

ESS101 Modelling and Simulation

# Hand-In Assignment

## Modelling and simulation of robot-arm

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Use this page as cover for your final report!

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# 1 Introduction

The assignment consists of modeling a laboratory process using different modelling techniques and implementing the developed models in a simulation environment. The laboratory process is a robot arm.



Figure 1: Photo of the robot arm.

## 1.1 Aim

The aim of this assignment is to show how different methods can be used for developing a mathematical model of a system.

## 1.2 Learning outcome

The learning outcomes from this assignment are

- learning methods and tools to develop mathematical models of dynamical systems by using basic physical laws or measurement data.
- implementing mathematical models in computer simulation tools.
- critically assessing model quality and model simplification.
- describing and motivating technical solutions by written documentation.

### **1.3 Outline**

The assignment is divided into two parts, focusing on physical modeling and system identification, respectively, with the objective of modeling a process by using physical insights and experimental data. The first part will be mostly theoretical, while the second part will require access and usage of the real process along with data acquisition hardware. The models developed in the assignment will be simulated in MATLAB/SIMULINK, available on the whole campus. In the second part of the assignment you will use the real lab process. The lab process is located in S2's lab on the fifth floor in the EDIT building. Booking lists will be available on the course homepage.

## 2 Physical modelling

### 2.1 Description of the system

The process available considered in this lab assignment is shown in Figure 2 and consists of a flexible link that is attached to two identical springs mounted on a rotating chassis. The chassis is moved by an electric DC motor, through an integrated gearbox, mounted on a fixed chassis (fundament).

A potentiometer measures the angular position of the arm relative to the rotating chassis, while a second potentiometer measures the angle of the chassis relative to the fundament.

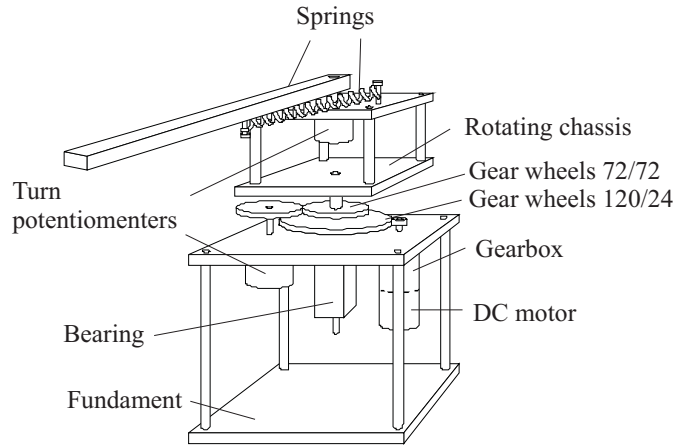


Figure 2: Schematic figure of the process.

### 2.2 Model

As shown in Figure 3 we can identify an electrical and a mechanical part, briefly described next.

#### 2.2.1 Mechanical System

The input to the mechanical system is the torque  $Q_m$  from the motor, and the outputs are the angular positions  $\Theta_1$  and  $\alpha$  (see Figure 5), which are measured by the potentiometers.

The mechanical system can be represented by two inertias  $J_1$  and  $J_2$  connected by a spring with constant  $K_{Lin}$ .  $J_1$  lumps together the inertias

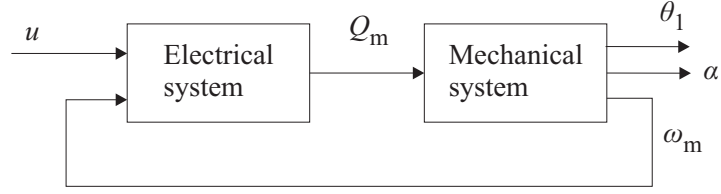


Figure 3: Block diagram illustrating the connection between the electrical system and the mechanical system.

