Laboration 2
MVE136, Random Signal Analysis

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Task 1

1.1

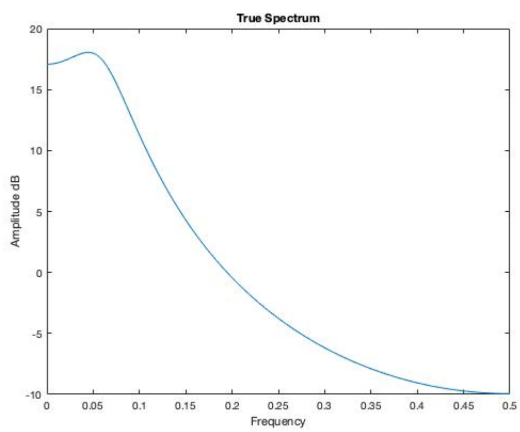

Figure 1: Graph showing the true spectrum of a random process

1.2

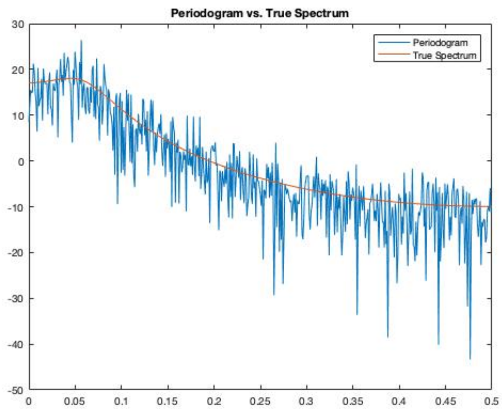

Figure 2: Graph showing the theoretical signal via Gaussian noise after go throw the filter

In figure 2 both the true spectrum and an estimate with periodogram are shown. The periodogram is a very noisy estimate but it can be seen that it follows the true spectrum.

1.3

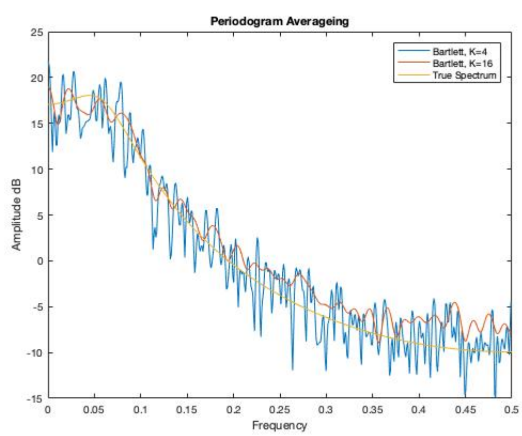

Figure 3: Grapgh showin the true spectrum and periodogram averaging with two different sizes of segments

The benefit of periodogram is to realize the difference between the amplitude via frequency and the window. Here it can be seen that the variance is reduce by K but also as drawback, the frequency resolution is reduced by $2.\pi$ K /N

Task 2

2.1

Length N=16	**Rectangular**	**Bartlett**	**Hann**	**Hamming**
Main Lobe (-3 dB)	0.10938	0.16406	0.1875	0.16406
Sidelobe	- 13.1 dB	- 26.3 dB	- 31.5 dB	- 39.4 dB
Leakage Factor	9.3 %	0.29 %	0.05 %	0.04 %

Length N=32	**Rectangular**	**Bartlett**	**Hann**	**Hamming**
Main Lobe (-3 dB)	0.054688	0.078125	0.085938	0.078125
Sidelobe	- 13.2 dB	- 26.5 dB	- 31.5 dB	- 41.8 dB
Leakage Factor	9.12 %	0.28 %	0.05 %	0.04 %

Length N=64	**Rectangular**	**Bartlett**	**Hann**	**Hamming**
Main Lobe (-3 dB)	0.027344	0.039062	0.042969	0.039062
Sidelobe	- 13.3 dB	- 26.5 dB	- 31.5 dB	- 42.5 dB
Leakage Factor	9.14 %	0.28 %	0.05 %	0.03 %

Length N=128	**Rectangular**	**Bartlett**	**Hann**	**Hamming**
Main Lobe (-3 dB)	0.013672	0.019531	0.021484	0.019531
Sidelobe	- 13.3 dB	- 26.5 dB	- 31.5 dB	- 42.6 dB
Leakage Factor	9.14 %	0.28 %	0.05 %	0.03 %

We can see that while the N is increasing, the main lobe become narrower with slightly decreasing in sidelobe power
In rectangular window, by changing the length of N the main-lobe change is realizable and sidelobe power has small change but leakage factor is high
For Bartlet window, the mainlobe changing is fast and became narrow. The change in the sidelobe is small but the Rectangular window sideloe has beggar power than Bartlett.
Also Hann window the mainlobe change fast and became narrow but still the wider between the four but also less power for sidelobe and less percentage of

leakage factor.
The mainlobe for Hamming window is seemlier to Bartlett but the sidelobe power in Hamming window is fare less and less leakage factor.

