networks1

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1

$$k_i = \sum_{j=1}^{N} A_{ij}$$

$$\langle k \rangle = \frac{1}{N} \sum_{i=1}^{N} k_i$$

2

The average degree of the neighbors of i is

$$k_{i,nn} = \frac{1}{k_i} \sum_{j=1}^{N} A_{ij} k_j = \frac{\sum_{j=1}^{N} A_{ij} k_j}{\sum_{h=1}^{N} A_{ih}} = \frac{\sum_{j=1}^{N} A_{ij} \sum_{r=1}^{N} A_{jr}}{\sum_{h=1}^{N} A_{ih}} = \frac{\sum_{j=1}^{N} \sum_{r=1}^{N} A_{ij} A_{jr}}{\sum_{h=1}^{N} A_{ih}}$$

and then

$$\langle k_{nn} \rangle = \frac{1}{N} \sum_{i=1}^{N} k_{i,nn}$$

3

We create an array of nodes, arbitrarily enumerated, and use it to create the matrix.

$$\{\overset{1}{426}, \overset{2}{345}, \overset{3}{365}, \overset{4}{153}, \overset{5}{245}, \overset{6}{165}, \overset{7}{358}, \overset{8}{121}, \overset{9}{452}, \overset{10}{143}, \overset{11}{131}, \overset{12}{272}, \overset{13}{546}, \overset{14}{369}, \overset{15}{171}, \overset{16}{782}, \overset{17}{888}\}$$

	426	**345**	**365**	**153**	**245**	**165**	**358**	**121**	**452**	**143**	**131**	**272**	**546**	**369**	**171**	**782**	**888**
426	0	1	1	0	1	1	0	1	1	0	0	0	0	0	0	1	0
345	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
365	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
153	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
245	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
165	1	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0
358	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0	0
121	1	0	0	0	0	0	0	0	0	1	1	0	0	0	0	1	0
452	1	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0
143	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
131	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
272	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	1	0
546	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
369	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
171	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
782	1	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	1
888	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0

4

$$
\begin{gathered}
k_1 = 7 \\
k_2 = 4 \\
k_3 = 2 \\
k_4 = 1 \\
k_5 = 2 \\
k_6 = 3 \\
k_7 = 3 \\
k_8 = 4 \\
k_9 = 3 \\
k_{10} = 1 \\
k_{11} = 1 \\
k_{12} = 3 \\
k_{13} = 1 \\
k_{14} = 1 \\
k_{15} = 1 \\
k_{16} = 4 \\
k_{17} = 1
\end{gathered}
$$

$$\langle k \rangle = 2.4705882352941178$$

5

$$
\begin{aligned}
k_{1,nn} &= 3.142857142857143\\
k_{2,nn} &= 3.0\\
k_{3,nn} &= 5.5\\
k_{4,nn} &= 4.0\\
k_{5,nn} &= 5.5\\
k_{6,nn} &= 3.6666666666666665\\
k_{7,nn} &= 2.3333333333333335\\
k_{8,nn} &= 3.25\\
k_{9,nn} &= 4.333333333333333\\
k_{10,nn} &= 4.0\\
k_{11,nn} &= 4.0\\
k_{12,nn} &= 2.6666666666666665\\
k_{13,nn} &= 3.0\\
k_{14,nn} &= 3.0\\
k_{15,nn} &= 3.0\\
k_{16,nn} &= 3.75\\
k_{17,nn} &= 4.0
\end{aligned}
$$

$$\langle k, nn\rangle = 3.6554621848739495$$

6

Denote a triangle by $\{i, j, h\}$, where $1 \leq i, j, h \leq N$ are three distinct nodes.

We have N choices for i, then $N-1$ choices for j and $N-2$ choices for h.

But we can choose every three nodes i, j, h in $3! = 6$ different ways, hence $T_{max} = \frac{N(N-1)(N-2)}{3!} = \binom{N}{3}$.

To calculate the actual number of triangles, we observe that if $T = \{i, j, h\}$ is a triangle, then $A_{kl} = 1$, where $k, l \in \{i, j, h\}$ and $k \neq l$. And then

$A_{ii} = 0$, because we do not have loops from a node to itself in a network.

$(A^2)_{ii} = \sum_{k=1}^{N} A_{ik}A_{ki}$, which is the number of paths of length 2 from node i to itself, which equals to the number of neighbors of node i.

$(A^3)_{ii} = \sum_{k=1}^{N} A_{ik}(A^2)_{ki} = \sum_{k=1}^{N} A_{ik}(\sum_{l=1}^{N} A_{kl}A_{li}) = \sum_{k=1}^{N}\sum_{l=1}^{N} A_{ik}A_{kl}A_{li}$, which is the count of all the paths from node i to itself of length 3, that is, the count of all the triangles that start and end at node i. But we count both paths $i \to j \to h$ and $i \to h \to j$, so we need to divide the count by 2. To count all the triangles, we must add the counts of all $(A^3)_{kk}$, where $1 \leq k \leq N$.

But if $\{i, j, h\}$ are nodes in a triangles, then $(A^3)_{ii} + (A^3)_{jj} + (A^3)_{hh}$ counts the same triangle three times, because the same triangle can start and end at node i or node j or node h, so we need to divide the count by 3, so the formula is

$$T = \frac{\sum_{i=1}^{N}(A^3)_{ii}}{3!}.$$

Indeed, in our class network we have 3 triangles (calculated in the code), and from the diagram we see that the triangles are $\{426, 345, 365\}$, $\{426, 345, 245\}$ and $\{426, 121, 782\}$.