431 Class 18

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2017-10-31

Today's Agenda

- Discussion of the Class 17 In-Class Survey
- Comparing More than Two Populations: The Analysis of Variance
- Pairwise Comparisons of Means after a Significant ANOVA
    - Multiple Comparisons
    - Bonferroni and Tukey HSD approaches

Today's R Setup

```
library(forcats); library(tidyverse)

source("Love-boost.R")
dm192 <- read.csv("data/dm192.csv") %>% tbl_df
class17a <- read.csv("data/class17a.csv") %>% tbl_df
class17b <- read.csv("data/class17b.csv") %>% tbl_df
```

Project Task C

See README for Class 18, and README for Project Task C, please.

The Google Form for the survey is at
https://goo.gl/forms/bB1xJ16NnLihP9Gu1

Everyone must fill out the survey, regardless of whether you are working in a group.

It's due at noon on Wednesday 2017-11-08, as is the Task C Word template.

In-Class Survey from Class 17

In-Class Survey (`class17a` data)

We chose (using a computer) a random number between 0 and 100.

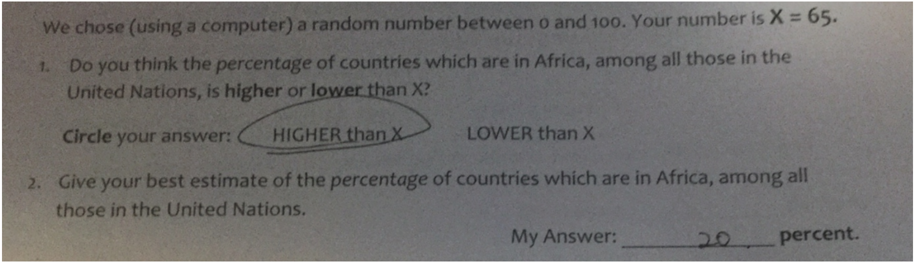
Your number is X = 10 (or 65).

1. Do you think the percentage of countries which are in Africa, among all those in the United Nations, is higher or lower than X?
2. Give your best estimate of the percentage of countries which are in Africa, among all those in the United Nations.

The facts

- There are 193 sovereign states that are members of the UN.
- The African regional group has 54 member states, so that's 28%.
- UN regions for countries are this Wikipedia link
- The `class17a` data set contains the answers to these questions from 185 students asked the same questions in the same way over the past four years (since 2014).

A troubling situation


We chose (using a computer) a random number between 0 and 100. Your number is X = 65.

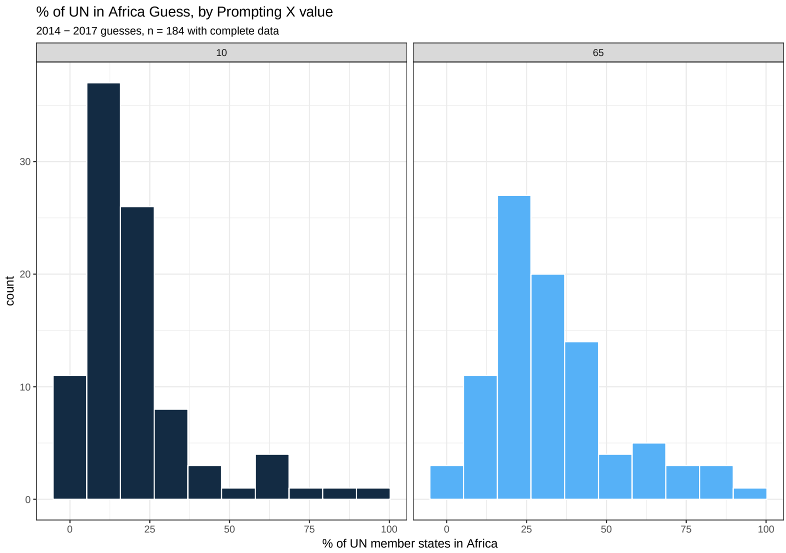
1. Do you think the *percentage* of countries which are in Africa, among all those in the United Nations, is **higher** or **lower** than X?

   Circle your answer: HIGHER than X LOWER than X

2. *Give your best estimate of the percentage of countries which are in Africa, among all those in the United Nations.*

   My Answer: 20 **percent.**

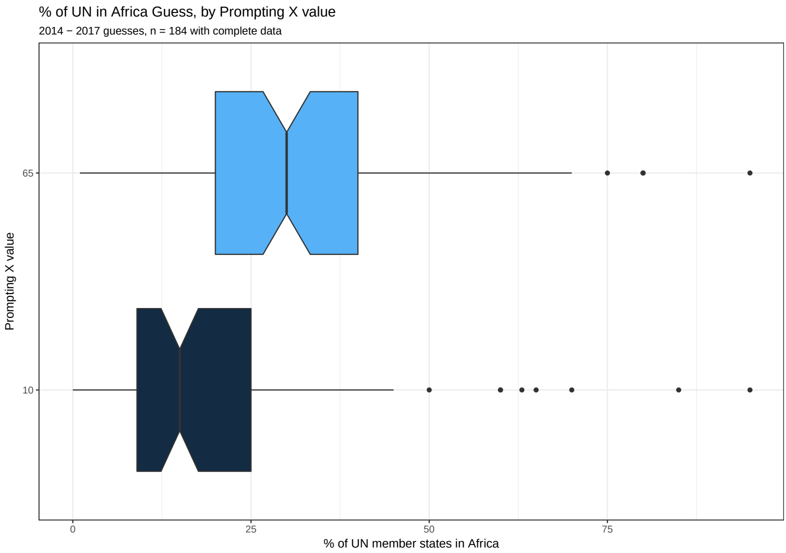
`class17a` Africa percentage guess by X = 10 or 65



`class17a` Analysis, Step-by-Step

1. What is the outcome under study?
2. What are the (in this case, two) treatment/exposure groups?
3. Were the data collected using matched / paired samples or independent samples?
4. Are the data a random sample from the population(s) of interest? Or is there at least a reasonable argument for generalizing from the sample to the population(s)?
5. What is the significance level (or, the confidence level) we require here?
6. Are we doing one-sided or two-sided testing/confidence interval generation?
7. If we have paired samples, did pairing help reduce nuisance variation?
8. If we have paired samples, what does the distribution of sample paired differences tell us about which inferential procedure to use?
9. If we have independent samples, what does the distribution of each individual sample tell us about which inferential procedure to use?