2.2

Length N=128	**Chebyshev**	**Hamming**
Main Lobe (-3 dB)	0.027344	0.019531
Sidelobe	- 100 dB	- 42.6 dB
Leakage Factor	0 %	0.03 %

Chebyshev has a wider mainlobe but low power in the sidelobe and 0 percent leaking.
Hamming has narrower mainlobe but higher power in the sidelobe and 0.03 percent leakege factor.
Hamming is better because of mainlobe is narrower.

2.3

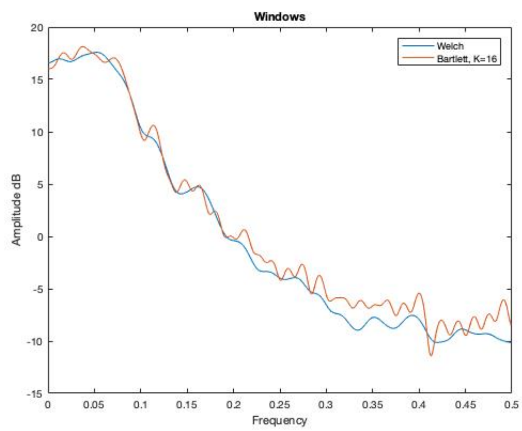

Figure 4: Graph showing Welch averaging copmpared to Bartlett

Figure 4 shows a comparison between periodogram averaging using Bartlett and Welch method. It can be seen from the graph that the variance is reduced using Welch and more closely resembles the true spectrum.

Task 3

3.1

Below is the the function `btmethod` used for the Blackman-Tukey's Method:

```
function [ PBT,fgrid ] = btmethod( x,M,NFFT)

w_lag = hamming(2*M+1); rx = xcorr(x,M,'biased');
BT = fft(w_lag'.*rx,NFFT);
fgrid = 0:1/NFFT:(NFFT-1)/(2*NFFT);
PBT_temp = BT(1:NFFT/2);
PBT = 10*log10(abs(PBT_temp));


end
```

3.2

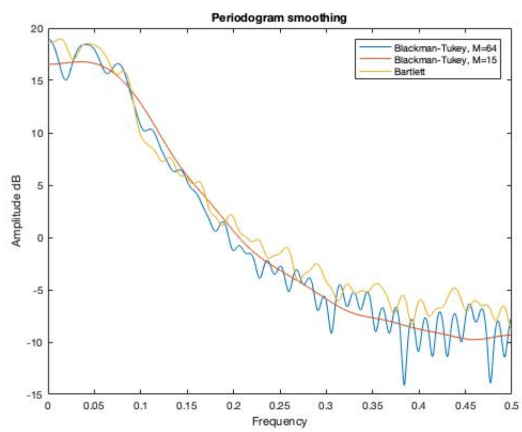

Figure 5: Graph showing Blackman-Tukey's Method compared to Bartlett

As we use Hamming window , it is clear in BT that reducing M will be smoother and better than Bartlett. In our case, M $\in$ (15,65) appear to be a good choice.

Task 4

4.1

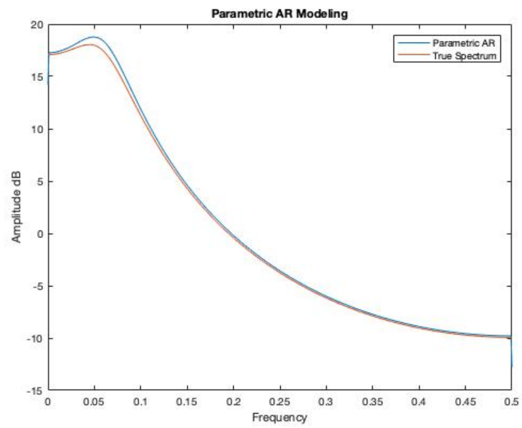

Figure 6: Graph showing Yule-Walker estimation compared to the true sepctrum

In figure 6 t can be seen that the Yule-Walker estimation closely resembles the true spectrum and has less noise than the previous methods where the variance was significantly higher.

