431 Class 07

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2017-09-19

This Week

1. Project Instructions
2. Assignment 1 debrief (Assignment 2 due 2017-09-22)
3. Course Notes
    - Identifying and quantifying outliers (Ch 7)
    - Using transformations to "normalize" data (Ch 9)
    - Summaries withing subgroups (Ch 10)
    - Associations, Using Linear Models (Ch 11)
4. Which states are in the midwest? (fill out survey)

Project Instructions

All materials and information related to the project will be maintained at

https://github.com/THOMASELOVE/431project

Regular updates will appear there throughout the semester.

Comments on Assignments 1 and 2

Answer Sketch for Assignment 1 posted on Friday afternoon. Visit this link for details. The PDF is password protected, rather than "invalid" as Github suggests when you try to display it.

1. R Markdown for Answer Sketch is also available.
2. Grading rubric for Assignment 1 is coming Thursday.
3. No, I did not expect you to write a 15 page response.
4. No, I do not plan to provide a new template for Assignment 2, but I strongly suggest you look over Claudia's slide deck, as linked in the Class 7 README.

Preliminaries for Today

```
library(NHANES); library(magrittr); library(viridis)
library(gridExtra); library(tidyverse)

source("Love-boost.R")

nh_temp <- NHANES %>%
    filter(SurveyYr == "2011_12") %>%
    filter(Age >= 21 & Age < 65) %>%
    mutate(Sex = Gender, Race = Race3,
           SBP = BPSysAve, DBP = BPDiaAve) %>%
    select(ID, Sex, Age, Race, Education,
           BMI, SBP, DBP, Pulse, PhysActive,
           Smoke100, SleepTrouble, HealthGen)
```

Random Sample of 500 to get nh_adults

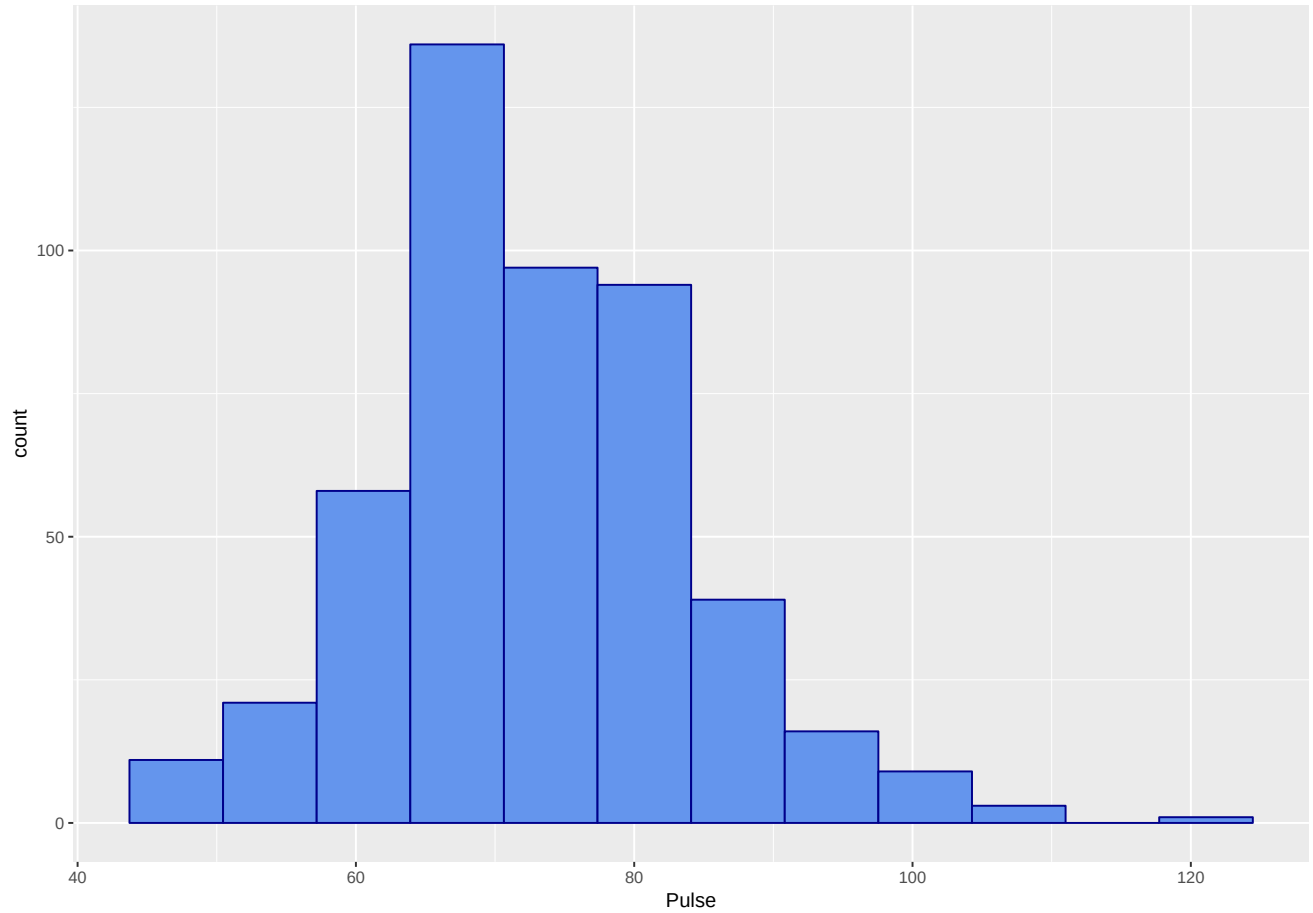
```
set.seed(431002)
# use set.seed to ensure that
# we all get the same random sample

nh_adults <- sample_n(nh_temp, size = 500)
```

What Summaries to Report (from Section 7.17)

It is usually helpful to focus on the shape, center and spread of a distribution. Bock, Velleman and DeVeaux provide some useful advice:

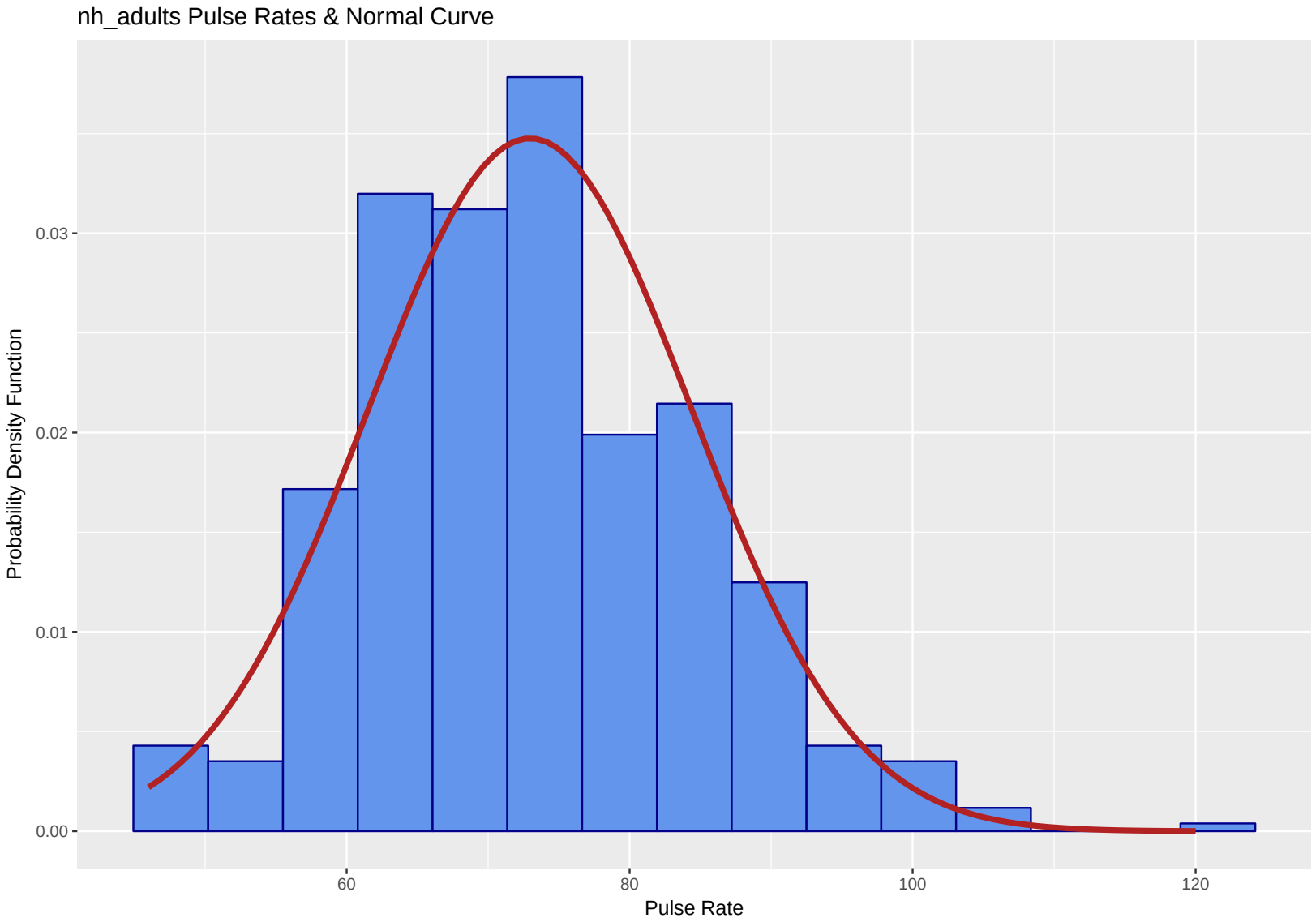
- If the data are skewed, report the median and IQR (or the three middle quantiles). You may want to include the mean and standard deviation, but you should point out why the mean and median differ. The fact that the mean and median do not agree is a sign that the distribution may be skewed. A histogram will help you make that point.
- If the data are symmetric, report the mean and standard deviation, and possibly the median and IQR as well.
- If there are clear outliers and you are reporting the mean and standard deviation, report them with the outliers present and with the outliers removed. The differences may be revealing. The median and IQR are not likely to be seriously affected by outliers.

Do the Pulse data appear skewed? Outlier-prone?



