Detection and Tracking of UAVs Using Interferometric Radar

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***Abstract*—In the modern era, the use of drones and unmanned aerial vehicles (UAVs) is rapidly increasing, both in civilian and commercial industries as well as in military applications. For example, drones are increasingly used in agriculture, logistics, professional photography, and delivery services. However, the growing popularity of drones also raises significant concerns about their potential misuse for illegal, criminal, or military purposes, such as smuggling contraband and carrying out terrorist attacks.**

The ability to detect and track drones is critical for preventing terrorist attacks and criminal activities. Radar systems are considered the leading solution in this field due to their capability to operate over long distances and, with proper adjustments, under extreme weather conditions and at any time. Nevertheless, the detection and tracking of drones present a challenging problem because of their small size, unconventional flight patterns, and strong resemblance to birds.

I. Introduction

Drones are becoming increasingly prevalent, entering various industries such as photography, fashion, and collectibles. However, there is a growing concern regarding their misuse of criminal activities, such as cross-border smuggling, drug trafficking, and other illegal operations. Therefore, the ability to detect and track drones is critical. Microwave radar systems offer unique advantages over other drone detection and monitoring sensors, particularly over long distances, under all weather conditions, and day and night. These radars can provide diverse information such as range, velocity, direction of arrival, and micro-Doppler features, enabling precise classification. However, detecting and tracking drones is not simple due to their small size, relatively slow speed, low altitude, and flexible movements, such as hovering or circling, which make it challenging to construct a predictable mathematical model of their motion. Moreover, drones' radar cross-section (RCS) is similar to that of birds, making it difficult to distinguish between them. Traditional detection and tracking methods often fail to address these specific characteristics of drones. New drone detection and tracking methods have been developed in recent years. In this paper, the authors propose a novel method using an interferometric radar based on two-dimensional range and velocity measurements (radial and angular). The range and radial velocity are estimated using FFT on fast-time and slow-time data, while the angular velocity is derived from radar measurements. Detection relies on the range-radial velocity profile, but angular velocity is also necessary for tracking, as drones do not fly in straight lines along the radar's line of sight (LoS). The proposed method leverages two-dimensional range and velocity information to predict the drone's next position.

II. Problem Definition

Detection and tracking small uncrewed aerial vehicles (UAVs) is a technological challenge due to various characteristics, such as their unpredictable and inconsistent motion and physical structure. Small UAVs are typically smaller and fly at lower altitudes than other aerial targets, making their detection significantly more complex.

Additionally, their motion is often erratic, including circular movements and rapid changes in direction, which complicates the development of mathematical models capable of predicting their trajectories and movements. Furthermore, the radar cross-section (RCS) of small UAVs is similar to that of birds, creating additional challenges in accurate identification and reducing false alarms.

Traditional methods are often insufficient for detection and tracking small UAVs, as they primarily focus on range and radial velocity and fail to address the complexities of two-dimensional motion. Another critical issue is the ability to perform real-time tracking of UAVs while managing radar data that may be heavily influenced by noise or measurement inaccuracies. Traditional solutions in the field fail to fully leverage UAVs' unique motion characteristics, leading to suboptimal accuracy and consistency in tracking performance.

III. The Solution Presented in the Paper

The primary solution proposed in the paper involves utilizing an interferometric radar to detect and track single UAVs by leveraging range and two-dimensional velocity measurements. The system combines radial and angular velocity estimation to improve tracking accuracy for targets with non-linear and unpredictable motion patterns.

A. System Overview

The radar system operates based on frequency-modulated continuous wave (FMCW) principles, where the transmitted signal is expressed as:

$$s_T(t) = \exp\left(-j2\pi\left[f_0 t_s + \frac{K}{2}t_s^2\right]\right), \tag{1}$$

where f_0 is the carrier frequency, $K = \frac{B}{T}$ is the chirp rate (defined by bandwidth B and sweep time T), and t_s represents the time from the start of the chirp.

