Test functions for optimization

In applied mathematics, **test functions**, known as **artificial landscapes**, are useful to evaluate characteristics of optimization algorithms, such as:

- Convergence rate.
- Precision.
- Robustness.
- General performance.

Here some test functions are presented with the aim of giving an idea about the different situations that optimization algorithms have to face when coping with these kinds of problems. In the first part, some objective functions for single-objective optimization cases are presented. In the second part, test functions with their respective Pareto fronts for multi-objective optimization problems (MOP) are given.

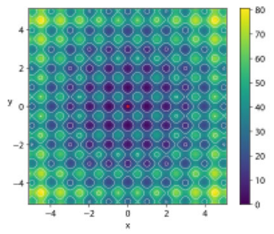
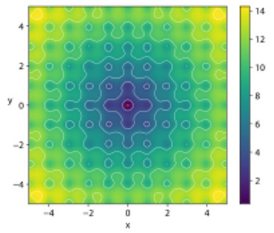
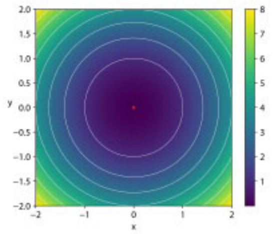
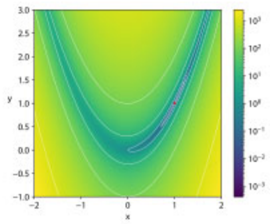
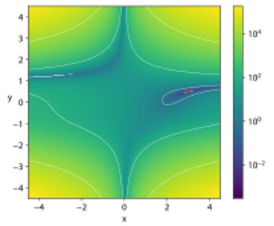
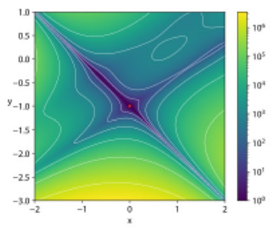
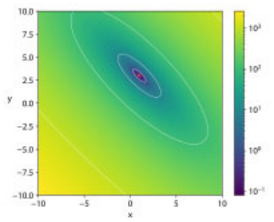
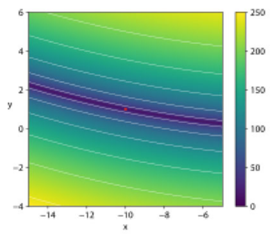
The artificial landscapes presented herein for single-objective optimization problems are taken from Bäck,[1] Haupt et al.[2] and from Rody Oldenhuis software.[3] Given the number of problems (55 in total), just a few are presented here.

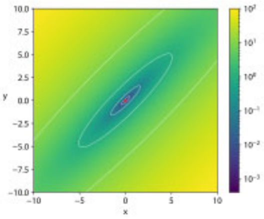
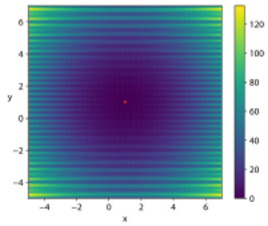
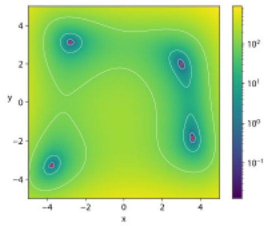
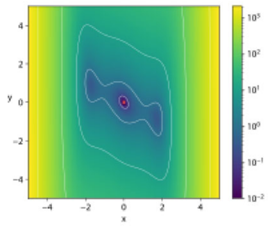
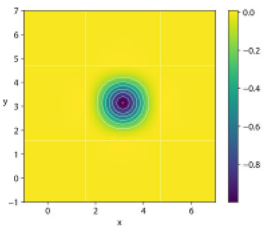
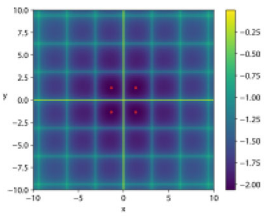
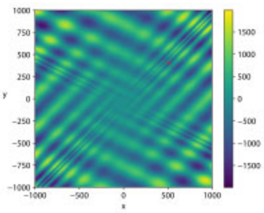
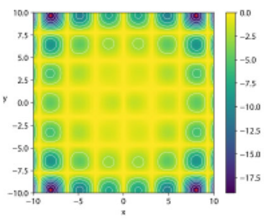
The test functions used to evaluate the algorithms for MOP were taken from Deb,[4] Binh et al.[5] and Binh.[6] The software developed by Deb can be downloaded,[7] which implements the NSGA-II procedure with GAs, or the program posted on Internet,[8] which implements the NSGA-II procedure with ES.