For Pulse, *skew*1 = 0.08, and kurtosis = 0.4

Pulse Rates + Normal Model

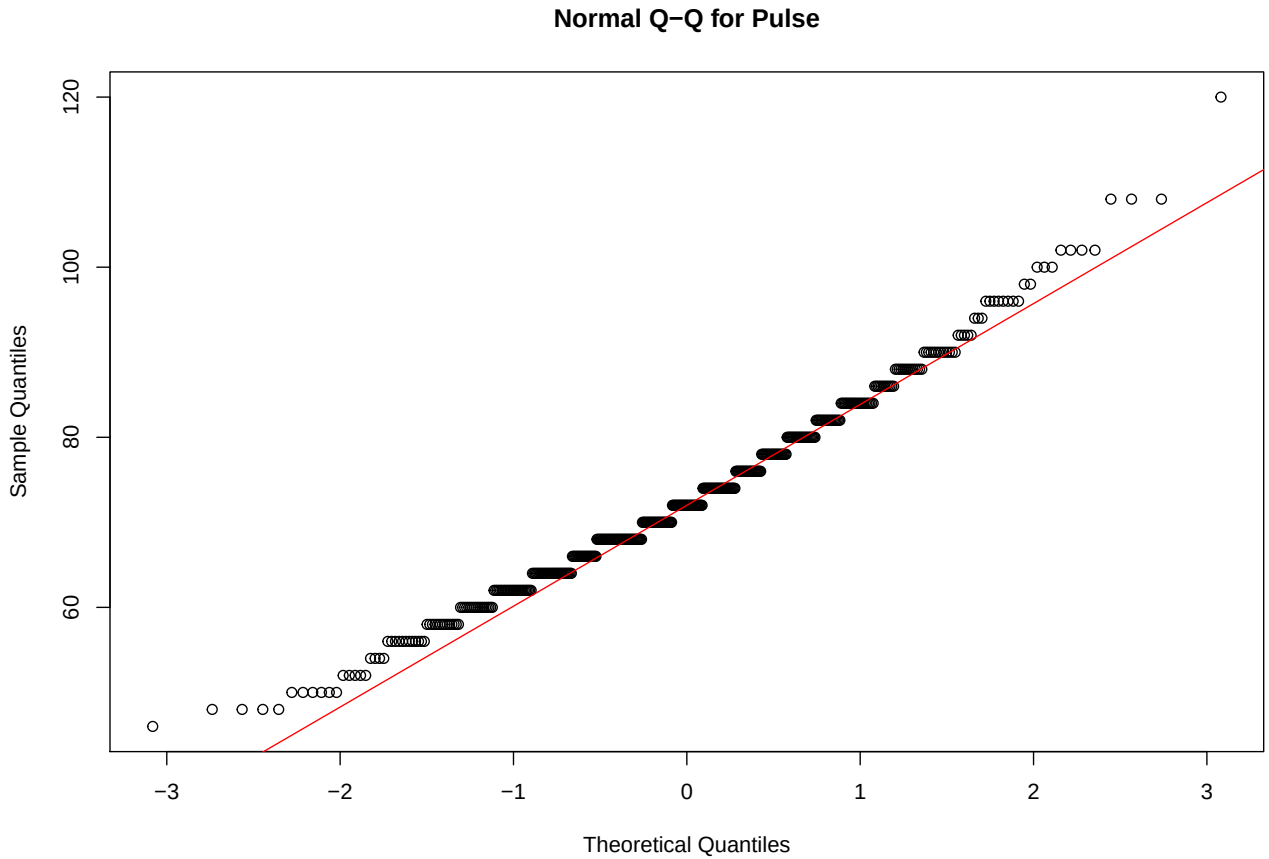


Pulse Rates + Normal Model (Code)

```
nh_adults %>% filter(!is.na(Pulse)) %>%
ggplot(., aes(x = Pulse)) +
    geom_histogram(aes(y = ..density..), bins=15,
                   fill = "cornflowerblue",
                   color = "blue4") +
    stat_function(fun = dnorm,
          args = list(
             mean = mean(nh_adults$Pulse, na.rm=TRUE),
             sd = sd(nh_adults$Pulse, na.rm=TRUE)),
          lwd = 1.5, col = "firebrick") +
    labs(title = "nh_adults Pulse Rates & Normal Curve",
          x = "Pulse Rate",
          y = "Probability Density Function")
```

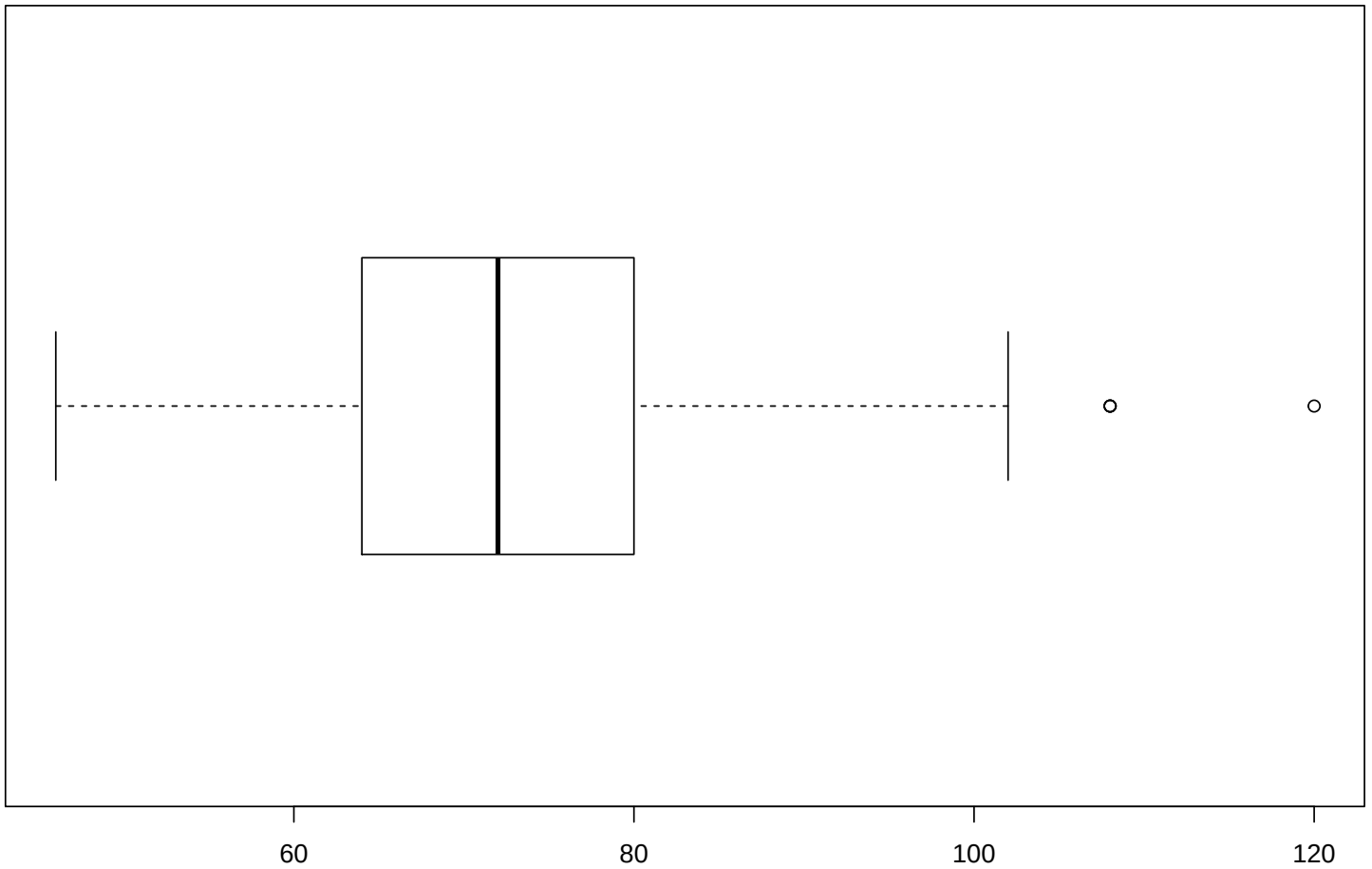
Normal Q-Q plot for Pulse

```
qqnorm(nh_adults$Pulse, main = "Normal Q-Q for Pulse")
qqline(nh_adults$Pulse, col = "red")
```



Identifying outliers (with a boxplot)

```
boxplot(nh_adults$Pulse, horizontal = TRUE)
```



How the Boxplot identifies Outlier Candidates

Calculate the upper and lower (inner) fences. Points outside that range are candidate outliers. If IQR = 75th percentile - 25th percentile, then

- Upper fence = 75th percentile + 1.5 IQR
- Lower fence = 25th percentile - 1.5 IQR

Let us consider the SBP data again. There, we have

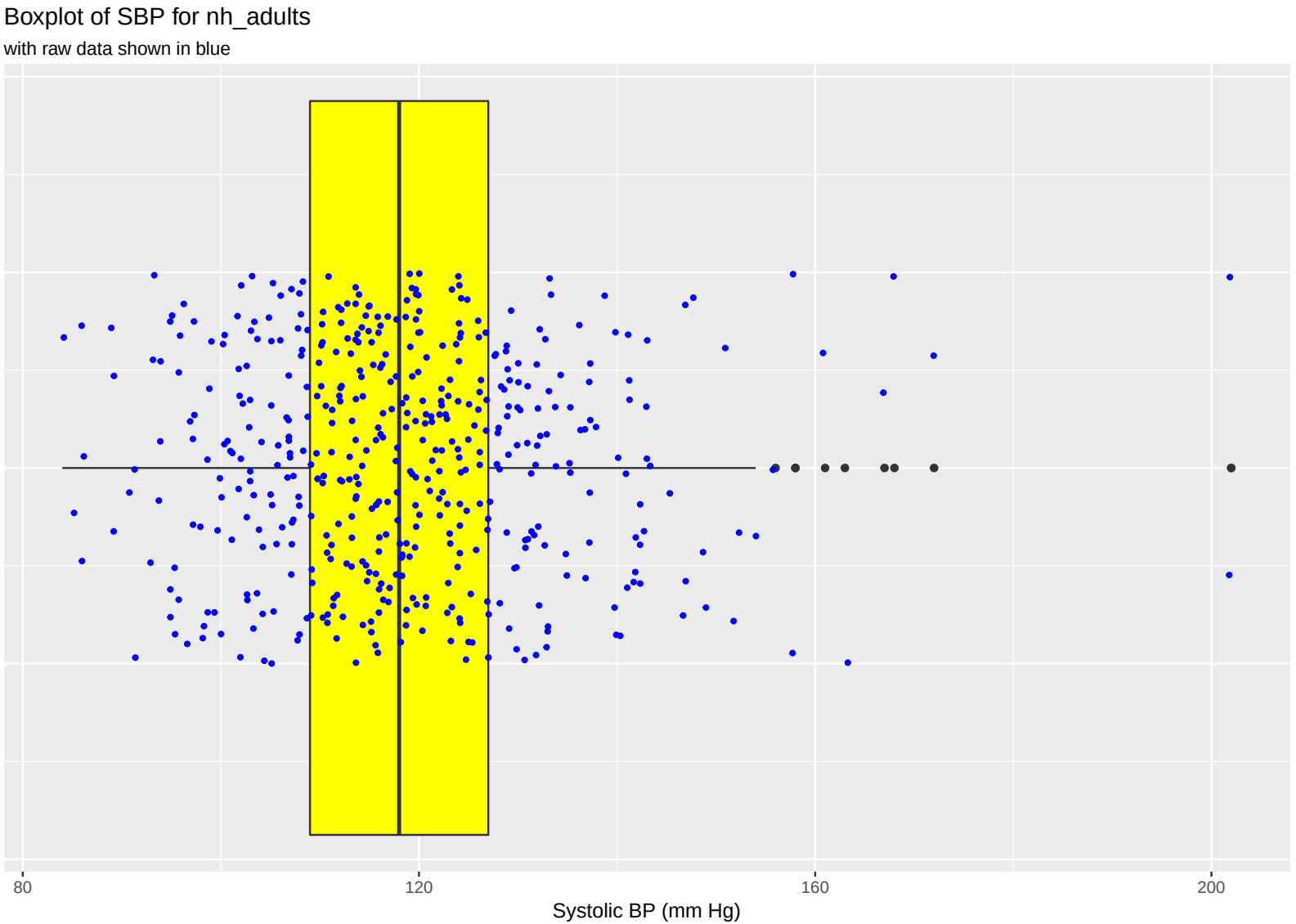
```
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
   84.0   109.0   118.0   118.6   127.0   202.0
   NA's
     15
```

