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MULTIPLE REGRESSION ANALYSIS BETWEEN THE MECHANICAL AND PHYSICAL PROPERTIES OF COHESIVE SOILS

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ABSTRACT

In order to accurately determine the relationships between a mechanical property and some physical properties in both natural and artificially mixed cohesive soils, multiple regression analysis was performed. In this paper, two kinds of regression models representing the shear strength of soils are examined. The first model is based on the Atterberg limit and the second is based on the Cam-Clay model. The concept of this study is that the mechanical properties of the soils are characterized by soil types and soil states. The following conclusions were obtained: (1) Both regression models established for the shear strength of soils are not only found to be valid for the unconfined compressive strength, but also for the modulus of deformation and consolidation yield stress. (2) The validity of the concept employed here can be quantitatively proved by multiple regression analysis. (3) In a fully saturated soil, the correlation between the two regression models can be found both by statistical analysis and by theoretical examination. (4) The regression equations are obtained in good predictive accuracy for unconfined compressive strength, the modulus of deformation, consolidation yield stress and compression index.

Key words: cohesive soil, density, fully saturated soil, sensitivity, shear strength, soli structure, statistical analysis, unconfined compression test, water content (**IGC:** D3/D6/D0)

INTRODUCTION

The results of soundings in the site or of mechanical property tests of soil samples in the laboratory can be applied directly to foundation design. On the other hand, the results of physical property tests of soil samples can be easily obtained in the laboratory and play a role of supplement for soundings or mechanical property tests. If the relationships between a mechanical and a physical property become clear, it is so useful and important. Several studies have been done on this problem as follows: the relationships between shear strength and liquidity index (Ohsaki, 1961; Mikasa, 1967), those between internal friction angle and plasticity index (Bjerrum and Simons, 1960), those between sensitivity ratio

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and liquidity index (Skempton, 1953; Bjerrum, 1954), those between compression index and liquid limit (Skempton, 1944).

But these relationships would be limited in their application to foundation design, because of the following two reasons: (1) These relationships can not be explained in the necessary and sufficient conditions that the mechanical properties are generally characterized by both the soil types and states. (2) These relationships are not expressed together with the predictive accuracies that are needed for the designers to estimate a safety factor in foundation design.

In this paper, multiple regression analysis is used to attempt to accurately determine the relationships between a mechanical property and some major physical properties in cohesive soils. The data from both natural soils and artificially mixed soils, of which the clay content was more than 20%, was analyzed statistically. The following points were the purposes in the analysis:

(1) To establish reasonable regression models of cohesive soils, before the analysis.

(2) To determine the regression equations expressing the relationship between a mechanical property and some physical properties of soils.

(3) To examine the contributions (partial correlation coefficient) of physical properties to a mechanical property.

(4) To examine the influencing relationships (F-values) between each physical property.

(5) To determine the predictive accuracies (multiple correlation coefficient) of regression equations obtained from the analysis.

CONCEPT AND METHOD OF ANALYSIS

Multiple regression analysis was adopted to obtain the relationships between a mechanical property and some major physical properties on both natural soils and artificially mixed soils. Mechanical properties are given as dependent variables in Table 1, physical properties are given as independent variables in Table 2.

Regression Models

Mechanical properties are characterized generally by soil types and states (Mikasa, 1964, 1967) as shown by the following function F of Eq. (1).

Table 2. Physical properties as independent variables

	physical property	
soil type	clay content	C (%)
	silt content	M_0 (%)
	sand content	S (%)
	liquid limit	w_L (%)
	plastic limit	w_p (%)
	plasticity index	I_p (%)
	plasticity ratio	P_r
density	unit weight	γ_t (gf/cm^3)
	void ratio	e
water content	water content	w (%)
	degree of saturation	S_r (%)
	liquidity index	I_L

(1 gf/cm^3 = 9.81 kN/m^3)

Table 1. Mechanical properties as dependent variables

	mechanical property			nat. soil	art. soil
strength	unconfined compressive strength	q_u	(kgf/cm^2)	○	
	remolded strength	q_{ur}	(kgf/cm^2)	○	○
	undrained internal friction angle	ϕ_u	(degree)	○	
deformation	failure strain	ε_u	(%)	○	
	remolded failure strain	ε_{ur}	(%)		○
	modulus of deformation	E_{50}	(kgf/cm^2)	○	
	remolded modulus of deformation	E_{50r}	(kgf/cm^2)		○
compressibility	compression index	C_c		○	
	consolidation yield stress	p_c	(kgf/cm^2)	○	
soil structure	sensitivity ratio	S_t		○	

(1 kgf/cm^2 = 98.1 kN/m^2)

Mechanical properties of soil
$= F$ (types; states) (1)

Where the soil types are characterized by clay mineral or chemical composition, grain size distribution and the other properties of the soil. The soil states are characterized by density, water content and soil structure, and the equation in the following form is generally obtained.

Mechanical properties of soil
$= F$ (types; density, water content, structure) (2)

In this paper, analysis was made on the assumption that soil structure depends only on physical properties, and the validity of this assumption was examined on the basis of the analysis. Thus, the following form can be used for multiple regression analysis.

Mechanical properties of soil
$= F$ (types; density, water content) (3)

Assuming now the fully saturated state (S_r= 100%) of soils, the density is dependent on water content, thus the following form is obtained.

Mechanical properties of soil
$= F$ (types; density or water content) (4)

Based on the above mentioned concepts, the regression models 1 and 2 are established as follows.

Regression model 1

The authors have made clear the relationships between shear strength and the liquidity index. The liquidity index I_L is given by the Atterberg limits and water content w as shown in Eq. (5).

$$I_L = (w - w_p)/(w_L - w_p) \quad (5)$$

where w_p: plastic limit
w_L: liquid limit

In the liquid limit test, the relationship between the water content w and the number of blows N is given as shown in Eq. (6).

$$w = -I_f \log N + c \quad (6)$$

Where "log" is defined as a common logarithm.
I_f: flow index
c : constant

Cohesion c_u is assumed to be related linearly to the number of blows N (Ohsaki, 1961) as shown in Eq. (7).

$$N = kc_u \quad (7)$$

where k: constant

Substituting Eq. (7) to Eq. (6), the following Eq. (8) is obtained. If Eq. (8) is valid for the wide range of w from the plastic limit w_p to the liquid limit w_L, Eqs. (9) and (10) can be obtained.

$$w = -I_f \log (kc_u) + c \quad (8)$$

$$w_L = -I_f \log (kc_{uL}) + c \quad (9)$$

$$w_p = -I_f \log (kc_{up}) + c \quad (10)$$

where c_u : cohesion at natural water content
c_{uL}: cohesion at liquid limit
c_{up}: cohesion at plastic limit

By substituting Eqs. (8), (9) and (10) to Eq. (5), the following Eq. (11) or (12) is obtained.

$$I_L = \frac{\log kc_{up} - \log kc_u}{\log kc_{up} - \log kc_{uL}} = \frac{\log (c_{up}/c_u)}{\log (c_{up}/c_{uL})} \quad (11)$$

or

$$c_u = c_{up} \frac{1}{(c_{up}/c_{uL})^{I_L}} = c_{up} (c_{up}/c_{uL})^{-I_L} \quad (12)$$

Using the relationship of $c_u = q_u/2$, the following Eq. (13) is obtained,

$$q_u = q_{up}(q_{up}/q_{uL})^{-I_L} \quad (13)$$

where q_u is unconfined compressive strength, and rewriting the Eq. (13), the following Eq. (14) is obtained.

$$\ln q_u = -\ln(q_{up}/q_{uL}) I_L + \ln q_{up} \quad (14)$$

Where "ln" is defined as a natural logarithm. Now, substituting $a_1 = -\ln(q_{up}/q_{uL})$ and $a_2 = \ln q_{up}$ to Eq. (14), the following from is obtained.

$$\ln q_u = a_1 I_L + a_2 \quad (15)$$

Where a_1 and a_2 are determined by the multiple regression analysis using Eq. (24). In a fully saturated case, Eq. (15) is valid under the requirement of Eq. (4), but in an unsaturated case, the equation as the following form will be established to satisfy the condition shown by Eq. (3).

$$\ln q_u = b_1 X_1 + b_2 I_L + b_3 \tag{16}$$

Where X_1 represents the variable related to the soil density, which will be described in the later examinations, and b_1, b_2 and b_3 are determined by the multiple regression analysis using Eq. (24).

