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International Journal of Forecasting 20 (2004) 697–711
international journal of forecasting
www.elsevier.com/locate/ijforecast

The value of statistical forecasts in the UK association football betting market

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Abstract

In this paper, we evaluate the economic significance of statistical forecasts of UK Association Football match outcomes in relation to betting market prices. We present a detailed comparison of odds set by different bookmakers in relation to forecast model predictions, and analyse the potential for arbitrage across firms. We also examine extreme odds biases. A detailed re-examination of match result odds and a new examination of correct score odds for the period 1993 to 1996 suggest that the market is inefficient.

Keywords: Statistical forecast; Score odd; Firm

1. Introduction

In this paper, we analyse the fixed odds betting market for UK Association Football (soccer) matches. We extend the analysis of Pope and Peel (1988), who examine the bias of odds and the possibility of arbitrage across different bookmakers, based on data for the early 1980s. Our main focus is on assessing the economic value of forecasts derived from the statistical model of Dixon and Coles (1997), and thus on testing the efficiency of the market with respect to that model. We analyse the value of predictions of match outcomes, extending Dixon and Coles. We also examine forecasts of exact scores in relation to the odds offered on the exact scores of each match, e.g. odds of 18–1 offered against a final score of 2–0. As far as we are aware, there are no published analyses of the correct score betting market. In addition, we investigate inter-bookmaker arbitrage possibilities and extreme odds biases.

For both the match outcome and correct score odds, we find evidence of *reverse* long-shot bias, as documented by Woodland and Woodland (1994) for the baseball fixed odds betting market. However, we also report evidence of informational inefficiencies over and above those due to simple long-shot bias. It appears possible to use predictions from a forecasting model based on prior public information to identify bets having better-than-fair odds.

The rest of the paper is organised as follows: in Section 2 we summarise the main characteristics of the forecasting model used as the benchmark against which to test the odds; in Section 3 we describe the

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doi:10.1016/j.ijforecast.2003.12.007

data sources and provide descriptive statistics; in Section 4 we present our results; and in Section 5 we conclude.

2. Forecasting models

2.1. The basic model

We examine the forecasts from the Dixon and Coles (1997) forecasting model of football match outcomes. This model accounts for the abilities of both teams in a match, ability being proxied by summary measures of recent performance. Two dimensions of ability are recognised: ability to attack (score goals) and ability to defend (not concede goals). In addition, *ceteris paribus* teams playing at home appear to have an advantage—the so-called 'home-effect'.

The forecasting model is based on Maher (1982), with modifications to enable the inclusion of non-complete data sets, data from different divisions, and to allow for dynamic changes in a team's performance. The number of goals scored by the home and away teams are independent Poisson variables, with means determined by the respective attack and defence abilities of each side. (Other distributions have also been considered, see Dixon and Coles (1997) for a discussion.) More explicitly, in a match between teams indexed i and j, if $X_{i,j}$ and $Y_{i,j}$ are the number of goals scored by the home and away sides, respectively, then

$$X_{i,j} \sim \mathrm{Poisson}(\alpha_i\beta_j\gamma) \;\; Y_{i,j} \sim \mathrm{Poisson}(\alpha_j\beta_i), \tag{1}$$

where $X_{i,j}$ and $Y_{i,j}$ are independent and α_i, $\beta_i>0\forall i$. The parameters α_i and β_i measure the 'attack' rate and 'defence' rate of team i, respectively, and $\gamma>0$ is a parameter which allows for the home effect. For the English League system, which comprises the Premier League and Divisions 1 to 3 of the Football League, $n=92$, and therefore the model has $185-1=184$ identifiable parameters, with 2 being common across teams (after restricting the parameter space appropriately, see Dixon & Coles (1997)).

With matches indexed $k=1, \ldots, N$, and corresponding scores (x_k, y_k), the likelihood is

$$L(\alpha_i,\beta_i,\gamma;\ i=1,\ldots,n) = \prod_{k=1}^{N} e^{-\lambda_k}\lambda_k^{x_k}e^{-\mu_k}\mu_k^{y_k} \tag{2}$$

where,

$$\lambda_k = \alpha_{i(k)}\beta_{j(k)}\gamma;\ \ \mu_k = \alpha_{j(k)}\beta_{i(k)},$$

and $i(k)$ and $j(k)$ denote, respectively, the indices of the home and away teams playing in match k.

2.2. Model enhancement and estimation

We refer the reader to Dixon and Coles (1997) for more details of the model, including a description of the dynamic updating procedure. One point to note, however, is that we obtain the model estimate of outcome probabilities by adding probabilities over scores corresponding to each respective outcome. Thus $p_{\mathrm{H}}(i, j, t)$, the probability of a home win between teams i and j at time t, is estimated as

$$p_{\mathrm{H}}(i,j,t) = \sum_{l,m\in B_{\mathrm{H}}} P(X_{i,j}=l, Y_{i,j}=m) \tag{3}$$

where $B_{\mathrm{H}}=\{(l,m): l>m\}$, and the score probabilities are determined from model (2) at time t. Similar expressions hold for the probabilities of an away win or a draw.

Finally, it is important to note that the structure of the forecasting model was developed using data from 1991 to 1994. Therefore, all analysis in this paper is implemented in pseudo real-time without hindsight bias. For example, to obtain parameter and probability estimates for a match occurring on December 14, 1996, we only use data known on or before December 13, 1996.

3. Data

3.1. Data sources

There are two main elements to the data we analyse: the full-time score results and the bookmakers' quoted odds. The score results data are used in evaluating trading rule profitability. They are also fundamental in deriving and fitting the statistical forecasting model. Each team's history of league and cup match full-time scores over the three-season period 1993–1996 was collected from Williams (1992, 1993, 1994) and the 90-minutes magazine.

The results database comprises home and away team scores for 6629 matches.[1]

3.2. Outcome odds

The odds consist of *outcome* odds, i.e. those for a home win, draw, or away win, and *score* odds, i.e. odds for the exact score result, such as 0–0, or 1–0, and so on. The odds are "fixed-odds" and do not vary over time in response to betting volumes. Discussions with representatives from several bookmaking firms suggest that the typical approach to setting the posted odds is for a panel of professional odds compilers, consisting of people with considerable experience in setting odds, to meet on a weekly basis. They use their judgement and expert knowledge of the current state of each football team subjectively to set the odds on home team win, away team win and drawn match outcomes for matches to be played just under one week ahead (Singh & Younger, 1994). (Private correspondence with (independent) representatives from Coral, Ladbrokes and William Hill bookmakers, confirms this description of the odds setting process.)[2] Our database consists of posted odds spanning three seasons from 1993 to 1996. Of these we have odds for three independent UK bookmakers for 1533 matches, and odds from at least one bookmaker for 2647 matches.

