Laboration 1

MVE136, Random Signal Analysis

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Task 2

2.1

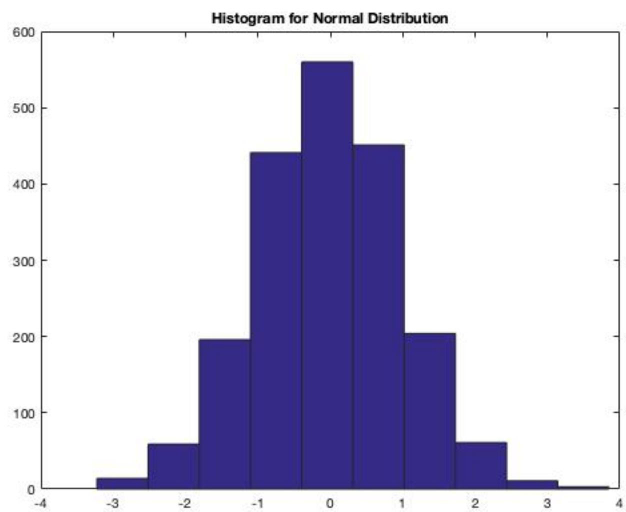

Figure 1: Histogram for 2000 random variables with Normal Distribution

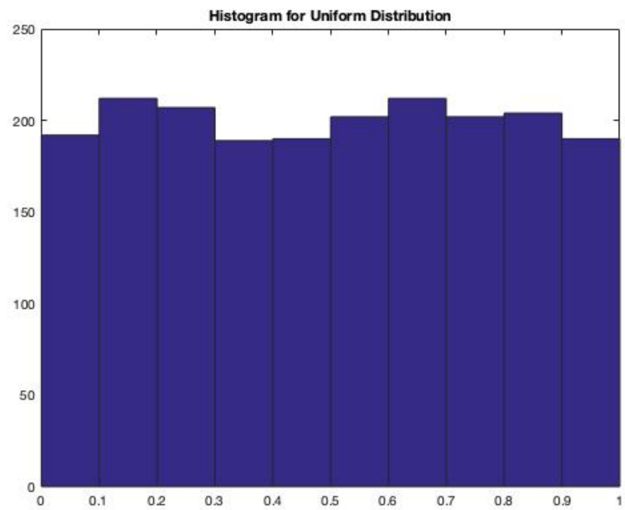

Figure 2: Histogram for 2000 random variables with Uniform Distribution

2.2

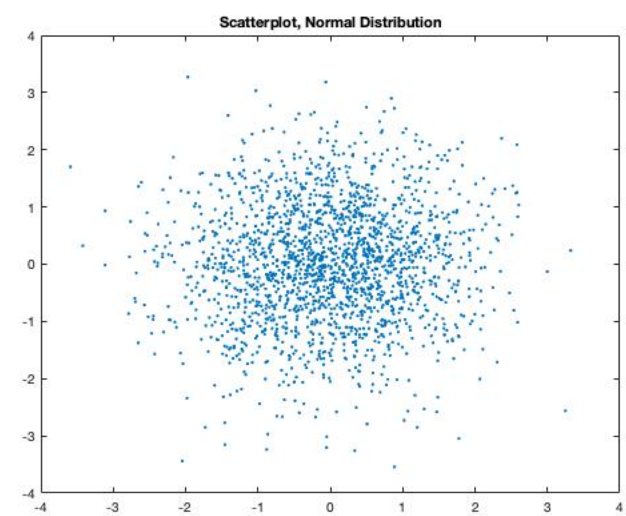

Figure 3: Scatterplot showing the joint distribution of two vectors with 2000 random variables each with Normal distribution

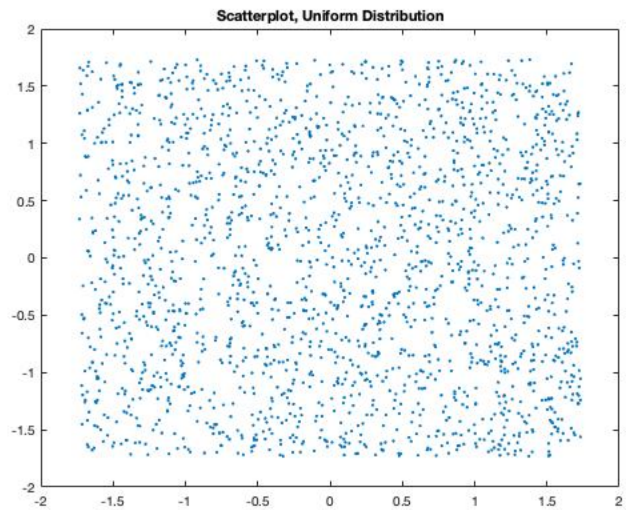

Figure 4: Scatterplot showing the joint distribution of two vectors with 2000 random variables each with Uniform distribution

