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I assure That I did The exam on my own with out getting help from any other person and That I formulated all The solutions myself

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$Pr(X(1)Y(2)>0)$

$X(t), Y(t)$ Zero mean WSS Gaussian process and Independent.

$S_{XX}(f) = S_{YY}(f) = e^{-|f|}$

$Pr(X(1)Y(2)>0) = \phi\left(\frac{X-\mu}{\sigma}\right)$

$\mu = E[X(1)Y(2)] = E[X(1)]E[Y(2)] = 0$

$\sigma^2 = E[X(1)^2 Y(2)^2]$

$= E[X(1)^2]E[Y(2)^2]$

$= R_{XX}(0) R_{YY}(0) = R_{XX}(0)^2$

$= \int_{-\infty}^{\infty} S_{XX}(f) S_{XY}(f) e^{+j2\pi f(0)} df$

$= \int_{-\infty}^{\infty} S_{XX}(f) S_{YY}(f) df$

$= \int_{-\infty}^{\infty} e^{-|f|} e^{-|f|} df$

$= \int_{-\infty}^{\infty} e^{-2|f|} df$

$Pr(X(1)Y(2)>0) = Pr(N(\mu, \sigma^2) > 0)$

$= Pr(N(0, \int_{-\infty}^{\infty} e^{-2|f|} df) > 0).$

Task 2:

$X(t), Y(t)$ independent poission processs $\lambda = 1$

$$Pr(X(1) = Y(2)] = Pr[X(1) - Y(2) = 0]$$

$$= Pr[P_0(X(1)=0) \cdot P_0(Y(2)=0)]$$

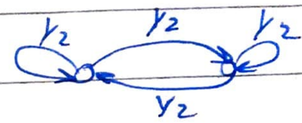
$$= \sum_{k=0}^{\infty} \left(\frac{1^k e^{-1}}{k!}\right) \cdot \left(\frac{1^k e^{-1}}{k!}\right)$$

$$Pr(X(1) = Y(2)] = \sum_{k=0}^{\infty} \frac{e^{-2}}{(k!)^2}$$

Task 3:

Markov chain.

$\pi(0) = [1 \quad 0] \qquad P = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$



$\pi(1) = \pi(0)\, P = [0{,}5 \quad 0{,}5]$

$\pi(2) = \pi(1) . P = \pi(0) . P^2 = [0{,}5 \quad 0{,}5]$

$P = P^2 = P^n$

$\pi(n) = \pi(0)\, P^n = \pi P$ stationary distripution.

$E[X(1)] = \sum x\, P(x) = 0 . \frac{1}{2} + 1 . \frac{1}{2} = \frac{1}{2}$

$E[X(2)] = \sum x\, P(x) = 0 . \frac{1}{2} + 1 . \frac{1}{2} = \frac{1}{2}$

$E[X(n)] = 1/2$

$E[T] = E[X(n-1)\, X(n)] = E[X(m)\, X(n)]$

$= E[X(n-1)] . E[X(n)]$

$= (0 . P(X(n-1) + 1 . P(n-1)) . 1/2$

Since its stationary distripution

$\pi(0) = \pi(n) = \pi(n-1) = \pi(m).$

$E[T] = \frac{1}{4}$

Task 4:

~~For~~ $x(t)$, $y(t)$ WSS random process.

show $R_{xy}(t, t+\tau)$ depend on t.

$$E[x(t)] = \mu_x \quad , \quad E[y(t)] = \mu_y.$$

$$R_{xy}(t, t+\tau) = E[x(t) . y(t+\tau)].$$

$$= \operatorname{cov}(x(t) . y(t+\tau)) + \mu_x \mu_y$$

Task 5:

$$X(t) \xrightarrow{\oplus\, N(t)} y(t)$$

$$y(t) \rightarrow \boxed{h} \rightarrow Z(t)$$

$$D = E[(X(t) - Z(t))^2] = E[X(t)^2 + Z(t)^2 - 2Z(t)X(t)]$$

$$= E[X(t)^2] + E[Z(t)^2] - 2E[Z(t)X(t)]$$

$$= R_{xx}(0) + E[(h(t)*X(t) + h(t)*N(t))^2] + 2E[h(t)*X(t) + h(t)*X(t)\,N(t)]$$

$$= R_{xx}(0) + E[h(t)^2 * X(t)^2] + E[h(t)* N(t)^2] + 2E[h(t)^2 * X(t)N(t)]$$
$$- 2E[h(t)\,X^2(t)] - 2E[h(t)\,X(t)\,N(t)]$$

$$= R_{xx}(0) + h(t)^2 * R_{xx}(0) + h(t)^2 * R_{NN}(0) - 2h(t) * R_{xx}(0)$$

$$= \int_{-\infty}^{\infty} \left(S_{xx}(f) + |H(f)|^2 S_{xx}(f) + |H(f)|^2 S_{NN}(f) - 2H(f)S_{xx}(f)\right) df$$

depending on winer filter. after tack derivative $\delta = 0$

$$H(f) = \frac{S_{xx}(f)}{S_{xx}(f) + S_{NN}(f)}$$

Put it back to equation.