Eq. (15) or (16) is the final form representing the regression model 1 as to the shear strength of the soil which is expressed mainly by the liquidity index.

Regression model 2

Eqs. (17) and (18) are given from the Cam-Clay model (Roscoe, Schofield and Thurairajah, 1963).

$$q_u = M p_c \tag{17}$$

$$e = \Gamma - \lambda \ln p_c \tag{18}$$

Where M, Γ and λ are constants characterized by the soil types, p_c is the consolidation yield stress, and e is the void ratio. From Eqs. (17) and (18), the following form is obtained.

$$q_u = M \exp\left(\frac{\Gamma - e}{\lambda}\right) \tag{19}$$

Rewriting Eq. (19), the following from,

$$\ln q_u = \ln M + \Gamma/\lambda - e/\lambda \tag{20}$$

is obtained. The compression index λ is dependent on the liquid limit w_L, so that λ can be expressed in linear form as $\lambda = a w_L$, referring to the linear relationship of $C_c = b w_L$ (Skempton, 1944) and also considering the relationship of $\lambda = 0.434 C_c$. Thus Eq. (21) is obtained.

$$\ln q_u = \ln M + \Gamma / a w_L - e / a w_L \tag{21}$$

Substituting the following relationships to Eq. (21), Eq. (22) is obtained: $b_1 = \Gamma/a$, $b_2 = 1/a$, $b_3 X_2 + b_4 = \ln M$.

$$\ln q_u = b_1 / w_L + b_2 e / w_L + b_3 X_2 + b_4 \tag{22}$$

Where X_2 represents the physical property which expresses M, and b_1, b_2, b_3 and b_4 are determined by the multiple regression analysis using Eq. (24). Eq. (22) is the finally established form as the regression model 2 based on the Cam-Clay model. However, it must be remarked that Eq. (22) is valid only for fully saturated soil.

Independent variables X_1 and X_2 in Eqs. (16) and (22) will be determined respectively by examining the magnitude of the contribution of independent variables to a dependent variable during the course of the analysis. The magnitude of the contribution can be shown by F-values during analysis. The regression models 1 and 2 could be adopted for mechanical properties other than shear strength as will be shown later in this paper, because shear strength is usually related lineally to the other mechanical properties (JSSMFE, 1966; Hirata, 1985).

Method of Analysis

Regression models are based on linear regression as shown in Eq. (23). Transforming the dependent variables into logarithmic form, Eq. (24) will be adopted to non-linear regression (loglinear regression).

$$y = a_1 x_1 + a_2 x_2 + \cdots\cdots + a_m x_m + a_{m+1} \tag{23}$$

$$Y = \ln y = b_1 x_2 + b_2 x_2 + \cdots\cdots + b_m x_m + b_{m+1} \tag{24}$$

Where y is the dependent variable(mechanical property), x_i is the independent variable (physical property), m is the number of independent variables, Y is the dependent variable transformed to the logarithm.

Multiple correlation coefficient, partial correlation coefficient and F-value are used to examine the process and result of multiple regression analysis, and are defined as follows (Kobayashi, 1982; Flury and Riedwyl, 1988).

Multiple correlation coefficient

The multiple correlation coefficient R expresses the magnitude of predictive accuracy of the regression equation as shown in Eq. (25).

$$\left.\begin{aligned} R &= S_{y_0 y} / \sqrt{S_{y_0 y_0} \cdot S_{yy}} \\ S_{y_0 y_0} &= \sum_{i=1}^{n} (y_{0i} - \bar{y}_0)^2 \\ S_{yy} &= \sum_{i=1}^{n} (y_i - \bar{y})^2 \\ S_{y_0 y} &= \sum_{i=1}^{n} (y_{0i} - \bar{y}_0)(y_i - \bar{y}) \end{aligned}\right\} \tag{25}$$

where

$S_{y_0 y_0}$: sum of square of measured values y_0
S_{yy} : sum of square of predicted values y
$S_{y_0 y}$: partial sum of square of y_0 and y

$\bar{y}_0$: average value of y_0
$\bar{y}$: average value of y
n : number of data samples

Partial correlation coefficient

The partial correlation coefficient r expresses the magnitude of the relationship among the variables excluding the influence of the other variables. In this paper, the partial correlation coefficient was mainly used to estimate how strongly physical properties contribute to a mechanical property excluding the influence of the other physical properties.

The partial correlation coefficient could be more practically explained by using the following example of regression equation.

$$y=a_1x_1+a_2x_2+a_3x_3+a_4$$

When the relationship between y and x_1 excluding the influence of x_2 and x_3 is taken into account, as the first step, regression analysis for the following form is employed,

$$y=b_2x_2+b_3x_3+b_4$$

and as the result of the analysis, $\hat{b}_2$, $\hat{b}_3$, $\hat{b}_4$ are obtained as predicted values of b_2, b_3, b_4. Then y' is calculated as follows.

$$y'=y-(\hat{b}_2x_2+\hat{b}_3x_3+\hat{b}_4)$$

As the second step, the regression analysis with the following form is employed,

$$x_1=c_2x_2+c_3x_3+c_4$$

and as the result of the analysis, $\hat{c}_2$, $\hat{c}_3$, $\hat{c}_4$ are obtained as predicted values of c_2, c_3, c_4. Then x_1' is calculated as follows.

$$x_1'=x_1-(\hat{c}_2x_2+\hat{c}_3x_3+\hat{c}_4)$$

Finally, the partial correlation coefficient is obtained by calculating the multiple correlation coefficient R of Eq. (25) between y' and x_1', and is expressed by the symbol $r_{yx_1 \cdot x_2x_3}$ as follows.

$$r_{yx_1 \cdot x_2x_3}=S_{y'x_1'}/\sqrt{S_{y'y'} \cdot S_{x_1'x_1'}} \qquad (26)$$

F–value

F–value is expressed as follows in the case of the example used above.

$$F_{yx_1 \cdot x_2x_3}=r_{yx_1 \cdot x_2x_3}{}^2(n-k)/(1-r_{yx_1 \cdot x_2x_3}{}^2) \qquad (27)$$

where

$r_{yx_1 \cdot x_2x_3}$: partial correlation coefficient between the dependent variable y and the independent variable x_1.
$n-k$: degree of freedom, n is the number of data samples, and k is the number of independent variables plus one.

F–value is the alternative expression of partial correlation coefficient r shown by Eq. (26). During the regression analysis, F–values are effectively used to select the important independent variables which contribute significantly to dependent variables. Those independent variables are selected in step by step analytical procedure as explained in later chapter; RESULTS OF MULTIPLE REGRESSION ANALYSIS.

DATA USED IN THE ANALYSIS

The data samples of both natural and artificially mixed soils with a clay content of more than 20% were used in multiple regression analysis. The outlines of these data are as follows.

Data of Natural Soils

The data of natural soils was collected from site investigation results in Osaka and Hyogo (Yao, Hirata and Saito, 1985a). The grain size distributions of soil samples are given in Fig. 1, and the total number of data samples is 734. The method of the physical property tests and that of the mechanical property tests conformed to the JSSMFE standard.

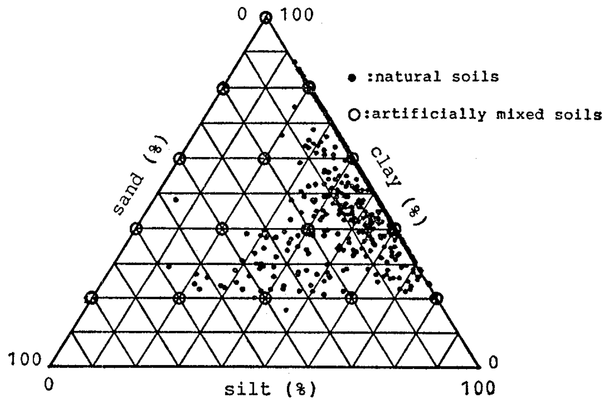

Fig. 1. Grain size distribution of natural and artificially mixed soils

The unconfined compression test was carried out with a constant rate of axial strain of 1%/min. The average value of the degree of saturation in the sample was 99.46% and the value of standard deviation was 2.00%.