The UAV is detected based on the range (R) and radial velocity (v_r), estimated through processing the received signal:

$$s_R(t) = \exp\left(-j2\pi\left[f_0(t_s - \tau) + \frac{K}{2}(t_s - \tau)^2\right]\right), \quad (2)$$

where τ is the time delay proportional to the UAV's distance.

B. Range and Radial Velocity Estimation

The range R and radial velocity v_r are determined by applying a fast and slow Fourier transform (FFT) on the beat frequency signal:

$$R(t) \approx R_0 + v_r n T, \quad (3)$$

$$v_r = \frac{f_d \lambda}{2}, \quad (4)$$

where f_d is the Doppler frequency shift, $\lambda = c/f_0$ is the wavelength, and c is the speed of light.

C. Angular Velocity Calculation

The angular velocity ω is derived using the interferometric frequency shift (f_a), which is obtained from the phase difference between the beat signals of two antennas.

The second receiving antenna introduces a geometrical delay τ_0, defined as:

$$\tau_0 = \frac{D \sin\phi}{c}, \quad (5)$$

where: - D is the baseline distance between the two antennas, - ϕ is the angle of arrival relative to the antenna broadside, - c is the speed of light.

This delay contributes to the phase difference in the received signals, leading to a time-varying interferometric frequency shift. The interferometric beat signal is expressed as:

$$y_c(t) = s_{B1}(t) s_{B2}^*(t), \quad (6)$$

where $s_{B1}(t)$ and $s_{B2}(t)$ are the beat signals from the first and second antennas, respectively.

The interferometric frequency shift is calculated as the time derivative of the phase term:

$$f_a = \frac{\partial}{\partial t}\left[(f_0 + K t_s)\frac{D \sin\phi}{c}\right]. \quad (7)$$

For small angles ($\phi \approx 0$), simplifying assumptions yield:

$$f_a \approx \frac{D}{\lambda_t}\omega, \quad (8)$$

where: - $\lambda_t = c/(f_0 + K t_s)$ is the wavelength corresponding to the instantaneous frequency of the radar signal.

Rearranging for ω, the angular velocity is expressed as:

$$\omega = \frac{f_a \lambda_t}{D}. \quad (9)$$

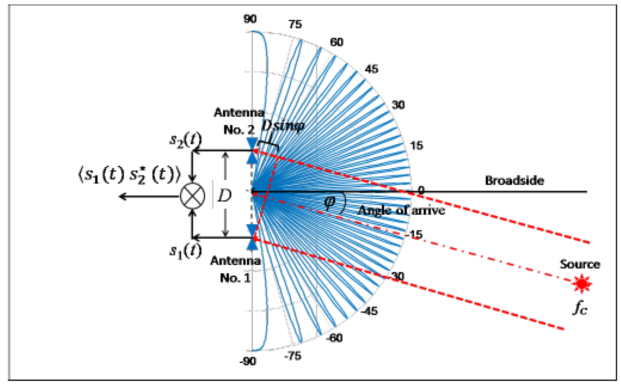

Fig. 1. Geometry of the interferometric radar system.

1) Practical Implementation: To improve robustness against noise, the interferometric frequency shift f_a can be obtained by subtracting the estimated time-varying spectra of the beat signals received by the two antennas. This ensures more accurate computation of ω under practical conditions.

D. Tracking Algorithm with Kalman Filter

The UAV's trajectory is estimated by combining the range and two-dimensional velocity information (radial and angular velocities). Due to the presence of noise and potential inaccuracies in the measurements, a Kalman filter is employed to enhance the accuracy of the tracking process.

The state vector of the UAV at time k is defined as:

$$x_k = \begin{bmatrix} \phi \\ R \\ \omega \\ v_r \end{bmatrix}, \quad (10)$$

where ϕ is the angular position, R is the range, ω is the angular velocity, and v_r is the radial velocity.

