Exercise 2

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In time $t = 0$, we have $X_0 = B_1 - (a + b0)B_{\frac{1}{1-0}} = B_1 - aB_1 = (1 - a)B_1$. But for X_t to be a Brownian Motion, there must exist some constant $y \in \mathbb{R}$ (specifically $y = 0$), such that a.s. $X_0 = y$, but $(1 - a)B_1$ is a random variable, not a constant. Unless $a = 1$, and then a.s. $X_0 = B_1 - 1B_1 = 0$.

$$X_{t+s} - X_t = B_1 - (1 + b(t + s))B_{\frac{1}{1-(t+s)}} - (B_1 - (1 + bt)B_{\frac{1}{1-t}}) =$$

$$= (1 + bt)B_{\frac{1}{1-t}} - (1 + b(t + s))B_{\frac{1}{1-(t+s)}}$$