties are based on disturbed samples. Based on test results for Norwegian marine clays in Bjerrum (1954),

$$[3] \qquad S_t = \exp(k \times \mathrm{LI})$$

where S_t is the sensitivity and k is the model constant, found to be around 2 for the Norwegian clays (Wood 1990). Similar correlations were summarized by Eden and Kubota (1961) for Leda clays in the Ottawa area, but with different k values. Combining eq. [3] with eq. [1], the following equation is obtained (Wood 1990):

$$[4] \qquad \begin{aligned} s_u &= s_{u,L}^{re} R S_t \exp[-\log(R) \times \mathrm{LI}] \\ &= s_{u,L}^{re} R \exp\{[k - \log(R)] \times \mathrm{LI}\} \end{aligned}$$

A very similar correlation relation between σ'_v and LI is proposed by Wood (1990) for clays at the critical state

$$[5] \qquad \begin{aligned} \sigma'_v &= \sigma'_{v,L} R S_t \exp[-\log(R) \times \mathrm{LI}] \\ &= \sigma'_{v,L} R \exp\{[k - \log(R)] \times \mathrm{LI}\} \end{aligned}$$

where $\sigma'_{v,L}$ ($\approx$ 8 kN/m^2) is the σ'_v value at the liquid limit. Based on 150 data points from clays with $S_t \leq 10$, Stas and Kulhawy (1984) suggested that

$$[6] \qquad \sigma'_p/P_a = 10^{1.11-1.62\mathrm{LI}}$$

while for clays with $S_t \geq 10$, they found that σ'_p and LI are nearly uncorrelated. The Naval Facilities Engineering Command (NAVFAC 1982) also developed a relation among S_t, σ'_p, and LI, indicating that for a given S_t, there is a stronger correlation between σ'_p and LI.

Correlation with stress history is also essential. The OCR is usually believed to decrease with increasing LI (e.g., Kulhawy and Mayne 1990). The "stress history and normalized soil engineering properties" (SHANSEP) concept (Ladd and Foott 1974) correlates s_u with OCR

$$[7] \qquad s_u/\sigma'_v = \alpha \mathrm{OCR}^m$$

where α is the s_u/σ'_v of the normally consolidated (NC) clay and m is the SHANSEP exponent. Jamiolkowski et al. (1985) summarized that s_u/σ'_v is roughly $0.23 \times \mathrm{OCR}^{0.8}$. Mesri (1993) reported that s_u is roughly $0.26 \times \sigma'_p$.

There are at least two limitations in many of the aforementioned pairwise correlations. The first limitation has been discussed earlier: pairwise correlations are unable to address the scenarios where there is multivariate information. Another limitation is that many of the above correlations only provide point estimates of soil parameters. From the design point of view, deterministic point estimates may be insufficient because the resulting design safety level should depend on the magnitude of uncertainties associated with the estimation procedure. This is especially true for reliability-based design or comparable simplified methodologies, such as load-resistance factor design (LRFD), limit-state design, and partial-factor design.

Database

This study compiles two databases from the literature: a database of 345 structured clay data points with simultaneously known $\{\mathrm{LI}, s_u, s_u^{re}, \sigma'_p, \sigma'_v\}$ information and another database of 1792 data points with partially known $\{\mathrm{LI}, s_u, s_u^{re}, \sigma'_p, \sigma'_v\}$ information. The former is for the purpose of constructing the multivariate probability distribution of $\{\mathrm{LI}, s_u, s_u^{re}, \sigma'_p, \sigma'_v\}$, while the latter is for subsequent validation. To ensure meaningful validation, the sources for these two databases are completely independent. In the following, the former database will be referred as the calibration database, while the latter is referred as the validation database.

Calibration database

Table 1 shows the basic information for the calibration database. There are 345 data points with complete $\{\mathrm{LI}, s_u, s_u^{re}, \sigma'_p, \sigma'_v\}$ information from 37 sites. The geographical regions cover Canada, USA, Sweden, Japan, Thailand, UK, Brazil, and India. The clay properties cover a wide range of sensitivity (1 ~ several hundred; a few sites > 1000), OCR (1 ~ 4; one site up to 12), and LI (0.1 ~ 3.8). The clay types are also broad, covering marine clays, sandy–silty clays, Leda clays, etc. Most are quick clays with $S_t > 8$, and highly overconsolidated (OC; fissured) and organic clays are nearly absent in this database.

For all cases, the reported s_u and s_u^{re} values were tested based on various types of tests, including unconfined compression (UC) and field vane (FV). These values cannot be

Table 1. Basic information for the cases in the calibration database.

Site (region)	Reference	LI	s_u	s_u^{re}	σ'_v	σ'_p
Ariake Bay (Japan), UC tests, $S_t = 9.9 \sim 42.4$	Ohtsubo et al. (1995)	1.28	2.58	0.22	9.56	7.94
		1.27	4.74	0.16	12.82	29.41
		1.45	5.42	0.39	16.44	17.14
		1.20	5.82	0.59	20.06	27.63
		1.26	6.97	0.65	24.04	23.37
		1.36	6.83	0.44	27.30	26.53
		1.29	11.08	0.44	31.65	34.10
		1.24	10.36	0.52	34.54	29.30
		1.24	13.10	0.39	38.53	40.84
		1.31	15.88	0.63	41.79	42.82
		1.22	15.77	0.68	45.05	46.16
		1.44	16.66	1.01	50.48	52.61
		1.55	19.19	1.47	54.82	77.45
		1.22	25.00	0.59	59.53	75.70
		1.22	29.38	1.17	63.87	82.35
		1.05	40.89	1.43	69.31	127.24
		0.89	49.35	2.94	73.65	181.98
Gosport (UK), UC tests, $S_t = 2.4 \sim 3.1$	Skempton (1948)	0.59	8.93	3.67	39.01	29.61
		0.38	34.55	13.67	130.66	128.31
		0.55	9.87	3.22	35.72	31.49
		0.42	20.68	6.75	110.45	88.36
		0.46	12.22	4.13	33.37	46.06
Åsrum (Canada), UC tests, $S_t = 35.8 \sim 189.8$	Parry and Wroth (1981)	2.02	10.56	0.14	7.70	29.98
		2.16	8.36	0.07	9.72	27.95
		1.55	10.25	0.09	10.13	27.55
		2.83	11.04	0.09	10.53	34.84
		3.37	13.72	0.12	10.53	38.08
		2.09	7.88	0.22	13.37	38.08
		1.91	20.50	0.21	15.80	42.94
		2.59	13.87	0.18	19.85	51.04
		2.07	15.14	0.16	23.50	56.72
		2.44	8.83	0.15	27.95	61.17
		2.43	5.77	0.06	12.93	29.66
		4.85	11.39	0.06	15.97	39.93
		1.67	8.65	0.11	18.63	39.54
		3.47	10.17	0.09	22.43	36.88
		1.75	12.60	0.10	25.86	36.12
		3.26	7.44	0.13	31.18	38.02
		1.46	12.14	0.14	35.74	38.78
Fremont (USA), UC tests, $S_t = 1 \sim 48.7$	Wu (1958)	0.94	55.75	1.71	147.96	158.74
		0.17	42.09	1.08	151.55	163.78
		1.13	52.56	1.08	86.12	98.34
		0.36	18.35	18.11	199.70	199.70
		0.74	10.55	2.99	142.37	142.37
		0.82	10.07	2.36	116.57	122.31
		0.60	13.07	5.07	159.57	159.57
Sagina (USA), UC tests, $S_t = 2.2 \sim 3.2$	Wu (1958)	0.51	103.83	32.25	68.00	135.79
		0.43	114.53	52.82	102.99	102.99
Sault Ste. Marie (USA), UC tests, $S_t = 5.9 \sim 21.1$	Wu (1958)	0.83	51.37	8.76	163.32	163.32
		0.86	57.21	8.76	179.54	179.54
		0.72	64.21	5.25	203.86	203.86
		1.13	40.86	4.09	223.55	223.55

Table 1 (*continued*).

Site (region)	Reference	LI	s_u	s_u^{re}	σ'_v	σ'_p
		1.67	45.53	3.50	243.24	243.24
		1.25	49.04	2.33	264.09	264.09
Detroit (USA), UC tests, $S_t = 1.8 \sim 1.82$	Wu (1958)	0.55	48.99	27.24	90.14	115.53
		0.53	48.99	24.82	104.42	109.18
		0.54	55.44	23.21	123.46	123.46
		0.49	68.33	19.18	156.79	156.79
		0.57	85.26	27.24	204.39	217.09
		0.54	89.29	31.27	217.09	234.54
		0.51	74.78	28.04	231.37	236.13
		0.71	59.47	19.98	240.89	194.87
		0.64	61.89	13.54	250.41	191.70
		0.85	65.11	7.90	256.76	194.87
Bombay, Maharashtra (India), FV tests, $S_t = 2.8 \sim 4.0$	Kulkarani et al. (1967)	0.87	14.62	3.79	38.31	38.22
		0.62	19.37	4.83	64.69	45.39
		0.57	18.69	6.62	80.59	59.24
Bangkok (Thailand) FV tests, $S_t = 1 \sim 29.7$	Moh et al. (1969)	0.52	10.01	10.01	19.43	235.05
		0.82	9.01	9.01	42.74	75.76
		0.14	36.04	1.21	69.93	141.81
New Liskeard (Canada), FV tests = $8.3 \sim 18.0$	Eden and Bozozuk (1962)	1.35	12.33	0.88	28.48	52.56
		1.44	13.21	0.88	30.83	35.53
		1.37	15.86	0.88	36.12	49.92
		1.32	19.09	1.17	42.28	71.94
		1.48	20.55	1.47	47.86	71.64
		1.62	22.02	2.35	53.73	69.29
		1.43	21.43	1.76	59.90	78.98
		2.12	19.38	2.35	64.89	77.81
		2.03	22.32	2.06	71.35	88.67
Gloucester (Canada), FV tests, $S_t = 13.0 \sim 100.4$	Konrad and Law (1987)	1.17	12.95	1.00	35.68	58.38
		1.79	18.90	0.19	45.41	64.86
		1.16	21.87	0.31	51.89	107.03
		2.03	24.84	0.56	61.62	103.78
		1.53	36.16	0.36	72.43	120.00
		1.37	54.56	1.16	83.24	141.62
		1.63	43.06	0.62	95.14	161.08
		1.44	54.74	1.77	115.68	175.14
Ottawa Sewage Treatment Plant (Canada), FV tests, $S_t = 23.0 \sim 504.9$	Konrad and Law (1987)	1.04	130.94	3.54	99.84	447.31
		1.02	134.17	1.34	106.69	452.20
		1.78	144.60	1.45	107.67	454.16
		1.27	148.20	5.49	116.48	459.05
		1.22	150.72	6.55	124.31	463.95
		1.34	162.95	3.70	133.12	467.86
		1.43	155.76	2.08	137.03	471.78
		2.09	157.55	0.98	140.95	474.72
		2.01	135.61	0.68	149.76	479.61
		3.76	136.33	0.27	155.63	483.52
Pernambuco (Brazil), FV tests, $S_t = 4.9 \sim 17.1$	Coutinho (2007)	0.74	38.62	4.14	66.28	154.65
		0.55	44.48	6.90	79.07	177.91
		0.71	47.59	7.59	84.88	103.49

Table 1 (*continued*).