In the JSSMFE (1979) it is remarked that liquidity index is underestimated as compared with that of the original soil, because the Atterberg limit tests were performed on soil fraction smaller than 420μm, while the water content test was performed on the soil sample of original grain size To compensate for this underestimate, the correcting coefficient T given by Eq. (28) is introduced here, based on the fact that a grain size of 420μm is approximately in the middle between the upper and lower bounds of sand grain size on the logarithmic scale. Eq. (28) means that half of the sand contest S of the soils used in the Atterberg limit tests are removed from the original soils. Thus the plastic limit and liquid limit should be corrected respectively by Eqs. (29) and (30). The plasticity index I_p and liquidity index I_L are obtained using the values of Eqs. (29) and (30).

$$T = (C+M_o+S/2)/100 \tag{28}$$

$$w_p = Tw_{pt} \tag{29}$$

$$w_L = Tw_{Lt} \tag{30}$$

where T : correcting coefficient
C : clay content (%)
M_o : silt content (%)
S : sand content (%)
w_p : corrected plastic limit (%)
w_{pt} : measured plastic limit (%)
w_L : corrected liquid limit (%)
w_{Lt} : measured liquid limit (%)

Data of Artificially Mixed Soils

Artificially mixed soils are composed of clay (either bentonite or kaolinite), silt (rubble dust) and sand (Toyoura sand) as shown in Fig. 1 (Yao, Hirata and Saito, 1985b). The grain size distribution curves are shown in Fig. 2, and the physical properties are given in Table 3. Artificially mixed soils in Fig. 1 were thoroughly remolded at a water content of the liquidity index $I_L=0$ (i.e., $w=w_p$), and were compacted into a mold of 50 mm in diameter and 100 mm in height. Soils with a sand content of 0 % and at a water cotent with liquidity indexes of $I_L=0.25$, 0.50 and 0.75 were used to make the test specimens to examine the influence of the magnitude of water contents. The physical property tests conformed also to the JSSMFE standard. The unconfined compression test was performed with a constant rate of axial strain of 5%/min. The total number of data samples was 120. The average value of the degree of saturation was 90.69% and the value of standard deviation was 5.02%.

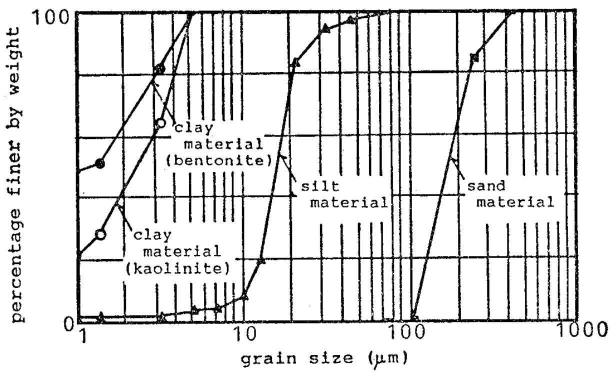

Fig. 2. Grain size distribution curves of mixed materials

Table 3. Physical properties of mixed materials

material	D_{max} (μm)	D_{10} (μm)	U_c	U_c'	w_L (%)	w_p (%)	I_p (%)	P_r	A_c
clay (kaolinite)	5	—	—	—	49.5	32.1	17.3	0.5	0.43
clay (bentonite)	5	—	—	—	158.8	47.2	111.6	2.4	1.82
silt	74	10.3	1.8	1.10	39.5	26.5	12.9	0.5	4.78
sand	420	121.1	1.7	0.96	—	—	—	—	—

RESULTS OF MULTIPLE REGRESSION ANALYSIS

The regression equations obtained by multiple regression analysis are given in Table 4 for natural soils, and in Table 14 for artificially mixed soils. In these tables, the total number of data samples n, the multiple correlation coefficient R and the regression equation numbers (REN is used for natural soils and REA is used for artificially mixed soils) are given together with the regression equation. The partial correlation coefficients r are also given respectively below each partial regression coefficient.

In order to show the regression procedure, F-values that express shifts in the magnitude of contribution to the dependent variable during analysis, are given respectively for some trpical results in Table 5 to 13 for natural

Table 4. Results of multiple regression analysis for natural soils

	regression equation	n	R	REN
q_u	$\ln q_u = -2.025 I_L + 1.456$ $r\,(-0.648)$	673	0.648	1
	$\ln q_u = -0.546 e - 1.823 I_L + 2.131$ $r\,(-0.371 \quad -0.626)$	673	0.707	2
	$\ln q_u = 0.039 w_p + 78.777/w_L - 154.233 e/w_L + 1.136$ $r\,(0.239 \quad 0.455 \quad -0.733)$	673	0.734	3
q_{ur}	$\ln q_{ur} = -3.361 I_L + 0.376$ $r\,(-0.760)$	637	0.760	4
	$\ln q_{ur} = -0.681 e - 2.985 I_L + 1.151$ $r\,(-0.439 \quad -0.742)$	637	0.812	5
	$\ln q_{ur} = 0.033 w_p + 103.035/w_L - 232.047 e/w_L + 0.608$ $r\,(0.212 \quad 0.564 \quad -0.845)$	637	0.846	6
ϕ_u	$\phi_u = -0.564 w_p + 5.577 e + 13.982$ $r\,(-0.266 \quad 0.174)$	119	0.267	7
ε_u	$\varepsilon_u = -0.017 w_L + 2.111 I_L + 4.980$ $r\,(-0.123 \quad 0.163)$	672	0.239	8
E_{50}	$\ln E_{50} = -2.362 I_L + 5.458$ $r\,(-0.590)$	673	0.590	9
	$\ln E_{50} = -0.217 e - 2.282 I_L + 5.725$ $r\,(-0.109 \quad -0.569)$	673	0.597	10
	$\ln E_{50} = 0.033 w_p + 48.917/w_L - 160.815 e/w_L + 5.654$ $r\,(0.142 \quad 0.210 \quad -0.605)$	673	0.623	11
S_t	$\ln S_t = 1.163 I_L + 1.194$ $r\,(0.460)$	637	0.460	12
	$\ln S_t = 0.019 w_p + 1.262 I_L + 0.647$ $r\,(0.204 \quad 0.489)$	637	0.494	13
p_c	$\ln p_c = -1.853 I_L + 2.202$ $r\,(-0.557)$	558	0.557	14
	$\ln p_c = -0.844 e - 1.627 I_L + 3.318$ $r\,(-0.489 \quad -0.554)$	558	0.689	15
	$\ln p_c = 0.047 w_p + 115.265/w_L - 170.452 e/w_L + 1.588$ $r\,(0.236 \quad 0.541 \quad -0.727)$	558	0.733	16
C_c	$C_c = 0.010 w_L - 0.063$ $r\,(0.799)$	558	0.799	17
	$C_c = 0.633 e - 0.215$ $r\,(0.820)$	558	0.820	18
	$C_c = 0.005 w_L + 0.388 e - 0.242$ $r\,(0.414 \quad 0.499)$	558	0.853	19

n : total number of data samples
R : multiple correlation coefficient
REN: No. of regression equation for natural soils
r : partial correlation coefficient

Table 5. F-values for $\ln q_u\,(e,\ I_L)$: REN 2

	Initiation	1st step	2nd step
C	3.3006	66.3336	1.7891
M_0	5.0461	8.4816	0.8043
S	0.0299	66.6607	6.9641
$1/w_L$	0.3157	73.7648	1.5888
w_L	0.1796	41.7440	56.3577
w_p	2.7009	10.6578	64.6694
I_p	0.0179	55.1328	16.8982
P_r	0.6147	84.6820	5.5513
γ_t	133.3190	86.6571	0.5280
e/w_L	453.2770	6.0972	30.1400
e	143.8190	107.1370	*104.1370
w	132.5280	97.4184	5.1358
S_r	4.1715	3.6347	0.2747
I_L	484.8230	*484.8230	*432.3700