4.2

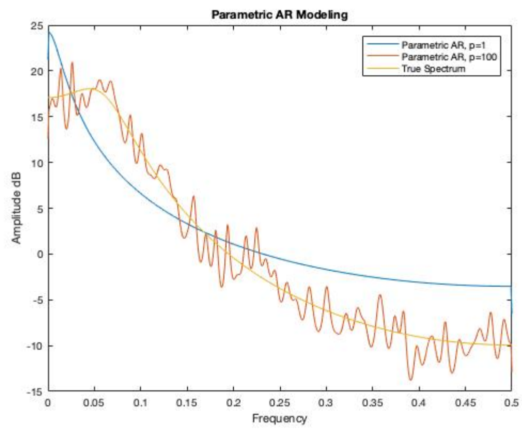

Figure 7: Graph showing how the value p affects the estimation.

Figure 7 illustrates the effect the value p has on the estimation. To low (p=1) the estimation does not follow the true spectrum. If it is to high (p=100) the variance increases and compared to figure 6 where the variance was lower.

Task 5

5.1

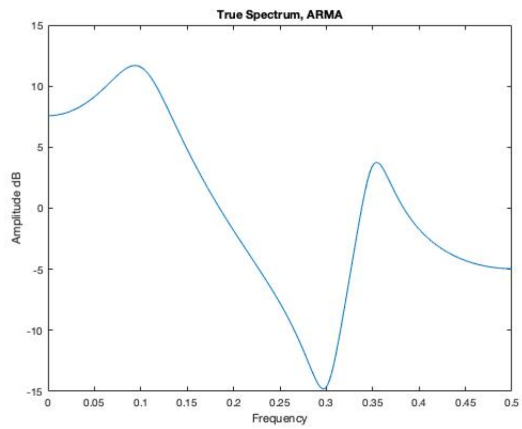

Figure 8: Graph showing the true spectrum of an ARMA model

5.2

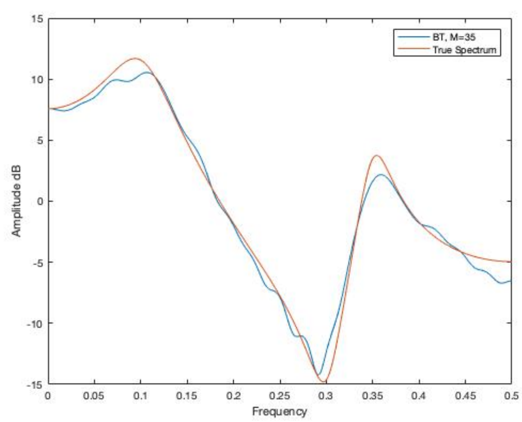

Figure 9: Graph showing the true spectrum of an ARMA model and an estimation using Blackman-Tukey's Method

From figure 9 the estimation using Blackman-Tukey's Method is shown compared to the true spectrum. From the figure it can be seen that the peaks are harder to estimate with this method.

5.3

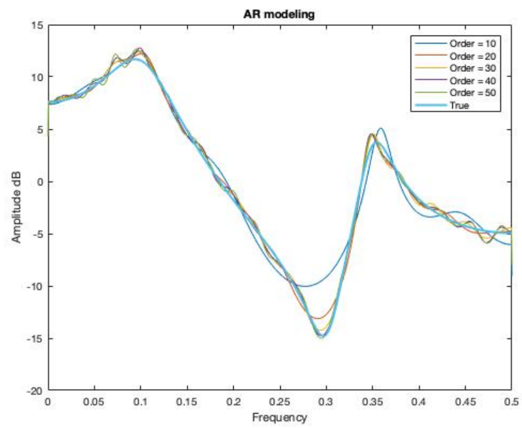

Figure 10: Estimations on the ARMA model using Yule-Walker with different orders

Figure 10 shows how the order of Yule-Walker effects the estimation on an ARMA model. From the graph a conclusion can be drawn that the best choice of order is around p=40 or p=50. However there are clear oscillations in the beginning and end of the graph with a higer p-value. From the graph it can also be shown that the valley is the most difficult part to estimate. By the peak the other p-values are close to the true spectrum, not like in the valley where the difference is bigger.