Site (region)	Reference	LI	s_u	s_u^{re}	σ'_v	σ'_p
		0.75	43.79	6.55	89.53	125.58
		0.42	47.24	8.28	95.35	129.07
		0.71	48.97	8.97	105.81	175.58
		0.66	51.03	10.34	109.30	136.05
		0.71	37.24	4.48	117.44	160.47
		0.71	39.66	3.10	127.91	119.77
		0.79	42.07	3.10	141.86	150.00
		0.63	44.48	3.79	147.67	181.40
		0.65	46.90	3.10	153.49	141.86
		0.67	25.91	3.34	28.83	37.72
		0.97	18.29	2.75	29.72	35.94
		0.93	18.00	3.63	30.61	29.72
		0.65	17.4	3.34	34.16	25.27
		1.03	17.70	3.05	37.72	43.95
		0.85	16.82	2.75	37.72	33.27
		1.00	19.17	3.05	37.72	28.83
		1.06	17.70	1.88	43.06	40.39
		1.15	15.94	1.29	45.73	35.05
		1.15	19.17	1.58	53.74	43.06
		0.77	22.10	1.29	61.74	48.40
		0.62	22.39	1.88	69.75	67.08
		0.73	29.7	2.46	73.31	64.41
		0.64	37.05	3.34	82.21	77.76
National Research Council, Ottawa* (Canada), FV tests, $S_t = 32.6 \sim 730.9$	Eden and Hamilton (1957)	1.00	58.09	1.46	52.85	185.15
		1.00	47.60	1.46	55.84	176.90
		1.20	61.47	0.93	58.09	170.91
		1.20	70.84	0.93	62.22	162.66
		1.60	70.46	0.46	64.46	157.79
		1.40	51.72	0.64	69.34	151.42
		2.00	56.22	0.27	71.96	148.79
		1.30	51.72	0.77	74.21	145.79
		3.30	58.47	0.08	76.46	143.92
National Museum, Ottawa* (Canada), FV tests, $S_t = 16.1 \sim 1084.7$	Eden and Hamilton (1957)	0.60	81.40	5.07	77.05	216.70
		0.90	106.75[2]	1.89	81.93	190.10
		0.90	110.74	1.89	86.36	175.91
		1.00	85.92	1.46	91.68	170.59
		1.00	58.87	1.46	96.56	175.03
		1.00	87.69	1.46	103.21	182.56
		1.50	113.40	0.54	113.85	195.42
		2.50	110.74	0.16	123.16	205.17
		3.80	65.08	0.06	135.13	217.59
Beauharnois, Quebec* (Canada), FV tests, $S_t = 22.5 \sim 93.6$	Eden and Hamilton (1957)	1.50	38.23	0.54	87.37	92.83
		1.60	31.98	0.46	90.88	91.66
		1.30	29.25	0.77	94.78	95.17
		1.50	44.08	0.54	98.69	99.47
		1.20	40.18	0.93	102.59	103.37
		1.70	37.45	0.40	106.49	107.66
		1.30	44.47	0.77	109.61	111.17
		0.90	63.58	1.89	112.73	115.07
		0.80	56.56	2.51	116.24	118.19
		1.20	58.90	0.93	118.97	121.70

Table 1 (*continued*).

Site (region)	Reference	LI	s_u	s_u^{re}	σ'_v	σ'_p
		1.20	48.76	0.93	122.48	125.60
		0.90	60.85	1.89	130.67	133.01
Hawkesbury, Ontario* (Canada), FV tests, $S_t = 31.4 \sim 71.8$	Eden and Hamilton (1957)	1.70	23.00	0.40	23.00	62.81
		1.50	25.21	0.54	26.10	65.90
		1.60	25.65	0.46	28.75	68.56
		1.50	26.10	0.54	31.85	71.65
		1.20	29.19	0.93	34.94	74.75
		1.50	30.08	0.54	38.04	77.85
		1.20	38.04	0.93	40.25	80.94
		1.30	41.13	0.77	44.23	83.60
		1.20	42.46	0.93	46.88	86.25
		1.30	55.29	0.77	53.08	92.88
		1.50	36.71	0.54	58.83	99.08
		1.30	45.56	0.77	61.92	102.17
		1.40	44.67	0.64	68.56	108.37
Boston, MIT campus (USA), FV tests, $S_t = 6.5 \sim 9.4$	Horn and Lambe (1964)	0.51	107.07	13.68	194.01	712.71
		0.57	91.94	11.71	208.11	465.11
		0.58	90.62	11.71	211.24	559.14
		0.47	63.66	9.73	223.78	581.08
		0.37	56.43	7.10	247.29	465.11
		0.81	54.45	5.79	256.69	314.67
Gloucester* (Canada), FV tests, $S_t = 48.2 \sim 413.9$	Eden and Crawford (1957)	1.60	22.28	0.46[2]	14.26	63.29
		1.88	28.52	0.31	24.07	59.72
		1.80	23.18	0.35	27.63	98.05
		2.13	34.76	0.23	40.11	85.57
		2.21	39.22	0.21	53.48	106.07
		1.58	61.50	0.47	64.18	131.92
		1.63	48.13	0.45	70.42	142.62
		1.43	66.85	0.61	85.57	164.90
		1.34	72.20	0.72	97.16	181.84
		2.65	57.94	0.14	104.29	165.79
National Museum, Ottawa* (Canada), FV tests, $S_t = 35.6 \sim 1237.7$	Eden and Crawford (1957)	0.81	85.44	2.40	74.26	215.60
		0.91	114.98	1.86	84.64	170.88
		0.86	124.57	2.09	87.84	164.49
		1.00	118.98	1.46	89.43	167.68
		1.04	107.80	1.33	93.42	170.88
		1.06	86.24	1.27	103.01	180.46
		1.40	121.37	0.64	111.79	191.64
		1.83	111.79	0.33	119.78	204.42
		3.80	74.26	0.06	131.75	211.60
		1.00	74.56	1.46	49.22	193.28
		0.99	65.15	1.51	52.12	180.97
National Research Council, Ottawa* (Canada), FV tests, $S_t = 39.2 \sim 868.7$	Eden and Crawford (1957)	1.00	57.19	1.46	55.02	172.29
		1.19	65.15	0.96	57.19	165.77
		1.27	69.49	0.82	61.53	164.32
		1.48	77.46	0.56	64.43	154.19
		1.41	63.70	0.63	69.49	156.36
		1.93	90.49	0.29	72.39	148.40

Table 1 (*continued*).

Site (region)	Reference	LI	s_u	s_u^{re}	σ'_v	σ'_p
		1.36	68.77	0.69	75.28	140.44
		3.14	78.18	0.09	77.46	146.95
Rio de Janeiro (Brasil), FV tests, $S_t = 2.0 \sim 4.8$	Anderson et al. (1982)	1.11	4.47	1.30	5.11	20.18
		1.39	5.69	1.52	6.46	20.72
		1.26	8.28	1.74	8.34	21.26
		1.63	8.47	1.96	10.76	23.68
		1.20	6.78	2.39	12.91	26.64
		1.24	5.38	2.72	14.53	28.52
		1.21	9.12	2.72	16.68	30.67
		1.18	9.84	3.04	18.57	33.63
		1.42	13.75	3.26	20.45	36.05
		1.18	14.40	3.80	22.87	38.48
Shellhaven (UK), FV tests, $S_t = 2.1 \sim 7.0$	Skempton and Henkel (1953)	0.63	12.62	2.04	23.50	31.39
		0.66	16.66	3.47	38.41	37.53
		0.59	17.89	3.73	39.28	39.28
		0.72	15.25	2.99	45.42	44.54
		0.63	16.31	3.20	48.93	48.05
		0.76	16.66	3.27	50.68	49.81
		0.45	14.73	2.10	58.57	56.82
		0.66	15.78	2.25	60.33	60.33
		0.76	18.24	2.90	67.34	68.22
		0.67	18.59	2.95	68.22	69.97
		0.82	19.47	3.09	70.85	70.85
		0.80	20.35	3.23	74.36	73.48
		0.67	21.05	3.34	76.99	75.24
		0.42	16.84	8.02	82.25	82.25
Bäckebol, Goteborg (Sweden), FV tests, $S_t = 11.0 \sim 31.9$	Massarch et al. (1975)	0.88	17.70	1.61	26.66	36.10
		1.34	16.26	0.51	36.44	44.54
		1.11	16.55	0.59	41.50	51.63
Järva Krog, Stockholm (Sweden), FV tests, $S_t = 20.1 \sim 25.8$	Massarch et al. (1975)	1.10	18.60	0.72	48.79	71.97
		1.23	21.05	1.05	61.73	69.54
		1.46	25.83	1.12	74.39	79.51
Skå Edeby, Stockholm (Sweden), FV tests, $S_t = 8.0 \sim 23.0$	Massarch et al. (1975)	0.95	11.12	1.39	12.45	24.12
		1.82	6.91	0.38	21.01	24.90
		1.76	10.52	0.75	31.52	38.13
		1.60	13.82	0.60	44.75	40.08
		1.52	15.02	0.72	59.14	59.53
Kalix (Sweden,) FV tests, $S_t = 10.0 \sim 17.1$	Massarch et al. (1975)	0.83	13.51	0.79	14.98	40.07
		0.87	14.81	0.99	16.48	31.84
		0.78	15.83	1.58	23.22	37.83
Ursvik (Sweden), FV tests, $S_t = 14.1 \sim 27.2$	Massarch et al. (1975)	1.33	5.62	0.40	11.28	29.57
		2.77	6.90	0.38	20.23	49.42
		1.77	7.22	0.33	25.68	50.19
		1.34	8.99	0.33	31.91	62.26
		1.29	11.72	0.45	38.13	55.64
		1.18	11.24	0.43	44.75	91.83
		1.32	16.06	0.94	58.37	100.78

Table 1 (*continued*).

Site (region)	Reference	LI	s_u	s_u^{re}	σ'_v	σ'_p
Bangkok-Siracha Highway km 28, Bangkok (Thailand), FV tests, $S_t = 5.3 \sim 9.6$	Eide and Holmberg (1972)	0.93	8.64	1.40	11.77	26.31
		0.90	9.11	1.17	12.92	26.77
		0.87	10.27	1.17	15.69	30.46
		0.79	10.27	1.40	16.84	31.84
		0.92	12.14	1.40	19.15	34.61
		0.91	12.14	1.40	19.61	35.31
		0.87	12.14	1.87	20.08	35.77
		0.85	12.14	1.87	21.00	36.69
		0.83	12.38	2.10	21.46	37.15
		0.82	12.38	2.10	21.92	37.84
		0.72	14.71	2.57	26.08	43.15
		0.76	14.94	2.80	26.54	43.84
		0.82	15.41	2.80	27.23	44.54
		0.86	15.64	2.80	27.46	45.00
		0.86	16.35	2.80	27.69	45.00
		0.87	16.58	2.33	28.15	45.46
		0.89	17.05	2.33	28.38	46.38
		0.86	17.51	1.87	29.31	47.07
		0.82	17.98	1.87	29.77	47.54
		0.78	17.98	1.87	30.23	48.00
		0.80	17.98	2.10	30.69	48.69
		0.81	18.21	2.10	31.15	49.15
		0.76	18.45	2.33	31.61	49.61
Bäckebol, Goteburg (Sweden), FV tests, $S_t = 15.0 \sim 24.0$	Massarch and Broms (1976)	0.77	18.18	1.21	55.43	81.77
		1.19	26.62	1.11	66.63	75.07
Megurie (Japan), FV tests, $S_t = 6.8 \sim 22.2$	Ohtsubo et al. (2007)	0.98	5.95	0.88	3.70	13.87
		1.31	4.29	0.59	7.40	12.95
		1.78	5.07	0.39	13.87	9.25
		1.51	5.00	0.39	17.57	17.57
		1.31	5.95	0.39	21.27	20.35
		1.34	6.43	0.59	24.05	21.27
		1.63	7.62	0.39	27.75	24.05
		1.42	10.48	0.68	31.45	24.97
		1.45	7.86[2]	0.68	35.14	29.60
		1.27	12.38	0.78	39.77	29.60
		1.21	13.10	0.88	44.39	30.52
		1.38	13.81	0.98	49.02	36.07
		1.45	17.38	1.18	51.79	55.49
		1.51	13.10	1.37	58.27	60.12
		1.22	18.57	0.98	61.97	48.09
		1.18	17.14	0.88	66.59	72.14
		0.93	26.19	1.18	71.21	97.11
Okishin (Japan), FV tests, $S_t = 4.2 \sim 17.3$	Ohtsubo et al. (2007)	1.18	6.19[2]	1.47	14.30	23.84
		1.13	10.95	1.52	44.50	46.09
		1.37	14.76	1.41	63.58	58.81
		1.41	15.71	1.41	81.06	76.29
		1.13	19.05	1.10	96.95	92.19
		1.15	21.90	1.39	114.44	108.08
		0.82	28.57	1.98	147.82	155.76
		0.90	26.67	2.21	165.30	162.12
		1.76	29.05	2.34	184.37	235.23

Table 1 (*concluded*).