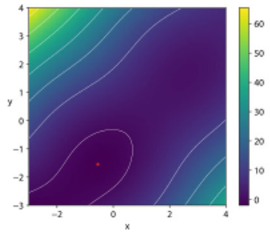
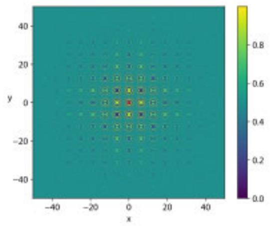
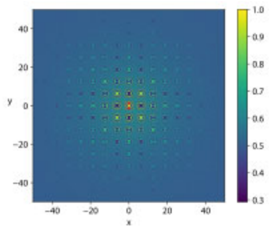
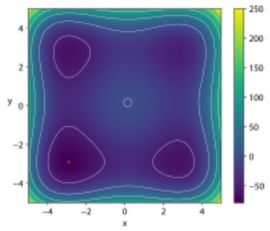
Just a general form of the equation, a plot of the objective function, boundaries of the object variables and the coordinates of global minima are given herein.

Contents

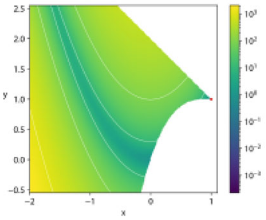
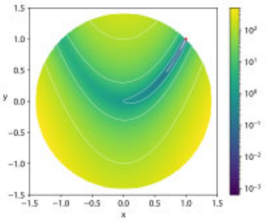
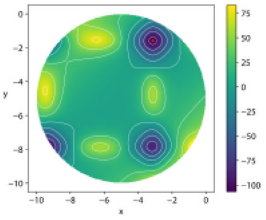
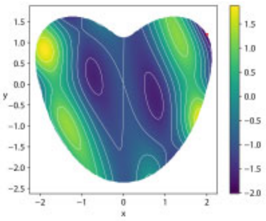
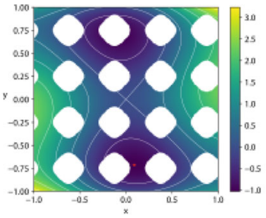
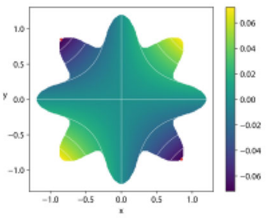
Test functions for single-objective optimization

Name	Plot	Formula	Global minimum
Rastrigin function		$$f(\mathbf{x}) = An + \sum_{i=1}^{n} \left[x_i^2 - A\cos(2\pi x_i)\right]$$ where: $A = 10$	$f(0,\dots,0) = 0$
Ackley function		$$f(x,y) = -20\exp\left[-0.2\sqrt{0.5\left(x^2+y^2\right)}\right]$$ $-\exp\left[0.5\left(\cos 2\pi x + \cos 2\pi y\right)\right] + e + 20$	$f(0,0) = 0$
Sphere function		$$f(\boldsymbol{x}) = \sum_{i=1}^{n} x_i^2$$	$f(x_1,\dots,x_n) = f(0,\dots,0)$
Rosenbrock function		$$f(\boldsymbol{x}) = \sum_{i=1}^{n-1} \left[100\left(x_{i+1} - x_i^2\right)^2 + \left(1 - x_i\right)^2\right]$$	$\text{Min} = \begin{cases} n = 2 & \rightarrow \quad f(1,1) = \\ n = 3 & \rightarrow \quad f(1,1,1) \\ n > 3 & \rightarrow \quad f(\underbrace{1,\dots,}_{n \text{ times}} \end{cases}$
Beale function		$$f(x,y) = \left(1.5 - x + xy\right)^2 + \left(2.25 - x + xy^2\right)^2$$ $+\left(2.625 - x + xy^3\right)^2$	$f(3,0.5) = 0$
Goldstein–Price function		$$f(x,y) = \left[1 + (x+y+1)^2\left(19 - 14x + 3x^2 - 14y + 6xy + 3y^2\right)\right]$$ $\left[30 + (2x-3y)^2\left(18 - 32x + 12x^2 + 48y - 36xy + 27y^2\right)\right]$	$f(0,-1) = 3$
Booth function		$$f(x,y) = (x + 2y - 7)^2 + (2x + y - 5)^2$$	$f(1,3) = 0$
Bukin function N.6		$$f(x,y) = 100\sqrt{\left\lvert y - 0.01x^2\right\rvert} + 0.01\left\lvert x + 10\right\rvert.$$	$f(-10,1) = 0$
Matyas function		$$f(x,y) = 0.26\left(x^2 + y^2\right) - 0.48xy$$	$f(0,0) = 0$