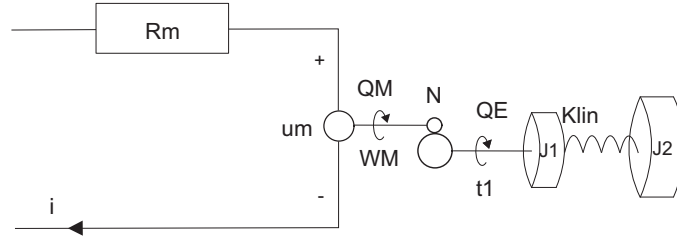


Figure 4: Illustration of the physical model.

of the motor, gearbox, gear wheels and the rotating chassis, while  $J_2$  only includes the arm.

### 2.2.2 Electrical System

A simplified model for the electrical part can be used, consisting of a resistance  $R_m$  and the induced back e.m.f voltage  $u_m$  (see Figure 5). The torque  $Q_m$  from the electric motor can be calculated as  $Q_m = iK_m$ , where  $K_m$  is the torque constant of the motor and  $i$  is the current. Note that the torque  $Q_m$  of the motor is not the same as  $Q_E$  because of the gear box and gear wheel ratios.

## 2.3 Assignments

The main assignment is to make a mathematical model of the robot arm process, implement it in SIMULINK and simulate it. In order to facilitate and structure the work, solve the following assignments.

**Assignment 2.1:** Use the three phase method in order to formulate a mathematical model in the state space form of the whole system.



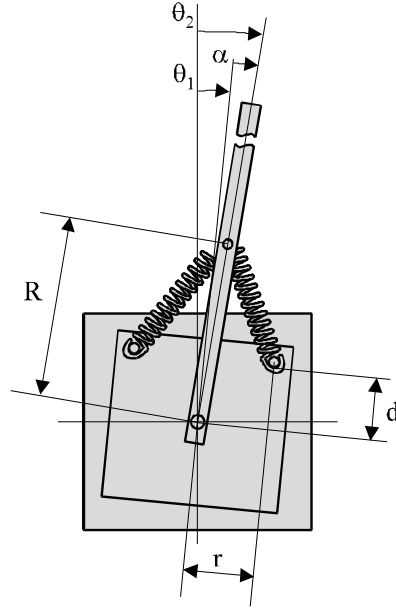


Figure 5: Angles and distances of the robot arm (from above).

**Assignment 2.2:** Assign the following values to the system parameters in your model. Implement the model in SIMULINK and simulate it. Simulate a step response and compare the steady state value of  $\dot{\theta}_1$  with the value predicted by the model derived in Assignment 2.1. By analyzing your model, can you find an explanation for the slope of the step response at time 0 that you have obtained in simulation?

| $J_1(Kg\ m^2)$       | $J_2(Kg\ m^2)$       | N  | $K_{Lin}$ | $R_m$ | $K_m(Nm/A)$             |
|----------------------|----------------------|----|-----------|-------|-------------------------|
| $1.7 \times 10^{-3}$ | $4.8 \times 10^{-3}$ | 64 | 1.19      | 3.21  | $8.39 \times 10^{-3}$ . |

**Assignment 2.3:** Change the values of two system parameters of your choice and re-do simulations as in Assignment 2.2. Compare and comment the results.

## 3 System identification

In this part of the assignment the robot arm process will be modeled by using system identification techniques. The main assignment is to develop a “good” parametric model of the robot arm.

To be able to do this part you will have access to the real lab process.

### 3.1 Sequence of work

System identification consists of several steps:

1. Choice of sampling interval
2. Choice of input signal
3. Data collection
4. Posttreatment of data
5. Choice of model structure
6. Model validation

Normally the last two steps need to be repeated several times until you obtain satisfactory results. It can even happen that you need to redo your data collection due to a bad choice of sampling interval or input signal. **In this case you may want to contact TAs who may provide you good data sets.** Data collection, including choice of sampling interval or input signal, are time consuming steps. Based on the set of data that you have collected it is relatively easy and straight forward to get a large set of models to evaluate using the SYSTEM IDENTIFICATION TOOLBOX (SITB) in MATLAB.