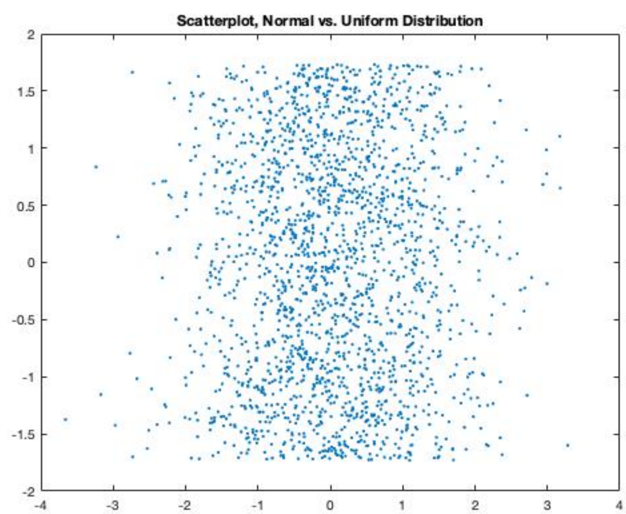

Figure 5: Scatterplot showing the joint distribution of two vectors with 2000 random variables each with Normal distribution along the x-axis and Uniform distribution along the y-axis

In figure 3 and 4 the joint distribution of to vectors containing random variables are shown. Figure 3 shows a scatterplot where the variables are of Normal distribution. The shape of "equiprobability" lines is a circle with the center at (0,0). Figure 4 shows a scatterplot with Uniform distribution. In this plot the "equiprobability" lines are not distinguishable. In figure 5 ascatterplot with to vectors where one has Normal distribution and one have Uniform distribution are illustrated. The Uniform distribution is along the y-axis. It can be seen that the plot has a Uniform distribution in this direction. Along the x-axis is the Normal distribution.

2.3

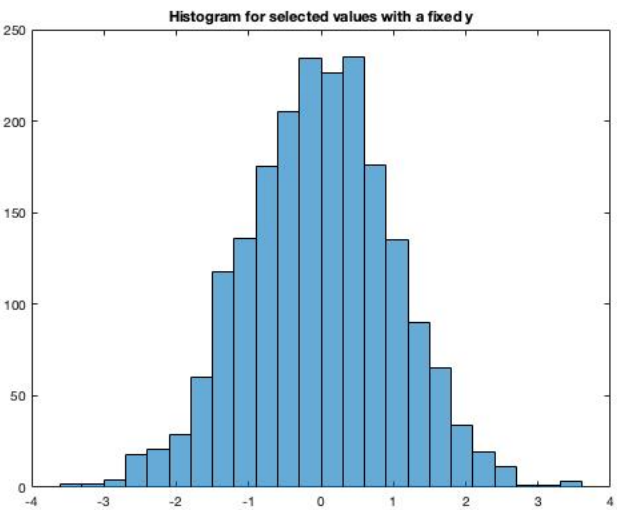

Figure 6: Histogram of selected samples of Normal distribution with a fixed y

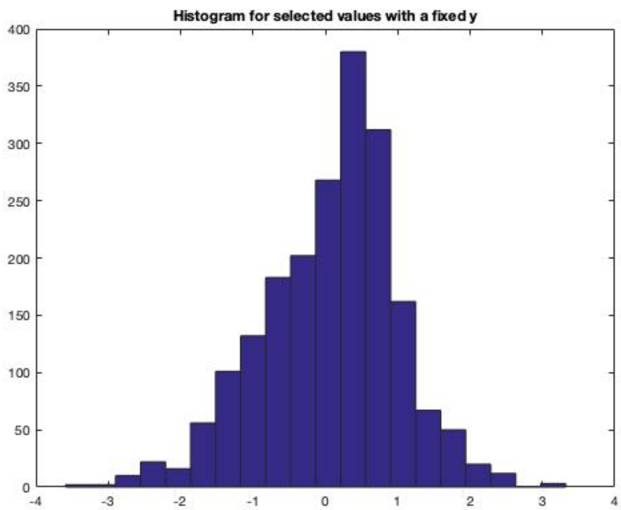

Figure 7: Histogram of selected samples of Uniform distribution with a fixed u

Figure 6 shows a histogram of Normal distribution with a fixed y. The shape is similar as without the condition of the fixed y. The difference is the values on the y-axis. This is expected since not all values are within the limited y. Figure 7 illustrates a histogram for random variables with Uniform distribution but with a fixed u.