Table 6. F-values for $\ln q_u\,(w_p,\ 1/w_L,\ e/w_L)$: REN 3

	Initiation	1st step	2nd step	3rd step
C	3.3006	103.0550	8.6930	10.4832
M_0	5.0461	17.4400	2.8402	2.8123
S	0.0299	81.7994	0.3038	0.7093
$1/w_L$	0.3157	147.9770	*147.9770	*174.2430
w_L	0.1796	110.8220	0.9884	24.0475
w_p	2.7009	18.3536	40.6151	*40.6151
I_p	0.0179	132.9660	9.1386	27.5854
P_r	0.6147	193.2120	52.4098	24.2547
γ_t	133.3190	138.5170	2.8615	22.0752
e/w_L	453.2770	*453.2770	*699.8960	*774.5030
e	143.8190	161.3590	13.4881	46.6955
w	132.5280	161.9580	15.1544	59.5024
S_r	4.1715	18.3063	1.4378	1.0552
I_L	484.8230	25.0677	4.3214	1.2959

Table 7. F-values for $\ln q_{ur}\,(e,\ I_L)$: REN 5

	Initiation	1st step	2nd step
C	3.7128	57.4071	0.8312
M_0	0.7927	0.2189	31.9237
S	4.6769	156.3160	41.2348
$1/w_L$	3.4679	148.7420	7.2141
w_L	0.1881	59.5438	93.5772
w_p	0.3435	58.5555	10.8731
I_p	0.1166	66.4413	49.0977
P_r	0.7150	70.8508	0.2784
γ_t	211.7700	148.7310	4.2843
e/w_L	720.4270	6.6029	48.8392
e	201.3270	150.9400	*150.9400
w	186.7240	136.8020	6.0014
S_r	16.9245	13.6716	1.1886
I_L	869.5470	*869.5470	*778.1030

Table 8. F-values for $\ln q_{ur}\,(w_p,\ 1/w_L,\ e/w_L)$: REN 6

	Initiation	1st step	2nd step	3rd step
C	3.7128	112.0430	0.9645	0.5610
M_0	0.7927	2.8793	7.1758	6.9675
S	4.6769	196.0700	2.7054	3.2069
$1/w_L$	3.4679	366.2650	*366.2650	*295.4870
w_L	0.1881	196.1030	3.2650	1.0279
w_p	0.3435	79.8744	29.6475	*29.6475
I_p	0.1166	204.5790	0.1055	1.7392
P_r	0.7150	231.9440	14.7861	2.4666
γ_t	211.7700	287.6680	0.0701	7.2836
e/w_L	720.4270	*720.4270	*1489.5000	*1580.7600
e	201.3270	280.8150	0.0043	6.1218
w	186.7240	290.0570	1.6809	18.3904
S_r	16.9245	55.1166	11.6323	10.6697
I_L	869.5470	77.0799	14.8507	6.8224

Table 9. F-values for $\ln E_{50}\,(e,\ I_L)$: REN 10

	Initiation	1st step	2nd step
C	1.3764	9.6503	2.3777
M_0	0.3641	0.5064	0.1533
S	5.5121	11.9633	4.7688
$1/w_L$	22.8023	2.0151	6.7374
w_L	17.3540	0.3116	29.0940
w_p	17.3211	0.3025	21.4382
I_p	15.4126	1.5212	10.1060
P_r	10.9505	4.9760	0.0265
γ_t	30.6444	4.7893	1.6608
e/w_L	376.4110	17.6356	26.2034
e	33.1410	7.9924	*7.9924
w	32.5544	8.1715	0.1771
S_r	2.4610	1.6338	0.3655
I_L	358.3140	*358.3140	*321.0900

Table 10. F-values for $\ln E_{50}\,(w_p,\ 1/w_L,\ e/w_L)$: REN 11

	Initiation	1st step	2nd step	3rd step
C	1.3764	24.1081	8.0684	8.9132
M_0	0.3641	3.5136	0.8605	0.8155
S	5.5121	20.2011	4.4611	5.2686
$1/w_L$	22.8023	17.0755	*17.0755	*30.8518
w_L	17.3540	14.2113	0.3081	7.7134
w_p	17.3211	0.2615	13.7252	*13.7252
I_p	15.4126	19.4558	3.2252	9.1524
P_r	10.9505	31.6207	14.2472	5.6385
γ_t	30.6444	16.6648	0.5336	5.7635
e/w_L	376.4110	*376.4110	*367.2560	*386.7120
e	33.1410	21.1733	4.0182	13.7471
w	32.5544	25.2540	8.6256	26.0565
S_r	2.4610	11.7666	5.5567	5.1043
I_L	358.3140	5.7546	1.6989	0.5428

Table 11. F-values for $\ln p_c$ (e, I_L): REN 15

	Initiation	1st step	2nd step
C	14.2256	110.4360	5.7362
M_0	6.4432	15.2609	1.1086
S	9.1299	127.8880	24.3782
$1/w_L$	5.2462	152.9410	1.6706
w_L	3.2416	73.0560	70.9743
w_p	0.1343	27.6216	75.8936
I_p	4.1086	85.1616	32.8047
P_r	9.1404	122.9810	3.0803
r_t	160.3740	149.0060	0.1102
e/w_L	227.0170	0.2004	30.7790
e	177.9340	174.4650	*174.4650
w	169.2680	165.8630	1.7534
S_r	14.5566	12.5769	0.2300
I_L	249.6210	*249.6210	*245.7150

Table 12. F-values for $\ln p_c$ (w_p, $1/w_L$, e/w_L): REN 16

	Initiation	1st step	2nd step	3rd step
C	14.2256	144.5940	9.9852	9.5321
M_0	6.4432	23.2416	1.8018	1.2004
S	9.1299	135.8150	2.8803	3.4054
$1/w_L$	5.2462	249.1410	*249.1410	*228.6540
w_L	3.2416	141.6810	0.1862	14.7117
w_p	0.1343	47.7759	32.6690	*32.6690
I_p	4.1086	155.9280	1.1297	14.3845
P_r	9.1404	208.0920	24.9356	9.7018
r_t	160.3740	202.0170	0.6151	15.2933
e/w_L	227.0170	*227.0170	*566.8900	*620.7970
e	177.9340	235.5660	6.7244	37.7199
w	169.2680	232.7480	8.0094	51.4208
S_r	14.5566	28.1633	2.0851	1.4482
I_L	249.6210	16.2274	0.3237	0.0070

soils and in Table 15 to 18 for artificially mixed soils. In the column of Initiation of these tables, F-values for all independent variables are given. In the column of the 1st step, F-values for all independent variables excluding one independent variable marked by * that is included in the regression equation already are given. In the column of the 2nd step, F-values for all independent variables excluding two independent variables marked by * that are succeedingly included in the regression equation already are shown. In the column of the 3rd step, final F-values obtained by similar procedures are given.

In Fig. 3 to 8, the relationships between

Table 13. F-values for C_c (e, w_L): REN 19

	Initiation	1st step	2nd step
C	242.8720	6.1236	2.5250
M_0	79.3593	1.9933	0.0771
S	118.9110	1.6926	3.1572
$1/w_L$	—	—	—
w_L	979.4710	114.5670	*114.5670
w_p	605.1300	69.5421	2.7529
I_p	879.1560	100.2890	1.6210
P_r	403.3830	34.1377	1.7919
r_t	814.1970	0.4260	0.1211
e/w_L	—	—	—
e	1138.4700	*1138.4700	*183.8990
w	1103.2700	0.2647	7.9267
S_r	21.2529	2.1582	2.6365
I_L	2.3547	79.4070	0.8828

Table 14. Results of multiple regression analysis for artificially mixed soils

	regression equation	n	R	REA
q_{ur}	$\ln q_{ur} = -3.262 I_L - 0.566$ r (-0.837)	120	0.837	1
	$\ln q_{ur} = 0.014 e - 3.283 I_L - 0.578$ r (0.012 -0.769)	120	0.837	2
	$\ln q_{ur} = -111.141 e/w_L - 2.278 I_L + 1.383$ r (-0.755 -0.811)	120	0.933	3
	$\ln q_{ur} = 0.030 w_p + 89.952/w_L - 255.344 e/w_L + 0.958$ r (0.274 0.717 -0.924)	120	0.934	4
ε_{ur}	$\varepsilon_{ur} = -0.014 w_p - 3.513 e + 0.441 S_r - 21.123$ r (-0.015 -0.393 0.460)	120	0.670	5
E_{50r}	$\ln E_{50r} = -2.806 I_L + 1.751$ r (-0.690)	120	0.690	6
	$\ln E_{50r} = 0.365 e - 3.350 I_L + 1.443$ r (0.233 -0.676)	120	0.711	7
	$\ln E_{50r} = -155.702 e/w_L - 1.428 I_L + 4.481$ r (-0.768 -0.543)	120	0.886	8
	$\ln E_{50r} = 0.013 w_p + 51.896/w_L - 242.904 e/w_L + 4.419$ r (0.089 0.400 -0.861)	120	0.885	9

n : total number of data samples
R : multiple correlation coefficient
REA: No. of regression equation for artificially mixed soils
r : partial correlation coefficient