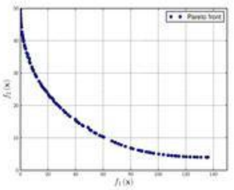
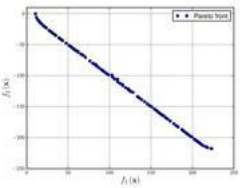
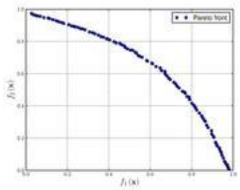
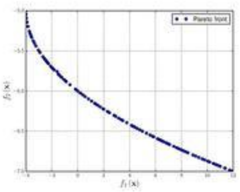
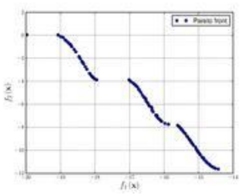
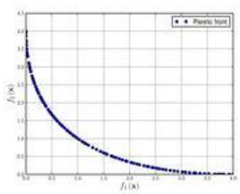
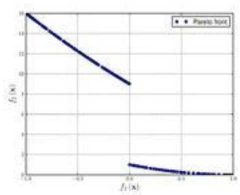
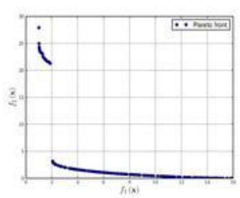
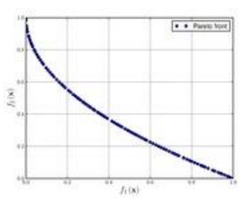
			
Lévi function N.13		$f(x,y) = \sin^2 3\pi x + (x-1)^2 \left(1 + \sin^2 3\pi y\right)$ $+(y-1)^2 \left(1 + \sin^2 2\pi y\right)$	$f(1,1) = 0$
Himmelblau's function		$f(x,y) = (x^2 + y - 11)^2 + (x + y^2 - 7)^2.$	$\text{Min} = \begin{cases} f(3.0, 2.0) \\ f(-2.805118, 3.131312) \\ f(-3.779310, -3.28318 \\ f(3.584428, -1.848126) \end{cases}$
Three-hump camel function		$f(x,y) = 2x^2 - 1.05x^4 + \frac{x^6}{6} + xy + y^2$	$f(0,0) = 0$
Easom function		$f(x,y) = -\cos(x)\cos(y)\exp\left(-\left((x-\pi)^2 + (y-\pi)^2\right)\right)$	$f(\pi,\pi) = -1$
Cross-in-tray function		$f(x,y) = -0.0001\left[\left\lvert\sin x \sin y \exp\left(\left\lvert 100 - \frac{\sqrt{x^2+y^2}}{\pi}\right\rvert\right)\right\rvert + 1\right]^{0.1}$	$\text{Min} = \begin{cases} f(1.34941, -1.34941) & = \\ f(1.34941, 1.34941) & = \\ f(-1.34941, 1.34941) & = \\ f(-1.34941, -1.34941) & = \end{cases}$
Eggholder function[9]		$f(x,y) = -(y+47)\sin\sqrt{\left\lvert\frac{x}{2} + (y+47)\right\rvert} - x\sin\sqrt{\lvert x - (y+47)\rvert}$	$f(512, 404.2319) = -959.64$
Hölder table function		$f(x,y) = -\left\lvert\sin x \cos y \exp\left(\left\lvert 1 - \frac{\sqrt{x^2+y^2}}{\pi}\right\rvert\right)\right\rvert$	$\text{Min} = \begin{cases} f(8.05502, 9.66459) & = \\ f(-8.05502, 9.66459) & = \\ f(8.05502, -9.66459) & = \\ f(-8.05502, -9.66459) & = \end{cases}$
McCormick function		$f(x,y) = \sin(x+y) + (x-y)^2 - 1.5x + 2.5y + 1$	$f(-0.54719, -1.54719) = -1.$