Identifying outliers (with ggplot and a boxplot)

Here is the code (adapted from Course Notes 7.10.1)

```
nh_adults %>%
  filter(!is.na(SBP)) %>%
  ggplot(., aes(x = 1, y = SBP)) +
  geom_boxplot(fill = "yellow") +
  geom_point(col = "blue", size = 0.4) +
  coord_flip() +
  theme(axis.text.y = element_blank(),
        axis.ticks.y = element_blank()) +
  labs(title = "Boxplot of SBP for nh_adults",
       subtitle = "with raw data shown in blue",
       x = "", y = "Systolic BP (mm Hg)")
```

The Resulting Boxplot



Identifying outliers (with Z scores) (Section 8.2)

The maximum systolic blood pressure in the data is NA.

```
nh_adults %>%
  filter(!is.na(SBP)) %$%
  mosaic::favstats(SBP)
```

```
 min  Q1 median  Q3 max     mean       sd   n missing
  84 109    118 127 202 118.5918 15.30267 485       0
```

But how unusual is that value? One way to gauge how extreme this is (or how much of an outlier it is) uses that observation's **Z score**, the number of standard deviations away from the mean that the observation falls.

Z score for SBP = 202

Z score =

$$\frac{value - mean}{sd}$$

.

For the SBP data, the mean = 118.6 and the standard deviation is 15.3, so we have Z score for 202 =

$$\frac{202 - 118.6}{15.3} = 83.415.3 = 5.45$$

.

- A negative Z score indicates a point below the mean
- A positive Z score indicates a point above the mean
- The Empirical Rule suggests that for a variable that followed a Normal distribution, about 95% of observations would have a Z score in (-2, 2) and about 99.7% would have a Z score in (-3, 3).

How unusual is a value as extreme as Z = 5.45?

If the data really followed a Normal distribution, we could calculate the probability of obtaining as extreme a Z score as 5.45.

A Standard Normal distribution, with mean 0 and standard deviation 1, is what we want, and we want to find the probability that a random draw from such a distribution would be 5.45 or higher, *in absolute value*. So we calculate the probability of 5.45 or more, and add it to the probability of -5.45 or less, to get an answer to the question of how likely is it to see an outlier this far away from the mean.

```
pnorm(q = 5.45, mean = 0, sd = 1, lower.tail = FALSE)
```

```
[1] 2.518491e-08
```

```
pnorm(q = -5.45, mean = 0, sd = 1, lower.tail = TRUE)
```

```
[1] 2.518491e-08
```

But the Normal distribution is symmetric

```
2*pnorm(q = 5.45, mean = 0, sd = 1, lower.tail = FALSE)
```

```
[1] 5.036982e-08
```

The probability that a single draw from a Normal distribution with mean 0 and standard deviation 1 will produce a value as extreme as 5.45 is 0.00000005

The probability that a single draw from a Normal distribution with mean 118.6 and standard deviation 15.3 will produce a value as extreme as 202 is also 0.00000005, since the Normal distribution is completely characterized by its mean and standard deviation.

So, is 202 an outlier here? Do the SBP data look like they come from a Normal distribution?

Fences and Z Scores

Note the relationship between the fences (Tukey's approach to identifying points which fall within the whiskers of a boxplot, as compared to candidate outliers) and the Z scores.

min	Q1	median	Q3	max	mean	sd	n	missing
84	109	118	127	202	118.5918	15.30267	485	0

For the SBP data, the IQR is 127 - 109 = 18, so

- the upper inner fence is at 127 + 1.5*(18), or 154, and
- the lower inner fence is at 109 - 1.5*(18), or 82.
- Since the mean is 118.6 and the standard deviation is 15.3,
  - the Z score for the upper inner fence is 2.31, and
  - the Z score for the lower inner fence is -2.39
- It is neither unusual nor inevitable for the inner fences to fall at Z scores near -2.0 and +2.0.

Summing Up: Does a Normal Model fit well?

If a Normal model fits our data well, then we should see the following graphical indications:

1. A histogram that is symmetric and bell-shaped.
2. A boxplot where the box is symmetric around the median, as are the whiskers, without a serious outlier problem.
3. A normal Q-Q plot that essentially falls on a straight line.

As for numerical summaries, we'd like to see

4. The mean and median within 0.2 standard deviation of each other.
5. No real evidence of too many outlier candidates (more than 5% starts to get us concerned about a Normal model)
6. No real evidence of individual outliers outside the reasonable range for the size of our data (we might expect about 3 observations in 1000 to fall more than 3 standard deviations away from the mean.)

Should our data not be well-modeled by the Normal, what can we do?

The Ladder of Power Transformations (Section 9)

The key notion in re-expression of a single variable to obtain a more normal distribution or re-expression of an outcome in a simple regression model is that of a **ladder of power transformations**, which can apply to any unimodal data.

Power	Transformation
3	x^3
2	x^2
1	x (unchanged)
0.5	$x^{0.5} = \sqrt{x}$
0	ln x
-0.5	$x^{-0.5} = 1/\sqrt{x}$
-1	$x^{-1} = 1/x$
-2	$x^{-2} = 1/x^2$

Using the Ladder

- The ladder is most useful for strictly positive, ratio variables.
- Sometimes, if 0 is a value in the data set, we will add 1 to each value before applying a transformation like the logarithm.
- Interpretability is often an important criterion, although back-transformation at the end of an analysis is usually a sensible strategy.

Power	-2	-1	-0.5	0	0.5	1	2	3
Transformation	$1/x^2$	$1/x$	$1/\sqrt{x}$	ln x	$\sqrt{x}$	x	x^2	x^3

The nyfs1 data (see Chapter 7 of our Notes)

The nyfs1.csv data come from the 2012 National Youth Fitness Survey.

```
## first, we'll import the data into the nyfs1 data frame
nyfs1 <- read.csv("nyfs1.csv") %>% tbl_df()

dim(nyfs1)
```

