PDF mean μ and variance.

$$\mu = E[X(1) + \int_0^1 X(r)\,dr]$$

$$= E[X(1)] + E[\int_0^1 X(s)\,ds]$$

$$= 0 + \int_0^1 E[X(s)]\,dr$$

$$\mu = 0 + 0 = 0$$

$$\sigma^2 = Var[X(1) + \int_0^1 X(s)\,ds]$$

$$= E[(X(1) + \int_0^1 X(s)\,d\;)^2] - \mu^2$$

$$= R_{xx}(\tau) - 0$$

$$\sigma^2 = R_{xx}(s,t)$$

$$= E[(X(1) + \int_0^1 X(s)\,ds)(X(1) + \int_0^1 X(t)\,dt)]$$

$$= E[X(1)^2] + E[X(1)\int_0^1 X(s)\,ds] + E[X(1)\int_0^1 X(s)\,ds] + E[\int_0^1\int_0^1 X(s)X(t)\,ds\,dt]$$

$$= 1 + 2\int_0^1 E[X(1)X(s)]\,ds + \int_0^1\int_0^1 E[X(s)X(t)]\,ds\,dt$$

$$= 1 + 2\int_0^1 R_{xx}(1-s)\,ds + \int_0^1\int_0^1 R_{xx}(s-t)\,ds\,dt$$

$$= 1 + 2 + R_{min}(s,t)$$

$$= 3 + R_{min}(s,t)$$

PDF: $s \; N(0, (3 + R_{min}(s,t)]$