

# **SSY097 - Image Analysis**

## Lecture 3 - Scale-Invariant Local Features

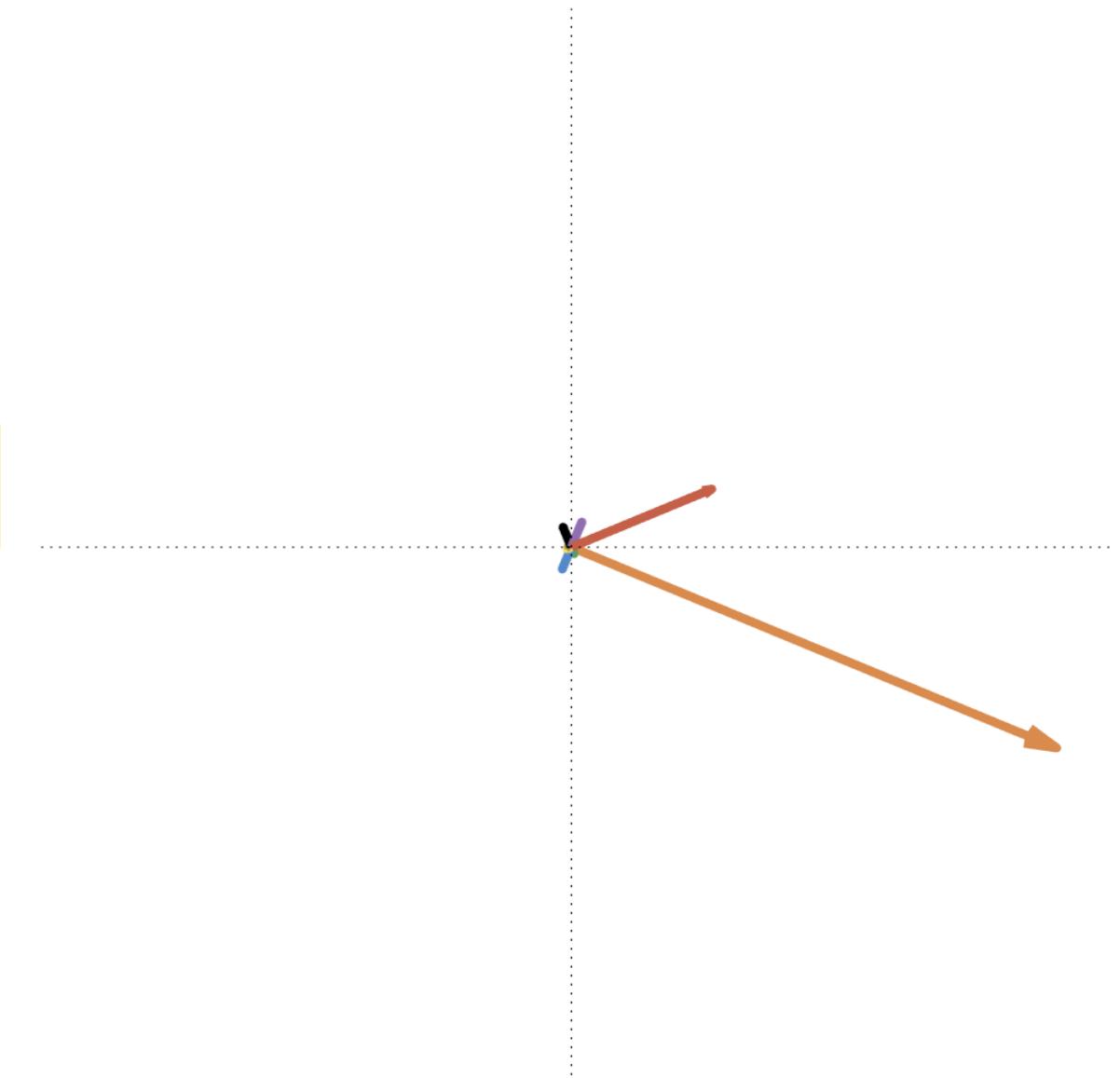
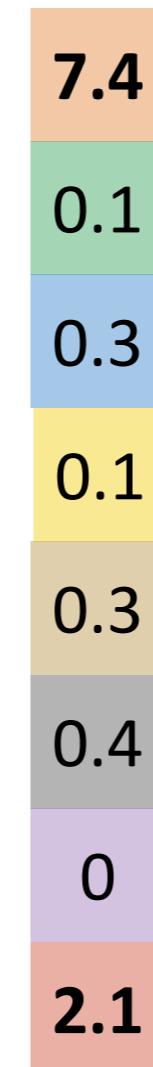
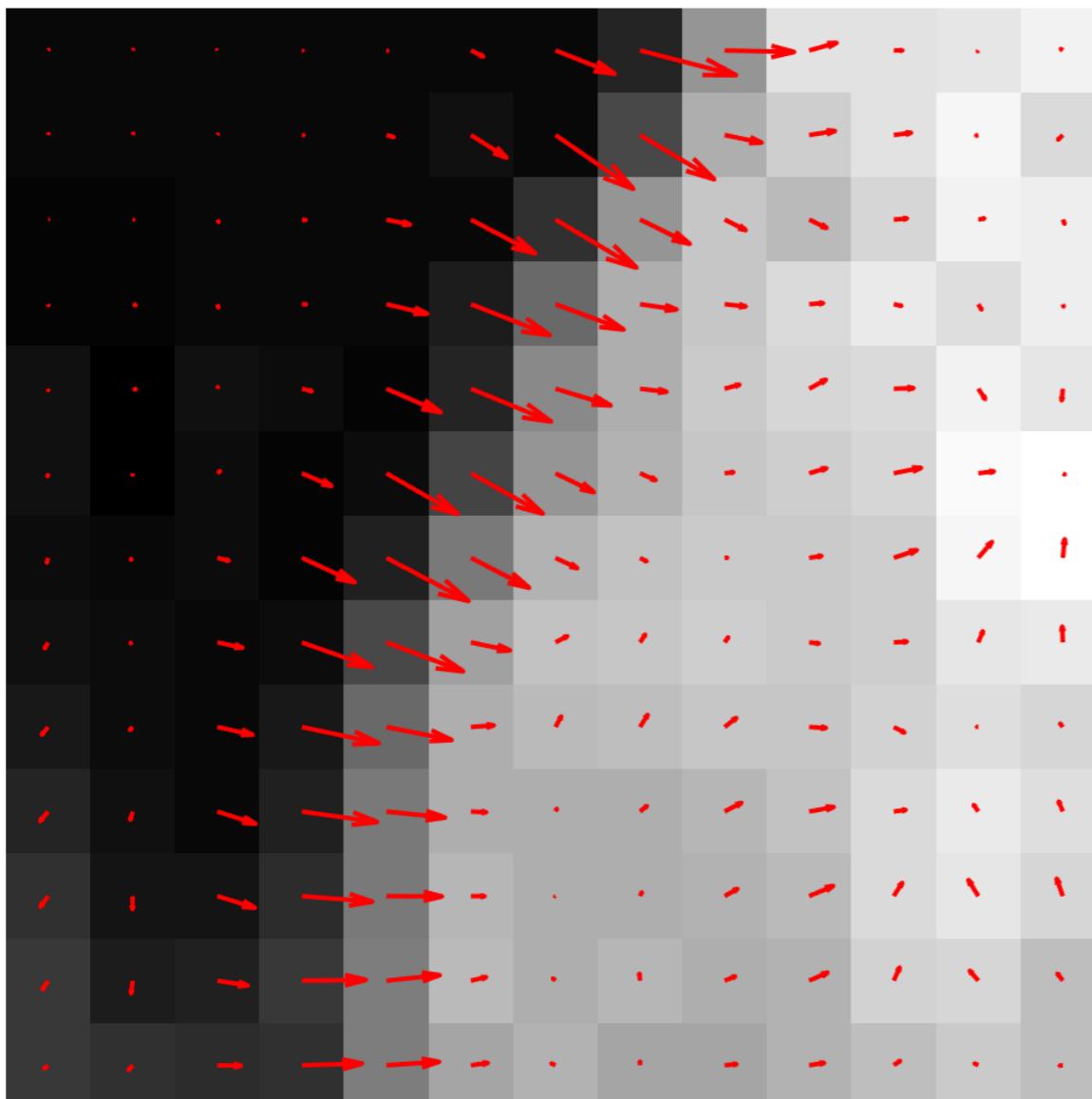
*Torsten Sattler  
(slides adapted from Olof Enqvist)*

# Last Lecture

Jan. 20	Introduction, Linear classifiers and filtering	
Jan. 23	<b>Filtering, gradients, scale</b>	Lab 1
Jan. 27	Local features	
Jan. 30	Learning a classifier	
Feb. 3	Convolutional neural networks	Lab 2
Feb. 6	More convolutional neural networks	
Feb. 10	Robust model fitting and RANSAC	
Feb. 13	Image registration	Lab 3
Feb. 17	Camera Geometry	
Feb. 20	More camera geometry	
Feb. 24	Generative neural networks	
Feb. 27	Generative neural networks	
Mar. 2	TBA	
Mar. 9	TBA	

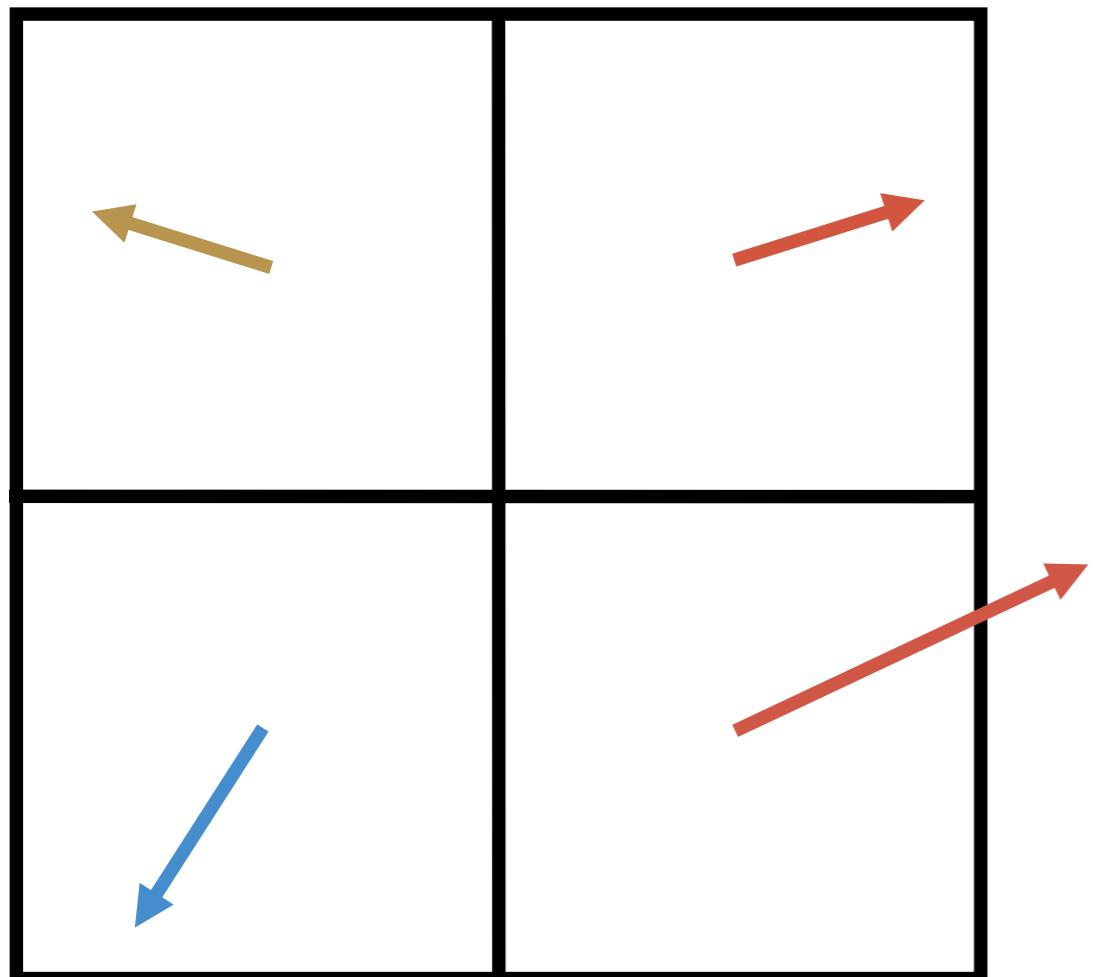
# Last Lecture

## Gradient histograms



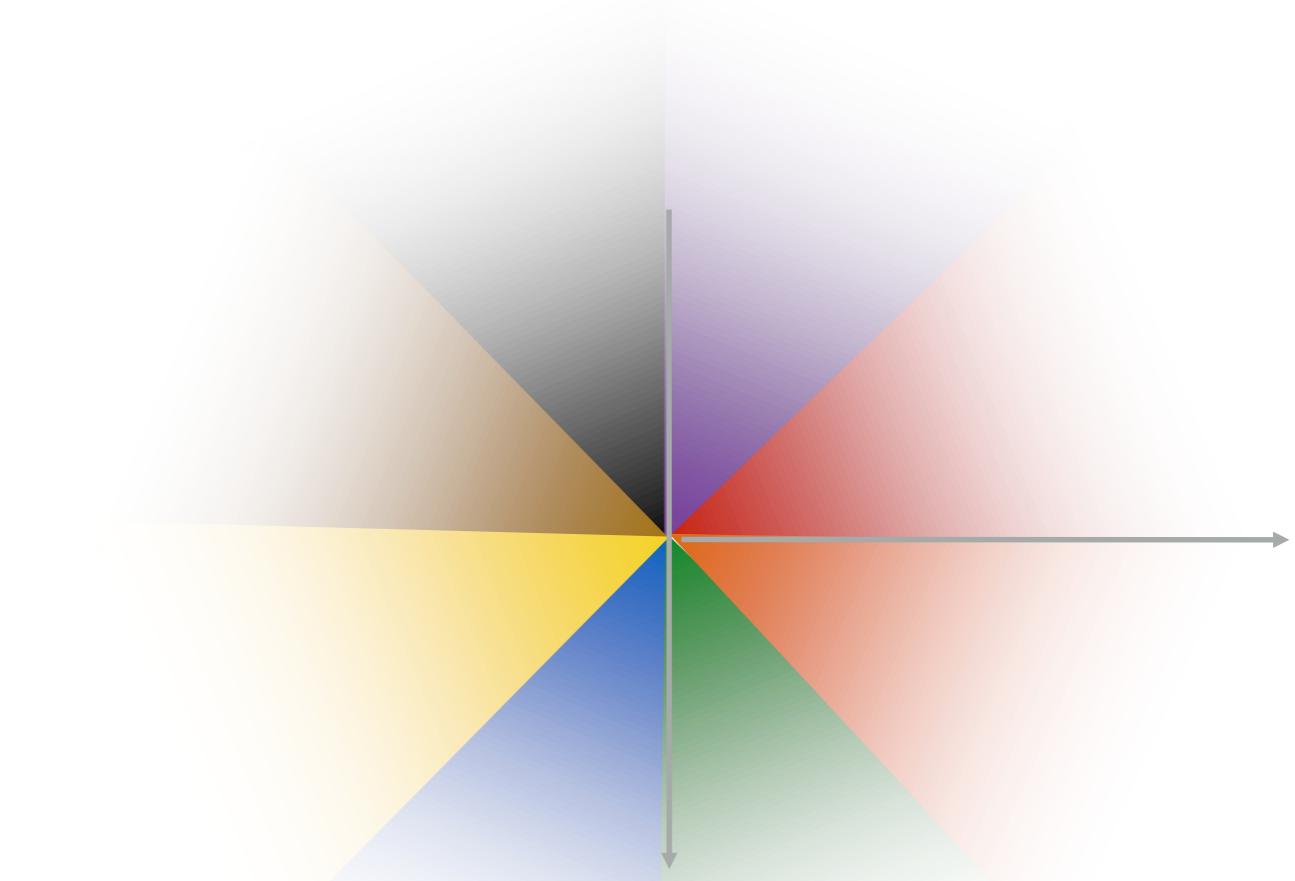
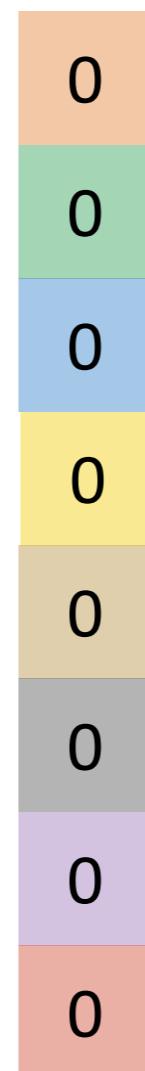
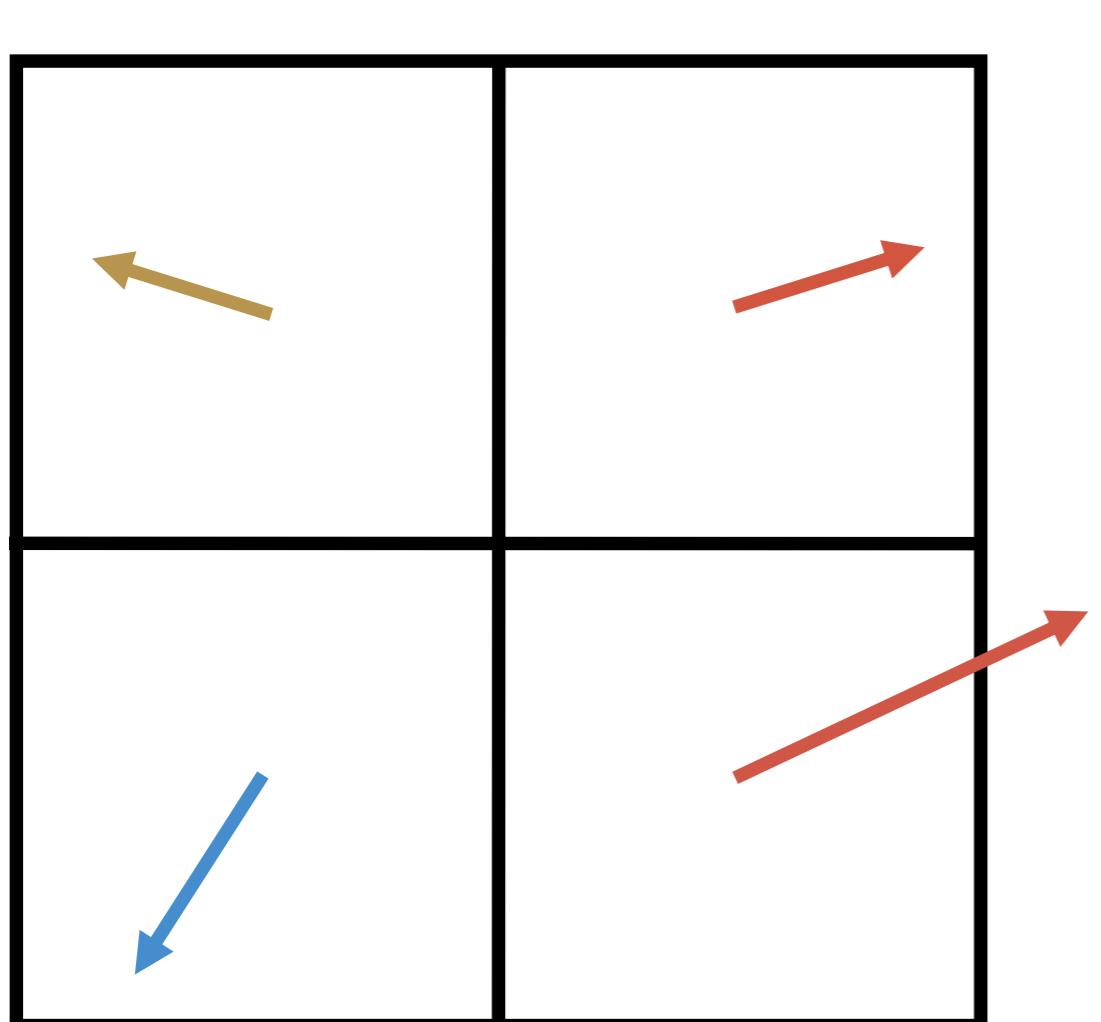
# Last Lecture

## Gradient histograms



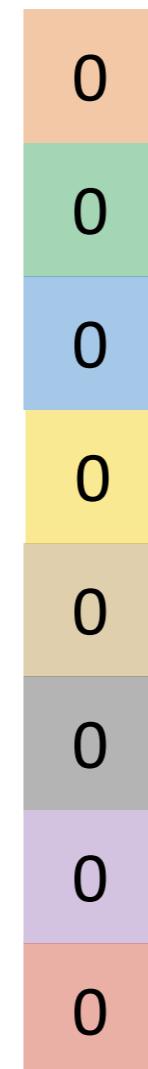
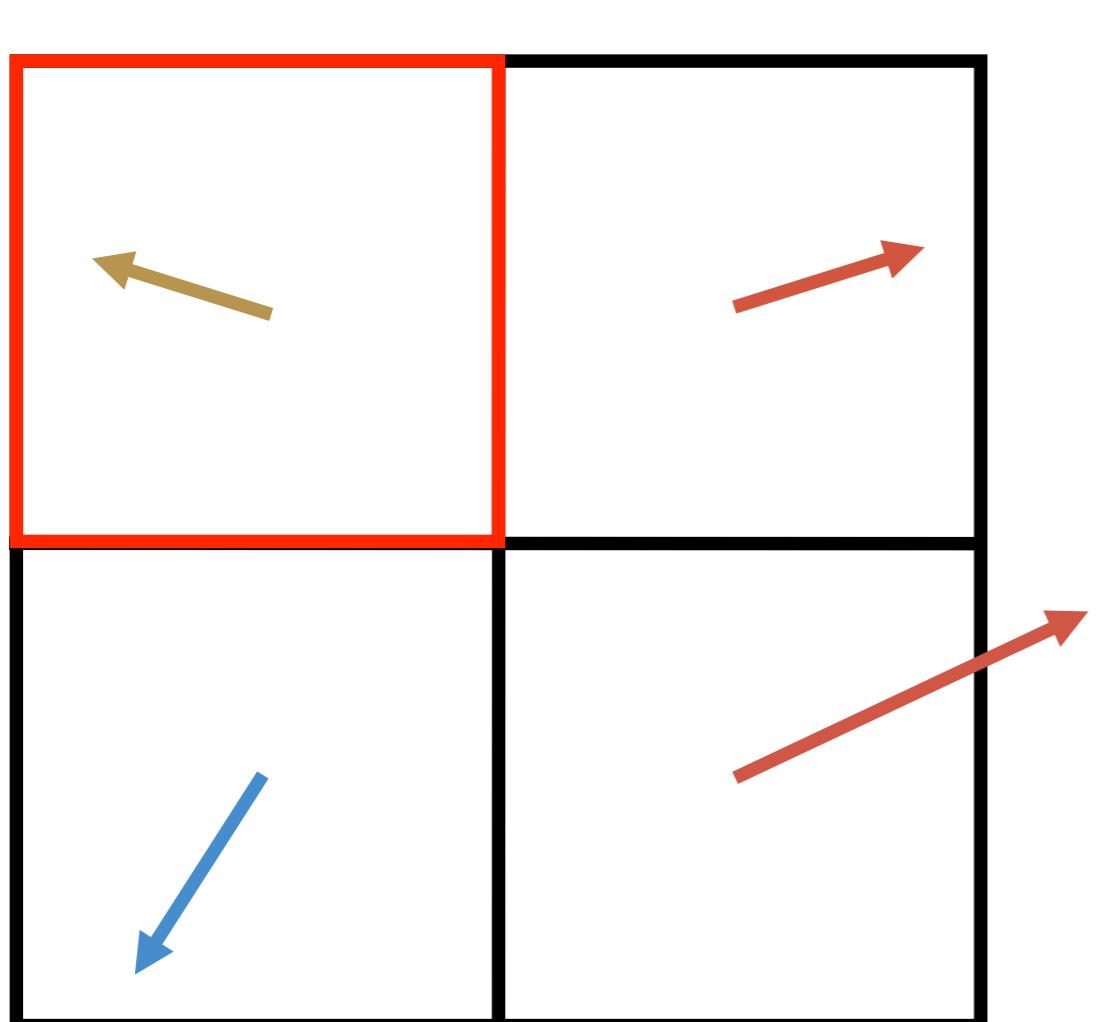
# Last Lecture

## Gradient histograms



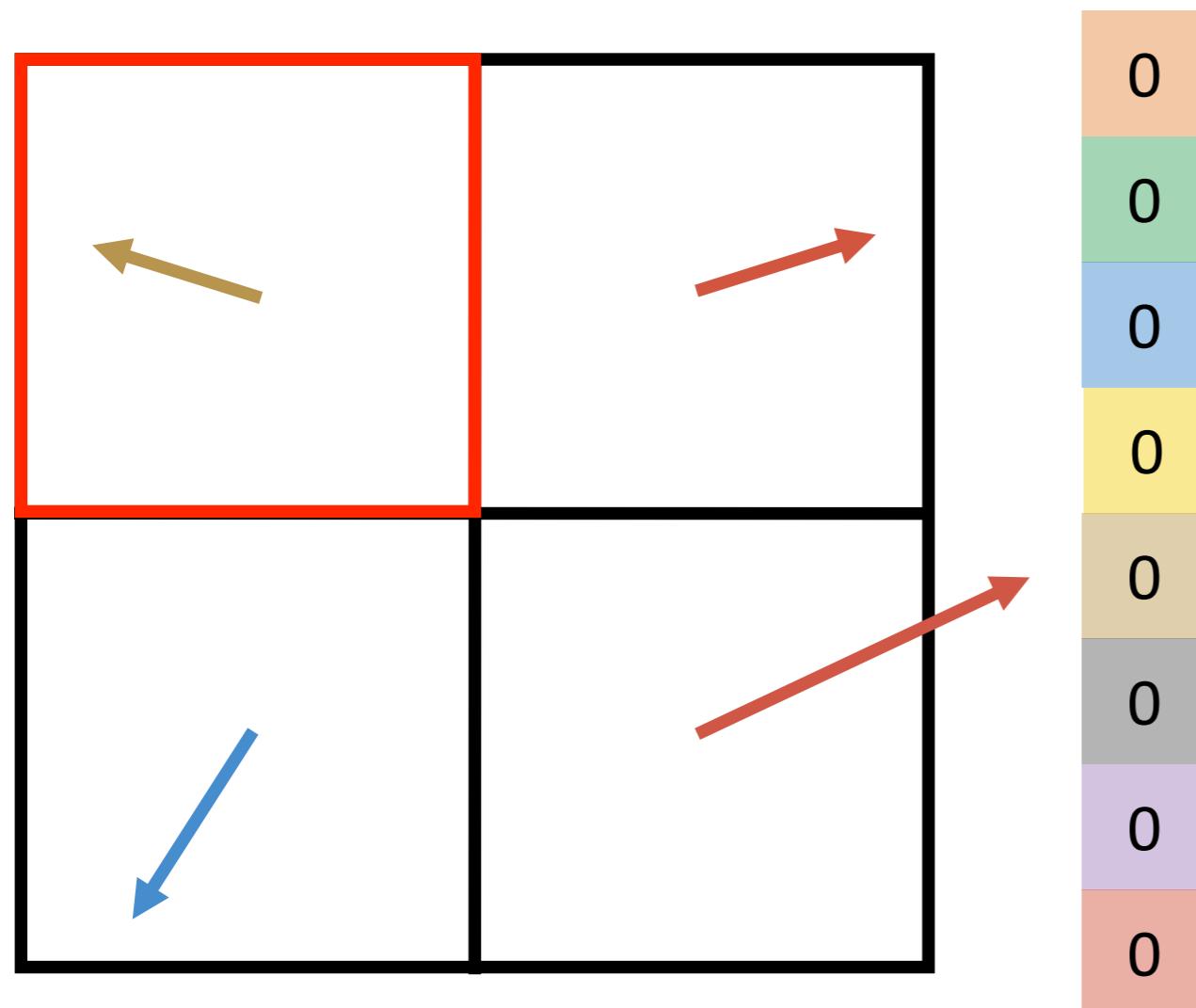
# Last Lecture

## Gradient histograms



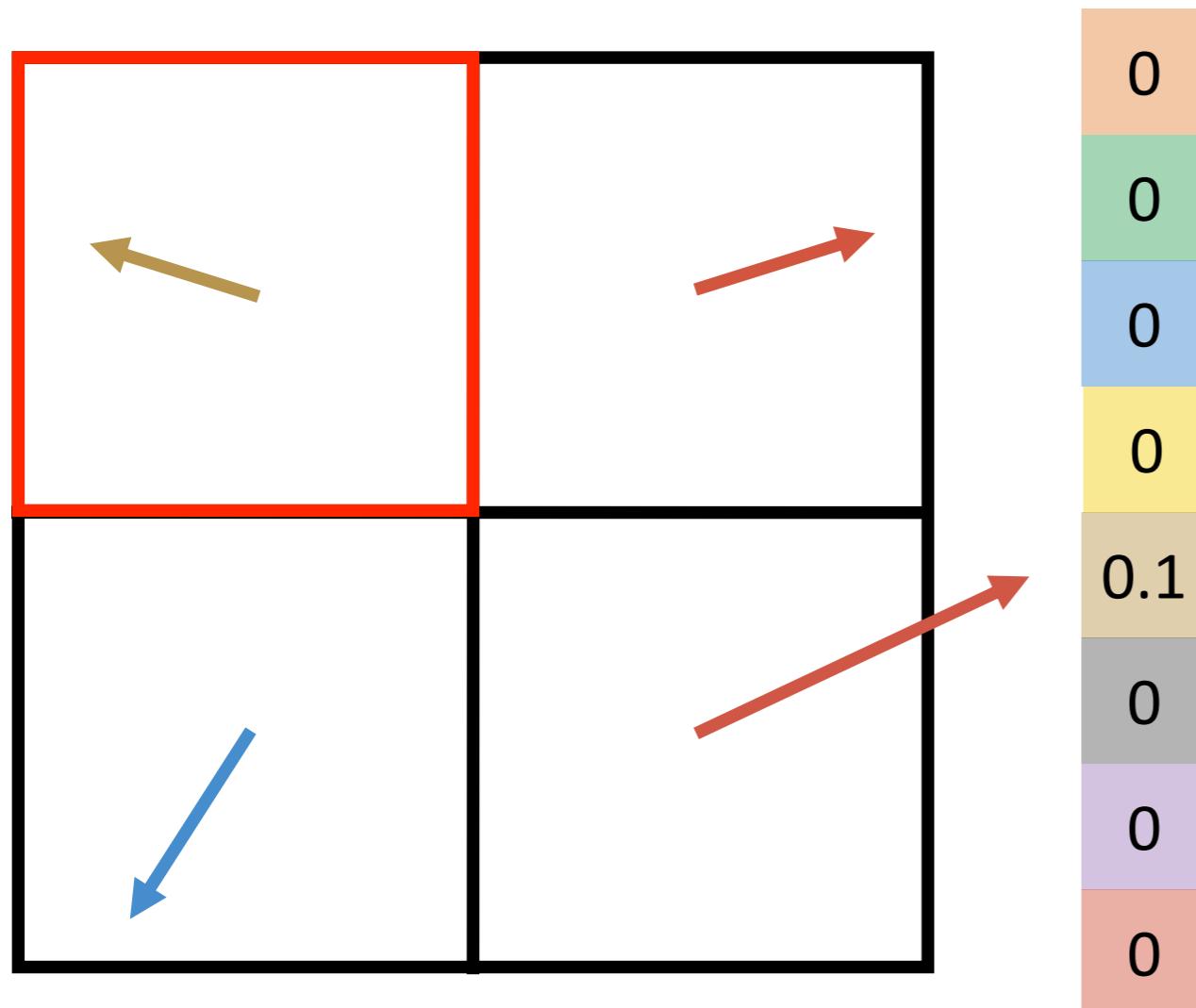
# Last Lecture

# Gradient histograms



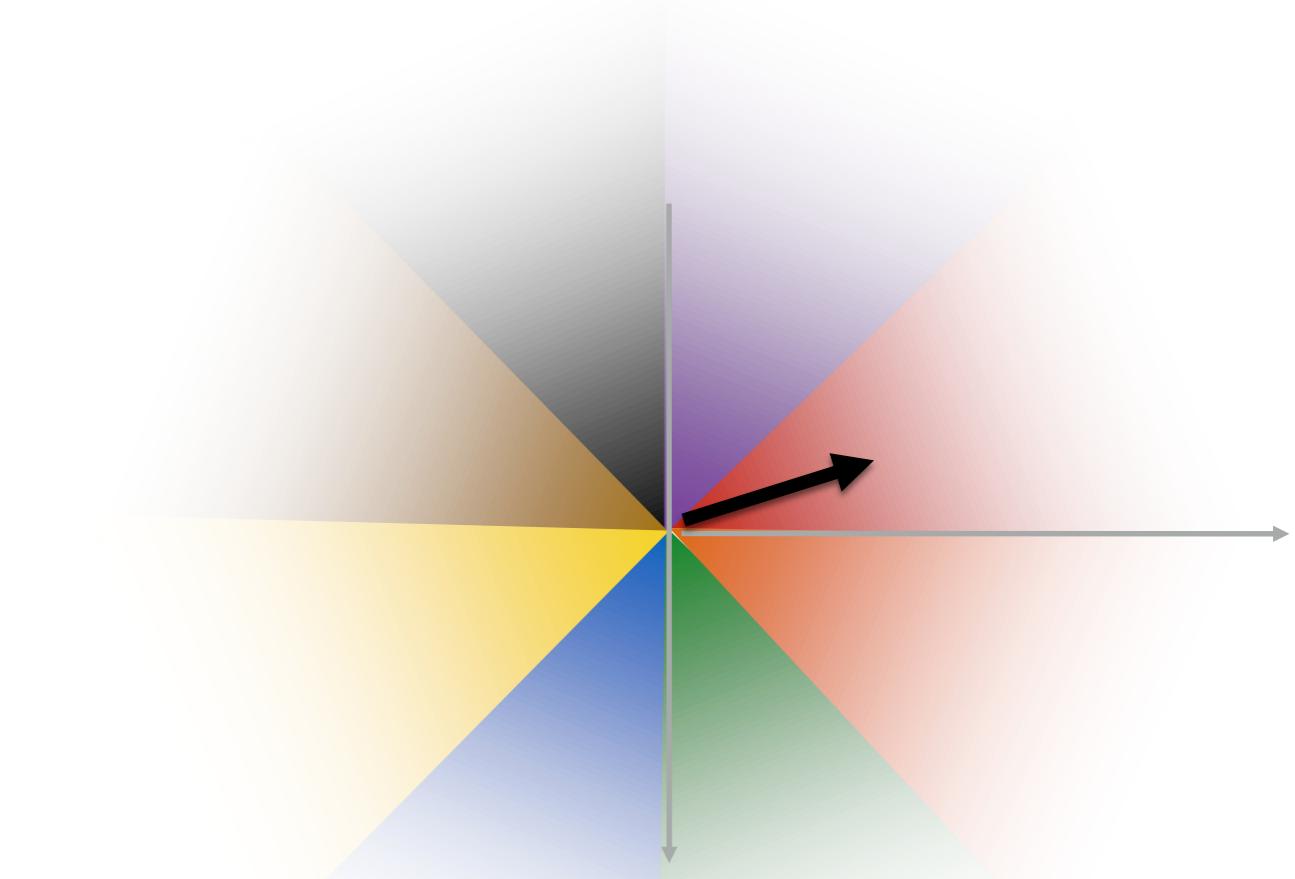
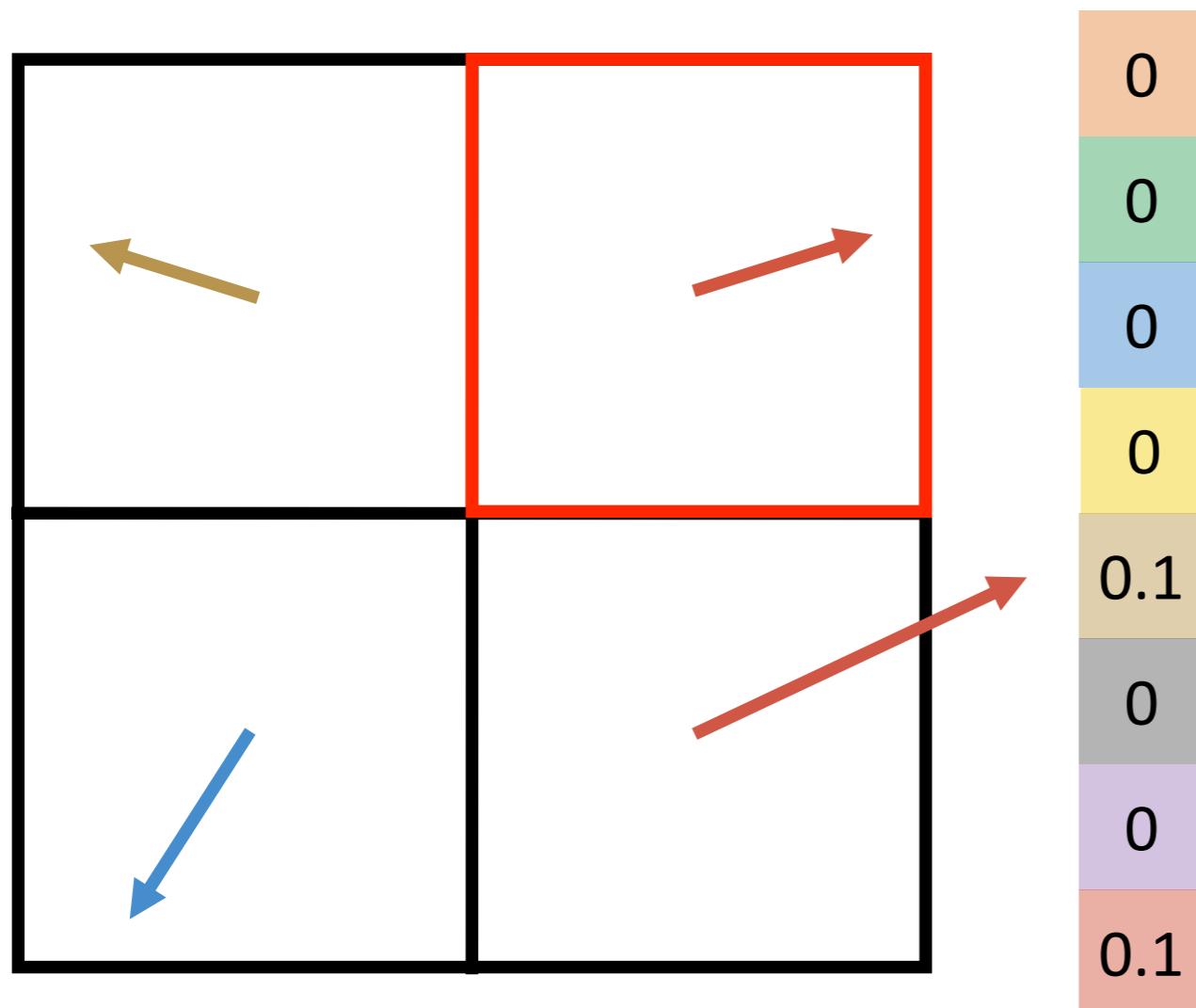
# Last Lecture

