Mapping

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We are using a for most of the elements, where the superscript in parentheses is r, the horizontal index of the block M_{1r}. For each block M_{1r}, every element on the block diagonal $(1,1),(2,2),\ldots,(i,i),\ldots,(n-r,n-r)$ has the subscript i, so any such element, and its occurrences in any other place in the matrix, are $a_i^{(r)}$. In addition, we have

- $a_0^{(r)}$, the element in the location $(r,1)$ of each block.
- a', the element in the location $(1,2)$ of the block $M_{1,n-2}$.
- a'', the element in the location $(n-1,1)$ of the block $M_{1,n-2}$.

For the block $M_{1,n-1}$, we will use $b_1,b_2,\ldots,b_{n-1}$.

The mapping between our notations and the notations of Berman are

- $a_{11} \to a_1^{(2)}$
- $a_{22} \to a_2^{(2)}$
- $a_{33} \to a_3^{(2)}$
- $b_{11} \to a_1^{(3)}$
- $b_{12} \to a'$
- $b_{22} \to a_2^{(3)}$
- $b_{31} \to a_0^{(3)}$
- $b_{41} \to a''$
- $c_1 \to a' a_0^{(2)} + b_1$
- $c_2 \to a_2^{(3)} a_0^{(2)} + b_2$
- $c_3 \to a_0^{(3)} a_3^{(2)} + b_3$
- $c_4 \to a'' a_3^{(2)} + b_4$

$$
N_{2,0}N_{2,2}N_{2,1}N_{2,3} = \begin{bmatrix}
1 & 0 & 0 & 0 & a_1^{(2)} & 0 & 0 & & & \\
0 & 1 & 0 & 0 & a_0^{(2)} & a_2^{(2)} & 0 & a_0^{(2)}a_2^{(2)} & & \\
0 & 0 & 1 & 0 & 0 & -a_1^{(2)} & a_3^{(2)} & & -a_1^{(2)}a_3^{(2)} & \\
0 & 0 & 0 & 1 & 0 & 0 & -a_2^{(2)} & & a_1^{(2)}a_2^{(2)} & \\
 & & & & 1 & 0 & 0 & a_2^{(2)} & 0 & \\
 & & & & 0 & 1 & 0 & a_0^{(2)} & a_3^{(2)} & a_0^{(2)}a_3^{(2)} \\
 & & & & 0 & 0 & 1 & 0 & -a_1^{(2)} & \\
 & & & & & & & 1 & 0 & a_3^{(2)} \\
 & & & & & & & 0 & 1 & a_0^{(2)} \\
 & & & & & & & & & 1
\end{bmatrix}
$$

$$
N_3'N_{2,0}N_{2,2}N_{2,1}N_{2,3} = \begin{bmatrix}
1 & 0 & 0 & 0 & a_1^{(2)} & 0 & 0 & & a' & a'a_0^{(2)} \\
0 & 1 & 0 & 0 & a_0^{(2)} & a_2^{(2)} & 0 & a_0^{(2)}a_2^{(2)} & & \\
0 & 0 & 1 & 0 & 0 & -a_1^{(2)} & a_3^{(2)} & & -a_1^{(2)}a_3^{(2)} & \\
0 & 0 & 0 & 1 & 0 & 0 & -a_2^{(2)} & & a_1^{(2)}a_2^{(2)} & \\
 & & & & 1 & 0 & 0 & a_2^{(2)} & 0 & \\
 & & & & 0 & 1 & 0 & a_0^{(2)} & a_3^{(2)} & a_0^{(2)}a_3^{(2)} \\
 & & & & 0 & 0 & 1 & 0 & -a_1^{(2)} & \\
 & & & & & & & 1 & 0 & a_3^{(2)} \\
 & & & & & & & 0 & 1 & a_0^{(2)} \\
 & & & & & & & & & 1
\end{bmatrix}
$$

$$
N_3''N_3'N_{2,0}N_{2,2}N_{2,1}N_{2,3} = \begin{bmatrix}
1 & 0 & 0 & 0 & a_1^{(2)} & 0 & 0 & & a' & a'a_0^{(2)} \\
0 & 1 & 0 & 0 & a_0^{(2)} & a_2^{(2)} & 0 & a_0^{(2)}a_2^{(2)} & & \\
0 & 0 & 1 & 0 & 0 & -a_1^{(2)} & a_3^{(2)} & & -a_1^{(2)}a_3^{(2)} & \\
0 & 0 & 0 & 1 & 0 & 0 & -a_2^{(2)} & a'' & a_1^{(2)}a_2^{(2)} & a''a_3^{(2)} \\
 & & & & 1 & 0 & 0 & a_2^{(2)} & 0 & \\
 & & & & 0 & 1 & 0 & a_0^{(2)} & a_3^{(2)} & a_0^{(2)}a_3^{(2)} \\
 & & & & 0 & 0 & 1 & 0 & -a_1^{(2)} & \\
 & & & & & & & 1 & 0 & a_3^{(2)} \\
 & & & & & & & 0 & 1 & a_0^{(2)} \\
 & & & & & & & & & 1
\end{bmatrix}
$$