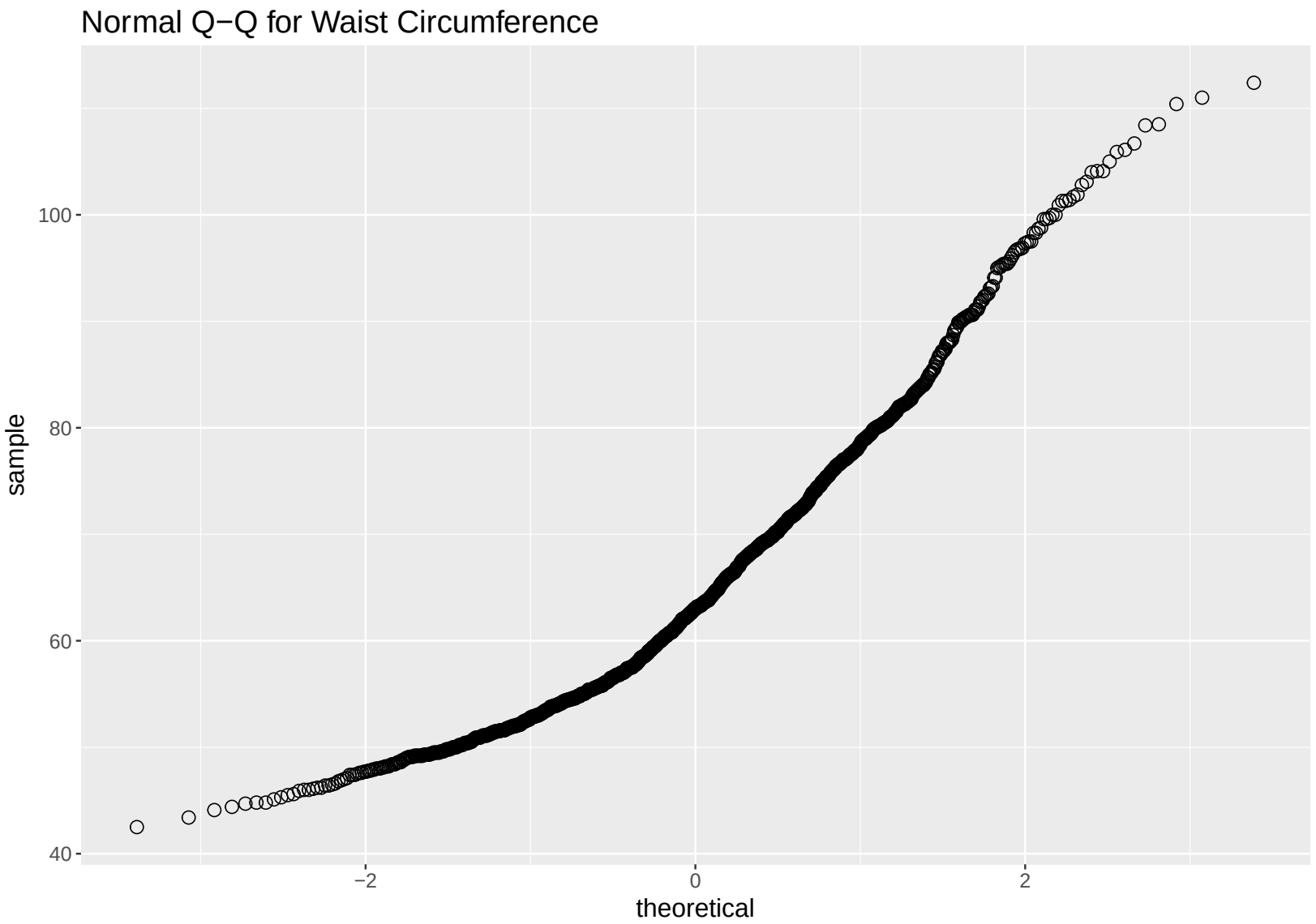
```
[1] 1416    7
```

Structure of our Tibble

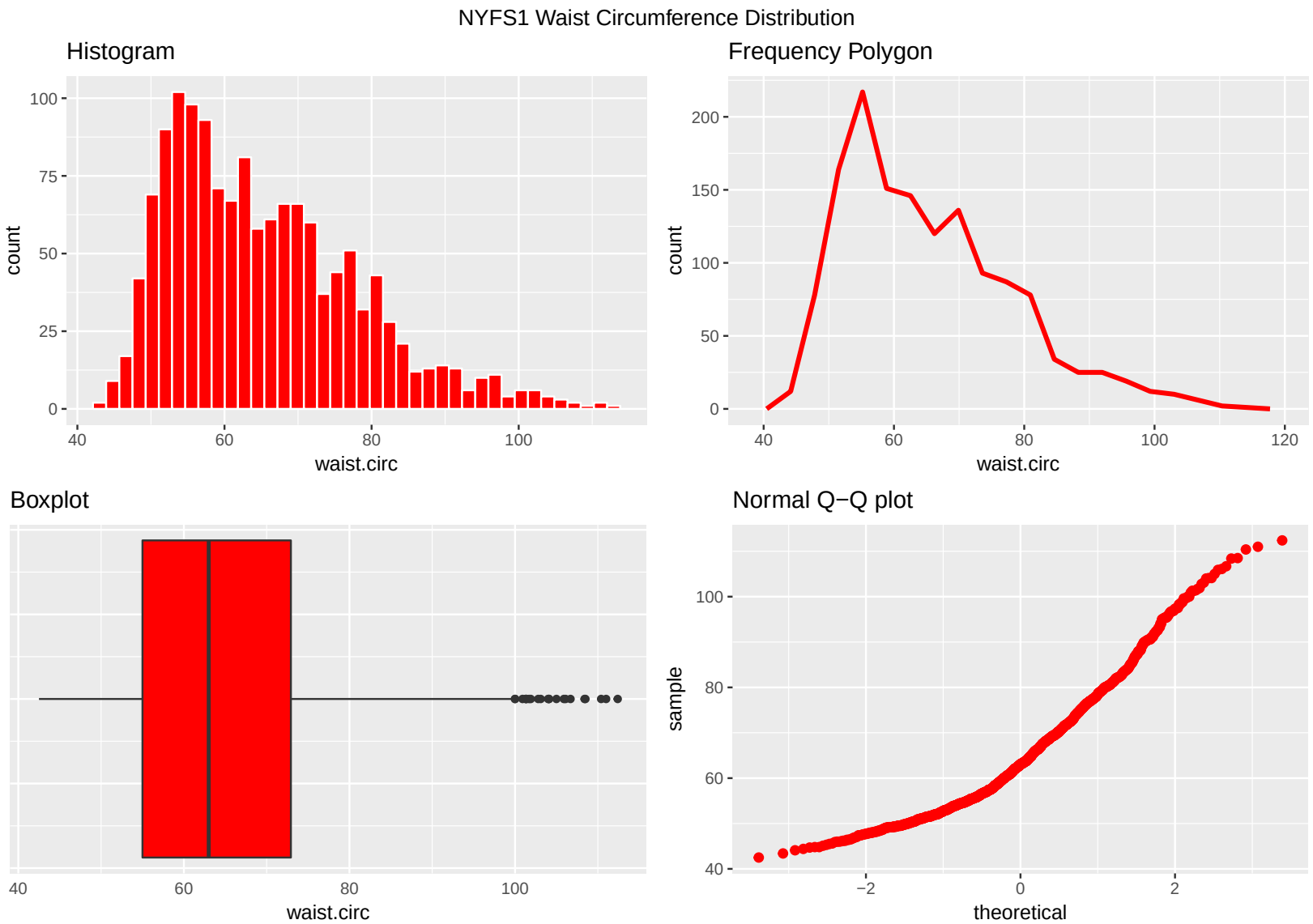
```
str(nyfs1)
```

```
Classes 'tbl_df', 'tbl' and 'data.frame':   1416 obs. of  7 va
 $ subject.id       : int  71918 71919 71921 71922 71923 71924
 $ sex              : Factor w/ 2 levels "Female","Male": 1 1 2
 $ age.exam         : int  8 14 3 12 12 8 7 8 3 9 ...
 $ bmi              : num  22.3 19.8 15.2 25.9 22.5 14.4 15.9 1
 $ bmi.cat          : Factor w/ 4 levels "1 Underweight",..: 4
 $ waist.circ       : num  71.9 79.4 46.8 90 72.3 56.1 54.5 59.
 $ triceps.skinfold: num  19.9 15 8.6 22.8 20.5 12.9 6.9 8.8 1
```