Table 15. F-values for ln q_{ur} $(e/w_L, I_L)$: REA3

	Initiation	1 st step	2 nd step
C	0.0806	9.1389	6.1256
M_0	10.3883	1.1461	2.0333
S	20.6713	8.6415	2.4486
$1/w_L$	0.3908	59.2115	11.2489
w_L	0.2216	16.8603	11.4434
w_p	9.1430	6.4209	2.9428
I_p	0.0086	18.2226	13.2176
P_r	2.1004	31.7494	18.2803
γ_t	46.6368	0.9956	14.1844
e/w_L	194.2390	154.9370	*154.9370
e	42.5928	0.0173	7.9464
w	44.8130	0.0295	8.5808
S_r	6.0383	2.1235	0.8651
I_L	274.9380	*274.9380	*225.2200

Table 16. F-values for lnq_{ur} $(w_p, 1/w_L, e/w_L)$: REA 4

	Initiation	1 st step	2 nd step	3 rd step
C	0.0806	0.8090	51.5708	39.6135
M_0	10.3883	8.6895	12.7345	6.4424
S	20.6713	26.7087	13.2824	9.7432
$1/w_L$	0.3908	202.3090	*202.3090	*122.4820
w_L	0.2216	81.4053	0.2063	4.5889
w_p	9.1430	50.9650	9.4477	*9.4477
I_p	0.0086	81.6924	0.0384	4.6168
P_r	2.1004	115.4490	4.7682	4.4824
γ_t	46.6368	130.3080	0.4423	0.4902
e/w_L	194.2390	*194.2390	*725.1320	*680.4130
e	42.5928	76.5303	2.4692	0.0945
w	44.8130	82.6755	1.6650	0.0192
S_r	6.0383	20.2508	2.8707	6.3503
I_L	274.9380	225.2200	20.3792	11.7779

Table 17. F-values for lnE_{50r} $(e/w_L, I_L)$: REA8

	Initiation	1 st step	2 nd step
C	0.2653	8.8739	5.9113
M_0	13.8576	4.3945	9.6809
S	15.8512	1.7046	0.8630
$1/w_L$	2.2936	82.8847	1.9186
w_L	4.3947	49.5806	0.6198
w_p	2.3286	9.1993	1.1421
I_p	8.1918	59.5417	1.4803
P_r	22.5211	109.4920	5.0998
γ_t	11.3193	14.0776	0.3196
e/w_L	269.6670	168.5570	*168.5570
e	11.4491	6.7008	0.9407
w	13.1793	5.8058	0.2947
S_r	16.1111	3.0309	54.4823
I_L	107.3850	*107.3850	*49.0191

Table 18. F-values for ln$E_{50r}$$(w_p, 1/w_L, e/w_L)$: REA 9

	Initiation	1 st step	2 nd step	3 rd step
C	0.2653	0.1723	20.5355	24.2848
M_0	13.8576	16.6270	17.2389	16.6727
S	15.8512	21.0162	0.0594	0.2076
$1/w_L$	2.2936	45.3908	*45.3908	*22.0799
w_L	4.3947	10.6601	11.7242	13.8704
w_p	2.3286	20.5085	0.9227	*0.9227
I_p	8.1918	8.1440	13.6689	13.8141
P_r	22.5211	6.9862	16.4512	16.9400
γ_t	11.3193	18.3636	21.0774	21.0877
e/w_L	269.6670	*269.6670	*406.3330	*333.4860
e	11.4491	10.4463	17.3369	17.0172
w	13.1793	13.4123	11.3828	10.7818
S_r	16.1111	89.3236	60.3837	72.3494
I_L	107.3850	49.0191	4.5533	3.5710

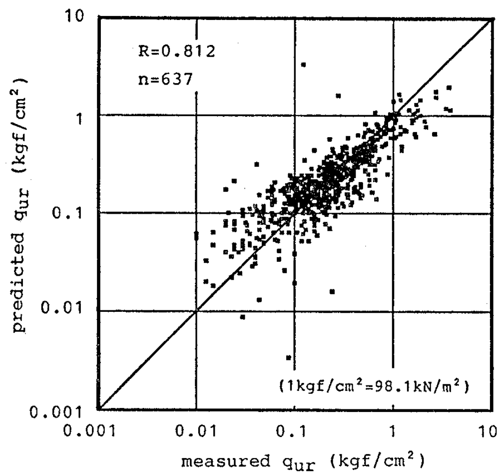

Fig. 3. Relationship between measured and predicted q_{ur}: REN5

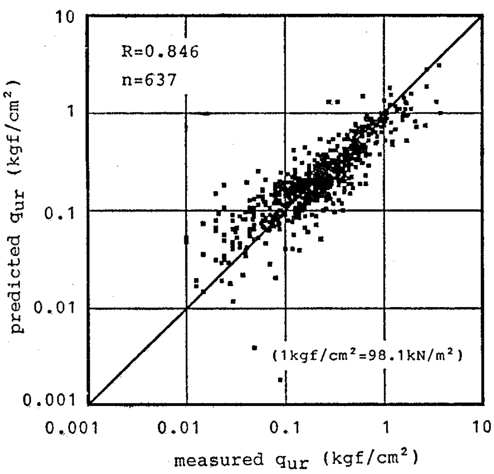

Fig. 4. Relationship between measured and predicted q_{ur}: REN6

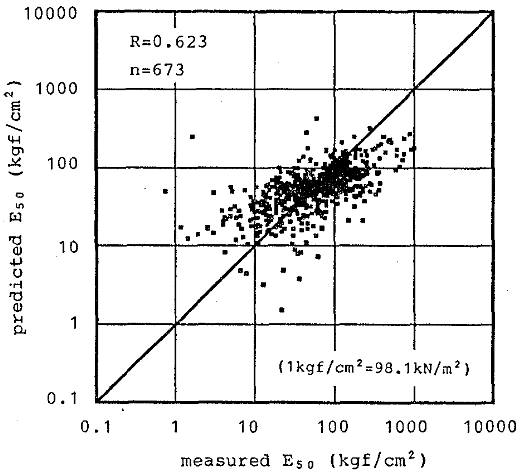

Fig. 5. Relationship between measured and predicted E_{50}: REN11

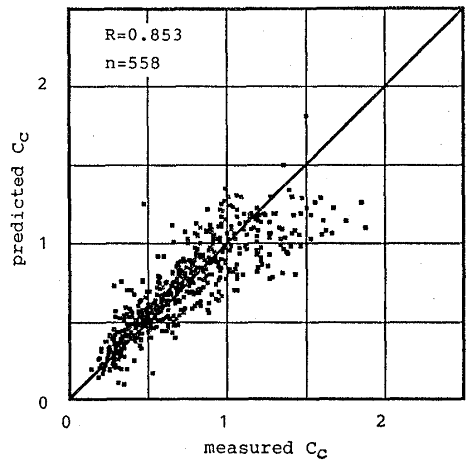

Fig. 6. Relationship between measured and predicted C_c: REN19

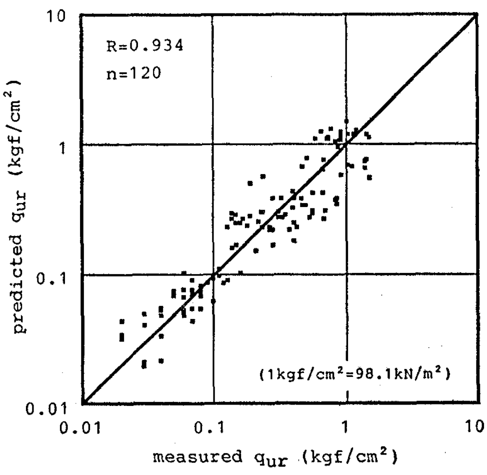

Fig. 7. Relationship between measured and predicted q_{ur}: REA4

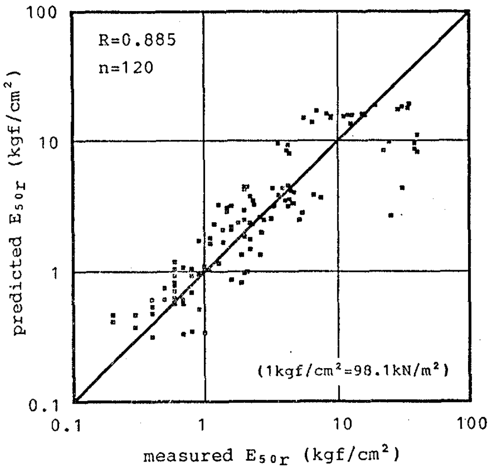

Fig. 8. Relationship between measured and predicted E_{50r}: REA9

the measured and predicted values are given respectively for some typical results.

