

# MIMO Simulation Session — Solutions

## Part A — User Selection Simulation

### Preliminary Questions

1. **What is the maximal number of users the BS can simultaneously transmit to using Zero-Forcing Beam Forming (ZFBB)?**

The maximal number of users that a base station (BS) can simultaneously transmit to using Zero-Forcing Beam Forming (ZFBB) is equal to the number of transmit antennas, denoted as  $t$ . This is because, in ZFBB, the transmitter uses the knowledge of the channel state information (CSI) to form the precoding matrix. The precoding matrix is formed in such a way that the interference at each of the user's receivers is eliminated or 'zero-forced', hence the name. Therefore, in an ideal scenario with perfect CSI, each antenna can serve a single user without causing any interference to the other users.

2. **What is the SNR observed by user  $i$  under ZFBB?**

The Signal to Noise Ratio (SNR) observed by user  $i$  under ZFBB can be computed based on the power allocated to user  $i$ , the gain from the base station to user  $i$ , and the noise power. SNR considers both the signal power and noise power, hence it's a good measure of the signal quality. Mathematically,  $SNR_i = |h_i|^2 * P_i / \sigma^2$ , where  $|h_i|^2$  is the channel gain from BS to user  $i$ ,  $P_i$  is the power allocated to user  $i$ , and  $\sigma^2$  is the noise power. In a ZFBB system, the channel gain and power allocation can be influenced by the precoding matrix.

3. **How does one generate a zero-mean  $\sigma^2$ -variance complex Gaussian random variable? You may find this link from Wikipedia helpful.**

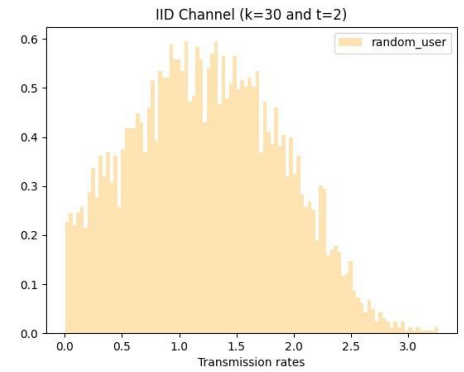
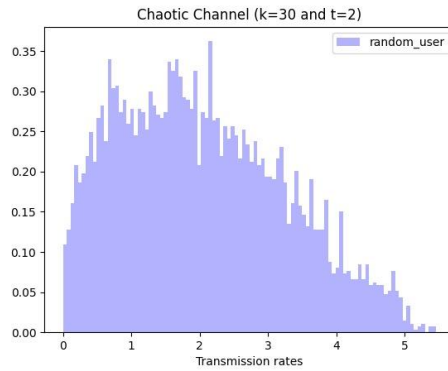
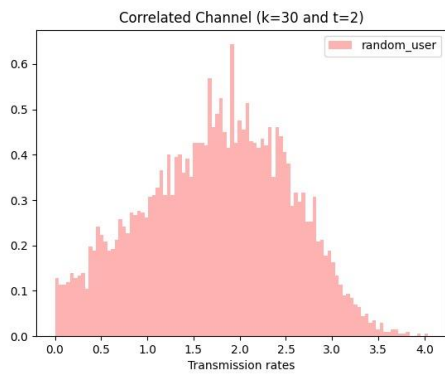
A zero-mean  $\sigma^2$ -variance complex Gaussian random variable can be generated by combining two independent Gaussian random variables with zero mean and variance  $\sigma^2/2$  for the real and imaginary parts. This is because the variance of a complex Gaussian random variable is the sum of the variances of its real and imaginary parts. Many programming environments (e.g., Python, MATLAB) have built-in functions to generate Gaussian random variables.