Task 3

3.1 a

Theoretical caluctlation

$$z = \alpha x + \sqrt{1 - \alpha^2}$$
$$\mu_x = E[x] = 0, \quad \mu_y = E[y] = 0$$

$$E[Z] = \alpha E[x] + \sqrt{1-\alpha^2}E[y] = 0$$
$$\mu_z = 0$$
z is also N(0,1)
$$R_x z = E[xz] = E[(\alpha x + \sqrt{1-\alpha^2} \quad y) * (\alpha x)]$$
$$R_x z = \alpha^2 E[x^2] + \alpha\sqrt{1-\alpha^2} \quad E[yx]$$
we knew x and y independent and E[xˆ2]=1
$$R_x z = \alpha^2$$
$$R_x z = \alpha$$

3.1 b

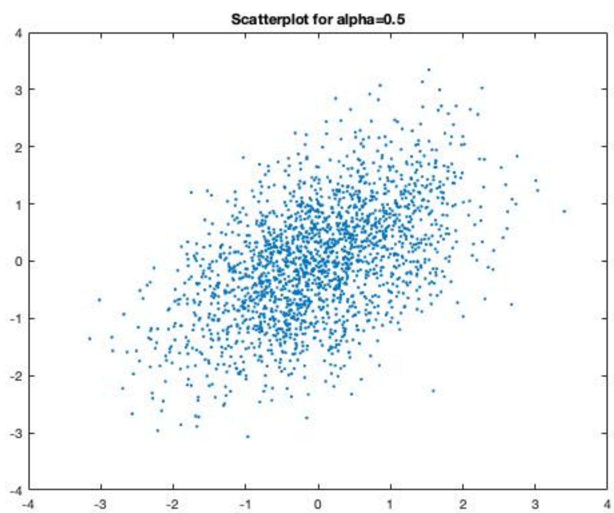

Figure 8: Scatterplot when α=0.5

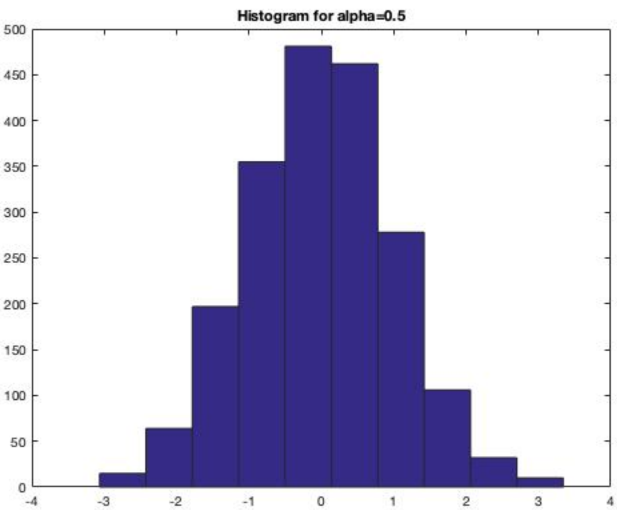

Figure 9: Histogram when α=0.5

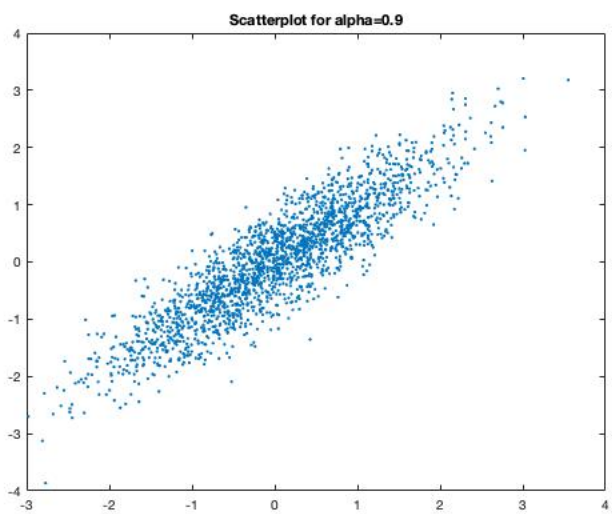

Figure 10: Scatterplot when α=0.9

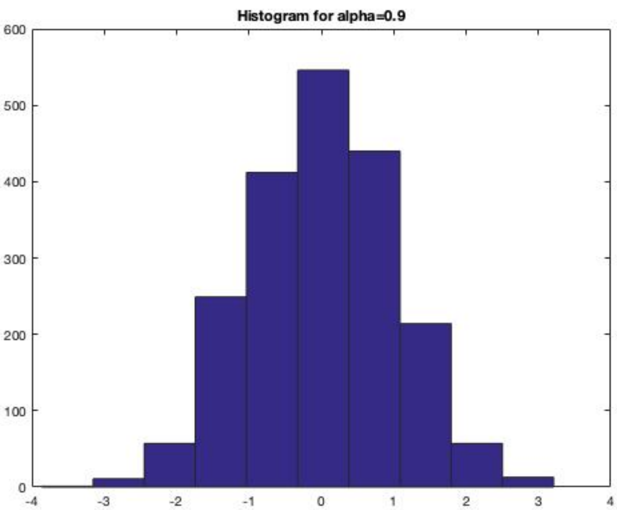

Figure 11: Histogram when α=0.9