DISCUSSION

Comparisons with Single Regression Analysis in the Past

On unconfined compressive strength

Fig. 9 shows the relationships between the remolded unconfined compressive strength q_{ur} and the liquidity index I_L, which were obtained by Mikasa (1967) and by the authors. Mikasa's curve was obtained as the standard curve of the remolded cohesive soil by using the relationship of $q_{ur}=2c_{ur}$. This standard curve had been determined by three points which are given by experimental values of cohesion at the remolded state c_{ur} of 0.1, 1.0 and 10 tf/m^2 (0.981, 9.81 and 98.1 kN/m^2) correspond to liquidity index I_L of 1.0, 0.4 and 0.1, respectively. The curve of REN4 shows the regression result obtained from natural soils, and the curve of REA1 shows the regression result obtained from artificially mixed soils. Mikasa's curve is located at the middle of the two curves of REN4 and REA1 by the authors.

On sensitivity ratio

Fig. 10 shows the relationship between the sensitivity ratio S_t and the liquidity index I_L, which Mikasa (1967) compared to the data measured in Osaka clay with the curves found

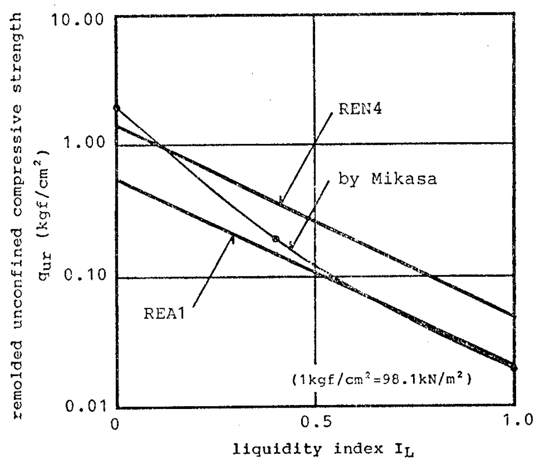

Fig. 9. Regression curves and Mikasa's standard curve for q_{ur}-I_L relationship

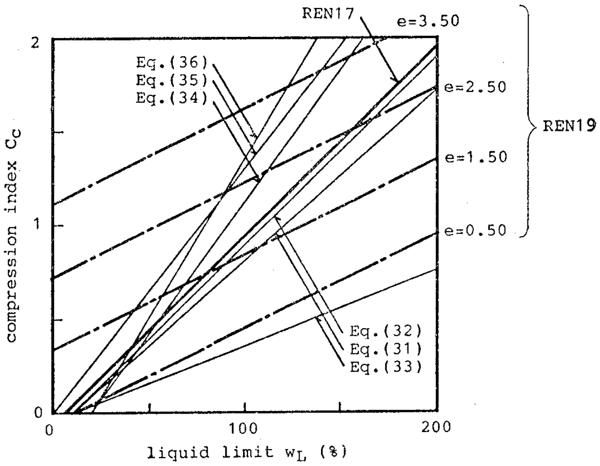

Fig. 11. Regression curves for C_c-w_L relationship

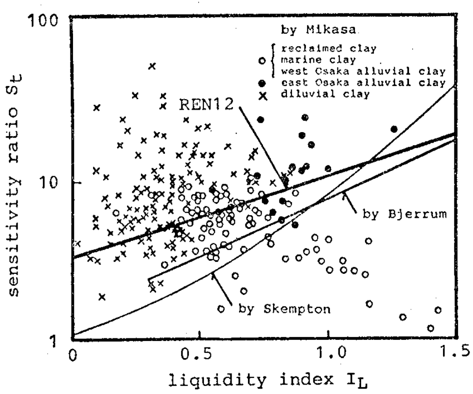

Fig. 10. Regression curves for S_t-I_L relationship

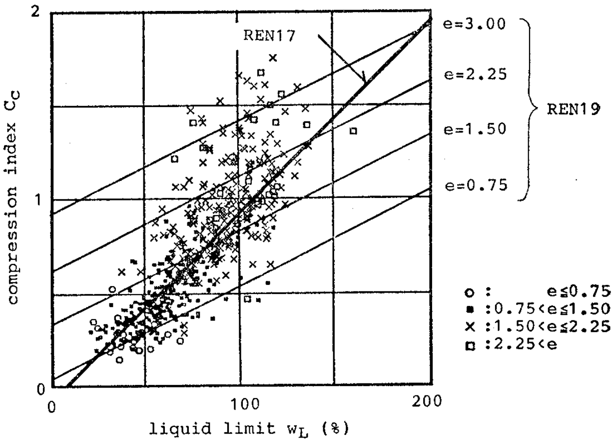

Fig. 12. Measured data and regression curves for C_c-w_L relationship

by Skempton (1953) and by Bjerrum (1954). The curve of REN12 by the authors is also illustrated in this figure. As shown in this figure, the curve of REN12 is found to increase directly with increasing I_L, as same trend as those curves by Skempton and by Bjerrum, and also locates almost above those curves.

On Compression index

i) The relationship between the compression index and the liquid limit

Skempton (1944) had found Eq. (31) in the relationship between the compression index C_c and the liquid limit w_L, and afterwards Eqs. (32) to (36) were reported for different soft grounds in Japan. Fig. 11 shows the curves of these formula obtained in the past and the curves of REN17 and REN19 obtained by the authors. The curve of REN17 shows the regression result of analyzing C_c with only w_L as well as the regression result in the past. The curve of REN19 shows the multiple regression result of analyzing C_c with both w_L and the void ratio e.

As shown in this figure, the curve of REN17 is necessarily close to the curve of Eq. (32) for Osaka alluvial clay, and is also close to the curve of Eq. (31) by Skempton. But each curve of Eqs. (33) to (36) has a distinct tendency. This means that the factors of density and water content of the soil expressed in Eq. (4) are not taken into account in the regression analysis.

REN19 is analyzed by using the independent variable e characterizing soil density in addition to w_L. The curves of REN19 can describe approximately the curves of Eqs. (31) to (36) and others reported in the past, by varying the values of the parameter e of REN19 based on the measured data in Osaka and Hyogo (Fig. 12). This means that the multiple regression analysis presented here is valid.