## Defining A Channel

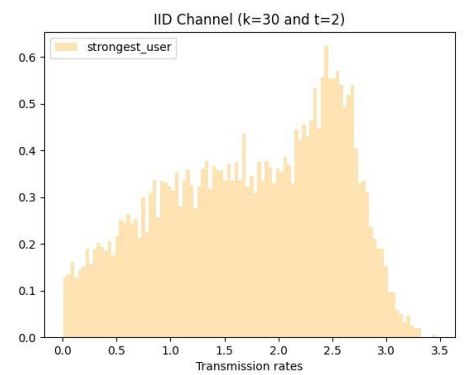
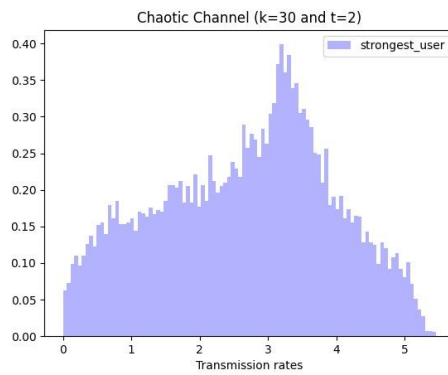
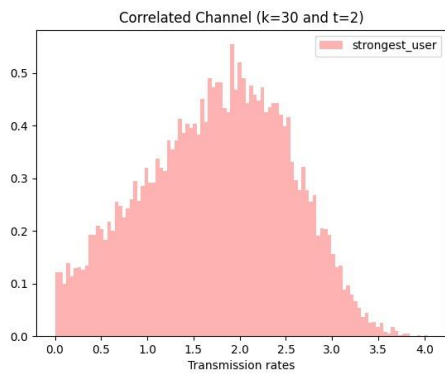
1. The **i.i.d complex Gaussian channel model** describes a MIMO system where the fading coefficients between each pair of transmit and receive antennas are independent and identically distributed complex Gaussian random variables.
2. In the **chaotic channel model**, the channel gain between each pair of transmit and receive antennas is a complex Gaussian random variable with a random mean and variance. The parameters of this Gaussian distribution are uniformly distributed, meaning that they can take any value between their specified ranges with equal probability. These parameters are drawn only once, meaning the channel gains remain constant over time, but vary across different pairs of antennas.
3. The **complete correlative channel model** is a special case where the channel gains between all pairs of transmit and receive antennas are identical, i.e., they are drawn from the same complex Gaussian distribution with zero mean and unit variance. This channel model is typically used to model scenarios where all the antennas are co-located and experience the same channel conditions. This kind of channel model is quite rare in practice due to the assumption of complete correlation.

## Simulation:

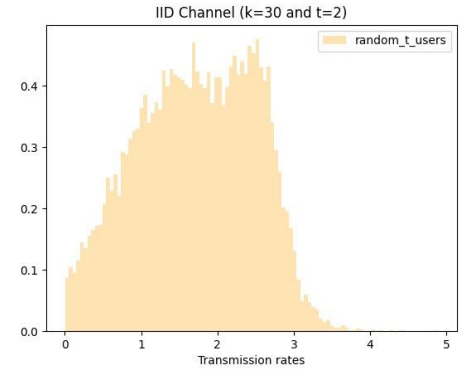
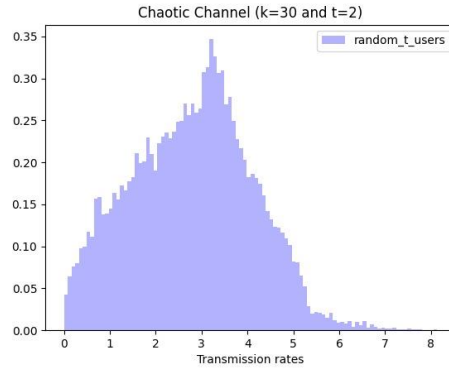
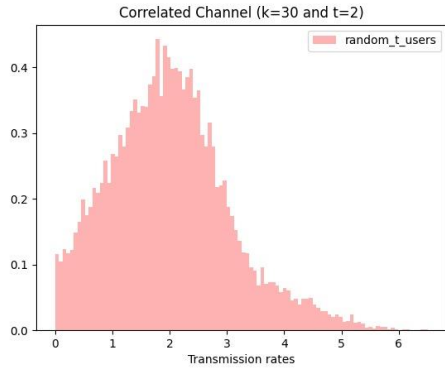
### Random User:



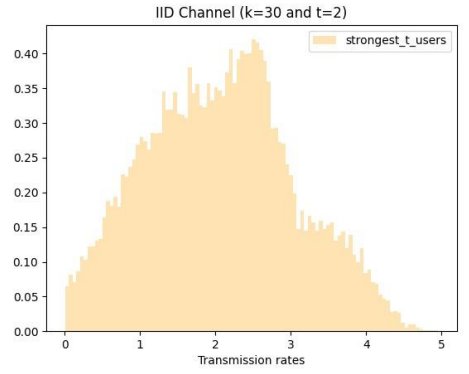
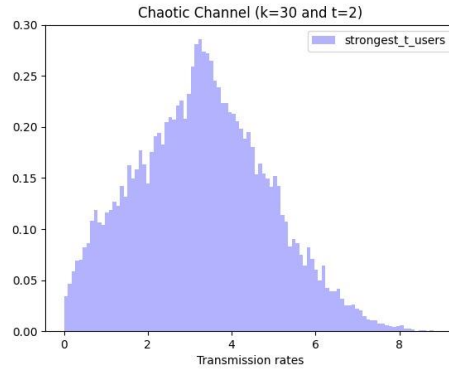
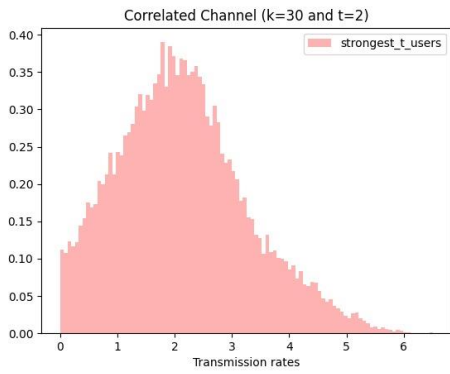
### Strongest User:



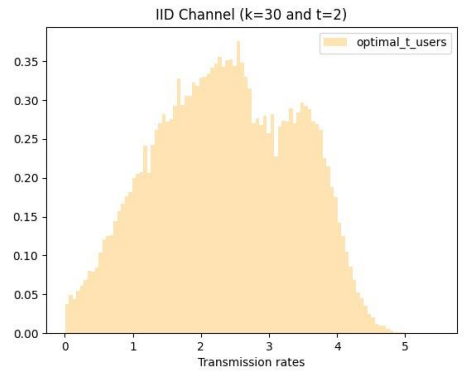
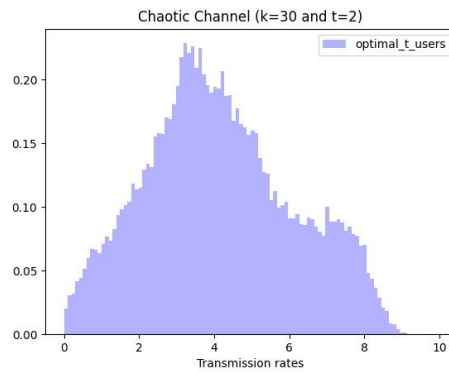
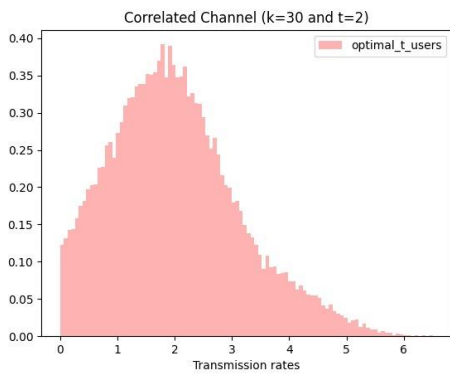
### Random T Users:



