Proposition 0.0.1. *Let $x = \sum_{i=1}^{n-1} \lambda_i e_{i,i+1}$, where $\lambda_i \in \mathbb{Q}_p$ not all zero. Then* $\dim \mathcal{C}_{\gamma_3}(x) = \#\{i : \lambda_i = 0\} + 1$

Proof. Let $y = \sum_{i=1}^{n-1} \mu_i e_{i,i+1} \in \mathcal{C}_{\gamma_3}(x)$, where $\lambda_i \in \mathbb{Q}_p$. For every $1 \leq i \leq n-1$, denote by c_i the constraint equation $[\lambda_i e_{i,i+1}, \mu_{i+1} e_{i+1,i+2}] - [\lambda_{i+1} e_{i+1,i+2}, \mu_i e_{i,i+1}] = (\lambda_i \mu_{i+1} - \lambda_{i+1}\mu_i) e_{i,i+2} = 0$. Suppose $\lambda_j = 0$, for some $2 \leq j \leq n-1$, and $\lambda_i \neq 0$, for all $i < j$, then by constraints $c_1, c_2, \ldots, c_{j-2}$, we have that $\mu_{j-1} = \frac{\lambda_{j-1}}{\lambda_{j-2}} \mu_{j-2} = \frac{\lambda_{j-1}}{\lambda_{j-2}} \frac{\lambda_{j-2}}{\lambda_{j-3}} \mu_{j-3} = \frac{\lambda_{j-1}}{\lambda_{j-3}} \mu_{j-3} = \cdots = \frac{\lambda_{j-1}}{\lambda_1} \mu_1$ $\square$

We observe that for each $2 \leq j \leq n-2$, μ_j is obviously determined by the two constraints c_{j-1} and c_j, which means that we have several options for $\lambda_{j-1}, \lambda_j, \lambda_{j+1}$. We look at the two equations:

$$c_{j-1} = (\lambda_{j-1}\mu_j - \lambda_j \mu_{j-1}) e_{j-1,j+1} = 0$$

$$c_j = (\lambda_j \mu_{j+1} - \lambda_{j+1}\mu_j) e_{j,j+2} = 0$$

Obviously, if $\lambda_{j-1} = \lambda_j = \lambda_{j+1} = 0$, then both c_{j-1} and c_j are invalid constraints, which means that μ_j can assume any value, we usually denote this by $\mu_j = *$. Suppose that we have only two zeros, then if $\lambda_{j-1} = \lambda_j = 0$ and $\lambda_{j+1} \neq 0$ or if $\lambda_j = \lambda_{j+1} = 0$ and $\lambda_{j-1} \neq 0$, then we must also have that $\lambda_{j+1}\mu_j = 0$ or $\lambda_{j-1}\mu_j = 0$, respectively, which means that $\mu_j = 0$. On the other hand, if $\lambda_{j-1} = \lambda_{j+1} = 0$ and $\lambda_j \neq 0$, then we must have $\lambda_j \mu_{j-1} = \lambda_j \mu_{j+1} = 0$, which means that $\mu_{j-1} = \mu_{j+1} = 0$, and since μ_j depends only on c_{j-1} and c_j, we have that $\mu_j = *$. Suppose that only one of the three λ coefficients is zero, then if $\lambda_{j-1} = 0$, we must have that $\mu_{j-1} = 0$, and $\mu_{j+1} = \frac{\lambda_{j+1}}{\lambda_j} \mu_j$. If $lambda_j = 0$, then we must have that $\lambda_{j-1}\mu_j = \lambda_{j+1}\mu_j = 0$, which means that $\mu_j = 0$. If $\lambda_{j+1} = 0$, then we must have that $\mu_{j+1} = 0$, and $\mu_j = \frac{\lambda_j}{\lambda_{j-1}} \mu_{j-1}$. If the three λ coefficients are non-zero, then we must have that $\mu_j = \frac{\lambda_j}{\lambda_{j-1}} \mu_{j-1}$, and $\mu_{j+1} = \frac{\lambda_{j+1}}{\lambda_j} \mu_j = \frac{\lambda_{j+1}}{\lambda_j} \frac{\lambda_j}{\lambda_{j-1}} \mu_{j-1} = \frac{\lambda_{j+1}}{\lambda_{j-1}} \mu_{j-1}$. We conclude that every chain of non-zero consecutive coefficients $\lambda_j, \lambda_{j+1}, \ldots \lambda_{j+l-1}$, where l is clearly the length of the chain, has that $\mu_{k+1} = \frac{\lambda_{k+1}}{\lambda_j} \mu_j$, which means that