Task 6

6.A.1

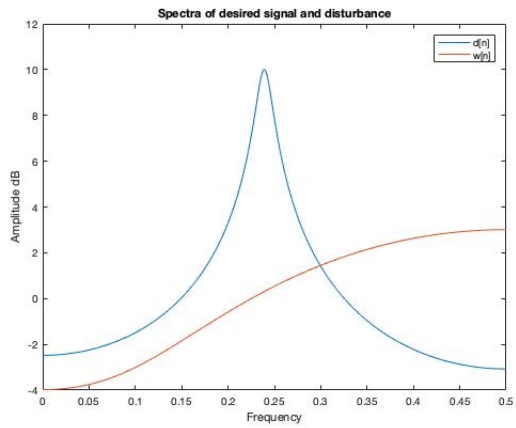

Figure 11: Spectra of d[n] and w[n]

The graphs shows that the desired signal can select as band pass characteristics, the noise act alike high-pass filter because the response value is increasing with the increase in frequency.

6.A.2

Given in assignment:

$$x[n] = d[n] + w[n]$$
$$e_d[n] = d[n] - 0.13d[n-1] + 0.9d[n-2]$$
$$w[n] = e_w[n] - 0.8e_w[n-1] + 0.2e_w[n-2]$$
$$r_d[k] = E[d[n]d[n-k]]r_w[k] = E[w[n]w[n-k]]$$

for e˙d[n] and e˙w[n]: mean = 0, $\sigma_{ed}^2 = \sigma_{ew}^2 = 1$

Show that r_d[0]= 5.2879, r_d[1]= 0.3618 and $r_d[k]$=0.13$r_d[k-1]$-0.19$r_d[k-2]$ for all k≥2:

$$
\begin{aligned}
r_d[0] = E[d[n]d[n-0]] = E[d^2[n]] = \\
= E[(e_d[n] + 0.13d[n-1] - 0.9d[n-2])^2] = \\
= E[e_d^2[n] + 0.26d[n-1]e_d[n] - 0.38d[n-2]e_d[n] + \\
+0.0169d^2[n-1] - 0.234d[n-1]d[n-2] + 0.81d^2[n-2]] = \\
= E[e_d^2[n]] + 0.26E[d[n-1]e_d[n]] - 0.38E[d[n-2]e_d[n]] + \\
+0.0169E[d^2[n-1]] - 0.234E[d[n-1]d[n-2]] + 0.81E[d^2[n-2]] = \\
\sigma_{ed}^2 + 0.0169E[d^2[n-1]] - 0.234E[d[n-1]d[n-2] + 0.81E[d^2[n-2]] = \\
1 + 0.0169r_d[0] - 0.234r_d[1] + 0.81r_d[0]
\end{aligned}
$$

$$
\begin{aligned}
r_d[0](1 - 0.0169 - 0.81) = 1 - 0.234r_d[1] \\
0.173r_d[0] = 1 - 0.234r_d[1] \qquad (1)
\end{aligned}
$$

$$
\begin{aligned}
r_d[1] = E[d[n]d[n-1]] = \\
= E[e_d[n]d[n-1] + 0.13d^2[n-1] - 0.9d[n-1]d[n-2]] = \\
= 0.13E[d^2[n-1]] - 0.9E[d[n-1]d[n-2]] = \\
= 0.13r_d[0] - 0.9r_d[1]
\end{aligned}
$$

$$
\begin{aligned}
1.9r_d[1] = 0.13r_d[0] \\
r_d[1] = \frac{0.13}{1.9}r_d[0] \qquad (2)
\end{aligned}
$$

Combining (1) and (2) gives:

$$
\begin{aligned}
0.173r_d[0] = 1 - 0.234\frac{0.13}{1.9}r_d[0] \\
r_d[0](0.173 + 0.234\frac{0.13}{1.9}) = 1 \\
r_d[0] = 5.29
\end{aligned}
$$

Inserting in (2) gives:

$$
r_d[1] = 0.3618
$$

$$\begin{aligned} r_d[k] = E[d[n]d[n-k]] = \\ = E[e_d[n]d[n-k] + 0.13d[n-1]d[n-k] - 0.9d[n-2]d[n-k]] = \\ = 0.13E[d[n-1]d[n-k]] - 0.9E[d[n-2]d[n-k]] = \\ = 0.13E[d[n-k]d[n-1]] - 0.9E[d[n-k]d[n-2]] = \\ = 0.13r_d[k-1] - 0.9r_d[k-2] \end{aligned}$$