Site (region)	Reference	LI	s_u	s_u^{re}	σ'_v	σ'_p
		0.69	32.38	5.61	205.03	225.70
Isahaya (Japan), FV tests, $S_t = 5.9 \sim 198.1$	Ohtsubo et al. (2007)	1.58	49.52	0.25	12.97	9.73
		1.55	43.33	0.65	18.38	17.30
		1.42	43.81	0.56	24.86	21.62
		1.46	52.86	0.54	33.51	25.95
		1.47	52.38	0.71	40.00	29.19
		1.27	55.71	0.73	47.57	38.92
		1.33	51.43	1.35	58.38	51.89
		1.28	42.86	1.15	67.03	50.81
		1.22	23.81	1.21	76.76	68.11
		0.93	85.24	1.31	86.49	75.68
		0.76	56.19	9.55	95.14	100.54
Ushiya (Japan), FV tests, $S_t = 3.9 \sim 33.1$	Ohtsubo et al. (2007)	1.13	1.97	0.50	8.13	17.50
		1.21	3.35	0.22	11.25	30.63
		1.21	3.60	0.16	14.38	15.63
		1.52	4.53	0.39	16.25	21.88
		1.14	4.94	0.57	20.00	27.50
		1.29	4.48	0.65	22.50	22.50
		1.44	4.45	0.44	25.63	23.75
		1.31	7.41	0.44	28.75	31.88
		1.24	7.16	0.52	32.50	28.13
		1.29	9.73	0.39	35.63	37.50
		1.37	11.30	0.63	39.38	38.13
		1.14	10.72	0.68	41.88	39.38
		1.41	13.55	1.01	46.88	45.00
		1.71	16.85	1.47	51.25	73.13
		1.17	19.54	0.59	55.00	70.63
		1.15	23.72	1.17	59.38	73.75
		1.06	31.80	1.43	65.00	105.63
Louiseville, Montréal (Canada), FV tests, $S_t = 6.1 \sim 23.9$	Silvestri and Aubertin (1988)	0.64	19.17	3.14	11.27	80.72
		1.30	21.16	1.09	16.18	91.34
		1.27	25.14	1.05	21.90	101.96
		1.00	29.12	1.37	27.61	111.77
		1.41	32.44	1.49	32.52	122.39
		1.00	36.42	1.80	37.42	133.01
		1.25	40.07	1.76	43.95	142.81
		1.00	48.03	2.15	55.39	164.05
I-95 Interchange, Portsmouth (USA), FV tests, $S_t = 23.9 \sim 73.5$	Ladd et al. (1972)	1.59	12.41	0.47	26.75	74.13
		1.58	11.49	0.48	30.57	62.67
		1.64	11.03	0.43	32.10	61.14
		2.52	11.03	0.15	32.86	58.85
		1.91	12.26	0.30	44.33	64.19
		1.76	14.87	0.37	53.50	71.84
		1.75	15.48	0.37	55.02	74.13
		1.89	11.03	0.31	56.55	74.13
		2.11	11.80	0.23	61.90	80.24
		2.05	12.72	0.25	68.02	86.36
		1.57	13.79	0.49	75.66	94.76

Note: FV, field vane; UC, unconfined compression.

*s_u^{re} values are adjusted based on the relation developed by Locat and Demers (1988) as shown in their Table 1.

directly compared because s_u typically depends on stress state, strain rate, sampling disturbance, etc. By following the recommendations made by Bjerrum (1972) and Mesri and Huvaj (2007), these s_u values are all converted to the mobilized s_u values, denoted by s_u(mob), which is the in situ undrained shear strength mobilized in embankment and slope failures. The conversion of s_u(UC) to s_u(mob) is not necessary, as Mesri and Huvaj (2007) suggested that $s_u(\text{mob}) \approx s_u(\text{UC})$. For field vane tests, the PI-based correction factor proposed by Bjerrum (1972) is used to convert s_u(FV) into the back-calculated s_u values for a collection of actual failure cases. These back-calculated values were called design values of s_u by Bjerrum (1972), denoted herein by s_u(design). Our interpretation is that $s_u(\text{mob}) \approx s_u(\text{design})$ because most of the actual failure cases are embankments. The conversion from s_u(cone penetration test) to s_u(mob) is also suggested by Mesri and Huvaj (2007).

There are seven sites in two literature papers (see the asterisks in Table 1) whose reported s_u^{re} values are unreasonably high compared to the LI–s_u^{re} relations summarized by previous studies (e.g., Mitchell 1976, Leroueil et al. 1983, Locat and Demers 1988). For these data points, the s_u^{re} values are replaced by the s_u^{re} values estimated from the LI–s_u^{re} empirical equation developed by Locat and Demers (1988). The reason for this inconsistency is unknown, but it is possible that the remolded state was not yet reached when those s_u^{re} values were recorded.

Validation database

In literature, most data points do not have complete {LI, s_u, s_u^{re}, σ'_p, σ'_v} information, but only have partial information, i.e., a subset of {LI, s_u, s_u^{re}, σ'_p, σ'_v} is known. For example, it could be simply {LI, s_u^{re}} or normalized parameters {σ'_p/σ'_v, s_u/σ'_v}, etc. These data points are left out of the calibration database as the development of the multivariate probability distribution of {LI, s_u, s_u^{re}, σ'_p, σ'_v} requires complete information. Nonetheless, these data points can be taken to validate the conclusions derived from the calibration database.

For the validation database, there are 1792 data points from 177 sites. Table 2 lists the numbers of available data points for each correlation of interest. The geographical coverage is quite wide: 49 sites in Canada, 33 in Norway, 20 in USA, 15 in UK, 17 in Japan, 15 in Sweden, three in Ireland, four in Singapore, three in Italy, others in Hong Kong, Thailand, New Zealand, Australia, Mexico, South Africa, Malaysia, Brazil, India, Poland, Iraq, etc. The clay properties cover a wide range of sensitivity (1 ~ several hundred), OCR (1 ~ 14; two sites > 20), and LI (0 ~ 6.5). The clay types are also quite broad. Organic and highly OC (fissured) clays are absent in this database. For the validation database, the s_u and s_u^{re} values of different test types are similarly converted to s_u(mob). The detailed information and references for the validation database can be found in Hsue (2010).

Construction of multivariate normal distribution

The main purpose of this section is to demonstrate how the multivariate data points in the calibration database, after a proper nonlinear transform, can be modeled as a multivariate normal distribution. The multivariate normal probability density function is available analytically and can be defined uniquely by a mean vector, $\boldsymbol{\mu}$, and a covariance matrix, $\mathbf{C}$:

Table 2. Available numbers of data points in the validation database.

Correlation	Number of data points
LI versus S_t	856
LI versus s_u^{re}	358
{LI, S_t} versus s_u	431
{LI, S_t} versus σ'_p	309
LI versus OCR	416
OCR versus s_u/σ'_v	249
s_u versus σ'_p	355

$$[8] \quad f(\boldsymbol{X}) = |\mathbf{C}|^{-1/2}(2\pi)^{-n/2}e^{-(1/2)(\boldsymbol{X}-\boldsymbol{\mu})'\mathbf{C}^{-1}(\boldsymbol{X}-\boldsymbol{\mu})}$$

in which $\boldsymbol{X} = (X_1, X_2, \ldots, X_n)'$ is a normal random vector with n components; the prime symbol represents the matrix–vector transpose. For $n = 3$, the mean vector and covariance matrix are given by

$$[9] \quad \boldsymbol{\mu} = \begin{Bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{Bmatrix} \quad \mathbf{C} = \begin{bmatrix} \sigma_1^2 & \delta_{12}\sigma_1\sigma_2 & \delta_{13}\sigma_1\sigma_3 \\ \delta_{12}\sigma_1\sigma_2 & \sigma_2^2 & \delta_{23}\sigma_2\sigma_3 \\ \delta_{13}\sigma_1\sigma_3 & \delta_{23}\sigma_2\sigma_3 & \sigma_3^2 \end{bmatrix}$$

in which μ_i and σ_i are the mean and standard deviation of X_i, respectively, and δ_{ij} is the product-moment (Pearson) correlation between X_i and X_j. If X_i is a standard normal random variable with zero mean and unity standard deviation (i.e., $\mu_i = 0$ and $\sigma_i = 1$), eq. [9] reduces to

$$[10] \quad \boldsymbol{\mu} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad \mathbf{C} = \begin{bmatrix} 1 & \delta_{12} & \delta_{13} \\ \delta_{12} & 1 & \delta_{23} \\ \delta_{13} & \delta_{23} & 1 \end{bmatrix}$$

It is clear that the full multivariate dependency structure of a normal random vector only depends on a covariance matrix ($\mathbf{C}$) containing bivariate information (correlations) between all possible pairs of components, namely X_1 and X_2, X_1 and X_3, and X_2 and X_3. The practical advantage of capturing multivariate dependencies in any dimension (i.e., any number of random variables) using only bivariate dependency information is obvious. The most evident disadvantage is that most soil parameters are not normally distributed.

Multivariate nonnormal distribution

Most soil parameters are not normally distributed as they are typically nonnegative. Let Y_1, Y_2, and Y_3 denote three nonnormally distributed soil parameters. One well-known cumulative distribution function (CDF) transform approach for constructing a valid multivariate distribution for these soil parameters is

1. Define $X_i = \Phi^{-1}[F_i(Y_i)]$, in which $\Phi^{-1}()$ is the inverse standard normal cumulative distribution function and $F_i()$ is the cumulative distribution function of Y_i. By definition, X_1, X_2, and X_3 are *individually* standard normal random variables. That is, the histogram of any component, X_i, will look normal (bell-shaped).

2. Assume (X_1, X_2, X_3) follows a multivariate normal distribution as defined by eq. [9] with $\boldsymbol{\mu}$ and $\mathbf{C}$ given by eq. [10]. It is crucial to note here that *collectively* (X_1, X_2, X_3) does not necessarily follow a multivariate normal distribution even if each component is normally distributed. For example, if the scatter plot of X_i versus X_j shows a distinct nonlinear trend, then the multivariate normal distribution assumption is incorrect.

For illustration, consider a common example involving Y_1, Y_2, and Y_3 as lognormal random variables. It can be shown that step 1 results in

$$[11] \quad \begin{aligned} X_i &= \Phi^{-1}[F_i(Y_i)] = [\ln(Y_i) - \lambda_i]/\xi_i \\ \lambda_i &= \ln(m_i) - 0.5\xi_i^2 \\ \xi_i^2 &= \ln(1 + s_i^2/m_i^2) \end{aligned}$$

where m_i is the mean of Y_i, s_i is the standard deviation of Y_i, λ_i is the mean of $\ln(Y_i)$, and ξ_i is the standard deviation of $\ln(Y_i)$. Step 2 results in

$$[12] \quad \rho_{ij} = \frac{\exp(\xi_i\xi_j\delta_{ij}) - 1}{\sqrt{[\exp(\xi_i^2) - 1][\exp(\xi_j^2) - 1]}}$$

in which ρ_{ij} is the product-moment (Pearson) correlation between Y_i and Y_j.

Simulation

It is simple to obtain realizations of *independent* standard normal random variables (U_1, U_2, U_3) using library functions in many software programs. Realizations of *correlated* standard normal random variables (X_1, X_2, X_3) can be obtained using $\boldsymbol{X} = \mathbf{L}\boldsymbol{U}$, in which $\mathbf{L}$ is the lower triangular Cholesky factor satisfying $\mathbf{C} = \mathbf{L}\mathbf{L}'$. Finally, from eq. [11], each soil parameter is obtained using $Y_i = \exp(\lambda_i + \xi_i X_i)$.

Application to calibration database

The aforementioned CDF transform approach is applied to the calibration database, where Y_1 = LI, $Y_2 = s_u$, $Y_3 = s_u^{re}$, $Y_4 = \sigma'_p$, and $Y_5 = \sigma'_v$. Each set of {LI, s_u, s_u^{re}, σ'_p, σ'_v} constitutes one data point. The marginal distributions F_1, F_2, …, F_5 can be found by comparing the histograms of the 345 data points with the fitted distributions. Although not shown, more accurate comparisons can be done by showing the quantile–quantile (Q–Q) plots. Figure 1 shows the histograms of Y_1, Y_2, …, Y_5 as well as the fitted lognormal distributions. In general, all plots show that the lognormal marginal distributions perform satisfactorily. Table 3 lists the statistics of the chosen lognormal distributions.

