Proposition 0.0.1. *Let* $x = \sum_{i=1}^{n-1} \lambda_i e_{i,i+1}$*, where* $\lambda_i \in \mathbb{Q}_p$ *not all zero. Then* $\dim \mathcal{C}_{\gamma_3}(x) = \#\{i : \lambda_i = 0\} + 1$

Proof. Let $y = \sum_{i=1}^{n-1} \mu_i e_{i,i+1} \in \mathcal{C}_{\gamma_3}(x)$, where $\lambda_i \in \mathbb{Q}_p$. For every $1 \leq i \leq n-1$, denote by c_i the constraint equation $[\lambda_i e_{i,i+1}, \mu_{i+1} e_{i+1,i+2}] - [\lambda_{i+1} e_{i+1,i+2}, \mu_i e_{i,i+1}] = (\lambda_i \mu_{i+1} - \lambda_{i+1}\mu_i) e_{i,i+2} = 0$. Let $1 \leq j < k \leq n-1$ be two indices, such that $\lambda_{j-1} = 0$, and $\lambda_{k+1} = 0$, and $\lambda_j, \lambda_{j+1}, \dots, \lambda_k$ are all non-zero[1], then by constraints $c_j, c_{j+1}, \dots, c_{m-1}$, we have that $\mu_m = \frac{\lambda_m}{\lambda_{m-1}} \mu_{m-1} = \frac{\lambda_m}{\lambda_{m-1}} \frac{\lambda_{m-1}}{\lambda_{m-2}} \mu_{m-2} = \frac{\lambda_m}{\lambda_{m-2}} \mu_{m-2} = \cdots = \frac{\lambda_m}{\lambda_j} \mu_j$, for every $j+1 \leq m \leq k-1$, which means that all μ coefficients of y, with indices from $j+1$ to k, depend on the first coefficient, namely μ_j. We denote the free choice of μ_j by $\mu_j = *$. One easily checks that we can choose freely any coefficient μ_m from $j+1$ to k, instead of μ_j, and all other coefficients in that range will depend on our choice of μ_m. By constraint c_{j-1}, we have that $\lambda_{j-1}\mu_j - \lambda_j \mu_{j-1} = 0$, but $\lambda_{j-1} = 0$, hence $\lambda_j \mu_{j-1}$ must vanish, but $\lambda_j \neq 0$, which obviously means that $\mu_{j-1} = 0$. Similarly, we have that $\mu_{k+1} = 0$, due to constraint c_k. By constraint c_{k+1}, we have that $\lambda_{k+1}\mu_{k+2} - \lambda_{k+2}\mu_{k+1} = 0$, but $\lambda_{k+1} = \mu_{k+1} = 0$, hence, $\lambda_{k+1}\mu_{k+2}$ must vanish, but $\lambda_{k+1} = 0$, which means that we need to look at constraint c_{k+2}, that is, $\lambda_{k+2}\mu_{k+3} - \lambda_{k+3}\mu_{k+2}$. We check the different options. If $\lambda_{k+2} = 0$, then $\lambda_{k+3}\mu_{k+2}$ must vanish. Therefore, if $\lambda_{k+3} \neq 0$, then $\mu_{k+2} = 0$, but if $\lambda_{k+3} = 0$, then $\mu_{k+2} = *$. If $\lambda_{k+2} \neq 0$, then again $\mu_{k+2} = *$.

Suppose that $j > 1$, then by constraint c_{j-1}, we have that $\lambda_{j-1}\mu_j - \lambda_j \mu_{j-1} = 0$, but $\lambda_{j-1} = 0$, so $\lambda_j \mu_{j-1}$ must vanish, but $\lambda_j \neq 0$, so it must be that $\mu_{j-1} = 0$. Same way, if $k < n-1$, then it must be that $\mu_{k+1} = 0$. Let $1 \leq j$ □

We observe that for each $2 \leq j \leq n-2$, μ_j is obviously determined by the two constraints c_{j-1} and c_j, which means that we have several options for $\lambda_{j-1}, \lambda_j, \lambda_{j+1}$. We look at the two equations:

$$c_{j-1} = (\lambda_{j-1}\mu_j - \lambda_j \mu_{j-1}) e_{j-1,j+1} = 0$$

$$c_j = (\lambda_j \mu_{j+1} - \lambda_{j+1}\mu_j) e_{j,j+2} = 0$$

Obviously, if $\lambda_{j-1} = \lambda_j = \lambda_{j+1} = 0$, then both c_{j-1} and c_j are invalid constraints, which means that μ_j can assume any value, we usually denote

[1]We extend our notation of indices, to include also the case where $j = 1$ or $k = n-1$, and define that $\lambda_{j-1} = \lambda_0 = 0$ or $\lambda_{k+1} = \lambda_n = 0$, respectively

this by $\mu_j = *$. Suppose that we have only two zeros, then if $\lambda_{j-1} = \lambda_j = 0$ and $\lambda_{j+1} \neq 0$ or if $\lambda_j = \lambda_{j+1} = 0$ and $\lambda_{j-1} \neq 0$, then we must also have that $\lambda_{j+1}\mu_j = 0$ or $\lambda_{j-1}\mu_j = 0$, respectively, which means that $\mu_j = 0$. On the other hand, if $\lambda_{j-1} = \lambda_{j+1} = 0$ and $\lambda_j \neq 0$, then we must have $\lambda_j\mu_{j-1} = \lambda_j\mu_{j+1} = 0$, which means that $\mu_{j-1} = \mu_{j+1} = 0$, and since μ_j depends only on c_{j-1} and c_j, we have that $\mu_j = *$. Suppose that only one of the three λ coefficients is zero, then if $\lambda_{j-1} = 0$, we must have that $\mu_{j-1} = 0$, and $\mu_{j+1} = \frac{\lambda_{j+1}}{\lambda_j}\mu_j$. If $lambda_j = 0$, then we must have that $\lambda_{j-1}\mu_j = \lambda_{j+1}\mu_j = 0$, which means that $\mu_j = 0$. If $\lambda_{j+1} = 0$, then we must have that $\mu_{j+1} = 0$, and $\mu_j = \frac{\lambda_j}{\lambda_{j-1}}\mu_{j-1}$. If the three λ coefficients are non-zero, then we must have that $\mu_j = \frac{\lambda_j}{\lambda_{j-1}}\mu_{j-1}$, and $\mu_{j+1} = \frac{\lambda_{j+1}}{\lambda_j}\mu_j = \frac{\lambda_{j+1}}{\lambda_j}\frac{\lambda_j}{\lambda_{j-1}}\mu_{j-1} = \frac{\lambda_{j+1}}{\lambda_{j-1}}\mu_{j-1}$. We conclude that every chain of non-zero consecutive coefficients $\lambda_j, \lambda_{j+1}, \ldots \lambda_{j+l-1}$, where l is clearly the length of the chain, has that $\mu_{k+1} = \frac{\lambda_{k+1}}{\lambda_j}\mu_j$, which means that