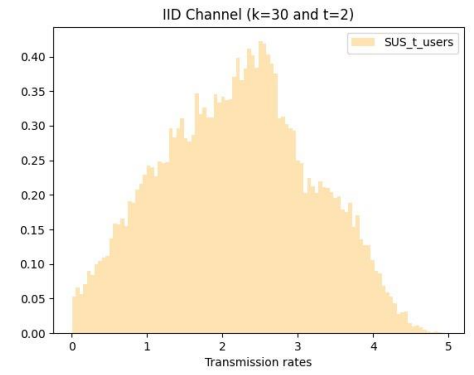
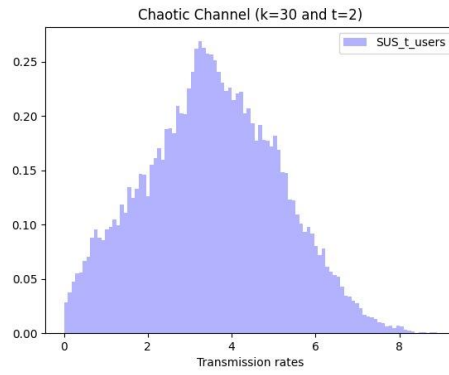
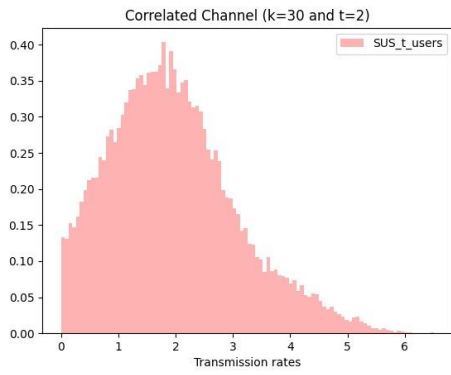
Strongest T Users:



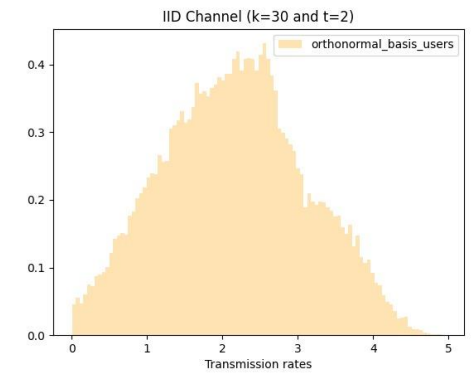
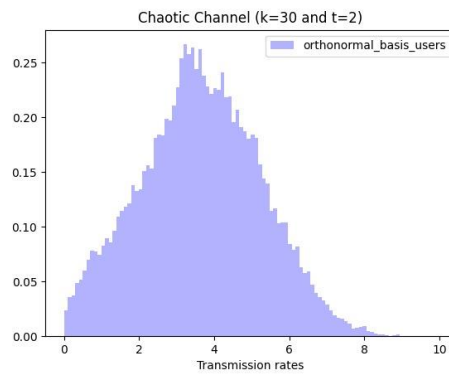
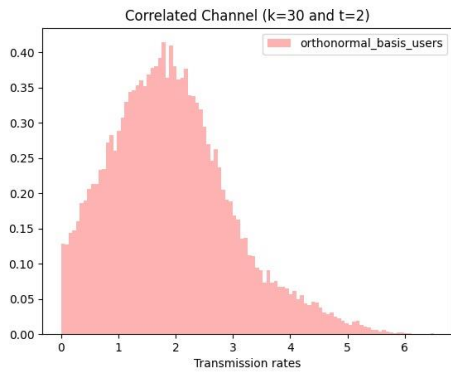
Optimal T Users:



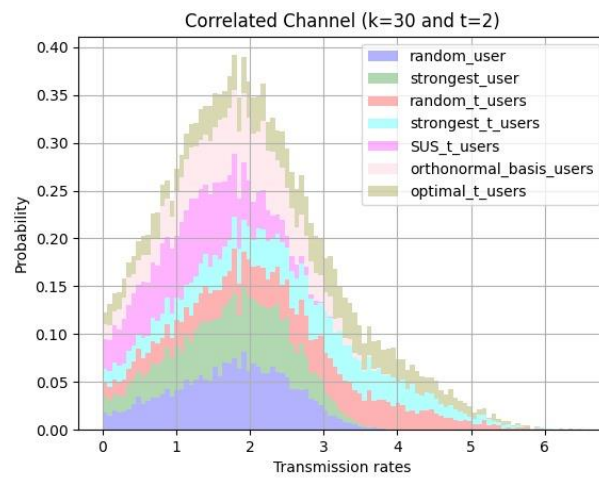
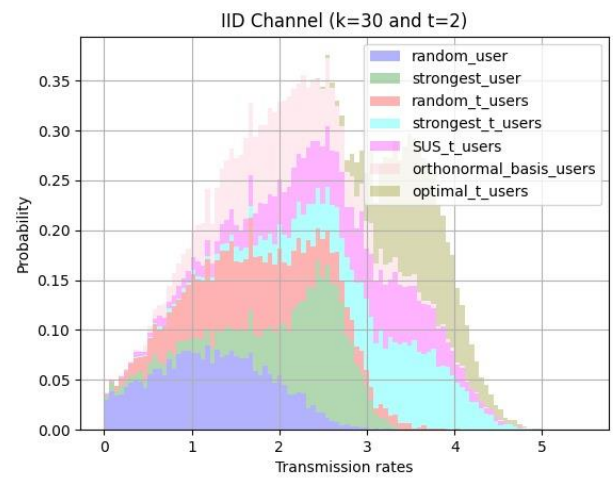
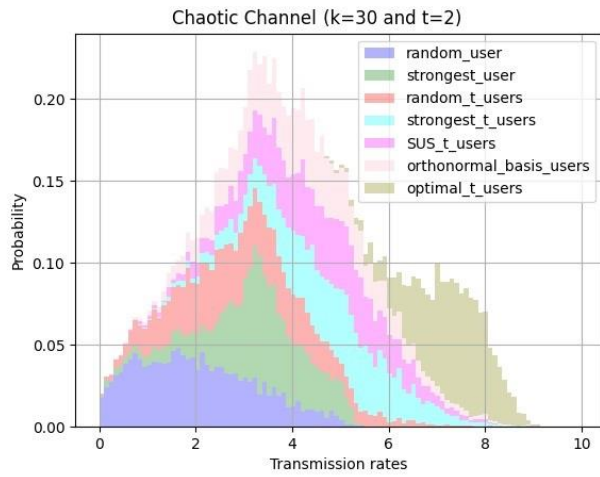
### SUS T Users - Bonus:



### Orthonormal Users (DEMOS) - Bonus:



### All Graphs Together:



## Post-Simulation Question

1.

<b>Disadvantages</b>	<b>Advantages</b>	
Not consider the varying channel conditions or user priorities and lead to not efficient transmission result by picking randomly bad user.	Simple and easy to implement. Provides a fair chance for all users to be selected.	<b>Random User</b>
Ignores other users channel conditions, which may lead to unfairness or inefficient resource allocation. To do so we must calculate which user is strongest which takes a lot of time and resources.	May potentially maximize the system capacity or reliability.	<b>Single Strongest User</b>
may not consider the varying channel conditions or user priorities, resulting in suboptimal performance for certain users.	Allowing for a balance between fairness and resource efficiency.	<b>T-Random Users</b>
Users with weaker channel conditions may be consistently excluded, resulting in degraded performance for those users.	Balancing between capacity and fairness.	<b>T-Strongest Users</b>
A very complicated and difficult calculation. high runtime complexity.	Equal distribution of the transmission power between users.	<b>Optimal group</b>
Not always pick the best approximation, doesn't good for nay channel (like dynamic channels). Usually hard to implement.	reduce interference and improve system performance by equally distribution of the transmission power between users.	<b>SUS Algorithm (Bonus)</b>
Hard to implement, may not perform optimally in dynamic scenarios.  By using Gram-Schmidt may require longer convergence time and sufficient exploration to reach optimal or near-optimal user selections.	By using with orthonormal basis which pick in some channel's optimal users. In addition, improve system performance by equally distribution of the transmission power between users.  Balances exploration and exploitation in user selection, allowing for the discovery of better user subsets over time.	<b>Demos Algorithm (Bonus)</b>

2.

In this question, we compare the Gaussian channel and the chaotic channel. We see that in all the algorithms/methods we used for user selection, the mean of the Gaussian channel is lower than that of the chaotic channel. This indicates that more users with weak channels are selected.

In both channels, we notice that random or the strongest algorithms yield a lower mean compared to the optimal, sus, or orthonormal algorithms. Additionally, in both channels, we notice that the strongest and random algorithms yield a lower mean compared to the optimal, sus, or orthonormal algorithms.

We can see that the optimal subgroups algorithm is not a good option in the chaotic channel, in contrast to the Gaussian channel.

Furthermore, we see that the variance of the Gaussian channel is always lower than that of the chaotic channel. This is because the Gaussian channel's variance remains constant, unlike the chaotic channel.