Based on the 345 data points, the scatter plots between $X_i = [\ln(Y_i) - \lambda_i]/\xi_i$ and $X_j = [\ln(Y_j) - \lambda_j]/\xi_j$ are shown in Fig. 2. It can be seen that the trends are fairly linear. Hence, for this particular soil database, there is no strong evidence to reject the underlying multivariate normality. The product-moment correlation matrix for (X_1, X_2, X_3, X_4, X_5) can be readily estimated, as shown in Table 4. If the underlying multivariate normality does hold exactly, the product-moment correlation matrix of (Y_1, Y_2, Y_3, Y_4, Y_5) can be computed using eq. [12], which is shown in Table 5. The product-moment correlation matrix of (Y_1, Y_2, Y_3, Y_4, Y_5) can also be computed directly using the samples of (Y_1, Y_2, Y_3, Y_4, Y_5), which is shown in Table 6. It is evident that the two covariance matrices in Tables 5 and 6 are similar, suggesting that the underlying multivariate normality is plausible.

Simulation results

The CDF transform approach is employed to simulate 100 000 samples of {LI, s_u, s_u^{re}, σ'_p, σ'_v} that have the similar multivariate probability distribution in the second-moment sense. The first 10 000 simulated samples, together with the calibration database, are shown in Fig. 3 for illustration (P_a in the figure is 101.3 kN/m^2). Not only the correlations among the original random variables {LI, s_u, s_u^{re}, σ'_p, σ'_v} are shown, but the correlations among their derived (normalized) quantities, including $S_t = s_u/s_u^{re}$, OCR = σ'_p/σ'_v, and s_u/σ'_v, are also shown. The CDF transform approach in constructing the multivariate probability distribution performs adequately. This is evident insofar as pairwise correlations are concerned. The simulated samples closely mimic the correlation behaviors of the calibration database, even for those with nonlinear trends, e.g., LI–s_u^{re} and LI–S_t correlations.

Validation of constructed multivariate distribution

The constructed multivariate distribution by the CDF transform approach must be validated. In this study, two types of qualitative validations are performed

1. Comparison between the simulated data points from the constructed multivariate distribution with the validation database. As mentioned earlier in the "Database" section, there is another validation database besides the calibration database. The data points in the validation database do not overlap with those in the calibration database. It is of interest to see if the validation database follows the same pattern as the data points simulated by the multivariate distribution.
2. Comparison between the simulated data points from the multivariate distribution with the empirical correlation equations in the literature.

LI–S_t and LI–s_u^{re} correlations

Figure 4 shows the validation results for the LI–S_t and LI–s_u^{re} correlations. The behaviors of the validation database are similar to those of the simulated data. For the LI–S_t correlation, the validation is also taken with respect to empirical equations provided by Bjerrum (1954) and Eden and Kubota (1961). These two empirical equations are consistent with the simulated data trends for LI < 3.

For the LI–s_u^{re} correlation, Leroueil et al. (1983) and Locat and Demers (1988) provided empirical equations, while Mitchell (1976) provided empirical upper and lower bounds. Most equations seem consistent with the simulated data. Notice that the upper and lower bounds proposed by Mitchell (1976) do not cover the trends summarized by Leroueil et al. (1983) and Locat and Demers (1988) for LI > 2. This indicates that the bounds may not be applicable to clays with higher LI values.

LI–s_u, LI–σ'_p, and LI–OCR correlations

Figure 5 shows the comparisons between the simulated data and the validation data points for the LI–s_u correlation. The comparisons are summarized in the following more standard

Fig. 1. Histograms of Y_1, Y_2, …, Y_5 and the fitted lognormal distributions.

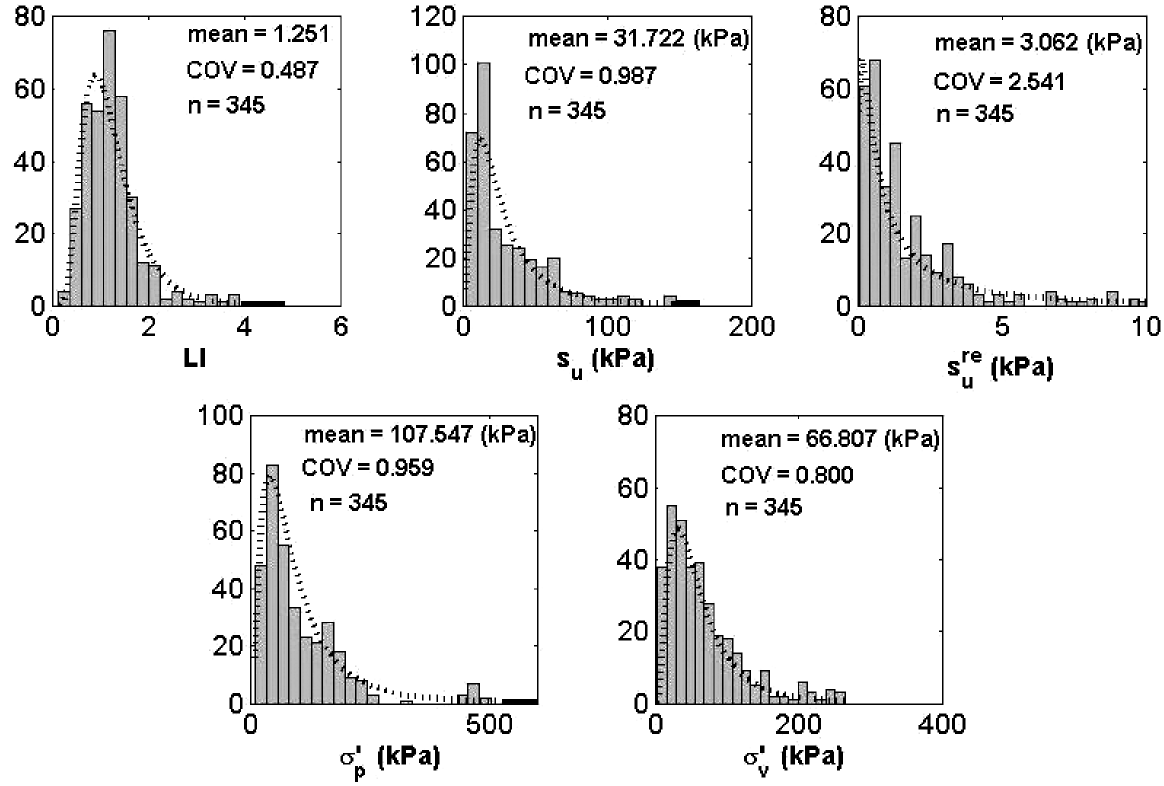


Table 3. Statistics of the chosen lognormal distributions for (Y_1, Y_2, Y_3, Y_4, Y_5).

	m_i	c.o.v. of Y_i, s_i/m_i	λ_i	ξ_i
Y_1 = LI	1.251	0.487	0.119	0.466
$Y_2 = s_u$	31.722 kN/m^2	0.987	3.033	0.931
$Y_3 = s_u^{re}$	3.062 kN/m^2	2.541	0.198	1.263
$Y_4 = \sigma'_p$	107.547 kN/m^2	0.960	4.344	0.804
$Y_5 = \sigma'_v$	66.807 kN/m^2	0.800	3.897	0.813

S_t ranges proposed by Mitchell (1976): $1 \le S_t \le 4$ (slightly to medium sensitive), $4 \le S_t \le 16$ (very sensitive to slightly quick), $16 \le S_t \le 64$ (medium quick to very quick), and $64 \le S_t$ (extra quick). The behaviors of the validation database are similar to those of the simulated data. The trends for eq. [4] given by Wood (1990) are also shown in Fig. 5. These trends (or bounds) in general agree with the simulated data points and the validation data (except for the case with $S_t > 64$), although the data points are more scattered than the bounds.

Figure 6 shows the comparisons for the LI–σ'_v correlation to the trends for eq. [5] given by Wood (1990). Again, the data points (simulated and validation) are more scattered than the bounds. For S_t less than 16, the validation data points seem to have σ'_v values somewhat less than the bounds, while the simulated data points are more consistent with the bounds. Figure 7 shows the comparisons for the LI–σ'_p correlation to the results summarized by Stas and Kulhawy (1984). Consistency between the validation data points and the simulated data points is again found. The empirical equations summarized by Stas and Kulhawy (1984) for the four S_t ranges are also shown in the figure. They are in general consistent with the validation and simulated data points.

Even when the S_t range is known, the correlation between LI and OCR is found to be weak. Figure 8 shows the simulated LI–OCR data as well as the validation data points for the four S_t ranges: again, consistency is evident. There seem to be LI-dependent upper bounds for the OCR values, as tabulated in the figure.

OCR–(s_u/σ'_v) and s_u–σ'_p correlations

It is well know that s_u/σ'_v is strongly correlated with OCR and that s_u is strongly correlated with σ'_p: the former is based on the SHANSEP concept (Ladd and Foott 1974), while the latter was confirmed by several previous studies such as Mesri (1993). Figure 9 compares the simulated data with the validation database for these two relations, indicating that the validation data behave similarly to the simulated data. Jamiolkowski et al. (1985) summarized that $s_u/\sigma'_v \approx 0.23 \times \mathrm{OCR}^{0.8}$, and Mesri (1993) reported that $s_u \approx 0.26 \times \sigma'_p$. These two empirical equations are shown in the figure. Again, consistency is found, except that the gradient of the simulated data seems higher. This may be due to the fact that the simulated data points are for structured clays.

Fig. 2. Examination of multivariate normality between $X_i = [\ln(Y_i) - \lambda_i]/\xi_i$ and $X_j = [\ln(Y_j) - \lambda_j]/\xi_j$ for the calibration database.

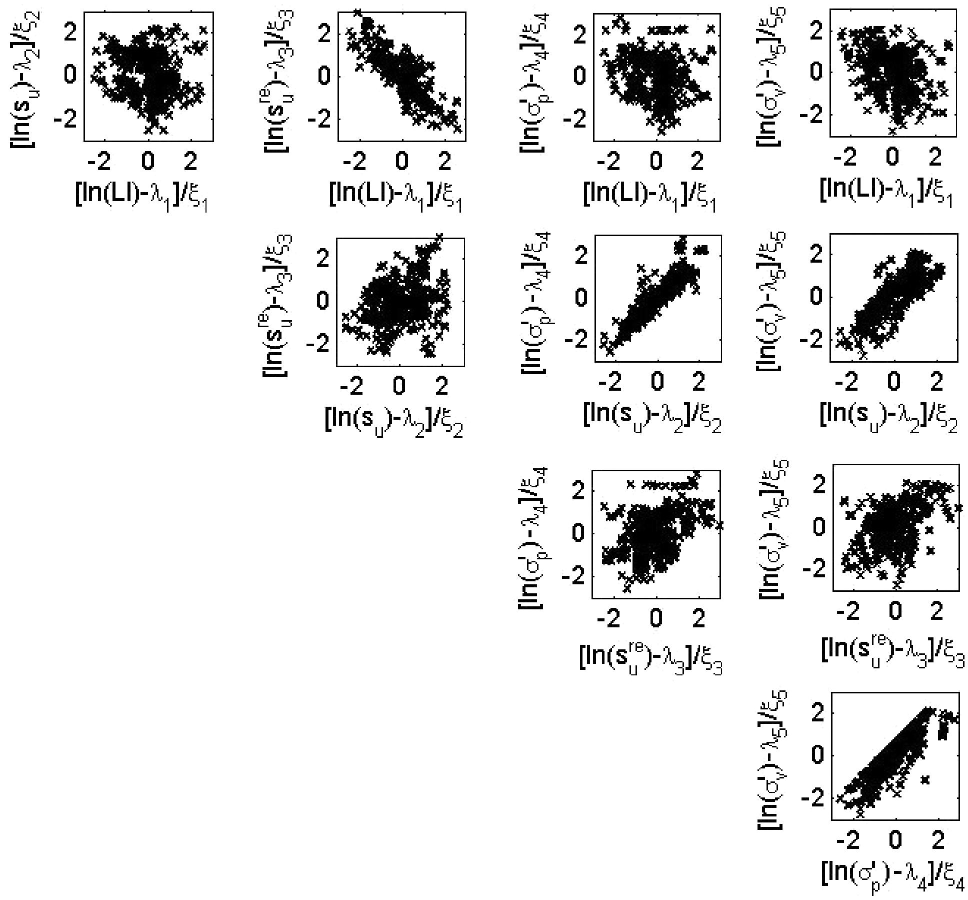


Table 4. Correlations among (X_1, X_2, X_3, X_4, X_5).