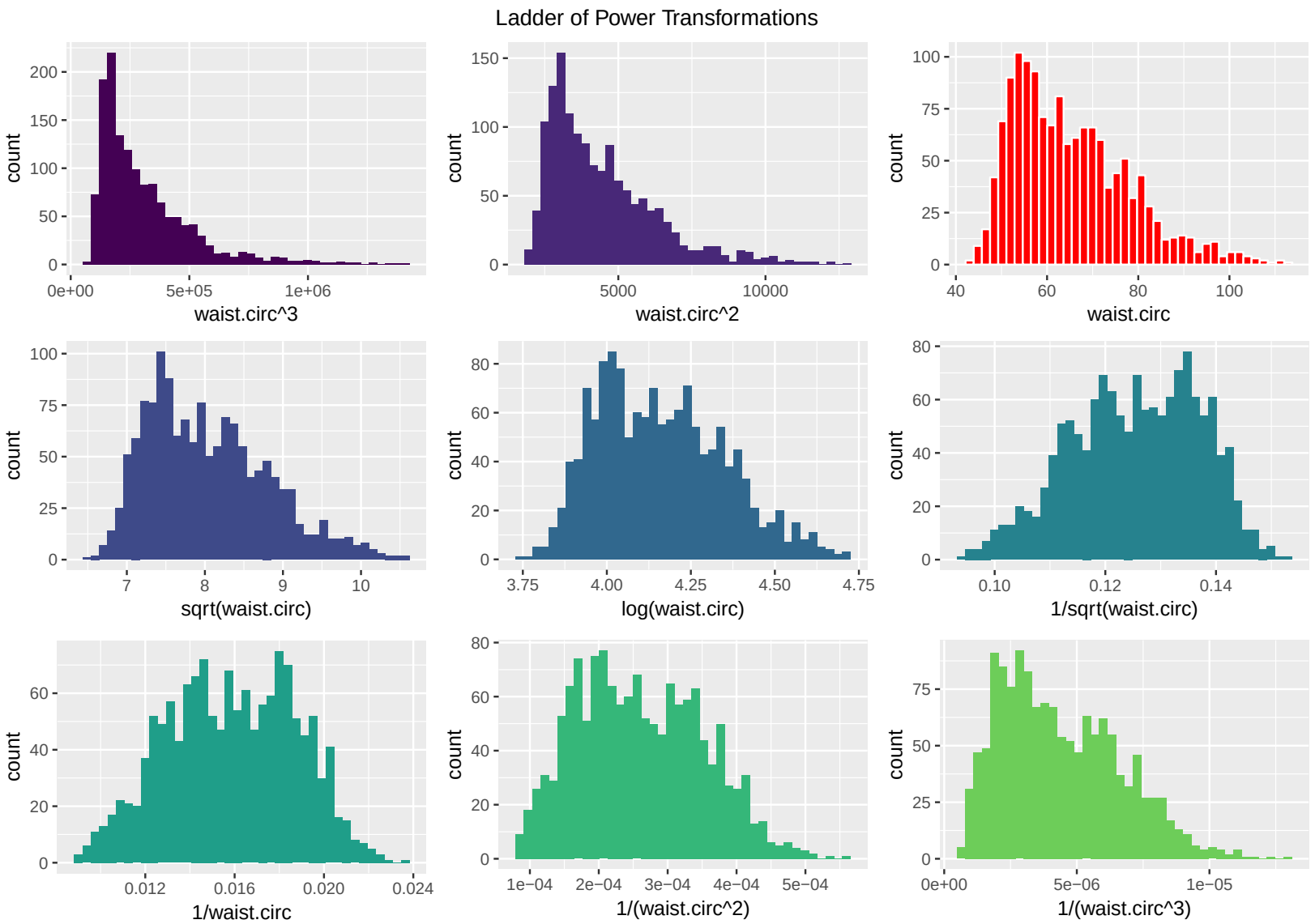
Normal Q-Q Plot of Waist Circumferences



The Waist Circumference Data



Waist Circumference Histograms: Ladder



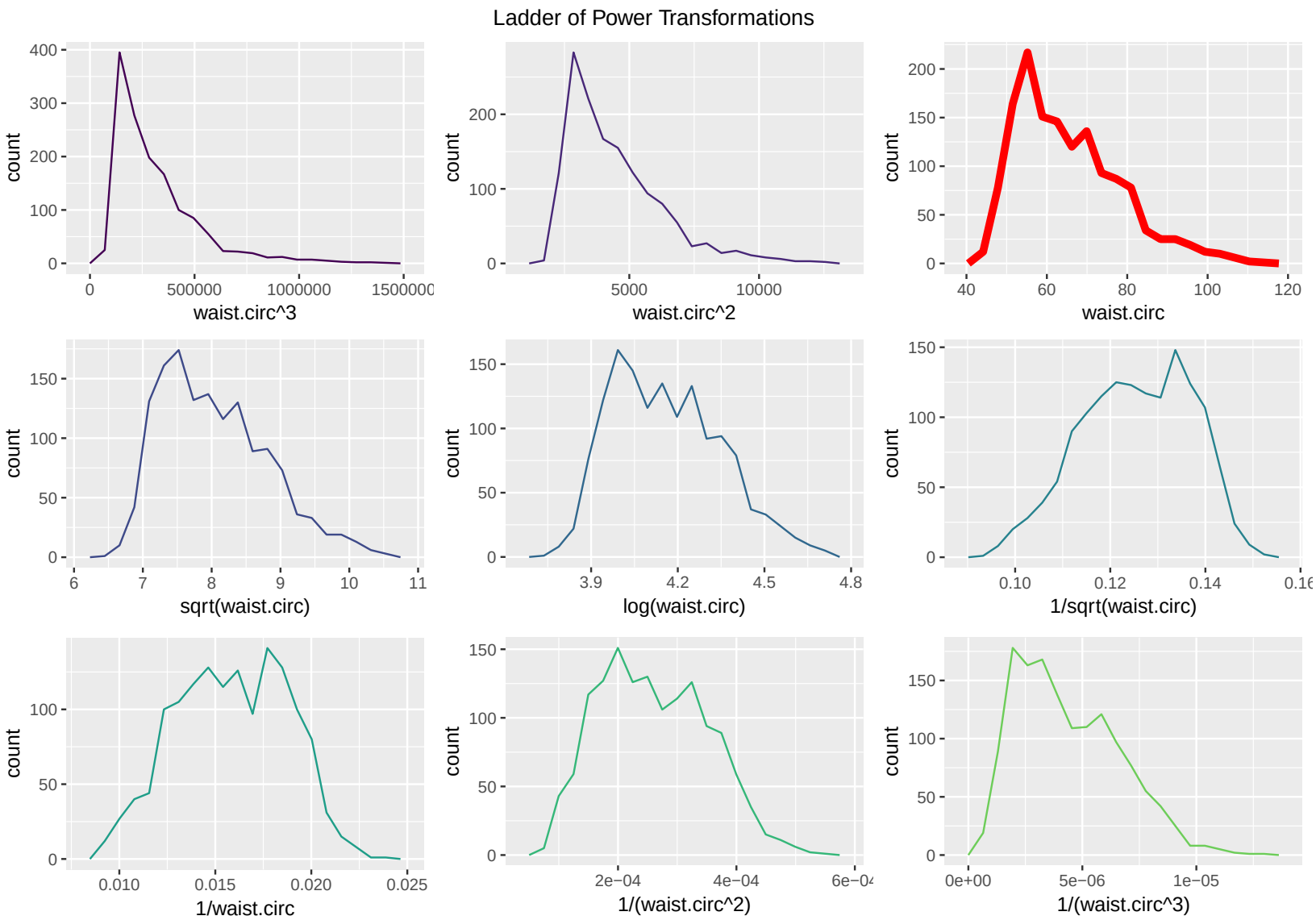
$skew_1$ and Power Transformations

Let's add in the $skew_1$ values we observe for each of the available transformations of the waist circumference data.

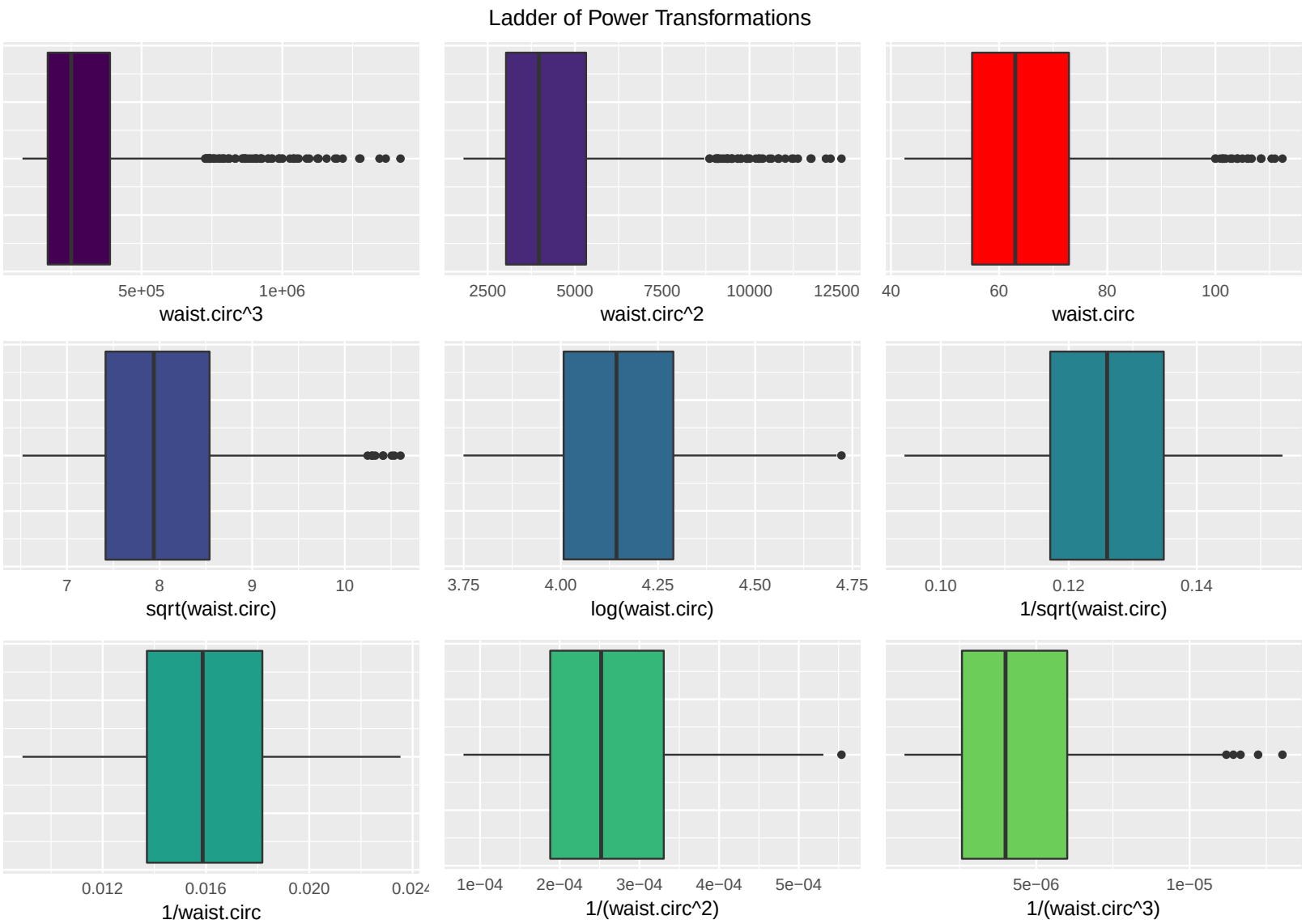
For waist circumference,

Power	Transformation	$skew_1$
3	x^3	0.31
2	x^2	0.25
1	x	0.18
0.5	$\sqrt{x}$	0.14
0	ln x	0.09
-0.5	$1/\sqrt{x}$	-0.05
-1	$1/x$	0
-2	$1/x^2$	0.09
-3	$1/x^3$	0.17

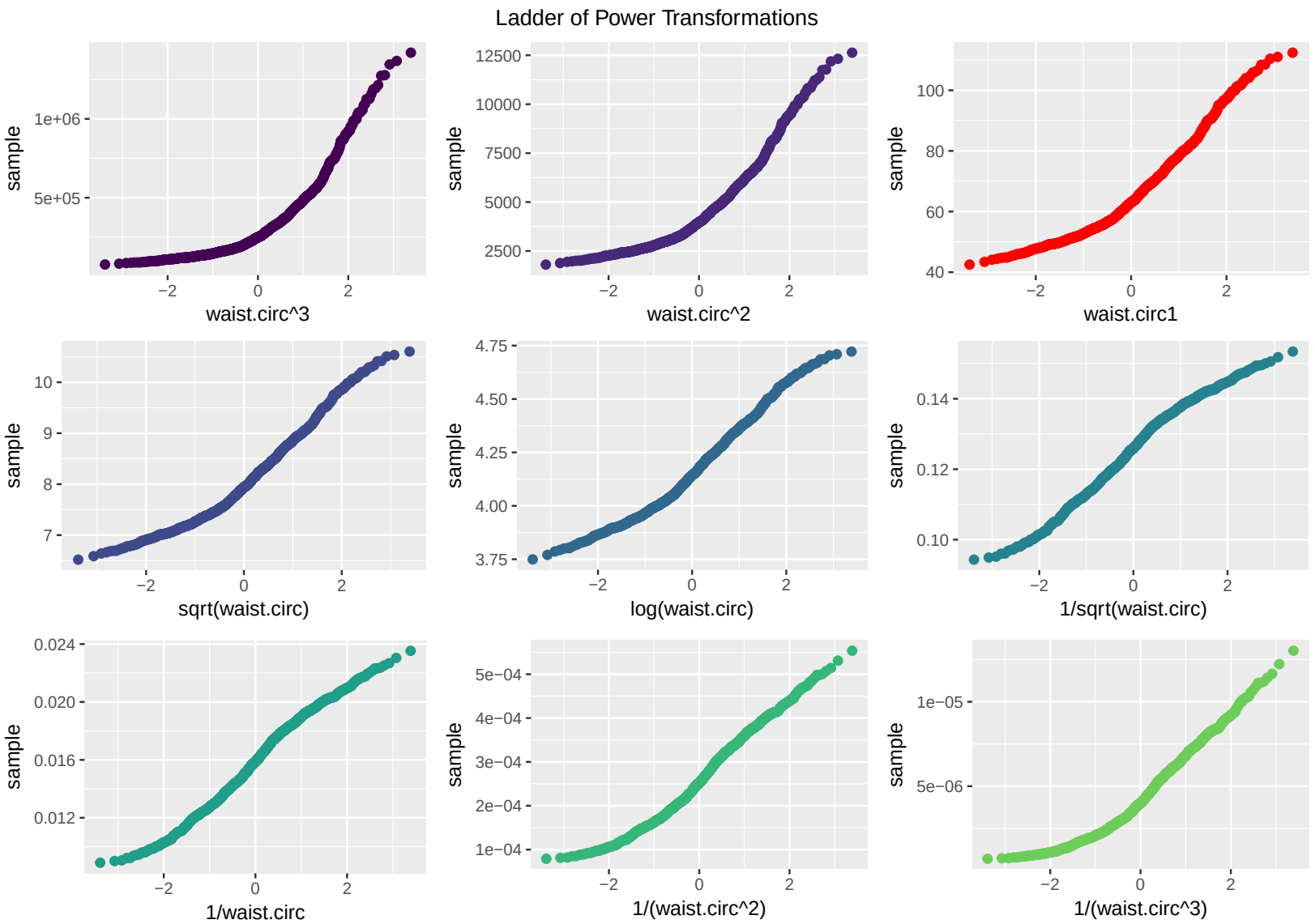
But we should be looking at Frequency Polygons. . .