The Kalman filter operates in two main steps: 1. Prediction Step: Predict the next state based on the current state and a state transition model:

$$x_{k|k-1} = F x_{k-1} + \nu, \quad (11)$$

where F is the state transition matrix:

$$F = \begin{bmatrix} 1 & 0 & t & 0 \\ 0 & 1 & 0 & t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (12)$$

and ν represents the process noise with covariance Q:

$$Q = q\begin{bmatrix} \frac{1}{3}t^3 & 0 & \frac{1}{2}t^2 & 0 \\ 0 & \frac{1}{3}t^3 & 0 & \frac{1}{2}t^2 \\ \frac{1}{2}t^2 & 0 & t & 0 \\ 0 & \frac{1}{2}t^2 & 0 & t \end{bmatrix}. \quad (13)$$

2. Update Step: Update the state estimate based on the measurement Z_k and observation model:

$$Z_k = H x_k + \omega, \quad (14)$$

where H is the observation matrix:

$$H = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \tag{15}$$

and ω represents the measurement noise with covariance R.

The updated state is calculated as:

$$x_k = x_{k|k-1} + K_k(Z_k - Hx_{k|k-1}), \tag{16}$$

where K_k is the Kalman gain:

$$K_k = P_{k|k-1}H^T(HP_{k|k-1}H^T + R)^{-1}. \tag{17}$$

The updated covariance matrix is:

$$P_k = (I - K_kH)P_{k|k-1}. \tag{18}$$

E. Illustration of the Method

The following figure illustrates the geometry of the interferometric radar setup and the tracking process.

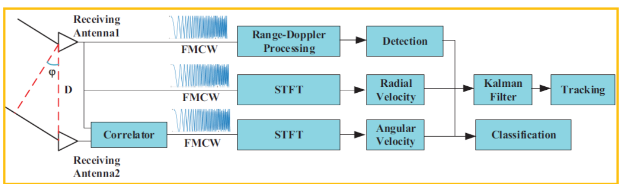

Fig. 2. Illustration of the tracking and detecting process based on 2-D velocity estimation and Kalman filtering.

This methodology effectively combines interferometric radar capabilities with advanced signal processing to achieve accurate real-time UAV detection and tracking.

IV. My Proposed Modification to this Solution

The original approach demonstrated effective detection and tracking of single UAVs using interferometric FMCW radar. However, its applicability to scenarios with multiple overlapping targets is limited. My proposed modification significantly enhances this capability by integrating advanced signal processing techniques, robust mathematical frameworks, and the geometry of interferometric radar systems, allowing for simultaneous and precise tracking of multiple UAVs even in challenging environments.

A. Objective and Mathematical Framework

The proposed approach focuses on addressing the challenges of multi-target tracking by accurately estimating the two-dimensional positions and velocities of each target. This includes both *radial velocity* and *angular velocity* using *stretch processing* and *time-frequency analysis*. Additionally, a *Nearest Neighbor Data Association (NNDA)* algorithm ensures robust tracking in situations with overlapping or ambiguous signals.

1) Transmission Model: The radar transmits an FMCW signal characterized as:

$$s_T(t) = \exp\left(j2\pi\left[f_0t + \frac{K}{2}t^2\right]\right), \tag{19}$$

where:

- f_0 is the carrier frequency,
- $K = \frac{B}{T}$ is the chirp rate, with B as the bandwidth and T as the sweep time.

2) Received Signals and Range Estimation: For a target located at range R and angle ϕ, the beat signals received by the two antennas are:

$$s_{B1}(t) = \exp\left(j2\pi\left[f_0\tau_1 + \frac{K}{2}(2\tau_1t - \tau_1^2)\right]\right), \tag{20}$$

$$s_{B2}(t) = \exp\left(j2\pi\left[f_0\tau_2 + \frac{K}{2}(2\tau_2t - \tau_2^2)\right]\right), \tag{21}$$

where τ_1 and τ_2 represent the time delays at each antenna.

The range for each antenna is calculated as:

$$R_1 = \frac{cf_1}{4\pi K}, \quad R_2 = \frac{cf_2}{4\pi K}, \tag{22}$$

where c is the speed of light, K is the chirp rate, and f_1, f_2 are the beat frequencies for the two antennas.