		X_1	X_2	X_3	X_4	X_5
$C =$	X_1	1.000	$\delta_{12} = -0.083$	$\delta_{13} = -0.824$	$\delta_{14} = -0.176$	$\delta_{15} = -0.280$
	X_2	$\delta_{21} = -0.083$	1.000	$\delta_{23} = 0.276$	$\delta_{24} = 0.915$	$\delta_{25} = 0.801$
	X_3	$\delta_{31} = -0.824$	$\delta_{32} = 0.276$	1.000	$\delta_{34} = 0.365$	$\delta_{35} = 0.453$
	X_4	$\delta_{41} = -0.176$	$\delta_{42} = 0.915$	$\delta_{43} = 0.365$	1.000	$\delta_{45} = 0.850$
	X_5	$\delta_{51} = -0.280$	$\delta_{52} = 0.801$	$\delta_{53} = 0.453$	$\delta_{54} = 0.850$	1.000

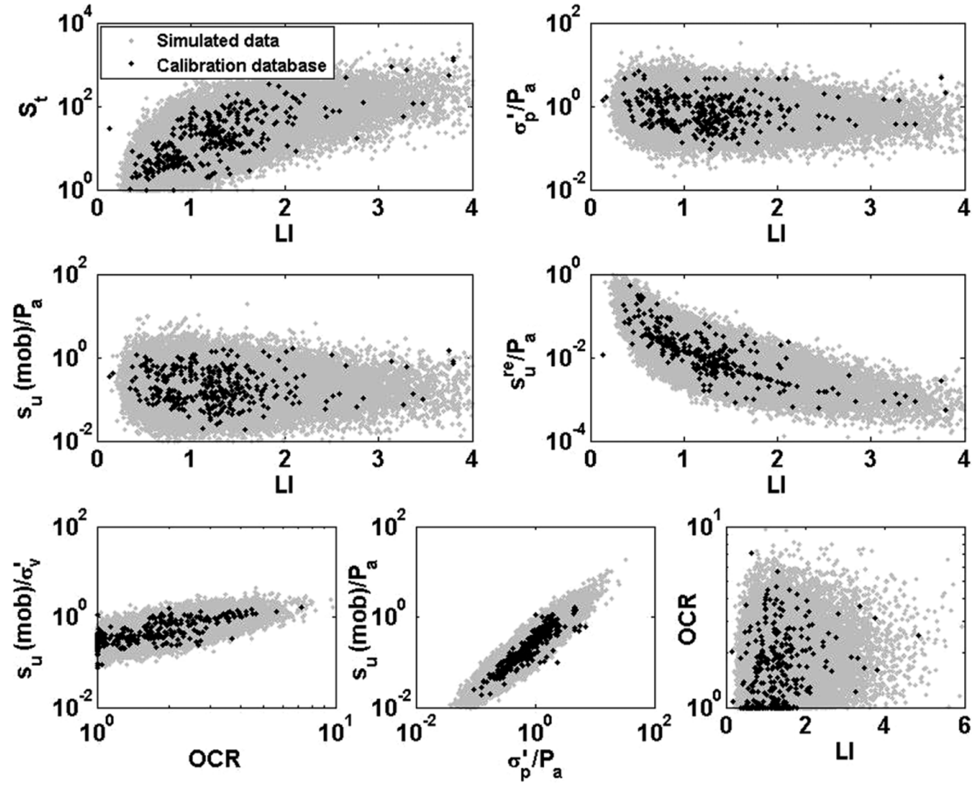
Table 5. Correlations among (Y_1, Y_2, Y_3, Y_4, Y_5) as predicted by eq. [12].

		Y_1	Y_2	Y_3	Y_4	Y_5
$R_1 =$	Y_1	1.000	$\rho_{12} = -0.047$	$\rho_{13} = -0.327$	$\rho_{14} = -0.094$	$\rho_{15} = -0.142$
	Y_2	$\rho_{21} = -0.047$	1.000	$\rho_{23} = 0.185$	$\rho_{24} = 0.871$	$\rho_{25} = 0.715$
	Y_3	$\rho_{31} = -0.327$	$\rho_{32} = 0.185$	1.000	$\rho_{34} = 0.256$	$\rho_{35} = 0.334$
	Y_4	$\rho_{41} = -0.094$	$\rho_{42} = 0.871$	$\rho_{43} = 0.256$	1.000	$\rho_{45} = 0.780$
	Y_5	$\rho_{51} = -0.142$	$\rho_{52} = 0.715$	$\rho_{53} = 0.334$	$\rho_{54} = 0.780$	1.000

Table 6. Actual correlations among (Y_1, Y_2, Y_3, Y_4, Y_5).

		Y_1	Y_2	Y_3	Y_4	Y_5
	Y_1	1.000	$\rho_{12} = -0.069$	$\rho_{13} = -0.389$	$\rho_{14} = -0.141$	$\rho_{15} = -0.210$
	Y_2	$\rho_{21} = -0.069$	1.000	$\rho_{23} = 0.162$	$\rho_{24} = 0.885$	$\rho_{25} = 0.737$
$R_2 =$	Y_3	$\rho_{31} = -0.389$	$\rho_{32} = 0.162$	1.000	$\rho_{34} = 0.239$	$\rho_{35} = 0.306$
	Y_4	$\rho_{41} = -0.141$	$\rho_{42} = 0.885$	$\rho_{43} = 0.239$	1.000	$\rho_{45} = 0.809$
	Y_5	$\rho_{51} = -0.210$	$\rho_{52} = 0.737$	$\rho_{53} = 0.306$	$\rho_{54} = 0.809$	1.000

Fig. 3. Comparisons between the calibration database and the first 10 000 simulated data points.



Updated mean values and coefficients of variation of soil parameters

Given the multivariate normal distribution, it is possible to derive the mean and coefficient of variation (c.o.v.) of any parameter for a structured clay given the information of the subset of other parameters systematically for further Bayesian inference. If $\boldsymbol{U}$ and $\boldsymbol{V}$ are multivariate normal distribution, given the information of $\boldsymbol{U}$, it can be shown that $\boldsymbol{V}$ is still a multivariate normal. Moreover, the updated mean vector and covariance for $\boldsymbol{V}$ are given by the following analytical form:

$$[13]\quad \begin{aligned} &\mathrm{E}(\boldsymbol{V}|\boldsymbol{U}) = \mathrm{E}(\boldsymbol{V}) + \mathrm{COV}(\boldsymbol{V}, \boldsymbol{U})\, \mathrm{VAR}^{-1}(\boldsymbol{U})[\boldsymbol{U} - \mathrm{E}(\boldsymbol{U})] \\ &\mathrm{VAR}(\boldsymbol{V}|\boldsymbol{U}) = \mathrm{VAR}(\boldsymbol{V}) - \mathrm{COV}(\boldsymbol{V}, \boldsymbol{U})\, \mathrm{VAR}^{-1}(\boldsymbol{U})\, \mathrm{COV}(\boldsymbol{V}, \boldsymbol{U})^{\mathrm{T}} \end{aligned}$$

where E, VAR, and COV denote mean vector, covariance matrix of a vector, and covariance matrix between two different vectors, respectively. Consider the example where $\ln(\mathrm{LI}) = \ln(Y_1)$, $\ln(\sigma'_\mathrm{v}) = \ln(Y_5)$, and $\ln(S_\mathrm{t}) = \ln(Y_2) - \ln(Y_3)$ are known. Given the above information, it can be shown that $\ln(s_\mathrm{u}) = \ln(Y_2)$ is still normal with the following updated mean and standard deviation:

Fig. 4. Verification for the LI–S_t and LI–s_u^{re} correlations. The upper two plots show the verification with the validation database, while the lower two plots show the verification with literature.

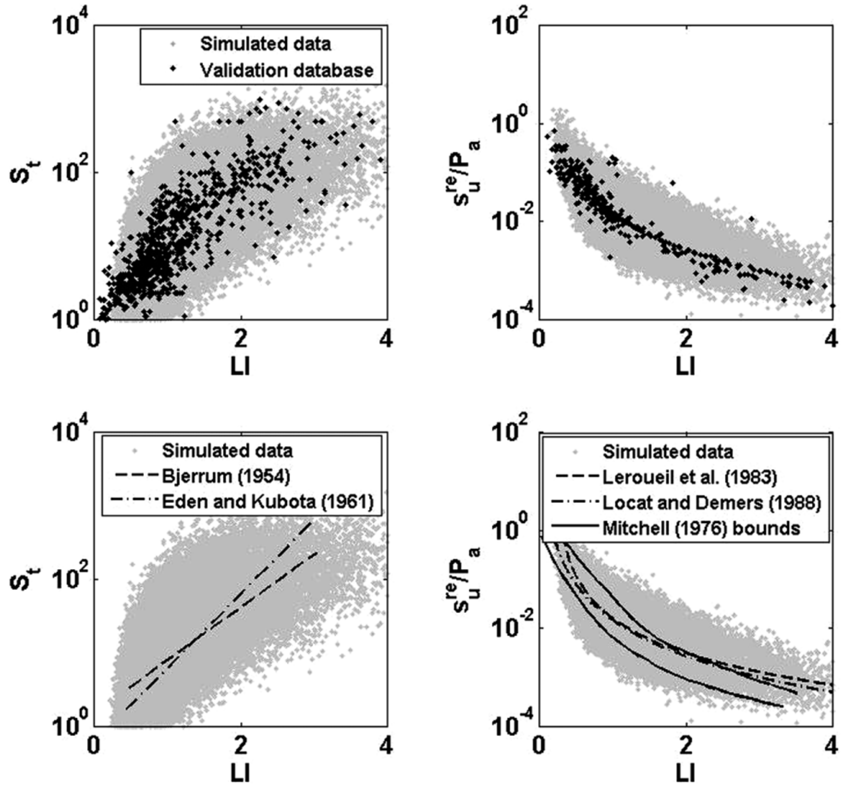


Fig. 5. Comparison between simulated data and validation data using the LI–s_u(mob) correlation.

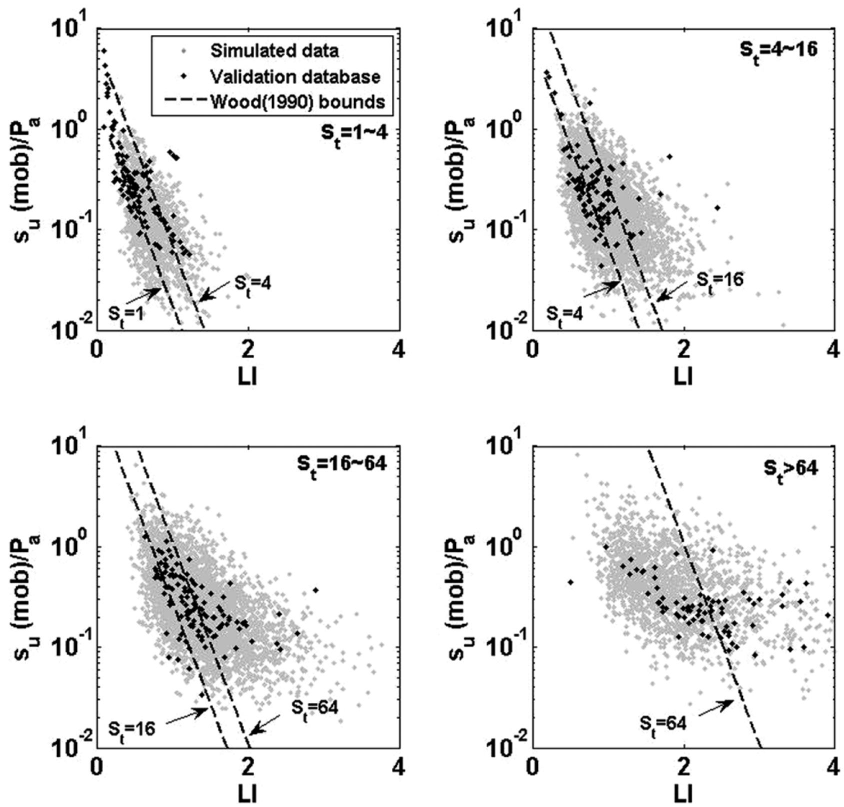


Fig. 6. Validation for the LI–σ'_v correlation.