## Gradient histograms



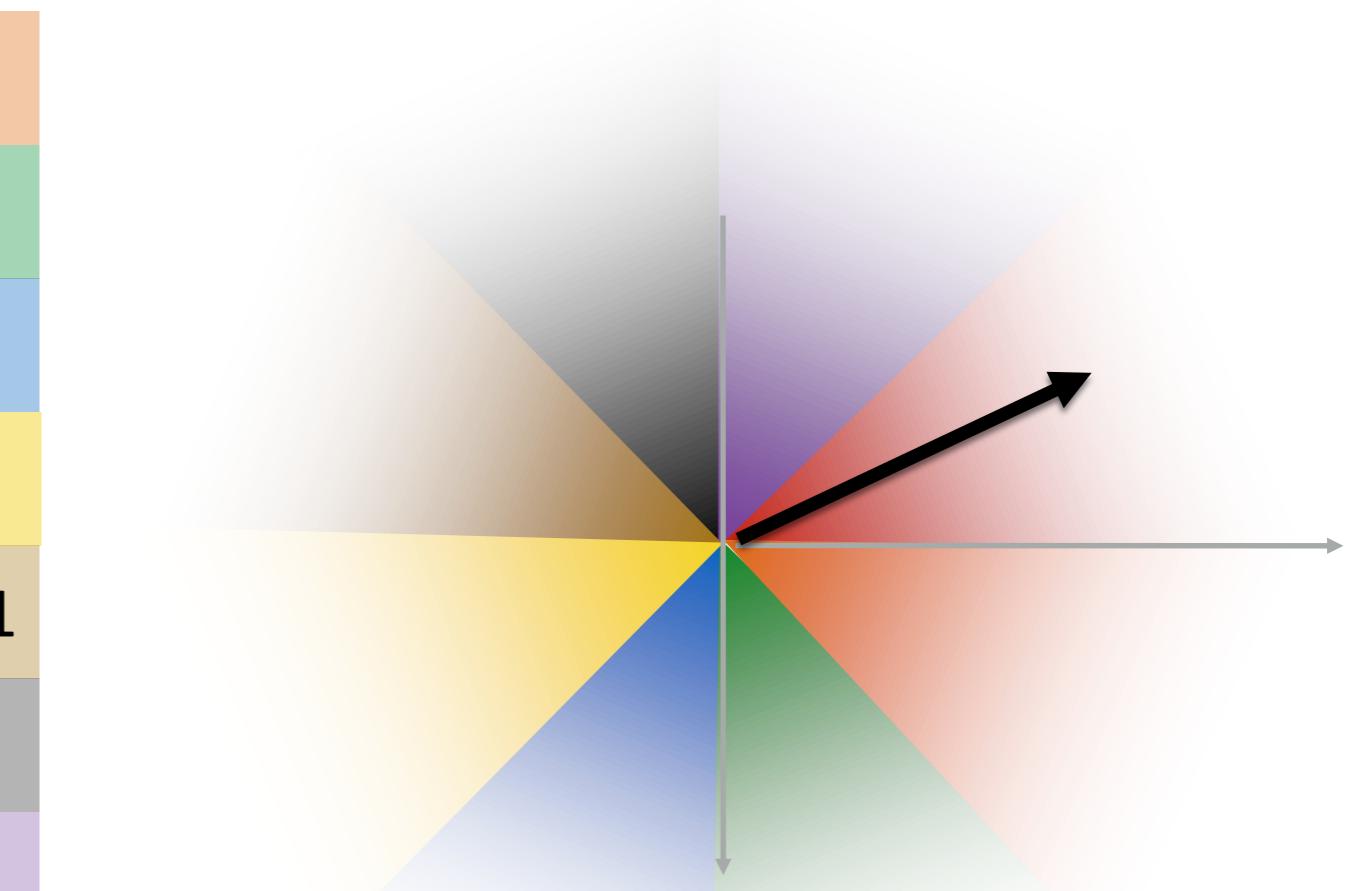
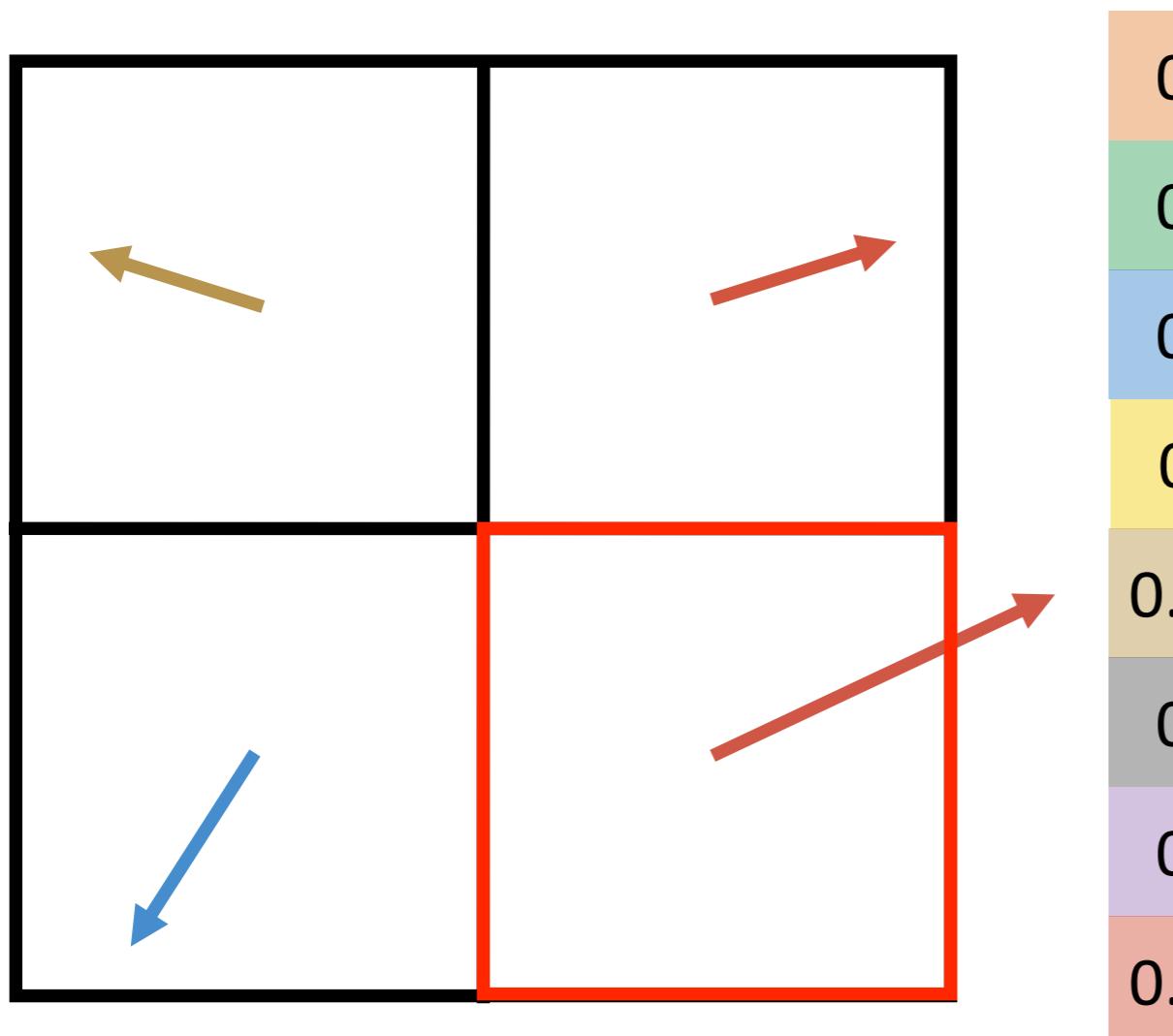
# Last Lecture

## Gradient histograms



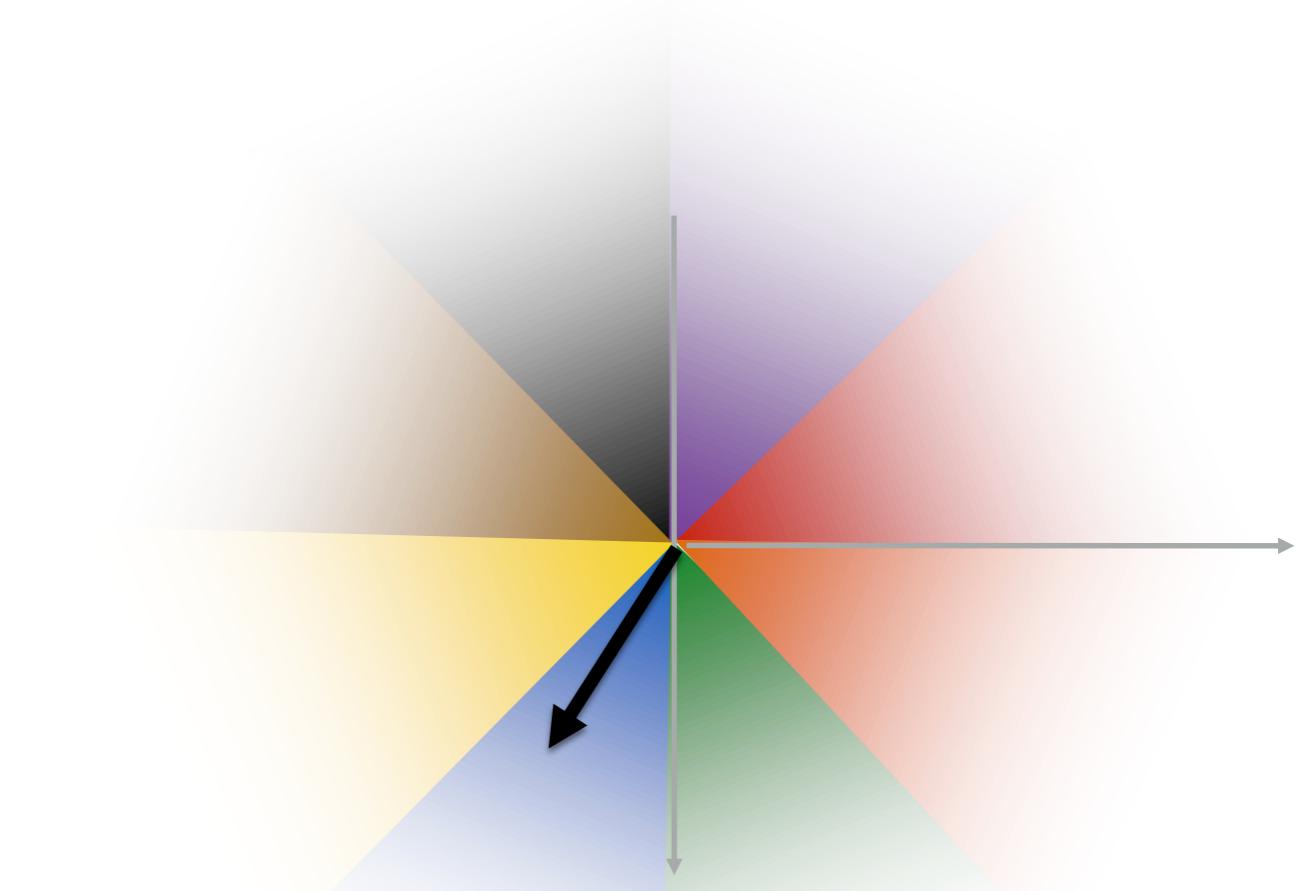
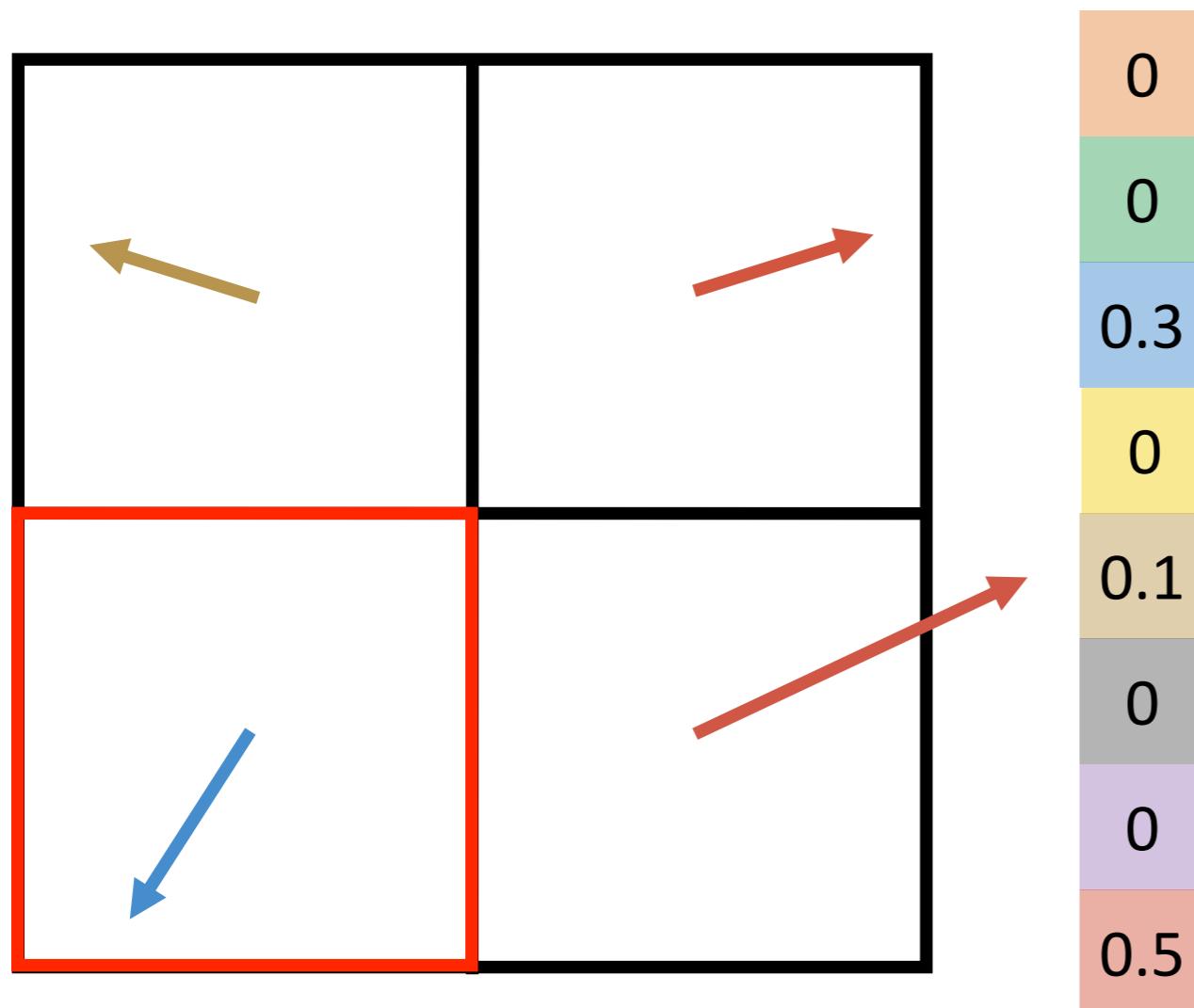
# Last Lecture

