ctmc3

Haim Lavi, 038712105

August 2025

1

$dX_t = 2X_t dB_t$, $X_0 = 3$.

We divide both sides of the equation by X_t,

$\frac{dX_t}{X_t} = 2dB_t$.

When integrating the right hand expression, from 0 to t, we get,

$\int_0^t 2dB_2 = 2\int_0^t B_s = 2B_t$.

We calculate the left hand integral by Ito, Setting $Y_t := \ln X_t$, we get,

$dY_t = d\ln X_t = \frac{1}{X_t}dX_t + \frac{1}{2} \cdot -\frac{1}{X_t^2}(dX_t)^2 = \frac{dX_t}{X_t} - \frac{1}{2X_t^2}(2X_t dB_t)^2 = \frac{dX_t}{X_t} - \frac{4X_t^2(dB_t)^2}{2X_t^2} = 2(dB_t)^2$.

Using the identity $(dB_t)^2 = dt$, and the equation $\frac{dX_t}{X_t} = 2dB_t$, we get,

$dY_t = d\ln X_t = \frac{dX_t}{X_t} - 2dt = 2dB_t - 2dt$.

We integrate both sides, and add the constant $Y_0 = \ln X_0$,

$Y_t = 2B_t - 2t + Y_0 \Rightarrow \ln X_t - \ln X_0 = 2B_t - 2t \Rightarrow \ln\frac{X_t}{X_0} = 2B_t - 2t \Rightarrow \frac{X_t}{X_0} = e^{2B_t - 2t} \Rightarrow X_t = X_0 e^{2Bt-2t} = 3e^{2B_t-2t}$.

2

$X_t = \cos(t + B_t)$.

We reconstruct the equation using Ito.

Denote $F(t, B_t) = X_t = \cos(t + B_t)$.

We calculate partial derivatives,

$\frac{\partial F}{\partial t} = -\sin(t + B_t)$.

$\frac{\partial F}{\partial B_t} = -\sin(t + B_t)$.

$\frac{\partial^2 F}{\partial {B_t}^2} = -\cos(t + B_t)$.

We get,

$dF(t, B_t) = dX_t = -\sin(t + B_t)dt + \frac{1}{2} \cdot -\cos(t + B_t)dt - \sin(t + B_t)dB_t$.