3.3. Correct score odds

The correct score odds database contains prices for the same set of matches for which we have outcome odds. Odds are set using a predetermined table which assigns a set of score odds from 0–0 upwards conditional on each set of match outcome odds. (The conditional odds table is presumably compiled using a simple statistical model, perhaps based on independent Poisson distributions.) The ceiling on these odds is 100–1, so that all scores outside the range of the table are quoted as 100–1. This truncation creates a potential problem in estimating a bookmaker's "take" or "overround". This issue is considered in later sections.

4. Results

4.1. Comparisons of outcome odds

Table 1 contains descriptive statistics for the odds posted by each firm. It shows that the distributions of odds are very similar for each bookmaker. The median odds are identical for each firm and the standard deviations differ only slightly. However, there are some notable differences, mainly relating to Firm B which offers slightly shorter odds on home wins and slightly longer odds on draws than Firms A and C. The statistical significance of these differences is confirmed by Table 2 which reports the results of

Table 1
Summary statistics of prices from the three bookmakers

	Mean	S.D.	25%	Median	75%
Firm A (home)	0.499	0.111	0.400	0.500	0.579
Firm B (home)	0.506	0.110	0.421	0.500	0.600
Firm C (home)	0.502	0.116	0.400	0.500	0.579
Model (home)	0.452	0.103	0.383	0.451	0.517
Firm A (draw)	0.305	0.017	0.294	0.308	0.312
Firm B (draw)	0.298	0.018	0.286	0.308	0.308
Firm C (draw)	0.300	0.019	0.294	0.308	0.308
Model (draw)	0.287	0.034	0.268	0.290	0.309
Firm A (away)	0.311	0.102	0.231	0.308	0.383
Firm B (away)	0.311	0.100	0.222	0.308	0.381
Firm C (away)	0.310	0.106	0.231	0.308	0.400
Model (away)	0.260	0.084	0.203	0.253	0.308

The mean, standard deviation (S.D.), and quantiles (25%, median, and 75%) are given. It can be seen that there is virtually no difference between bookmakers in terms of these summaries. Also given are the corresponding quantities obtained from the statistical model. The probabilities from the model and the bookmakers are also similar in value once the 11% take has been accounted for.

[1] There is a wide variety of available information that is potentially relevant for forecasting results, such as the times of goals in prior matches, goal scorers, teams' league positions and so on. An individual team's performance in any particular game could be affected by many external factors such as newly signed players or the sacking of a manager. Although this information is also potentially available to the punter, it is less easily formalised and its qualitative value is subjective. Therefore, the forecasting model is based only on past results data. For an extension of the models that incorporate goal time data, see Dixon and Robinson (1998). A discussion of public and semipublic information, i.e. information expected to be commonly available to at least some betting market participants, is provided in Forrest and Simmons (2000).

[2] This ignores the possibility that a match may not be completed, for example due to adverse weather conditions. Under such circumstances a bet is declared void and stakes are returned. In practice this happens infrequently for English matches, and can be ignored.

Table 2
Results of pairwise comparisons of odds for each two bookmakers

	Larger	Equal	Smaller	p-value	Corr. coef.
Firm A vs. Firm B (home)	0.269	0.257	0.473	0.037	0.963
Firm A vs. Firm C (home)	0.262	0.416	0.322	0.812	0.987
Firm B vs. Firm C (home)	0.445	0.284	0.271	0.123	0.963
Firm A vs. Firm B (draw)	0.497	0.386	0.117	0.000	0.712
Firm A vs. Firm C (draw)	0.398	0.473	0.129	0.000	0.806
Firm B vs. Firm C (draw)	0.178	0.388	0.434	0.000	0.709
Firm A vs. Firm B (away)	0.377	0.248	0.376	0.984	0.953
Firm A vs. Firm C (away)	0.313	0.390	0.296	0.914	0.984
Firm B vs. Firm C (away)	0.366	0.269	0.364	0.865	0.960

The first three columns represent the proportion of times that the first bookmaker's odds were larger than, equal to, or smaller than odds quoted by the second bookmaker. The fourth column is the p-value of a t-test for equality of the means of the quoted odds. The fifth column is the estimated correlation coefficient obtained by standard least squares estimation.

pairwise comparisons of odds across firms. The correlation coefficients in the final column of Table 2 also show that the differences are primarily attributable to disagreement over the draw probabilities. This is consistent with findings of Pope and Peel (1988) on the lack of ability of experts to forecast draws.[3]

Fig. 1 shows graphically the comparisons of the odds distributions across bookmakers along with the model probability estimates. The histograms for the three bookmakers are not smooth due to the discrete nature of the posted odds. The histograms support the observation that Firms A and C produce similar odds and that Firm B is more idiosyncratic. The posted draw odds are almost constant. One possible explanation for the bimodal distributions for home and away odds is as follows: suppose that the variation in draw odds is underestimated; if the odds are set as suggested in Boyle (1994), the win/lose pairs will tend to be shifted towards 0 and 1, leading to a bimodal distribution.[4]

In contrast to the bookmakers' odds, the model probabilities display far greater dispersion for the draw outcome. Unsurprisingly, the distributions of model probabilities are relatively smooth and are not bimodal for home and away odds. Note that the lack of bimodality in the estimated model distributions is *not* due to restrictions in the parametric model. That is, our model is flexible enough to display bimodality if it really existed in the data. (There are many ways that this could arise. For example, if the league consisted of half very good and half very bad teams, then a histogram of estimates would tend to be very bimodal.) The fact that it does not is evidence that the bimodality in the posted odds is an artifact of the odds-setting process.