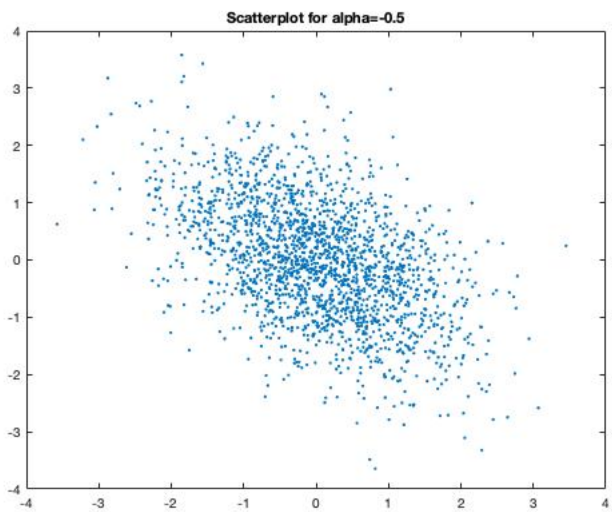

Figure 12: Scatterplot when α=-0.5

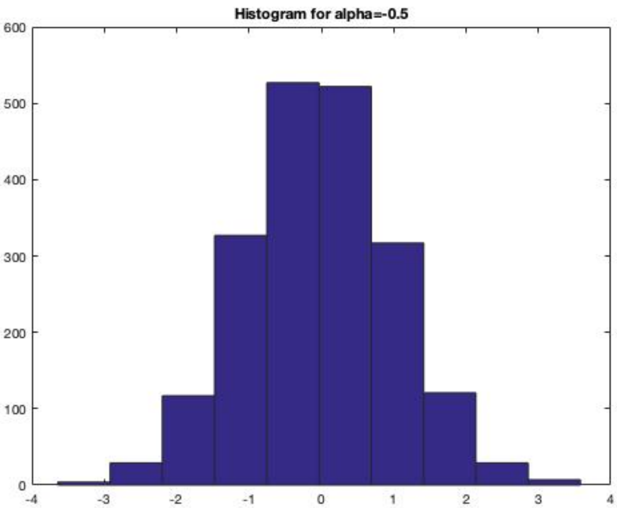

Figure 13: Histogram when α=-0.5

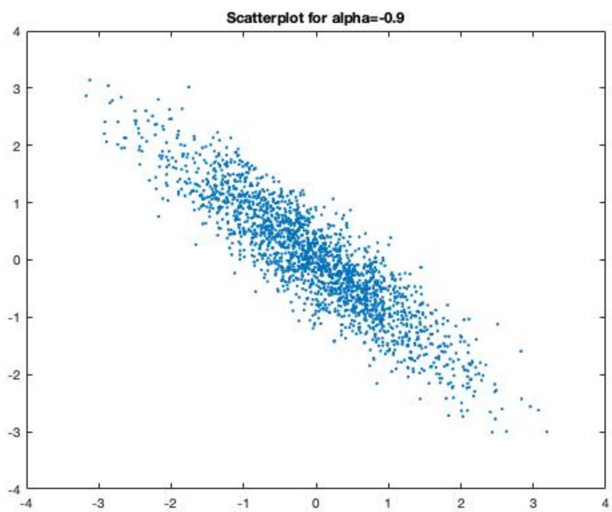

Figure 14: Scatterplot when α=-0.9

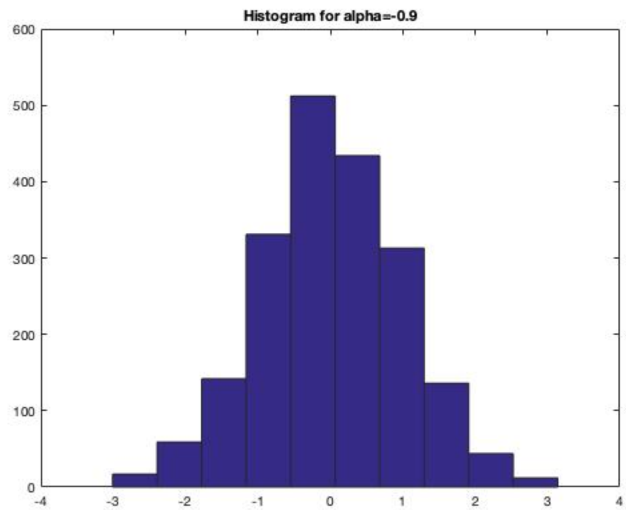

Figure 15: Histogram when α=-0.9

Figure 11 shows a histogram of Normal distribution of z with α =-0.9. The shape show clearly at α=0.9 the mean value is zero. The simple way to interpret correlation between two random variables is by decreasing the correlation coefficient and here by making $\alpha = 0$

3.2

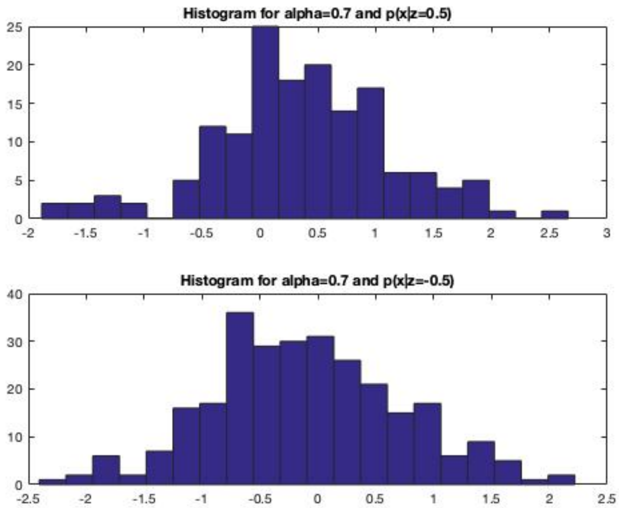

Figure 16: Two histograms for α=0.7 and p(x—y=0.5) (above) and p(x—y=-0.5) (below).