Waist Circumference Boxplots: Ladder



Waist Circumference Normal Q-Q: Ladder



Can we test to see if our data follow a Normal model?

Yes, but don't. Graphical approaches are far better.

What would such a test look like?

```
shapiro.test(nyfs1$waist.circ)
```

```
    Shapiro-Wilk normality test

data:  nyfs1$waist.circ
W = 0.94391, p-value < 2.2e-16
```

The very small p value indicates that the test finds strong indications **against** adopting a Normal model.

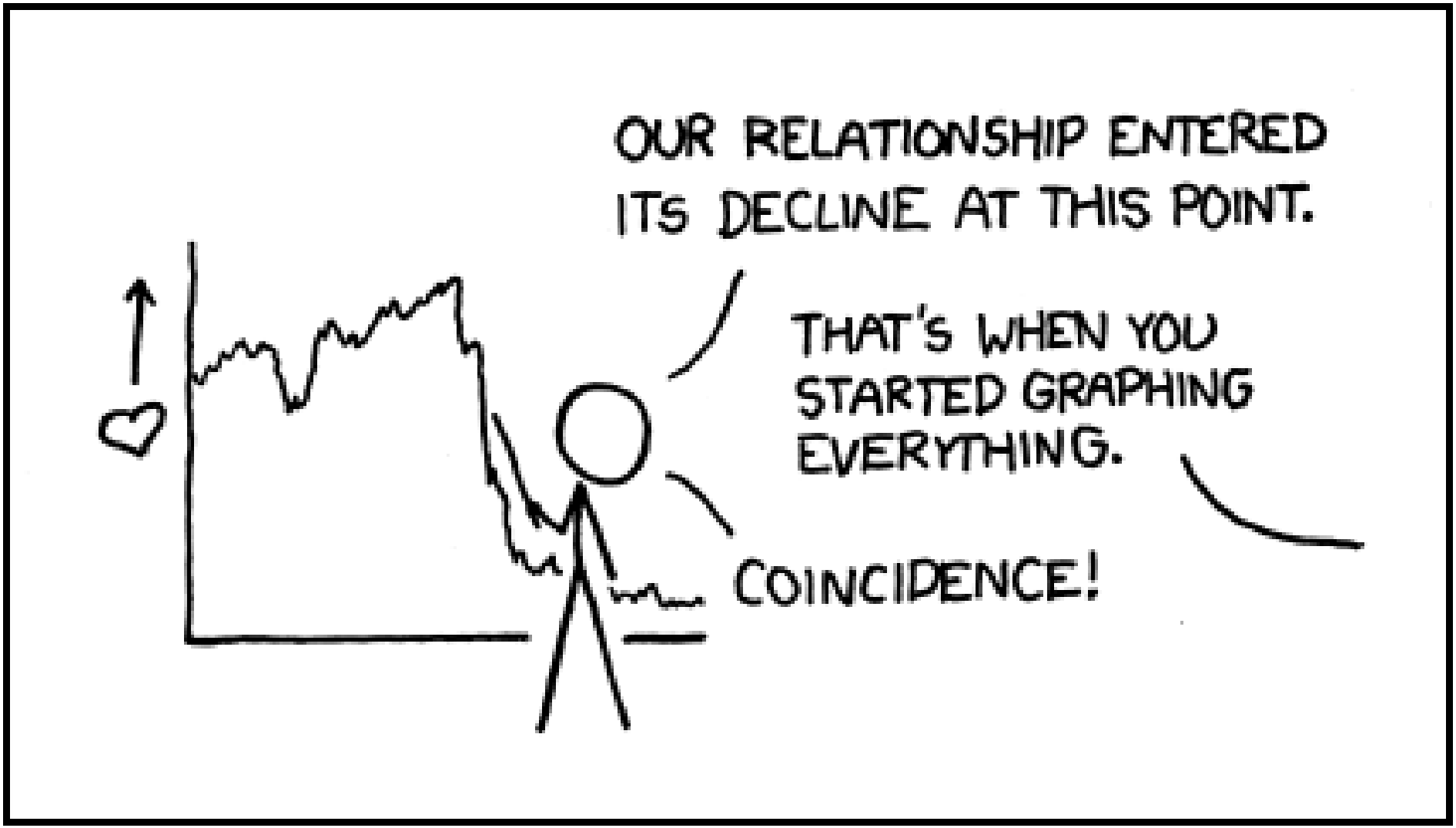
Why not test?

Because the sample size is so large, and the test is so poor at detecting non-normality compared to our eyes, that it finds problems we don't care about, and ignores problems we do care about. For waist circumference. . .

Power	Transformation	Shapiro-Wilk p value
3	x^3	< 2.2 e-16
2	x^2	< 2.2 e-16
1	x	< 2.2 e-16
0.5	$\sqrt{x}$	< 2.2 e-16
0	ln x	1.5 e-14
-0.5	$1/\sqrt{x}$	< 2.2 e-16
-1	$1/x$	1.8 e-09
-2	$1/x^2$	1.4 e-12
-3	$1/x^3$	< 2.2 e-16

DON'T DO THIS - instead, graph everything!

From XKCD


OUR RELATIONSHIP ENTERED ITS DECLINE AT THIS POINT.
THAT'S WHEN YOU STARTED GRAPHING EVERYTHING.
COINCIDENCE!

Which states are in the Midwest?

The Midwest Survey (prior to 431 Class 7)

Please complete the survey to the best of your ability. A quick response is desirable. Thank you! Dr. Love

* Required

I have a strong opinion about which U.S. states are part of the Midwest. *

Please indicate the degree to which you agree with the statement above.

	1	2	3	4	5	
Strongly Disagree	○	○	○	○	○	Strongly Agree

Map of the United States

For each of the 25 states listed below, please indicate whether or not you consider it to be part of the Midwest. *

	Yes, in Midwest	No, not in Midwest.
Missouri (MO)	☐	☐
California (CA)	☐	☐
Colorado (CO)	☐	☐
Arkansas (AR)	☐	☐
Pennsylvania (PA)	☐	☐
Wisconsin (WI)	☐	☐
Iowa (IA)	☐	☐
Indiana (IN)	☐	☐
Ohio (OH)	☐	☐

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APR. 29, 2014 AT 9:26 AM

Which States Are in the Midwest?

By Walt Hickey

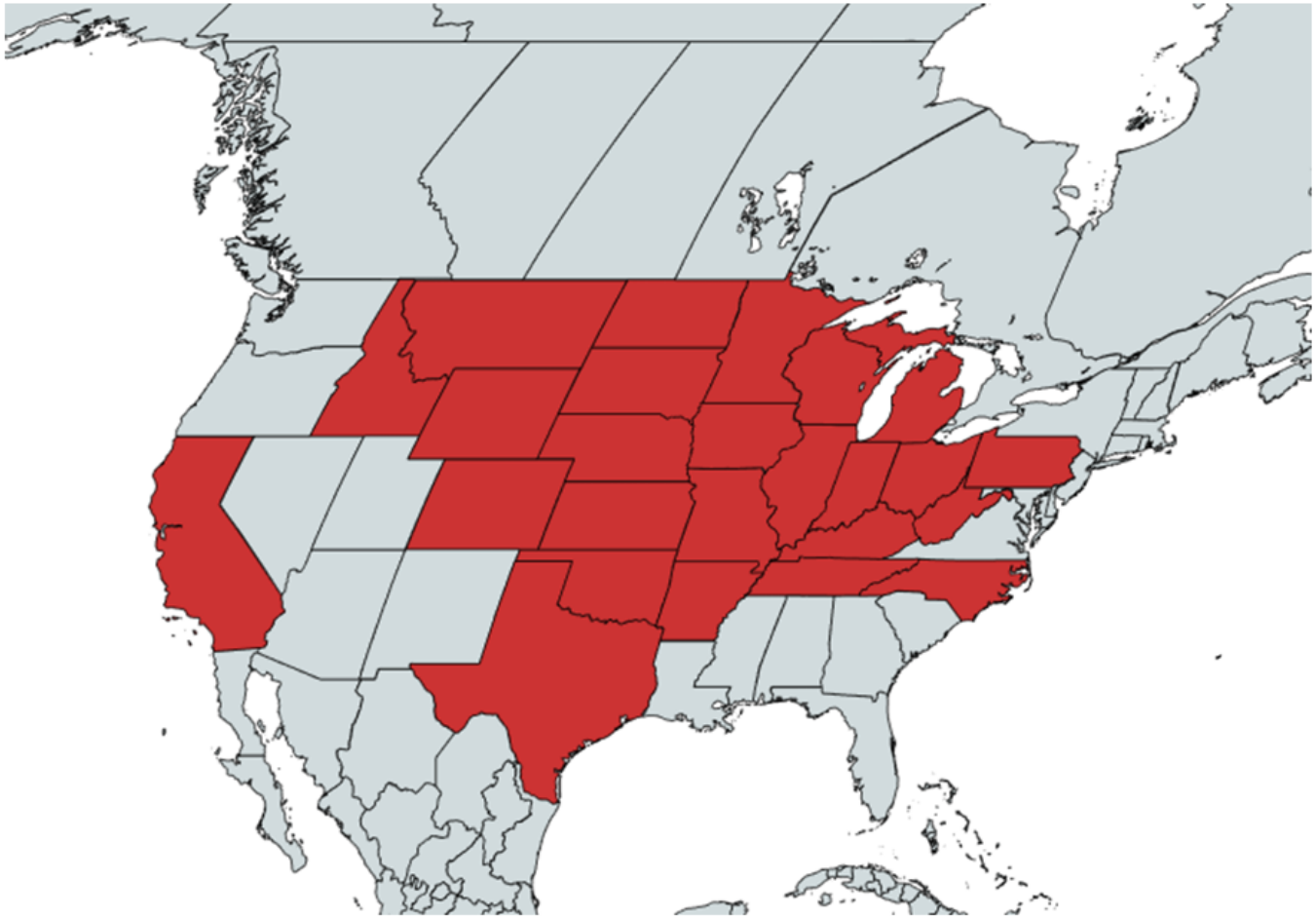
Filed under Regionalism

Get the data on GitHub

Which states are in the Midwest?

25 states I asked about are shown in red...

1. Arkansas (AR)
2. California (CA)
3. Colorado (CO)
4. Idaho (ID)
5. Illinois (IL)
6. Indiana (IN)
7. Iowa (IA)
8. Kansas (KS)
9. Kentucky (KY)
10. Michigan (MI)
11. Minnesota (MN)
12. Missouri (MO)
13. Montana (MT)
14. Nebraska (NE)
15. North Carolina (NC)
16. North Dakota (ND)
17. Ohio (OH)
18. Oklahoma (OK)
19. Pennsylvania (PA)
20. South Dakota (SD)
21. Tennessee (TN)
22. Texas (TX)
23. West Virginia (WV)
24. Wisconsin (WI)
25. Wyoming (WY)



Which states are in the Midwest?