Further insight into the relationship between odds and model probabilities can be gained by examining the respective kernel density estimates (Silverman, 1986), shown in Fig. 2 for Firm A. Relative to the model estimated distributions, we see that the density for the home odds is high in the probability range 0.3–0.4 and low in the range 0.4–0.5. For away odds it is low in the range 0.2–0.35 and high in the range 0.35–0.45. In contrast, the near constancy of the draw odds means that the density of intermediate probabilities is higher and that for both tails is lower than the model based estimates. It could be that the bookmakers systematically underestimate the variance of draw probabilities. If the model was without error, then these differences between the distributions of implied probabilities from the odds and estimated probabilities from the model are consistent with expectational bias and inefficiency. However, it is also possible that the observed biases are rational for the bookmakers if punters display judgmental biases in opposite directions. At this stage we cannot assess

[3] It should be pointed out that the results reported in Tables 1 and 2 are based on all the available data. We do not restrict the analysis to a common sample across all firms and matches. However, unreported analysis of a common sample of matches for which odds from all firms were available leads to qualitatively similar conclusions.

[4] Boyle (1994) suggests that bookmakers' odds are fixed according to the following algorithm. First the odds for a draw are decided upon by (subjectively) considering the relative form of the opposing teams and choosing from a table with "narrow width". Then a pair of odds is chosen from a predetermined table of odds pairs, corresponding to this draw probability. The odds pairs are odds for home and away wins; one such table is given in Boyle (1994).

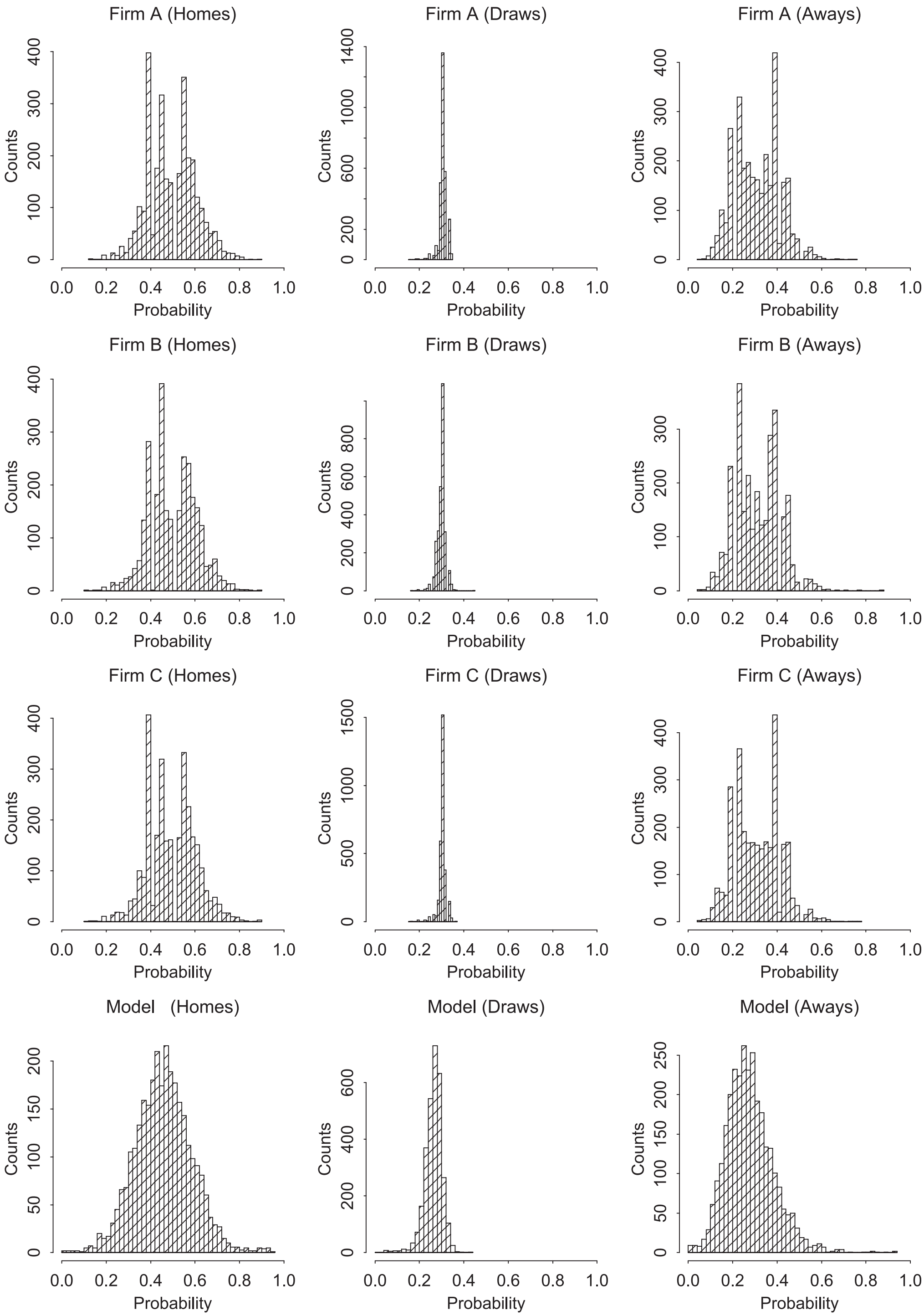

Fig. 1. Histogram estimates of aggregated model probability estimates and odds for each bookmaker. Note that the ordinates are not the same for each histogram. The histograms for bookmakers A, B, and C are based on 2647, 2458, and 1631 matches, respectively.