#### 3.1.1 Choice of sampling interval

Criteria for choosing the sampling interval are discussed in Section 10.2 in the textbook. A step response can be performed to get a feeling of the systems time constants, pure time delays and static gain. A typical choice of sampling frequency is 20 times the systems bandwidth,  $\omega_S = 20\omega_B$ . The bandwidth  $\omega_B$  is approximately equal to  $1/\tau$ , where  $\tau$  is the systems time constant, i.e.

$$\omega_S = 20\omega_B \approx \frac{20}{\tau}.$$

The systems time constant can be estimated from the step response as the time it take for the step response to reach 63% of its final value (disregard all pure time delays). An alternative method is to choose the sampling interval such that you get 7-10 samples during the transient in the step response.

### 3.1.2 Choice of input signal

It is important to choose a signal which excites as many modes of the system as possible. The energy of the input signal should be spread over as many frequencies as possible. The best (ideal) choice would be using a white noise as input signal because the spectra, for a white noise, is constant over all frequencies.

To create noise signals in MATLAB use the functions `rand` and `randn`. Another standard choice is to use a so-called telegraph signal which randomly switches between two levels. In MATLAB can the following function be used to create a telegraph signal:

```
function u=telegraph(N,a)
%
% N = number of samples
% a = switching probability
%
x=rand(N,1); u(1)=(-1)^(round(x(1))); for i=2:N
    u(i)=u(i-1)*(-1)^(x(i)<a);
end
```

As input to the process, the telegraph signal is suitable as input signal.

### 3.1.3 Data collection

In general this step requires a lot of work with the communication to the process, data acquisition, I/O cards etc. Fortunately, for our system this has already been configured.

**Note** The measured output from the system is  $\theta_1$ .

### 3.1.4 Posttreatment of data

Once data has been collected, the first thing to do is visualize it to find out deficiencies. It might be that the signal levels drift away or that there are obvious fault values (so-called outliers) among the data. It is also important to remove trends and means by using the commands in SYSTEM IDENTIFICATION TOOLBOX (SITB).

### 3.1.5 Choice of model structure

To choose a model structure that is suitable for identification is perhaps the most difficult decision to make. The choice involves many aspects, including model structure (ARX, OE, ARMAX, or BJ) and order. Using SITB makes testing a large set of model structures and orders relatively easy. Using knowledge from physical insight might ease the selection of model structure and order.

### 3.1.6 Model validation

Model validation can be done by, for instance, comparing spectral analysis estimates, plotting poles and zeros, studying the residuals and simulating the model and comparing it with the output from the real process. To be able to perform the last test it is important not to use the same data used for identifying the model parameters. Therefore you have to

1. Create a new input signal vector
2. Simulate the developed model with the new input signal and plot the result
3. Collect a new data set using the new input signal
4. Plot the results and compare!

Testing the model against “new” data is called cross validation and is probably the most effective way to determine whether a model is good enough or not. In general, the model should be exposed to as many tests as possible in order to minimize the risk of a “bad” model.

## 3.2 Data Acquisition

Start MATLAB and on the desktop you will find MoS folder. Open `calib.m` and `calibration.mdl`, these files are used for calibrating the system. After calibration, open `blackBox.mdl` and `greybox.mdl` to collect data.

The sampling time is set by choosing step size for the numerical solver. Make sure that the step size is set to fixed. Generate an input signal vector `u` and a time vector `t` (both column vectors). The time vector shall include the sampling times, and must agree with the chosen step size. When this is done, simulate the SIMULINK model. After the simulation is completed, the output signal `y` is available in the MATLAB workspace.

