Exam in SSY097

June 7, 2017

Allowed materials: Pencil, eraser.

The exam consists of six problems. Make sure that you have them all.

- Motivate all answers carefully.
- Use a new paper for each new numbered problem.
- Only write on one side of the papers only.
- Write your anonymous number on each new page.
- Avoid using a red pen.
- If you want the result registered as SSY096, write this on the cover page.

Grades

≥ 8 **points** Grade: 3

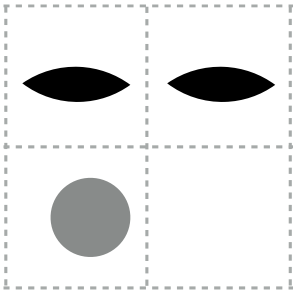
≥ 11 **points** Grade: 4

≥ 14 **points** Grade: 5

1 SIFT, 3 points

A SIFT-like descriptor (as the one in Lab 1) was computed for the following image patch Q. The regions used are indicated with grey dashed lines so these lines are not part of the actual image. Note that unlike for original SIFT only four regions are used.

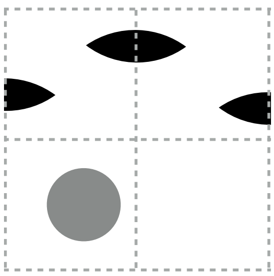
(a) The SIFT-like descriptor for this image patch is a single vector. How many elements does this vector have?



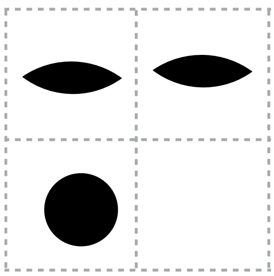
Q

(b) Show (for example using vector bouquets) what the SIFT-like descriptor for Q will look like.

(c) Which of the two following image patches, A or B, will produce a SIFT-like descriptor most similar to that for Q. Motivate your answer carefully.



A



B

2 Statistical learning, 3 points

(a) When training a general image classifier, the training set can consist of millions of images and the loss function is a sum

$$L(\theta) = \sum_{i=1}^{n} L_i(\theta). \tag{1}$$

where the sum is over the set of training images. Write down the update rule for stochastic gradient descent in this case.

(b) What is the main advantage of stochastic gradient descent compared to ordinary gradient descent in this case?

(c) A simple classifier uses the following formula to produce a the probability of foreground.

$$y = I \cdot w + w_0, \tag{2}$$

where the dot refers to the dot product. The probability of foreground is then modeled as

$$p = \frac{e^y}{1 + e^y}. \tag{3}$$

To train the classifier parameters in w and w_0, we try to minimize the negative log likelihood over a training set of positive examples

$$I_1,\ I_2,\ \ldots,\ I_n, \tag{4}$$

and negative examples

$$I_{n+1},\ I_{n+2},\ \ldots,\ I_{2n}. \tag{5}$$

Derive the update rules for w and w_0 if we use stochastic gradient descent with learning rate μ on this classifier. Consider both positive and negative training examples. Motivate your answer by showing your derivations.

3 Image registration, 3 points

Let's say we want to warp a source image to a target image. Let $\tilde{u}_1, \dots, \tilde{u}_n$ be points in the source image and $u_1, \dots, u_n$ points in the target image, where

$$\tilde{u}_k = \begin{pmatrix} \tilde{x}_k \\ \tilde{y}_k \end{pmatrix} \quad \text{and} \quad u_k = \begin{pmatrix} x_k \\ y_k \end{pmatrix}. \tag{6}$$

In order to align the images we use Ransac to estimate an *affine* transformation.

(a) Explain how to construct a minimal solver for this problem. Show how to use the u_j's and $\tilde{u}_j$'s to get a system on $M\theta = b$ form, (where θ are all the unknowns) and explain how it can be solved in Matlab. Be sure to define all variables that you use in your explanation.

(Let's say we have 6 correspondences in total.) What can you say about the certainty of a solution that yields

(b) 3 inliers

(c) 4 inliers

(d) The transformation used for warping should always be backwards, that is, from target/warped coordinates to source coordinates. Explain what would happen if we used forward warping (from source to target/warped) with the transformation

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} \tilde{x} \\ \tilde{y} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{7}$$

Assume that undefined pixels are set to 0.

4 Triangulation, 3 points

Given a set of Sift points with pixel coordinates

$$u_i = \begin{pmatrix} x_i \\ y_i \end{pmatrix} \tag{8}$$

and corresponding camera matrices P_i, we want to triangulate a 3D point U using Ransac.

(a) Explain how to construct a *minimal* solver for this problem. Show how to get a system $M\theta = b$ form, (where θ are all the unknowns) and explain how it can be solved in Matlab. Be sure to define all variables that you use in your explanation. (There are multiple correct solvers. Choose one.)

(b) For a camera matrix P, a 3D point, U and image point (x, y), write down explicit formulas for the reprojection residual. The formulas should be on the form

$$r_x = \ldots \tag{9}$$

$$r_y = \ldots \tag{10}$$

Make sure to clearly define any new variables that you introduce and simplify the formulas as much as possible.

5 Ransac, 3 points

An affine transformation in 3D can be parameterized using a general 3×3-matrix, A, and a 3-vector t. Consider using Ransac with a minimal solver to estimate an affine transformation between two point sets in 3D. Assume that the rate of outliers is p, with $0 \leq p \leq 1$.

(a) How many Ransac iterations, K, are required before the probability that we never sampled an outlier-free subset is less than 0.0001. Answer with a formula $K = \ldots$ (As p is unknown, we cannot get a numerical value).

(b) Compare this formula to the rule of thumb suggested in the lecture notes for choosing the number of Ransac iterations. What is the difference for large outlier rates, p? *Hint: Use a Maclaurin expansion.*

6 IRLS, 3 points*

Given a set of residuals, the Huber loss can be written as

$$L(\theta) = \sum_{i=1}^{n} h(r_i^2(\theta)) \tag{11}$$

where

$$h(s) = \begin{cases} s & \text{if } s < \delta \\ 2\delta\sqrt{s} - \delta^2, & \text{otherwise.} \end{cases} \tag{12}$$

(a) Assuming that we know how to solve the least squares problem, describe how to use iteratively reweighted least squares (IRLS) to minimize the Huber loss.

(b) Show that if IRLS has converged we are at a stationary point with respect to the Huber loss.