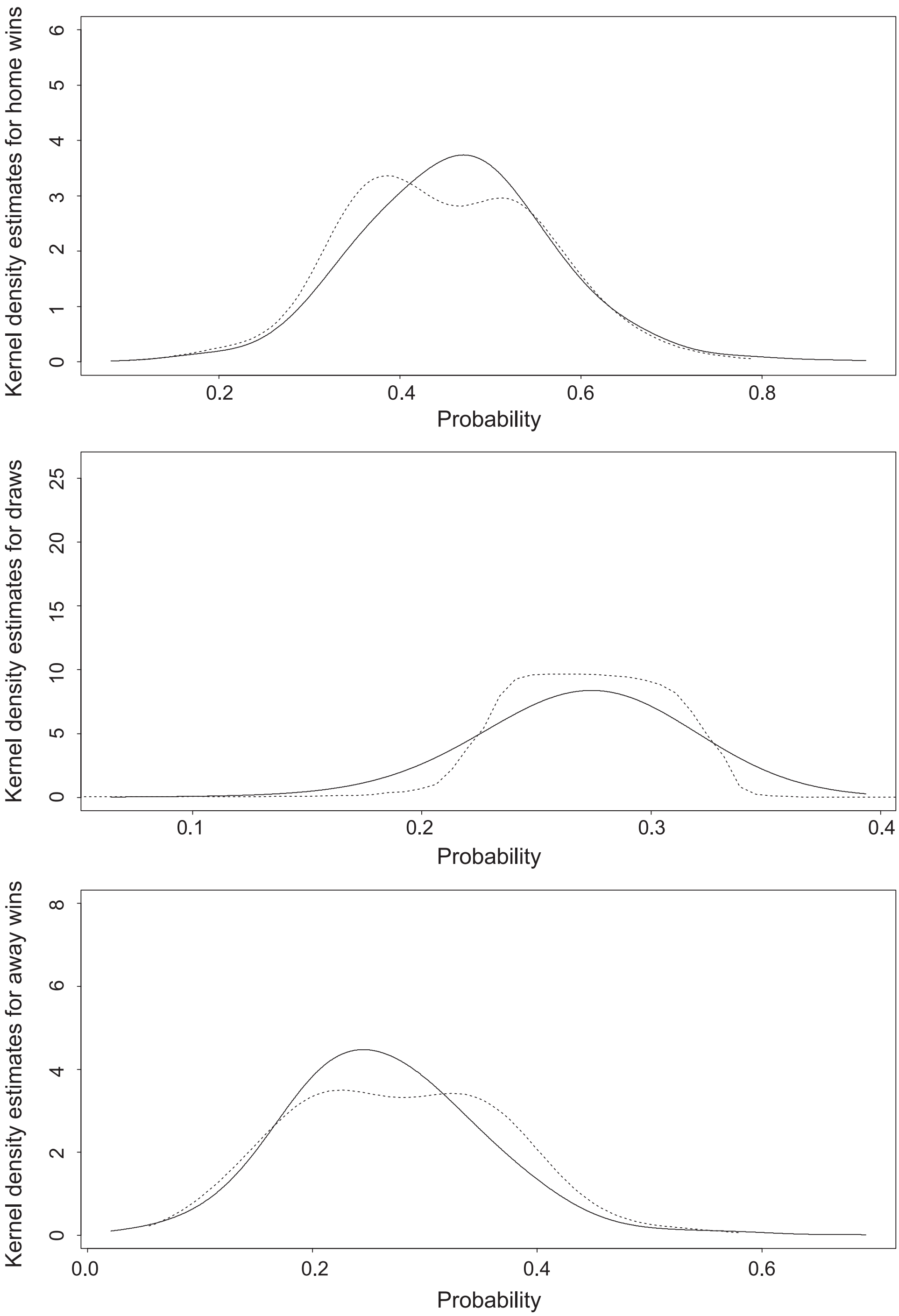

Fig. 2. Kernel density estimates of aggregated match probabilities for model estimates (solid line) and odds from bookmaker Firm A (dotted line). The top, middle and lower plots are for home wins, draws, and away wins, respectively. The odds for each match have been rescaled multiplicatively (by the sum of probabilities for a home/draw/away for that match) to account for the bookmaker's take.

whether it is our model or the odds which are more reliable: in the next section we present evidence to suggest that it is the former.

4.2. Comparisons of score odds

An analysis of the score odds reveals similar results. Fig. 3 shows histograms for a selection of implied probabilities for score odds from the three bookmakers and the model. Again, the differences across the bookmakers are small relative to that between the model and the bookmakers, but the differences are even more pronounced than was the case for the outcome odds. The discretisation is very noticeable. For example, for the score 1–0 in the fifth row of Fig. 3, the dispersion is much less for all three bookmakers than for the model estimates.

There are many ways in which we can analyse these data to determine which distributions better predict or reflect the true outcomes. For example, in the case of 1–0, we can look at the proportion of matches having predicted probabilities less than 0.10 (say) that actually result in that score. From Fig. 3, the bookmakers' odds imply that the probability of a 1–0 score is greater than 10% for all matches. In contrast, the proportion of matches having model estimates of less than 10% and resulting in a 1–0 score was approximately 6%. Although the estimates are subject to error, this strongly suggests that the bookmakers are setting odds that are too short for the 1–0 correct score in these matches. Again, it is important to note that the model probabilities are derived on a true *ex ante* basis. The inconsistency between the implied probabilities from the forecasting model and the odds cannot be explained by estimation or sampling biases.

4.3. Efficiency of odds

Throughout this section, we examine the form and/or existence of inefficiency in posted odds by considering the observed returns that would have been obtained had a particular trading rule been adopted. The sample space for bets on match outcomes is finite (and equal to home win, draw, away win). Therefore, we can estimate the bookmaker's take, or over-round, by summing the implied probabilities over all outcomes. (There are many ways of defining the take, or over-round. We use a simple definition of the percentage return that long-term random betting would give. For a discussion, see Haigh (2000).) Inefficiency in the match outcome betting market will be indicated if expected returns are greater than the expected return on an uninformed, random betting strategy. The pretax expected return to randomly placed bets on match outcomes is negative and equal to the bookmaker's take of approximately 11%. (For outcomes (home/draw/away) this is approximately constant. Across matches and bookmakers it varies between 11.2% and 11.9%.)

For the correct score odds, a bookmaker's take is less clearly defined because the sum of the implied odds is infinite. Therefore, as a guide for evaluating the efficiency of the correct score odds, we obtained Monte Carlo estimates of the bookmaker's take as follows: for each match we calculate the return from placing a unit stake on all scores where the maximum of the home and away score is less than 4. This resulted in an expected value of −26% (with standard error 1%). Therefore, to an approximation the average take on correct score bets is 26%.

