

Let assume we have in LTI system we have-

filter  $H(z)$  with impulse response  $\{h(n)\}_{n=-\infty}^{\infty}$

and input signal  $e(n)$  and output signal  $x(n)$

$$e(n) \xrightarrow{\quad} \boxed{H(z)} \rightarrow x(n)$$

$$x(n) = h(n) * e(n) = \sum_{k=-\infty}^{\infty} h(k) e(n-k)$$

we can see that  $x(n)$  is stationary stochastic

Process, where the output power:

$$P_x(e^{j\omega}) = |H(e^{j\omega})|^2 P_e(e^{j\omega})$$

if we consider  $e(n)$  as white noise, the output spectrum is:

$$P_x(e^{j\omega}) = \sigma_e^2 |H(e^{j\omega})|^2.$$

Since the process is stochastic, we can say that the filter is limited and the transfer function for filter will be:

$$H(z) = \frac{B(z)}{A(z)}$$

in  $z$  domain.

$$B(z) = 1 + b_1 z^{-1} + \dots + b_q z^{-q}$$

$$A(z) = 1 + a_1 z^{-1} + \dots + a_p z^{-p}$$

we can express the output signal  $x(n)$  like:

$$x(n) + a_1 x(n-1) + x(n-2) \dots + a_p x(n-p) = e(n) + b_1 e(n-1) + e(n-2) + \dots + b_q e(n-q).$$

Thus,  $x(n)$  is ARMA process and the output is:

$$P_x(e^{j\omega}) = \sigma_e^2 \frac{|B(e^{j\omega})|^2}{|A(e^{j\omega})|^2}.$$

so by set filter coefficient we can get power shape spectrum.

when  $q=0 \Rightarrow H(z) = \frac{1}{A(z)}$  and this is AR processes.

$$x(n) + a_1 x(n-1) + \dots + a_p x(n-p) = e(n)$$

and the power spectrum AR is

$$P_x(e^{j\omega}) = \frac{\sigma_e^2}{|A(e^{j\omega})|^2}$$

Similarly, when  $p=0$ ,  $H(z) = B(z)$  so we can get MA process.

$$x(n) = e(n) + b_1 e(n-1) + \dots + b_q e(n-q).$$

with power spectrum.

$$P_x(e^{j\omega}) = \sigma_e^2 |B(e^{j\omega})|^2$$