## Gradient histograms



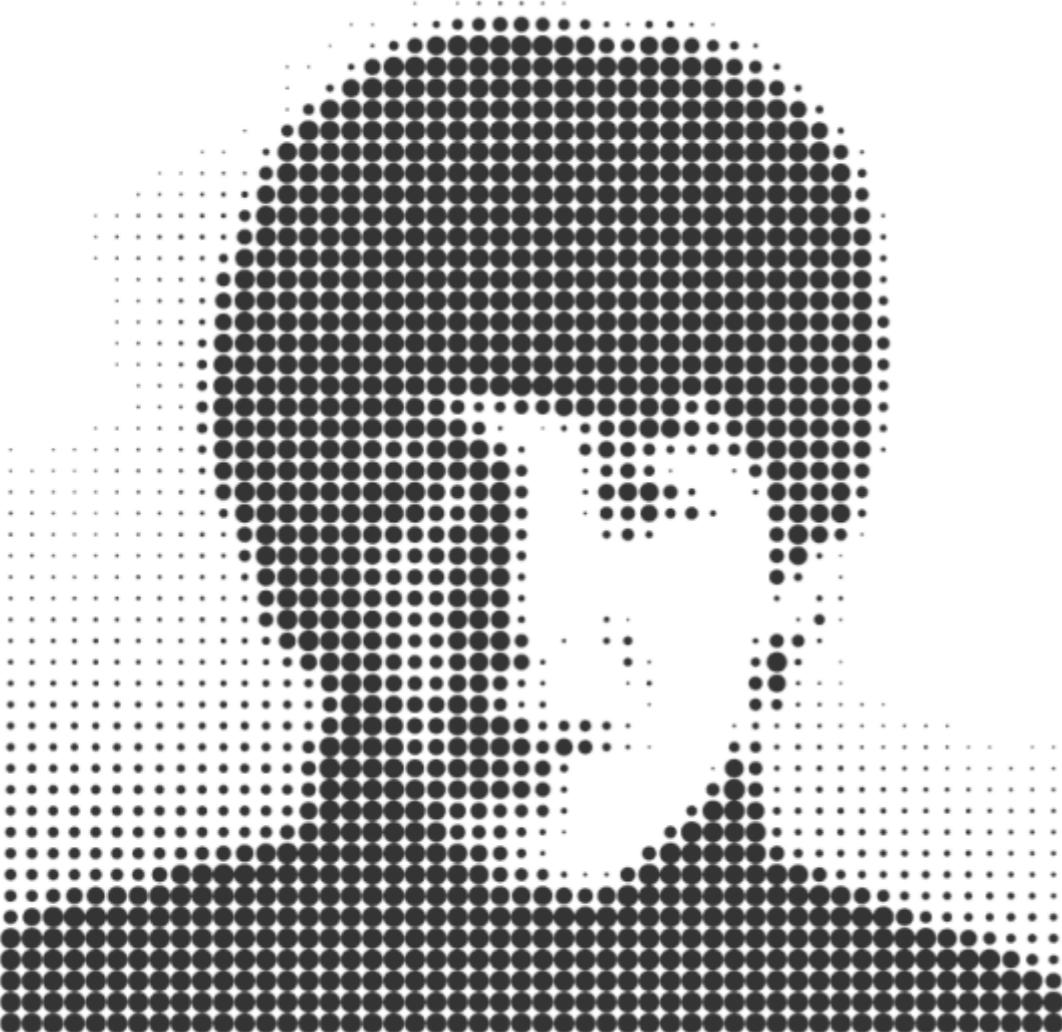
# Last Lecture

## Gradient histograms



# Last Lecture

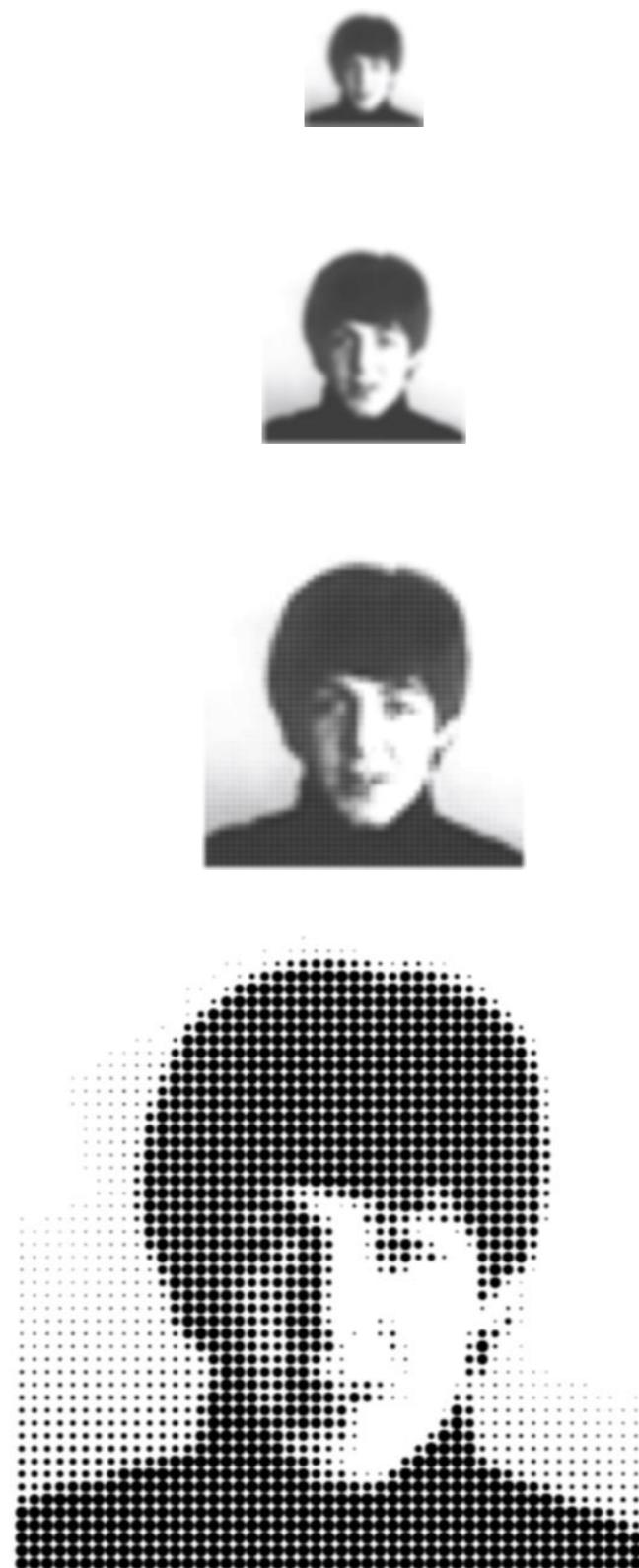
Object detection at different scales



# Last Lecture

Scale space

$\sigma$



$L(x, y, 8^2)$

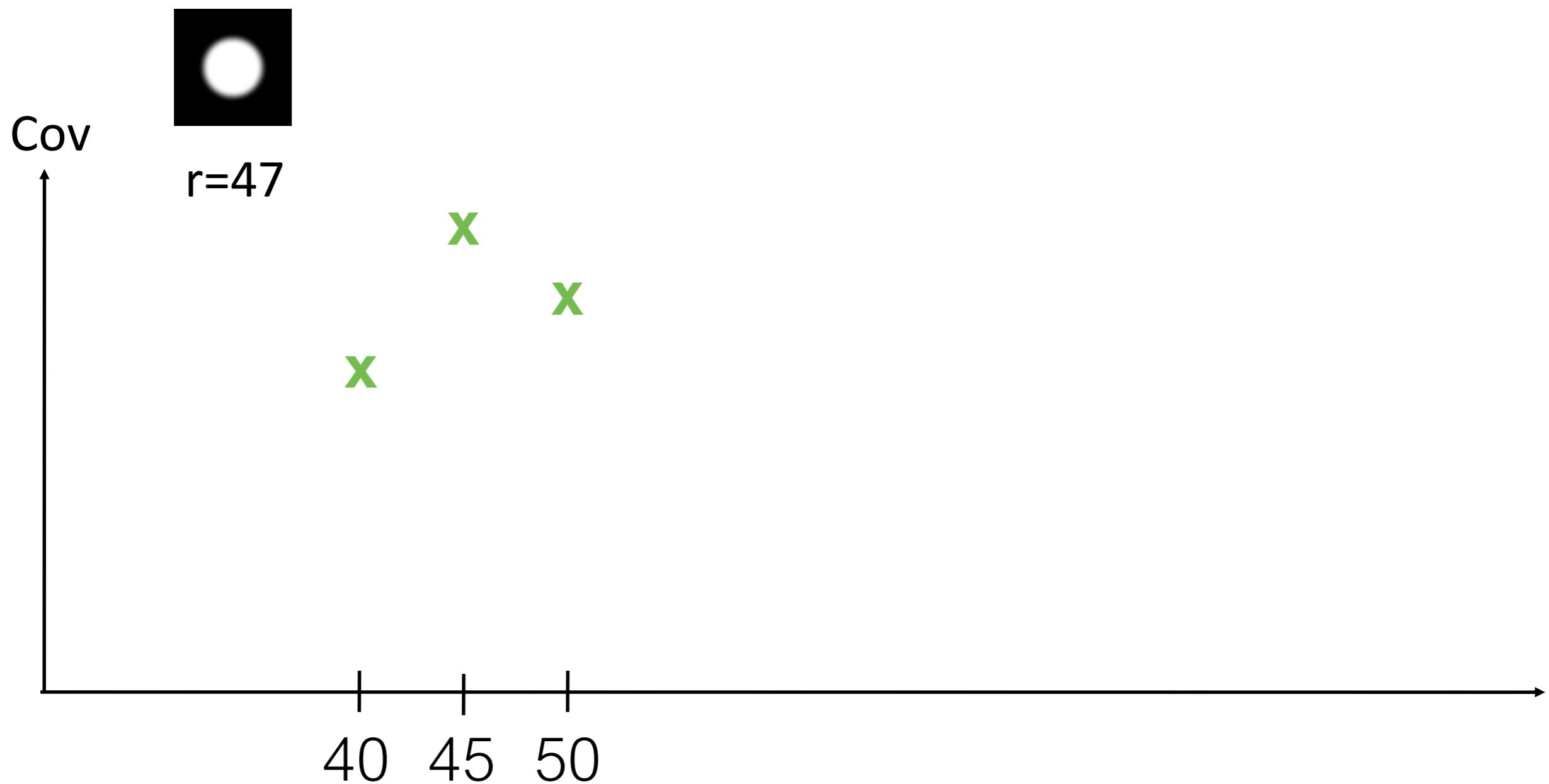
$L(x, y, 4^2)$

$L(x, y, 2^2)$

$L(x, y, 1^2)$

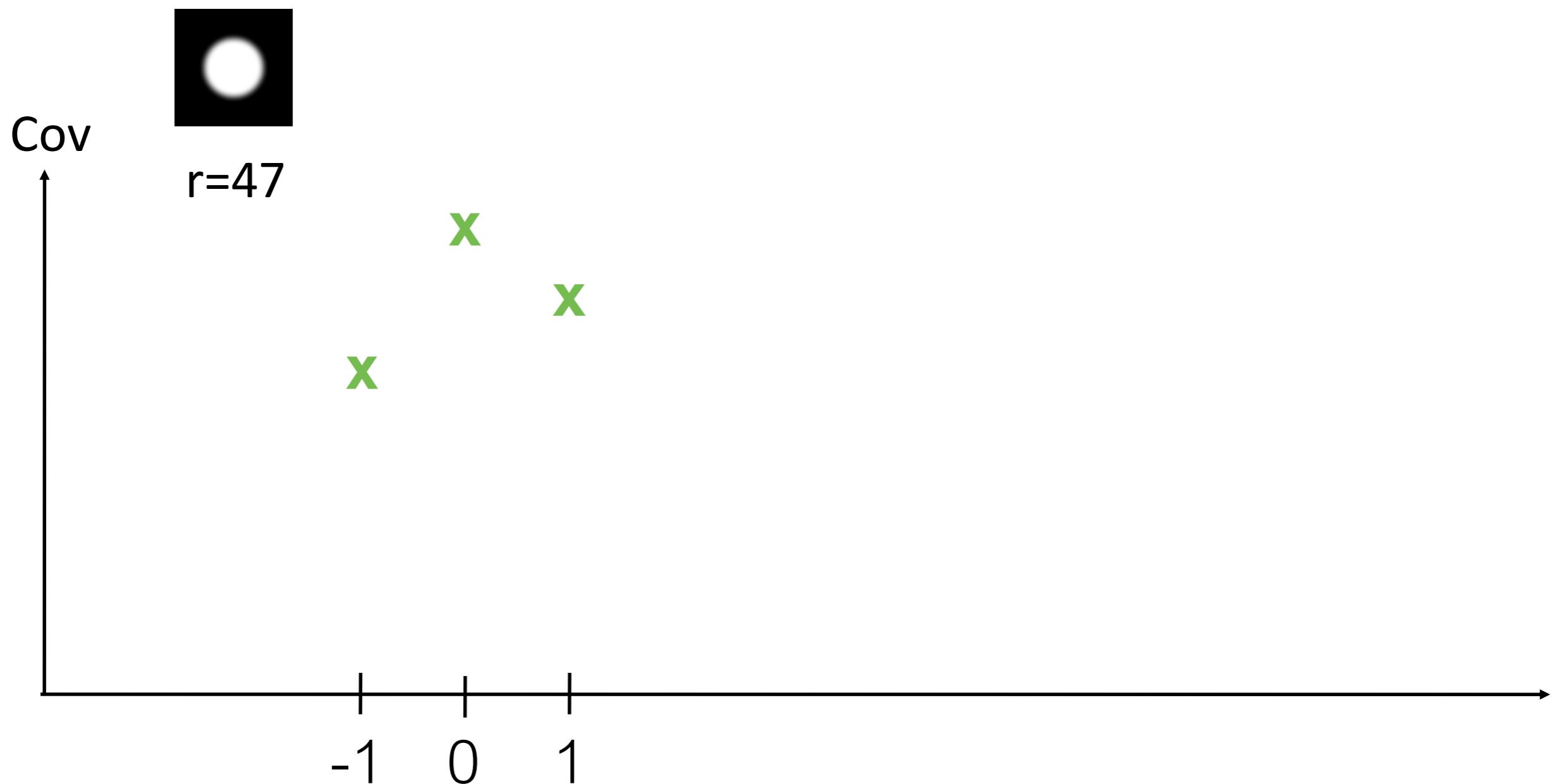
# Last Lecture

Scale refinement



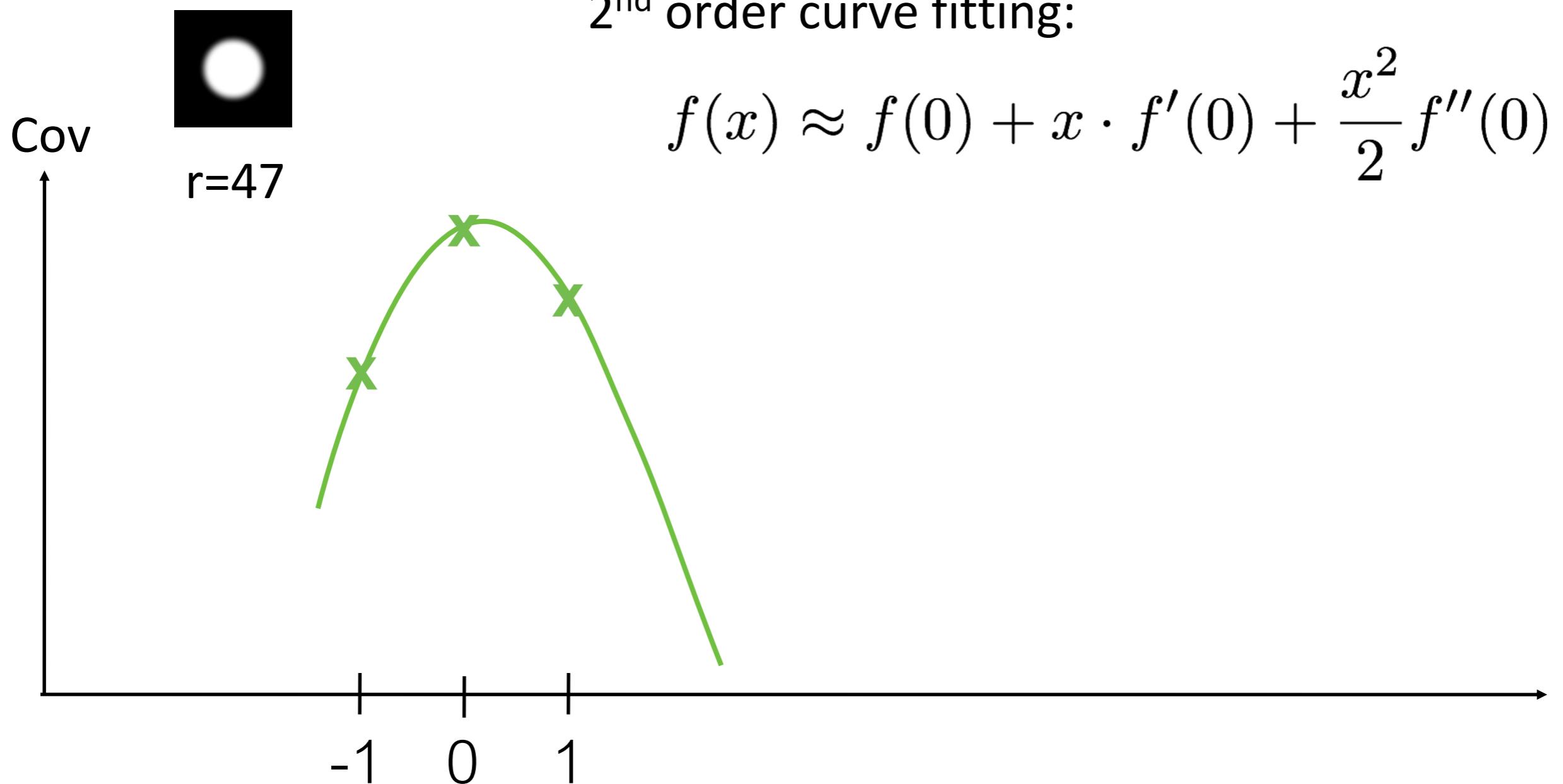
# Last Lecture

Scale refinement



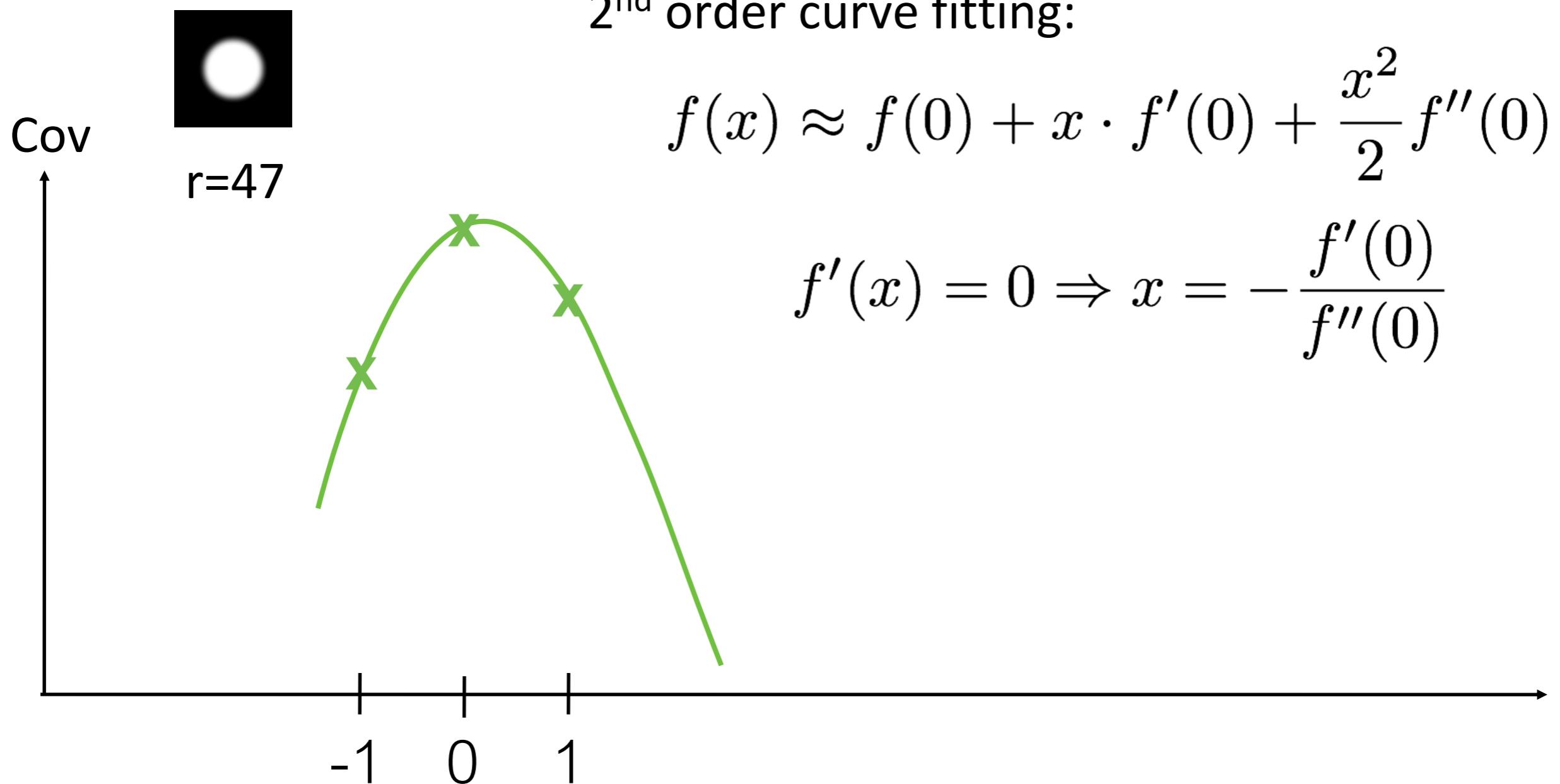
# Last Lecture

Scale refinement



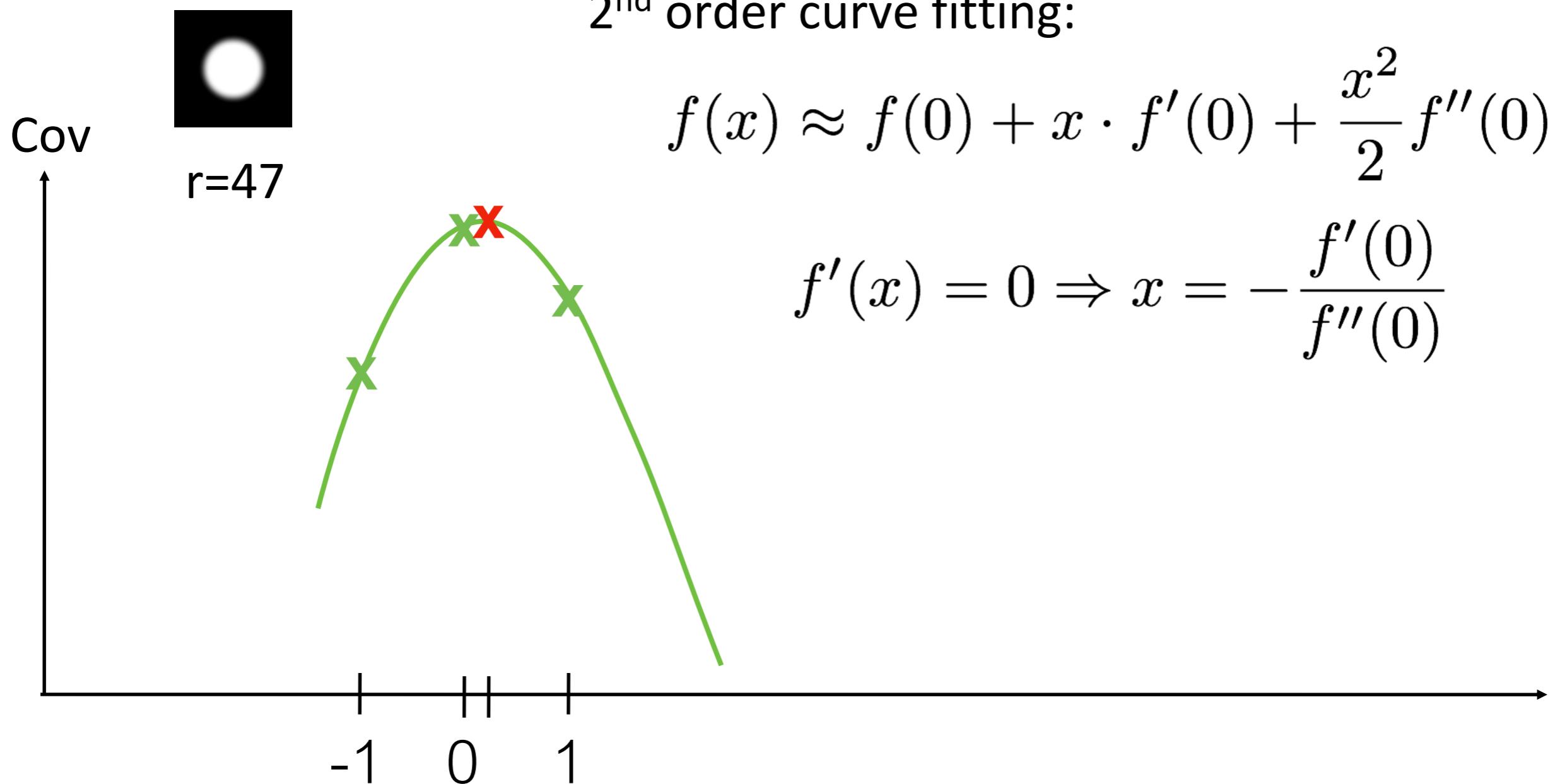
# Last Lecture

Scale refinement



# Last Lecture

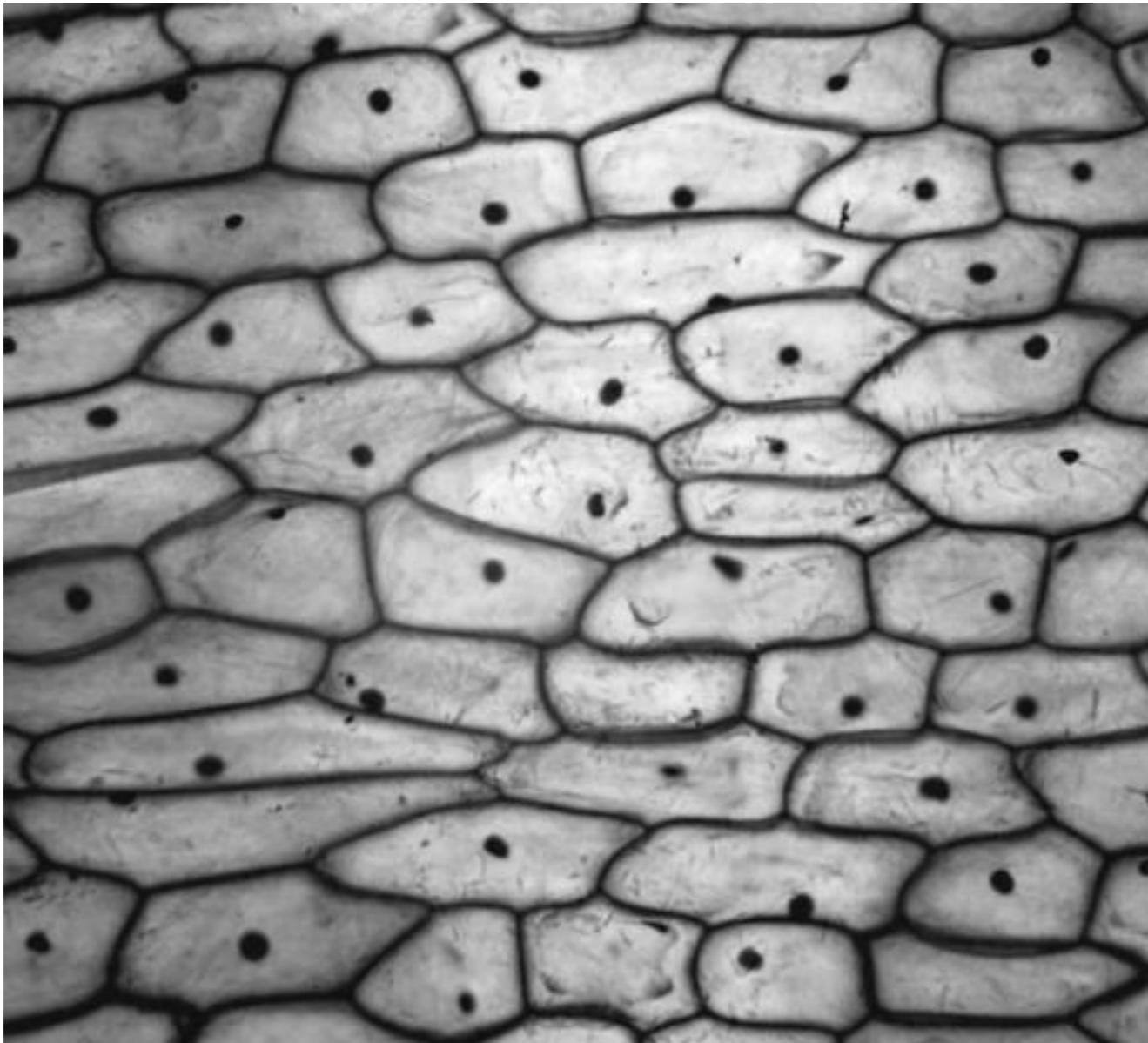
Scale refinement



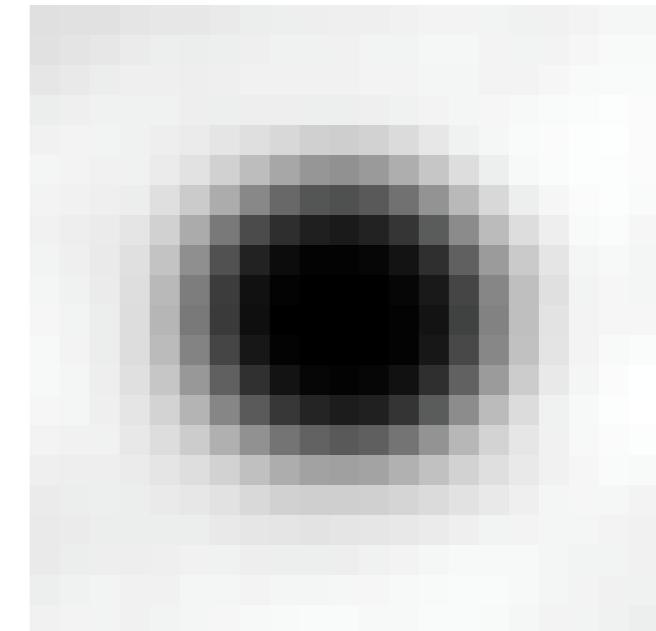
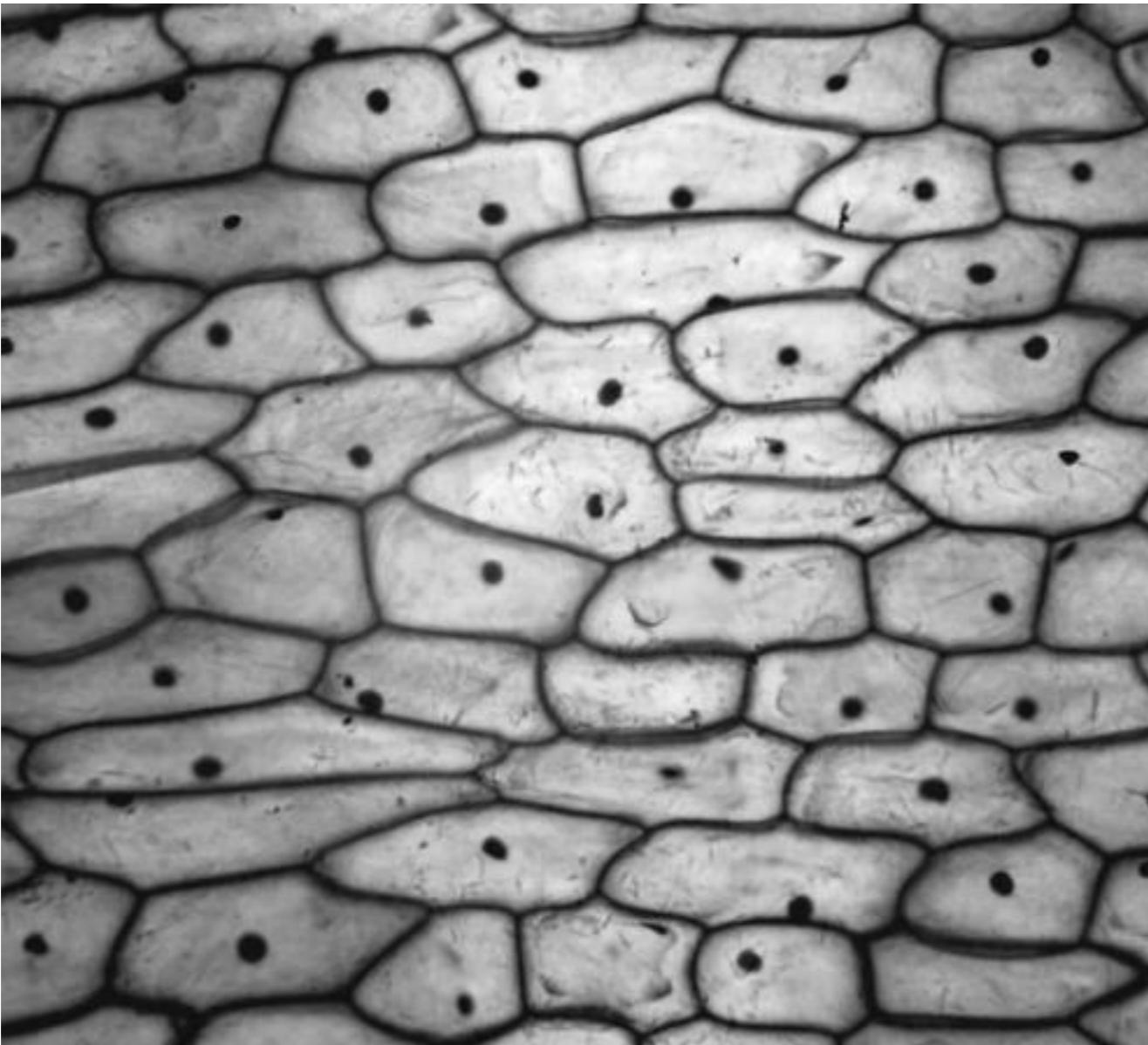
# Today

- Scale-Invariant Local Features (SIFT)
- Geometric Invariances

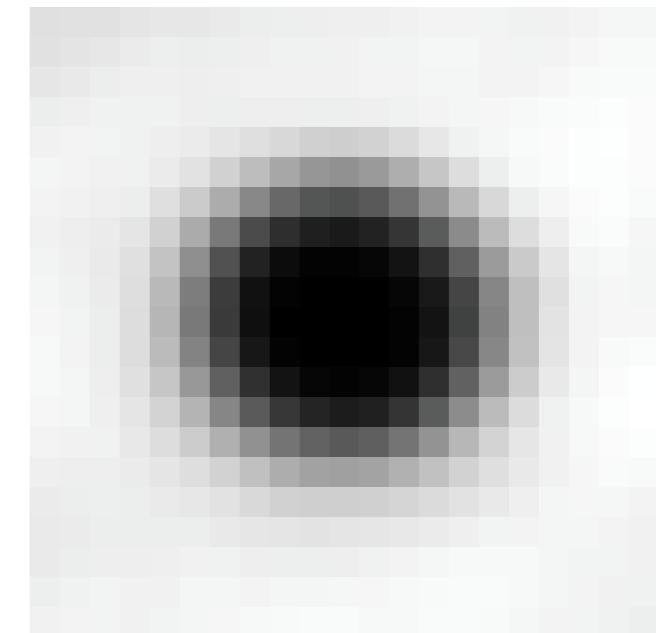
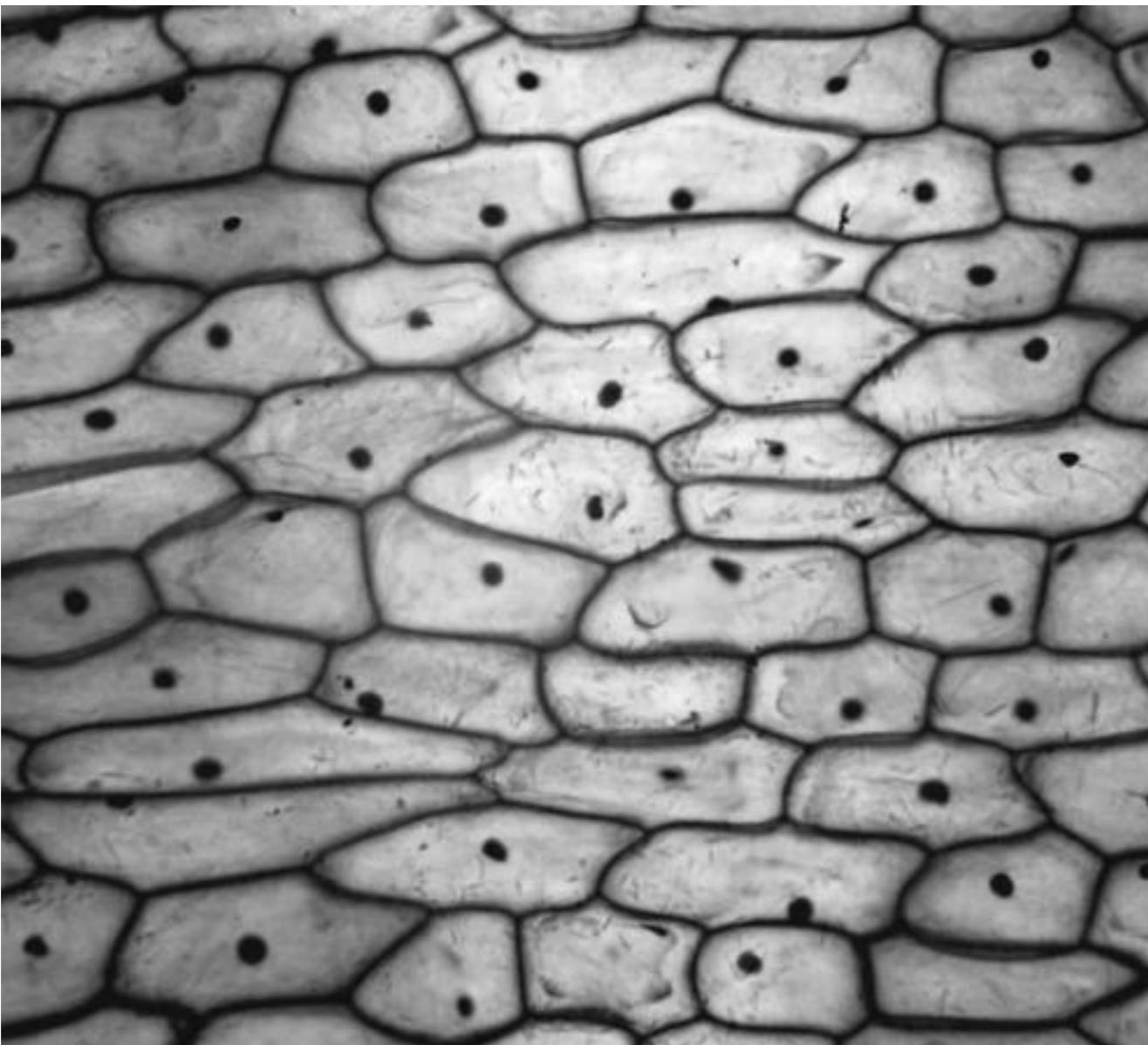
# Object Detection via Classification



# Object Detection via Classification

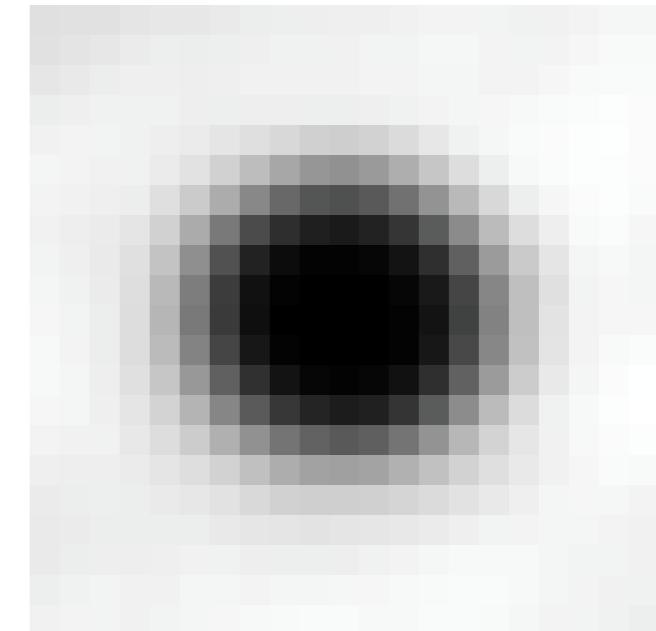
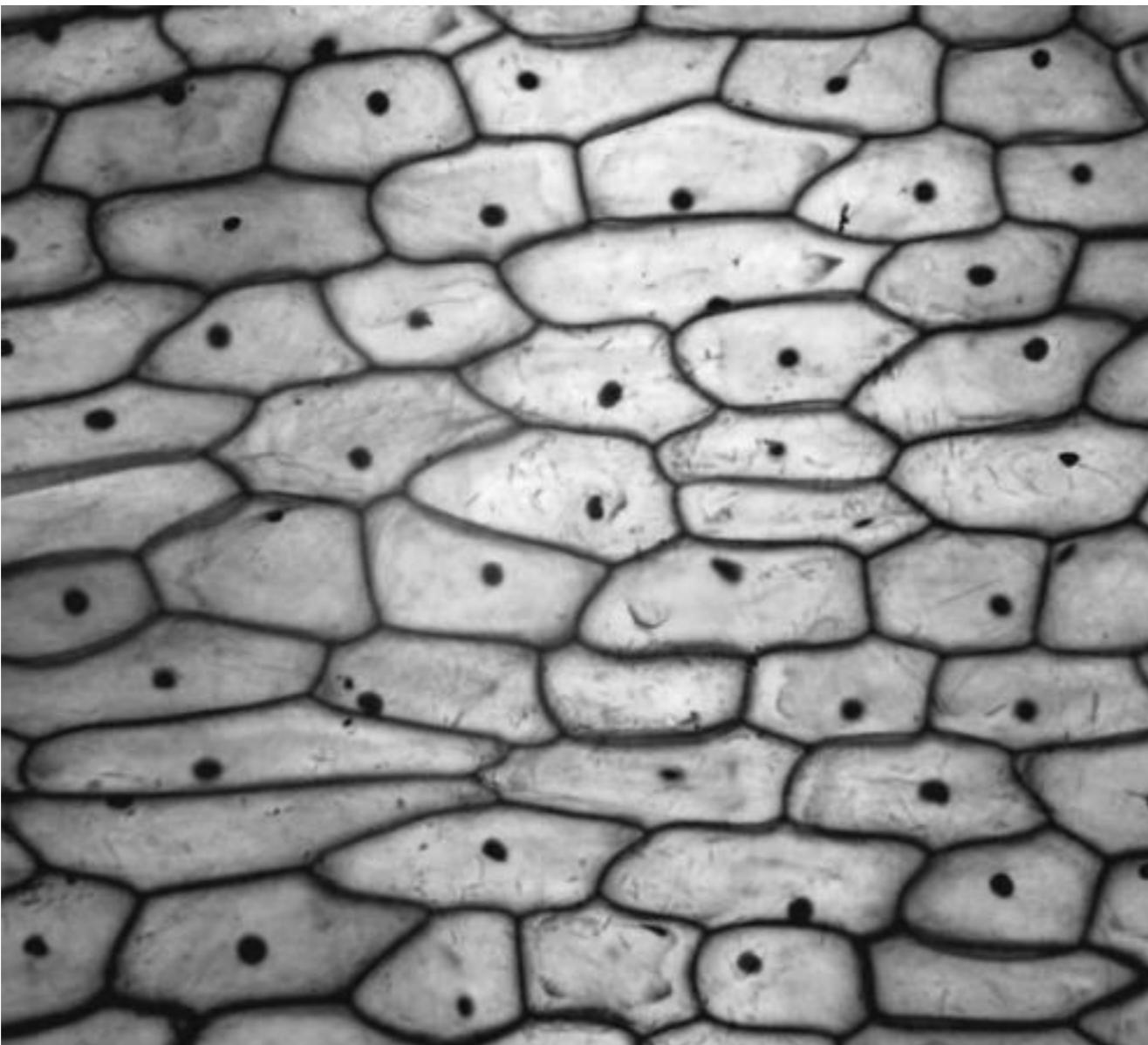