The usual way to test the profitability of trading rules is to use a holdout sample. We adopt an alternative method. We calculate confidence intervals for the observed return using a bootstrapping technique (see, for example, Efron & Hinkley, 1978). As long as the trading rule is prespecified before the data is examined and therefore has some pre-analysis justification, any significant results can be interpreted as representative. The procedure adopted throughout this section is to calculate trading rule returns as a percentage of the total stake, where the stake on a particular match is equal to the probability (implied by the posted odds) of the outcome. Allowing the stake to vary across matches in this way reduces the variance of returns for small samples. Again, we reiterate that the forecasting model is implemented on a true ex ante basis. For each match, only data known prior to the match is assumed to be known both for model development and estimation. Based on this approach to assessing efficiency, the expected return (with associated 90% confidence interval) for placing bets on all home wins is −8.8% (−11.6% to −6.5%). The corresponding statistics for draws are −4.9% (−8.6% to −1.3%) and for away wins −14.2% (−17.8% to −9.8%). These expected losses serve as a relevant benchmark for assessing the

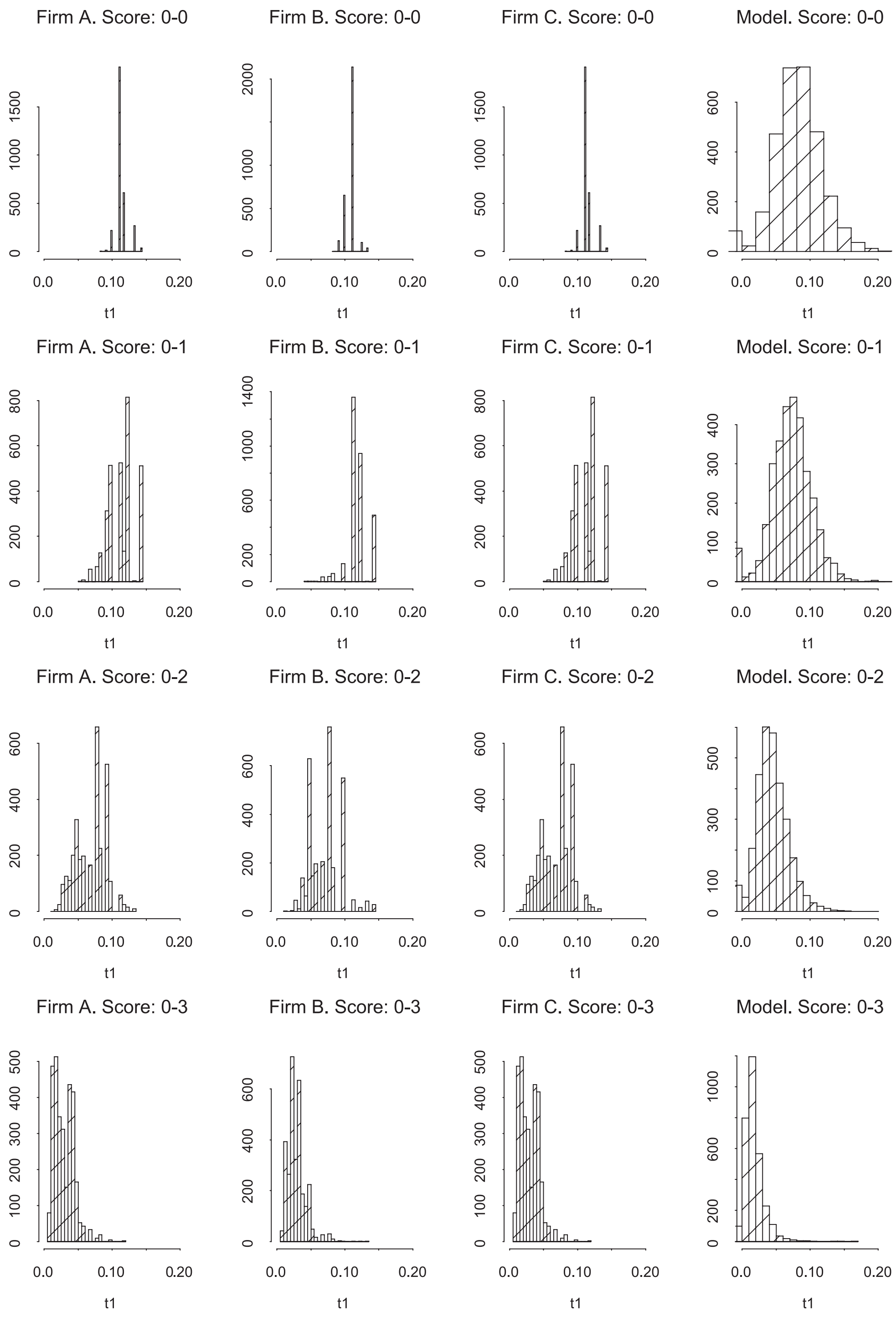

Fig. 3. Histograms of score odds and model probabilities.