Show that r_w[1]=-0.96, r_w[2]=0.2, $r_w[k]$=0 for all k¿2 and $r_x[k] = r_d[k] + r_w[k]$

$$\begin{aligned} r_w[0] = E[w[n]w[n-0]] = E[w^2[n]] = \\ = E[(e_w[n] - 0.8e_w[n-1]0.2e_w[n-2])w[n]] = \\ = E[e_d[n]w[n] - 0.8e_w[n-2]w[n] + 0.2e_w[n-2]w[n]] = \\ = E[w[n]e_w[n]] - E[0.8w[n]e_w[n-1]] + E[0.2w[n]e_w[n-2]] = \\ = \text{Noise uncorrelated } E[w[n]e_w[n] = \\ = \sigma_{ew}^2] = \\ = \sigma_{ew}^2(1 + 0.8^2 + 0.2^2) = 1.68 \end{aligned}$$

$$\begin{aligned} r_w[1] = E[w[n]w[n-1]] = \\ = E[w[n-1]e_w[n]] + E[0.8w[n-1]e_w[n-1]] + E[0.2w[n-1]e_w[n-2]] = \\ = 0 - 0.8\sigma_{ew}^2 + E[0.2e_w[n-2](e_w[n-1] - 0.8e_w[n-2] + 0.2e_w[n-3])] = \\ = 0.8\sigma_{ew}^2 + 0 - 0.2\sigma_{ew}^2 - 0 = -0.96 \end{aligned}$$

$$\begin{aligned} r_w[2] = E[w[n]w[n-2]] = \\ = E[e_w[n]w[n-2]] + E[-0.8e_w[n-1]w[n-2]] + E[0.2e_w[n-2]w[n-2]] = \\ = 0.2\sigma_{ew}^2 = 0.2 \end{aligned}$$

$$\begin{aligned} r_w[k] = E[w[n]w[n-k]] = \\ = E[e_w[n]w[n-k]] + \\ + E[-0.8e_w[n-1]w[n-k]] + E[0.2e_w[n-2]w[n-k]] = [k > 2] = 0 \end{aligned}$$

$$
\begin{aligned}
r_x[k] = E[x[n]x[n-k]] = \\
= E[(d[n] + w[n])(d[n-k] + w[n-k])] = \\
= E[d[n]d[n-k] + d[n]w[n-k] + d[n-k]w[n] + w[n]w[n-k]] = \\
= r_d[k] + E[d[n]w[n-k]] + E[d[n-k]w[n]] + r_w[k] = \\
= [d[n] \text{ and } w[n] \text{ are uncorrelated}] = r_d[k] + r_w[k]
\end{aligned}
$$

6.A.3

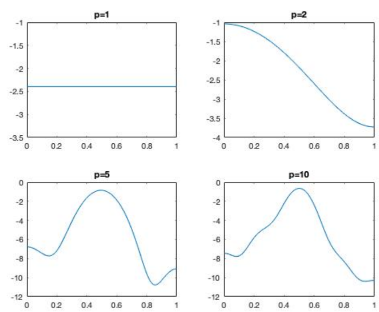

Figure 12: Caption

Figure 11 shows that the disturbance w[n] has high-pass properties therefore the filters shown in figure 12 is as expected. These filters out the higher frequencies as desired. The filters are more effective with increasing p.

6.B.4

Filter Coefficient	MSE
h_1	1.2749
h_2	1.1787
h_5	0.7596
h_{10}	0.7455

Table 1: Table showing theoretical MSE for different filters.

Table 1 shows the values for MSE for different filters. It can be seen that the MSE decreases with the increase of filter parameters. However, between h_5 and h_{10} the decrease is significantly smaller than before. This leads to the conclusion that further increase of filter parameters will not have as much impact on the MSE.

6.B.5

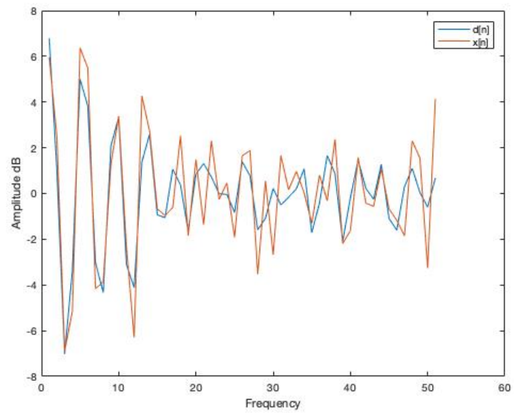

Figure 13: Graph showing d[n] and x[n]

6.B.6

Filter Coefficient	Empirical MSE
h_1	1.1179
h_2	1.1187
h_5	0.7336
h_{10}	0.7312

Table 2: Table showing the empirical MSE for different filters