To improve accuracy, the average range is used:

$$R = \frac{R_1 + R_2}{2}. \tag{23}$$

3) Angle Estimation: The angle of arrival is calculated using the geometric relationship between the antennas and the target:

$$\phi = \arcsin\left(\frac{R_1 - R_2}{D}\right), \tag{24}$$

where D is the baseline distance between the antennas.

4) Velocity Estimation: To estimate the velocities of the targets, the radial velocity for each antenna is derived from the Doppler frequency:

$$v_{r1} = \frac{\lambda f_{d1}}{2}, \quad v_{r2} = \frac{\lambda f_{d2}}{2}, \tag{25}$$

where $\lambda = \frac{c}{f_0}$ is the wavelength of the transmitted signal, and f_{d1}, f_{d2} are the Doppler frequencies for the two antennas.

The average radial velocity is:

$$v_r = \frac{v_{r1} + v_{r2}}{2}. \tag{26}$$

Finally, the angular velocity is estimated using:

$$\omega = \frac{v_{r1} - v_{r2}}{D\cos\phi}. \tag{27}$$

5) Data Association Using NNDA: The proposed approach incorporates the *Nearest Neighbor Data Association (NNDA)* algorithm to effectively match radar detections with individual targets. NNDA operates by calculating the distance between each predicted target state and all incoming measurements. The algorithm minimizes the assignment error to ensure accurate matching even in scenarios with multiple overlapping signals.

For each target, the distance to a measurement is calculated using:

$$d_{ij} = \sqrt{(x_i - z_j)^2 + (y_i - z_j)^2 + (v_{r,i} - v_{r,j})^2}, \tag{28}$$

where:

- (x_i, y_i) and (z_j) are the predicted and measured positions, respectively,
- $v_{r,i}$ and $v_{r,j}$ are the predicted and measured radial velocities.

The distances are stored in a cost matrix C, where each element $C[i, j]$ represents the distance between the i-th target and the j-th measurement:

$$C = \begin{bmatrix} d_{11} & d_{12} & \dots & d_{1M} \\ d_{21} & d_{22} & \dots & d_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ d_{N1} & d_{N2} & \dots & d_{NM} \end{bmatrix}. \tag{29}$$

Each target is assigned to the nearest measurement based on the minimum distance in the cost matrix:

$$z_j = \arg\min_j C[i, j]. \tag{30}$$

In cases of ambiguous assignments (e.g., multiple targets competing for the same measurement), additional criteria such as trajectory continuity are used to resolve conflicts. By leveraging NNDA, the algorithm ensures robust data association in dynamic and noisy environments.

6) Trajectory Tracking Using Kalman Filter: After measurements are assigned to targets using NNDA, the state of each target is updated using a *Kalman Filter (KF)*. The Kalman Filter combines information from radar measurements (range, radial velocity, and angular velocity) with the target's predicted state to estimate its position and velocity in 2D space.

The state vector for each target is:

$$\mathbf{x}_k = \begin{bmatrix} x_k \\ y_k \\ v_{x,k} \\ v_{y,k} \end{bmatrix}, \tag{31}$$

where (x_k, y_k) are the position coordinates, and $(v_{x,k}, v_{y,k})$ are the velocity components in the x and y directions.

The Kalman Filter operates in two steps:

a) Prediction Step: The state and covariance are predicted using the motion model:

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{F}\hat{\mathbf{x}}_{k-1}, \tag{32}$$

$$\mathbf{P}_{k|k-1} = \mathbf{F}\mathbf{P}_{k-1}\mathbf{F}^T + \mathbf{Q}, \tag{33}$$

Where F and Q are the same as above in single UAV detection, but this time H defined as:

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{34}$$

b) Update Step: The predicted state is corrected using new radar measurements:

$$\mathbf{K}_k = \mathbf{P}_{k|k-1}\mathbf{H}^T \left(\mathbf{H}\mathbf{P}_{k|k-1}\mathbf{H}^T + \mathbf{R}\right)^{-1}, \tag{35}$$

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k \left(\mathbf{z}_k - \mathbf{H}\hat{\mathbf{x}}_{k|k-1}\right). \tag{36}$$

V. Results

The following section presents the results of simulations conducted to evaluate the detection and tracking algorithms. The results are divided into two main parts: replication of the original methodology (single UAV detection and tracking) and the proposed extension for tracking multiple UAVs.