# Object Detection via Classification



Linear filter  
= example object

# Object Detection via Classification



Linear filter  
= example object

Know the object in advance

# Local Features

Establish pixel-level correspondences between two images



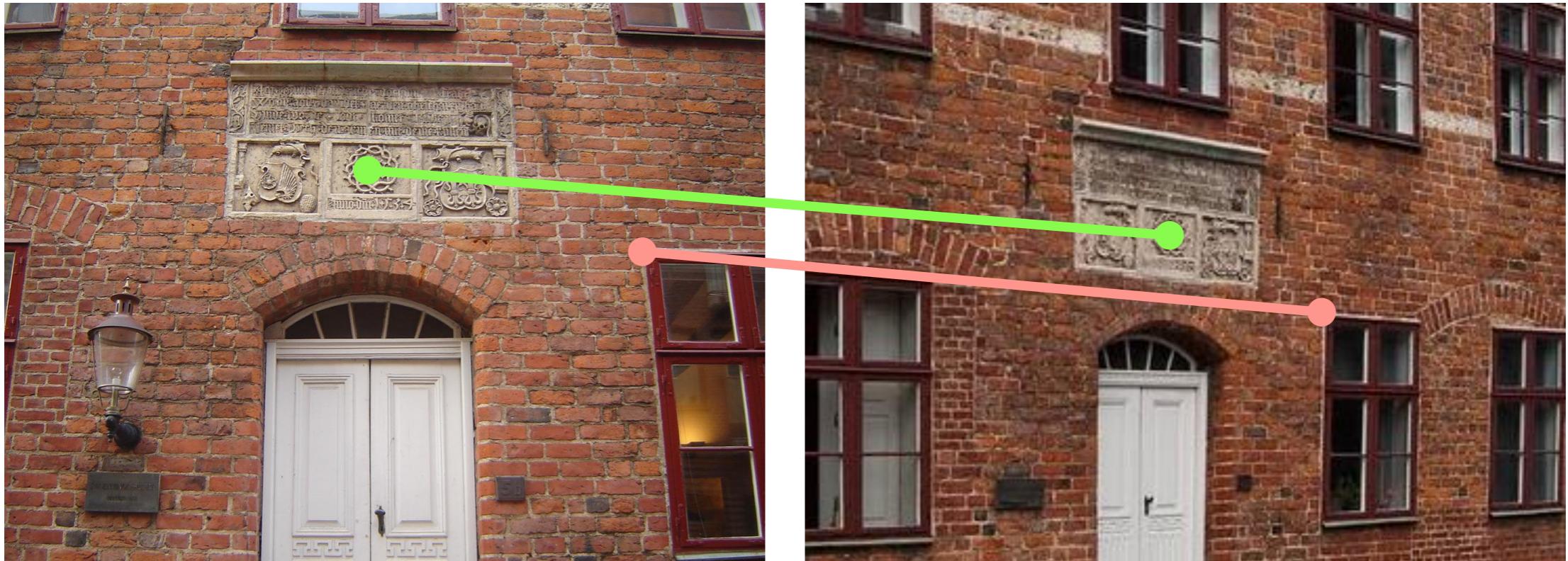
# Local Features

Establish pixel-level correspondences between two images



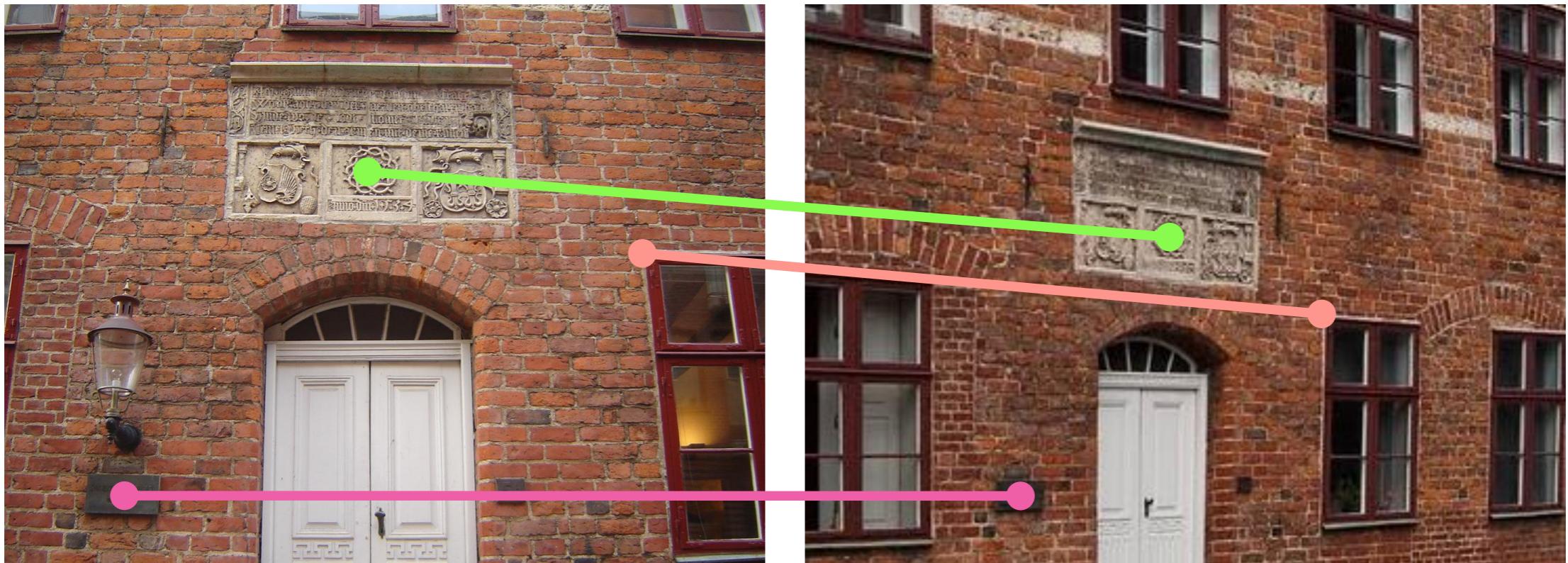
# Local Features

Establish pixel-level correspondences between two images



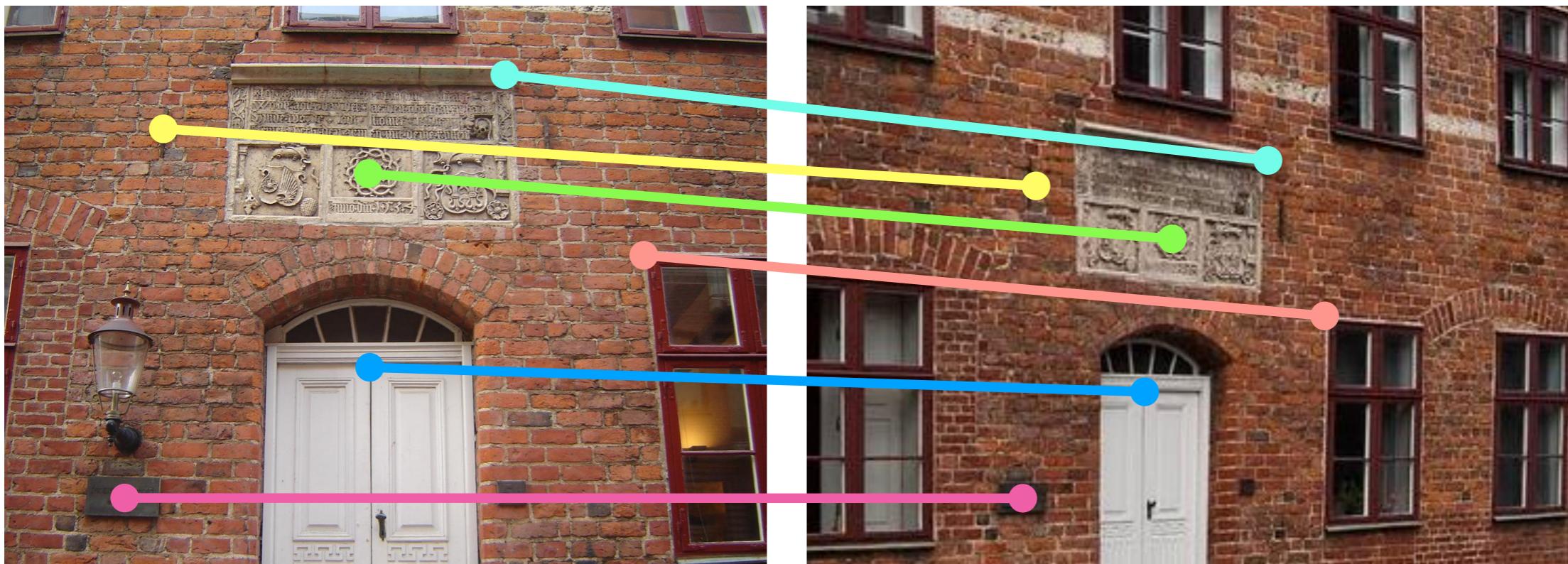
# Local Features

Establish pixel-level correspondences between two images



# Local Features

Establish pixel-level correspondences between two images

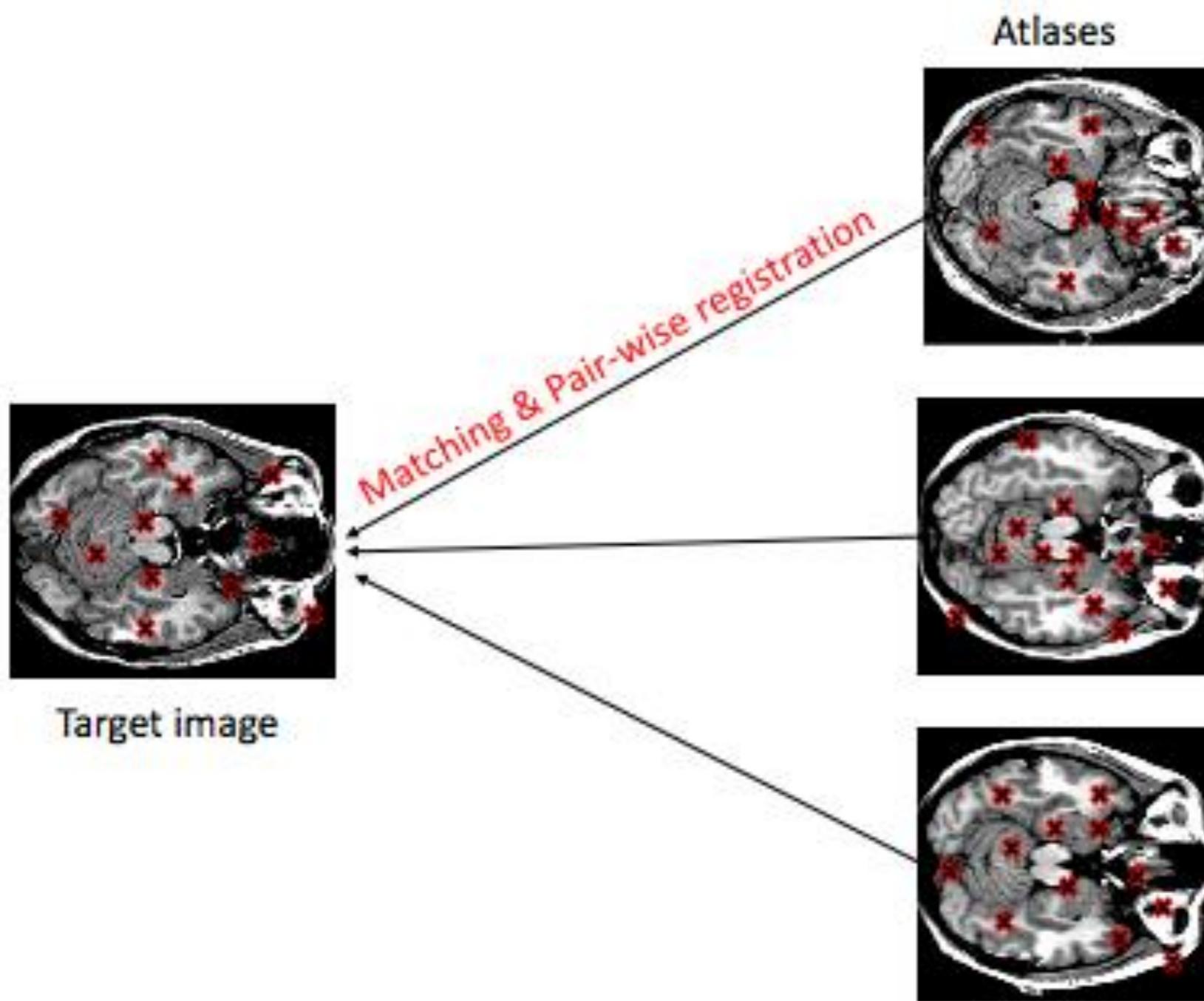


# Applications: Image Alignment



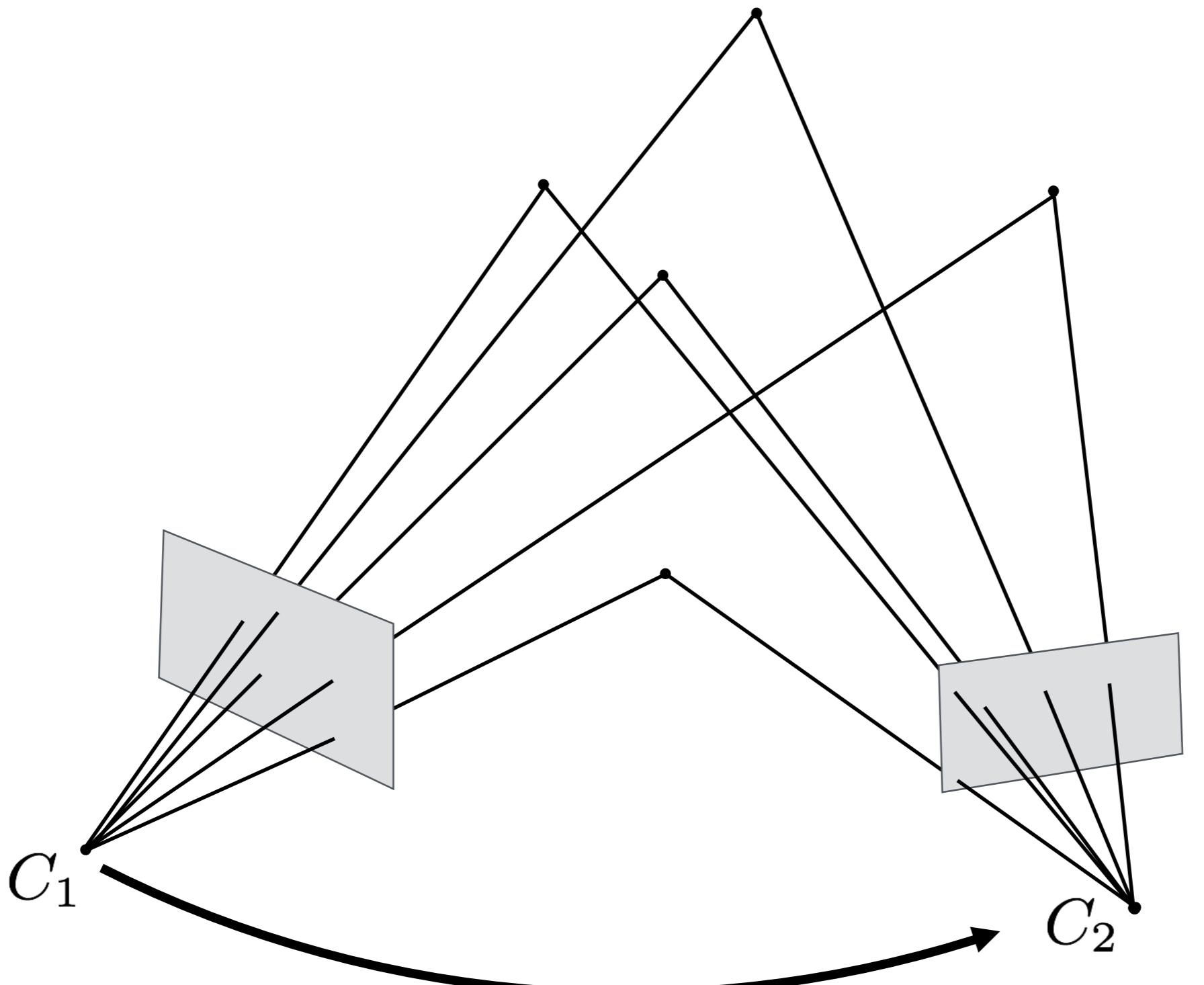
Lecture 8

# Applications: MRI Image Alignment



Alvén et al.

# Relative Pose Estimation



Lecture 9, 10

# Applications: 3D Reconstruction

## Structure-from-Motion Revisited

Johannes L. Schönberger, Jan-Michael Frahm

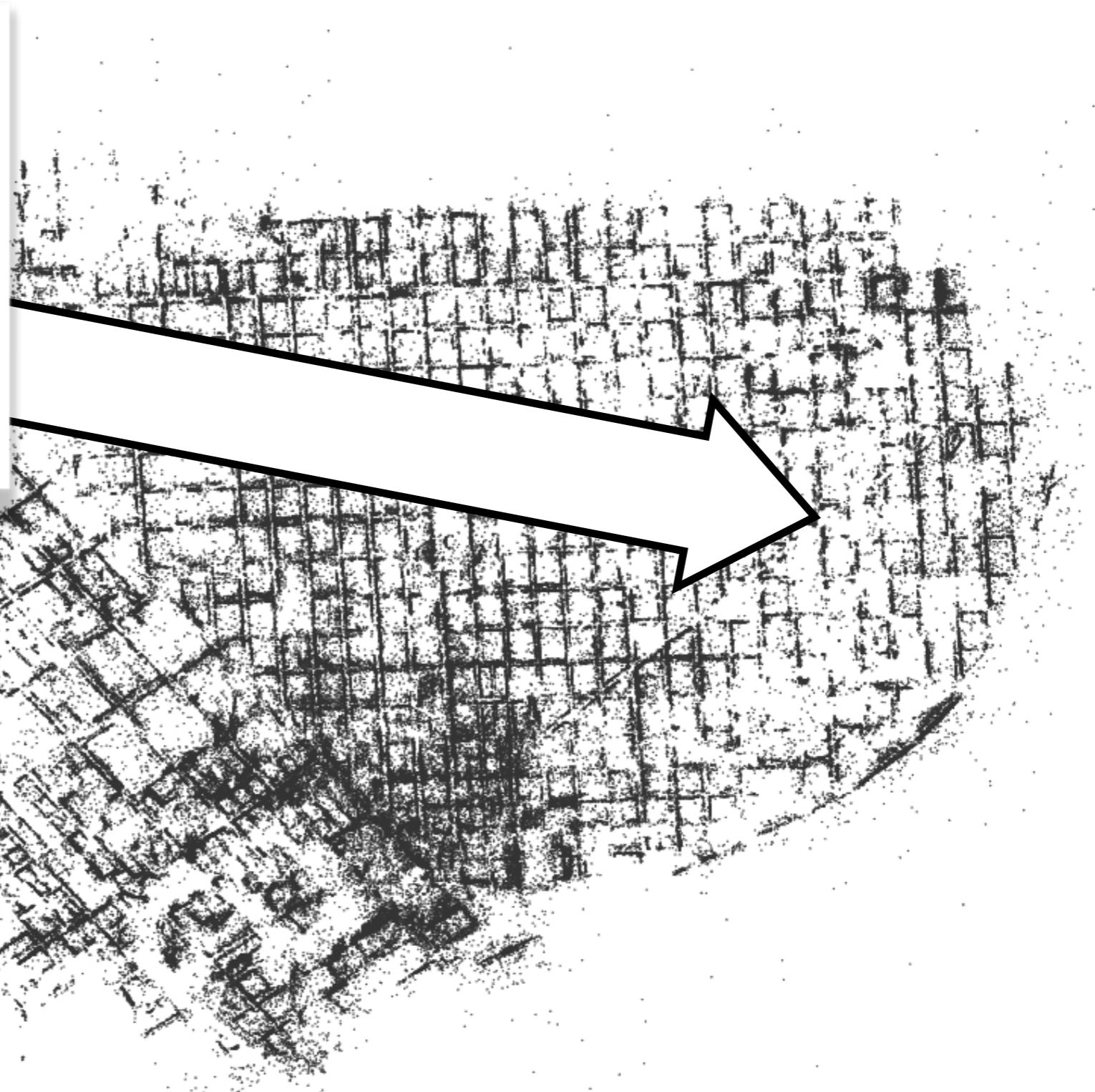
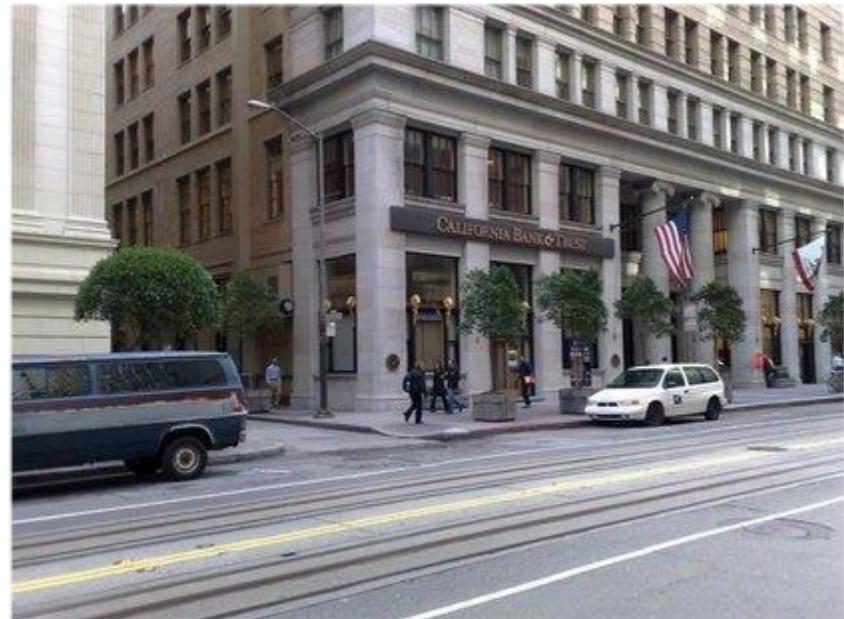
CVPR 2016

Code available at:  
<https://github.com/colmap/colmap>

[Schönberger & Frahm, Structure-from-Motion Revisited, CVPR 2016]

Lecture 10

# City-Scale Localization



[Svärm, Enqvist, Kahl, Oskarsson, City-Scale Localization for Cameras with Known Vertical Direction, IJCV 2017 / CVPR 2014]

[Zeisl, Sattler, Pollefeys, Camera Pose Voting for Large-Scale Image-Based Localization, CVPR 2015]

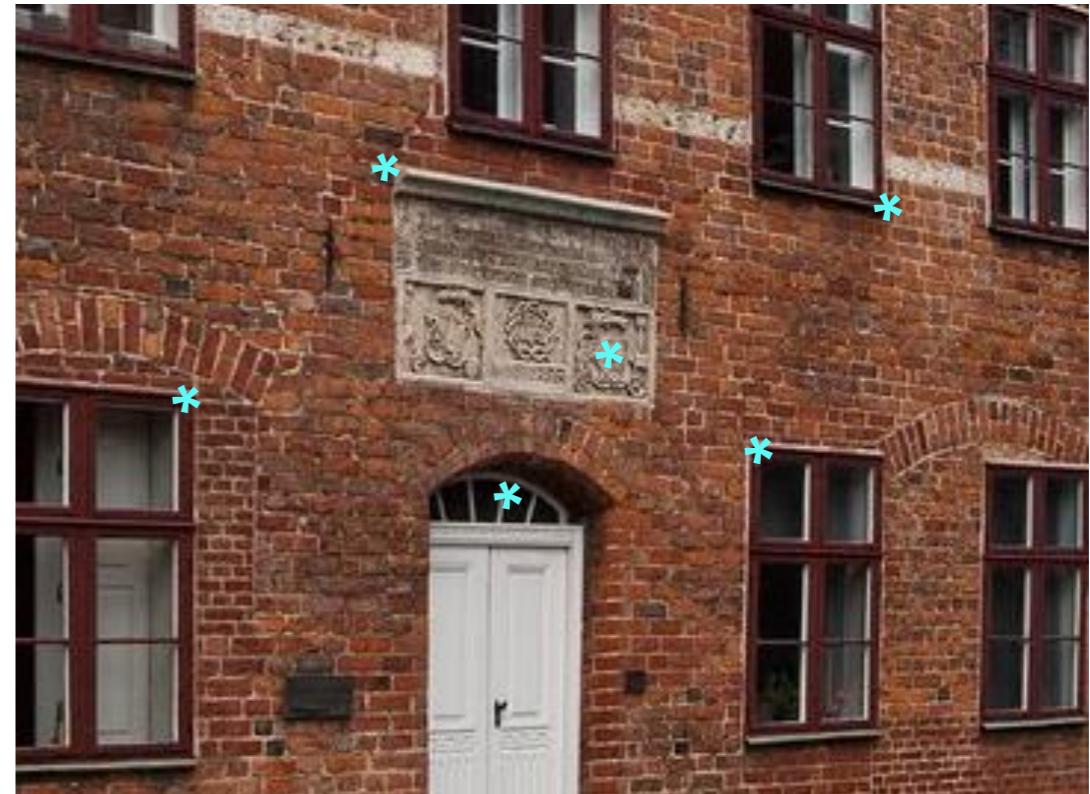
# Local Features

Two stage process:



# Local Features

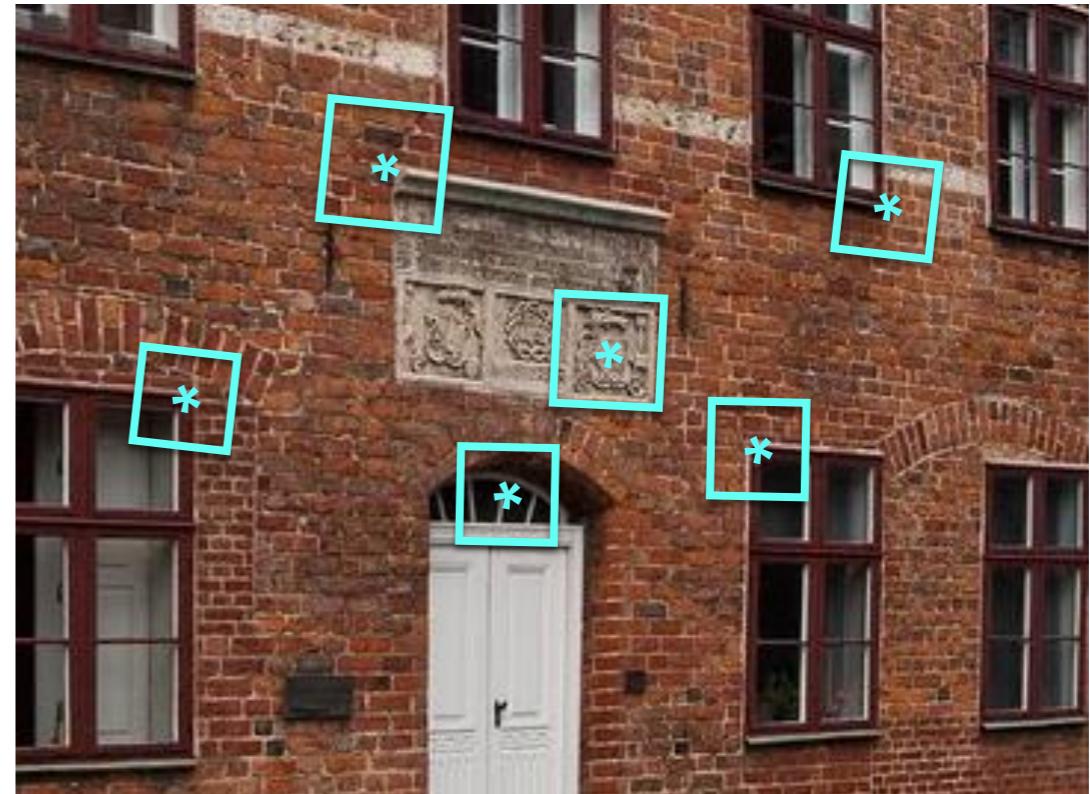
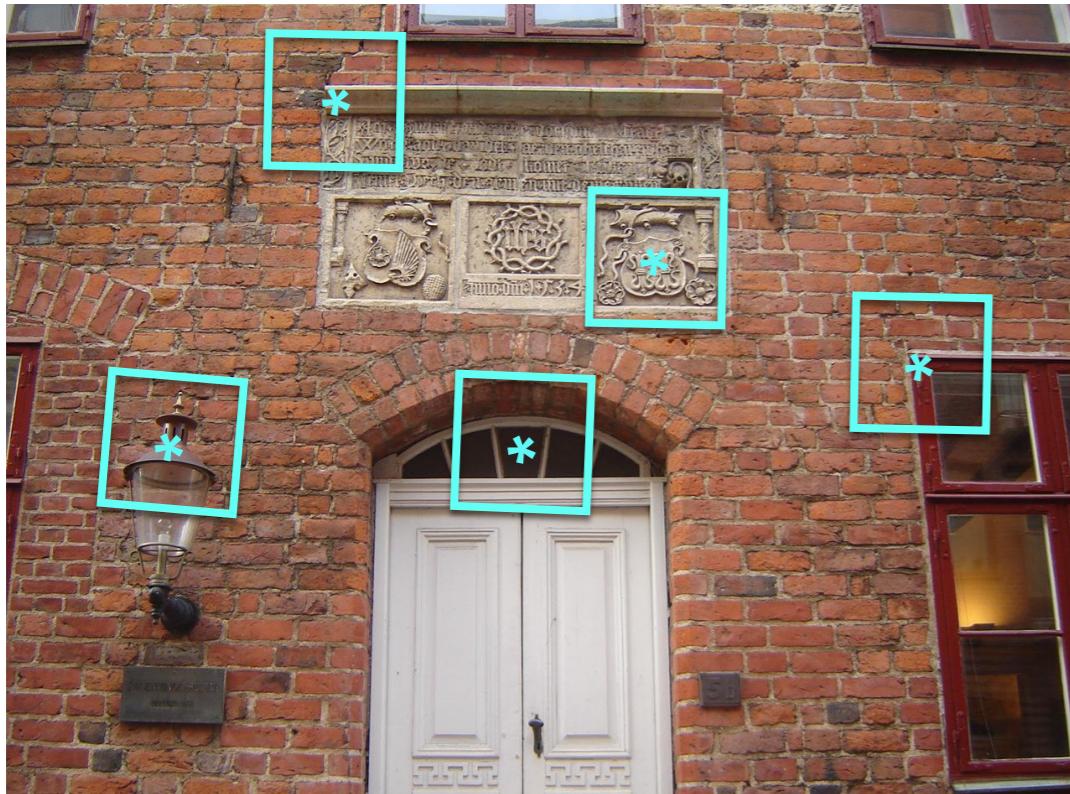
Two stage process:



1. **Keypoint Detector:** Find “interesting points” in the images

# Local Features

Two stage process:



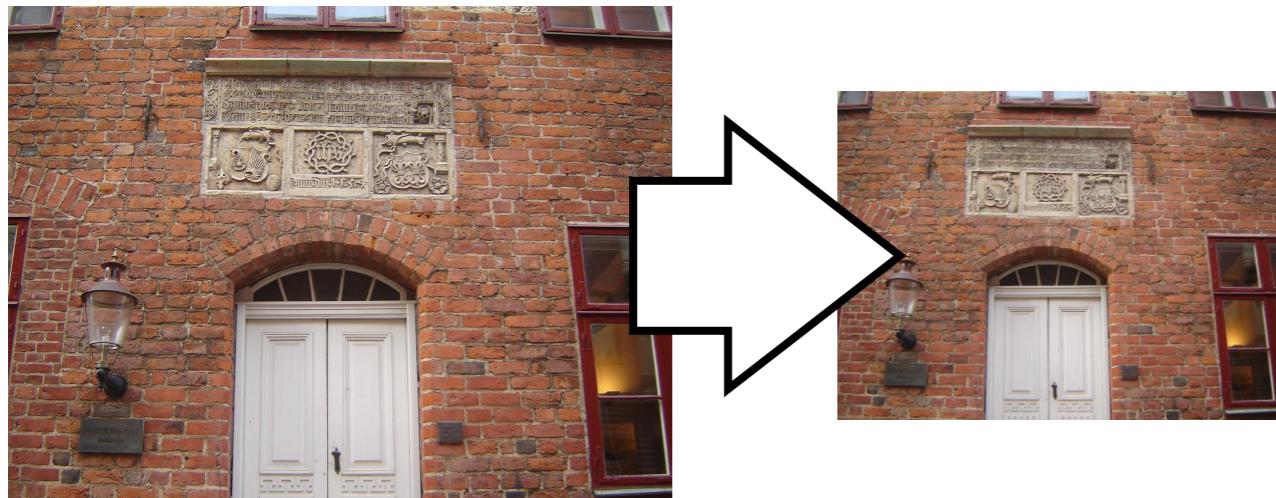
- 1. Keypoint Detector:** Find “interesting points” in the images
- 2. Keypoint Descriptor:** Describe regions around these points

# Local Features

Desired: Robustness / invariance to (among others)

# Local Features

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# Local Features

Desired: Robustness / invariance to (among others)



Changes in scale



in-plane rotation

# Local Features

Desired: Robustness / invariance to (among others)



Changes in scale



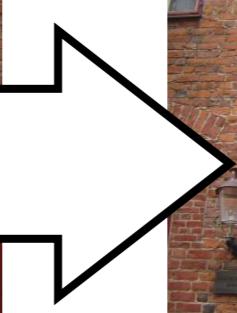
in-plane rotation



Changes in brightness

# Local Features

Desired: Robustness / invariance to (among others)



Changes in scale



in-plane rotation



Changes in brightness



Changes in viewpoint



# Scale-Invariant Feature Transform (SIFT) Features



[Lowe, Distinctive Image Features from Scale-Invariant Keypoints, IJCV 2004]

# Scale-Invariant Feature Transform (SIFT) Features



- Find a set of *interest* points

[Lowe, Distinctive Image Features from Scale-Invariant Keypoints, IJCV 2004]

# Scale-Invariant Feature Transform (SIFT) Features



- Find a set of *interest* points
  - At different scales
  - Robust to brightness changes

[Lowe, Distinctive Image Features from Scale-Invariant Keypoints, IJCV 2004]

# Scale-Invariant Feature Transform (SIFT) Features



- Find a set of *interest* points
- Extract scaled & oriented patch around interest points

[Lowe, Distinctive Image Features from Scale-Invariant Keypoints, IJCV 2004]

# Scale-Invariant Feature Transform (SIFT) Features



- Find a set of *interest* points
- Extract scaled & oriented patch around interest points
  - Invariance to rotations
  - Invariance to scale changes

[Lowe, Distinctive Image Features from Scale-Invariant Keypoints, IJCV 2004]

# Scale-Invariant Feature Transform (SIFT) Features



- Find a set of *interest* points
- Extract scaled & oriented patch around interest points
- Compute SIFT descriptor for each patch

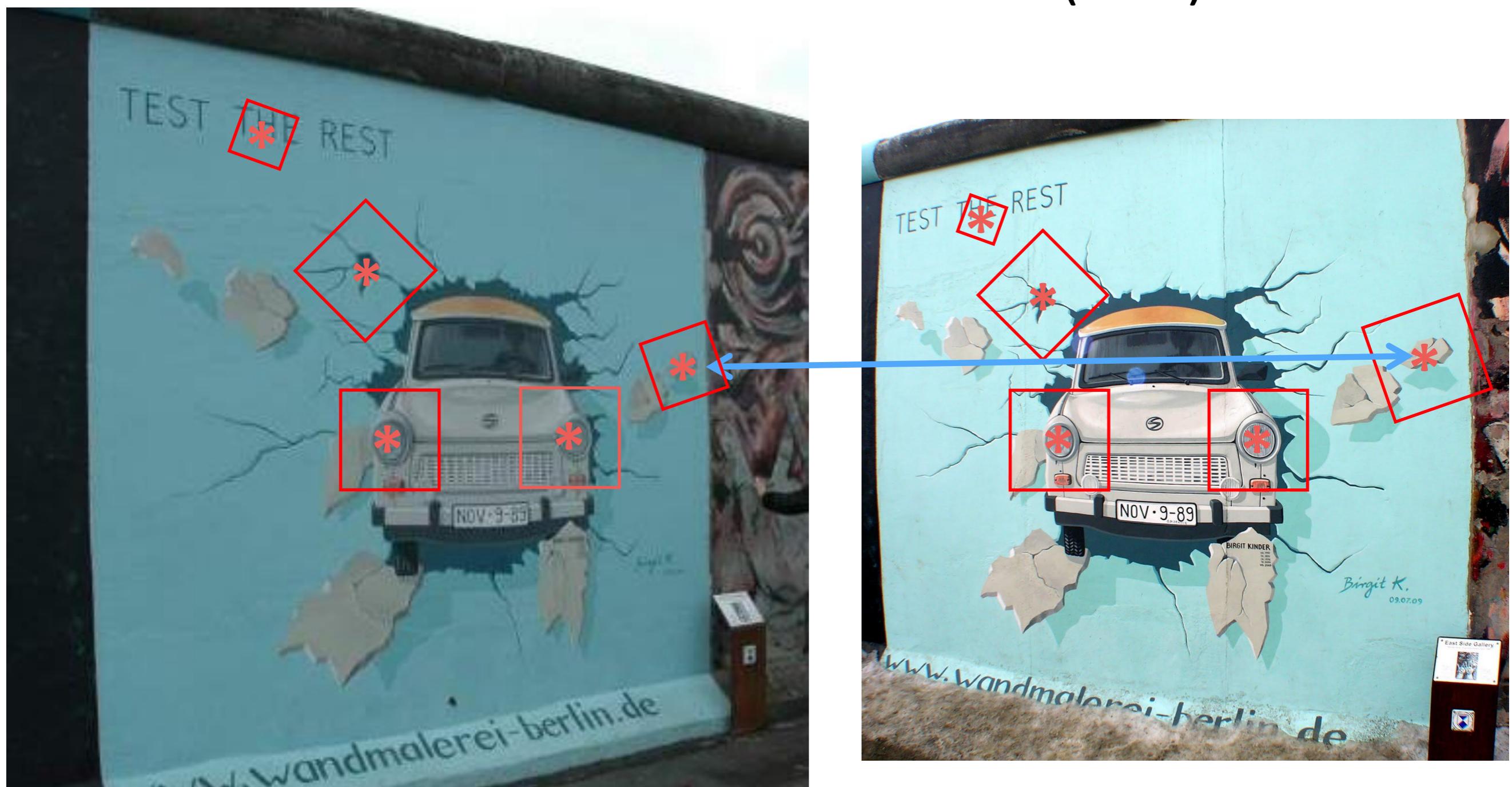
[Lowe, Distinctive Image Features from Scale-Invariant Keypoints, IJCV 2004]

# Scale-Invariant Feature Transform (SIFT) Features



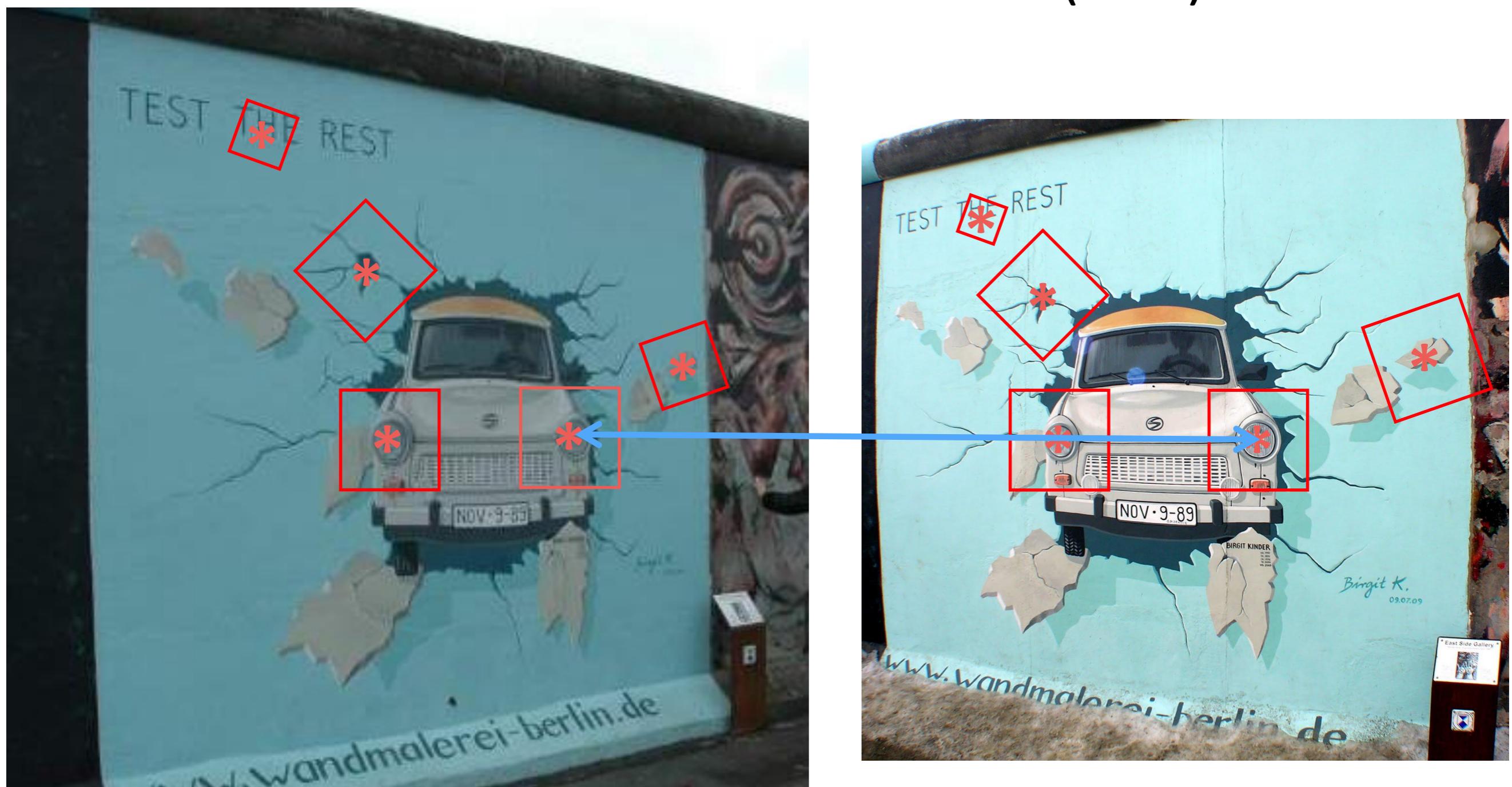
**Feature Matching:** Use descriptor similarity ( $L_2$  distance) to find corresponding points in different images

# Scale-Invariant Feature Transform (SIFT) Features



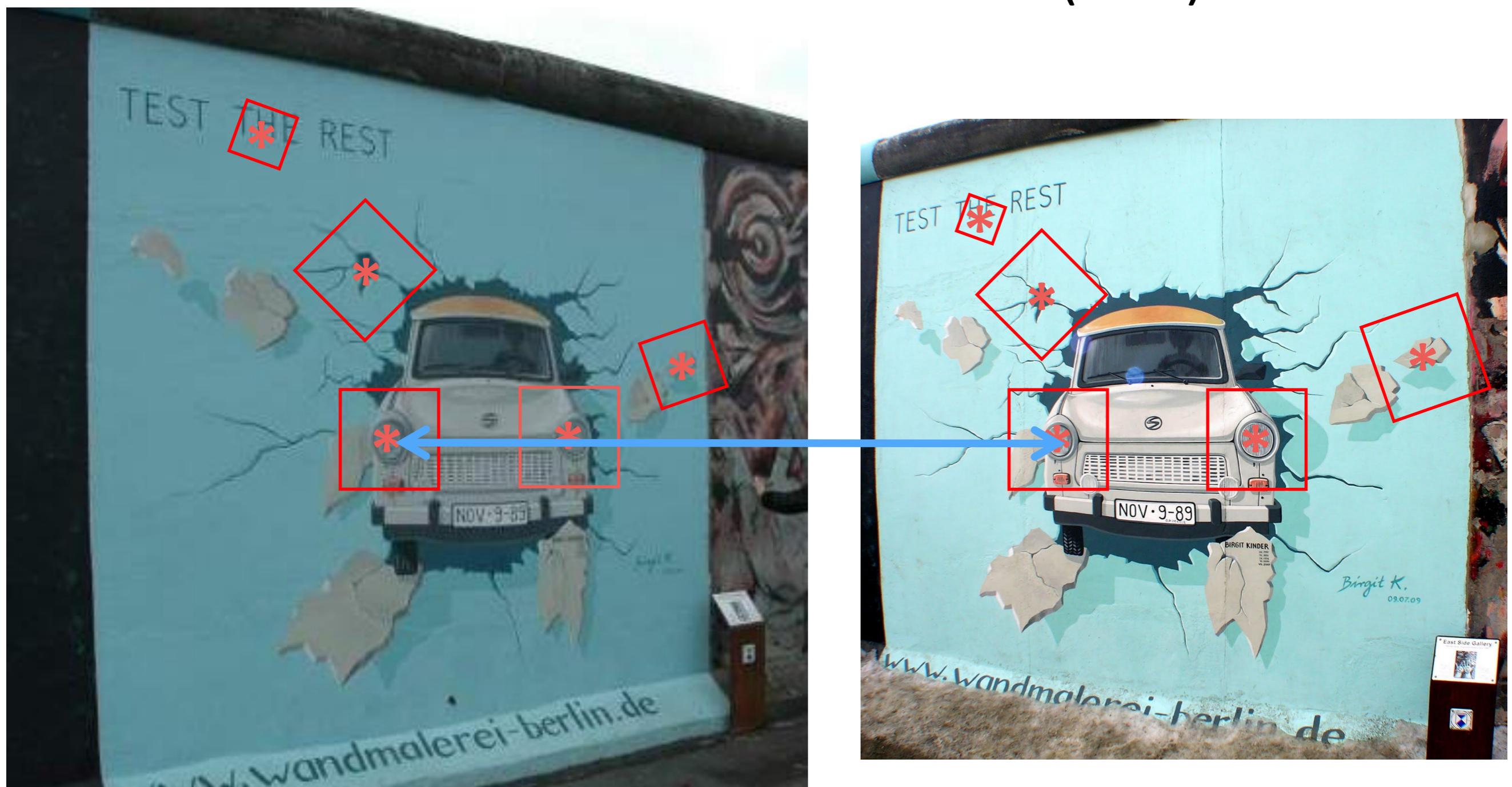
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# Scale-Invariant Feature Transform (SIFT) Features



