Question: 1. A Markov chain (X, n 2 0 with states 0,1,2, has the following transition matrix [1/2 1/3 1/61 ...

1. A Markov chain $\{X_n, n \geq 0\}$ with states 0,1,2, has the following transition matrix

$$P_1 = \begin{bmatrix} 1/2 & 1/3 & 1/6 \\ 0 & 1/3 & 2/3 \\ 1/2 & 0 & 1/2 \end{bmatrix}$$

If $P\{X_0 = 0\} = P(X_0 = 1) = 1/4$, find $E(X_3)$

Hint: you need to compute P_1^3 first and compute $\pi_0 P_1^3$ to derive the distribution for X_3, then compute the expectation.

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Expert Answer

swapkkkkk answered this
299 answers

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Given transition probability matrix is

$$P_1 = \begin{bmatrix} 1/2 & 1/3 & 1/6 \\ 0 & 1/3 & 2/3 \\ 1/2 & 0 & 1/2 \end{bmatrix}$$

Now first we find P_1^2

$$P_1^2 = \begin{bmatrix} 1/2 & 1/3 & 1/6 \\ 0 & 1/3 & 2/3 \\ 1/2 & 0 & 1/2 \end{bmatrix} \begin{bmatrix} 1/2 & 1/3 & 1/6 \\ 0 & 1/3 & 2/3 \\ 1/2 & 0 & 1/2 \end{bmatrix}$$

$$P_1^2 = \begin{bmatrix} 1/3 & 5/18 & 7/18 \\ 1/3 & 1/9 & 5/9 \\ 1/2 & 1/6 & 1/3 \end{bmatrix}$$

Now,

$$P_1^3 = P_1 \cdot P_1^2$$

$$= \begin{bmatrix} 1/2 & 1/3 & 1/6 \\ 0 & 1/3 & 2/3 \\ 1/2 & 0 & 1/2 \end{bmatrix} \begin{bmatrix} 1/3 & 5/18 & 7/18 \\ 1/3 & 1/9 & 5/9 \\ 1/2 & 1/6 & 1/3 \end{bmatrix}$$

$$= \begin{bmatrix} \cancel{13/36} & \cancel{19/54} & \cancel{46/108} \\ & & \\ & & \end{bmatrix}$$

$$P_1^3 = \begin{bmatrix} 13/36 & 11/54 & 47/108 \\ 4/9 & 4/27 & 11/27 \\ 5/12 & 8/36 & 13/36 \end{bmatrix}$$

Now given that

$P(X_0 = 0) = P(X_0 = 1) = 1/4$

$\therefore P(X_0 = 2) = 1 - 2P(X_0 = 0)$

$= 1 - 2/4$

$= 2/4 = 1/2$

$\therefore P(X_0 = 2) = 1/2$

$\pi_0 = (1/4 \quad 1/4 \quad 1/2)$

Now we find $\pi_0 P_1^3$

$$\pi_0 P_1^3 = (1/4 \quad 1/4 \quad 1/2) \begin{bmatrix} 13/36 & 11/54 & 47/108 \\ 4/9 & 4/27 & 11/27 \\ 5/12 & 8/36 & 13/36 \end{bmatrix}$$

$$\pi_0 P_1^3 = \left(\frac{59}{144} \quad \frac{43}{216} \quad \frac{169}{432} \right)$$

That is,

$P(X_3 = 0) = \frac{59}{144}$, $P(X_3 = 1) = 43/216$,

$P(X_3 = 2) = 169/432$

Now

$E(X_3) = 0(59/144) + 1(43/216) + 2(169/432)$

$= 43/216 + 169/216 = 212/216$

$\therefore E(X_3) = 212/216 = 53/54$

$$\boxed{\therefore E(X_3) = 53/54}$$

Comment

Question: For the next five problems, consider a continuous-time Markov chain on the state space {1,2) with...

For the next five problems, consider a continuous-time Markov chain on the state space $\{1, 2\}$ with

$$A = \begin{bmatrix} -2 & 2 \\ 1 & -1 \end{bmatrix}$$

Assume it starts from the distribution $x(0) = [0.3 \quad 0.7]$.