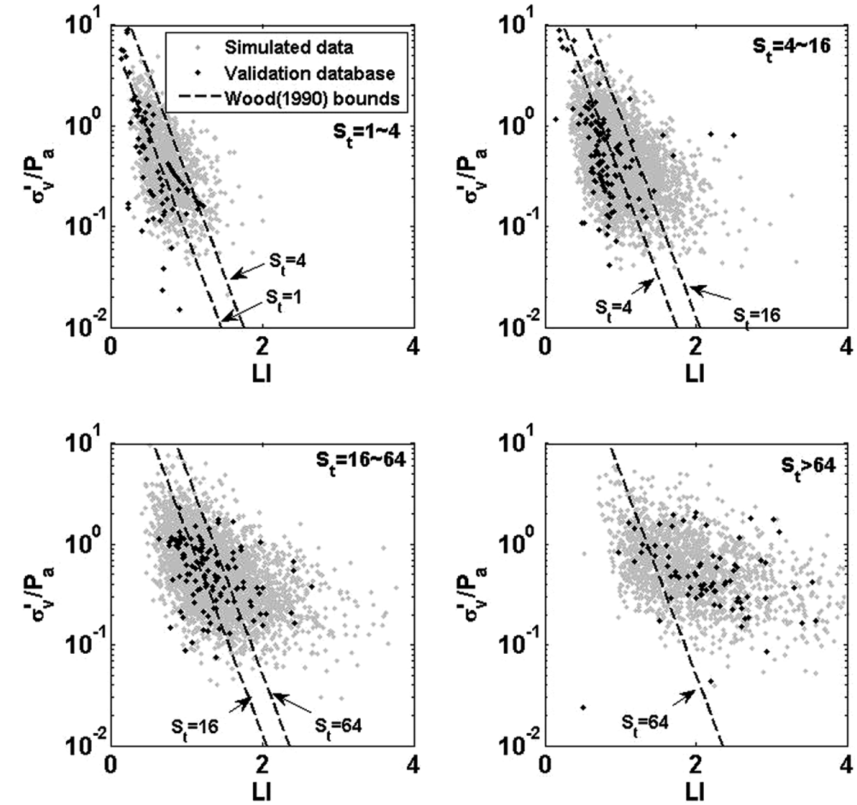


Fig. 7. Validation for the LI–σ'_p correlation.

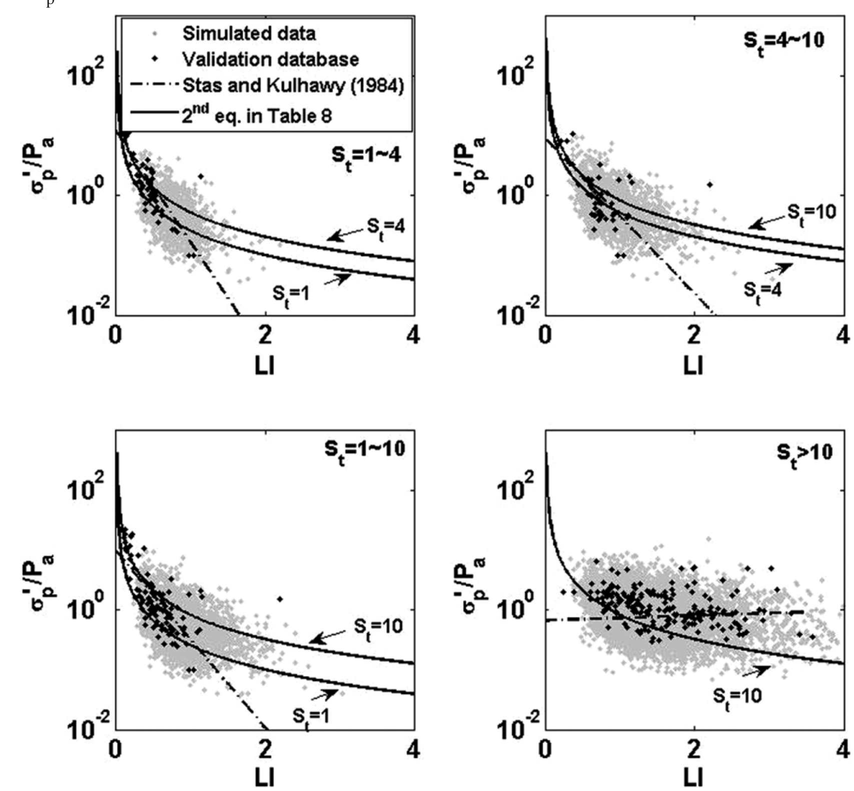


Fig. 8. Validation for the LI–OCR correlation.

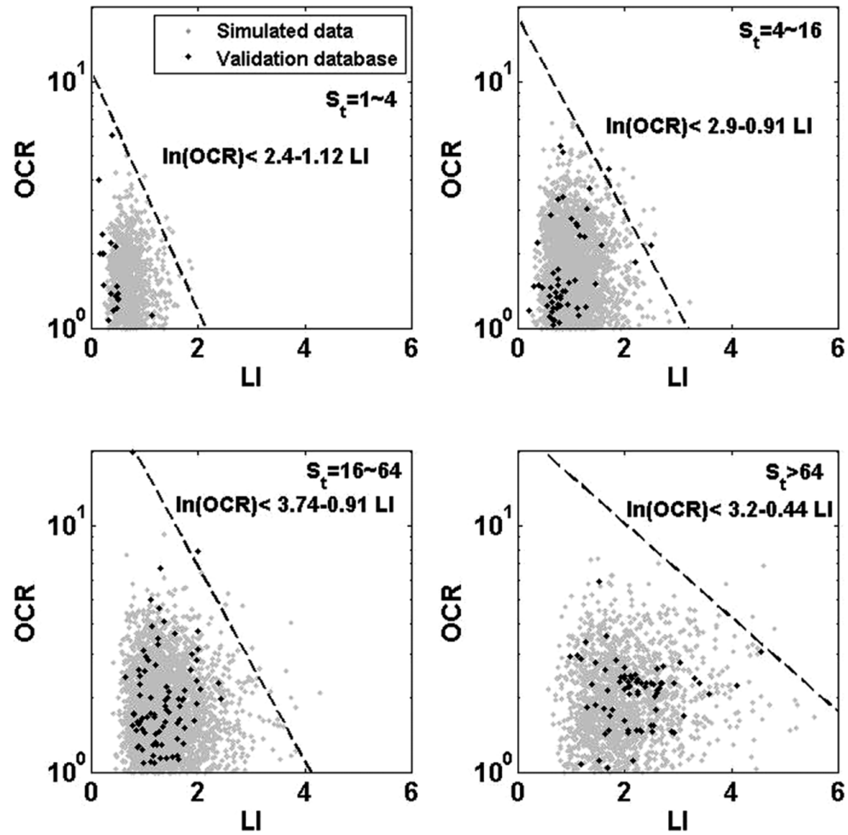
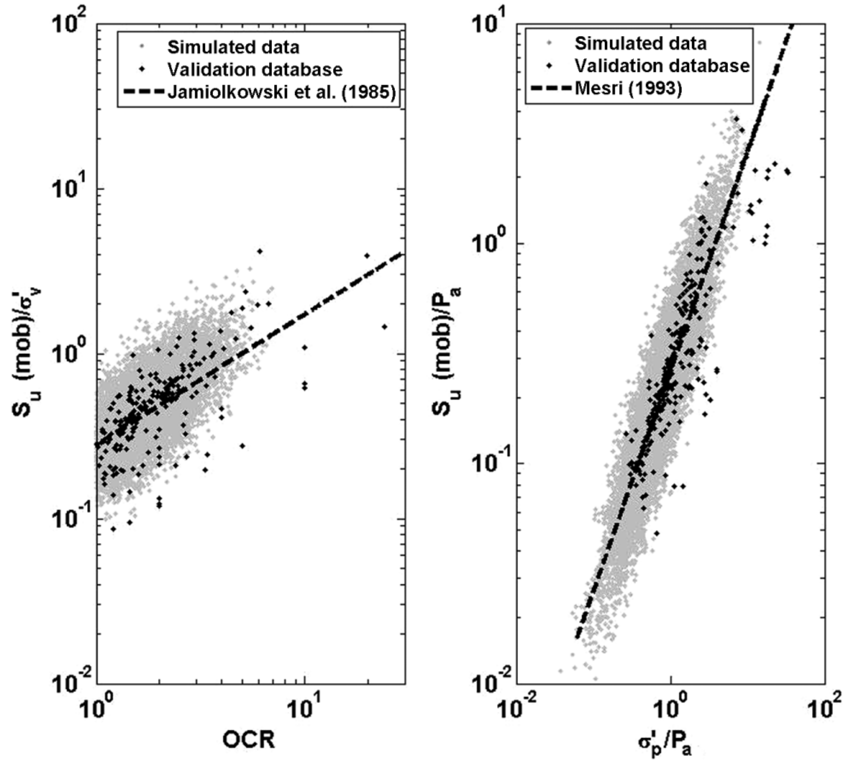


Fig. 9. Validation for the OCR–(s_u/σ'_v) and s_u–σ'_p correlations.



$$[14]\quad \lambda_2' = \lambda_2 + [\,\delta_{12}\xi_1\xi_2 \quad \delta_{25}\xi_2\xi_5 \quad \xi_2^2 - \delta_{23}\xi_2\xi_3\,]\begin{bmatrix} \xi_1^2 & \delta_{15}\xi_1\xi_5 & \delta_{12}\xi_1\xi_2 - \delta_{13}\xi_1\xi_3 \\ & \xi_5^2 & \delta_{25}\xi_2\xi_5 - \delta_{35}\xi_3\xi_5 \\ \mathrm{SYM} & & \xi_2^2 + \xi_3^2 - 2\delta_{23}\xi_2\xi_3 \end{bmatrix}^{-1}\begin{bmatrix} \ln(\mathrm{LI}) - \lambda_1 \\ \ln(\sigma_v') - \lambda_5 \\ \ln(S_t) - \lambda_2 + \lambda_3 \end{bmatrix}$$

$$= 3.033 + [\,-0.036 \quad 0.607 \quad 0.543\,]\begin{bmatrix} 0.217 & -0.106 & 0.449 \\ & 0.661 & 0.141 \\ \mathrm{SYM} & & 1.814 \end{bmatrix}^{-1}\begin{bmatrix} \ln(\mathrm{LI}) - 0.119 \\ \ln(\sigma_v') - 3.897 \\ \ln(S_t) - 2.835 \end{bmatrix}$$

$$= -0.6384\ln(\mathrm{LI}) + 0.7294\ln(\sigma_v') + 0.4007\ln(S_t) - 0.8696$$

$$\xi_2'^2 = \xi_2^2 - [\,\delta_{12}\xi_1\xi_2 \quad \delta_{25}\xi_2\xi_5 \quad \xi_2^2 - \delta_{23}\xi_2\xi_3\,]\begin{bmatrix} \xi_1^2 & \delta_{15}\xi_1\xi_5 & \delta_{12}\xi_1\xi_2 - \delta_{13}\xi_1\xi_3 \\ & \xi_5^2 & \delta_{25}\xi_2\xi_5 - \delta_{35}\xi_3\xi_5 \\ \mathrm{SYM} & & \xi_2^2 + \xi_3^2 - 2\delta_{23}\xi_2\xi_3 \end{bmatrix}^{-1}\begin{bmatrix} \delta_{12}\xi_1\xi_2 \\ \delta_{25}\xi_2\xi_5 \\ \xi_2^2 - \delta_{23}\xi_2\xi_3 \end{bmatrix} = 0.184$$

where λ_2' and ξ_2' are the updated mean and standard deviation of $\ln(Y_2)$, respectively. The updated mean m_2' and c.o.v. s_2'/m_2' for s_u (= Y_2) are

$$[15]\quad \begin{aligned} m_2' &= \exp(\lambda_2' + \xi_2'^2/2) = \mathrm{LI}^{-0.638} \times \sigma_v'^{\,0.729} \times S_t^{0.401} \times 0.460 \\ s_2'/m_2' &= \sqrt{\exp(\xi_2'^2) - 1} = 0.450 \end{aligned}$$