A. Replication of the Original Methodology

1) Range-Radial Velocity Map for Single Target: The first stage of the simulation replicates the detection methodology described in the original paper. A range-radial velocity map was generated to identify a single UAV. This map highlights the UAV as a distinct peak in the range and radial velocity dimensions, providing the foundation for trajectory tracking.

Figure 3 shows the range-radial velocity map generated for a single UAV. The distinct peak corresponding to the UAV is easily separable from noise and clutter, confirming the robustness of the radar system in isolating a single target.

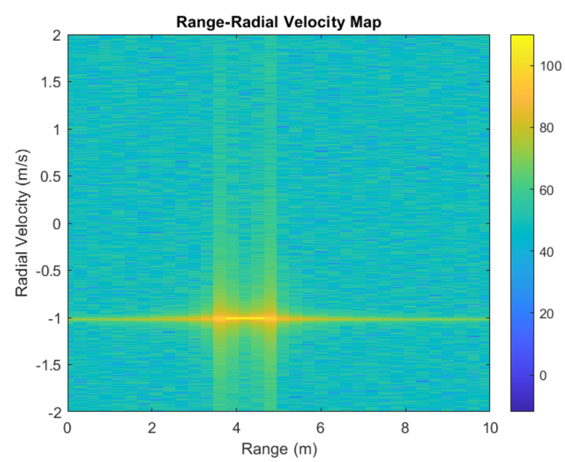

Fig. 3. Range-Radial Velocity Map for Single Target: Clear detection of a UAV based on range and radial velocity.

2) Single UAV Tracking: Once the UAV is detected, its trajectory is tracked using the Kalman filter. Figure 4 compares the true trajectory of the UAV with the estimated trajectory. Despite the presence of noise, the Kalman filter provides a smooth and accurate estimate of the UAV's path.

This result validates the effectiveness of the original methodology in detecting and tracking a single UAV.

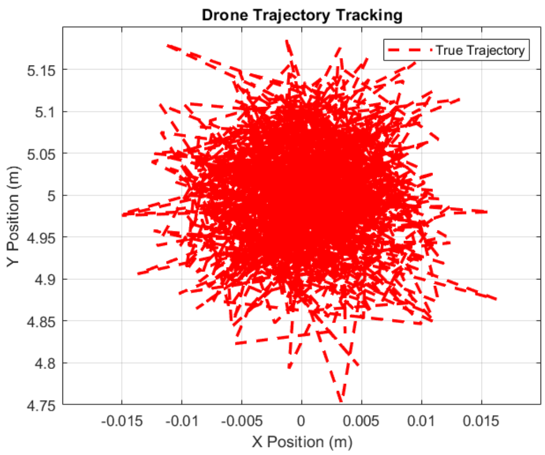

Fig. 4. Single UAV Tracking: Comparison of the true trajectory with the estimated trajectory using the Kalman filter.

B. Proposed Extension for Multiple UAV Tracking

1) Range-Radial Velocity Map for Multiple Targets: To extend the methodology to multiple UAVs, a range-radial velocity map was generated for a scenario involving two targets. Figure 5 shows the range-radial velocity map with two distinct peaks corresponding to the two UAVs. The peaks are clearly separated, allowing for accurate initialization of the tracking process.

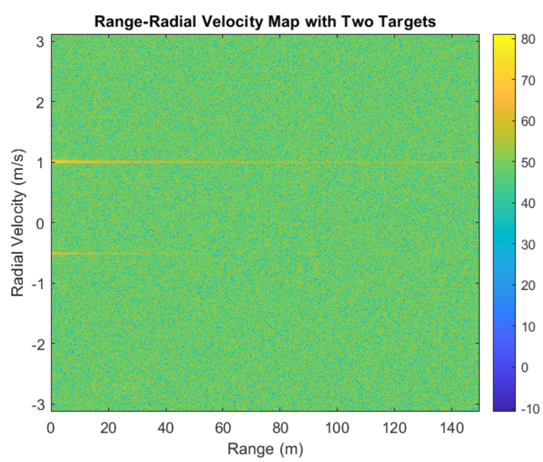

Fig. 5. Range-Radial Velocity Map for Two Targets: Detection of two UAVs in the range and radial velocity dimensions.