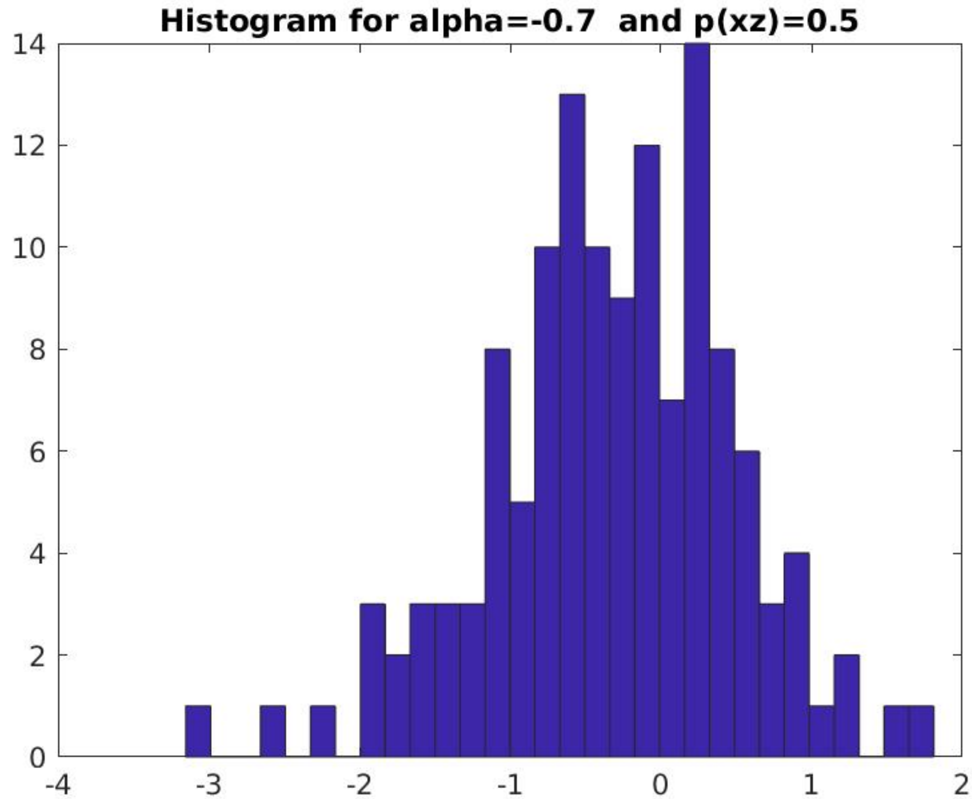

Figure 17: Histogram for α=-0.7 and p(x—y=0.5)

It is realized when α = 0.7 and p=0.5 strong positive correlation relationship between two x and z while α = 0.7 and p(x—y=-0.5) we have a negative relationship between x and z and the slope direction to minus in the same as the direction to plus when α = -0.7 and p(x—y=0.5).
So the Positive coefficients indicate that both x and z value are positive while negative coefficients indicate that one of them is negative.

Task 4

4.1

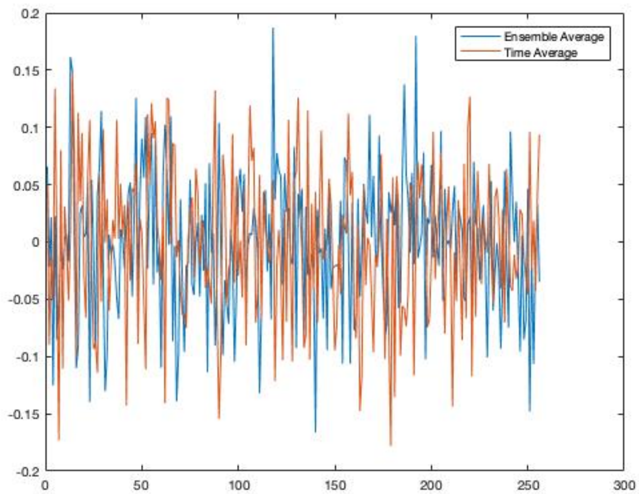

Figure 18: Graph showing the Ensemble average and Time average of a random process

Figure 18 show that both $\mu_x[n]$ and $x_k(n)$ are ergodic in the mean because of the limit $\mu_x[n]$ approaches the limit of $x_k(n)$