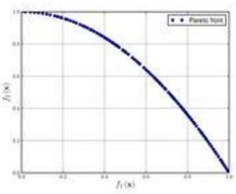
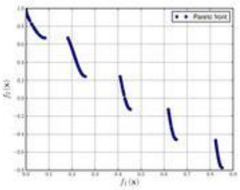
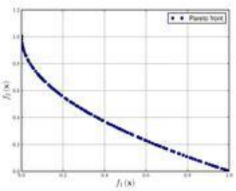
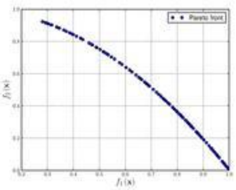
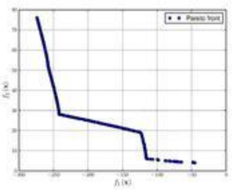
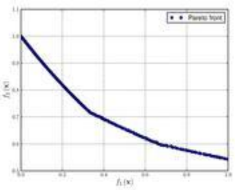
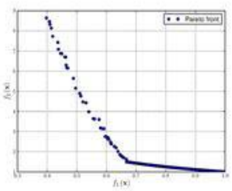
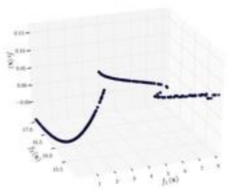
			
Schaffer function N. 2		$f(x,y) = 0.5 + \frac{\sin^2\left(x^2 - y^2\right) - 0.5}{\left[1 + 0.001\left(x^2 + y^2\right)\right]^2}$	$f(0,0) = 0$
Schaffer function N. 4		$f(x,y) = 0.5 + \frac{\cos^2\left[\sin\left(\lvert x^2 - y^2\rvert\right)\right] - 0.5}{\left[1 + 0.001\left(x^2 + y^2\right)\right]^2}$	$\text{Min} = \begin{cases} f\left(0, 1.25313\right) & = 0. \\ f\left(0, -1.25313\right) & = 0. \\ f\left(1.25313, 0\right) & = 0. \\ f\left(-1.25313, 0\right) & = 0. \end{cases}$
Styblinski–Tang function		$f(\boldsymbol{x}) = \frac{\sum_{i=1}^{n} x_i^4 - 16x_i^2 + 5x_i}{2}$	$-39.16617n < f(\underbrace{-2.903534, \ldots, -2.90353}_{n \text{ times}}$

Test functions for constrained optimization

Name	Plot	Formula	Global minimum
Rosenbrock function constrained with a cubic and a line[10]		$f(x,y) = (1-x)^2 + 100(y-x^2)^2,$ subjected to: $(x-1)^3 - y + 1 \leq 0$ and $x + y - 2 \leq 0$	$f(1.0, 1.0) = 0$
Rosenbrock function constrained to a disk[11]		$f(x,y) = (1-x)^2 + 100(y-x^2)^2,$ subjected to: $x^2 + y^2 \leq 2$	$f(1.0, 1.0) = 0$
Mishra's Bird function - constrained[12][13]		$f(x,y) = \sin(y)e^{\left[(1-\cos x)^2\right]} + \cos(x)e^{\left[(1-\sin y)^2\right]} + (x-y)^2,$ subjected to: $(x+5)^2 + (y+5)^2 < 25$	$f(-3.1302468, -1.5821422) =$ -
Townsend function (modified)[14]		$f(x,y) = -[\cos((x-0.1)y)]^2 - x\sin(3x+y),$ subjected to: $x^2 + y^2 < \left[2\cos t - \frac{1}{2}\cos 2t - \frac{1}{4}\cos 3t - \frac{1}{8}\cos 4t\right]^2 + [2\sin t]^2$ where: $t = \mathrm{Atan2(x,y)}$	$f(2.0052938, 1.1944509) = -$
Gomez and Levy function (modified)[15]		$f(x,y) = 4x^2 - 2.1x^4 + \frac{1}{3}x^6 + xy - 4y^2 + 4y^4,$ subjected to: $-\sin(4\pi x) + 2\sin^2(2\pi y) \leq 1.5$	$f(0.08984201, -0.7126564) = -$
Simionescu function[16]		$f(x,y) = 0.1xy,$ subjected to: $x^2 + y^2 \leq \left[r_T + r_S\cos\left(n\arctan\frac{x}{y}\right)\right]^2$ where: $r_T = 1, r_S = 0.2$ and $n = 8$	$f(\pm 0.84852813, \mp 0.84852813$

