SSY130 - Applied Signal Processing
Hand in Problem 2

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1. Signal and Observation

The original signal and observed signal of distance are shown in Figure 1.1. It is clear that there are much noise in observation, resulting in an unsmooth curve. The velocities of both signals, derivatives of distance, are shown in Figure 1.2. The approximate derivative of measured signal has obvious fluctuation due to the high frequency noise.

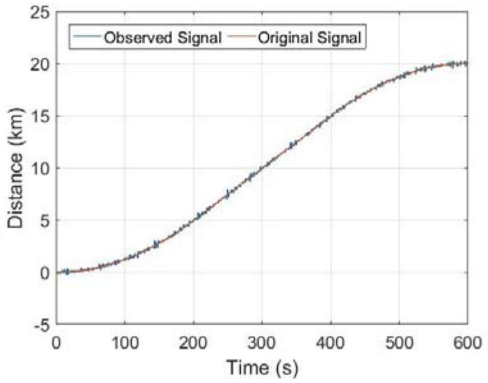

Figure 1.1 Original and Observed Signal

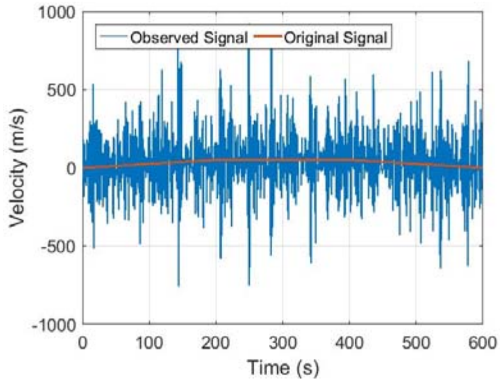

Figure 1.2 Derivatives of Two Signals

2. Design of Filter

As shown in code below, the designed filter consist of two parts which are differentiation filter (diff filter) and low pass filter. Here, the weights of both filter are 1. Convolve two filters to generate the designed filter. Coefficients of designed filter are shown in Figure 2.1. Filter's amplitude and phase response are shown in Figure 2.2. It is observed that, in pass band, designed filter has a high amplitude and a phase shift.

```
h1 = firpm(10, [0 0.1], [0 1]*pi, 'differentiator');    % Differentiation Filter
h2 = firpm(50, [0 0.1 0.2 1], [1 1 0 0]);                % Low Pass Filter
h = conv(h1, h2);                                        % Designed Filter
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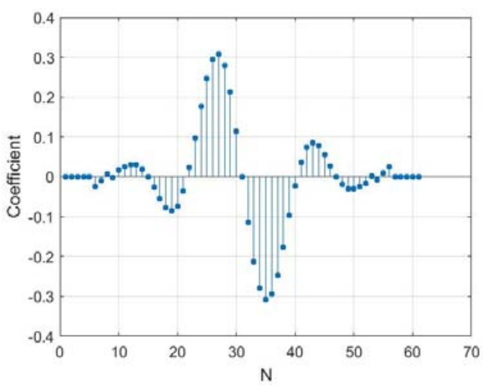

Figure 2.1 Coefficients of Designed Filter

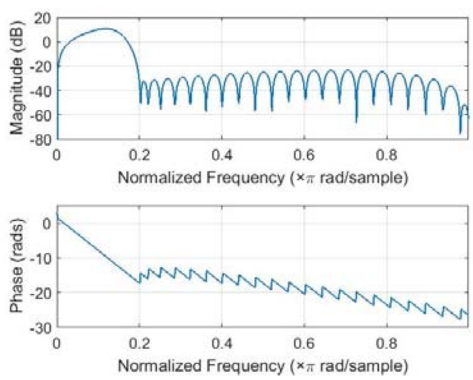

Figure 2.2 Frequency and Phase Response

3. Removing Delay

The result of filtering measured signal is shown in Figure 3.1. Comparing to original signal's velocity, there is a delay in filtered result. The delay D of signal filtered by a FIR filter can be calculated as Equation 3.1.

$$D = \frac{L-1}{2} \tag{3.1}$$