4.2

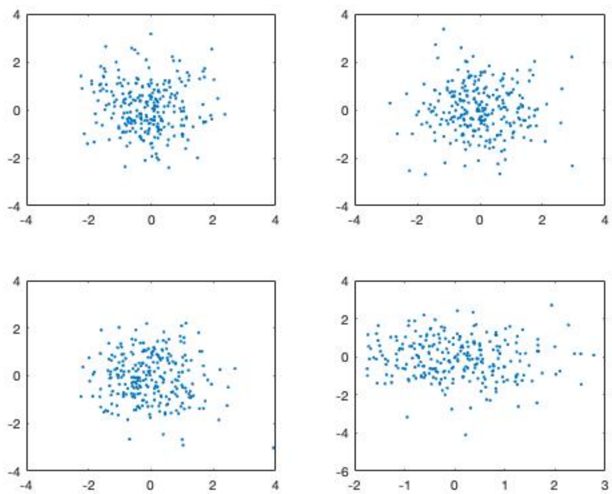

Figure 19: Scatterplots for four different values of n that are randomly generated

$$
\begin{aligned}
r_x(n1, n2) = 1/k \sum_{k}^{\infty} x_k(n_1)x_k(n_2) \\
= E[x(n_1)x(n_2)] = E[x(n_1)]E[x(n_2)] \\
= \mu_x(n_1)\mu_x(n_2) \\
\mu_x(n_1) = \lim_{k\to\infty} 1/k \sum_{k}^{\infty} x_k(n_1) \\
\mu_x(n_2) = \lim_{k\to\infty} 1/k \sum_{k}^{\infty} x_k(n_2) \\
= 0 \quad \textit{uncorrelated}
\end{aligned}
$$

When calculating the ensemble average according to the formula given the result is $\hat{r}_x(n1, n2) = -0.06$

Task 5

$$x[n] = x[n-1] + w[n]$$

where w[n] is N(0,1) WGN

5.1

It is known that the initial value of x is 0 and that the mean of w[n] for all n is zero since it is white noise.

$$\begin{aligned}\mu_x[n] = E[x[n]] = E[x[n-1] + w[n]] =\\ E[x[n-2] + w[n-1] + w[n]] =\\ E[x[0] + \sum_{a=1}^{n} w[a]] = E[\sum_{a=1}^{n} w[a]] = 0\end{aligned}$$

5.2

It is known that
$x[0] = 0, \quad x[1] = w[1], \quad E[x^2[1]] = \sigma_w^2 = 1, \quad E[x[n-l]w[n]] = 0$

$$\begin{aligned}P[n] = E[x^2[n]] = E[(x[n-1] + w[n])^2] =\\ E[x^2[n-1] + 2x[n-1]w[n] + w^2[n]] =\\ E[x^2[n-1] + w^2[n]] = P[n-1] + E[w^2[n]]\\ = P[n-1] + 1 = n\end{aligned}$$

5.3

$$\begin{aligned}r_x(n, n-1) = E[x[n]x[n-1]] =\\ E[(x[n-1] + w[n])x[n-1]] =\\ E[x^2[n-1] + w[n]x[n-1]] = E[x^2[n-1]]\end{aligned}$$

$$\begin{aligned}r_x(n, n-2) = E[x[n]x[n-2]] =\\ E[(x[n-1] + w[n])x[n-1]] =\\ E[(x[n-2] + w[n-1] + w[n])x[n-1]] =\\ E[x^2[n-2]]\end{aligned}$$

$$r_x(n, n-l) = E[x^2[n-l]]$$

So it is WSS since only change is l

$$\rho_x(n, n-l) = \frac{r_x(n, n-l)}{\sqrt{P_x(n)P_x(n-l)}} = \frac{E[x^2[n-l]]}{\sqrt{n(n-1)}} = \frac{n-l}{\sqrt{n(n-1)}}$$

5.4

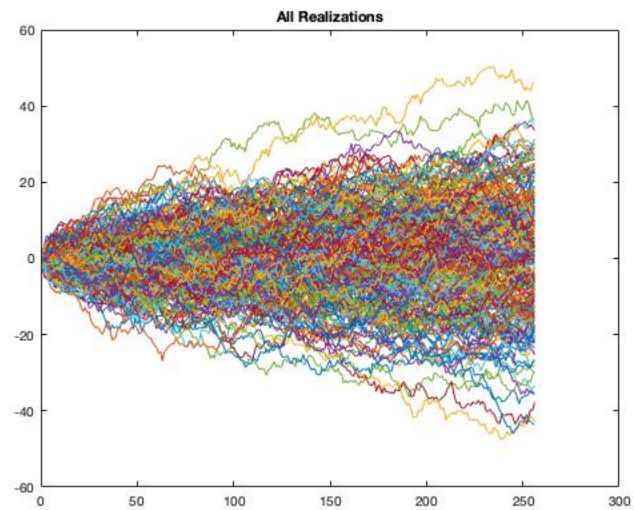

Figure 20: All realizations of a matrix when the columns is generated.