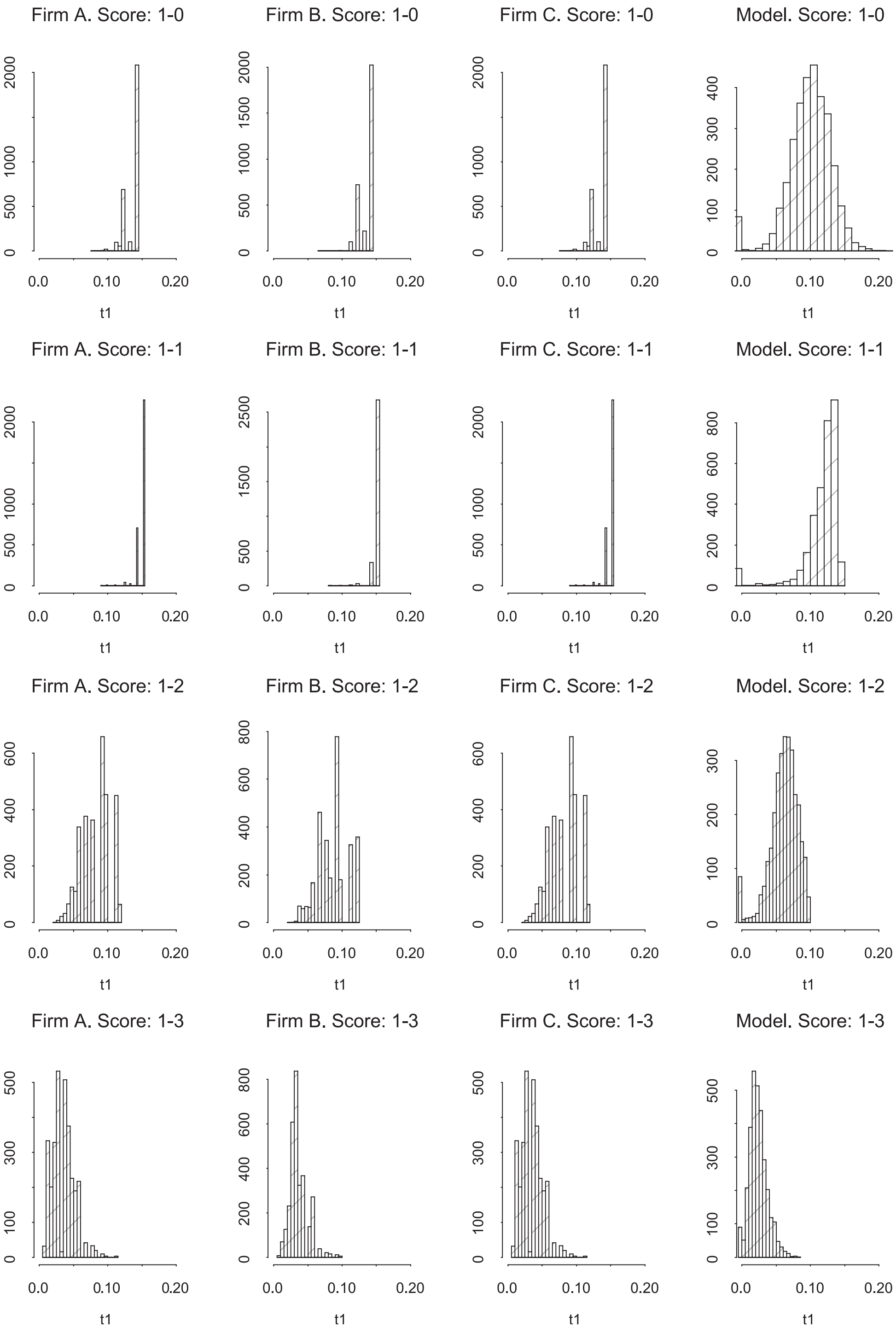

Fig. 3 (*continued*).

profitability of trading rules, and thus for assessing the existence and/or form of inefficiency in the market.

Before discussing the possibilities of inefficiency, we first illustrate, in Section 4.3.1, that it is adequate, for illustration purposes, to concentrate on just one bookmaker. This is perhaps counterintuitive, and is in contrast to the Pope and Peel (1988) analysis based on data from 1980 to 1982. In Sections 4.3.2 and 4.3.3, we obtain the main result and contribution of this paper, namely the evidence of inefficiency in the football fixed odds betting market. In Section 4.3.4, we consider possible explanations for the inefficiency, such as long-shot bias and over-reaction to news.

4.3.1. Cross-bookmaker arbitrage

Fig. 4 presents a histogram of the sum of the best odds for the full sample. There are no cases where the sum of the odds is less than one. This contrasts with the findings of Pope and Peel (1988, p. 324) who found many cases violating the no-arbitrage condition, at least on a pretax basis. (One may arbitrage across bookmakers by betting on all outcomes (home/draw/away), thereby guaranteeing a positive return if the respective prices and stakes are appropriately set.) Although their analysis involved four bookmakers compared to the three considered in the present study, this figure suggests that there has been far less divergence in odds in recent years than in earlier periods. This may be because bookmakers have learned to forecast more efficiently. Alternatively, it could be due to effective common limits on the ranges of odds being posted by different bookmakers. It is interesting to speculate whether such common limits could arise as a result of implicit or explicit collusion between the bookmakers (see also Hausch & Ziemba, 1995). (The general (but not perfect) agreement across bookmakers noted here is in line with our findings in Sections 4.1 and 4.2.)

4.3.2. Forecast model-based trading rules

Before discussing results from our analysis, we note that at first sight, the large betting tax-wedge of 9% suggests that trading rules need to provide a pretax expected profit greater than + 9% to provide an implementable, profitable betting strategy. However, by careful design, any trading rule that is expected to be profitable on a pretax basis and which uses only one bookmaker, can be converted into an expected post-tax profit. The potential of such bets is best explained by a simple example. Consider a situation where three home win bets each have a pretax expected return of 1.09 (i.e. an expected gain of 9% and hence a zero expected return after tax). An accumulator on all three outcomes will have an expected pretax gain of $1.09^3 = 1.30 > 1.09$,

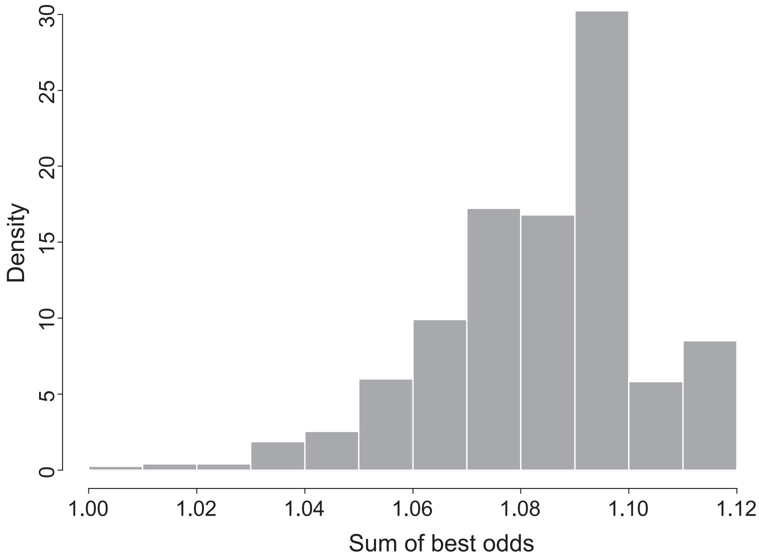

Fig. 4. Histogram density estimate of the best combination of bookmaker's odds. This gives some measure to how divergent the odds are across bookmakers: a level of less than 1 implies a pretax guaranteed positive return. The histogram is based on a sample of odds for 1553 matches.