Test functions for multi-objective optimization

Name	Plot	Functions	Constra
Binh and Korn function:[5]		$\text{Minimize} = \begin{cases} f_1(x,y) = 4x^2 + 4y^2 \\ f_2(x,y) = (x-5)^2 + (y-5)^2 \end{cases}$	$\text{s.t.} = \begin{cases} g_1(x,y) = (x - \\ g_2(x,y) = (x - \end{cases}$
Chankong and Haimes function:[17]		$\text{Minimize} = \begin{cases} f_1(x,y) = 2 + (x-2)^2 + (y-1)^2 \\ f_2(x,y) = 9x - (y-1)^2 \end{cases}$	$\text{s.t.} = \begin{cases} g_1(x,y) = \\ g_2(x,y) = \end{cases}$
Fonseca–Fleming function:[18]		$\text{Minimize} = \begin{cases} f_1(\boldsymbol{x}) = 1 - \exp\left[-\sum_{i=1}^{n}\left(x_i - \frac{1}{\sqrt{n}}\right)^2\right] \\ f_2(\boldsymbol{x}) = 1 - \exp\left[-\sum_{i=1}^{n}\left(x_i + \frac{1}{\sqrt{n}}\right)^2\right] \end{cases}$	
Test function 4:[6]		$\text{Minimize} = \begin{cases} f_1(x,y) = x^2 - y \\ f_2(x,y) = -0.5x - y - 1 \end{cases}$	$\text{s.t.} = \begin{cases} g_1(x,y) = 6 \\ g_2(x,y) = 7 \\ g_3(x,y) = 3 \end{cases}$
Kursawe function:[19]		$\text{Minimize} = \begin{cases} f_1(\boldsymbol{x}) = \sum_{i=1}^{2}\left[-10\exp\left(-0.2\sqrt{x_i^2 + x_{i+1}^2}\right)\right] \\ f_2(\boldsymbol{x}) = \sum_{i=1}^{3}\left[\lvert x_i\rvert^{0.8} + 5\sin\left(x_i^3\right)\right] \end{cases}$	
Schaffer function N. 1:[20]		$\text{Minimize} = \begin{cases} f_1(x) = x^2 \\ f_2(x) = (x-2)^2 \end{cases}$	
Schaffer function N. 2:		$\text{Minimize} = \begin{cases} f_1(x) = \begin{cases} -x, & \text{if } x \le 1 \\ x-2, & \text{if } 1 < x \le 3 \\ 4-x, & \text{if } 3 < x \le 4 \\ x-4, & \text{if } x > 4 \end{cases} \\ f_2(x) = (x-5)^2 \end{cases}$	
Poloni's two objective function:		$\text{Minimize} = \begin{cases} f_1(x,y) = \left[1 + (A_1 - B_1(x,y))^2 + (A_2 - B_2(x,y))^2\right] \\ f_2(x,y) = (x+3)^2 + (y+1)^2 \end{cases}$ $\text{where} = \begin{cases} A_1 = 0.5\sin(1) - 2\cos(1) + \sin(2) - 1.5\cos(2) \\ A_2 = 1.5\sin(1) - \cos(1) + 2\sin(2) - 0.5\cos(2) \\ B_1(x,y) = 0.5\sin(x) - 2\cos(x) + \sin(y) - 1.5\cos(y) \\ B_2(x,y) = 1.5\sin(x) - \cos(x) + 2\sin(y) - 0.5\cos(y) \end{cases}$	
Zitzler–Deb–Thiele's function N. 1:[21]		$\text{Minimize} = \begin{cases} f_1(\boldsymbol{x}) = x_1 \\ f_2(\boldsymbol{x}) = g(\boldsymbol{x})\,h\left(f_1(\boldsymbol{x}), g(\boldsymbol{x})\right) \\ g(\boldsymbol{x}) = 1 + \frac{9}{29}\sum_{i=2}^{30} x_i \\ h\left(f_1(\boldsymbol{x}), g(\boldsymbol{x})\right) = 1 - \sqrt{\frac{f_1(\boldsymbol{x})}{g(\boldsymbol{x})}} \end{cases}$	
Zitzler–Deb–Thiele's			

