Your Paper

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Denote $G_5 := G_5(\mathbb{Z}_p)$, and $G_5^+ := G_5^+(\mathbb{Q}_p)$.

$\zeta^{\wedge}_{L_{5,p}}(s) = \int_{G_5^+} |\det g|_p^s d\mu(G_5) = \int_{G_5^+} |\det uh|_p^s d\mu(G_5)$, where $h \in H$ and $u \in N_h$.

Each u is unipotent, hence $\zeta^{\wedge}_{L_{5,p}}(s) = \int_{G_5^+} |\det h|_p^s d\mu(G_5) = \int_{G_5^+} |\lambda_1^4\lambda_2^6\lambda_3^6\lambda_4^4|_p^s d\mu(G_5) =$

$= \int_{G_5^+} \left[|\lambda_1^4|_p|\lambda_2^6|_p|\lambda_3^6|_p|\lambda_4^4|_p\right]^s d\mu(G_5)$, by the inductive formula we have found for every $|h|$.

We denote $v_i = v(\lambda_i)$,

and so $\zeta^{\wedge}_{L_{5,p}}(s) = \int_{G_5^+} \left[p^{-4v_1}p^{-6v_2}p^{-6v_3}p^{-4v_4}\right]^s d\mu(G_5) = \int_{G_5^+} p^{-(4v_1+6v_2+6v_3+4v_4)s} d\mu(G_5)$.

We denote $\Lambda := p^{-(4v_1+6v_2+6v_3+4v_4)s}$. Now we use the natural matrix decomposition of the N_h matrix of Berman's, which means that

$\zeta^{\wedge}_{L_{5,p}}(s) = \int_{G_5^+} \Lambda d\mu(G_5) = \int_{\underline{\lambda}}\int_{\underline{a}}\int_{\underline{b}}\int_{\underline{c}} \Lambda d\mu(\underline{c})d\mu(\underline{b})d\mu(\underline{a})d\mu(\underline{\lambda})$. Since Λ depends only on $\lambda_1, \lambda_2, \lambda_3, \lambda_4$, which appear only in the computation of the outermost integral, we consider them as constants for all the inner integrals, which means that we have $\zeta^{\wedge}_{L_{5,p}}(s) = \int_{\underline{\lambda}} \Lambda \int_{\underline{a}}\int_{\underline{b}}\int_{\underline{c}} 1 d\mu(\underline{c})d\mu(\underline{b})d\mu(\underline{a})d\mu(\underline{\lambda})$, hence all the inner integrals evaluate to the measure of their domains of integration. now we compute the innermost integral by considering $\underline{a}$, $\underline{b}$ and $\underline{\lambda}$ as constants, and integrating only over $\underline{c}$. Considering the multiplication uh, we observe that for each element c_j, we must have that $\rho_j = c_j\lambda_1\lambda_2\lambda_3\lambda_4 \in \mathbb{Z}_p$, which means that $v(\rho_i) = v(c_i\lambda_1\lambda_2\lambda_3\lambda_4) \geq 0 \Rightarrow v(c_i) + v_1 + v_2 + v_3 + v_4 \geq 0 \Rightarrow v(c_i) \geq -(v_1 + v_2 + v_3 + v_4)$. But this means that $c_i \in p^{-(v_1+v_2+v_3+v_4)}\mathbb{Z}_p$, and since the domain of integration for this integral is $\underline{c} = \{c_1, c_2, c_3, c_4\}$, then $\mu(\underline{c}) = |c_j|_p^4 = p^{4(v_1+v_2+v_3+v_4)}$. Denote $C := p^{4(v_1+v_2+v_3+v_4)}$, we now have that $\zeta^{\wedge}_{L_{5,p}}(s) = \int_{\underline{\lambda}} C\Lambda \int_{\underline{a}}\int_{\underline{b}} 1 d\mu(\underline{b})d\mu(\underline{a})d\mu(\underline{\lambda})$.