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APR. 29, 2014 AT 9:26 AM

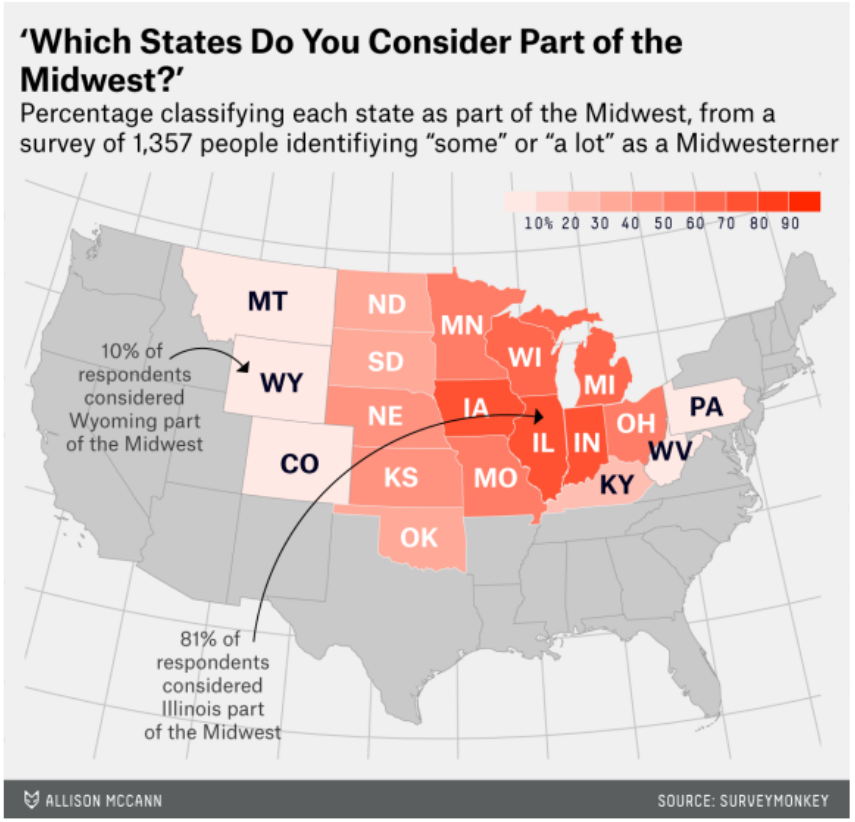
Which States Are in the Midwest?

By Walt Hickey

Filed under Regionalism

Get the data on GitHub

To get this broad-based view, we asked SurveyMonkey Audience to ask self-identified Midwesterners which states make the cut. We ran a national survey that targeted the Midwest from March 12 to March 17, with 2,778 respondents. Of those, 1,357 respondents identified "a lot" or "some" as a Midwesterner. We then asked this group to identify the states they consider part of the Midwest.



https://fivethirtyeight.com/datalab/what-states-are-in-the-midwest/

Some Additional Numerical Summaries

```
psych::describe(nyfs1$waist.circ)
```

```
   vars    n  mean    sd median trimmed   mad  min
X1    1 1416 65.29 12.85     63   64.05 12.75 42.5
     max range skew kurtosis   se
X1 112.4  69.9 0.85     0.38 0.34
```

The Trimmed Mean

The **trimmed mean** is specified by indicating the proportion of observations to be trimmed from each end of the outcome distribution before the mean is calculated. So here's how we get the mean of the middle 80% of the data.

```
mean(nyfs1$waist.circ, trim=.1)
```

```
[1] 64.04956
```

The Standard Error of the Sample Mean

The **standard error** of the sample mean is the standard deviation divided by the square root of the sample size.

```
sd(nyfs1$waist.circ)/sqrt(length(nyfs1$waist.circ))
```

```
[1] 0.3415844
```

The Median Absolute Deviation

An alternative to the IQR that is fancier, and a bit more robust, is the **median absolute deviation**, which, in large sample sizes, for data that follow a Normal distribution, will be (in expectation) equal to the standard deviation.

```
mad(nyfs1$waist.circ)
```

```
[1] 12.75036
```

```
sd(nyfs1$waist.circ)
```

```
[1] 12.85375
```

Summarizing within Groups using by (section 10)

```
by(nyfs1$waist.circ, nyfs1$bmi.cat, summary)
```

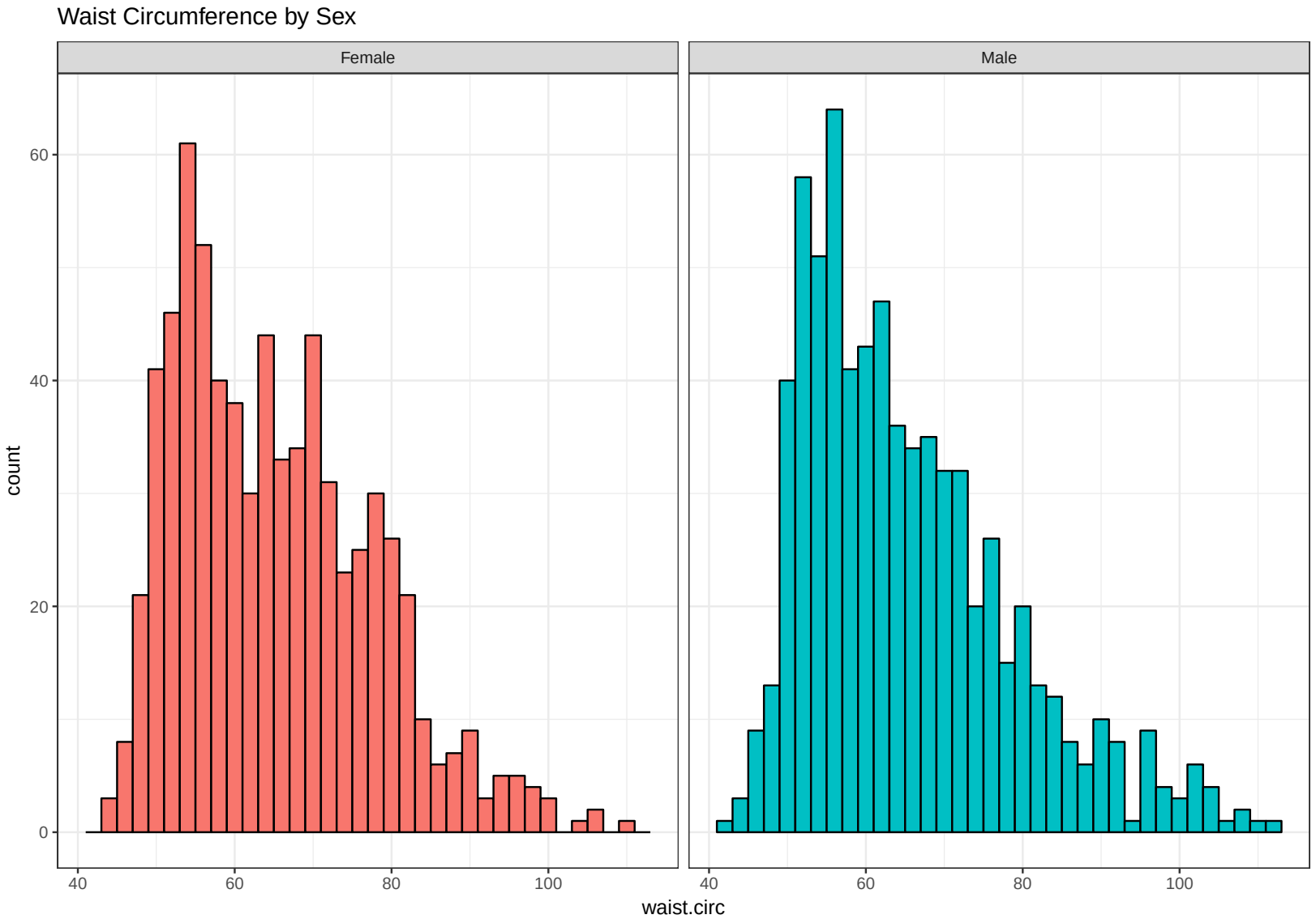
```
nyfs1$bmi.cat: 1 Underweight
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  42.50   49.23   53.90   54.94   62.38   68.50
------------------------------------------------
nyfs1$bmi.cat: 2 Normal weight
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  44.10   53.80   59.20   60.96   68.00   85.50
------------------------------------------------
nyfs1$bmi.cat: 3 Overweight
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  49.30   60.80   72.00   71.11   80.60   98.30
------------------------------------------------
nyfs1$bmi.cat: 4 Obese
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  52.10   66.65   79.90   79.85   91.60  113.40
```