Consider another example where $\ln(\mathrm{OCR}) = \ln(Y_4) - \ln(Y_5)$ and $\ln(S_t) = \ln(Y_2) - \ln(Y_3)$ are known. Given the above information, it can be shown that $\ln(s_u/\sigma_v') = \ln(Y_2) - \ln(Y_5)$ is still normal with the following updated mean and standard deviation:

$$[16]\quad \lambda' = \lambda_2 - \lambda_5 + [\,\delta_{24}\xi_2\xi_4 + \xi_5^2 - \delta_{25}\xi_2\xi_5 - \delta_{45}\xi_4\xi_5 \quad \xi_2^2 + \delta_{35}\xi_3\xi_5 - \delta_{23}\xi_2\xi_3 - \delta_{25}\xi_2\xi_5\,]$$

$$\times\begin{bmatrix} \xi_4^2 + \xi_5^2 - 2\delta_{45}\xi_4\xi_5 & \delta_{24}\xi_2\xi_4 + \delta_{35}\xi_3\xi_5 - \delta_{25}\xi_2\xi_5 - \delta_{34}\xi_3\xi_4 \\ \mathrm{SYM} & \xi_2^2 + \xi_3^2 - 2\delta_{23}\xi_2\xi_3 \end{bmatrix}^{-1}\begin{bmatrix} \ln(\mathrm{OCR}) - \lambda_4 + \lambda_5 \\ \ln(S_t) - \lambda_2 + \lambda_3 \end{bmatrix}$$

$$= 3.033 - 3.897 + [\,0.184 \quad 0.402\,]\begin{bmatrix} 0.196 & 0.174 \\ \mathrm{SYM} & 1.814 \end{bmatrix}^{-1}\begin{bmatrix} \ln(\mathrm{OCR}) - 0.446 \\ \ln(S_t) - 2.836 \end{bmatrix}$$

$$= 0.810\ln(\mathrm{OCR}) + 0.144\ln(S_t) - 1.634$$

$$\xi' = \xi_2^2 - [\,\delta_{24}\xi_2\xi_4 + \xi_5^2 - \delta_{25}\xi_2\xi_5 - \delta_{45}\xi_4\xi_5 \quad \xi_2^2 + \delta_{35}\xi_3\xi_5 - \delta_{23}\xi_2\xi_3 - \delta_{25}\xi_2\xi_5\,]$$

$$\times\begin{bmatrix} \xi_4^2 + \xi_5^2 - 2\delta_{45}\xi_4\xi_5 & \delta_{24}\xi_2\xi_4 + \delta_{35}\xi_3\xi_5 - \delta_{25}\xi_2\xi_5 - \delta_{34}\xi_3\xi_4 \\ \mathrm{SYM} & \xi_2^2 + \xi_3^2 - 2\delta_{23}\xi_2\xi_3 \end{bmatrix}^{-1}\begin{bmatrix} \delta_{24}\xi_2\xi_4 + \xi_5^2 - \delta_{25}\xi_2\xi_5 - \delta_{45}\xi_4\xi_5 \\ \xi_2^2 + \delta_{35}\xi_3\xi_5 - \delta_{23}\xi_2\xi_3 - \delta_{25}\xi_2\xi_5 \end{bmatrix}$$

$$= 0.108$$

where λ' and ξ' are the updated mean and standard deviation of $\ln(s_u/\sigma_v')$, respectively. The updated mean, m', and c.o.v., s'/m', for s_u/σ_v' are

$$[17]\quad \begin{aligned} m' &= \exp(\lambda' + \xi'^2/2) = \mathrm{OCR}^{0.810} \times S_t^{0.144} \times 0.206 \\ s'/m' &= \sqrt{\exp(\xi'^2) - 1} = 0.338 \end{aligned}$$

Tables 7–9 list the equations for the updated means and c.o.v.s for normalized s_u (s_u/σ_v'), normalized σ_p' (σ_p'/P_a), and $S_t = s_u/s_u^{re}$ under various combinations of information. For the estimation of s_u/σ_v', the second equation in Table 7 can be related to the SHANSEP template. This equation states that the mean value of $s_u(\mathrm{mob})/\sigma_v'$ for structured NC clay is 0.298. The exponent term for OCR for structured clays is 0.938, which is higher than the typical value of 0.7–0.8 (Kulhawy and Mayne 1990). The updated c.o.v. for $s_u(\mathrm{mob})/\sigma_v'$ is 0.392. Table 7 further indicates that if OCR = σ_p'/σ_v' is given, the information for LI is of marginal value insofar as estimation of s_u/σ_v' is concerned. We say LI is of "marginal value," because appending this additional piece of information to the SHANSEP model does not significantly reduce the c.o.v. of s_u/σ_v'. In contrast, appending S_t to the SHANSEP model reduces the c.o.v of s_u/σ_v' from 0.39 to 0.34. Hence, S_t has a "moderate value" in this context.

For σ_p', estimation can be made effectively based on information of s_u and σ_v', but such estimation equations are not

Table 7. Updated mean and coefficient of variation of normalized undrained shear strength under various combinations of information.

Information	Updated mean of s_u/σ'_v	Updated c.o.v. of s_u/σ'_v
LI	LI $^{0.322} \times 0.470$	0.583
σ'_p/σ'_v	$(\sigma'_p/\sigma'_v)^{0.938} \times 0.298$	0.392
LI, σ'_p/σ'_v	LI $^{0.154} \times (\sigma'_p/\sigma'_v)^{0.906} \times 0.296$	0.385
S_t, σ'_p/σ'_v	$S_t^{0.144} \times (\sigma'_p/\sigma'_v)^{0.810} \times 0.206$	0.338
LI, S_t, σ'_p/σ'_v	LI $^{-0.258} \times S_t^{0.208} \times (\sigma'_p/\sigma'_v)^{0.806} \times 0.177$	0.327

Table 8. Updated mean and coefficient of variation of normalized preconsolidation stress under various combinations of information.

Information	Updated mean of σ'_p/P_a	Updated c.o.v. of σ'_p/P_a
LI	LI $^{-0.303} \times 1.078$	0.934
LI, S_t	LI $^{-1.354} \times S_t^{0.509} \times 0.257$	0.699
LI, σ'_v/P_a	LI $^{0.116} \times \sigma'_v/P_a{}^{0.860} \times 1.521$	0.440
LI, S_t, σ'_v/P_a	LI $^{-0.377} \times S_t^{0.210} \times \sigma'_v/P_a{}^{0.736} \times 0.802$	0.398

Table 9. Updated mean and coefficient of variation of sensitivity under various combinations of information.

Information	Updated mean of S_t	Updated c.o.v. of S_t
LI	LI $^{2.066} \times 20.747$	1.194
LI, σ'_v/P_a	LI $^{2.355} \times (\sigma'_v/P_a)^{0.591} \times 27.599$	0.980
LI, σ'_p/P_a	LI $^{2.284} \times (\sigma'_p/P_a)^{0.719} \times 20.946$	0.868
LI, σ'_p/σ'_v	LI $^{1.978} \times (\sigma'_p/\sigma'_v)^{0.479} \times 16.566$	1.150
LI, s_u/σ'_v	LI $^{1.783} \times (s_u/\sigma'_v)^{0.880} \times 40.972$	0.966
LI, s_u^{re}/P_a	LI $^{0.896} \times (s_u^{re}/P_a)^{-0.524} \times 2.190$	1.052
LI, s_u^{re}/P_a, σ'_v/P_a	LI $^{0.413} \times (s_u^{re}/P_a)^{-0.947} \times (\sigma'_v/P_a)^{0.581} \times 0.564$	0.581
LI, s_u/σ'_v, σ'_p/P_a	LI $^{2.055} \times (s_u/\sigma'_v)^{0.582} \times (\sigma'_p/P_a)^{0.580} \times 32.794$	0.780
LI, s_u^{re}/P_a, σ'_p/P_a	LI $^{0.175} \times (s_u^{re}/P_a)^{-0.993} \times (\sigma'_p/P_a)^{1.073} \times 0.298$	0.382
s_u^{re}/P_a, σ'_v/P_a	$(s_u^{re}/P_a)^{-1.081} \times (\sigma'_v/P_a)^{0.975} \times 0.336$	0.594

Fig. 10. Comparison of relations between S_t, σ'_p, and LI in NAVFAC (1982) design manual.

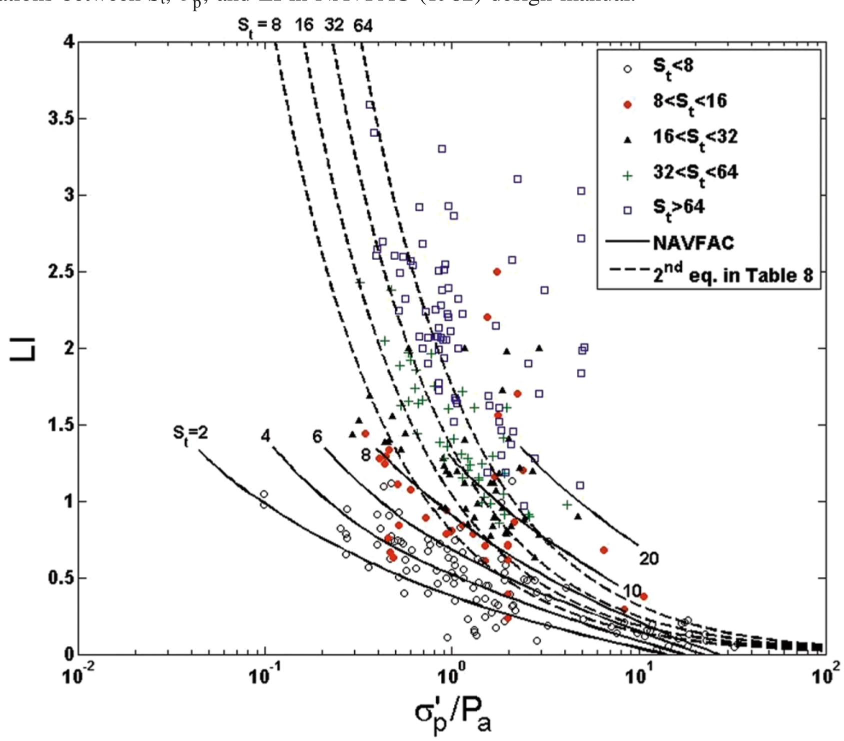


Fig. 11. Comparison between selected equations in Table 7 and the validation database.

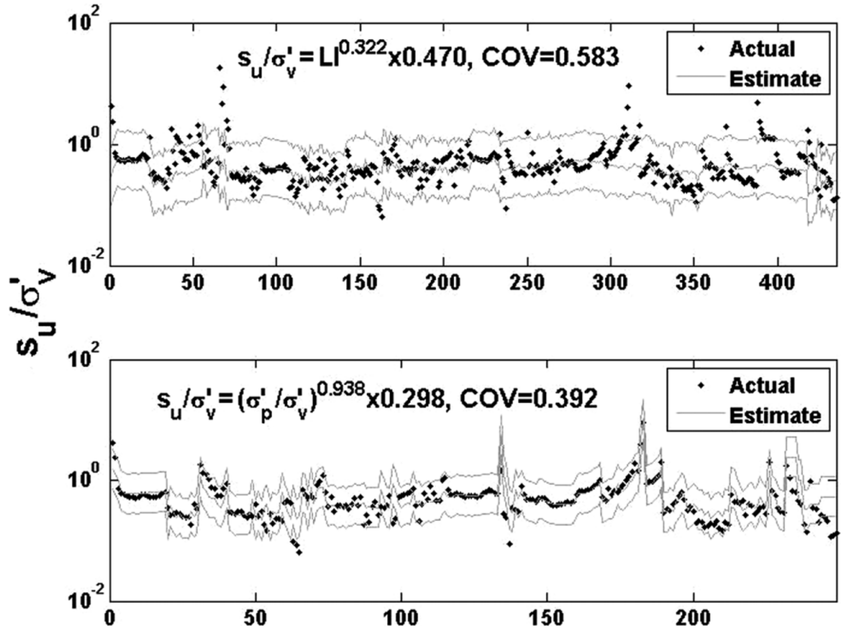
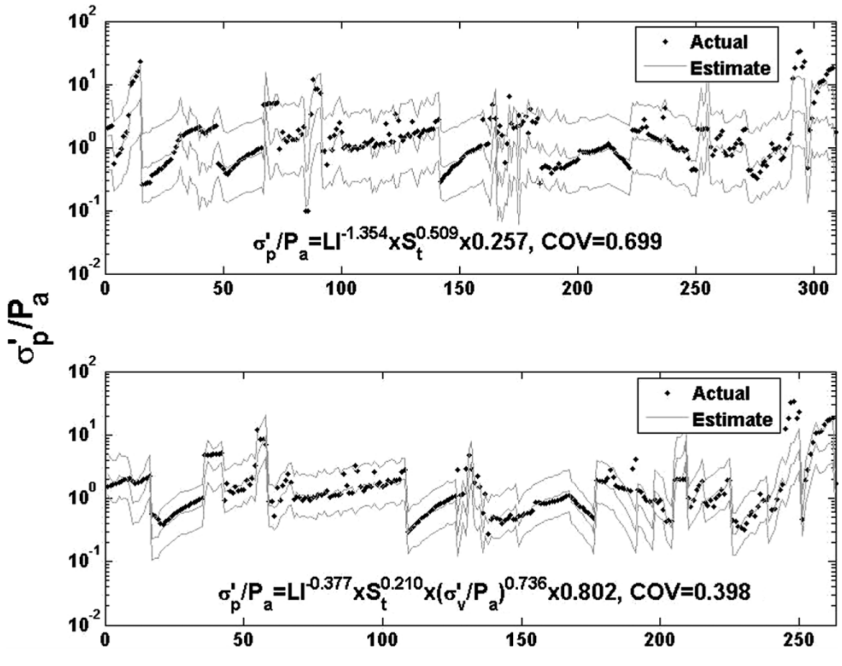


Fig. 12. Comparison between selected equations in Table 8 and the validation database.



presented in Table 8 because they are similar to the SHANSEP-style equations in Table 7. The first equation in Table 8 is similar to eq. [6] except that the LI term in eq. [6] is replaced by ln(LI). The latter form, i.e., ln(LI), is found to provide a much better fit to the data points for highly sensitive and quick clays. The second equation in Table 8 can be compared directly to the relation between S_t, σ'_p, and LI in the NAVFAC (1982) design manual. Figure 10 shows the relation between S_t, σ'_p, and LI in NAVFAC design manual (solid lines) together with the mean trend for the second equation in Table 8 (dashed lines). Data points from the validation database are also shown in the figure. It is clear that the second equation in Table 8 performs better for quick clays, i.e., data points with $S_t > 8$. The same equation is compared with the equations proposed by Stas and Kulhawy (1984) in Fig. 7. Table 9 shows that the updated c.o.v. of S_t is usually large and the best set of information in terms of value added is {LI, s_u^{re}/P_a, σ'_p/P_a } — the updated c.o.v. of S_t can be significantly reduced to less than 0.4.