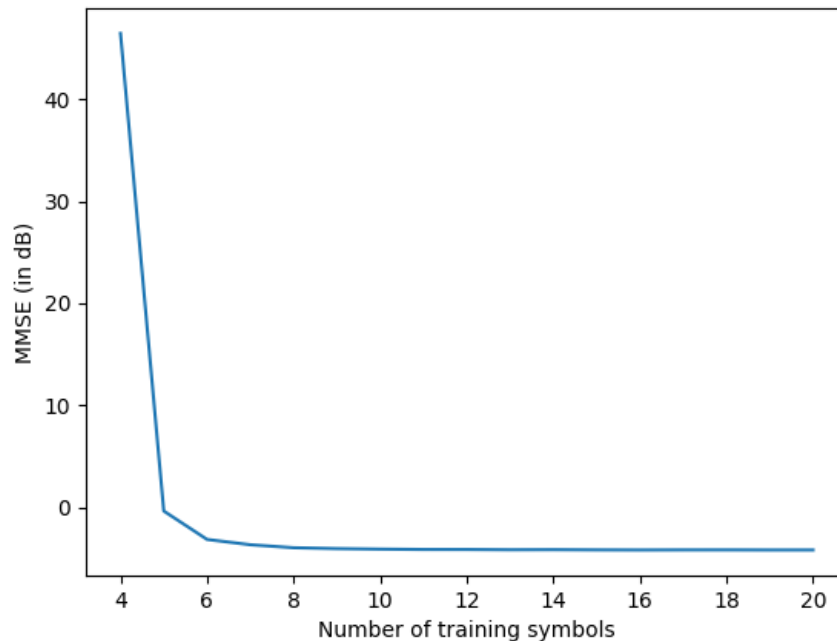


## Part B — CSI Collection

### Preliminary Questions

1. This formula represents the pseudo inverse matrix. The pseudo-inverse matrix is used to calculate the beamforming weights for each antenna element, which determines the signal phase and amplitude for transmission or reception.
2. We choose Gaussian channel because usually this is the channel that represents the best the “real- world” channel. In addition, this channel makes it easier to analyze and model wireless communication systems.
3. LS problem defining variables:
  - $A$  – The channel matrix, representing the wireless channel between the receiver and the transmitter.
  - $\hat{X}$  – The chooses symbol by the LS problem:  $Y = AX+W$ .
  - $X$  – The transmitted signal matrix, representing the transmitted signals from the user’s antennas.
  - $Y$  – The received signal matrix, representing the received signals at the user’s antennas.
  - $t$  – The number of antennas from the base station.
  - $n$  – The size of the transmitted vector (row of channel  $A$ )
  - $m$  – The columns size of channel  $A$ .

### Simulation



## Post-Simulation Questions

1. In our simulation, the assumption was that the channel matrix  $H$  is fixed during the channel estimation process. In this case, there is no optimal number of training symbols because the channel remains constant. The number of training symbols can affect the accuracy of the channel estimation, but there is no inherent optimality in terms of the number of training symbols.
2. Using orthogonal training symbols, such as in CDMA (Code Division Multiple Access), offers several advantages beyond minimizing the MMSE (Mean Square Error):
  - a. Multiple Access Interference (MAI) Reduction: Orthogonal training symbols ensure that different user's signals do not interfere with each other during the channel estimation process. This reduces the impact of interference and allows multiple users to share the same frequency band efficiently.
  - b. Ease of Decoding: Orthogonal training symbols simplify the decoding process since each user's signal can be separated from others using orthogonal codes or sequences. This simplifies the receiver design and improves the overall system performance.
  - c. Improved System Capacity: The use of orthogonal training symbols enables efficient resource allocation and increases the system capacity by allowing simultaneous transmissions from multiple users without significant interference.
  - d. Better Error Performance: Orthogonal training symbols help mitigate the effects of multi-path fading and improve the system's error performance. They provide diversity by spreading the transmitted energy across different paths, reducing the impact of fading on individual symbols.
  - e. Enhanced Security: The orthogonality of training symbols adds a level of security to the system. The use of orthogonal codes makes it difficult for unauthorized users to access and interfere with the communication.