MATHEMATICS

How to calculate the generating function of a Poisson distribution?

Asked 2 years, 5 months ago Active 2 years, 5 months ago Viewed 59 times

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$$\begin{aligned}G_X(s) = \mathbb{E}[s^X] &= \sum_{i=0}^{\infty} s^i \mathbb{P}\{X = i\} \\ &= \sum_{i=0}^{\infty} s^i \frac{\lambda^i}{i!} e^{-\lambda} \\ &= e^{-\lambda} \sum_{i=0}^{\infty} \frac{(\lambda s)^i}{i!} = e^{-\lambda} \cdot e^{\lambda s} \\ &= e^{\lambda(s-1)}.\end{aligned}$$

Hi there, I'm trying to understand the solution to the question in the title. Can anyone please explain why the final summation is equal to $e^{\lambda s}$?

Thank you!

statistics

1 That's simply the definition of the exp-function. en.wikipedia.org/wiki/Exponential_function – Konstantin Jan 24 '18 at 22:22

Ahhh, I see thank you! Will the process for other distributions like binomial/geometric involve a similar step where I need to just memorise the format of their power series? – Josh Jan 24 '18 at 22:28

Yes, you are right in some sense. Have a look at proofwiki.org/wiki/... and proofwiki.org/wiki/.... I hope these proofs belong to your thoughts. – Konstantin Jan 24 '18 at 22:33

1 Answer

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You would be presumed to have learned in calculus before studying probability that

$$e^a = \sum_{i=0}^{\infty} \frac{a^i}{i!}. \tag{1}$$

Without that, the function $i \mapsto \frac{\lambda^i}{i!} e^{-\lambda}$ for $i = 0, 1, 2, 3, \ldots$ would not be a probability mass function, since only line (1) above tells you that the sum of that over all values of i is 1.

Kai-Lai Chung's undergraduate introduction to probability, in at least one of its editions, says:

> "Everybody knows" that
>
> $$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}.$$

complete with quotation marks around those two words.

answered Jan 24 '18 at 22:47

Michael Hardy

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