$$N_{3,2}N_3''N_3'N_{2,0}N_{2,2}N_{2,1}N_{2,3} = \begin{bmatrix} 1 & 0 & 0 & 0 & a_1^{(2)} & 0 & 0 & & a' & a'a_0^{(2)} \\ 0 & 1 & 0 & 0 & a_0^{(2)} & a_2^{(2)} & 0 & a_0^{(2)}a_2^{(2)} & a_2^{(3)} & a_2^{(3)}a_0^{(2)} \\ 0 & 0 & 1 & 0 & 0 & -a_1^{(2)} & a_3^{(2)} & & -a_1^{(2)}a_3^{(2)} & \\ 0 & 0 & 0 & 1 & 0 & 0 & -a_2^{(2)} & a'' & a_1^{(2)}a_2^{(2)} & a''a_3^{(2)} \\ & & & & 1 & 0 & 0 & a_2^{(2)} & 0 & a_2^{(3)} \\ & & & & 0 & 1 & 0 & a_0^{(2)} & a_3^{(2)} & a_0^{(2)}a_3^{(2)} \\ & & & & 0 & 0 & 1 & 0 & -a_1^{(2)} & \\ & & & & & & & 1 & 0 & a_3^{(2)} \\ & & & & & & & 0 & 1 & a_0^{(2)} \\ & & & & & & & & & 1 \end{bmatrix}$$

$$N_{3,2}N_3''N_3'N_{2,0}N_{2,2}N_{2,1}N_{2,3}N_{3,1} = \begin{bmatrix} 1 & 0 & 0 & 0 & a_1^{(2)} & 0 & 0 & a_1^{(3)} & a' & a'a_0^{(2)} \\ 0 & 1 & 0 & 0 & a_0^{(2)} & a_2^{(2)} & 0 & a_0^{(2)}a_2^{(2)} & a_2^{(3)} & a_2^{(3)}a_0^{(2)} \\ 0 & 0 & 1 & 0 & 0 & -a_1^{(2)} & a_3^{(2)} & & -a_1^{(2)}a_3^{(2)} & \\ 0 & 0 & 0 & 1 & 0 & 0 & -a_2^{(2)} & a'' & a_1^{(2)}a_2^{(2)} - a_1^{(3)} & a''a_3^{(2)} \\ & & & & 1 & 0 & 0 & a_2^{(2)} & 0 & a_2^{(3)} \\ & & & & 0 & 1 & 0 & a_0^{(2)} & a_3^{(2)} & a_0^{(2)}a_3^{(2)} \\ & & & & 0 & 0 & 1 & 0 & -a_1^{(2)} & \\ & & & & & & & 1 & 0 & a_3^{(2)} \\ & & & & & & & 0 & 1 & a_0^{(2)} \\ & & & & & & & & & 1 \end{bmatrix}$$

$$N_{3,0}N_{3,2}N_3''N_3'N_{2,0}N_{2,2}N_{2,1}N_{2,3}N_{3,1} =$$

$$\begin{bmatrix}
1 & 0 & 0 & 0 & a_1^{(2)} & 0 & 0 & a_1^{(3)} & a' & a'a_0^{(2)} \\
0 & 1 & 0 & 0 & a_0^{(2)} & a_2^{(2)} & 0 & a_0^{(2)}a_2^{(2)} & a_2^{(3)} & a_2^{(3)}a_0^{(2)} \\
0 & 0 & 1 & 0 & 0 & -a_1^{(2)} & a_3^{(2)} & a_0^{(3)} & -a_1^{(2)}a_3^{(2)} & a_0^{(3)}a_3^{(2)} \\
0 & 0 & 0 & 1 & 0 & 0 & -a_2^{(2)} & a'' & a_1^{(2)}a_2^{(2)} - a_1^{(3)} & a''a_3^{(2)} \\
 & & & & 1 & 0 & 0 & a_2^{(2)} & 0 & a_2^{(3)} \\
 & & & & 0 & 1 & 0 & a_0^{(2)} & a_3^{(2)} & a_0^{(2)}a_3^{(2)} \\
 & & & & 0 & 0 & 1 & 0 & -a_1^{(2)} & a_0^{(3)} \\
 & & & & & & & 1 & 0 & a_3^{(2)} \\
 & & & & & & & 0 & 1 & a_0^{(2)} \\
 & & & & & & & & & 1
\end{bmatrix}$$

$$N_{3,0}N_{3,2}N_3''N_3'N_{2,0}N_{2,2}N_{2,1}N_{2,3}N_{3,1}N_4 =$$

$$\begin{bmatrix}
1 & 0 & 0 & 0 & a_1^{(2)} & 0 & 0 & a_1^{(3)} & a' & a'a_0^{(2)} + b_1 \\
0 & 1 & 0 & 0 & a_0^{(2)} & a_2^{(2)} & 0 & a_0^{(2)}a_2^{(2)} & a_2^{(3)} & a_2^{(3)}a_0^{(2)} + b_2 \\
0 & 0 & 1 & 0 & 0 & -a_1^{(2)} & a_3^{(2)} & a_0^{(3)} & -a_1^{(2)}a_3^{(2)} & a_0^{(3)}a_3^{(2)} + b_3 \\
0 & 0 & 0 & 1 & 0 & 0 & -a_2^{(2)} & a'' & a_1^{(2)}a_2^{(2)} - a_1^{(3)} & a''a_3^{(2)} + b_4 \\
 & & & & 1 & 0 & 0 & a_2^{(2)} & 0 & a_2^{(3)} \\
 & & & & 0 & 1 & 0 & a_0^{(2)} & a_3^{(2)} & a_0^{(2)}a_3^{(2)} \\
 & & & & 0 & 0 & 1 & 0 & -a_1^{(2)} & a_0^{(3)} \\
 & & & & & & & 1 & 0 & a_3^{(2)} \\
 & & & & & & & 0 & 1 & a_0^{(2)} \\
 & & & & & & & & & 1
\end{bmatrix}$$