function N. 2:[21]		$\text{Minimize} = \begin{cases} f_1(\boldsymbol{x}) = x_1 \\ f_2(\boldsymbol{x}) = g(\boldsymbol{x})\, h(f_1(\boldsymbol{x}), g(\boldsymbol{x})) \\ g(\boldsymbol{x}) = 1 + \frac{9}{29}\sum_{i=2}^{30} x_i \\ h(f_1(\boldsymbol{x}), g(\boldsymbol{x})) = 1 - \left(\frac{f_1(\boldsymbol{x})}{g(\boldsymbol{x})}\right)^2 \end{cases}$	
Zitzler–Deb–Thiele's function N. 3:[21]		$\text{Minimize} = \begin{cases} f_1(\boldsymbol{x}) = x_1 \\ f_2(\boldsymbol{x}) = g(\boldsymbol{x})\, h(f_1(\boldsymbol{x}), g(\boldsymbol{x})) \\ g(\boldsymbol{x}) = 1 + \frac{9}{29}\sum_{i=2}^{30} x_i \\ h(f_1(\boldsymbol{x}), g(\boldsymbol{x})) = 1 - \sqrt{\frac{f_1(\boldsymbol{x})}{g(\boldsymbol{x})}} - \left(\frac{f_1(\boldsymbol{x})}{g(\boldsymbol{x})}\right)\sin(10\pi f_1(\boldsymbol{x})) \end{cases}$	
Zitzler–Deb–Thiele's function N. 4:[21]		$\text{Minimize} = \begin{cases} f_1(\boldsymbol{x}) = x_1 \\ f_2(\boldsymbol{x}) = g(\boldsymbol{x})\, h(f_1(\boldsymbol{x}), g(\boldsymbol{x})) \\ g(\boldsymbol{x}) = 91 + \sum_{i=2}^{10}\left(x_i^2 - 10\cos(4\pi x_i)\right) \\ h(f_1(\boldsymbol{x}), g(\boldsymbol{x})) = 1 - \sqrt{\frac{f_1(\boldsymbol{x})}{g(\boldsymbol{x})}} \end{cases}$	
Zitzler–Deb–Thiele's function N. 6:[21]		$\text{Minimize} = \begin{cases} f_1(\boldsymbol{x}) = 1 - \exp(-4x_1)\sin^6(6\pi x_1) \\ f_2(\boldsymbol{x}) = g(\boldsymbol{x})\, h(f_1(\boldsymbol{x}), g(\boldsymbol{x})) \\ g(\boldsymbol{x}) = 1 + 9\left[\frac{\sum_{i=2}^{10} x_i}{9}\right]^{0.25} \\ h(f_1(\boldsymbol{x}), g(\boldsymbol{x})) = 1 - \left(\frac{f_1(\boldsymbol{x})}{g(\boldsymbol{x})}\right)^2 \end{cases}$	
Osyczka and Kundu function:[22]		$\text{Minimize} = \begin{cases} f_1(\boldsymbol{x}) = -25(x_1-2)^2 - (x_2-2)^2 - (x_3-1)^2 - (x_4-4)^2 - (x_5-1)^2 \\ f_2(\boldsymbol{x}) = \sum_{i=1}^{6} x_i^2 \end{cases}$	$\text{s.t.} = \begin{cases} g_1(\boldsymbol{x}) = x_1 + \\ g_2(\boldsymbol{x}) = 6 - \\ g_3(\boldsymbol{x}) = 2 - \\ g_4(\boldsymbol{x}) = 2 - \\ g_5(\boldsymbol{x}) = 4 - \\ g_6(\boldsymbol{x}) = (x_5 \end{cases}$
CTP1 function (2 variables):[4][23]		$\text{Minimize} = \begin{cases} f_1(x,y) = x \\ f_2(x,y) = (1+y)\exp\left(-\frac{x}{1+y}\right) \end{cases}$	$\text{s.t.} = \begin{cases} g_1(x,y) = \frac{}{0.85} \\ g_2(x,y) = \frac{}{0.72} \end{cases}$
Constr-Ex problem:[4]		$\text{Minimize} = \begin{cases} f_1(x,y) = x \\ f_2(x,y) = \frac{1+y}{x} \end{cases}$	$\text{s.t.} = \begin{cases} g_1(x,y) = \\ g_2(x,y) = \end{cases}$
Viennet function:		$\text{Minimize} = \begin{cases} f_1(x,y) = 0.5\left(x^2+y^2\right) + \sin\left(x^2+y^2\right) \\ f_2(x,y) = \frac{(3x-2y+4)^2}{8} + \frac{(x-y+1)^2}{27} + 15 \\ f_3(x,y) = \frac{1}{x^2+y^2+1} - 1.1\exp\left(-\left(x^2+y^2\right)\right) \end{cases}$	

See also

- Ackley function
- Himmelblau's function
- Rastrigin function
- Rosenbrock function
- Shekel function

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