**Feature Matching:** Use descriptor similarity ( $L_2$  distance) to find corresponding points in different images

# Scale-Invariant Feature Transform (SIFT) Features



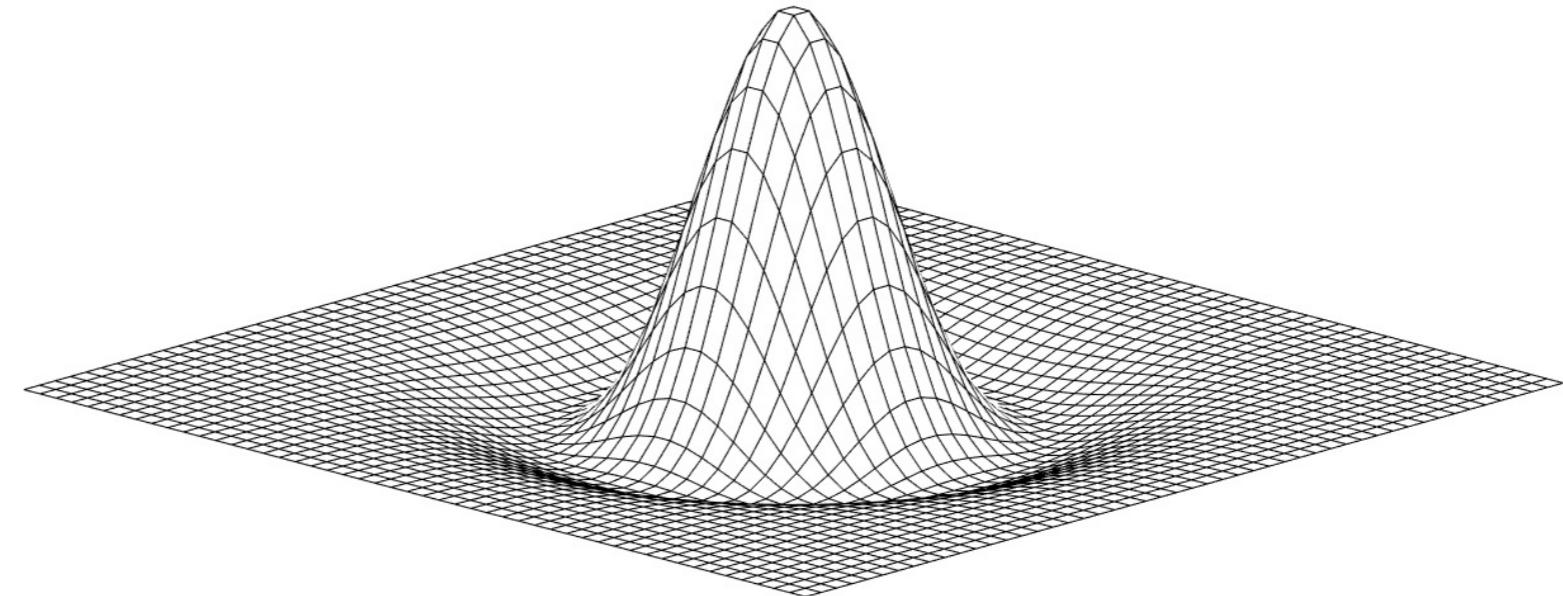
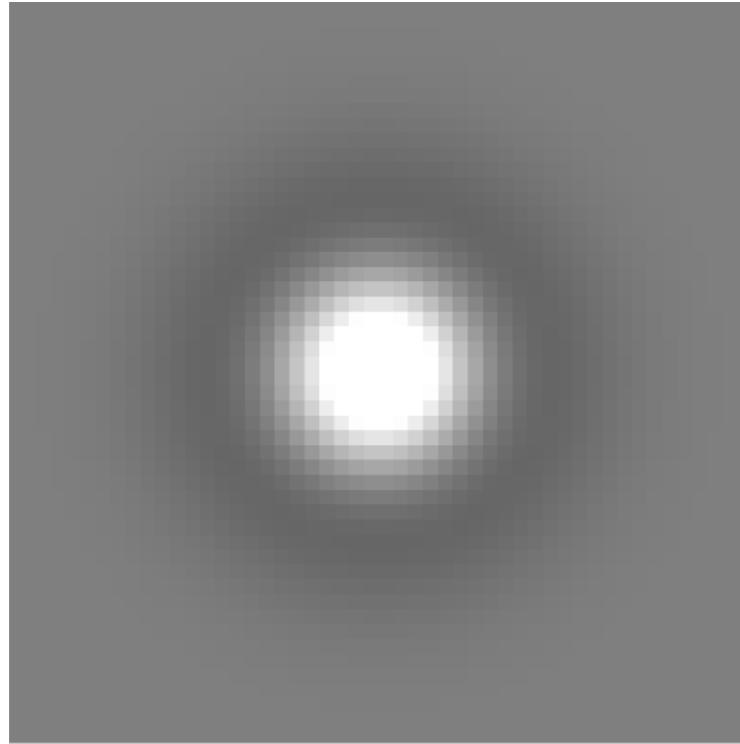
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# Scale-Invariant Feature Transform (SIFT) Features



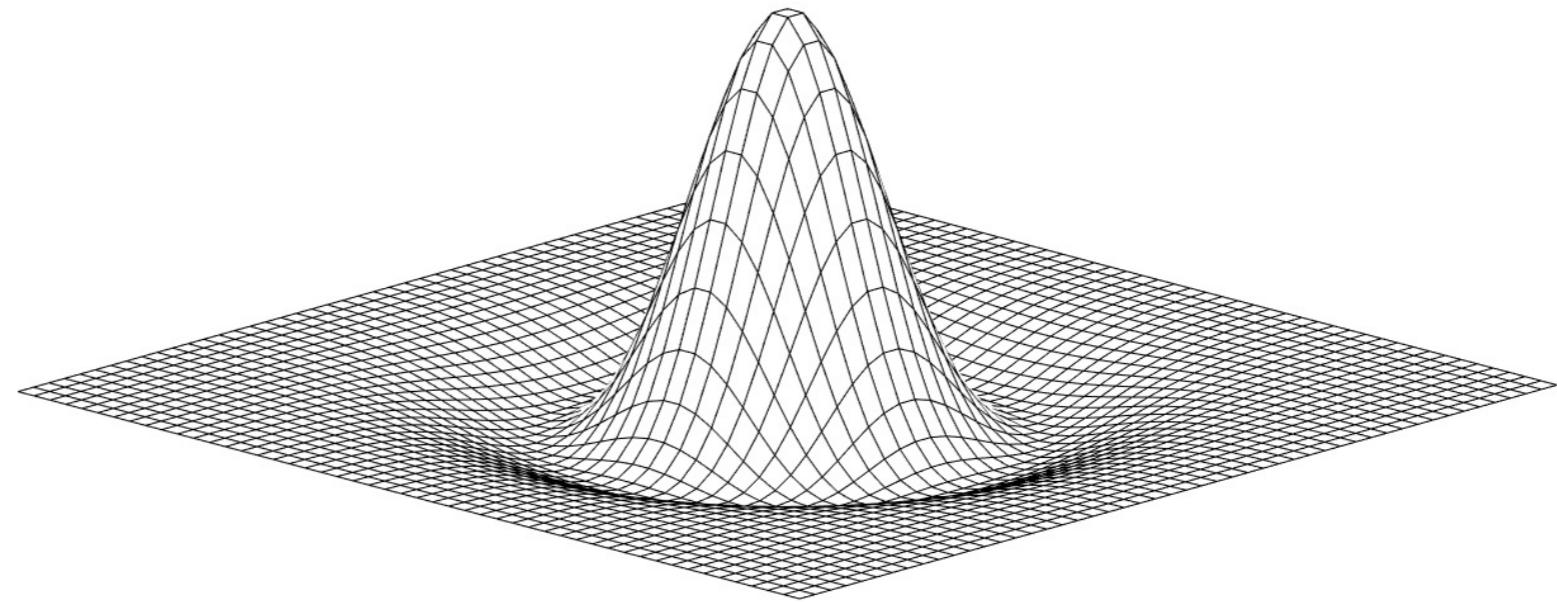
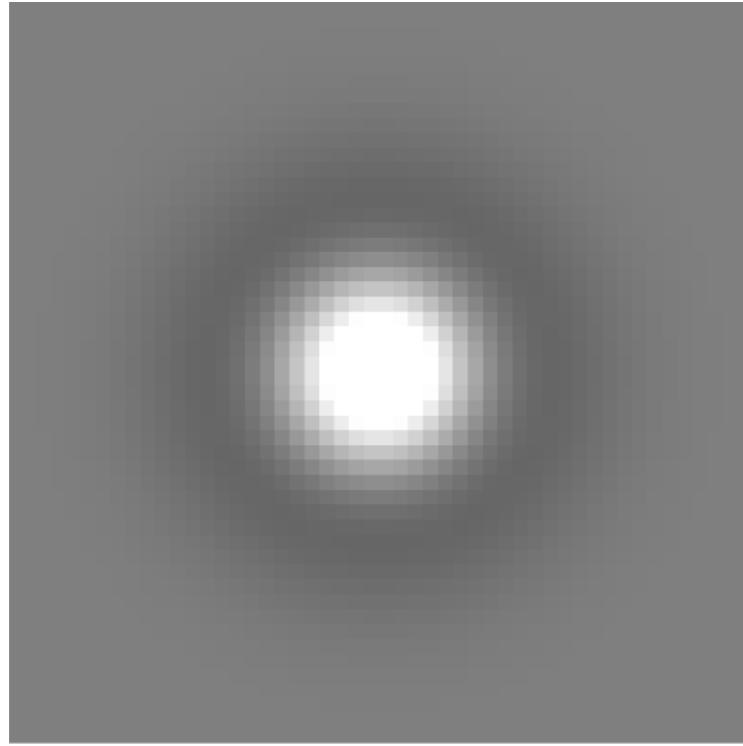
- Find a set of *interest* points
- Extract oriented patch around each interest point
- Compute SIFT descriptor for each patch

# Keypoint Detector: Difference of Gaussians (DoG)



- Apply DoG filter over the image
- Keypoints: Local extrema of DoG response
- Why? Efficient approximation to Laplacian of Gaussians
  - Produces very stable detections [Mikolajczyk, Detection of local features invariant to affine transformations, Ph.D. thesis, Institut National Polytechnique de Grenoble, France, 2002]

# Keypoint Detector: Difference of Gaussians (DoG)



- Difference of two Gaussian filters
- Intuitive explanation of extrema:
  - Local maxima at bright blobs
  - Local minima at dark blobs

# Robustness / Invariances?



in-plane rotation

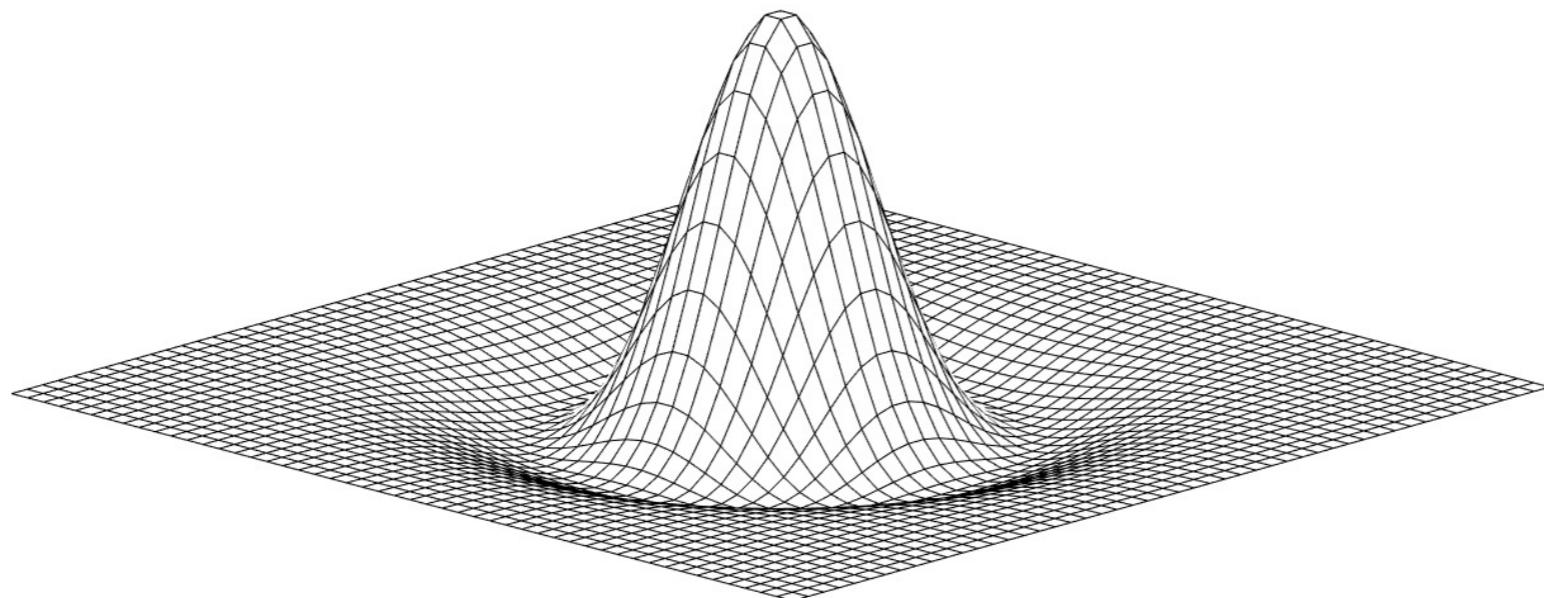
# Robustness / Invariances?



in-plane rotation



# Robustness / Invariances?



Filter is rotation symmetric



in-plane rotation



# Robustness / Invariances?



Changes in brightness

# Robustness / Invariances?

Local minima / maxima unaffected\* by:

- Additive changes in brightness
- Multiplicative changes in brightness

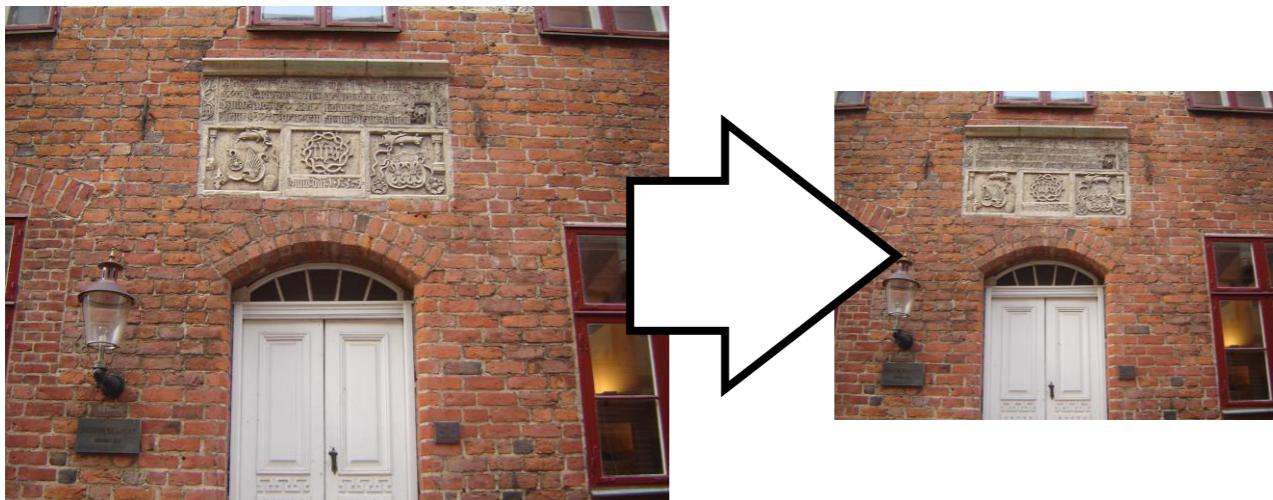
\* As long as inside dynamic range of camera



Changes in brightness



# Robustness / Invariances?



Changes in scale

# The Need for Multi-Scale Detections

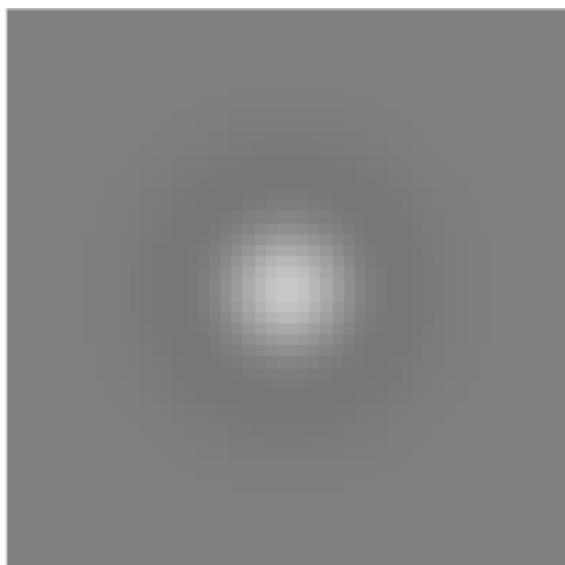
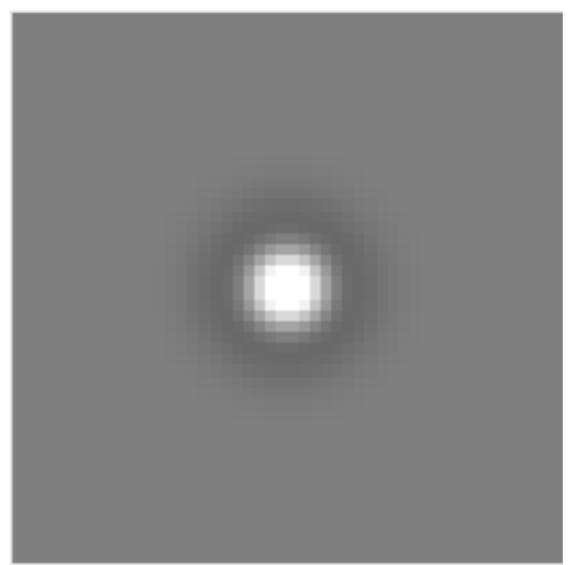
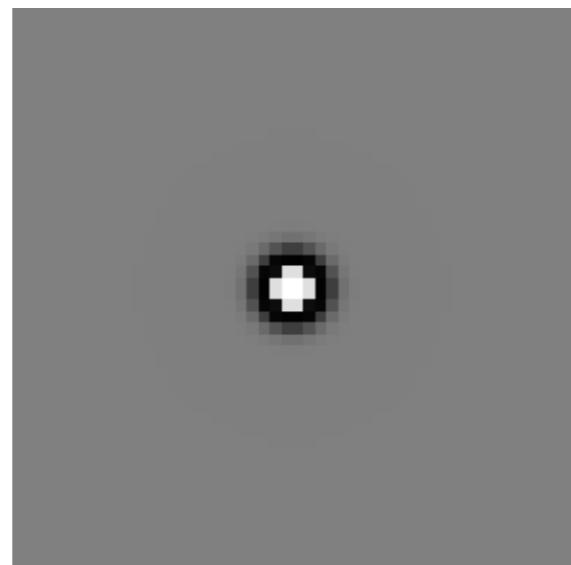


# The Need for Multi-Scale Detections



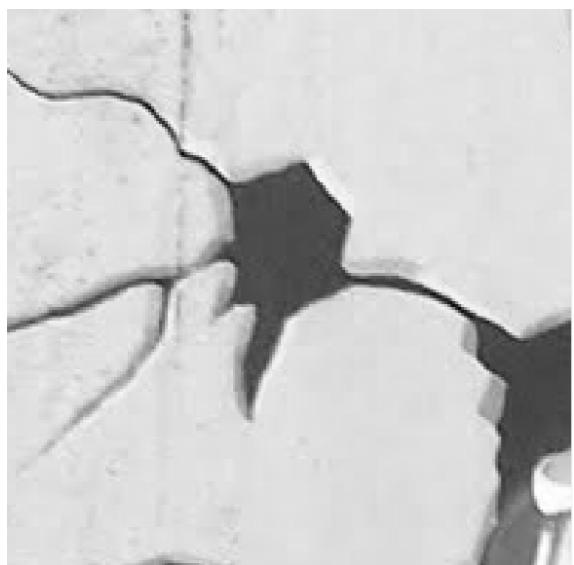
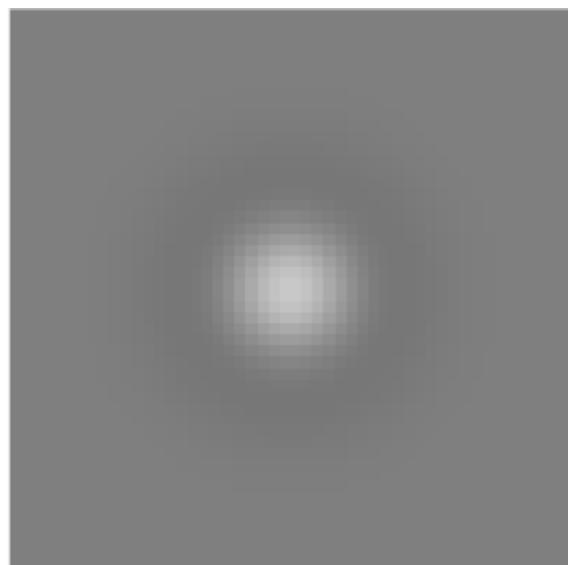
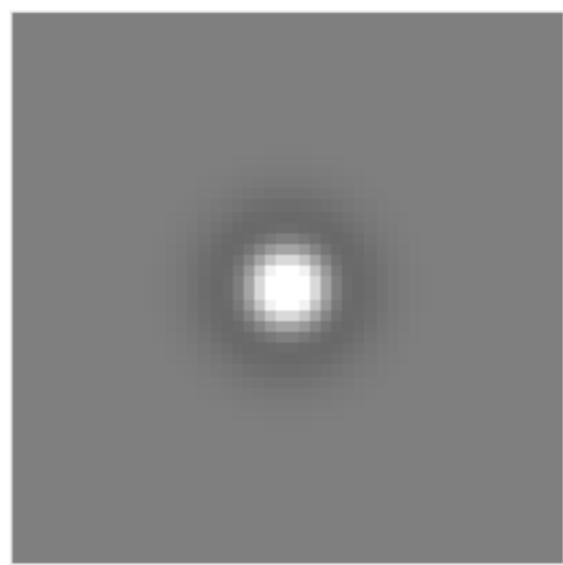
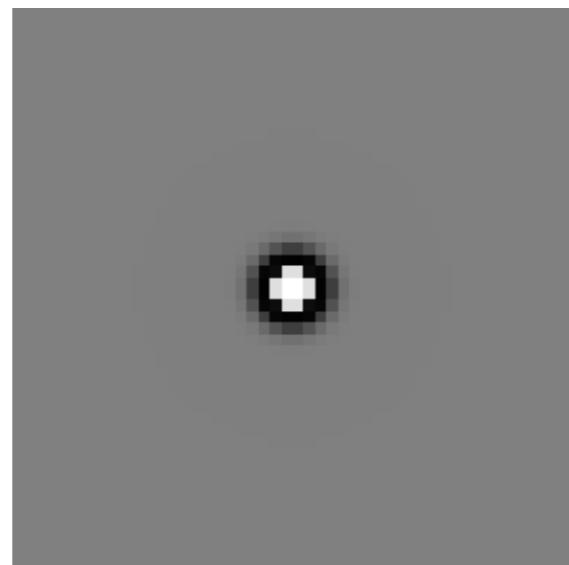
# Difference of Gaussians (DoG)

Use filters of different size



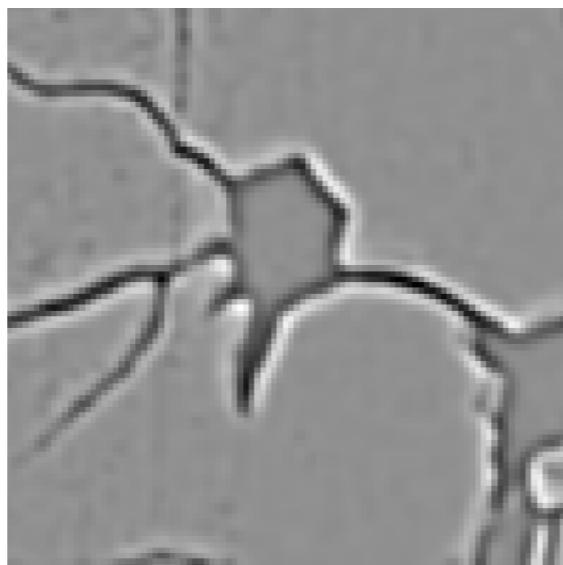
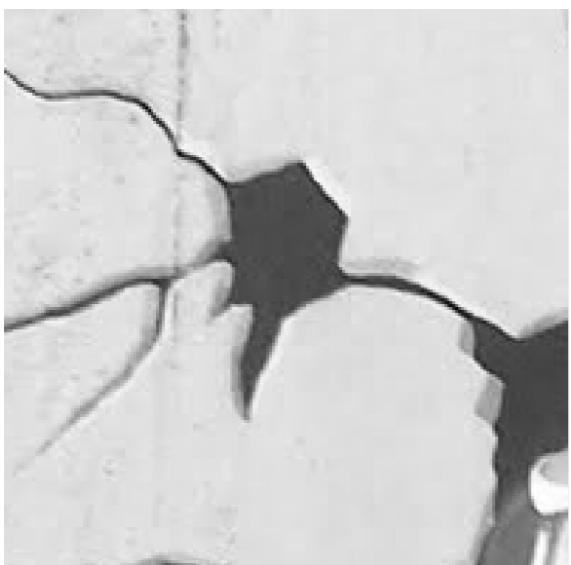
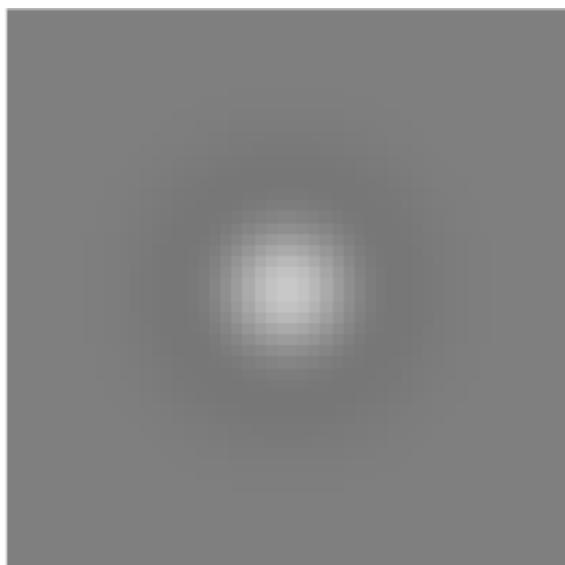
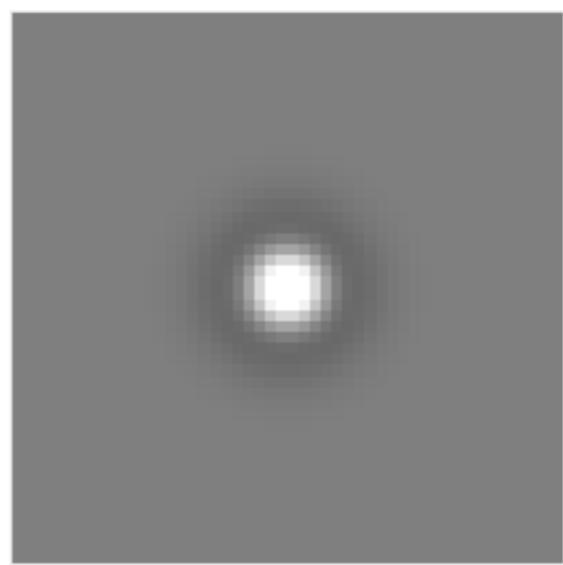
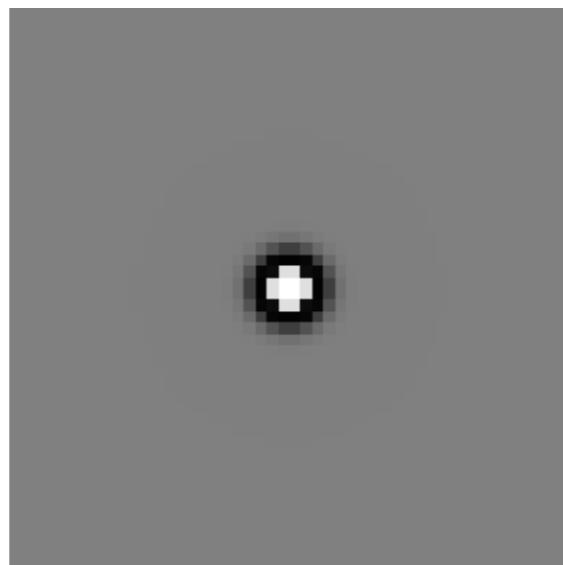
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Use filters of different size



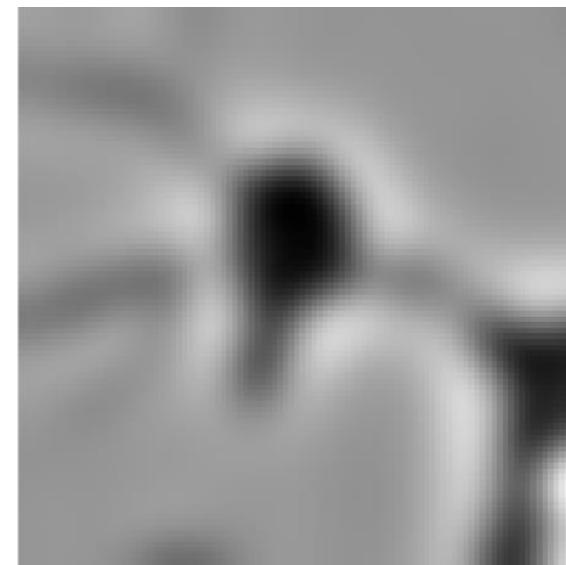
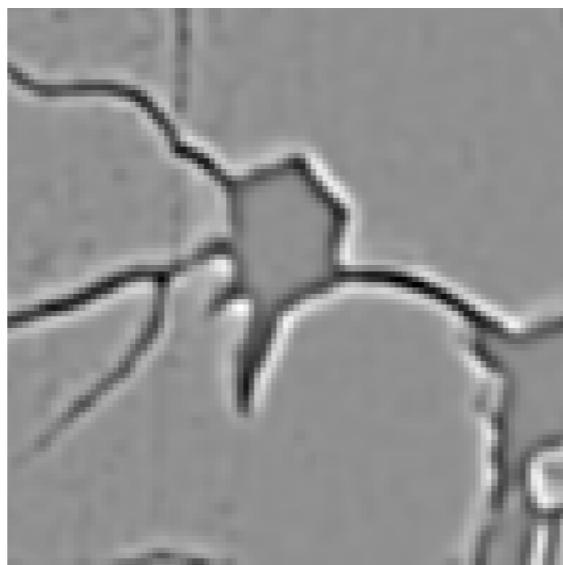
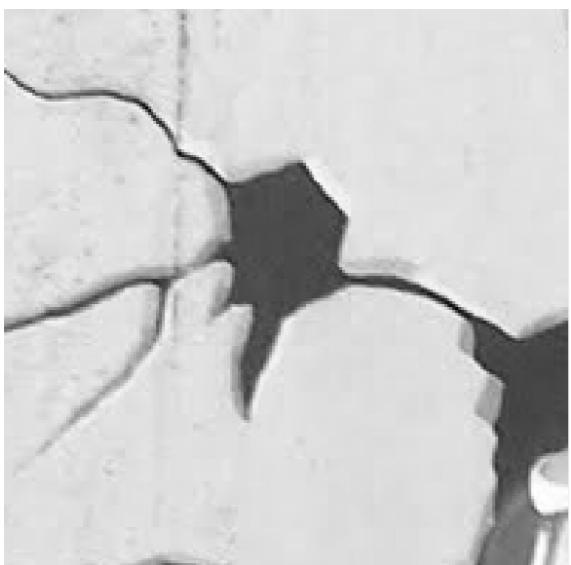
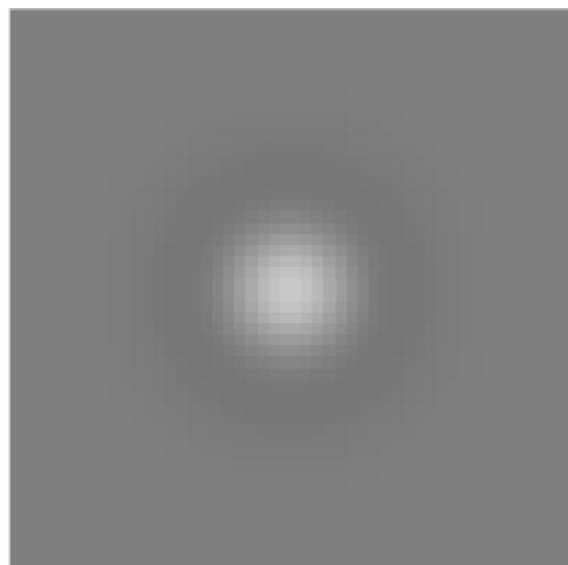
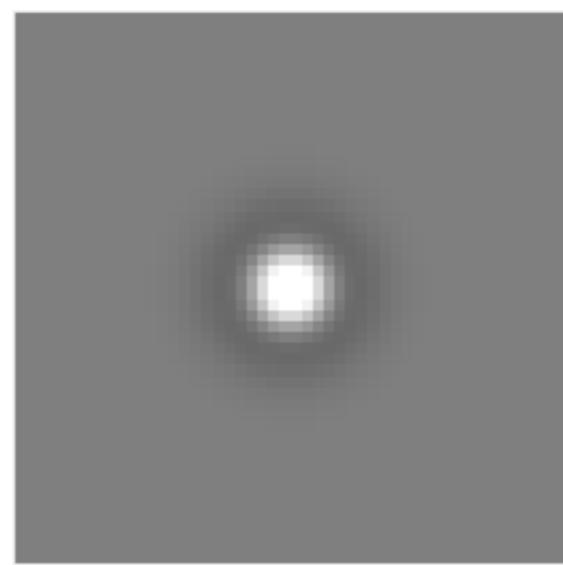
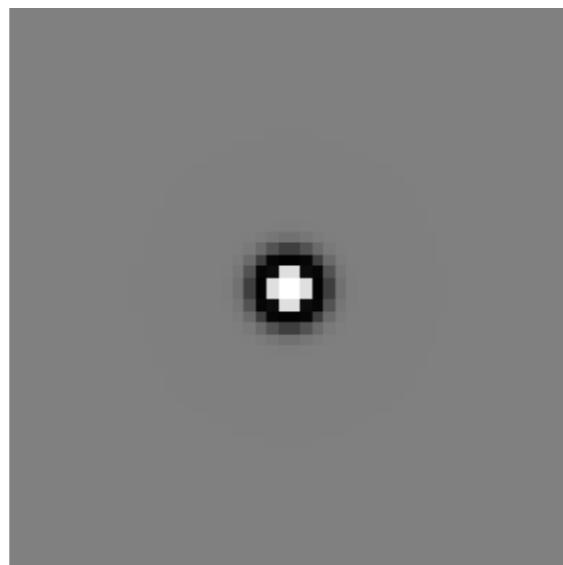
# Difference of Gaussians (DoG)