Skempton (1944)

$$C_c=0.009(w_L-10) \qquad (31)$$

Osaka alluvial clay (Murayama, Akai and Ueshita, 1958)

$$C_c=0.01(w_L-12) \qquad (32)$$

Rumoi clay (Taniguchi, 1962)

$$C_c=0.004(w_L-10) \qquad (33)$$

Ishikari clay (Taniguchi, Abe and Goto, 1960)

$$C_c=0.014(w_L-20) \qquad (34)$$

Ariake clay (Kyushu Branch of JSSMFE, 1959)

$$C_c=0.013w_L \qquad (35)$$

Nagoya clay (Tokai Branch of AIJ, Chubu Branch of JSSMFE and the Society for Research in Nagoya Ground, 1969)

$$C_c=0.017(w_L-20) \qquad (36)$$

ii) The relationship between the compression index and the void ratio

Eqs. (37) to (44) show the relationships between the compression index C_c and the void ratio e which are reported for different soft

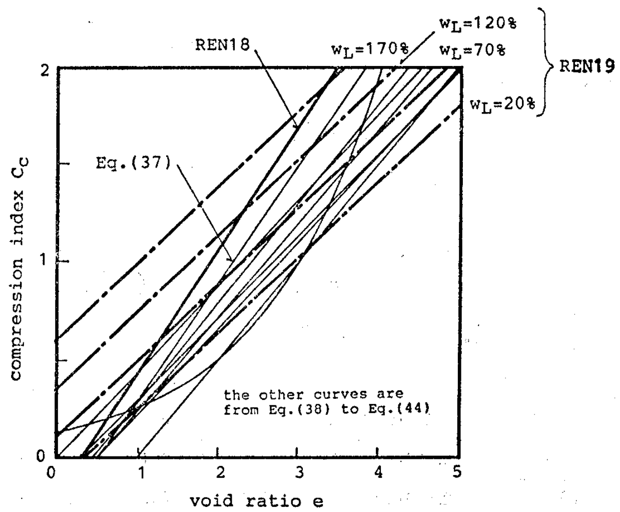

Fig. 13. Regression curves for C_c-e relationship

grounds in Japan. Fig. 13 shows the curves of these formula and the curves of REN18 and REN19 obtained by the authors. REN18 is the regression result of analyzing C_c with only e, and REN19 is the multiple regression result of analyzing C_c with both e and w_L. As shown in this figure, the curve of REN18 is necessarily close to the curve of Eq. (37) for Osaka alluvial clay. The curves of Eqs. (37) to (44), however, have distinct tendencies to each other, because they lack a factor characterizing the soil types as expressed in Eq. (4).

The curves of REN19 can describe approximately the curves reported in the past by varying the value of parameter w_L of REN19. This means that the multiple regression analysis presented here is valid.

Osaka alluvial clay (Murayama, Ueshita and Shibata, 1960)

$$C_c=0.6\,(e-0.5) \qquad (37)$$

Ariake clay (Kyushu Branch of JSSMFE, 1959)

$$C_c=0.48\,(e-0.50) \qquad (38)$$

Ariake Ohmuta clay (Uchida and Matumoto, 1959)

$$e=3.0+3.4\log C_c \qquad (39)$$

Chikuho Iizuka clay (Uchida and Matumoto, 1960)

$$C_c=0.42\,(e-0.30) \qquad (40)$$

Lake Biwa organic soil (Kinoshita, Horata and Taniyama, 1960)

$$C_c=0.44e \qquad (41)$$

Hachirogata clay(Fujita, Kudara and Harada, 1960)

$$C_c=0.44\,(e-0.5) \qquad (42)$$

Ohgaki clay (Japan Highway Public Co., 1963)

$$C_c=0.51e-0.23 \qquad (43)$$

Kushiro clay(Maeguchi, Sakai and Oyamada, 1965)

$$e=2C_c+1.0 \qquad (44)$$

Examination of the Multiple Regression Equation

On unconfined compressive strength

Comparing REN1 to REN3 with REN4 to

REN6 in Table 4, which represent the results of analyzing q_u and q_{ur} respectively of natural soils, the multiple correlation coefficients R of q_{ur} are found to be larger than those of q_u, in both regression models 1 and 2. This means that the characteristics of q_u include one more factor of soil structure which is significantly included in the factor of sensitivity ratio S_t than those of q_{ur} as shown in ihe following equation.

$$q_u = S_t q_{ur} \tag{45}$$

Comparing REN4 to REN6 in Table 4 with REA1 to REA4 in Table 14, which represent the results of analyzing q_{ur} of natural soils and artificially mixed soils respectively, it is found that R of q_{ur} for artificially mixed soils is larger than that of q_{ur} for natural soils. In Fig. 9, the curve of REA1 of q_{ur} for artificially mixed soils is located below the curve of REN4 of q_{ur} for natural soils. This means that there is difference in soil structure between artificially mixed soils and natural soils.

i) The regression model 1

As shown in REN2 and REN5 in Table 4 and in REA2 in Table 14, the absolute value of the partial correlation coefficient $r(=0.012)$ of e for artificially mixed soils is smaller than those of $r(=-0.371,\ -0.439)$ for the natural soils. Referring to the shifts of F-values during the analysis in Table 15, if the authors include e/w_L in the regression equation, the partial correlation coefficient $r(=-0.755)$ of e/w_L indicates a high absolute value. This means that the independent variable X_1 of Eq. (16) should be e for natural soils and e/w_L for artificially mixed soils.

Table 19 shows the predicted values of q_u and q_{ur} at the plastic and liquid limits, which are obtained by substituting $I_L=0$ and $I_L=1$ to the single regression equations of REN1, REN4 and REA1. As shown in this table, predicted values of q_u and q_{ur} at the plastic and liquid limits become smaller in descending order as follows: q_u of natural soils (REN1), q_{ur} of natural soils (REN4), q_{ur} of artificially mixed soils (REA1). This order is correlated with soil structure.

Table 19. Predicted q_{up} and q_{uL} (kgf/cm^2)

	q_{up}	q_{uL}
REN 1	4.289	0.566
REN 4	1.456	0.051
REA 1	0.568	0.022

(1 kgf/cm^2 = 98.1 kN/m^2)

ii) The regression model 2

Reviewing the F-values shown in the 2nd step in Table 6 or 8, the plastic limit w_p can account mainly for the variable X_2 in Eq. (22).

Referring to the partial correlation coefficient r of REN3, REN6 and REA3, e/w_L can contribute to characterizing q_u or q_{ur} much more than w_p or $1/w_L$.

Table 20 shows the values of M, Γ and λ that are obtained by the regression analysis of Eq. (19). As shown in this table, the variations of coefficients of physical properties which explain M, Γ and λ are small, thus the regression model 2 can be recognized.

On modulus of deformation

REN9 to REN11 in Table 4 show the analyzed results for the secant modulus of deformation E_{50} (modulus at $q_u/2$) of natural soils, and REA6 to REA9 in Table 14 show those for artificially mixed soils. As shown in these tables, the multiple correlation coefficients R of E_{50} or E_{50r} are high values for both regression models 1 and 2 as well as R of q_u. This shows that E_{50} or E_{50r} and q_u are correlated strongly to each other. The following regression result Eq. (46) was given by JSSMFE (1966).

$$E_{50} = 28.57 q_u - 1.14 \tag{46}$$

The authors obtained the following regression result:

$$E_{50} = 57.26 q_u - 7.30$$
$$n = 764 \quad R = 0.863 \qquad \text{(REN20)}$$

Table 20. M, Γ, λ of q_u or q_{ur} (kgf/cm^2)

	M	Γ	λ
REN 3	$3.114 \cdot \exp(0.039 w_p)$	0.511	$0.006 w_L$
REN 6	$1.836 \cdot \exp(0.033 w_p)$	0.444	$0.004 w_L$
REA 4	$2.606 \cdot \exp(0.030 w_p)$	0.351	$0.004 w_L$

(1 kgf/cm^2 = 98.1 kN/m^2)

The value of R for REN20 is very high. The difference of the partial regression coefficient of q_u between Eq. (46) and REN20, may be due to the influence of soil structure.

The theoretical relationship between E_{50} and q_u can be obtained as follows. In this study, the strain at failure ε_u of undisturbed soil is nearly constant. The value of ε_u has the average of 5.03% and the value of standard deviation is 3.44%. Assuming ε_u and ε_0 as constant, Eq. (47) is obtained.

$$E_{50}=q_u/2\varepsilon_0=aq_u \qquad (47)$$

where ε_0: strain at $q_u/2$

a : constant $(=1/2\varepsilon_0)$

Accordingly, the strong correlation between q_u and E_{50} can be explained, and the form of linear regressions in both Eq. (46) and REN20 can also be proved.

Table 21 shows the predicted values of E_{50} and E_{50r} at plastic and liquid limits, and Table 22 shows the predicted values of M, Γ, λ. As shown in these tables, both the influence of soil structure and the stability of the regression equation will be as same as in case of q_u.