class17a Africa percentage guess by X = 10 or 65



class17a Descriptive Statistics

```r
class17a %>%
  filter(!is.na(africa.pct)) %>%
  group_by(x.value) %>%
  summarise(n(), mean(africa.pct),
            sd = round(sd(africa.pct),2),
            median = median(africa.pct))
```

```
# A tibble: 2 x 5
  x.value `n()` `mean(africa.pct)`    sd median
    <int> <int>              <dbl> <dbl>  <int>
1      10    93           20.96774 17.63     15
2      65    91           33.00000 19.29     30
```

class17a comparisons (results: next slide)

```r
t.test(africa.pct ~ x.value,
       data = class17a) # Welch
t.test(africa.pct ~ x.value, data = class17a,
       var.equal = TRUE) # Pooled t
wilcox.test(africa.pct ~ x.value, conf.int = TRUE,
            data = class17a)
set.seed(43123)
bootdif(class17a$africa.pct, class17a$x.value)
```

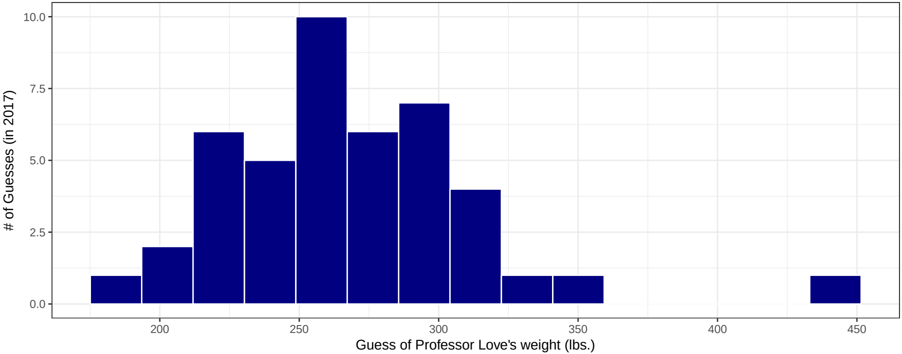
$$\Delta = \mu_{65} - \mu_{10}$$

Procedure	Est. Δ	95% CI for Δ	p
Welch t	12.0	(6.7, 17.4)	1.75e-05
Pooled t	12.0	(6.7, 17.4)	1.71e-05
Rank Sum	12.0	(8.0, 15.0)	6.06e-08
Bootstrap	12.0	(6.6, 17.5)	$< .05$

Conclusions?

In-Class Survey (`class17b` data)

3. Provide a point estimate for Dr. Love's current weight (in pounds.)



- 2017 Weight Guesses: $n = 44$, $\bar{x} = 267.7$ lbs., $s = 46.5$ lbs.
- Five Number Summary: 182 240 260 293 440

50% and 90% "Intervals" from Group Estimates

4. Now estimate one interval, which you believe has a 50% chance of including Dr. Love's current weight (again, in pounds.) Then do the same for a 90% interval.

We have $n = 44$ independent guesses, with $\bar{x} = 267.7$ lbs., $s = 46.5$ lbs. Let's first obtain quantiles, and use the crowd's wisdom.

```
quantile(class17b$love.lbs,
  probs = c(0.05, 0.10, 0.25, 0.50, 0.75, 0.90, 0.95))
```

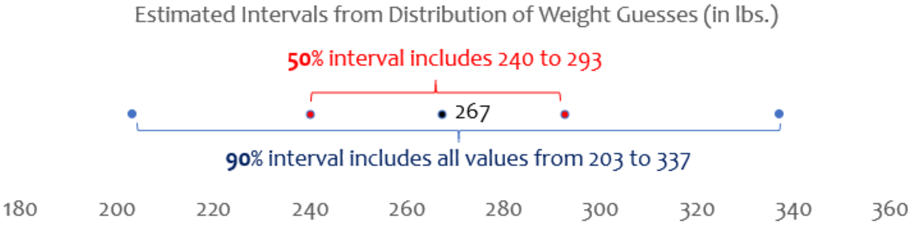
```
   5%   10%   25%   50%   75%   90%   95%
203.0 220.0 240.0 260.0 292.5 320.0 337.0
```

- What's a rational 50% interval for estimating my weight?
- How about a 90% interval?

One Possible, Rational, Set of Intervals

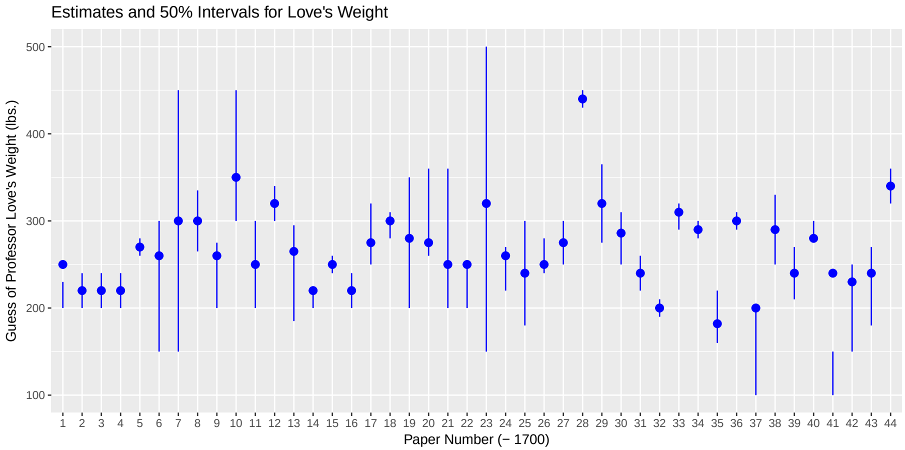
Suppose my estimate is 267 pounds.

- Then suppose I assign probability 0.50 to the interval (240, 293)
- And suppose I assign probability 0.90 to the interval (203, 337)



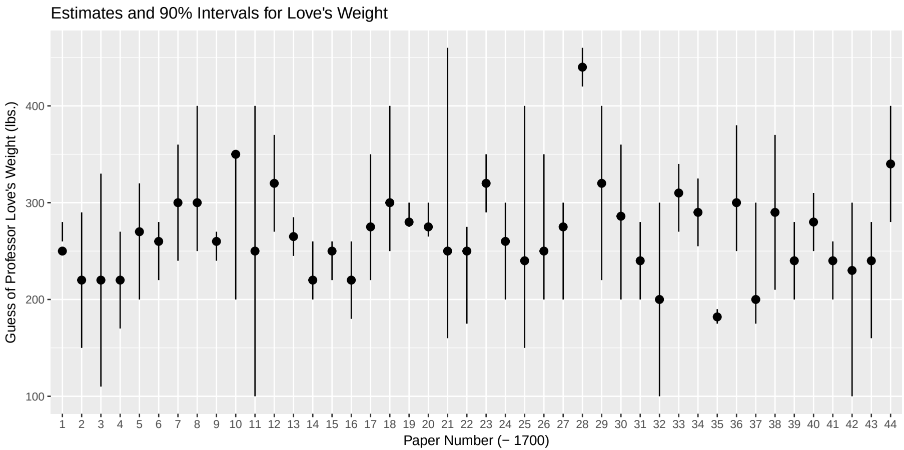
In-Class Survey (`class17b` data)

4a. Now estimate one interval, which you believe has a 50% chance of including Dr. Love's current weight (again, in pounds.)