Figures 11–13 show the comparison between selected equations summarized in Tables 7–9 and the validation database (Fig. 11 for the first and second equations in Table 7, Fig. 12 for the second and fourth equations in Table 8, and Fig. 13 for the second and seventh equations in Table 9). For Fig. 11, the prediction for s_u/σ'_v based on the first and second equations in Table 7 is verified. There are in total

Fig. 13. Comparison between selected equations in Table 9 and the validation database.

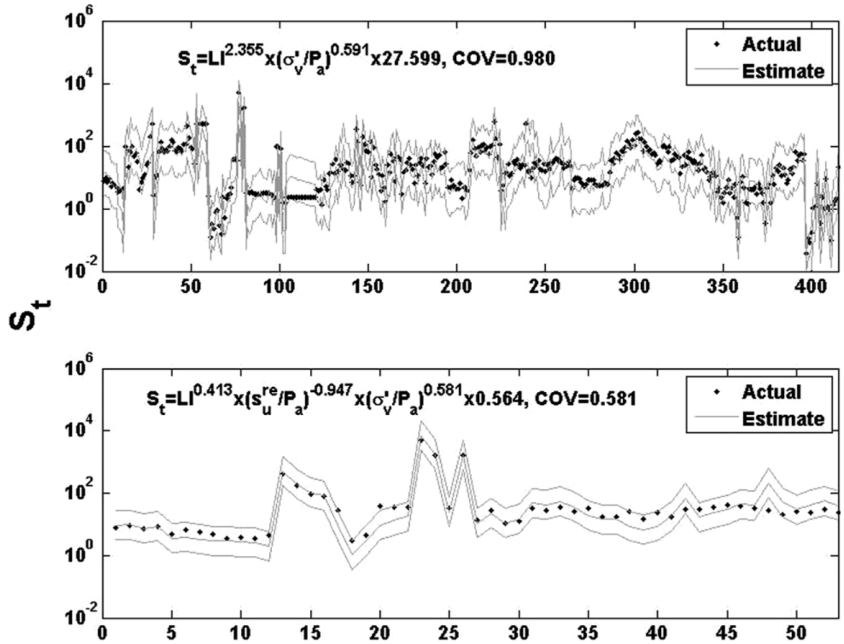
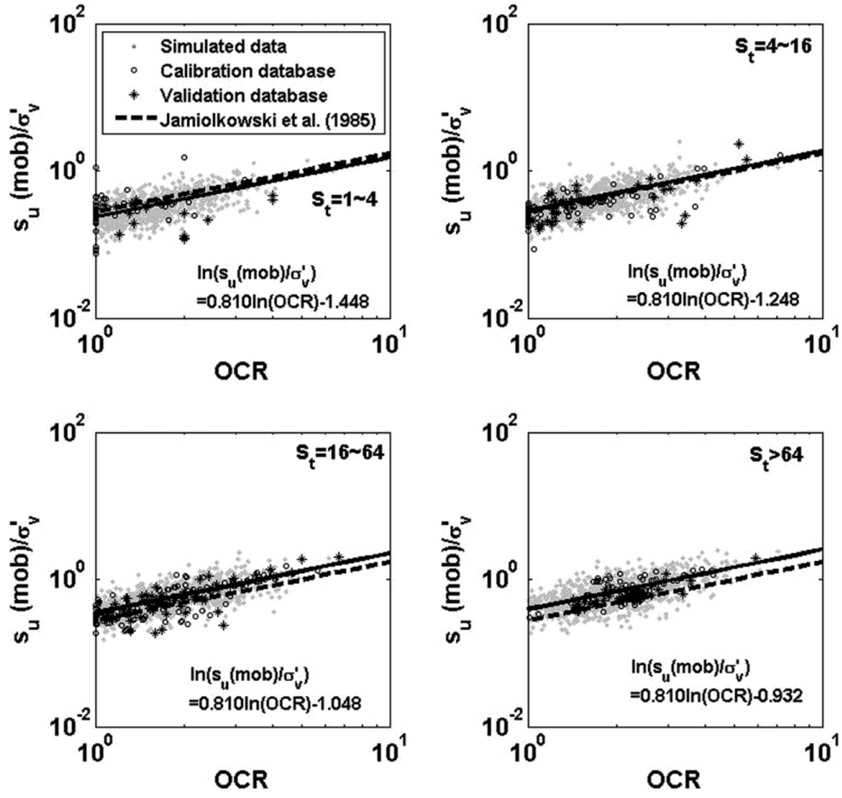
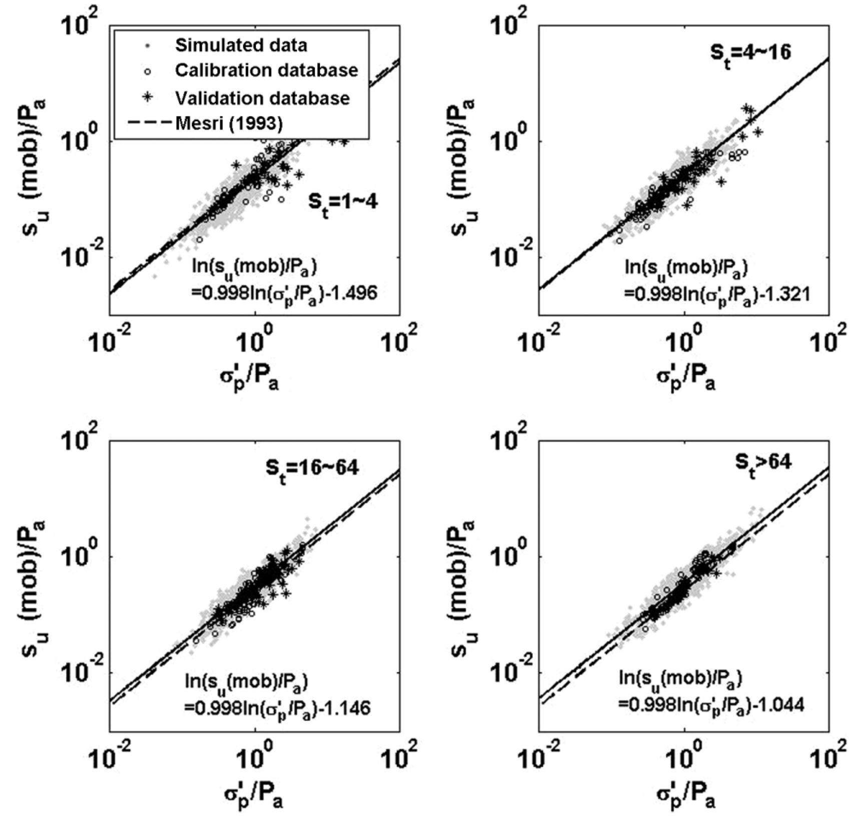


Fig. 14. Simulated data versus measured (calibration and validation) data within OCR–(s_u/σ'_v) plots restricted to various S_t ranges. The solid lines are the trends of the simulated data.



435 data points in the validation database with known s_u/σ'_v values. However, these s_u/σ'_v values will be treated as unknown during the verification. Among the 435 data points, 435 data points are with known LI and 249 with known {LI, σ'_p/σ'_v}. Based on the known information, the updated 95% confidence intervals and the updated median for s_u/σ'_v, i.e., [$\exp(\lambda'_{su/\sigma'_v} - 1.96 \times \xi'_{su/\sigma'_v})$ $\exp(\lambda'_{su/\sigma'_v})$ $\exp(\lambda'_{su/\sigma'_v} + 1.96 \times \xi'_{su/\sigma'_v})$] are calculated based on the equations in Table 7 and are plotted in Fig. 11. The results show that these two equations in Table 7 are effective. The same verification is done for the data points in the validation database with known σ'_p/P_a and S_t values, and the results are shown in Figs. 12

Fig. 15. Simulated data versus measured (calibration and validation) data within s_u–σ'_p plots restricted to various S_t ranges. Solid lines are the trends of the simulated data.



and 13. The plots in these figures also show that the equations in Tables 8 and 9 are effective.

Beyond pairwise correlations

The CDF transform approach is essentially founded on all possible pairwise correlations between any two components in the multivariate data. For example, in Table 4, it is clear that 10 separate correlations are needed to populate the correlation matrix for a five-dimensional random vector {LI, s_u, s_u^{re}, σ'_p, σ'_v}. It is interesting to explore if this relatively simple simulation method can capture higher-order dependency information in the database. It is difficult to visualize dependency between three or more components and there is no common statistical measure to quantify such higher-order dependencies. Nevertheless, it is of practical interest to study one aspect of higher-order dependency that can be found occasionally in practice; namely, a pairwise correlation equation may occasionally depend on the value of a third variable. In other words, the coefficients of the correlation equation are not constants, but functions of a third variable.

This section studies two such examples: OCR–(s_u/σ'_v) and s_u–σ'_p correlations. The goal is to study how these correlation equations for structured clays are affected by S_t. Figure 14 compares the simulated data and the measured (calibration and validation) data within OCR–(s_u/σ'_v) plots restricted to various S_t ranges. It is evident that for structured clays the trend lines (solid lines) move slightly upwards for increasing S_t. This behavior seems to be consistent with the calibration and validation databases. Although not as obvious, the s_u–σ'_p correlation for structured clays (Fig. 15) shows a comparable behavior with respect to various S_t ranges. This limited study appears to imply that third-order information is reasonably captured by the correlation-based CDF transform approach.

Conclusion

In this study, a calibration database of structured clays is compiled for the purpose of constructing the multivariate probability distribution of liquidity index (LI), undrained shear strength (s_u), remolded undrained shear strength (s_u^{re}), preconsolidation stress (σ'_p), and in situ vertical stress (σ'_v). Based on the proposed CDF transform approach, the logarithms of the above five parameters are effectively represented as a multivariate normal distribution. From the multivariate normal distribution, a set of Bayesian equations that are useful for predicting s_u, σ'_p, and S_t based on other information are derived for structured clays.

Another independent database of structured clays is compiled to validate the consistency of the summarized multivariate normal distribution and the resulting predictive equations for s_u, σ'_p, and S_t. The validation shows evidence of consistency. The behaviors of the multivariate normal distribution are also compared with empirical equations proposed in literature. The same consistency is found.

When multivariate geotechnical data exist in sufficient amounts, it is of significant practical usefulness to construct a multivariate probability distribution function (in the second-moment sense) using the proposed CDF transform approach. The usefulness is as follows: (*i*) it is possible to derive the mean and c.o.v. of pairwise correlations systematically using simulation and (*ii*) it may be possible to study this data theoretically using simulated data if new strong pairwise correlations can be found either among the original components or some derived components.

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