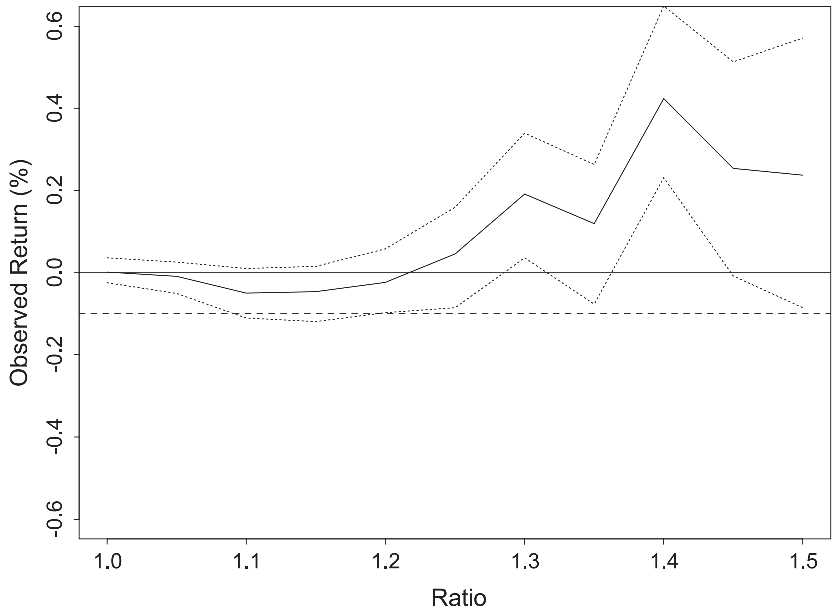

Fig. 5. Observed returns for the full odds set from 1994–1995 and 1995–1996 (bookmaker A only) plotted against the ratio of model probabilities to bookmakers odds. The lines are for a zero return (solid line) and the expected return under random betting (broken line). The dotted lines are 90% confidence intervals obtained by bootstrapping.

in other words, a positive return after tax. Increasing the number of matches in an accumulator increases the expected gain, although the variance of the bet is increased and the investment increases in risk. Of course, if the expected gain is less than 1, then accumulators have the opposite effect and reduce the overall expected gain. This strategy is an option in some cases, and is compulsory in certain situations. A common rule is that three-way or more accumulators must be placed for betting on the outcomes of matches and singles (or upwards) may be placed on correct scores.

First consider the match outcome prices. Fig. 5[5] shows the profitability of a betting strategy where a bet is placed whenever the ratio of the model probability to the odds for a match outcome exceeds a critical value measured on the abscissa. The average gross return and the 90% confidence interval obtained by bootstrapping,[6] together with the results of Section 4.2, suggest that the odds fail to capture all the information in the forecast model. The expected return conditional on the model forecast is systematically higher than the unconditional (negative) expected return. Above a ratio of 1.2 the expected return is positive, reaching a value well in excess of 20% for a ratio value of 1.4. Although the estimated standard error of returns increases with the ratio because the available number of bets decreases, the expected return is (marginally) significantly greater than zero for high ratios. The forecast model appears capable of generating economically and statistically significant abnormal returns, even allowing for the 9% betting tax.[7,8]

[5] Fig. 5 is based on Dixon and Coles (1997) with slight enhancements to the betting strategy.

[6] A simple bootstrapping procedure is used, whereby the entire data sample is repeatedly re-sampled with replacement and the Monte Carlo distributions obtained from the repeated samples.

[7] There is, of course, a degree of subjectivity in interpreting the plots in Figs. 5 and 6. However, there are two main reasons why we feel confident that the plots are consistent with the odds not fully reflecting the information in the model. Firstly, choosing across bookmakers leads to approximately uniform increases of around 2% in Fig. 5 (and subsequently 5% in Fig. 6). Second, we have used a relatively naive betting strategy in each case, which is far from optimal (this is an area where further work might be conducted).

[8] Subsequent analysis based on odds from Firms B and C reveal similar conditional returns plots, although the option to choose between bookmakers in a best-odds strategy does enhance observed returns and reduce the standard errors slightly. We have not presented these results for the reasons discussed earlier.

The forecast model-based trading rule assumes that the model leads to precise, unbiased estimates of true match outcome probabilities and that any difference between the posted odds and the model probabilities is due to misspecification in the odds. In general it is hard to test this assumption directly. (Dixon & Coles (1997) and Dixon & Robinson (1998) examine goodness of fit criteria.) However, Fig. 5 does go some way to showing that the model provides estimates, which more accurately reflect the true probabilities than do the posted odds.

4.3.3. Correct score odds

Fig. 6 shows the corresponding plots for correct scores rather than outcomes. There are substantially more “good value” bets (i.e. those with a ratio $r>1$), although the odds are naturally longer for correct scores. In particular, there are cases where the ratio reaches as much as 2.2. As in Fig. 5, we only display returns where there are at least 10 bets made, to mitigate sampling error. The conclusion from this plot is similar; the model estimates appear to be more accurate than the odds. However, one important feature to note is that, whereas with outcomes, bets are often restricted to multiple bets, correct score bets are accepted as singles. This allows the possibility of direct application of the strategy adopted in Fig. 5, at least for bets with a model odds ratio higher than about 1.4.

4.3.4. Trading rules based on extreme odds analysis

As mentioned above, our analysis located small consistent biases, and these are considered in this section with respect to match outcomes.

Bookmakers may be aware of predictable patterns in betting behaviour by punters, including behaviour based on cognitive or judgmental biases. If, on average, punters display systematic bias then bookmakers will seek to exploit the bias. For example, if punters display bias in favour of betting on long-shot outcomes, a rational bookmaker would offer less-than-fair odds on long-shot outcomes and fair odds on favourites, fair odds being defined as odds that equal the true probability of the outcome multiplied by a constant profit margin. Bookmaker rationality and punter irrationality would predict long-shot odds bias but not favourite odds bias in the opposite direction. On the other hand, it is

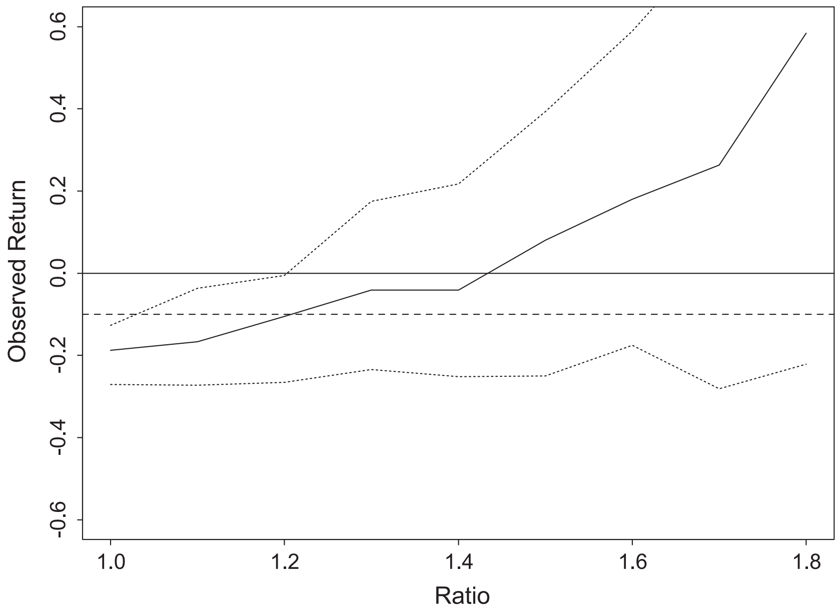

Fig. 6. Observed returns for full correct scores odds set 1994–1995 and 1995–1996 (one bookmaker) plotted against the ratio of model probabilities to bookmakers odds. The lines are for a zero return and the expected return under random betting (− 26%). The broken lines are 90% confidence intervals obtained by bootstrapping.

possible that the individual(s) setting the odds for bookmakers themselves display cognitive or judgmental biases. If this bias takes the form of long-shot bias, we would expect the resulting odds to have similar characteristics to the odds in horserace betting, namely less-than-fair odds on long-shot outcomes and more-than-fair odds on favourites.