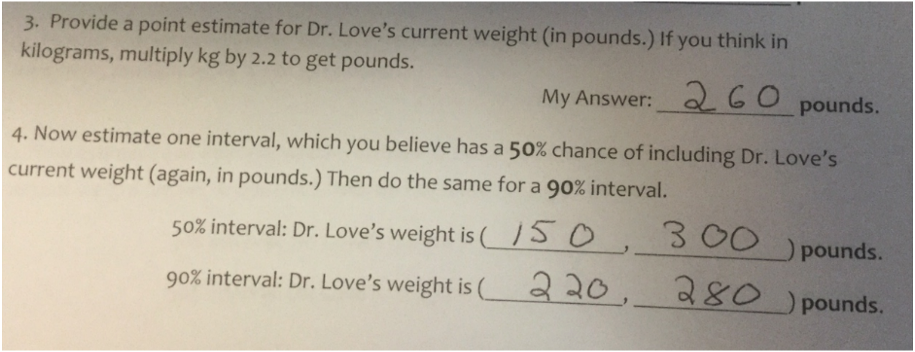


In-Class Survey (`class17b` data)

4b. Now do the same, but for a 90% interval...



Some Troubling Selections, 1



3. Provide a point estimate for Dr. Love's current weight (in pounds.) If you think in kilograms, multiply kg by 2.2 to get pounds.

My Answer: 260 pounds.

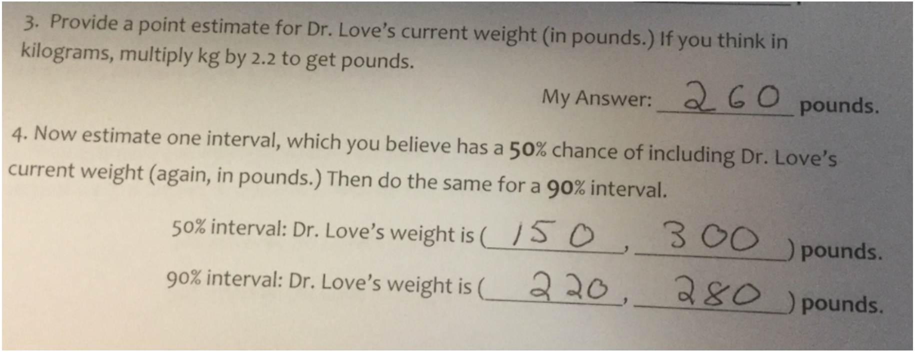
4. Now estimate one interval, which you believe has a **50%** chance of including Dr. Love's current weight (again, in pounds.) Then do the same for a **90%** interval.

50% interval: Dr. Love's weight is (150 , 300) pounds.

90% interval: Dr. Love's weight is (220 , 280) pounds.

- Why does this set of intervals not make sense?

Some Troubling Selections, 1



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My Answer: 260 pounds.

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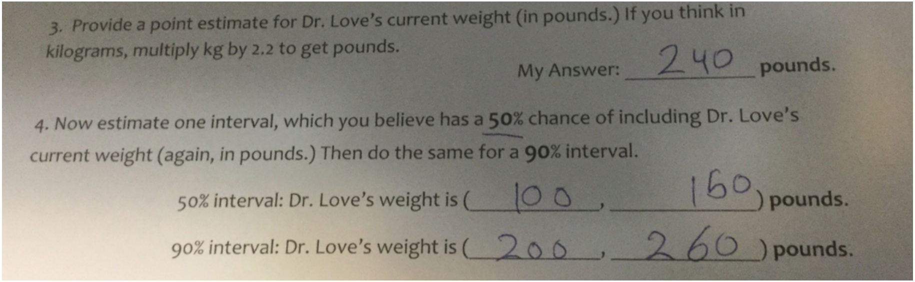
50% interval: Dr. Love's weight is (150 , 300) pounds.

90% interval: Dr. Love's weight is (220 , 280) pounds.

- Why does this set of intervals not make sense?
- There were **10** students (out of 44) who had a wider 50% interval than 90% interval.

Some Troubling Selections, 2


3. Provide a point estimate for Dr. Love's current weight (in pounds.) If you think in kilograms, multiply kg by 2.2 to get pounds.

My Answer: 240 pounds.

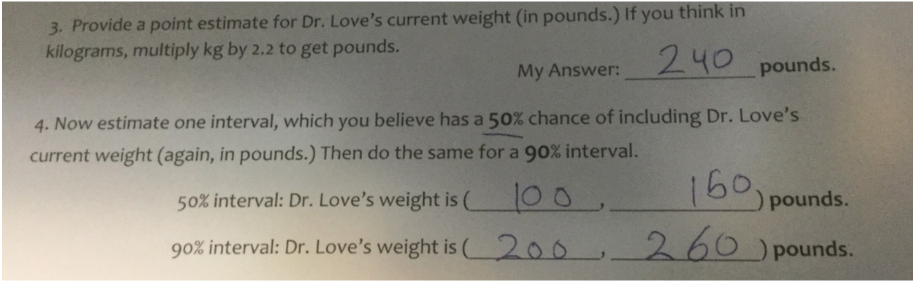
4. Now estimate one interval, which you believe has a **50%** chance of including Dr. Love's current weight (again, in pounds.) Then do the same for a **90%** interval.

50% interval: Dr. Love's weight is (100 , 160) **pounds.**

90% interval: Dr. Love's weight is (200 , 260) **pounds.**

- It wasn't clear enough that the interval estimate was meant to surround the point estimate.

Some Troubling Selections, 2



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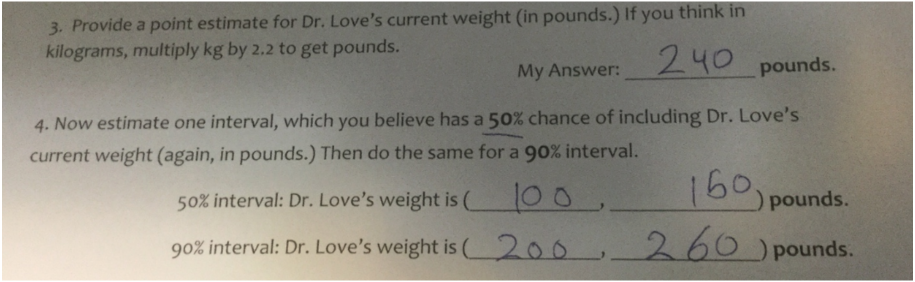
50% interval: Dr. Love's weight is (100 , 160) **pounds.**

90% interval: Dr. Love's weight is (200 , 260) **pounds.**

- It wasn't clear enough that the interval estimate was meant to surround the point estimate.
- There were **6** students out of 44 with this problem in their 50% interval, **2** in their 90% interval.