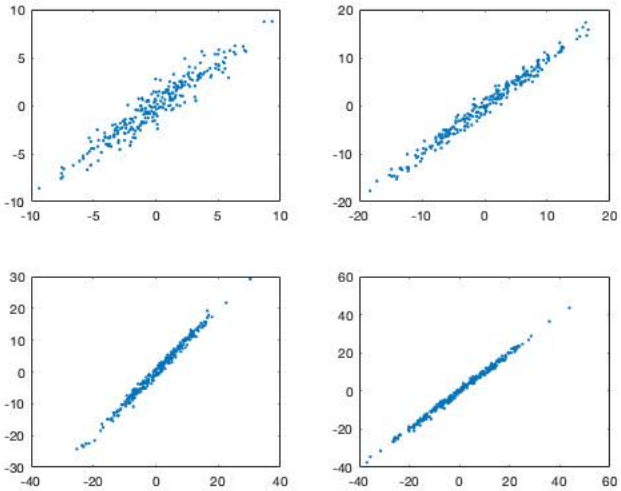

Figure 21: Scatterplots for four different values of n1 and n2. Top-left: (n1,n2)=(10,9), Top-right: (n1,n2)=(50,49). Bottom-left: (n1,n2)=(100,99). Bottom-right: (n1,n2)=(200,199)

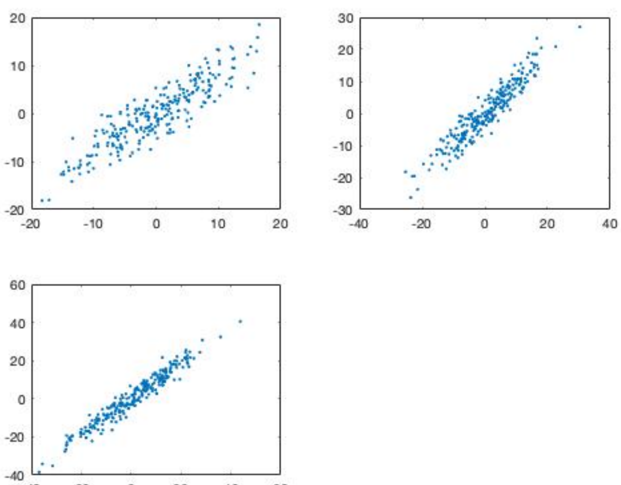

Figure 22: Scatterplots for three different values of n1 and n2. Top-left: (n1,n2)=(50,40), Top-right: (n1,n2)=(100,90). Bottom-left: (n1,n2)=(200,190).

5.5

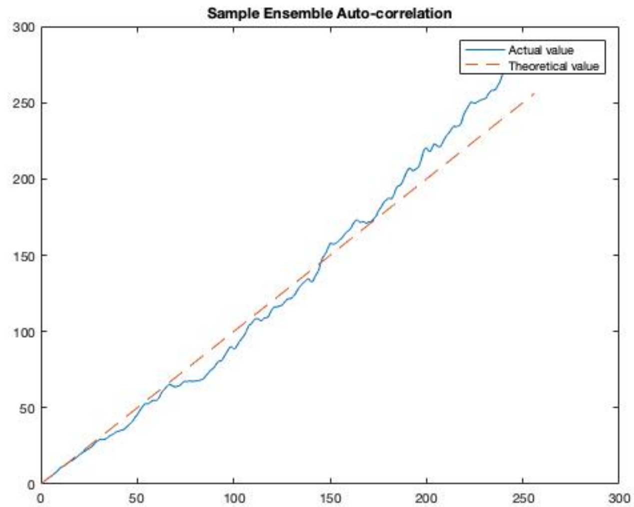

Figure 23: Graph showing the theoretical and generated auto-correlation

Task 6

6.1

$$\begin{aligned} P_x[n] = E[x^2[n]] = E[(0.9x[n-1] + w[n])^2] = \\ E[0.9^2x^2[n-1] + 2 \cdot 0.9x[n-1]w[n] + w^2[n]] = \\ E[0.9^2x^2[n-1] + w^2[n]] = E[0.9^2x^2[n-1]] + E[w^2[n]] = \\ 0.9^2E[x^2[n-1]] + 1 = 0.9^2P_x[n-1] + 1 = 1 + \sum_{i=1}^{n-1} 0.9^i \end{aligned}$$

6.2

$$
\begin{aligned}
r_x(n, n-l) = E[x[n]x[n-l]] = \\
E[(0.9x[n-1] + w[n])x[n-l]] = \\
E[(0.9^2x[n-2] + 0.9w[n-1] + w[n])x[n-l]] = \\
E[(0.9^l x[n-l] + \sum_{i=l+1}^{n} 0.9^{n-i}w[i])x[n-l]] = \\
E[0.9^l x^2[n-l]] = 0.9^l E[x^2[n-l]] = \\
0.9^l \sum_{i=0}^{n-1} 0.9^i
\end{aligned}
$$