Use filters of different size



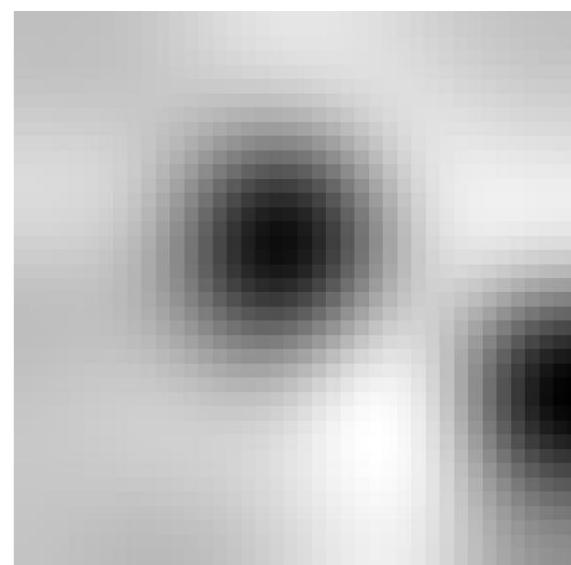
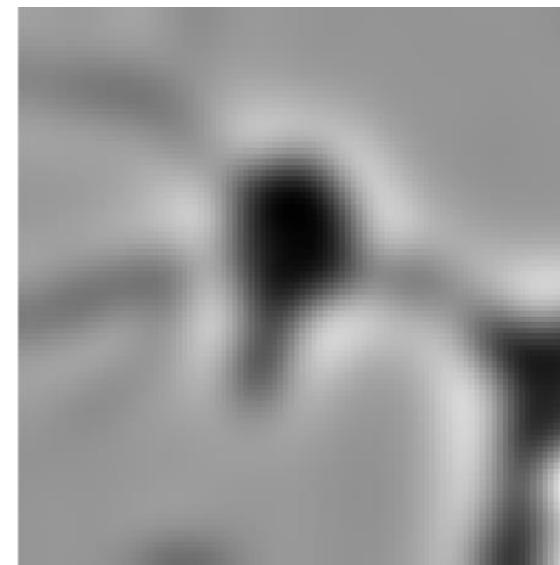
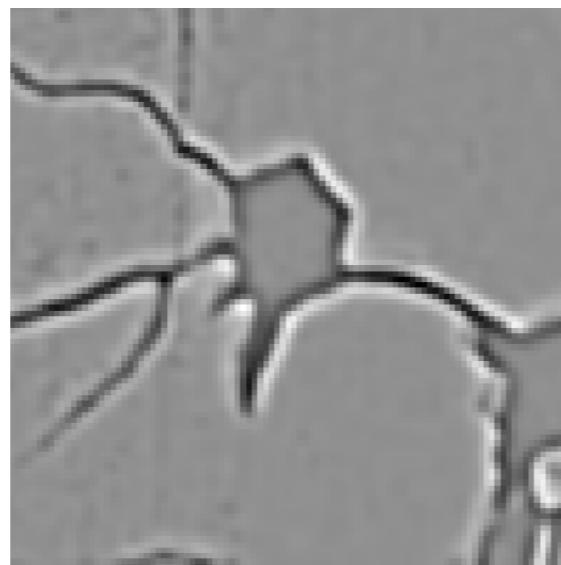
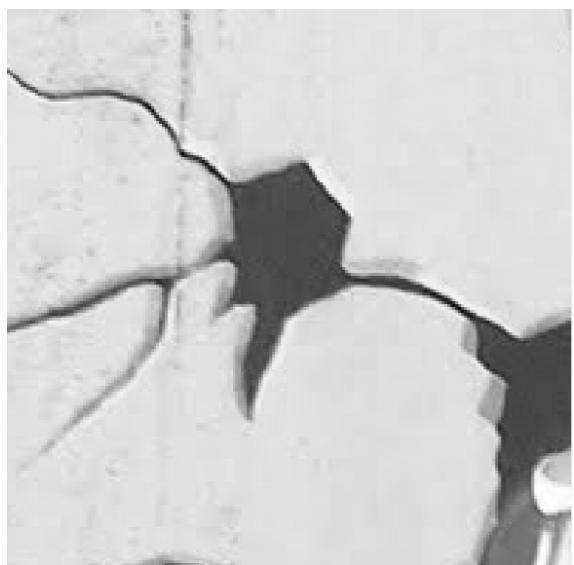
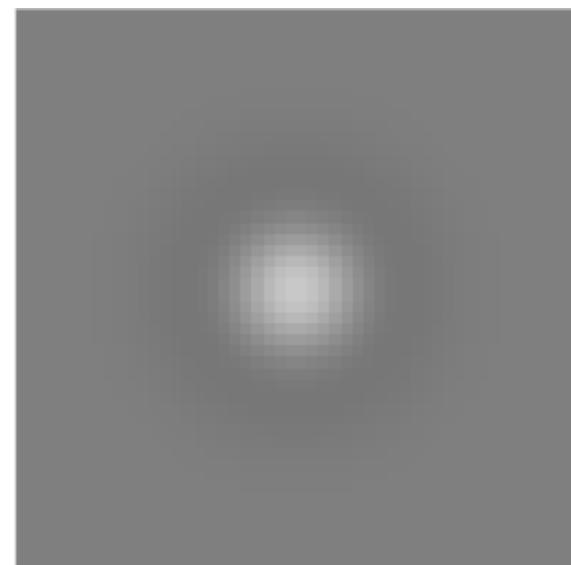
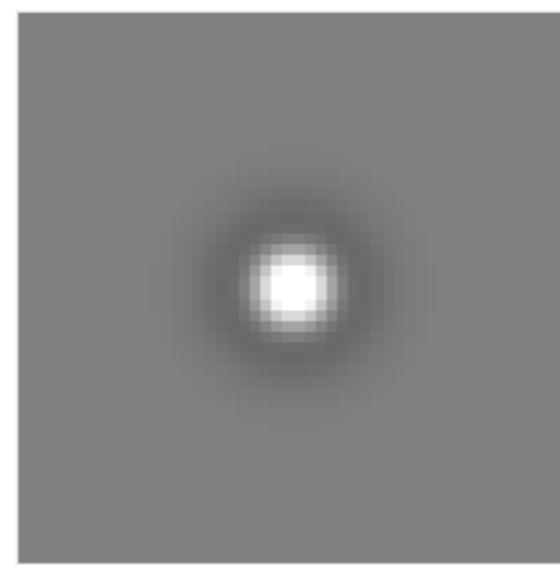
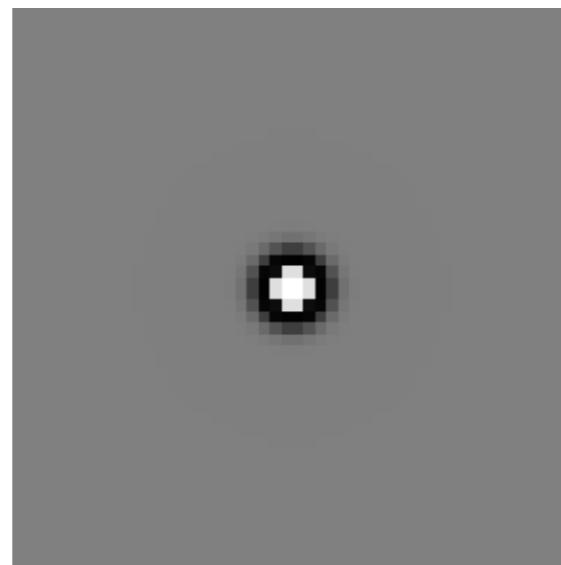
# Difference of Gaussians (DoG)

Use filters of different size



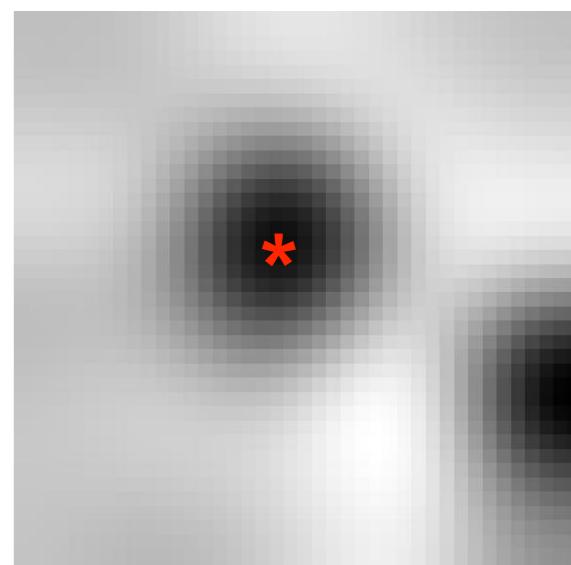
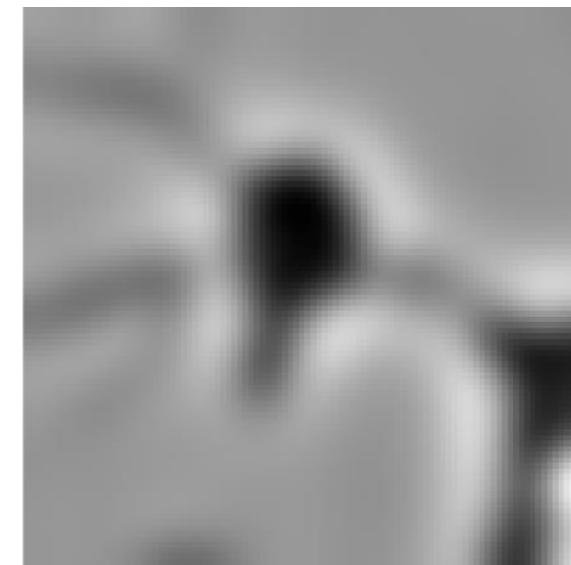
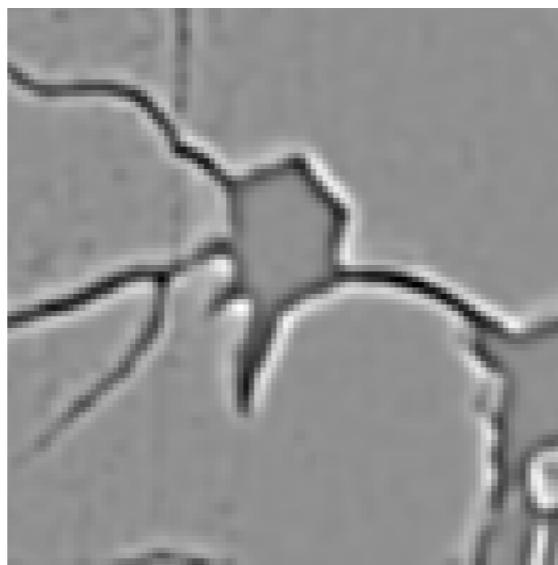
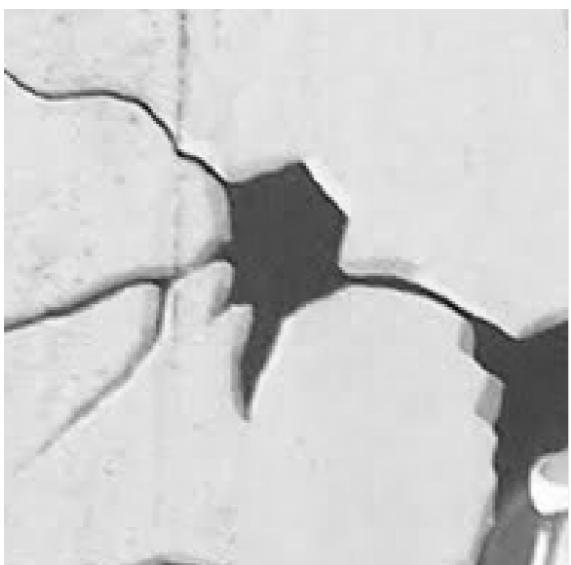
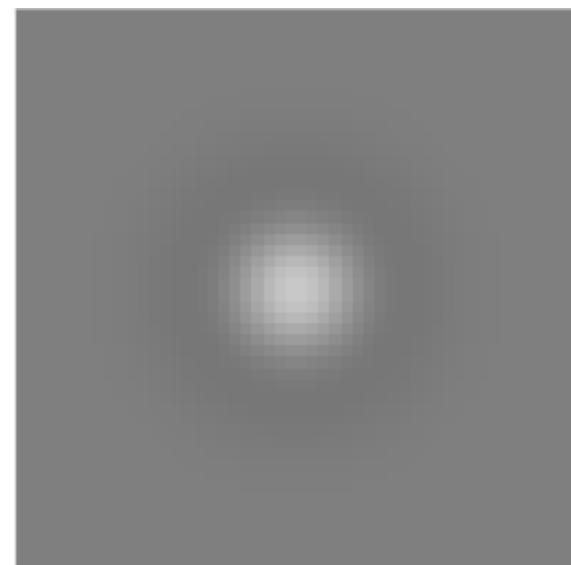
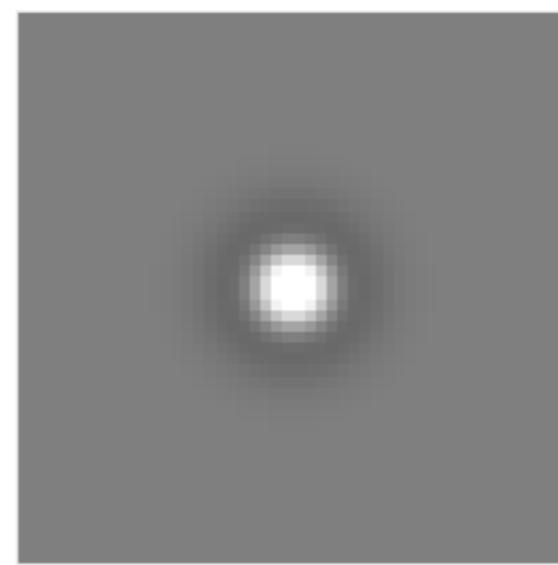
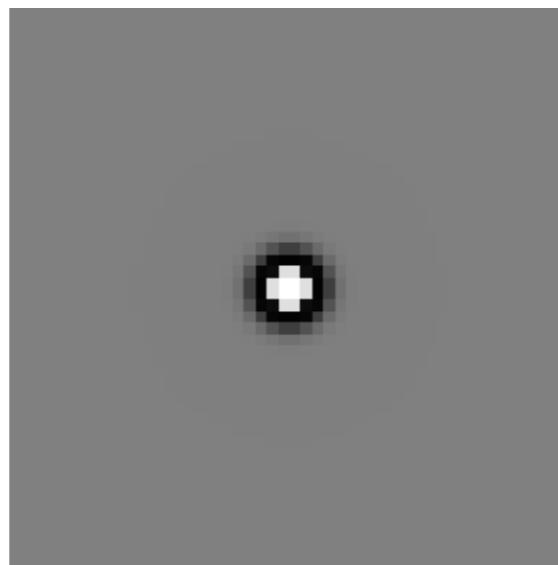
# Difference of Gaussians (DoG)

Use filters of different size

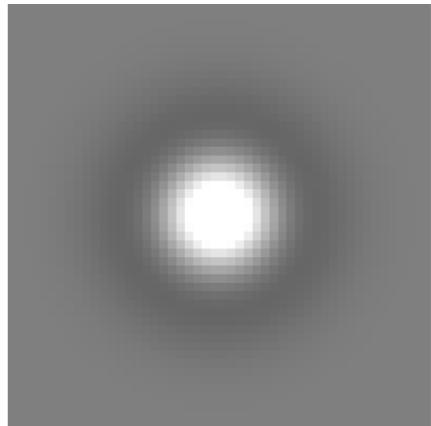
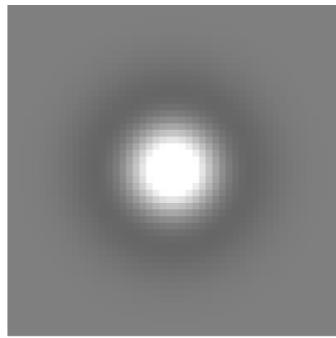
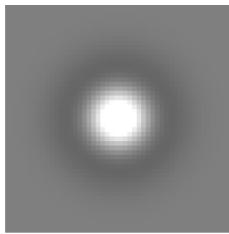


# Difference of Gaussians (DoG)

Use filters of different size

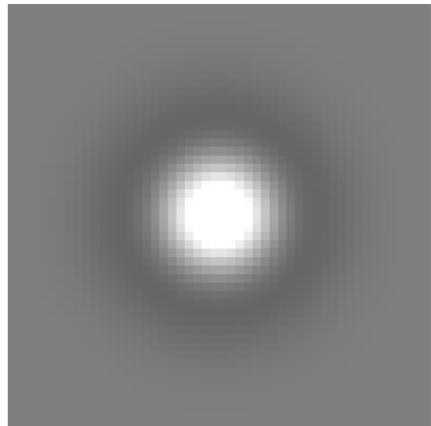
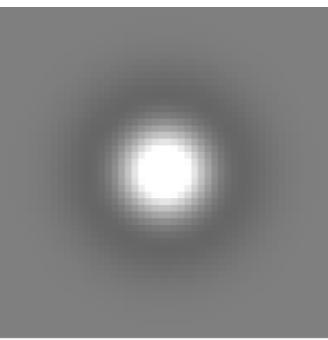
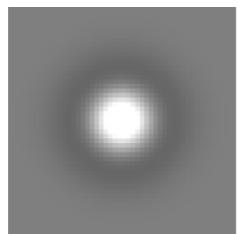


# Detect Scale and Position



# Detect Scale and Position

---

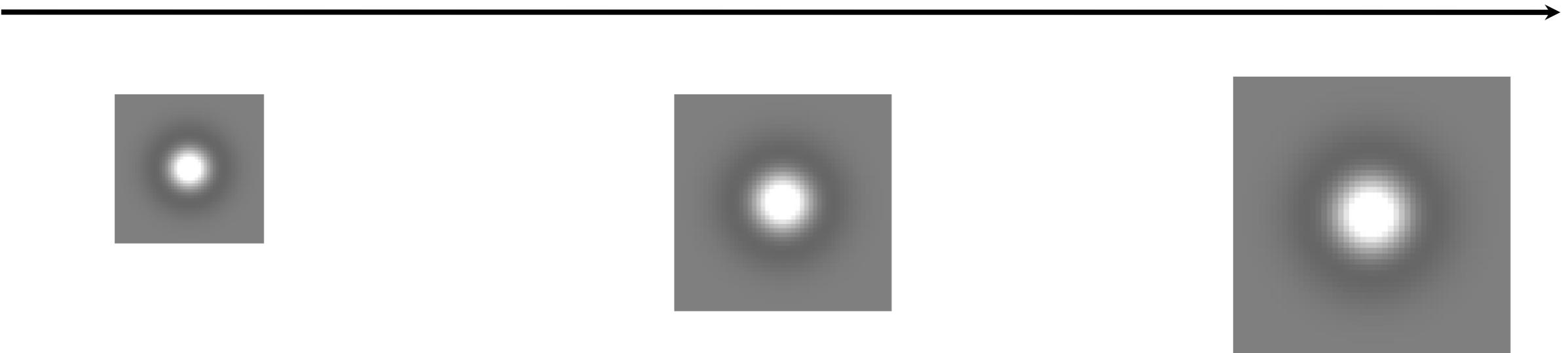


1	0	1	2	3	4	2
2	7	8	2	3	3	3
2	3	1	4	5	6	2
2	7	12	15	20	7	2
2	8	12	25	19	11	3
3	9	11	18	16	7	1
1	2	3	2	1	3	1

2	1	2	2	2	3	3
1	2	3	2	2	3	4
5	11	9	8	9	8	1
1	11	15	20	22	9	5
2	12	16	35	22	11	4
3	9	15	18	21	7	1
2	2	9	8	7	9	1

3	3	3	3	3	3	3
3	4	4	4	4	4	3
3	5	6	6	6	5	2
2	7	12	12	12	7	2
2	5	12	20	13	11	3
3	9	12	13	13	9	1
1	2	5	5	5	3	1

# Detect Scale and Position



1	0	1	2	3	4	2
2	7	8	2	3	3	3
2	3	1	4	5	6	2
2	7	12	15	20	7	2
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3	9	11	18	16	7	1
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2	1	2	2	2	3	3
1	2	3	2	2	3	4
5	11	9	8	9	8	1
1	11	15	20	22	9	5
2	12	16	35	22	11	4
3	9	15	18	21	7	1
2	2	9	8	7	9	1

3	3	3	3	3	3	3
3	4	4	4	4	4	3
3	5	6	6	6	5	2
2	7	12	12	12	7	2
2	5	12	20	13	11	3
3	9	12	13	13	9	1
1	2	5	5	5	3	1

Non-maximum suppression in image and scale!

# Sub-Pixel Refinement

2<sup>nd</sup> order curve fitting (1D):

$$f(x) \approx f(0) + x \cdot f'(0) + \frac{x^2}{2} f''(0)$$

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$$f(x, y, \sigma) \approx f(0, 0, 0) + (x \quad y \quad \sigma) \cdot \nabla f(0, 0, 0) + \frac{1}{2} (x \quad y \quad \sigma) \mathbf{H}(0, 0, 0) \begin{pmatrix} x \\ y \\ \sigma \end{pmatrix}$$

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Hessian matrix:

$$\mathbb{H}(0, 0, 0) = \begin{pmatrix} f''_{xx}(0, 0, 0) & f''_{xy}(0, 0, 0) & f''_{x\sigma}(0, 0, 0) \\ f''_{xy}(0, 0, 0) & f''_{yy}(0, 0, 0) & f''_{y\sigma}(0, 0, 0) \\ f''_{x\sigma}(0, 0, 0) & f''_{y\sigma}(0, 0, 0) & f''_{\sigma\sigma}(0, 0, 0) \end{pmatrix}$$

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Set derivative to 0:

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \nabla f(0, 0, 0) + H(0, 0, 0) \begin{pmatrix} x \\ y \\ \sigma \end{pmatrix}$$

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Solve small linear system:

$$\begin{pmatrix} x \\ y \\ \sigma \end{pmatrix} = -H(0, 0, 0)^{-1} \nabla f(0, 0, 0)$$

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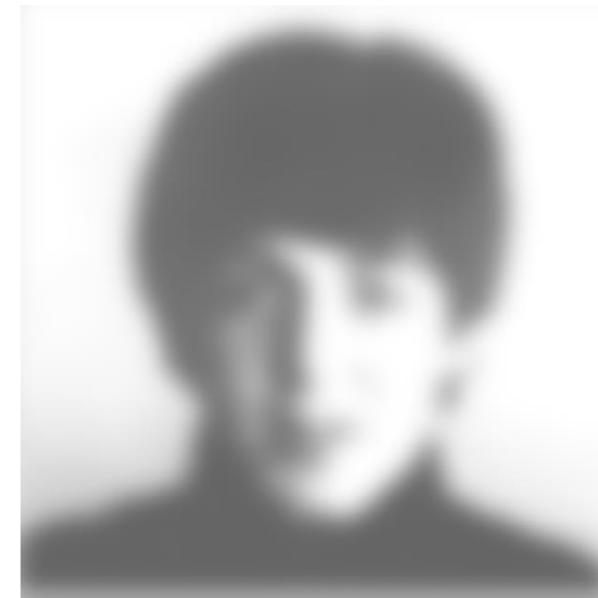
$$\begin{pmatrix} x \\ y \\ \sigma \end{pmatrix} = -H(0, 0, 0)^{-1} \nabla f(0, 0, 0)$$

offset from pixel

Potentially repeat for a few iterations (centered on current estimate!)

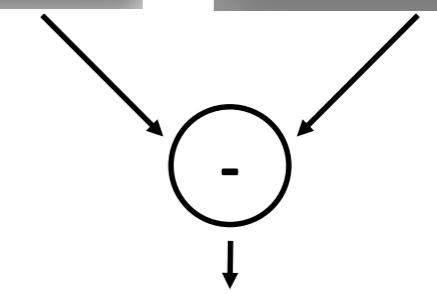
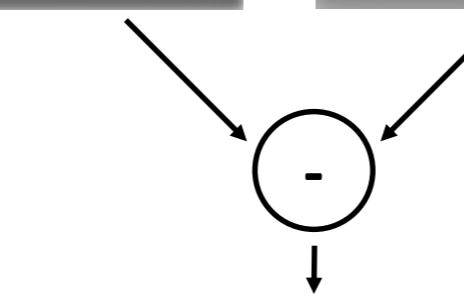
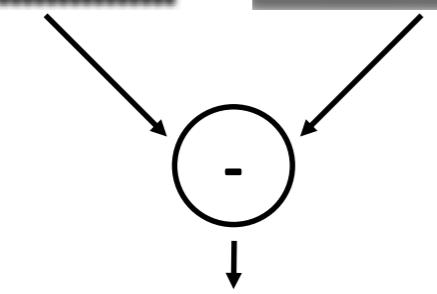
# Efficient implementation

Gaussian filtering

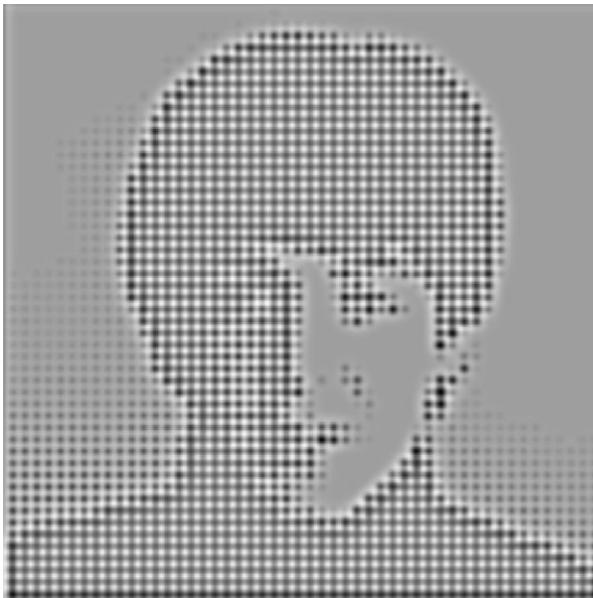


# Efficient implementation

Gaussian filtering



Difference  
of  
Gaussians



# Efficient implementation

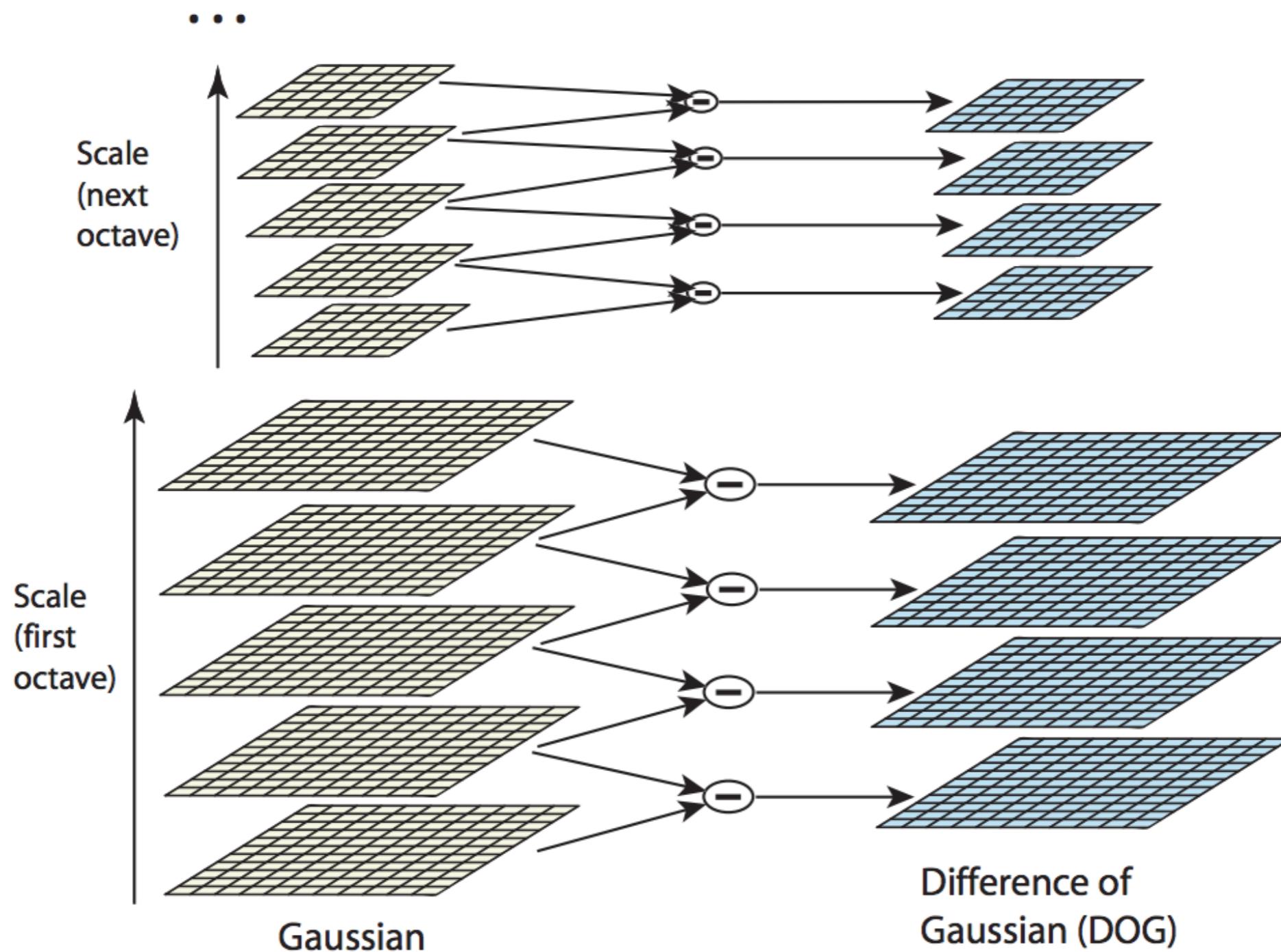


illustration from  
[Lowe, Distinctive Image Features from Scale-Invariant Keypoints, IJCV 2004]

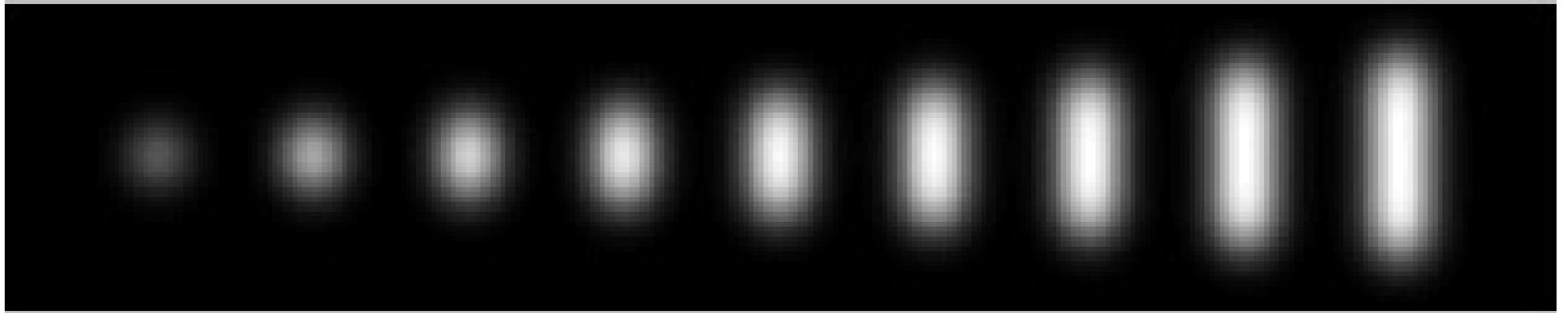
# Avoiding Edges



# Avoiding Edges



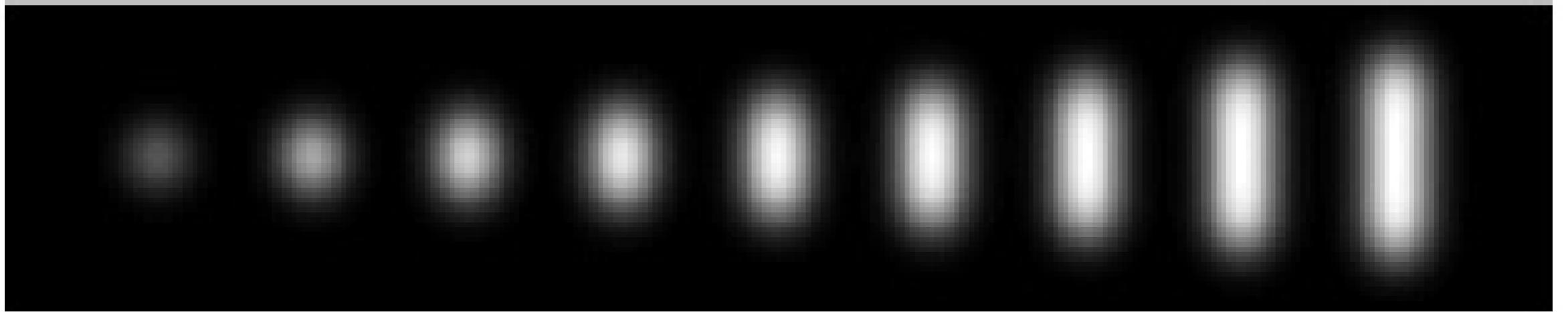
# Avoiding Edges



well-localized blob

edge

# Avoiding Edges



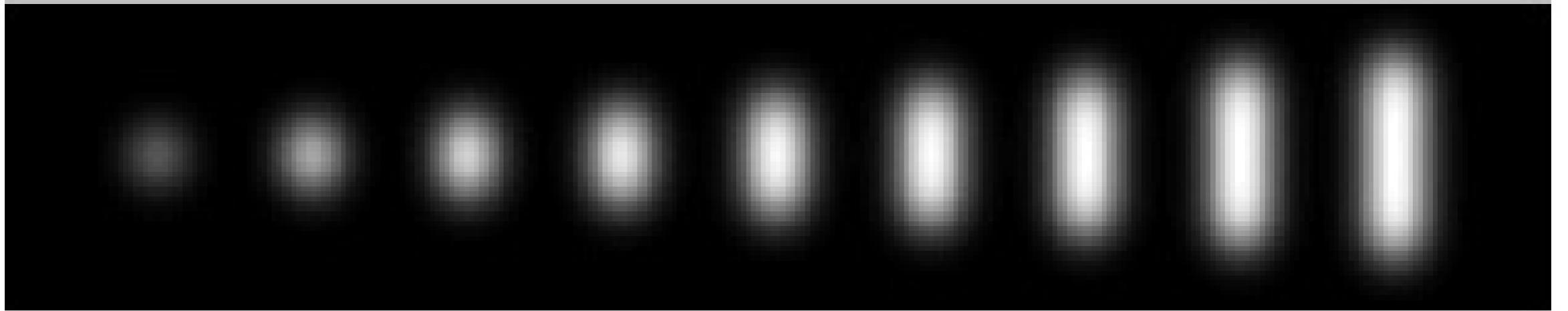
well-localized blob

edge

Consider 2D Hessian around detected keypoint (after refinement):

$$\mathbf{H} = \begin{pmatrix} D_{xx} & D_{xy} \\ D_{xy} & D_{yy} \end{pmatrix}$$