$$D = \int_{-\infty}^{\infty} \left|\frac{S_{xx}(f)}{S_{xx}(f)+S_{NN}(f)}\right|^2 \cdot [S_{xx}(f) + S_{NN}(f)] + S_{xx}(f) - 2\frac{S_{xx}(f)^2}{S_{xx}(f)+S_{NN}(f)}\, df$$

$$= \int_{-\infty}^{\infty} \frac{S_{xx}(f)^3 + S_{NN}(f)S_{xx}(f)^2 + S_{xx}(f)[S_{xx}(f)+S_{NN}(f)]^2 + 2S_{xx}(f)^2 S_{NN}(f)}{(S_{xx}(f)+S_{NN}(f))^2}\, df$$

$$= \int_{-\infty}^{\infty} 2S_{xx}(f)^3 - 2S_{xx}(f)^2 S_{NN}(f) / (S_{NN}(f) + S_{xx}(f))^2$$

$$= \int_{-\infty}^{\infty} \frac{S_{NN}(f)S_{xx}(f)^2 + S_{xx}(f)S_{NN}(f)^2}{(S_{xx}(f)+S_{NN}(f))^2}\, df$$

$$D = \int_{-\infty}^{\infty} \frac{S_{NN}(f)\, S_{xx}(f) \left[S_{xx}(f) + S_{NN}(f) \right]}{\left(S_{xx}(f) + S_{NN}(f)\right)^2}\, df$$

$$D = \int_{-\infty}^{\infty} \frac{S_{NN}(f)\, S_{xx}(f)}{S_{xx}(f) + S_{NN}(f)}\, df$$

$$D = \int_{-\infty}^{\infty} S_{NN}(f) \frac{S_{xx}(f)}{S_{xx}(f) + S_{NN}(f)}\, df = 0$$

~~D ≠ D to de~~

D to be equall zeros $\Rightarrow$ The integral between The noise and the filter will be zero. which mean the filter pass only the signal and elemenat the noise. and there is No error in filtring.

~~in~~ - ~~The~~ $\int_{-\infty}^{\infty} S_{NN}(f)\, df = \frac{N_o}{2}$

~~PSD for $Y(t)$ is $S_{YY}(f) = |H(f)|^2 S_{xx}$~~

PSD of $Z(t)$ is $S_{zz}(f) = |H(f)|^2 \cdot S_{YY}$

$$= \frac{S_{xx}(f)^2}{\left(S_{xx}(f) + S_{NN}(f)\right)^2} \cdot \left(S_{xx}(f) + S_{NN}(f)\right)$$

$$S_{zz}(f) = \frac{S_{xx}(f)^2}{S_{xx}(f) + S_{NN}(f)}$$

Task 6:

to compute the spectrum to the signal $X(t)$ in Theoritical way we need a infinitly numbr of realization of the signal.

- ~~Pr~~

- Practicaly that not posibole because of ~~Limitatin~~ Limitation in time and resource.

- So to Estimate the spectrum power on use non parametric method by doing Fourier Transform over limit amount of data but the dis advantage of this process is the losses in frequency resolution and the variance is high.

- or we use parametric method ~~to~~ depending on specifice modeling to the mesured signal. which is more accuret ~~spe the~~ spe
$\Rightarrow$ spectrum estimation approximitly right depending on model used confinince ~~num~~ amount of parameter. !

in AR model: will depend on signal mesured $X(n)$ and privious signal mesured $X(n-k)$ the Tack the expectation and calculate the Autocorrolation for this signal then PSD Lik:

$$X(n) + \sum_{k=1}^{P} a X(n-k) = e(n)$$

where $e(n)$ is the noise which is white noise
$E[e(n)^2] = \sigma_e^2$, P is the model order.

in Parametric metheod the spectrum estimation is reduce to model order P

$$E\left[\left(x(n) + \sum_{k=1}^{p} a_k\, x(n-k)\right) x(n-k)\right] = E\left[e(n)\, x(n-k)\right]$$

$$r_x(k) + \sum_{k=1}^{p} a_k\, r_x(k-\cancel{p}) = \sigma_e^2\, \delta(k).$$

$$\sigma_e^{\cancel{2}} = \begin{cases} 1 & k=0 \\ 0 & \cancel{o}ther\ wise. \end{cases}$$

Since we used sampled signal $\Rightarrow r_x(\cancel{k}) = \hat{r}_x(k) = \frac{1}{N} \sum x(n)\, x(n-k)$

$$\begin{bmatrix} \hat{r}_x(0) & \hat{r}_x(1) & \cdots & \hat{r}_x(p-1) \\ \hat{r}_x(1) & & & \vdots \\ \vdots & & & \vdots \\ \hat{r}_x(p-1) & & & \hat{r}_x(0) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_p \end{bmatrix} \overset{r}{=} \begin{bmatrix} \hat{r}_x(1) \\ \vdots \\ \vdots \\ \hat{r}_x(p) \end{bmatrix}$$

and Power :

$$\hat{P}_x(e^{jw}) = |H(e^{jw})|^2 . \hat{P}_e(e^{jw})$$

$$\hat{P}_e(e^{jw}) = \hat{\sigma}_e^2$$

$$P_x(e^{jw}) = |H\, e^{(jw)}|^2 . \hat{\sigma}_e^2$$

$$H(e^{jw}) = \frac{1}{A(e^{jw})}$$

$$P_x(e^{jw}) = \frac{\hat{\sigma}_e^2}{|A(e^{jw})|^2}$$