Here, L is the length of filter. The length of diff filter is 11 because it's a 10^{th} order filter. Low pass filter's order is 50, so that its length is 51. Thus, the length of designed filter is (11 + 51 - 1), i.e. 61, resulting in a delay of 30. As shown in Figure 3.2, the delay of filtered observed signal is removed.

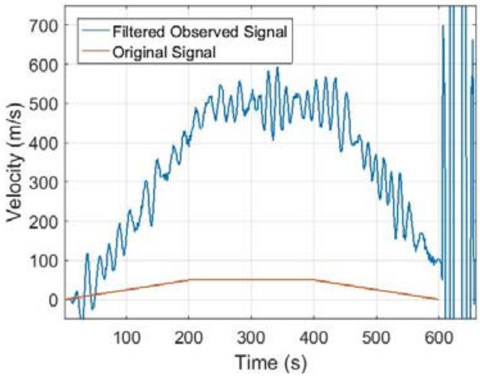

Figure 3.1 Filtered Observed Signal with Delay

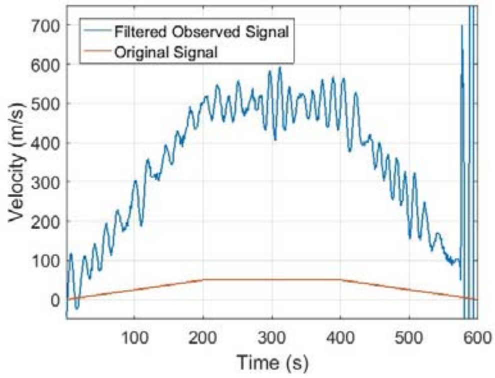

Figure 3.2 Filtered Observed Signal without Delay

4. Modifying Gain

Though the delay is removed, there is still an incorrect amplitude of filtered signal. To solve this problem, it should be determined first that which part causes the abnormal amplitude. The original signal is filtered by different filters, which is shown in Figure 4.1. It's clear that the diff filter has a small delay but a wrong gain, causing the wrong amplitude of signals filtered by designed filter. The delay has been removed in last section. The weight of diff filter has to be changed to a lower value. The max value of diff filtered result is 500m/s, the max value of original signal's speed is 50m/s. Thus the weight of diff filter should be modified to 0.1 (50/500), the filtered observed signal performs well as shown in Figure 4.2.

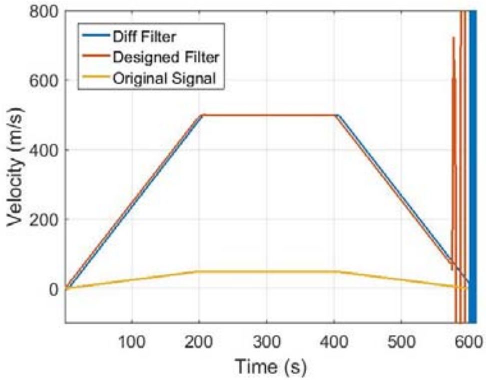

Figure 4.1 Filtering Original Signal with Different Filters

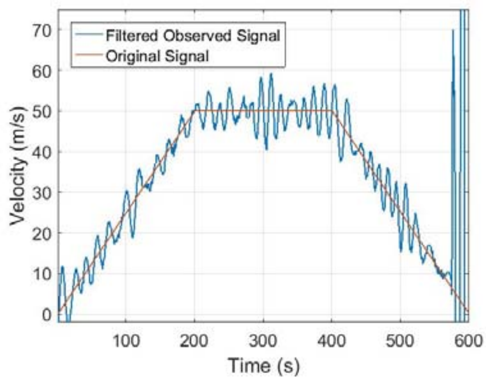

Figure 4.2 Filtered Observed Signal with Right Gain

5. Other Problems

(1) The amplitude specification of designed filter can be derived analytically. Because the coefficients of this FIR filter is known already. Frequency response of FIR filter can be calculated as Equation 5.1. Here, *M*, the length of FIR filter, is 61. *h(n)* contains coefficients of filter. The amplitude specification of designed filter is the mode of $H(\omega)$.

$$H(\omega) = \sum_{n=0}^{M-1} h(n)\, e^{-j\omega n} \tag{5.1}$$