On consolidation yield stress

As shown in REN14 to REN16 in Table 4, the multiple correlation coefficients R show large values for both regression models 1 and 2, as also in case of q_u. This means that a correlation between q_u and p_c exists as expressed by Eq. (17), and the value of M in this Eq. (17) is said to be a constant of approximately 0.3 (Mikasa, Nishigaki and Okajima, 1978; Nakase and Kamei, 1988).

Table 21. Predicted E_{50p} and E_{50L} (kgf/cm²)

	E_{50p}	E_{50L}
REN 9	234.628	22.109
REA 6	5.760	0.348

(1 kgf/cm² = 98.1 kN/m²)

Table 22. M, Γ, λ of E_{50} or E_{50r} (kgf/cm²)

	M	Γ	λ
REN 11	$285.431 \cdot \exp(0.033w_p)$	0.304	$0.006w_L$
REA 9	$83.013 \cdot \exp(0.013w_p)$	0.214	$0.004w_L$

(1 kgf/cm² = 98.1 kN/m²)

For some references, Table 23 shows the predicted values of p_c at plastic and liquid limits from regression model 1, and Table 24 shows the predicted values of M, Γ and λ from regression model 2.

On sensitivity ratio

As shown by the low multiple correlation coefficients R (=0.460, 0.494) in REN12 and REN13 in Table 4 or by the wide scattering plots in Fig. 14, the correlation between S_t and I_L is weak. Therefore it is proved quantitatively that sensitivity ratio or soil structure is almost independent of types, density and water content of soil.

In future, it would be necessary to analyze S_t by the factors of the geohistory or chemical properties which are not treated in this paper.

On internal friction angle

Internal friction angle (ϕ_u) in this paper is determined by undrained triaxial compression test. REN7 in Table 4 shows that the multiple

Table 23. Predicted p_{cp} and p_{cL} (kgf/cm²)

	p_{cp}	p_{cL}
REN 14	9.043	1.418

(1 kgf/cm² = 98.1 kN/m²)

Table 24. M, Γ, λ of p_c (kgf/cm²)

	M	Γ	λ
REN 16	$4.894 \cdot \exp(0.047w_p)$	0.676	$0.006w_L$

(1 kgf/cm² = 98.1 kN/m²)

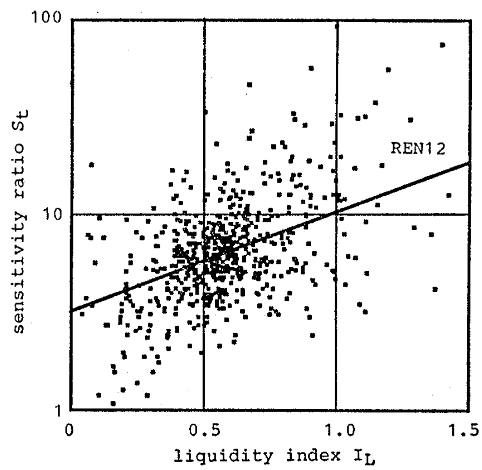

Fig. 14. Measured data and regression curve for S_t-I_L relationship

correlation coefficient R (=0.267) is small for the cohesive soils of which the clay content is more than 20%, but the authors obtained high value of R for the cohesive soil composed of much sand and gravel (Yao, Hirata and Saito, 1985a).

As a result, in Tables 4 and 14 the multiple correlation coefficients R show large values for q_u, q_{ur}, ε_{ur}, E_{50}, E_{50r}, p_c and C_c, but show small values for ε_u and ϕ_u.

Comparison of the Two Regression Models

In order to examine the relationship between each independent variable in regression models 1 and 2 and to ensure that the regression models 1 and 2 are essentially the same model, the following examination was carried out.

In the case of q_u or q_{ur} of natural soils, as shown in Table 6 or Table 8, an F-value of I_L is the highest in the Initiation state, that is, I_L is the most suitable independent variable. But at the 1st step, F-value of I_L reduces suddenly, if e/w_L is included in the regression equation. This means that there is a strong correlation between I_L and e/w_L. This correlation is explained as follows.

$$I_L=(w-w_p)/(w_L-w_p)$$
$$=(w-w_p)/I_p=w/I_p-w_p/I_p$$

Assuming $I_p=aw_L$, because of the strong correlation between I_p and w_L (Hirata, 1985).

$$I_L=w/aw_L-w_p/aw_L$$

Assuming $w_p=bw_L$, because of the strong correlation between w_p and w_L (Hirata, 1985).

$$I_L=w/aw_L-bw_L/aw_L=Aw/w_L-B$$

where $A=1/a$, $B=b/a=bA$

By using the relationship $w=S_re/G_s$,

$$I_L=AS_re/G_sw_L-B$$

Considering specific gravity G_s is approximately constant of 2.65, the following Eq. (48) is obtained.

$$I_L=A'S_re/w_L-B \tag{48}$$

where $A'=A/2.65$

Assuming $S_r=100\%$ as a fully saturated soil, Eq. (49) is obtained.

$$I_L=A'e/w_L-B \tag{49}$$

Thus, in case of natural soils, the relationship between I_L and e/w_L is proved to be dependent as shown by Eq. (49). Therefore in Tables 6 and 8, it can be understood that the F-value of I_L become very small at the 1st step, where e/w_L has already been included in the regression equation. Further, as shown in Table 4, the predictive accuracy of regression model 2 of which the independent variables are $1/w_L$ and w_p in addition to e/w_L, could be understood to be necessarily higher than those by model 1.

CONCLUSIONS

In this paper, the relationships between a mechanical property and some physical properties were studied by employing the multiple regression analysis for both natural and artificially mixed cohesive soils. The following conclusions were obtained:

(1) The regression models 1 and 2 proposed herein were proved to be valid for predicting shear strength of cohesive soils by examining F-values on analysis procedures and multiple correlation coefficients R of analysis results. These models were also valid for predicting modulus of deformation E_{50}, consolidation yield stress p_c, etc., because of their strong correlations with shear strength.

(2) The validity of the concept that the mechanical property is characterized by the soil types and states, can be quantitatively proved by performing the multiple regression analysis proposed herein.

(3) By examining the shifts of F-value on analysis procedure, mutual relationships between each physical property to explain a mechanical property can be understood and made clear. The dependent relationship between the regression models 1 and 2 in the fully saturated case was proved by both regression analysis and theoretical examination.

(4) The regresssion equations of unconfined compressive strength q_u, modulus of deformation E_{50}, consolidation yield stress p_c and compression index C_c were obtained with high predictive accuracy.

NOTATION

A_c=activity
C=clay content
C_c=compression index
c_u=cohesion
$c_{uL}=c_u$ at liquid limit
$c_{up}=c_u$ at plastic limit
$c_{ur}=c_u$ at remolded state
$D_{\max}$=maximum grain size
D_{10}=effective grain size
e=void ratio
E_{50}=secant modulus of deformation at $q_u/2$
$E_{50L}=E_{50}$ at liquid limit
$E_{50p}=E_{50}$ at plastic limit
$E_{50r}=E_{50}$ at remolded state
I_f=flow index
I_L=liquidity index
I_p=plasticity index
M=slope of critical state line when it is projected on to a constant volume plane
M_0=silt content
N=number of blows of liquid limittest
n=total number of data samples
p_c=consolidation yield stress
$p_{cL}=p_c$ at liquid limit
$p_{cp}=p_c$ at plastic limit
P_r=plasticity ratio $(=I_p/w_p)$
q_u=unconfined compressive strength
$q_{uL}=q_u$ at liquid limit
$q_{up}=q_u$ at plastic limit
q_{ur}=remolded unconfined compressive strength
R=multiple correlation coefficient
r=partial correlation coefficient
S=sand content
S_r=degree of saturation
S_t=sensitivity ratio
T=correcting coefficient of Eq. (28)
U_c=uniformity coefficient
U_c'=coefficient of curvature
w=water content
w_L=liquid limit
w_{Lt}=measured liquid limit
w_p=plastic limit
w_{pt}=measured plastic limit
Γ=void ratio of soil at critical state
γ_t=unit weight
ε_0=strain at $q_u/2$
ε_u=failure strain
$\varepsilon_{ur}=\varepsilon_u$ at remolded state
λ=slope of normal consolidation line
ϕ_u=undrained internal friction angle

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