Summarizing within Groups using group_by

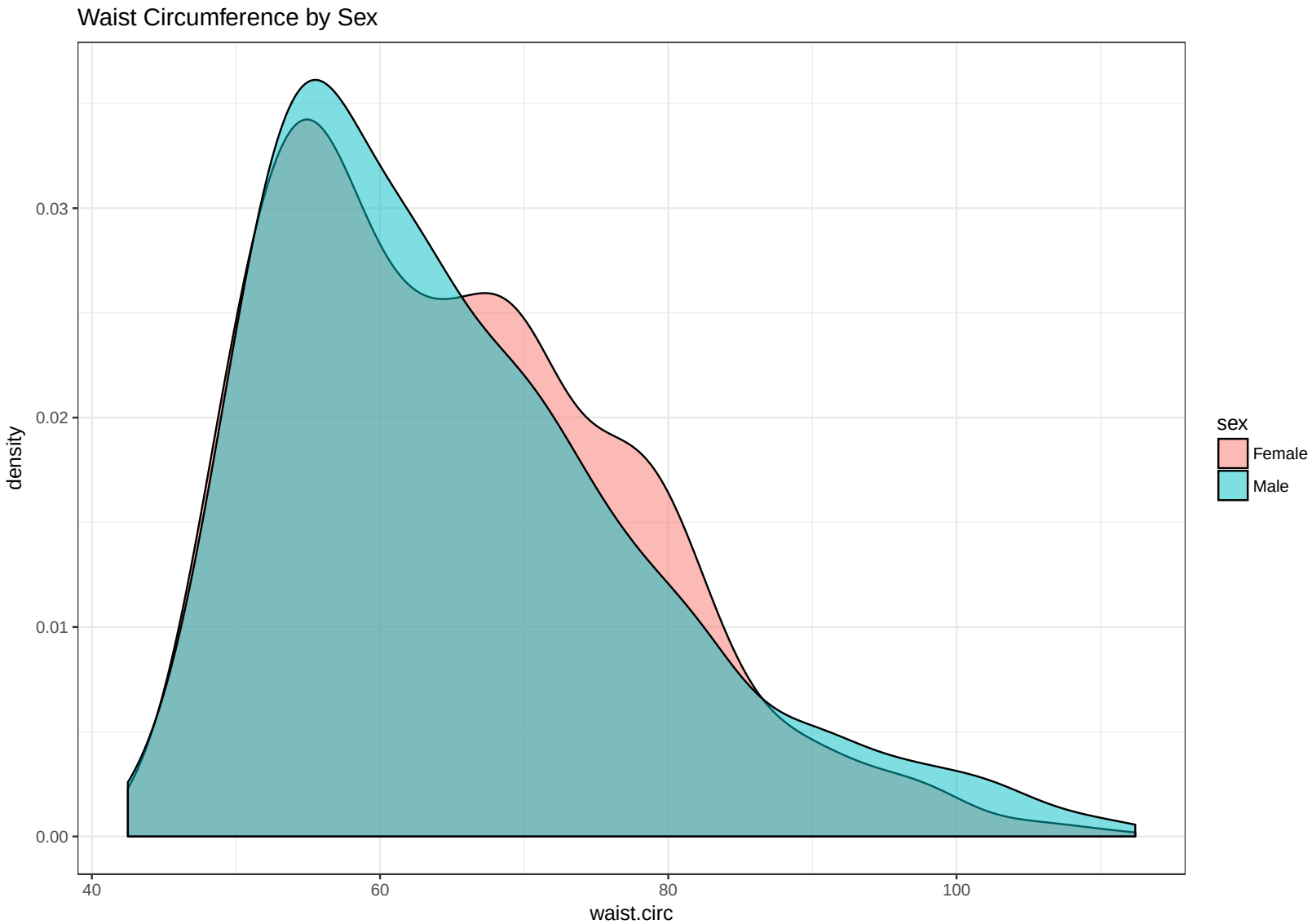
```
tempdat <- nyfs1 %>%
  group_by(bmi.cat) %>%
  summarize(mean = round(mean(waist.circ),2),
            Q50 = median(waist.circ),
            sd = round(sd(waist.circ),2),
            skew1 = round(skew1(waist.circ),2))
knitr::kable(tempdat)
```

bmi.cat	mean	Q50	sd	skew1
1 Underweight	54.94	53.9	7.63	0.14
2 Normal weight	60.96	59.2	9.10	0.19
3 Overweight	71.11	72.0	11.80	-0.08
4 Obese	79.85	79.9	15.01	0.00

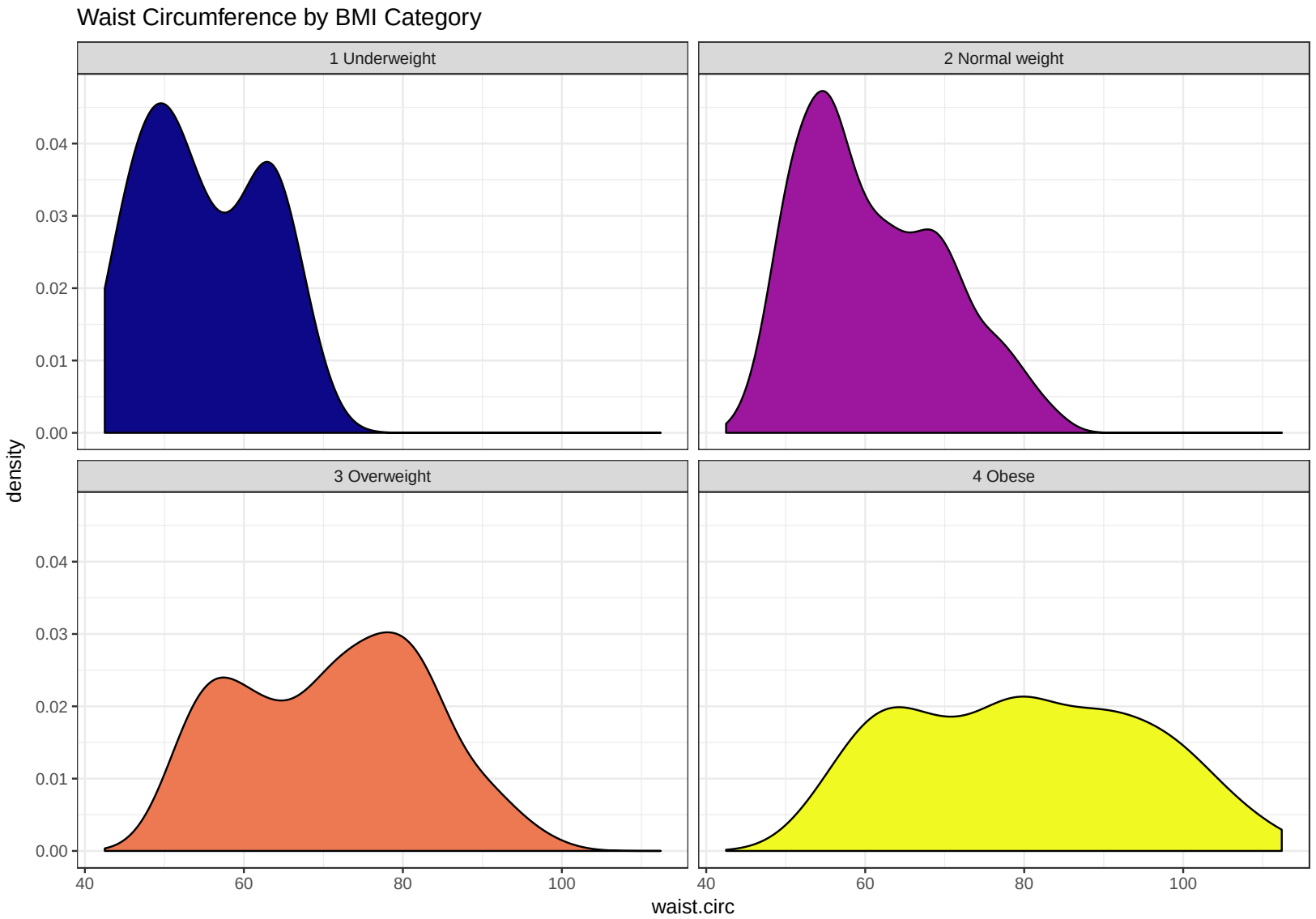
Comparing Histograms with Facetted Plots



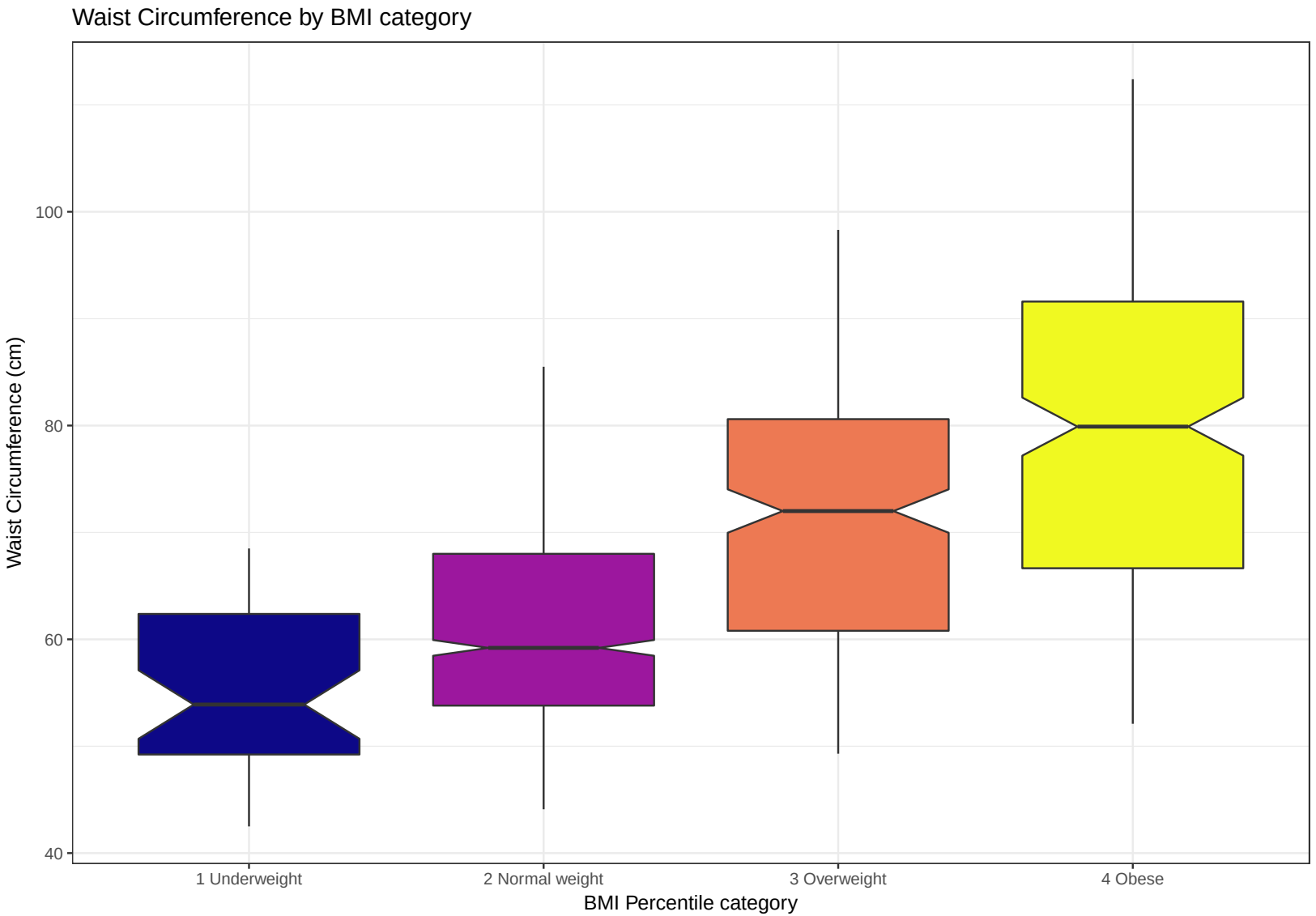
Density Plots, Overlapping



Density Plots, Facetted



Comparison Boxplots with Notches



Link to today's Google Form

You'll find the link at today's README (Class 7).

Please submit the form by 11 AM Thursday.