Some Troubling Selections, 2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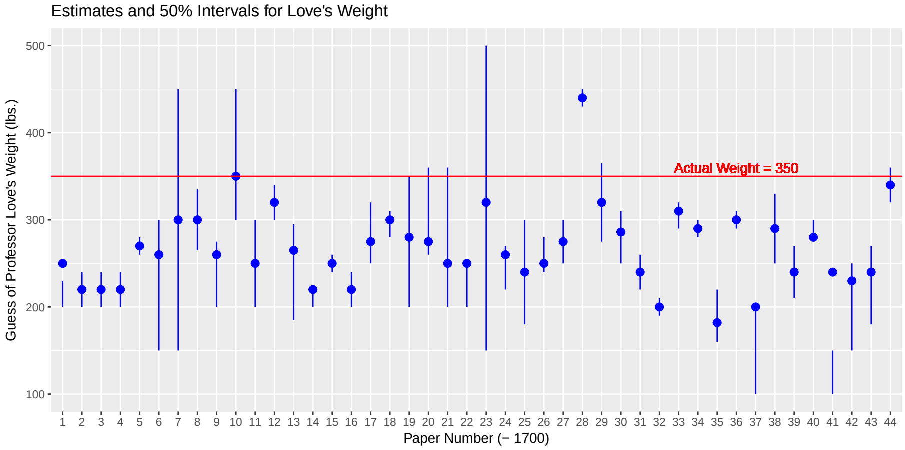
50% interval: Dr. Love's weight is (100 , 160) **pounds.**

90% interval: Dr. Love's weight is (200 , 260) **pounds.**

- It wasn't clear enough that the interval estimate was meant to surround the point estimate.
- There were **6** students out of 44 with this problem in their 50% interval, **2** in their 90% interval.
- For **15** students, the 90% interval did not contain the 50% interval.

The facts (with 50% intervals)

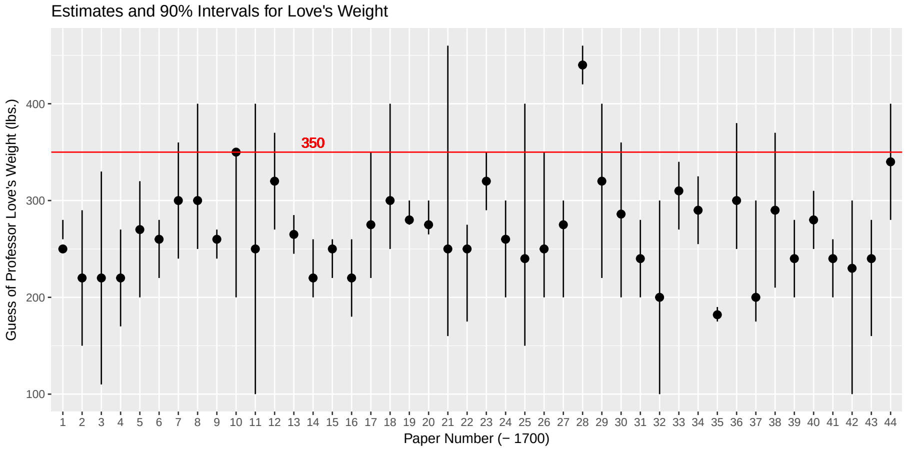
On 2017-10-26, Dr. Love actually weighed 350 lbs. or 158.8 kg or 25 stone, dressed but without shoes.



- **8** of the 44 50% intervals estimated by students included 350 lbs.

The facts (with 90% intervals)

On 2017-10-26, Dr. Love actually weighed 350 lbs.



- **16** of the 44 90% intervals estimated by students included 350 lbs.

Analysis of Variance (Section 28, Course Notes)

Analysis of Variance to Compare More Than Two Population Means using Independent Samples

Suppose we want to compare more than two population means, and we have collected three or more independent samples.

This is analysis of a continuous outcome variable on the basis of a single categorical factor — in fact, it's often called **one-factor** ANOVA or **one-way** ANOVA to indicate that the outcome is being split up into the groups defined by a single factor.

- H_0: population means in each group are the same
- H_A: H_0 isn't true; at least one μ differs from the others

When there are just two groups, then this boils down to an F test that is equivalent to the Pooled t test.

One-Way ANOVA

If we have a grouping factor with k levels, then we are testing:

- H_0: $\mu_1 = \mu_2 = ... = \mu_k$ vs.
- H_A: At least one of the population means $\mu_1, \mu_2, ..., \mu_k$ is different from the others.

Today's Example

We'll look at the `dm192` data again.

- Outcome is the `a1c` value (measured as a percentage),
- Factor is the insurance group (we'll compare 3 categories).

The dm192 data: Comparing Insurance Groups on Hemoglobin A1c

```r
dm.ins <- select(dm192, pt.id, insurance, a1c)
summary(dm.ins)
```

```
     pt.id                 insurance        a1c        
 Min.   :  1.00   commercial:39   Min.   : 5.400  
 1st Qu.: 48.75   medicaid  :67   1st Qu.: 6.300  
 Median : 96.50   medicare  :80   Median : 7.300  
 Mean   : 96.50   uninsured : 6   Mean   : 7.973  
 3rd Qu.:144.25                   3rd Qu.: 9.000  
 Max.   :192.00                   Max.   :16.100  
                                  NA's   :4       
```

- For now, we'll collapse the 6 uninsured in with Medicaid patients, and we'll drop the four cases without an A1c value.

Collapse medicaid and uninsured, drop missing a1c

```
dm.ins <- dm.ins %>%
    mutate(ins.3cat = fct_recode(insurance,
              "Commercial" = "commercial",
              "Medicare" = "medicare",
              "Medicaid/Unins." = "medicaid",
              "Medicaid/Unins." = "uninsured")) %>%
  filter(!is.na(a1c))
```

Summarize A1c by Insurance (3 categories)

```
dm.ins %>%
  group_by(ins.3cat) %>%
  summarise(n = n(), mean = round(mean(a1c),2),
            sd = round(sd(a1c),2), median = median(a1c))
```

```
# A tibble: 3 x 5
        ins.3cat     n  mean    sd median
          <fctr> <int> <dbl> <dbl>  <dbl>
1     Commercial    39  8.10  2.03    7.6
2 Medicaid/Unins.    73  8.12  2.35    7.5
3       Medicare    76  7.76  2.26    7.0
```

One-Way ANOVA for the `dm.ins` Data

If we have a grouping factor (insurance) with *3* levels, then we are testing:

