cases

haiml76

July 2025

Denote $p^{\min} := p^{\min\{0, v_2+v_{11}\}}$
Denote $\rho := (1-p^{-1})$

1 Case 1

$v_3 > v_1 \geq v_4 > v_2 \geq 0$
$\Rightarrow v_{11} \geq -v_2 - v_1$
$\Rightarrow \alpha := \min\{v_2, v_2 + v_{11} + v_1\} = \begin{cases} v_2, & v_{11} \geq -v_1 \\ v_2 - 1, v_2 - 2, \dots, & v_{11} < -v_1 \end{cases}$

For $v_{11} \geq -v_2$, we have $p^{\min}\mu(a_{11})(1+\alpha\rho) = p^{v_2}(1+v_2\rho)$.

For $-v_1 \leq v_{11} \leq -v_2 - 1$, we have $p^{\min}\mu(a_{11})(1+\alpha\rho) = \sum_{-v_1}^{-v_2-1} p^{v_2}\rho(1+v_2\rho) = p^{v_2}\rho(v_1-v_2)(1+v_2\rho) = p^{v_2}\rho(v_1-v_2) + p^{v_2}\rho^2 v_2(v_1-v_2)$.

For $-v_2-v_1 \leq v_{11} \leq -v_1 - 1$, we have $p^{\min}\mu(a_{11})(1+\alpha\rho) = \sum_{-v_2-v_1}^{-v_1-1} p^{v_2}\rho(1+\alpha\rho) = \sum_{i=1}^{v_2} p^{v_2}\rho(1+(v_2-i)\rho) = p^{v_2}v_2\rho + p^{v_2}v_2\rho^2 - p^{-v_2}\rho\binom{v_2+1}{2} = p^{v_2}v_2\rho + p^{v_2}\rho^2 - p^{v_2}\rho^2\binom{v_2+1}{2}$

So we have $\sum_{v_{11}\geq -v_2-v_1} = p^{v_2}(1+v_2\rho+(v_1-v_2)\rho+v_2(v_1-v_2)\rho^2+v_2\rho+v_2\rho^2-\frac{(v_2+1)v_2}{2}\rho^2)$. We compute the differences between v_1, v_2, v_3, v_4,

$v_3 \overset{d}{>} v_1 \overset{c}{\geq} v_4 \overset{b}{>} v_2 \overset{a}{\geq} 0$

Thus, we substitute $v_1 \to a+b+c$, $v_2 \to a$, $v_3 \to a+b+c+d$, $v_4 \to a+b$, and compute series with the new variables as indices, where $a, c \geq 0$ and $b, d \geq 1$.

2 Case 2

$v_1 \geq v_3 \geq v_4 > v_2 \geq 0$
$\Rightarrow v_{11} \geq -v_2 - v_3$
$\Rightarrow \alpha := \min\{v_2, v_2 + v_{11} + v_1\} = \begin{cases} v_2, & v_{11} \geq -v_1 \\ v_2 - 1, v_2 - 2, \dots, & v_{11} < -v_1 \end{cases}$

We have two sub cases.

2.1 Sub case I

$-v_2 - v_3 \geq -v_1 \Rightarrow v_1 \geq v_2 + v_3$

Here there is no phase transition, because the phase transition occurs at $-v_1 - 1 < -v_2 - v_3 \leq v_{11}$

For $v_{11} \geq -v_2$, we have $p^{\min}\mu(a_{11})(1+\alpha\rho) = p^{v_2}(1+v_2\rho)$.

For $-v_2 - v_3 \leq v_{11} \leq -v_2 - 1$, we have $p^{\min}\mu(a_{11})(1+\alpha\rho) = \sum_{-v_2-v_3}^{-v_2-1} p^{v_2}\rho(1+v_2\rho) = p^{v_2}v_3\rho(1+v_2\rho) = p^{v_2}v_3\rho + p^{v_2}v_3v_2\rho^2$.

So we have $\sum_{v_{11}\geq -v_2-v_2} = p^{v_2}(1+v_2\rho+v_3\rho+v_2v_3\rho^2)$. We compute the differences between v_1, v_2, v_3, v_4,

$v_1 \overset{d}{\geq} v_2 + v_3 \overset{e}{\geq} v_3 \overset{c}{\geq} v_4 \overset{b}{>} v_2 \overset{a}{\geq} 0$

But $e = v_2 + v_3 - v_3 = v_2 = a$

Thus, we substitute $v_1 \to 2a+b+c+d$, $v_2 \to a$, $v_3 \to a+b+c$, $v_4 \to a+b$, and compute series with the new variables as indices, where $a, c, d \geq 0$ and $b \geq 1$.