# Avoiding Edges



well-localized blob

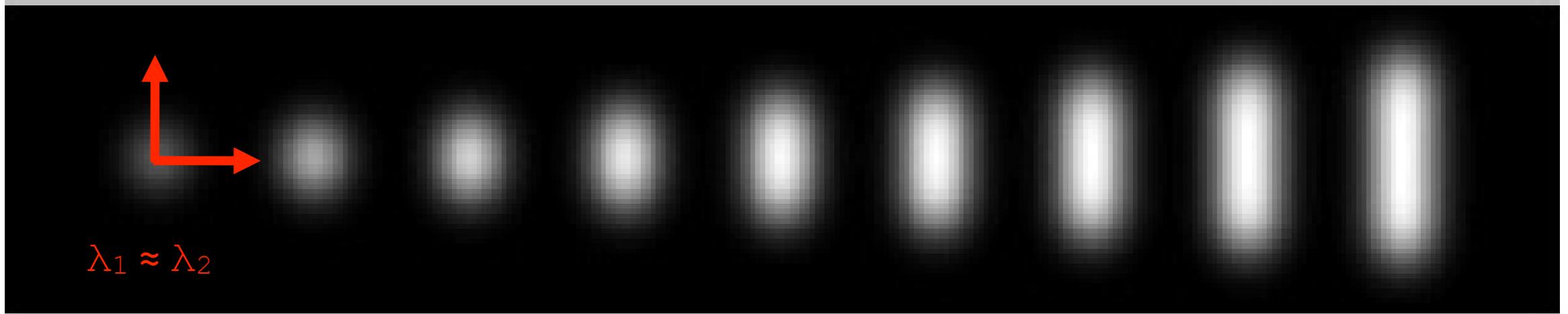
edge

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Eigenvalues  $\lambda_1 \geq \lambda_2$  of  $H$  proportional to *principal curvature*

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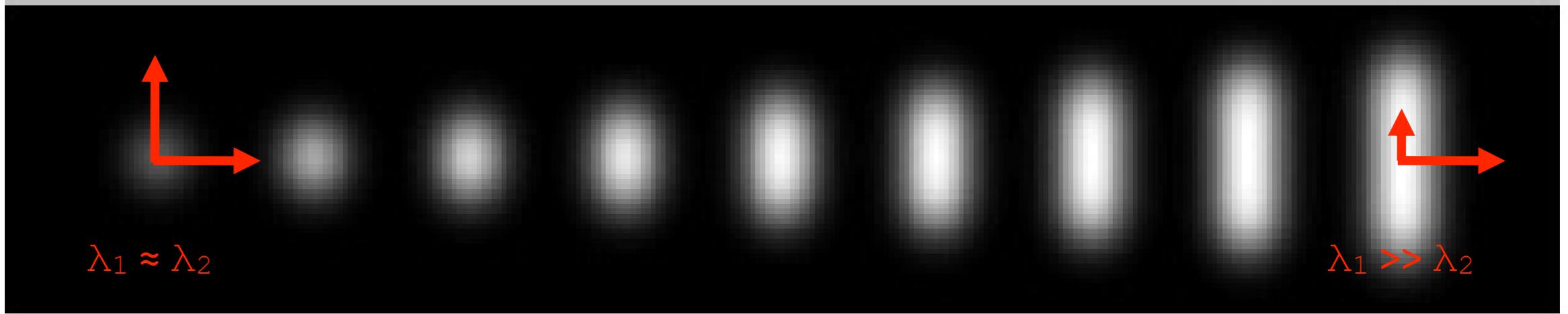
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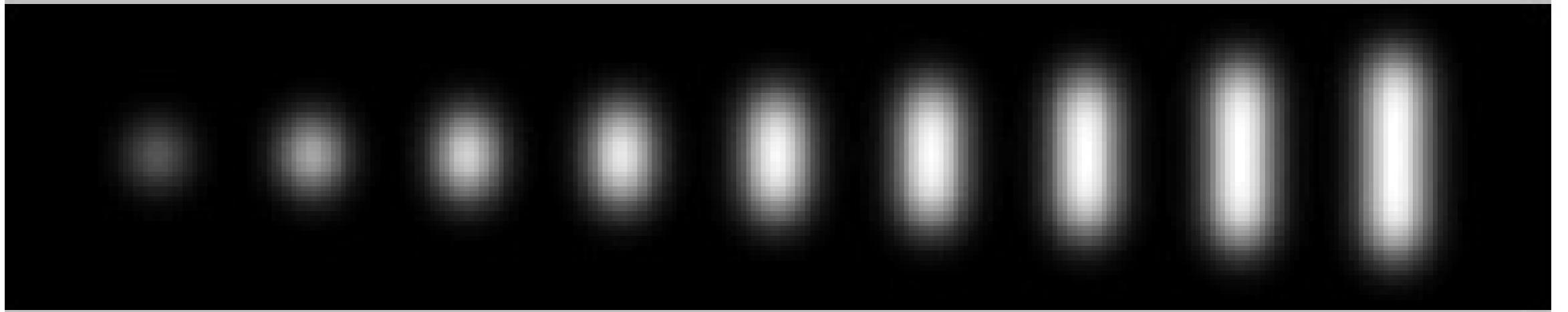
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well-localized blob

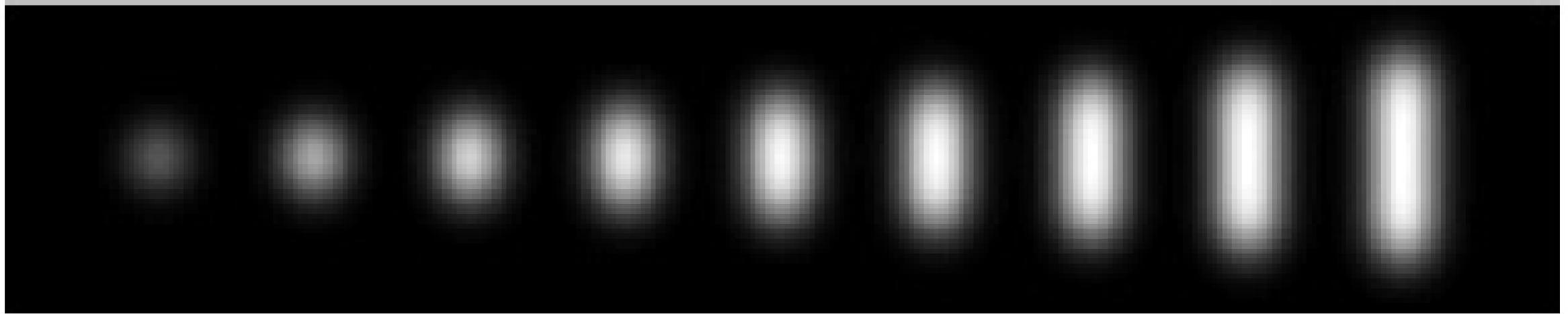
edge

Detecting edges: Look at ratio of Eigenvalues ( $\lambda_1 \geq \lambda_2$ )

$$\frac{\lambda_1}{\lambda_2} = r$$

$$\frac{(r+1)^2}{r} < \tau$$

# Avoiding Edges



well-localized blob

edge

Detecting edges: Look at ratio of Eigenvalues ( $\lambda_1 \geq \lambda_2$ )

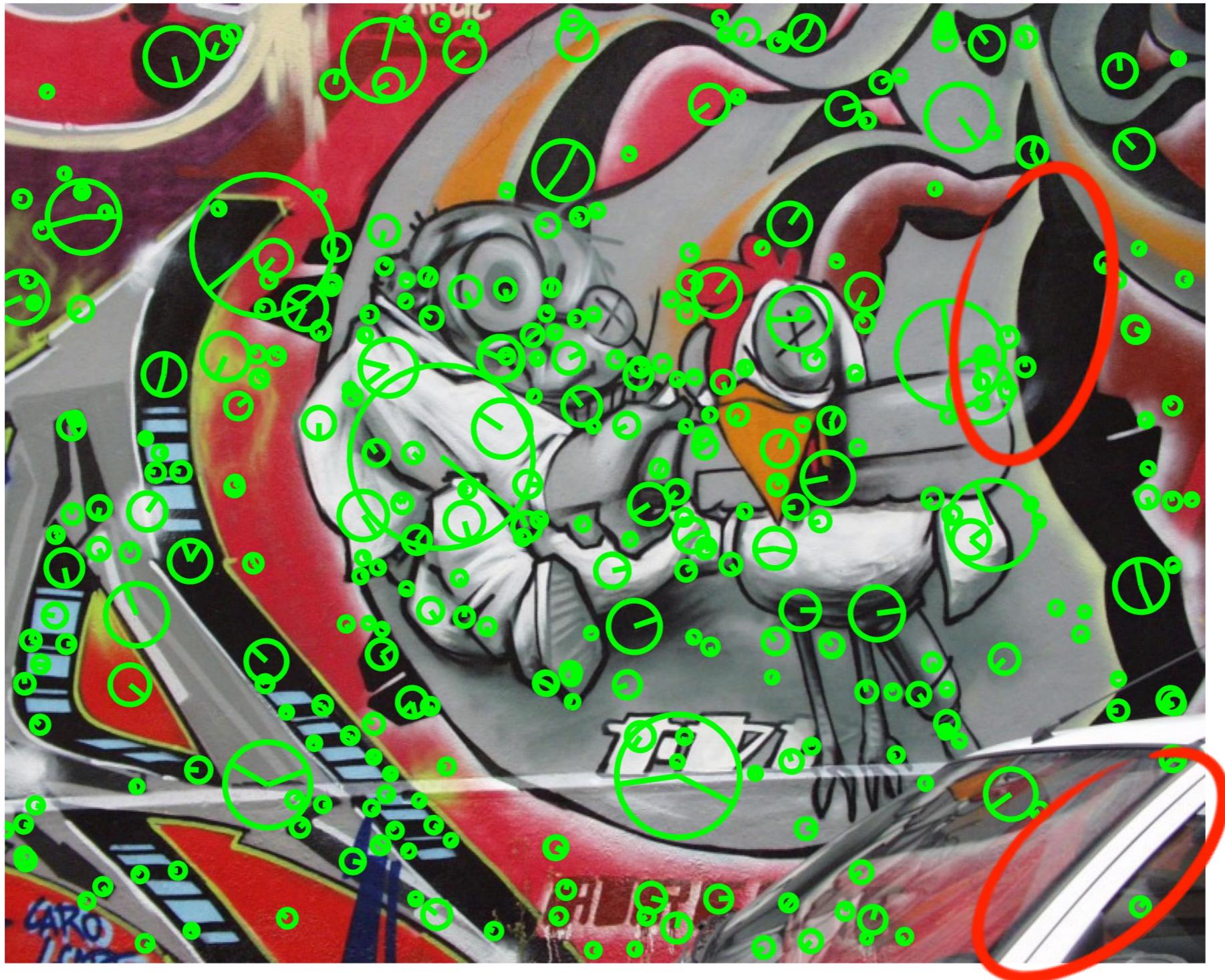
$$\frac{\lambda_1}{\lambda_2} = r \quad \frac{(r+1)^2}{r} < \tau$$

No need to explicitly compute Eigenvalues (see Lowe's paper)

# Avoiding Edges



# Avoiding Edges



# Invariances?



Changes in brightness



in-plane rotation

# Invariances?



Changes in scale



in-plane rotation

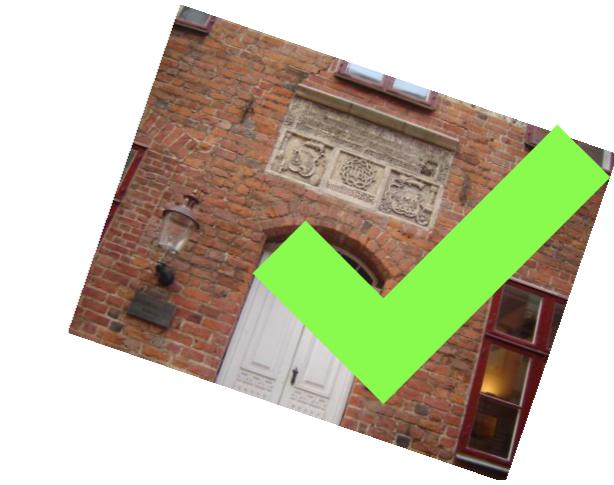


Changes in brightness

# Invariances?



Changes in scale



in-plane rotation



Changes in brightness

# Invariances?



Changes in scale



in-plane rotation



Changes in brightness



Changes in viewpoint



# Invariances?



Changes in scale



in-plane rotation



Changes in brightness



Not invariant, but reasonably robust



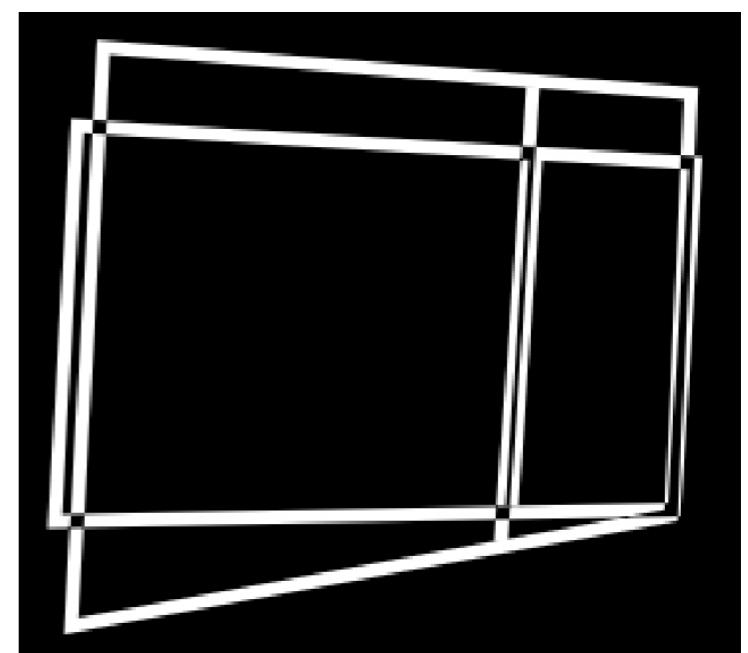
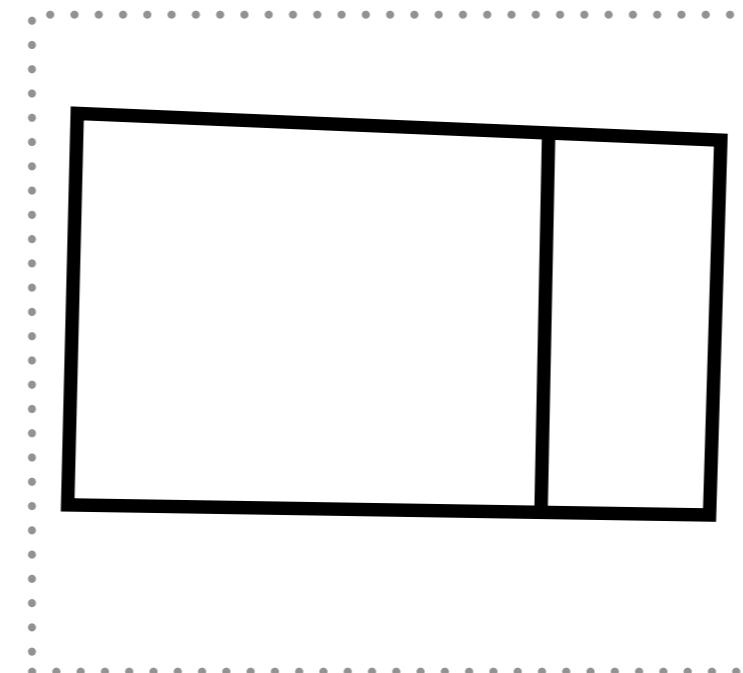
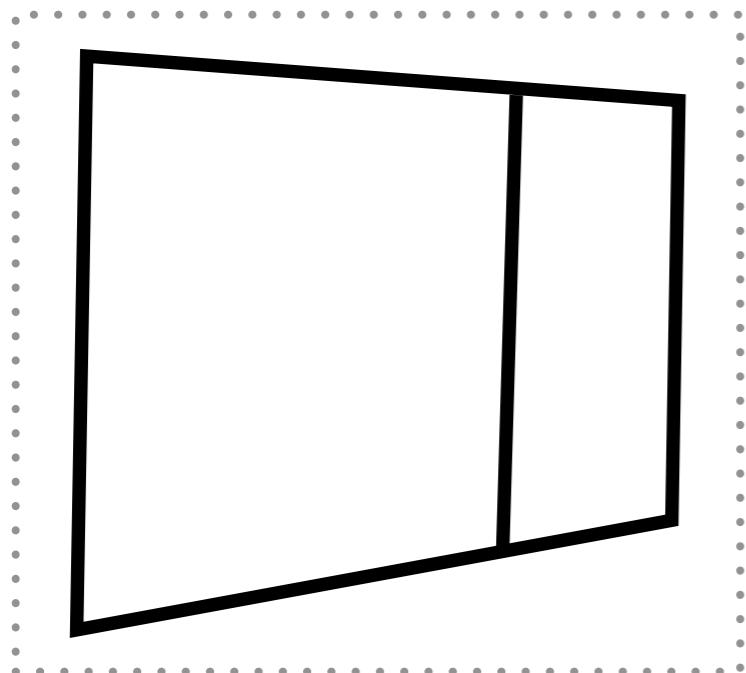
Changes in viewpoint

# Scale-Invariant Feature Transform (SIFT) Features

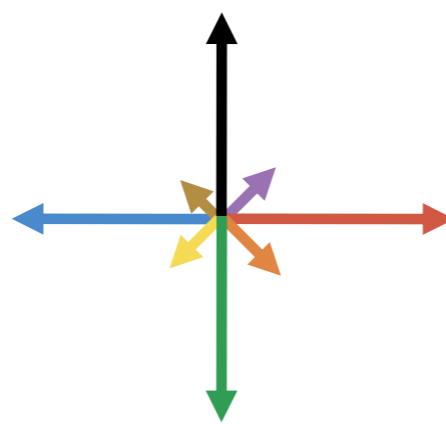
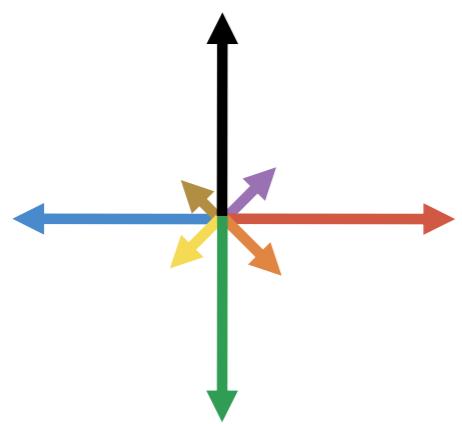
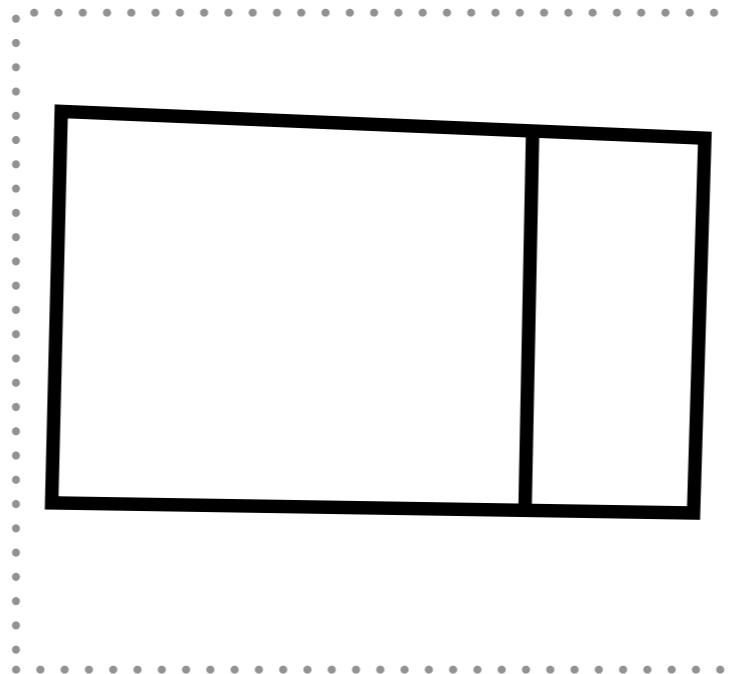
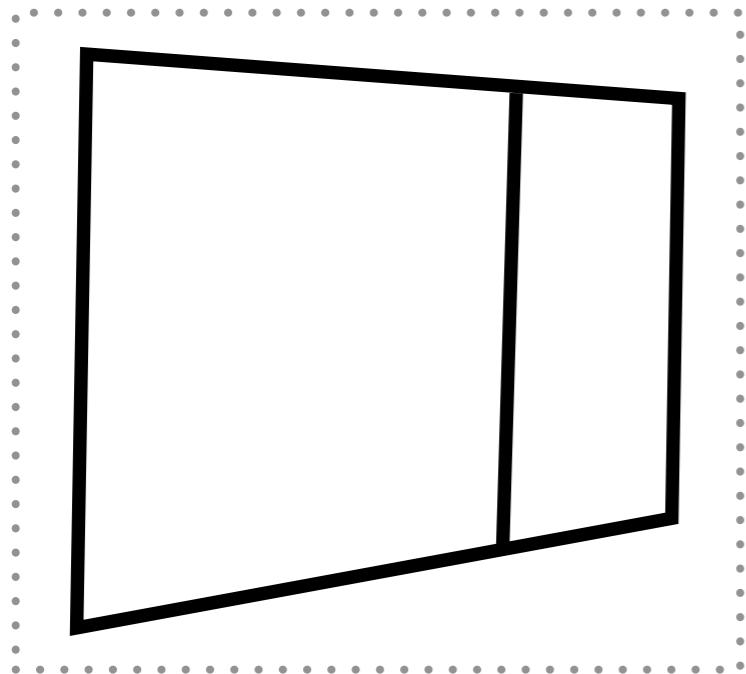


- Find a set of *interest* points
- Extract scaled & oriented patch around interest points
- **Compute SIFT descriptor for each patch**

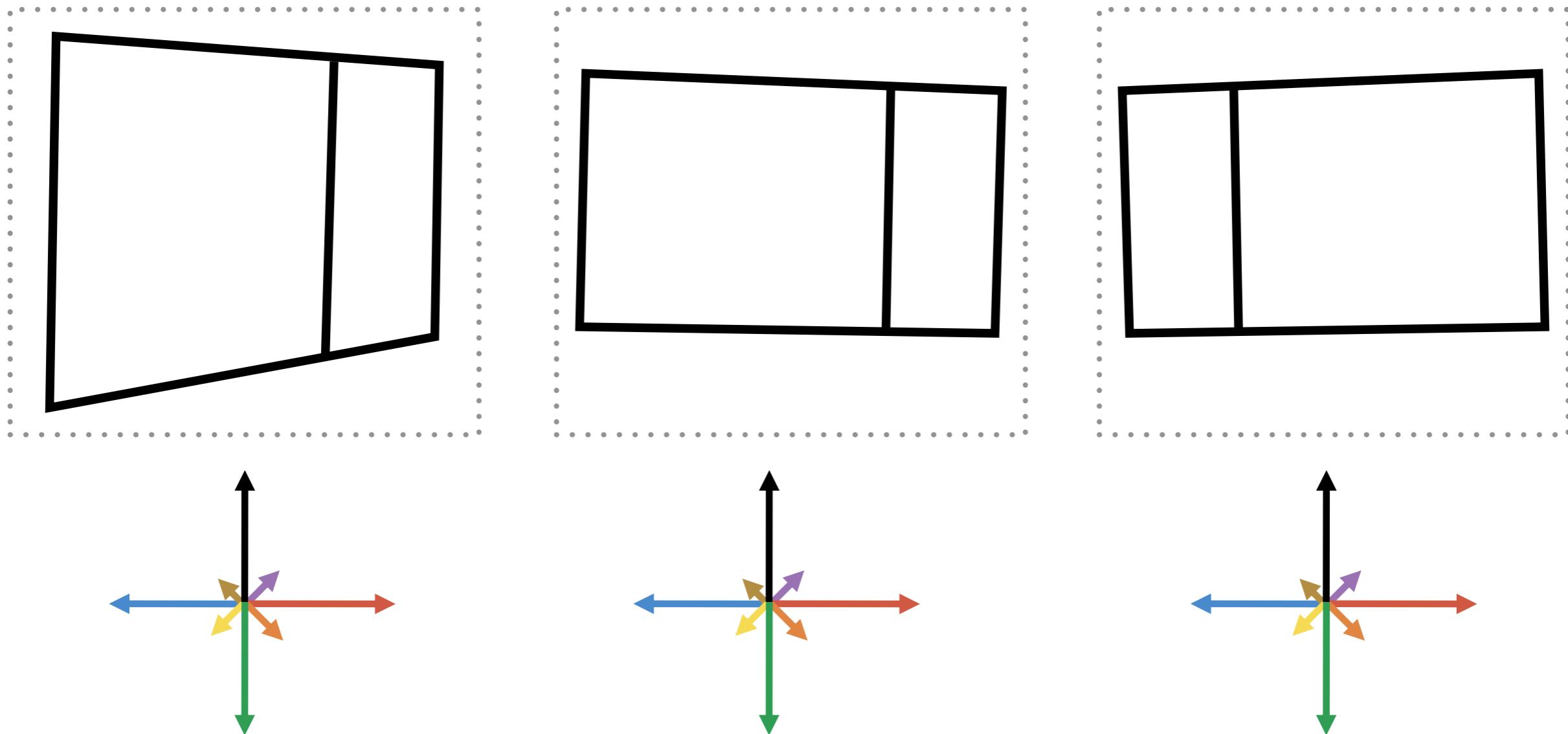
# Perspective Distortion



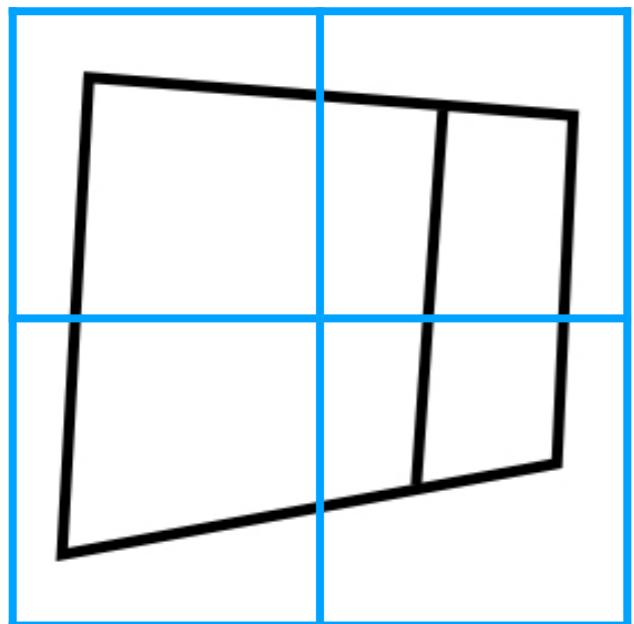
# Perspective Distortion



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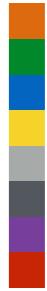
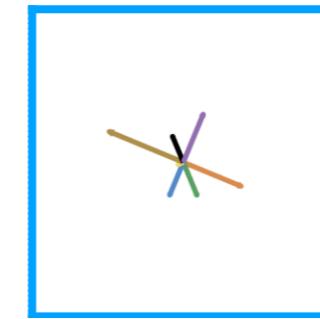
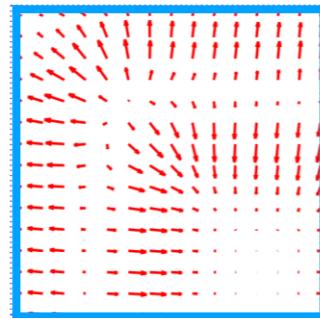
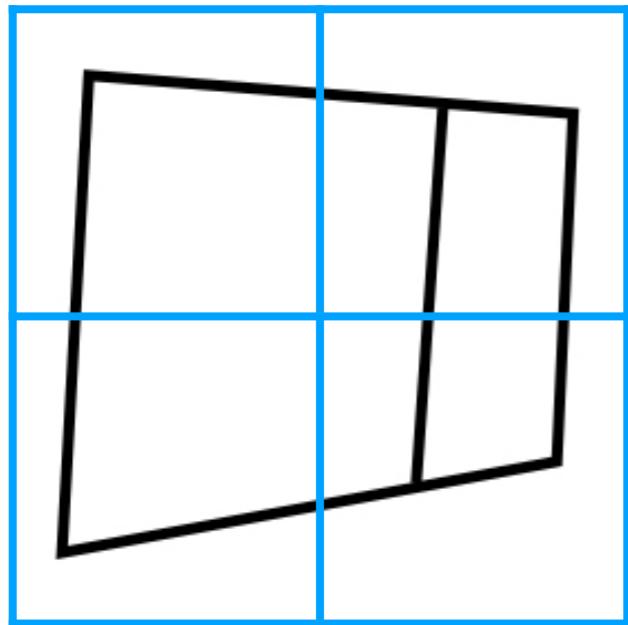


# SIFT Descriptor (Idea)



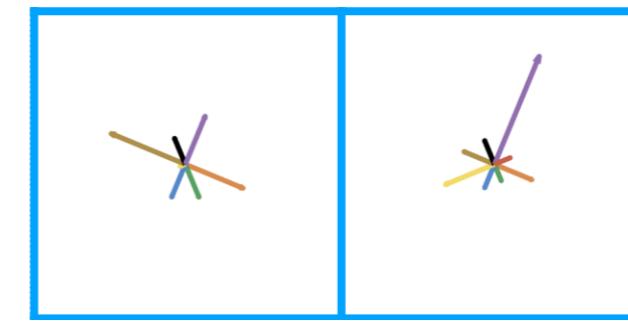
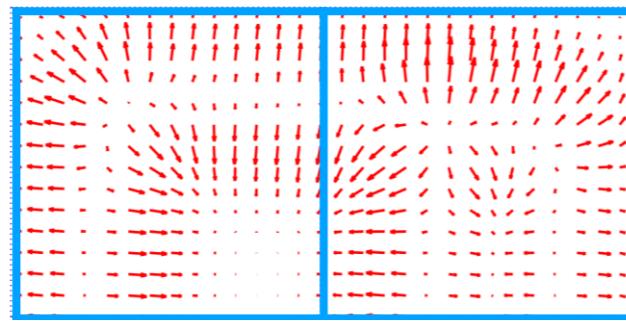
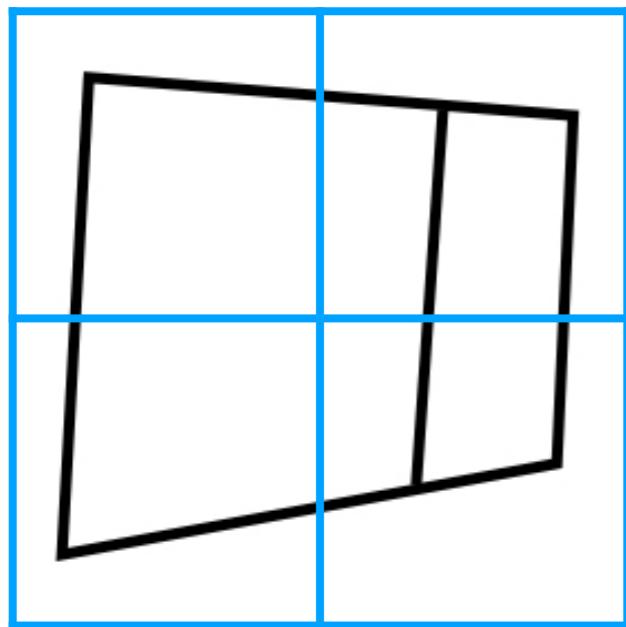
Split patch in 2x2 regions  
(SIFT actually uses 4x4 regions)

# SIFT Descriptor (Idea)



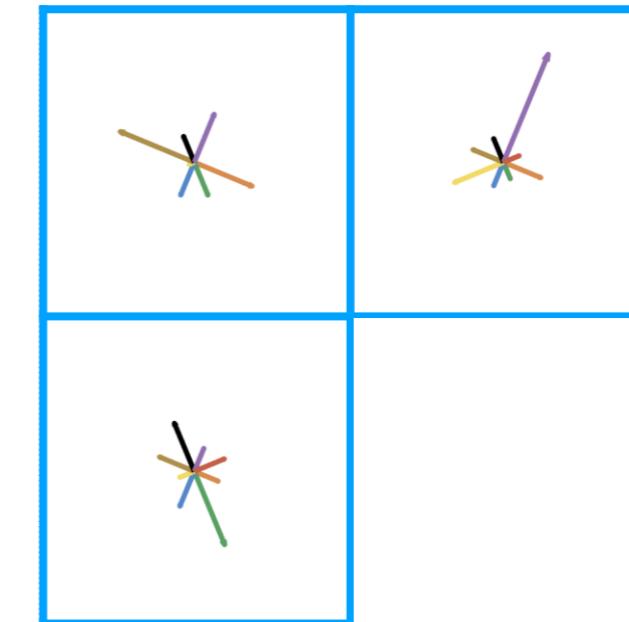
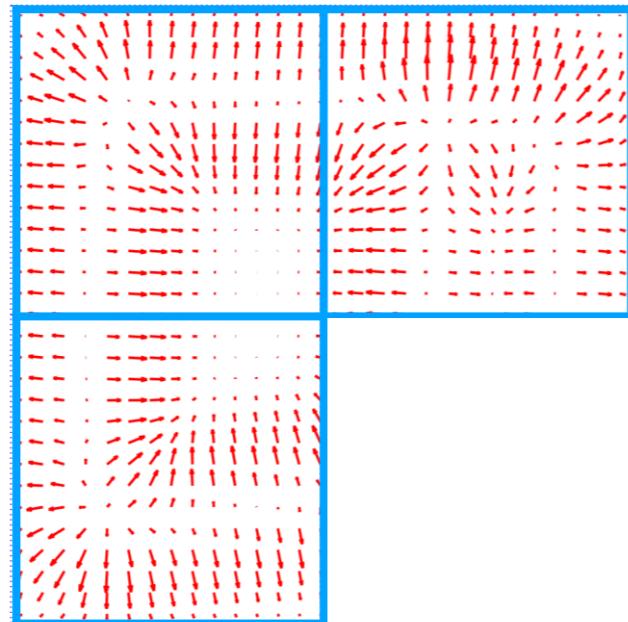
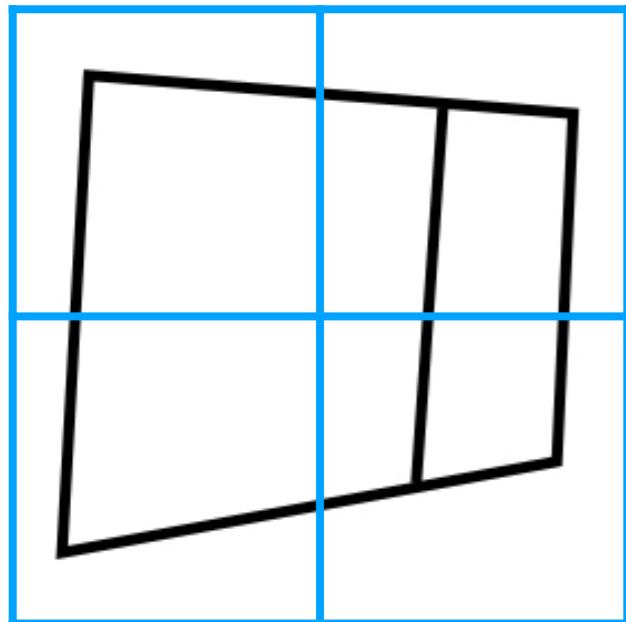
Compute a 8-bin gradient histogram for each region