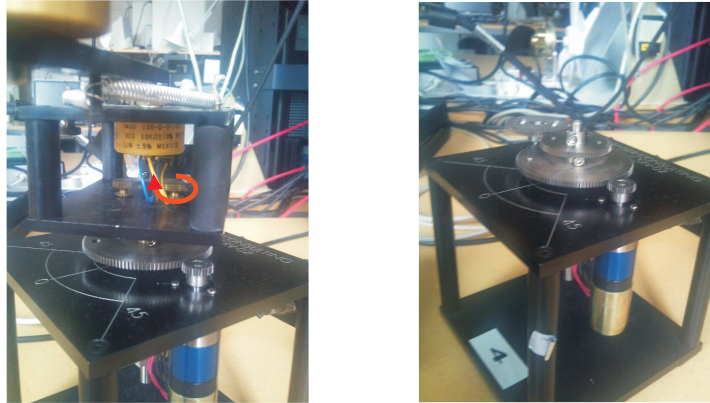


Figure 6: Removing the rotating part.

Warning: To get the correct sampling times it is important to not execute any commands (e.g., moving the mouse) during simulation, since Windows is not a real time system.

### 3.3 Assignments

**Assignment 3.1:** Show how any of the non-parametric system identification methods you have studied can be used to choose the sampling time.

**Assignment 3.2:** Show how any of the non-parametric system identification methods you have studied can be used to motivate the choice of model parametrization in the Assignment 3.3.

**Assignment 3.3:** (Black box modeling) Choose the best parametric model,  $G(z) = \frac{\theta_1(z)}{U(z)}$  and identify the parameters using the collected data.

**OBS!** The input signal must be in the interval 0-5 V in order not to destroy the lab process.

**Assignment 3.4:** Implement the obtained parametric model in SIMULINK and simulate it with the input signal you have collected. Compare your simulation results with the simulation results from the physical modeling. What are the differences and similarities? Which model is the best?

**Assignment 3.5:** (Grey box modeling) Consider the system obtained by removing the rotating part according to Figure. 6. The system can be described by Figure. 7. Identify the inertia,  $J$  which should be approximately equal to  $J_1$ , in the system.

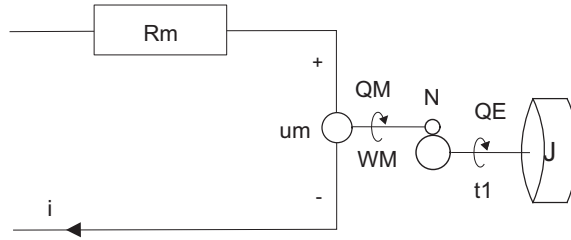


Figure 7: The system without the rotating part.

**Hint!** You may neglect the friction.

**Assignment 3.6:** *Experiment 1.* Identify again the inertia  $J$  on two different data sets, obtained by splitting the original data set used in Assignment 3.5. *Experiment 2.* Identify once more the inertia  $J$  on four different data sets, obtained by splitting the data sets used in Experiment 1. What's the trend you observe in the covariance of the estimated parameters in the two experiments? How do you explain it?

**Assignment 3.7:** The robot-arm operates in open-loop. I.e., there is not feedback control loop. How do you infer this by analyzing the input and output data?

**Assignment 3.8:** The system does not have any pure input delay. How do you infer this by analyzing the input and output data?

## 4 Numerical simulation

Consider the mathematical model in the state space you have developed in Assignment 2.1. Assume the input signal to be a sinusoidal with a frequency of your choice and simulate the system with the following methods. In particular,

**Assignment 4.1:** Implement the Forward Euler integration method and compare the obtained solution with the exact one. Determine the minimum step size that preserve the absolute stability and motivate our answer. Vary the step size and explain the obtained results.

**Assignment 4.2:** Implement the AB(2) method and compare the obtained solution with the exact one. Chose a step size and motivate your choice. Vary the step size and explain the obtained results.

**Assignment 4.3:** Implement the Trapezoidal rule and compare the obtained solution with the exact one. Chose a step size and motivate your choice. Vary the step size and explain the obtained results.

**Assignment 4.4:** Implement a two-stage RK method and compare the obtained solution with the exact one. Chose a step size and motivate your choice. Vary the step size and explain the obtained results.

## 5 Final report

The final report, including figures, cannot exceed 10 pages.

The report should include clear motivations to all the choices made when solving the assignment. You should also comment on what simplifications you have made. Be as clear as possible.