- H_0: $\mu_{Comm.} = \mu_{Medicare} = \mu_{Medicaid/Unins.}$ vs.
- H_A: At least one of the population means is different from the others.

```
anova(lm(a1c ~ ins.3cat, data = dm.ins))
```

```
Analysis of Variance Table

Response: a1c
           Df Sum Sq Mean Sq F value Pr(>F)
ins.3cat    2   5.55  2.7763  0.5466 0.5798
Residuals 185 939.60  5.0789
```

Elements of the ANOVA Table

The ANOVA table breaks down the variation in the outcome explained by the k levels of the factor of interest, and the variation in the outcome which remains (the Residual, or Error).

```
Analysis of Variance Table

Response: a1c
           Df Sum Sq Mean Sq F value Pr(>F)
ins.3cat    2   5.55  2.7763  0.5466 0.5798
Residuals 185 939.60  5.0789
```

- Df = degrees of freedom, Sum Sq = Sum of Squares,
- Mean Sq = Mean Square (Sum of Squares / df)
- F value = F test statistic, Pr(>F) = p value

The Degrees of Freedom

```
          Df
ins.3cat    2
Residuals 185
```

- The **degrees of freedom** attributable to the factor of interest (here, `ins.3cat`) is the number of levels of the factor minus 1.
  - Here, we have three insurance category levels, so df(`ins.3cat`) = 2.
- The total degrees of freedom are the number of observations (across all levels of the factor) minus 1.
  - We have 188 patients left in our `dm.ins` study after removing the four with missing A1c, so df(Total) = 187, although the Total row isn't shown here.
- Residual df = Total df - Factor df = 187 - 2 = 185.

The Sums of Squares

```
          Df Sum Sq
ins.3cat    2   5.55
Residuals 185 939.60
```

- The **sum of squares** (SS) represents variation explained.
- SS(Factor) is the sum across all levels of the factor of the sample size for the level multiplied by the squared difference between the level mean and the overall mean across all levels. SS(`ins.3cat`) = 5.55
- SS(Total) = sum across all observations of the square of the difference between the individual values and the overall mean.
    - Here SS(Total) = 5.55 + 939.60 = 945.15
- Residual SS = Total SS – Factor SS.

η^2, the Proportion of Variation Explained by ANOVA

```
          Df Sum Sq
ins.3cat   2   5.55
Residuals 185 939.60
```

- η^2 ("eta-squared") is equivalent to R^2 in a linear model.
    - η^2 = SS(Factor) / SS(Total) = the proportion of variation in our outcome (here, hemoglobin A1c) explained by the variation between levels of our factor (here, our three insurance groups)
    - In our case, $\eta^2 = 5.55 / (5.55 + 939.60) = 5.55 / 945.15 = 0.0059$
- So, insurance group accounts for about 0.59% of the variation in hemoglobin A1c observed in these data.

The Mean Square

```
           Df Sum Sq Mean Sq
ins.3cat    2   5.55  2.7763
Residuals 185 939.60  5.0789
```

- The Mean Square is the Sum of Squares divided by the degrees of freedom, so MS(Factor) = SS(Factor)/df(Factor).
- MS(ins.3cat) = SS(ins.3cat)/df(ins.3cat) = 5.55 / 2 = 2.78.
- MS(Residuals) = SS(Residuals) / df(Residuals) = 939.60 / 185 = 5.08.
    - MS(Residuals) estimates the residual variance, corresponds to σ^2 in the underlying linear model
    - MS(Residuals) = 5.0789, so Residual standard error = $\sqrt{5.0789} = 2.25$ percentage points.

The F Test Statistic and p Value

```
Analysis of Variance Table

Response: a1c
           Df Sum Sq Mean Sq F value Pr(>F)
ins.3cat    2   5.55  2.7763  0.5466 0.5798
Residuals 185 939.60  5.0789
```

- F value = MS(ins.3cat) / MS(Residuals) = 2.78 / 5.08 = 0.55
- For an F distribution with 2 and 185 degrees of freedom, this F value yields $p = 0.58$

What is our conclusion regarding our test of our ANOVA hypotheses?

- H_0: $\mu_{Commercial} = \mu_{MedicaidorUninsured} = \mu_{Medicare}$ vs.
- H_A: H_0 is not true

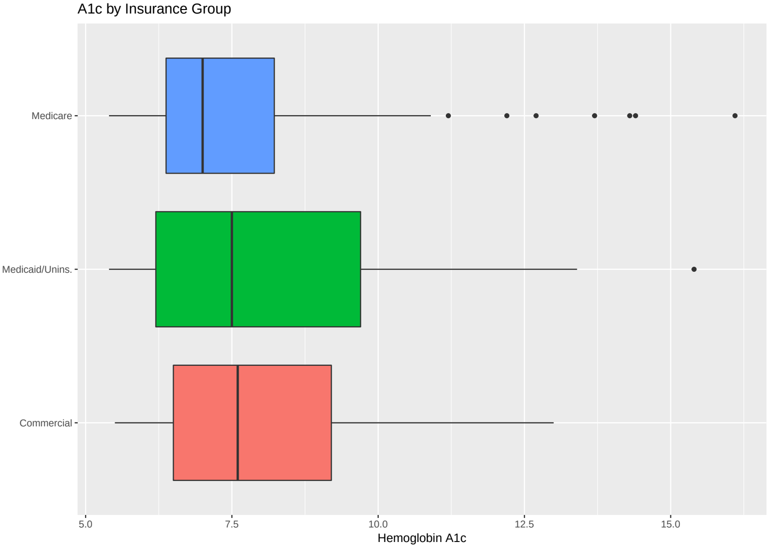
ANOVA Assumptions

The assumptions behind analysis of variance are the same as those behind a linear model. Of specific interest are:

- The samples obtained from each group are independent.
- Ideally, the samples from each group are a random sample from the population described by that group.
- In the population, the variance of the outcome in each group is equal. (This is less of an issue if our study involves a balanced design.)
- In the population, we have Normal distributions of the outcome in each group.

Happily, the F test is fairly robust to violations of the Normality assumption.

Can we assume population A1c levels are Normal?



Non-Parametric Alternative: Kruskal-Wallis Test

```r
kruskal.test(a1c ~ ins.3cat, data = dm.ins)
```

```
    Kruskal-Wallis rank sum test

data:  a1c by ins.3cat
Kruskal-Wallis chi-squared = 1.7809, df = 2,
p-value = 0.4105
```

Rank Sum test for

- H_0: Center of Commercial distribution = Center of Medicaid or Uninsured distribution = Center of Medicare distribution vs.
- H_A: H_0 not true.