(2) The amplitude and phase response of diff filter and designed filter are shown in Figure 5.1 and 5.2. Here, the weight of diff filter is 0.1. Both filters have the same amplitude in the range of pass band. Due to the low pass filter, the amplitude of designed filter outside the pass band is much lower than the amplitude of diff filter. Designed filter has a more obvious phase response in the range of pass band than diff filter's phase, indicating that designed filter gives a higher delay than diff filter.

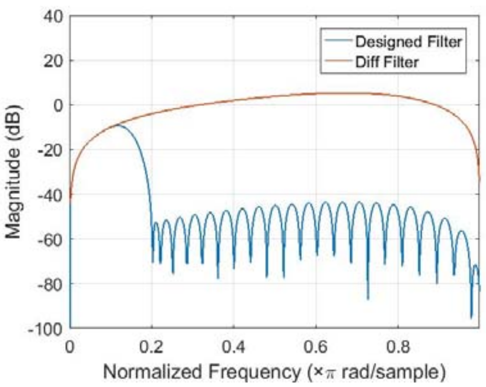

Figure 5.1 Amplitude Response of Two Filters

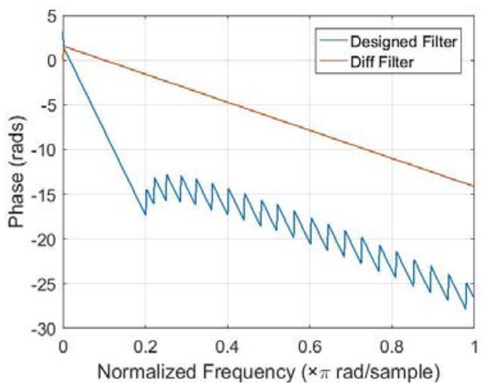

Figure 5.2 Phase Response of Two Filters

(3) This problem has been discussed in Section 3 above.

(4) As shown in Figure 5.3, results of original signal filtered by diff filter and designed filter are almost same (except the end part). Since original signal has little noise, low pass filter will not change the signal. However, the result of observed signal filtered by designed filter is much better than the approximate derivative of measured signal as shown in Figure 1.2. As shown in Figure 5.4, diff filter is able to present the approximate derivative of observed signal. After being filtered by designed filter, high frequency noise has been removed, resulting in a smoother curve of velocity.

(5) The start-up transient and ending transient are known as edge effects. Since at the beginning and the end part of signal, there are not enough data to be convolved with filter, causing the questionable results in boundary regions. Padding *M* zeros at both the beginning and the end of signal is able to solve this problem, where *M* is the number of filter's coefficients.

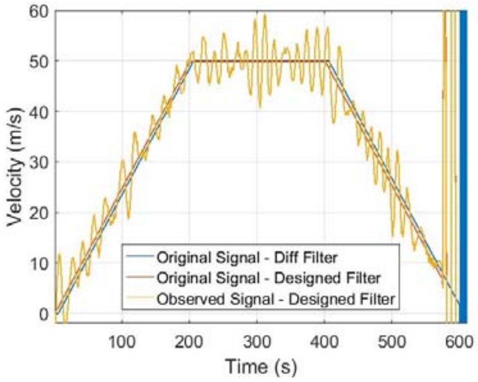

Figure 5.3 Filtering Signals with Different Filters

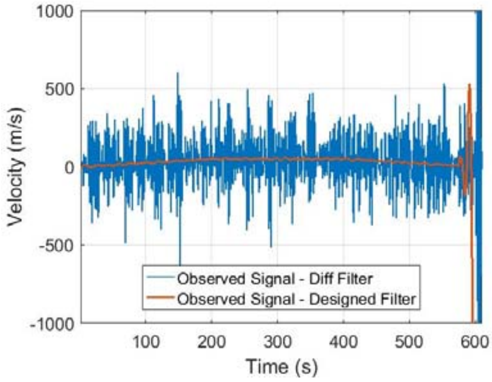

Figure 5.4 Filtered Observed Signal with Different Filters