Problem 15.4. Let T be the time this Markov chain spends at state 2 after getting there. Find the distribution of T, its expectation and variance.

Problem 15.5. Find the distribution $x(t)$ at time t.

Problem 15.6. Find its stationary distribution and rate of convergence.

Problem 15.7. Write the transition matrix for corresponding discrete-time Markov chain.

Problem 15.8. Find the stationary distribution for that discrete-time Markov chain.

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Expert Answer

Anonymous answered this
45 answers

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Problem 15.7 :

Transition matrix for corresponding discrete time Markov chain is as follows:

Pi,j = Ai,j /(-Ai,i) , i not equal to j

= 0 , o.w.

P= 0 1

1 0

Problem 15.8:

Stationary distribution of above transition matrix is as follows

π = (0.5 0.5)

Because , above transition matrix is doubly stochastic.

And when computing stationary distribution of such Markov chain just divid 1 by no. of state in Markov chain. And it is same for each state.

Comment

Question: Solve for the following probabilities (ranges of X values): a. &n...

Solve for the following probabilities (ranges of X values):	
a. $P(X \le 7)$ when m = 15	
b. $P(9 \le X \le 18)$ when m = 15	
c. $P(X \ge 15)$ when m = 15	
d. $P(12 \le X < 20)$ when m = 15	

(1) Solve the probabilities on the PROB worksheet. (To four decimal places) The P(X ≤7) when the poisson mean = 15 is ________.Assume a poisson distribution

(2) Solve the probabilities on the PROB worksheet. (To four decimal places) The P(9 ≤ X ≤ 18) when the poisson mean = 15 is ________.Assume a poisson distribution

(3) Solve the probabilities on the PROB worksheet. (To four decimal places) The P(X ≥ 15) when the poisson mean = 15 is________.Assume a poisson distribution

(4) Solve the probabilities on the PROB worksheet. (To four decimal places) The P(12 ≤ X < 20) when the poisson mean = 15 is ________.Assume a poisson distribution

(5) Which of the following Excel formulas can calculate P(10 ≤ X ≤ 20) when the poisson mean = 15 using the CUMULATIVE Distribution function (F(x))?

=poisson(20,15,1)-poisson(9,15,1)

=poisson(21,15,0)-poisson(10,15,0)

=poisson(21,15,1)-poisson(10,15,1)

=poisson(20,15,0)-poisson(9,15,0)

(6) Which of the following Excel formulas can calculate P(X ≥ 13) when the poisson mean = 15 using the PROBABILITY function (p(x))?

=sum(p(100):p(13)), where p(x) = poisson(x,15,0)

=1 - sum(p(13):p(100)), where p(x) = poisson(x,15,0)

=1 - sum(p(0):p(13)), where p(x) = poisson(x,15,0)

=1 - sum(p(0):p(12)), where p(x) = poisson(x,15,0)

Expert Answer

Anonymous answered this
572 answers

Was this answer helpful? 0 0

The poisson probability is given as:

$$P(X=x)=\frac{e^{-m}m^x}{x!}, x=0,1,2,.....\infty$$

1)

$$P(X\le 7)=P(X=0)+P(X=1)+....+P(X=7)$$

P(X=0)	3.05902E-07
P(X=1)	4.58853E-06
P(X=2)	3.4414E-05
P(X=3)	0.00017207
P(X=4)	0.000645263
P(X=5)	0.001935788
P(X=6)	0.00483947
P(X=7)	0.010370294
Total=	0.018002193

$$P(X\le 7)=0.0180$$

Excel formula used :
Poisson(x,15,0) where x =0,1,.....7
and then summing all the probabilities
Alternate formula:
Poisson(7,15,1) will give the direct result

2)

$$P(9\le X\le 18)=P(X=9)+P(X=10)+.........P(X=18)$$

P(X=9)	0.032407
P(X=10)	0.048611
P(X=11)	0.066287
P(X=12)	0.082859
P(X=13)	0.095607
P(X=14)	0.102436
P(X=15)	0.102436
P(X=16)	0.096034
P(X=17)	0.084736
P(X=18)	0.070613
Total	0.782025

$$P(9\le X\le 18)=0.7820$$

Excel formula used :
Poisson(x,15,0) where x =9,10,.....18
and then summing all the probabilities
Alternate formula:
Poisson(18,15,1)-Poisson(8,15,1) will give the direct result.