Another Way to get our ANOVA Results

H_0: H_0: $\mu_{Commercial} = \mu_{MedicaidorUninsured} = \mu_{Medicare}$ vs. H_A: H_0 not true.

```
summary(aov(a1c ~ ins.3cat, data = dm.ins))
```

```
             Df Sum Sq Mean Sq F value Pr(>F)
ins.3cat      2    5.6   2.776   0.547   0.58
Residuals   185  939.6   5.079
```

Regression on Indicator Variables = Analysis of Variance

Yet another way to obtain an even more complete analog to the pooled t test is to run a linear regression model to predict the outcome (here, a1c) on the basis of the categorical factor, insurance group. We run the following …

```
summary(lm(a1c ~ ins.3cat, data = dm.ins))
```

Linear Model Summary Output

```
Call:
lm(formula = a1c ~ ins.3cat, data = dm.ins)

Residuals:
    Min      1Q  Median      3Q     Max
-2.7219 -1.6000 -0.6432  1.0855  8.3355

Coefficients:
                        Estimate Std. Error t value Pr(>|t|)
(Intercept)              8.10000    0.36087  22.446   <2e-16 ***
ins.3catMedicaid/Unins.  0.02192    0.44699   0.049    0.961
ins.3catMedicare        -0.33553    0.44391  -0.756    0.451
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 2.254 on 185 degrees of freedom
Multiple R-squared:  0.005875,  Adjusted R-squared:  -0.004872
F-statistic: 0.5466 on 2 and 185 DF,  p-value: 0.5798
```

Linear Model Results

- **Residual standard error: 2.254** on 185 degrees of freedom
- **Multiple R-squared: 0.005875**, Adjusted R-squared: -0.004872
- F-statistic: 0.5466 on 2 and 185 DF, p-value: 0.5798

Indicator Variable Regression

The linear model uses two **indicator variables**, sometimes called **dummy variables**.

- Each takes on the value 1 when its condition is met, and 0 otherwise.
- With three insurance categories, we need two indicator variables (we always need one fewer indicator than we have levels of the factor).
- Here, we have a baseline category (which is taken to be `Commercial` in this case) and then indicators for `Medicaid or Uninsured` and for `Medicare`.

K-1 indicators specify K categories

These two indicator variables completely specify the insurance category for any subject, as follows:

Insurance Category	`var1`	`var2`
Commercial	0	0
Medicaid/Unins.	1	0
Medicare	0	1

- `var1` is `ins.3catMedicaid/Unins.`
- `var2` is `ins.3catMedicare`

The Regression Equation

What is the regression equation here?

```
Call: lm(formula = a1c ~ ins.3cat, data = dm.ins)

Coefficients             Estimate Std. Err.      t  Pr(>|t|)
(Intercept)               8.10000   0.36087  22.446   <2e-16 ***
ins.3catMedicaid/Unins.   0.02192   0.44699   0.049    0.961
ins.3catMedicare         -0.33553   0.44391  -0.756    0.451
```

Equation specifies the three sample means

- A1c $=$ 8.1 $+$ 0.02 [Medicaid or Uninsured] - 0.34 [Medicare]
- [group] is 1 if the patient is in that group, and 0 otherwise

The Model predictions are Sample Means

```
Coefficients       Estimate Std. Error t value Pr(>|t|)
(Intercept)         8.10000    0.36087  22.446   <2e-16 ***
Medicaid/Uninsured  0.02192    0.44699   0.049    0.961
Medicare           -0.33553    0.44391  -0.756    0.451
```

Model Predictions:

- A1c = 8.1 if in the Commercial group
- A1c = 8.1 + 0.02192 = 8.12 if in the Medicaid or Uninsured group
- A1c = 8.1 - 0.33553 = 7.76 if in the Medicare group

K-Sample Study Design, Comparing Means

1. What is the outcome under study?
2. What are the (in this case, $K > 2$) treatment/exposure groups?
3. Were the data in fact collected using independent samples?
4. Are the data random samples from the population(s) of interest? Or is there at least a reasonable argument for generalizing from the samples to the population(s)?
5. What is the significance level (or, the confidence level) we require here?
6. Are we doing one-sided or two-sided testing?
7. What does the distribution of each individual sample tell us about which inferential procedure to use?
8. Are there statistically meaningful differences between population means?
9. If an overall test is significant, can we identify pairwise comparisons of means that show significant differences using an appropriate procedure that protects against Type I error expansion due to multiple comparisons?

A New Comparison using `dm192`

Let's look at the `dm192` data again, but now we'll study `dbp` (diastolic blood pressure) as our outcome of interest.

- We'll first use ANOVA make a comparison between the four levels of insurance (Medicare, Commercial, Medicaid, Uninsured).
- Later, we'll compare the average `dbp` across the four practices (A, B, C and D) included in the `dm192` sample.

Analysis of Variance for dbp by insurance

H_0: $\mu_{Medicare} = \mu_{Commercial} = \mu_{Medicaid} = \mu_{Uninsured}$ vs. H_A: H_0 not true.

```
summary(aov(dbp ~ insurance, data = dm192))
```

```
             Df Sum Sq Mean Sq F value  Pr(>F)
insurance     3   1909   636.2   5.275 0.00163 **
Residuals   188  22672   120.6
---
Signif. codes:
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

So which of the pairs of means are significantly different?

The Problem of Multiple Comparisons

1. Suppose we compare Medicare to Commercial, using a test with $\alpha = 0.05$
2. Then we compare Medicare to Medicaid on the same outcome, also using $\alpha = 0.05$
3. Then we compare Medicare to Uninsured, also with $\alpha = 0.05$
4. Suppose we compare Commercial to Medicaid with $\alpha = 0.05$
5. Then we compare Commercial to Uninsured with $\alpha = 0.05$
6. Then we compare Medicaid to Uninsured with $\alpha = 0.05$

What is our overall α level across these six comparisons?

The Problem of Multiple Comparisons

What is our overall α level across these six comparisons?

- It could be as bad as $0.05 + 0.05 + 0.05 + 0.05 + 0.05 + 0.05$, or 0.30.
- Rather than our nominal 95% confidence, we have something as low as 70% confidence across this set of simultaneous comparisons.
- Does it matter if we *pre-plan* the comparisons or not?

The Bonferroni solution

1. Suppose we compare Medicare to Commercial, using a test with $\alpha = 0.05/6$
2. Then we compare Medicare to Medicaid on the same outcome, also using $\alpha = 0.05/6$

... and then we do the other four comparisons, also at $\alpha = 0.05/6$.

Then across these six comparisons, our overall α can be (at worst)

- $0.05/6 + 0.05/6 + 0.05/6 + 0.05/6 + 0.05/6 + 0.05/6 = 0.05$
- So by changing our nominal confidence level from 95% to 99.167% in each comparison, we wind up with at least 95% confidence across this set of simultaneous comparisons.
- This is a conservative (worst case) approach.

Bonferroni approach for Pairwise Comparisons

Goal: Simultaneous p values comparing each pair of insurance types:

- Medicare vs Commercial
- Medicare vs Medicaid
- Medicare vs Uninsured
- Commercial vs Medicaid
- Commercial vs Uninsured
- Medicaid vs Uninsured

Bonferroni results for `dbp` by `insurance`

