Name: <Haitham Babbili>
Personal number: <19880803-4915>

Question 3:

3.a :
Use cases

1. Triangulation of objects
2. Immunization of the populace

Alpha value gives the minimum number of vertices needed for algorithm to work. The alpha value also helps to determine the maximum number of edges needed to build a graph without having an overlap.

For graph to be partitioned, as a closed planar graph, there will be at least 3 vertices or nodes and 3 edges. This will form edges.
Also, if we need to colour a graph without repeating colours, at least 3 colours are needed. This is from self-stabilizing maximum matching.

With planar graph G(P,E), given P vertices and E edges
The planar graph should be maximal if E = 3P -6
This follows that our graph should have at least three vertices.

The assumption that the network is anonymous helps to remove any form of bias to any vertex or node. It also guarantees that the process can start from any node. It also helps to assume that it is an independent graph, with finite constraints.

3.b :
legal executions
i. Execution of the program can start from any node.
ii. Edges can be added to the graph until we have a maximum number of edges that the graph can support.
iii. It is not allowed for any edge to overlap on another enabling the graph to be divided into partitions or regions.
iv. A partition is a closed planar graph which requires only 3 edges and 3 vertices.

3.c :
Assumptions
i. There are finite number of P vertices or nodes, and E edges. Each vertice, or node, is unique.
ii. Every node is activated infinitely often
iii. There is a shared memory or register that keeps in details of all successive iterations or executions
iv. The graph is considered as an undirected graph
V. It is not allowed for any edge to overlap on another enabling the graph to be divided into partitions or regions.
vi. It is allowed to add more edges to a finite limit.

This is combination of graph colouring and maximal plana graph theorem.

pseudocode:

```
if i = 0 is the root processor then
    do forever
    lrn-1  := read(rn-1)
    if lrn-1 = 0 then
        color0 := 1
    else
        color0 := 2
    end if
    write r0 := color0
else(i not= 0):
    do forever
    lri-1 := read(ri-1)
    if lri-1 > 0 then
        colori := 0
        write ri := colori
    else
        colori := 1
        write ri := colori
    end if
end if
```

3.d :

Given that T =(P,E) is a planar graph with P vertices and E edges.

A planar graph T with R regions, the boundary of each region will form a triangle.

Summation of all edges in the boundary is 3R, which also equal to 2E.

A vertice, not a leaf, has at least 2 edges.

3R = 2E.

Also, a planar graph, T, by Euler says, P - E + R = 2

This results in E = 3P - 6

Taking that we increase the number of egdes from E to E' such that T becomes T', where T'is a maximal graph.

Therefore, E' = 3P – 6 since E' is greater ot equal to E, then E is less than or equal to 3P-6