We test for inefficiencies of this form by examining simple trading rules conditional on the level of the odds. Fig. 7 shows the return from betting with bookmaker A on all matches which have odds within 0.05 from a specified probability plotted on the abscissa, ranging from 0.2 to 0.8. Note that the majority of high probability outcomes are home win outcomes, and the majority of low probability outcomes are away wins. (Separate plots for home wins and away wins have very similar shapes to Fig. 7 and so are not displayed. Draws have not been included since they all have approximately the same probability.) The figure shows that the returns to betting on low probability outcomes (below approximately 0.35) are greater than the unconditional return to a random betting strategy. Returns to betting on high probability outcomes (above approximately 0.6) are less than the unconditional return. Similar results are obtained for bookmakers B and C. (Details available from the authors on request.) Fig. 7 suggests that the fixed odds contain a *favourite* bias, i.e. a reverse long-shot bias. The odds on low probability (long-shot) outcomes are too generous, and those on high probability outcomes are too short. This conclusion is reinforced by remembering that the benchmark returns from betting on all homes or all always are − 8% and − 14%, respectively (see Section 4.3). Without information on the volume of bets placed on different outcomes, it is impossible to determine whether this bias reflects a rational response by the bookmakers to favourite bias by bettors, or irrational judgmental biases on the part of the bookmakers themselves. (For discussion of this phenomenon in other markets, see Ali, 1977; Quandt, 1986; Shin, 1992; Thaler & Ziemba, 1988; and Woodland & Woodland, 1994.)

Although the reversal of the long-shot bias is a significant feature, it falls a long way short of explain-

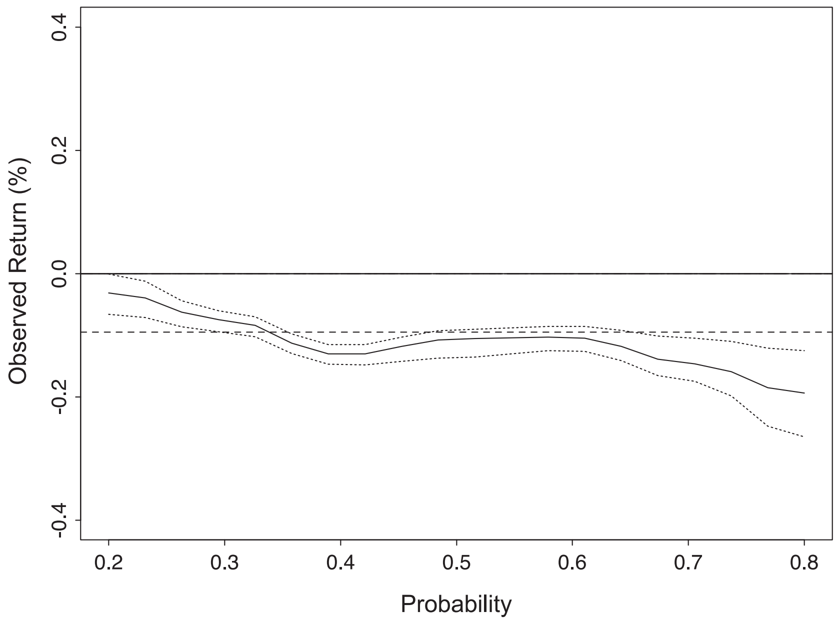

Fig. 7. Observed return plotted against probability for betting on all matches with odds as the probability shown on the abscissa. This is for bookmaker A; the pattern is similar for B and C. The other lines shown are for a zero return (horizontal solid line), return under random betting (broken line), and 90% confidence intervals obtained by bootstrapping (dotted lines).

ing the levels of inefficiency observed earlier. Another apparently common judgmental bias takes the form of under-reaction or over-reaction to recent news (Brown & Sauer, 1993; Camerer, 1989; DeBondt & Thaler, 1995). We examined the possibility of under-reaction or over-reaction to news in the betting market by conditioning the analysis on various measures of surprise contained in recent match results. For example, we examined returns from betting *against* teams which in the previous week had probability p of winning and which won, for various values of p. We experimented with several surprise indicators, and found weak evidence for inefficiency. However, the magnitudes were too small to account for the levels observed in Figs. 5 and 6, or to lead to profitable trading rules.

5. Conclusions

This examination of the fixed odds football betting market, including correct score odds, reveals significant changes in patterns of posted odds compared to earlier time periods. Using an advanced statistical forecasting model, we find that the market is inefficient in that model probabilities can be exploited to earn positive abnormal trading rule returns. Although we have been able to characterise some of the overall inefficiency in terms of systematic judgmental bias, in particular a reverse long-shot bias, much of the observed inefficiency is difficult to explain in terms of known cognitive biases.

We have concentrated exclusively on the ‘‘prices set’’ side of the market, reflecting bookmakers’ expectations. Of course, in order to gain a more complete understanding of the market, it would also be interesting to study the other side of the market, i.e. the ‘‘volume of bets’’ conditional on the prices. Examination of betting volume data, were it available, would be potentially informative about bettors’ consensus expectations. Further, it would also be potentially interesting to examine the properties of expert tipsters’ forecasts in relation to the model considered in this paper. Such a study would complement the work of Forrest and Simmons (2000) who find, using ordered logit models, that tipsters do not fully capture public information about teams’ comparative strengths and weaknesses.

Acknowledgements

The authors are, respectively, from the Department of Actuarial Science and Statistics, City University, London and the Management School, Lancaster University, LA1 4YX, UK. We are grateful to Simon Garside, Sara Morris, Mike Robinson and Barry Rowlinson for assistance in digitising the data.

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