3)

$$P(X\ge 15)=1-P(X<15)=1-P(X\le 14)$$

$$=1-[P(X=0)+P(X=1)+...+P(X=14)]$$

P(X=0)	3.06E-07
P(X=1)	4.59E-06
P(X=2)	3.44E-05
P(X=3)	0.000172
P(X=4)	0.000645
P(X=5)	0.001936
P(X=6)	0.004839
P(X=7)	0.01037
P(X=8)	0.019444
P(X=9)	0.032407
P(X=10)	0.048611
P(X=11)	0.066287
P(X=12)	0.082859
P(X=13)	0.095607
P(X=14)	0.102436
Total=	0.465654

$$P(X\ge 15)=1-P(X<15)=1-P(X\le 14)=1-0.4657=0.5343$$

Excel formula used to calculate probability upto 14 :
Poisson(x,15,0) where x =0,1,.....14
and then summing all the probabilities and subtracting from 1

Alternate formula:
1-Poisson(14,15,1) will give the direct result

4)

$$P(12\le X\le 20)=P(X=12)+P(X=13)+....+P(X=20)$$

12	P(X=12)	0.082859
13	P(X=13)	0.095607
14	P(X=14)	0.102436
15	P(X=15)	0.102436
16	P(X=16)	0.096034
17	P(X=17)	0.084736
18	P(X=18)	0.070613
19	P(X=19)	0.055747
20	P(X=20)	0.04181
	Total	0.732277

$$P(12\le X\le 20)=0.7323$$

Excel formula used :
Poisson(x,15,0) where x =12,13,.....20
and then summing all the probabilities
Alternate formula:
Poisson(20,15,1)-Poisson(11,15,1) will give the direct result.

(5) Which of the following Excel formulas can calculate P(10 ≤ X ≤ 20) when the poisson mean = 15 using the CUMULATIVE Distribution function (F(x))?
=poisson(20,15,1)-poisson(9,15,1)

(6) Which of the following Excel formulas can calculate P(X ≥ 13) when the poisson mean = 15 using the PROBABILITY function (p(x))?

=1 - sum(p(0):p(12)), where p(x) = poisson(x,15,0)

Comment

Question: Let X1 and X2 be independent Poisson random variables with respective means lambda1 = 2 and...

Let $X1$ and $X2$ be independent Poisson random variables with respective means $\lambda 1 = 2$ and $\lambda 2 = 3$. Find

(a) $P(X1 = 3, X2 = 5)$.

(b) $P(X1 + X2 = 1)$. Hint. Note that this event can occur if and only if $\{X1 = 1, X2 = 0\}$ or $\{X1 = 0, X2 = 1\}$.

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Expert Answer

rajsahoo answered this
16,299 answers

Was this answer helpful? 0 0

$P(x) = e^{-\mu} * \mu^x / x!$

a) $P(X_1 = 3 \text{ and } X_2 = 5) = P(X_1 = 3) * P(X_2 = 5)$

$= e^{-2} * 2^3 / 3! * e^{-3} * 3^5 / 5!$

$= 0.2240 * 0.1088$

$= 0.0244$

b) $P(X_1 + X_2 = 1) = P(X_1 = 1) * P(X_2 = 0) + P(X_1 = 0) * P(X_2 = 1)$

$= e^{-2} * 2^1 / 1! * e^{-2} * 3^0 / 0! + e^{-2} * 2^0 / 0! * e^{-3} * 3^1 / 1!$

$= 0.2707 * 0.0498 + 0.1353 * 0.1494$

$= 0.0337$

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