```
pairwise.t.test(dm192$dbp, dm192$insurance,
                p.adjust="bonferroni")
```

```

    Pairwise comparisons using t tests with pooled SD

data:  dm192$dbp and dm192$insurance

          commercial medicaid medicare
medicaid  0.31337    -        -
medicare  1.00000    0.00082  -
uninsured 1.00000    1.00000  0.91293

P value adjustment method: bonferroni
```

Tukey's Honestly Significant Differences

Most appropriate for **pre-planned** comparisons, with a balanced (or near-balanced) design.

Goal: Simultaneous (less conservative) confidence intervals and *p* values for our six pairwise comparisons:

- Medicare vs Commercial
- Medicare vs Medicaid
- Medicare vs Uninsured
- Commercial vs Medicaid
- Commercial vs Uninsured
- Medicaid vs Uninsured

Tukey HSD Confidence Intervals

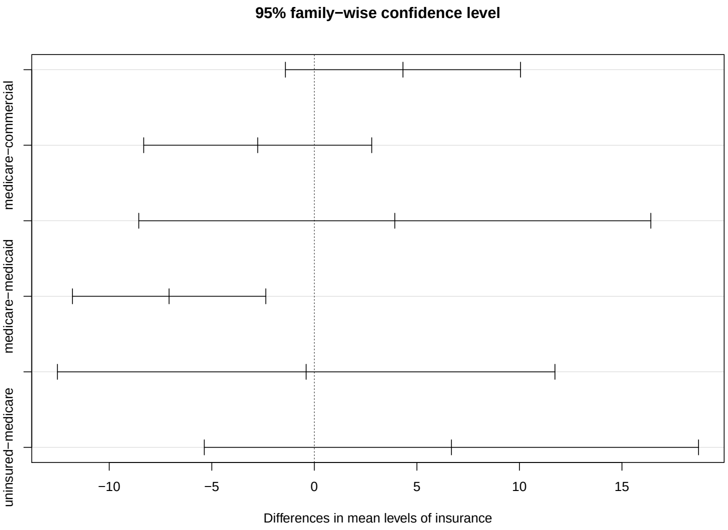
```
TukeyHSD(aov(dbp ~ insurance, data = dm192))
```

```
  Tukey multiple comparisons of means
    95% family-wise confidence level

Fit: aov(formula = dbp ~ insurance, data = dm192)

$insurance
                          diff        lwr       upr     p adj
medicaid-commercial   4.321087  -1.412308 10.054482 0.2095130
medicare-commercial  -2.760256  -8.319617  2.799104 0.5723295
uninsured-commercial  3.923077  -8.560153 16.406307 0.8475052
medicare-medicaid    -7.081343 -11.795510 -2.367177 0.0007847
uninsured-medicaid   -0.398010 -12.528463 11.732443 0.9997788
uninsured-medicare    6.683333  -5.365840 18.732506 0.4774431
```

Plot of Tukey HSD results (default, no relabeling)



Need to build smaller names for `insurance` levels

The `forcats` package can help

```
levels(dm192$insurance)
```

```
[1] "commercial" "medicaid"   "medicare"   "uninsured"
```

```
dm192$ins <- fct_recode(dm192$insurance,
                        "C" = "commercial",
                        "Md" = "medicaid",
                        "Mr" = "medicare",
                        "U" = "uninsured")
levels(dm192$ins)
```

```
[1] "C"  "Md" "Mr" "U"
```

Tukey 90% HSD CI

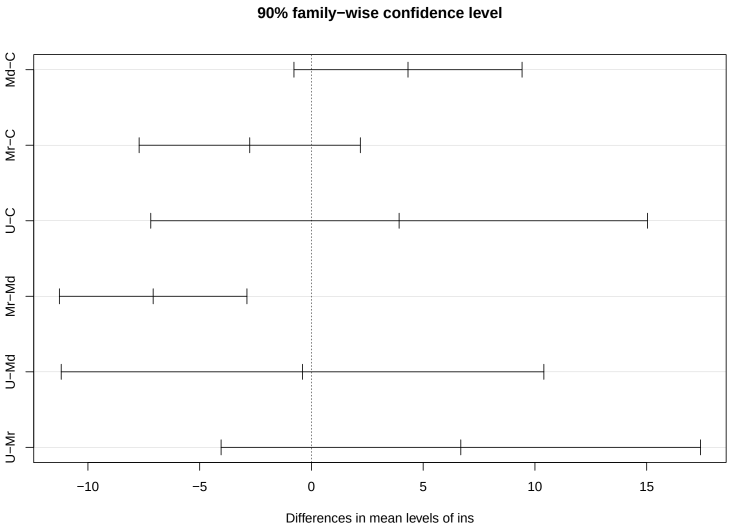
```
TukeyHSD(aov(dbp ~ ins, data = dm192), conf.level = 0.9)
```

```
  Tukey multiple comparisons of means
    90% family-wise confidence level

Fit: aov(formula = dbp ~ ins, data = dm192)

$ins
            diff         lwr       upr     p adj
Md-C    4.321087  -0.7822561  9.424430 0.2095130
Mr-C   -2.760256  -7.7086896  2.188177 0.5723295
U-C     3.923077  -7.1883505 15.034504 0.8475052
Mr-Md  -7.081343 -11.2774623 -2.885224 0.0007847
U-Md   -0.398010 -11.1954283 10.399408 0.9997788
U-Mr    6.683333  -4.0417366 17.408403 0.4774431
```

Plot of 90% Tukey HSD Intervals



Conclusions for `dbp` by `insurance`

The `dbp` levels are statistically significantly higher in some insurance groups than in others.

In particular, with 90% confidence across all six pairwise comparisons of insurance types, we see a statistically significant difference between Medicare and Medicaid, with Medicare patients showing `dbp` levels that are 7.1 mm Hg lower on average than Medicaid patients (90% simultaneous CI: 2.9 to 11.3 mm Hg.)

Looking at dbp by practice

```
summary(aov(dbp ~ practice, data = dm192))
```

```
             Df Sum Sq Mean Sq F value  Pr(>F)    
practice      3   2694   898.0   7.714 6.9e-05 ***
Residuals   188  21887   116.4                    
---
Signif. codes:  
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Bonferroni *p* values for `dbp` by `practice`

```r
pairwise.t.test(dm192$dbp, dm192$practice,
                p.adjust="bonferroni")
```

```

    Pairwise comparisons using t tests with pooled SD

data:  dm192$dbp and dm192$practice

  A      B      C
B 1.0000 -      -
C 0.0044 0.0014 -
D 0.0174 0.0063 1.0000

P value adjustment method: bonferroni
```