6.3

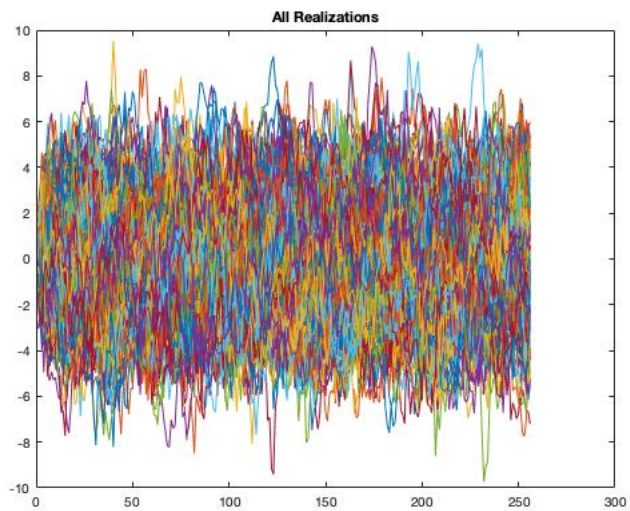

Figure 24: All realizations of a matrix when the columns is generated.

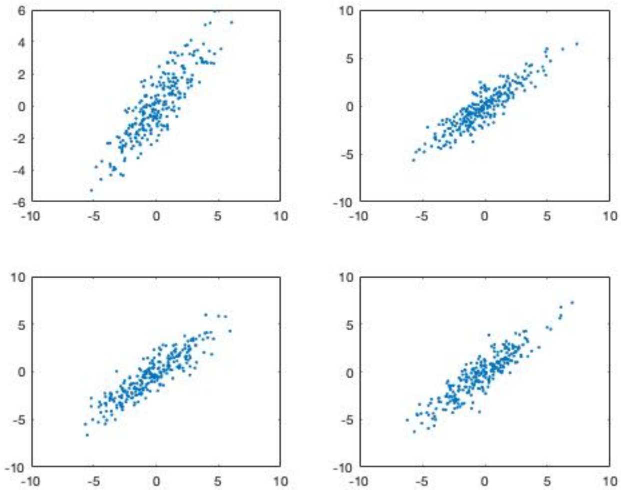

Figure 25: Scatterplots for four different values of n1 and n2. Top-left: (n1,n2)=(10,9), Top-right: (n1,n2)=(50,49). Bottom-left: (n1,n2)=(100,99). Bottom-right: (n1,n2)=(200,199)

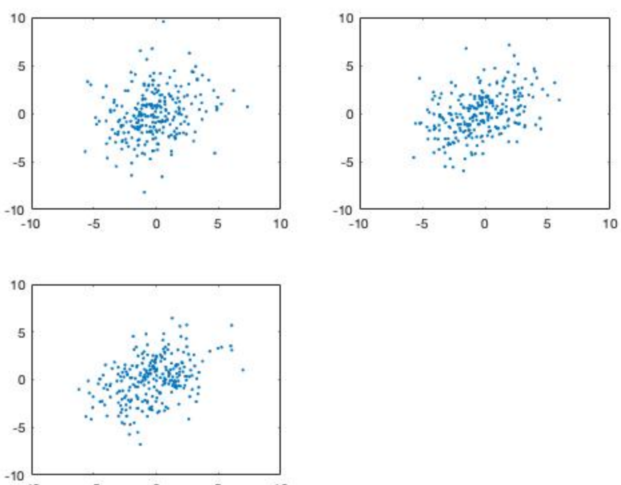

Figure 26: Scatterplots for three different values of n1 and n2. Top-left: (n1,n2)=(50,40), Top-right: (n1,n2)=(100,90). Bottom-left: (n1,n2)=(200,190).

6.4

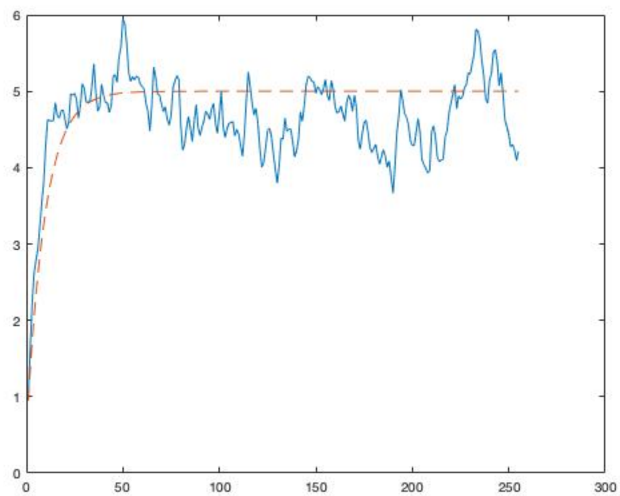

Figure 27: Graph showing the theoretical and generated auto-correlation when l=1