This map demonstrates the radar system's capability to detect multiple UAVs, even in noisy conditions, and provides the required parameters for the subsequent tracking phase.

2) Multiple UAV Tracking: Building on the range-radial velocity map, the proposed algorithm was used to simultaneously track the trajectories of two UAVs. The algorithm employs a nearest-neighbor data association method combined with the Kalman filter to differentiate between overlapping measurements and maintain accurate trajectory estimates.

Figure 6 illustrates the estimated trajectories of the two UAVs. Despite overlapping signals and noise, the algorithm successfully tracks both UAVs independently.

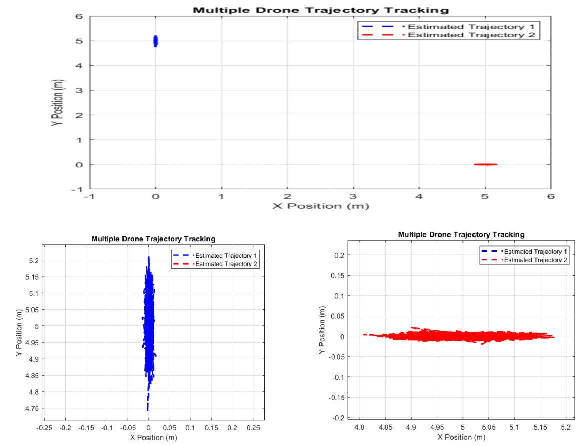

Fig. 6. Multiple UAV Tracking: Estimated trajectories of two UAVs using the proposed algorithm.

The results confirm the robustness of the proposed method for handling complex scenarios involving multiple targets.

C. Discussion

The simulations highlight the strengths of the proposed extensions to the original methodology. The range-radial velocity map serves as a reliable tool for detecting UAVs, while the Kalman filter ensures accurate trajectory tracking in both single and multiple UAV scenarios.

For single UAV tracking, the system effectively compensates for noise, providing smooth and reliable trajectory estimates. In the case of multiple UAVs, the proposed data association and filtering methods enable accurate tracking even under challenging conditions such as overlapping signals and noise.

These results suggest that the proposed extensions significantly enhance the radar system's capability, providing a scalable solution for UAV detection and tracking. Future work may focus on real-world testing, incorporating dynamic flight patterns and environmental clutter.

VI. Conclusions

The detection and tracking of UAVs have become critical challenges in modern radar systems, particularly with the increasing prevalence of small UAVs in civilian and military applications. The first solution, as presented in the original paper, demonstrates the capability of an interferometric radar system to effectively track single UAVs. By leveraging range, radial velocity, and angular velocity measurements, and employing advanced signal processing techniques such as FFT and Kalman filtering, the method achieves robust tracking for targets with non-linear and erratic flight patterns.

However, the original approach is limited to scenarios involving single targets, making it unsuitable for environments where multiple UAVs are present. The second paper addresses this limitation by introducing a modified system capable of simultaneous tracking of multiple UAVs. This is achieved through stretch processing, interferometric measurements, and the integration of a nearest-neighbor data association algorithm. By utilizing both radial and angular velocity information, along with improved data association and trajectory prediction methods, the proposed solution extends the applicability of interferometric radar systems to more complex scenarios.

In summary, the combined contributions of these two works highlight the evolution of interferometric radar technology. The first paper establishes a foundational method for single-target tracking, while the second paper advances this approach to handle multiple-target environments, offering a comprehensive framework for UAV detection and tracking in increasingly challenging operational scenarios. Future work may explore additional enhancements, such as machine learning techniques for improved classification or adaptive filtering to further mitigate noise and interference in real-time tracking systems.

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