Tukey HSD CI for dbp by practice

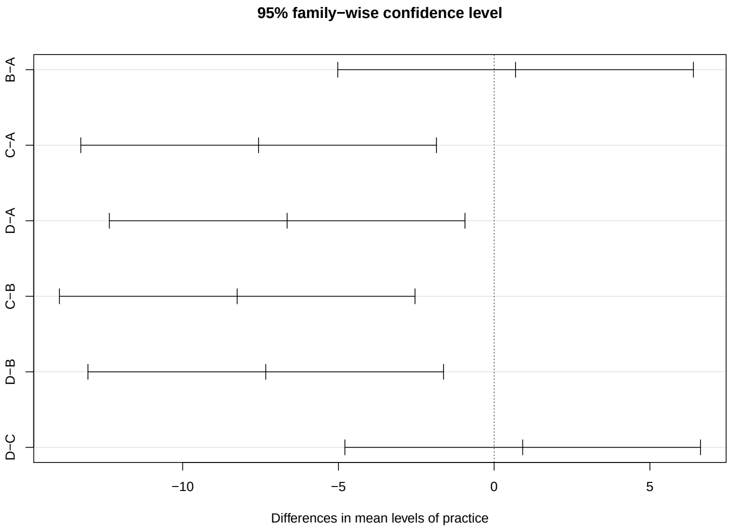
```r
TukeyHSD(aov(dbp ~ practice, data = dm192))
```

```
  Tukey multiple comparisons of means
    95% family-wise confidence level

Fit: aov(formula = dbp ~ practice, data = dm192)

$practice
          diff        lwr        upr     p adj
B-A  0.6875000  -5.021573  6.3965730 0.9894298
C-A -7.5625000 -13.271573 -1.8534270 0.0040503
D-A -6.6458333 -12.354906 -0.9367604 0.0152566
C-B -8.2500000 -13.959073 -2.5409270 0.0013559
D-B -7.3333333 -13.042406 -1.6242604 0.0057255
D-C  0.9166667  -4.792406  6.6257396 0.9756496
```

Plot of Tukey HSD Results (`dbp` by `practice`)



Conclusions for `dbp` by `practice`

The `dbp` levels are statistically significantly higher in some practices than in others.

In particular, with 95% confidence across all six pairwise comparisons of practices, we see statistically significant differences between A and C and between A and D, as well as between B and C and between B and D, with C and D showing significantly lower `dbp` than either A or B.

For example, comparing C to A, we see a difference of 7.6 mm Hg (with A higher than C), with 95% CI (via Tukey HSD) of (1.9, 13.3) mm Hg.

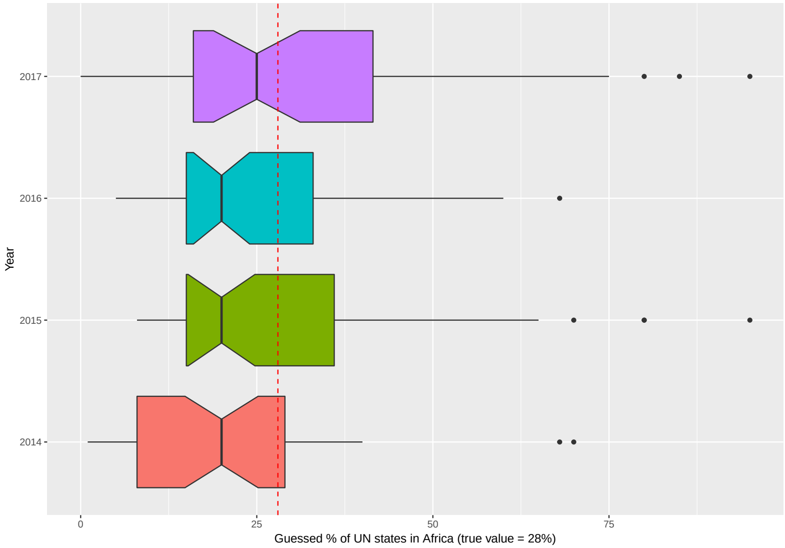
One Last Example

Here is the data from `class17a` again, Let's consider `africa.pct` by `year`

```
class17a %>%
  filter(!is.na(africa.pct)) %>%
  group_by(year) %>%
  summarise(n(), mean(africa.pct), sd(africa.pct))
```

```
# A tibble: 4 x 4
   year `n()` `mean(africa.pct)` `sd(africa.pct)`
  <int> <int>              <dbl>            <dbl>
1  2014    41           20.70732         15.58404
2  2015    49           28.79592         20.20518
3  2016    51           26.58824         15.96894
4  2017    43           31.09302         24.00081
```

Plot comparing four groups for `class17a`



ANOVA Question?

The question ANOVA answers is whether the means across the four subpopulations (here, years) are the same or not the same.

- Doesn't address the issue of which year best estimated the true value (28%) at all.

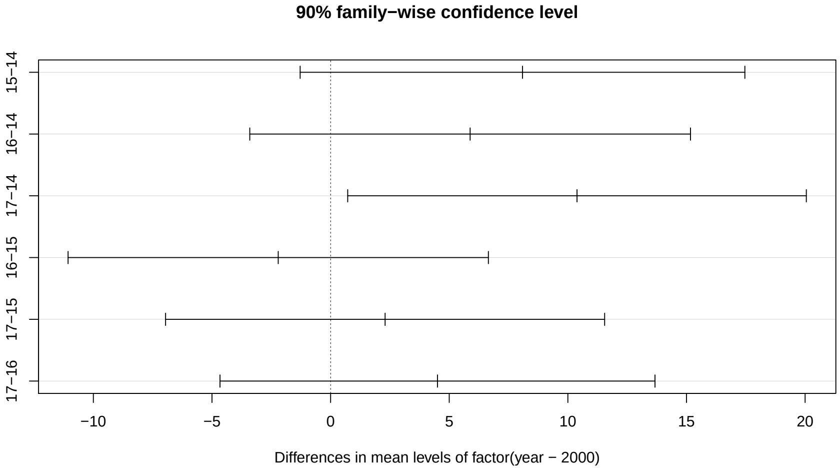
```
anova(lm(africa.pct ~ factor(year), data = class17a))
```

```
Analysis of Variance Table

Response: africa.pct
              Df Sum Sq Mean Sq F value  Pr(>F)
factor(year)   3   2509  836.45  2.2725 0.08174 .
Residuals    180  66254  368.08
---
Signif. codes:
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Tukey HSD 90% Comparisons, plotted

```
plot(TukeyHSD(aov(africa.pct ~ factor(year-2000),
                  data = class17a), conf.level = 0.90))
```



ANOVA summary

If you want to compare more than two population means, and are willing to assume Normality, the Analysis of Variance is attractive.

- Equivalent to fitting a linear model with a categorical predictor
- Compare the means, overall, using an F test
- Assess individual pairwise comparisons with Bonferroni and Tukey HSD procedures
- If Normality is a serious issue, consider Kruskal-Wallis test (431) or bootstrap (432) approaches