RandomGraphs

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1.1

1.2

We check that $1 - kp \leq (1-p)^k$, for all $k \in \mathbb{N}$, by induction.

For $k = 1$ it is trivial, for $k+1$, we have $(1-p)^{k+1} = (1-p)^k(1-p)$, and by the assumption, $(1-p)^k(1-p) \geq (1-kp)(1-p) = 1-kp-p+kp^2 = 1-(k+1)p+kp^2$, but $kp^2 > 0$, so $(1-p)^{k+1} = (1-p)^k(1-p) \geq 1-(k+1)p+kp^2 > 1-(k+1)p$, which proves the assumption.

But it means that for each potential edge e,

$\mathbb{P}[e \notin G \sim G(n,kp)] = 1 - kp \leq (1-p)^k = (\mathbb{P}[e \notin G \sim G(n,p)])^k = \mathbb{P}[e \notin G \sim \bigcup_{i=1}^k G(n,p)] \Rightarrow \mathbb{P}[e \in G \sim G(n,kp)] \geq \mathbb{P}[e \in G \sim \bigcup_{i=1}^k G(n,p)]$,

but by a theorem coming from staged exposure, it means that if A is an increasing monotonic property, then $\mathbb{P}[G \sim G(n,kp) \in A] \geq \mathbb{P}[G \sim \bigcup_{i=1}^k G(n,p) \in A] \Rightarrow \mathbb{P}[G \sim G(n,kp) \notin A] \leq \mathbb{P}[G \sim \bigcup_{i=1}^k G(n,p) \notin A] = (\mathbb{P}[G \sim G(n,p) \notin A])^k$