# SIFT Descriptor (Idea)



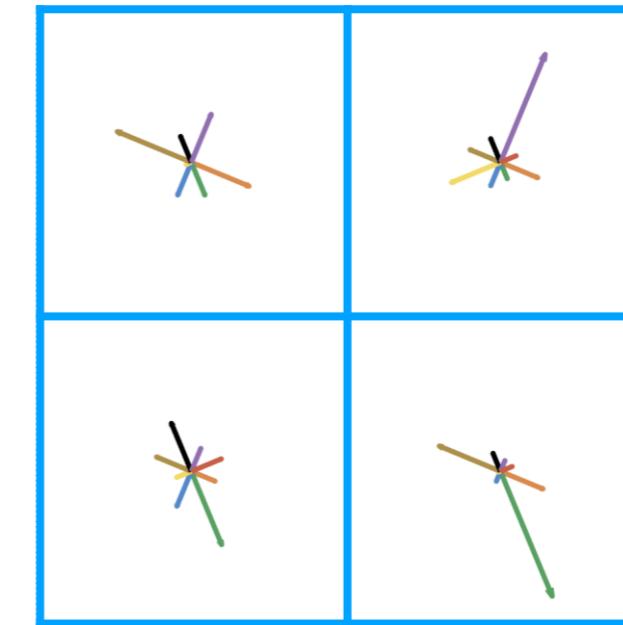
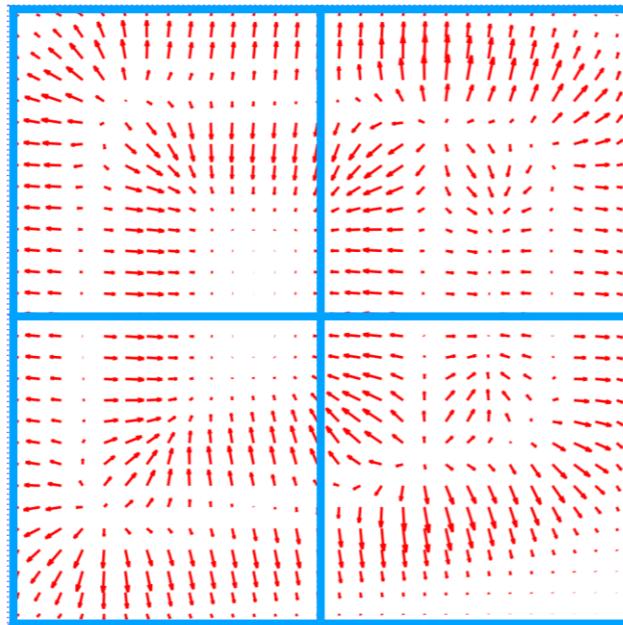
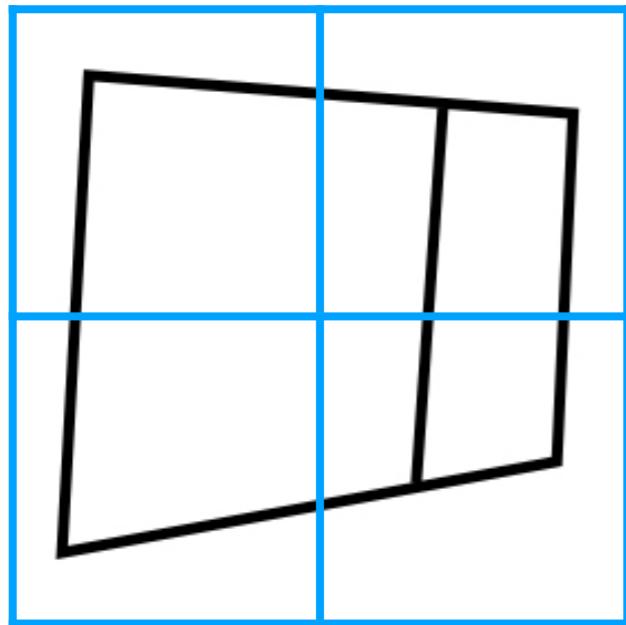
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# SIFT Descriptor (Idea)

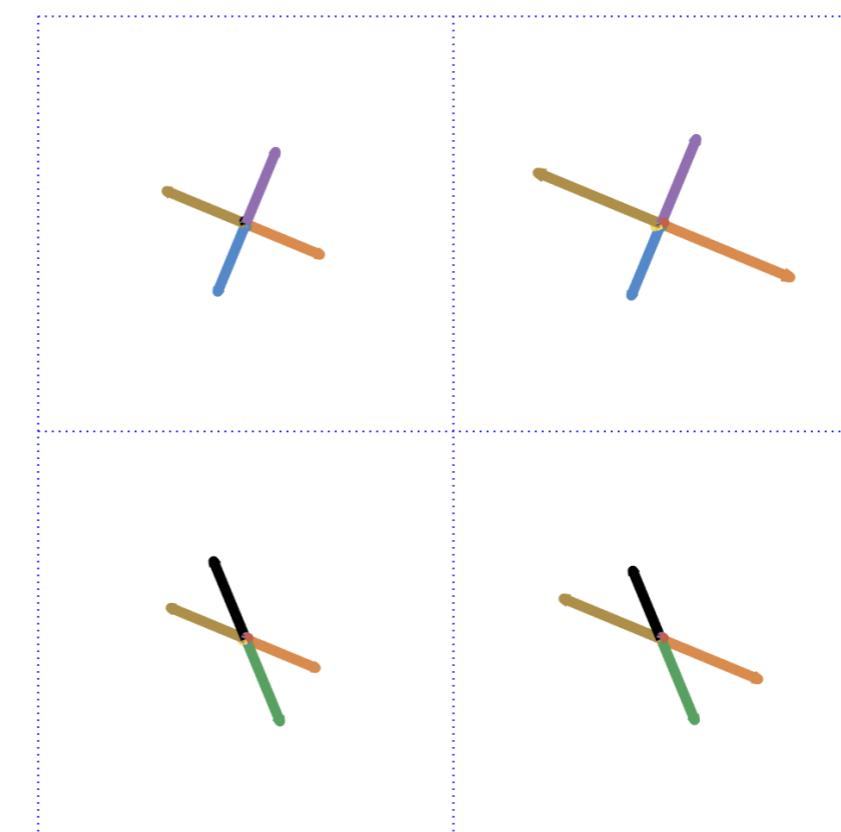
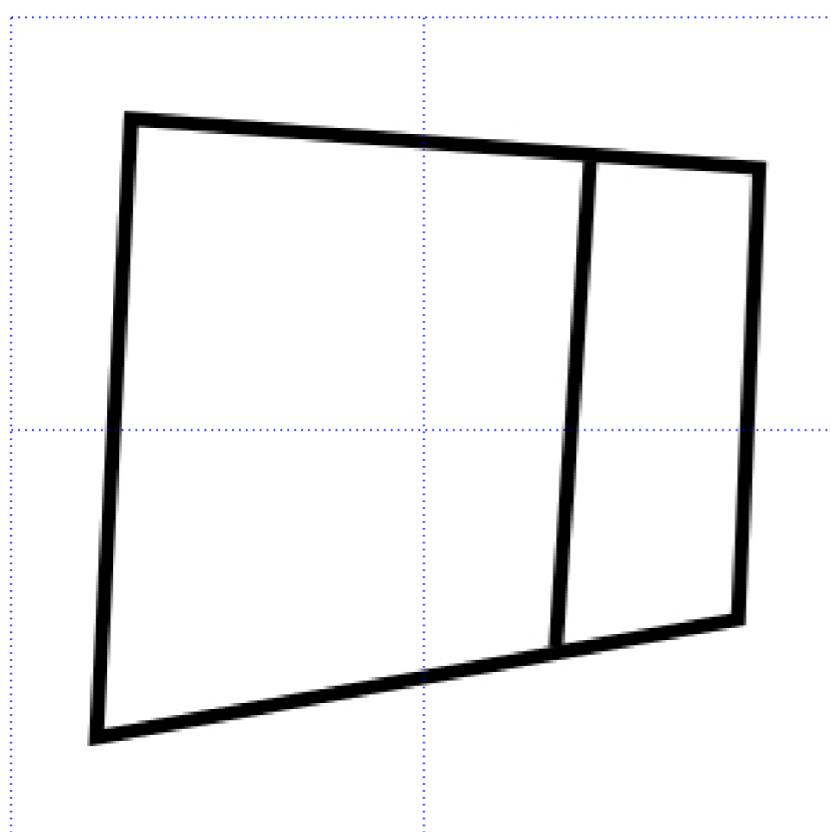
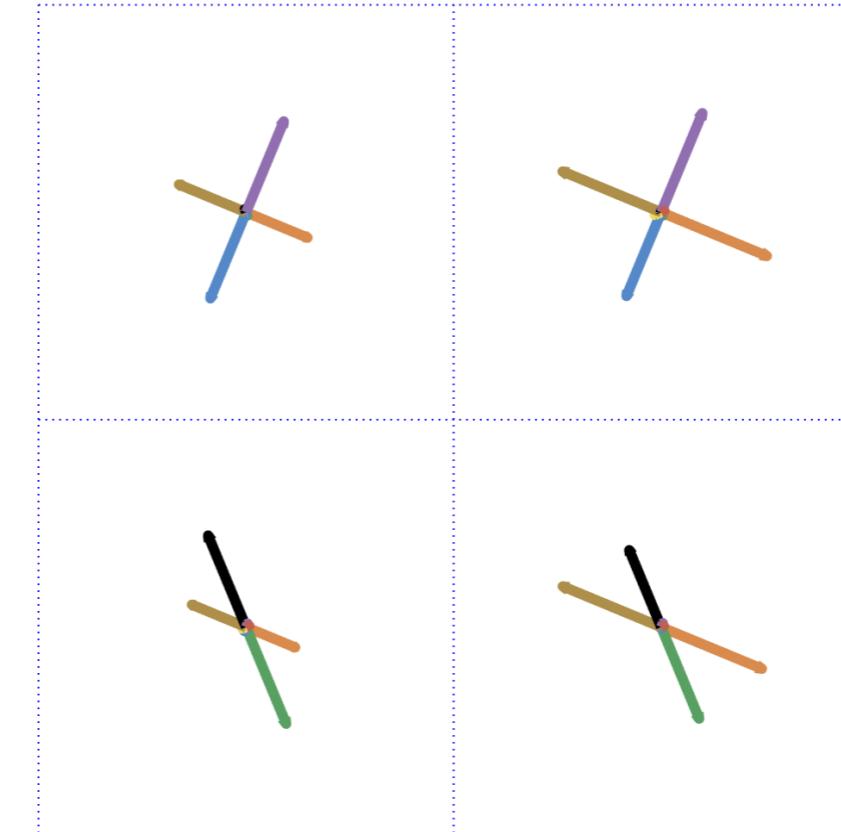
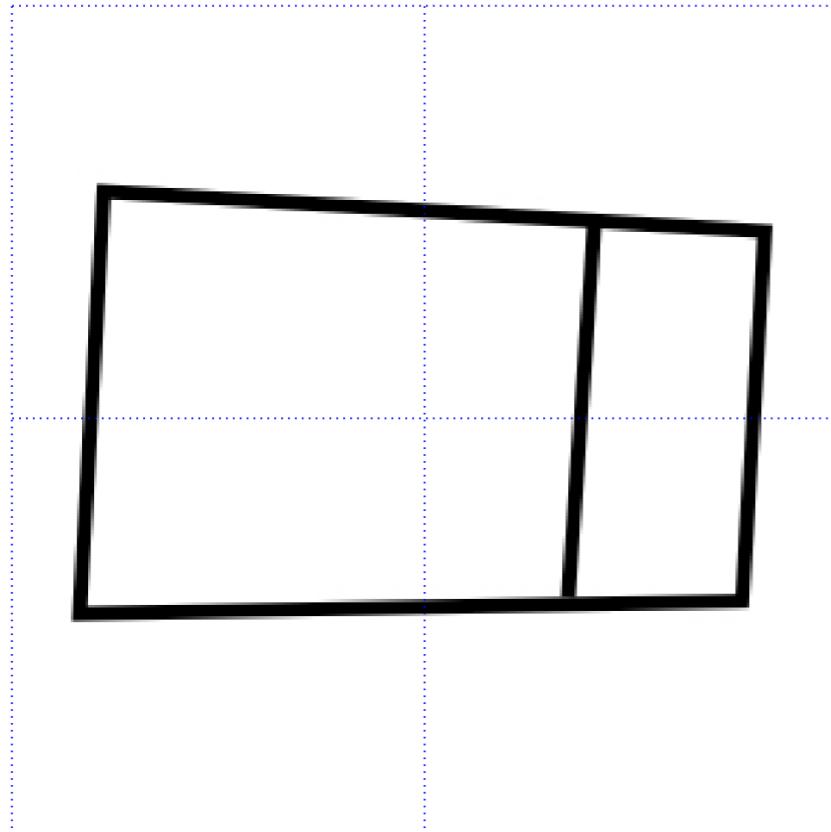


Compute a 8-bin gradient histogram for each region

# SIFT Descriptor (Idea)



$L_2$  Normalize the descriptor

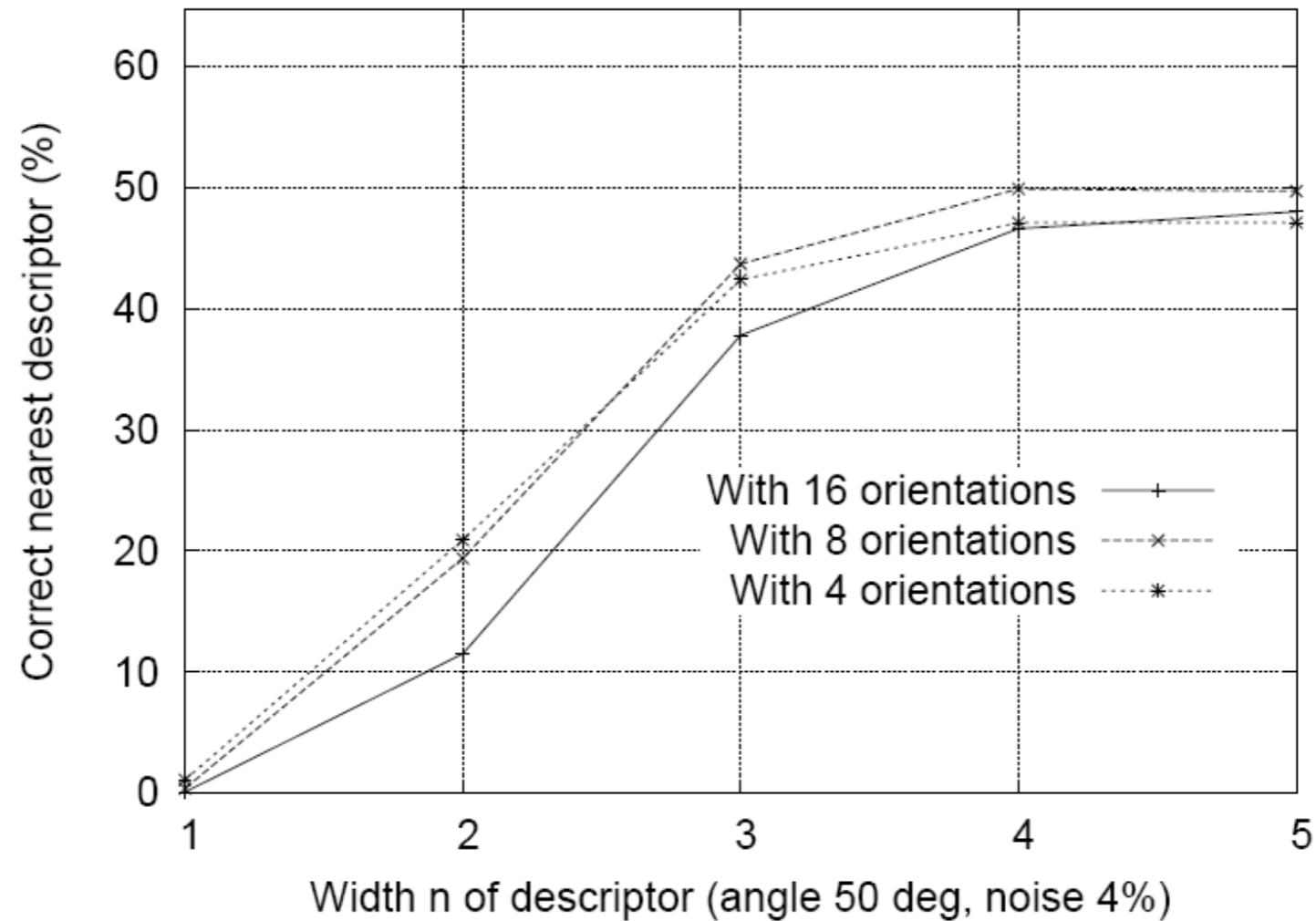


# SIFT Descriptor (Idea)

Why 4x4 regions and 8 bins per histogram?

# SIFT Descriptor (Idea)

Why 4x4 regions and 8 bins per histogram?



Careful parameter tuning

illustrations from

[Lowe, Distinctive Image Features from Scale-Invariant Keypoints, IJCV 2004]

# Invariances?



Changes in brightness

# Invariances?

- Additive changes in brightness



Changes in brightness

# Invariances?

- Additive changes in brightness
  - Do not affect the gradient



Changes in brightness

# Invariances?



- Additive changes in brightness
  - Do not affect the gradient



Changes in brightness

# Invariances?



Additive changes in brightness

- Do not affect the gradient
- Multiplicative changes in brightness
  - Affect gradient magnitude but not orientation



Changes in brightness

# Invariances?



Additive changes in brightness

- Do not affect the gradient

• Multiplicative changes in brightness

- Affect gradient magnitude but not orientation
- L2-normalize the descriptor



Changes in brightness

# Invariances?



Additive changes in brightness

- Do not affect the gradient

• Multiplicative changes in brightness

- Affect gradient magnitude but not orientation
- L2-normalize the descriptor

• Robustness against non-uniform changes

- L2-normalize

- Clamp every entry  $>0.2$  to 0.2

- Re-normalize

- Intuition: Gradient orientations more important than magnitudes



Changes in brightness

# Invariances?



Changes in brightness

# Invariances?

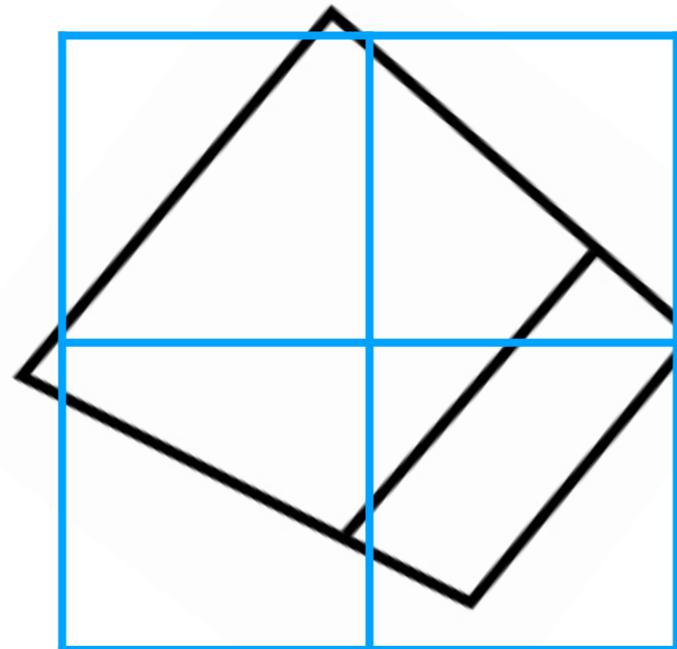
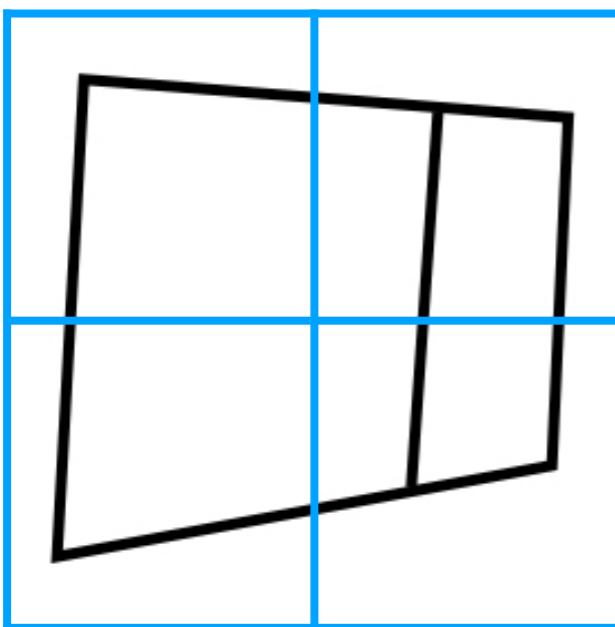


Changes in brightness



in-plane rotation

# Invariances?

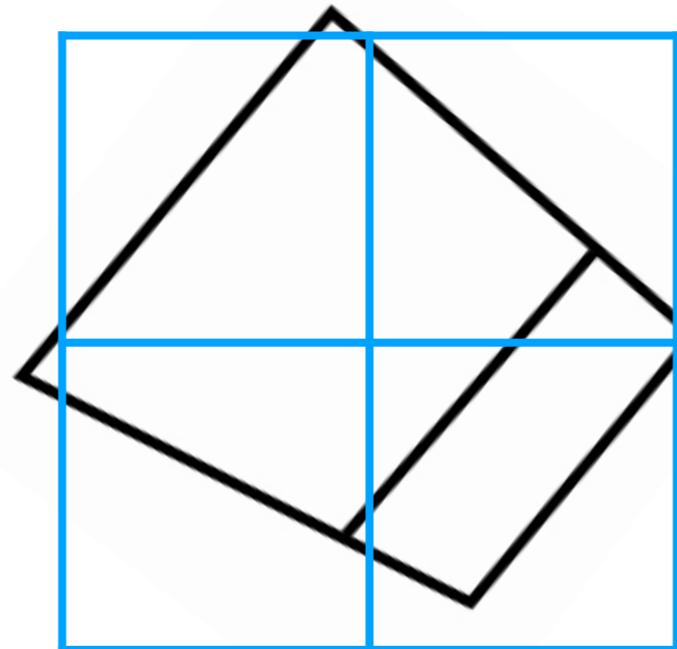
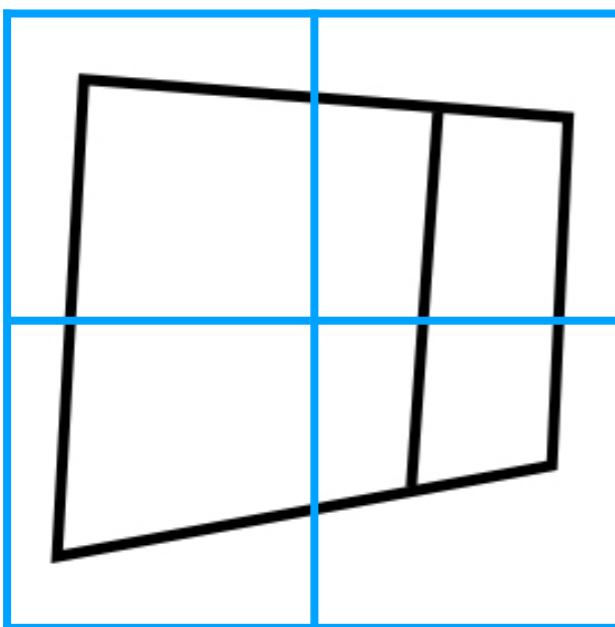


in-plane rotation



Changes in brightness

# Invariances?

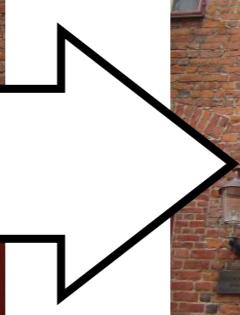


in-plane rotation



Changes in brightness

# Invariances?



Changes in scale



in-plane rotation



Changes in brightness

# Invariances?



Changes in scale



in-plane rotation



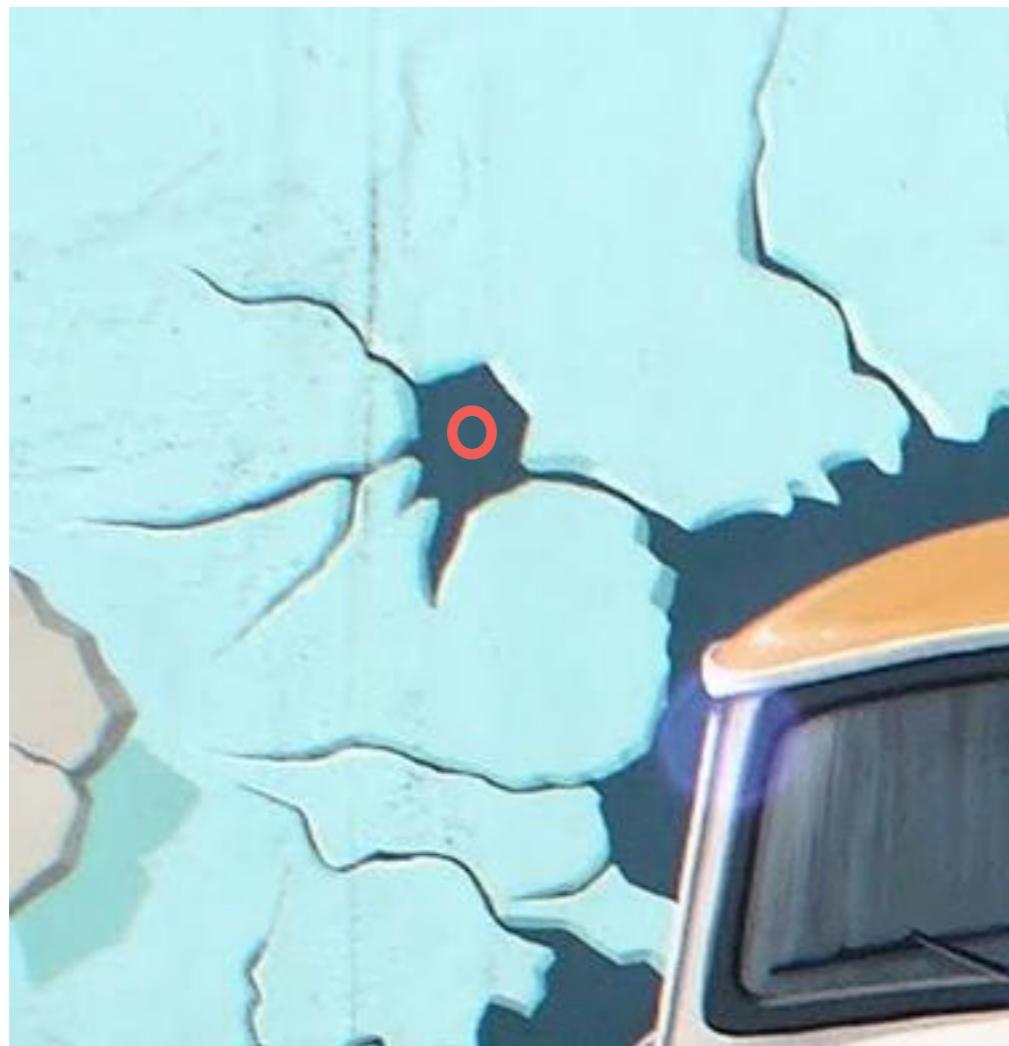
Changes in brightness

# Scale-Invariant Feature Transform (SIFT) Features



- Find a set of *interest* points
- Extract scaled & oriented patch around interest points
- Compute SIFT descriptor for each patch

# Scale Invariance



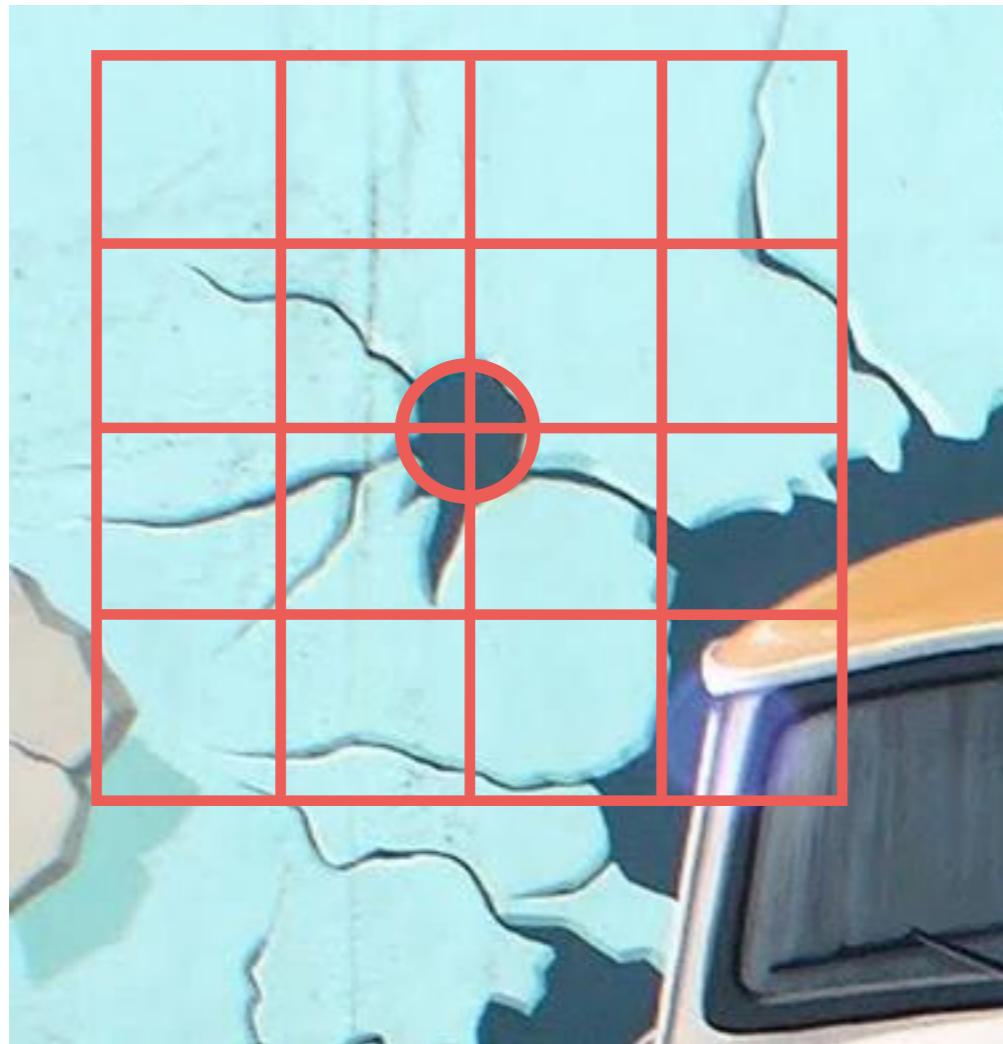
Detect a size,  $s$ , for each point

# Scale Invariance



Detect a size,  $s$ , for each point

# Scale Invariance



Detect a size,  $s$ , for each point

Scale patch size to fit.

Compute gradients in  $L(x, y, s^2)$

# Scale Invariance

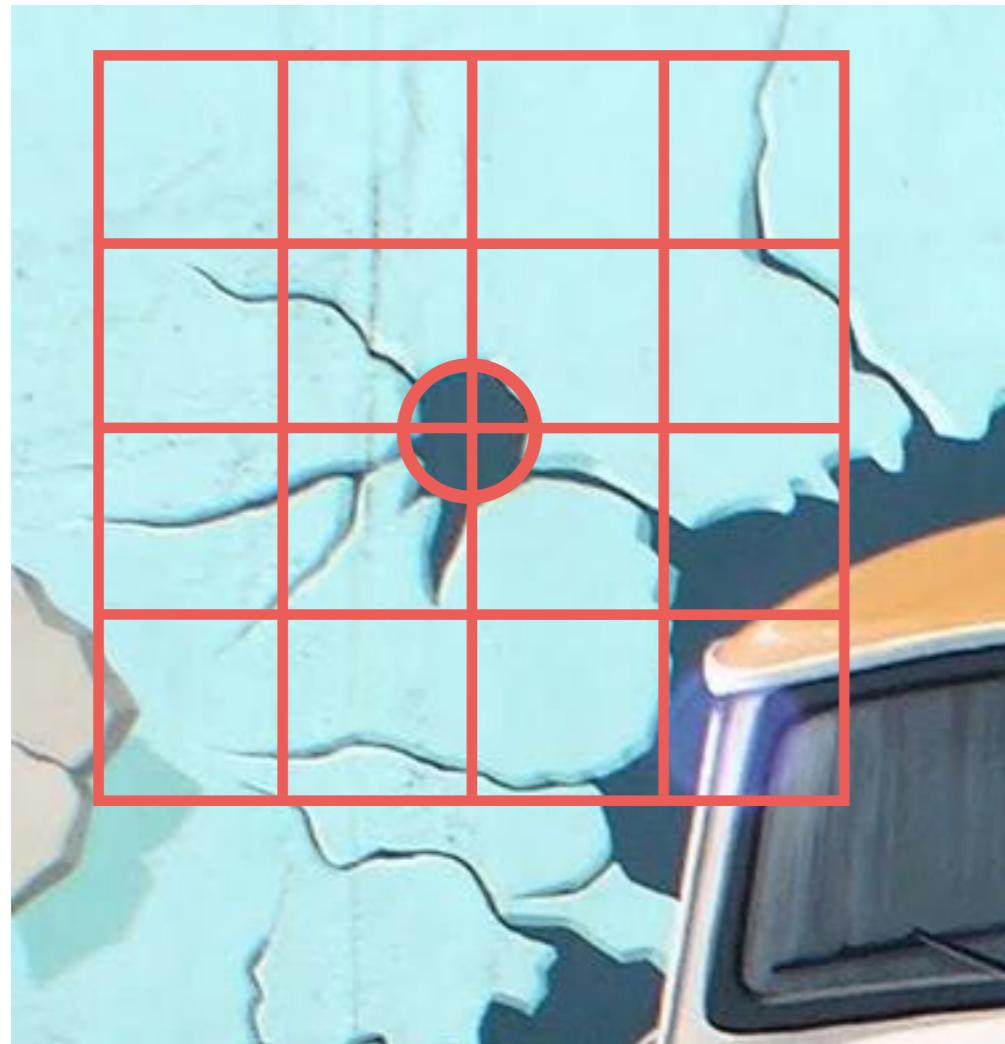


size=5



size=0.5

# Scale Invariance



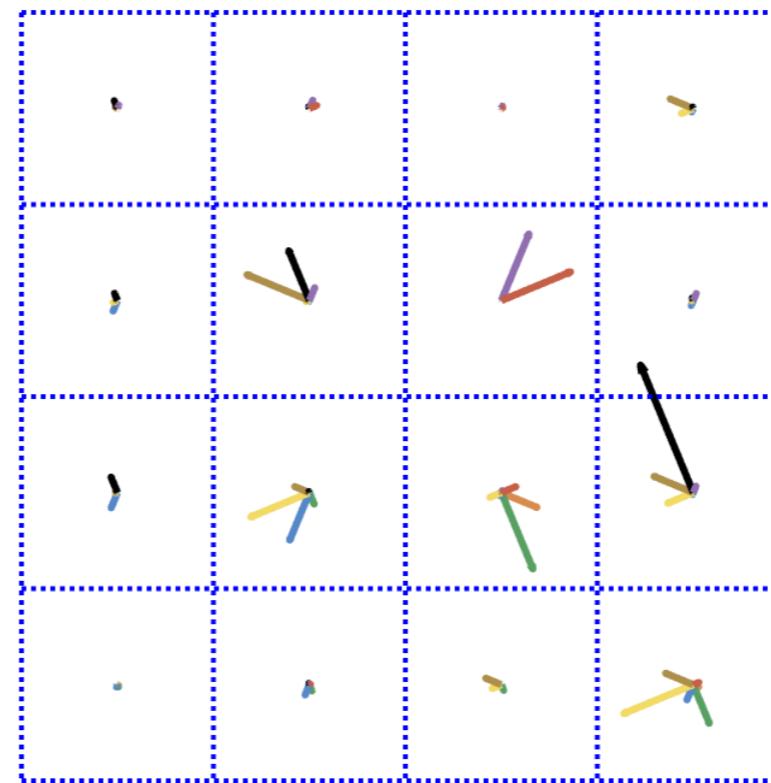
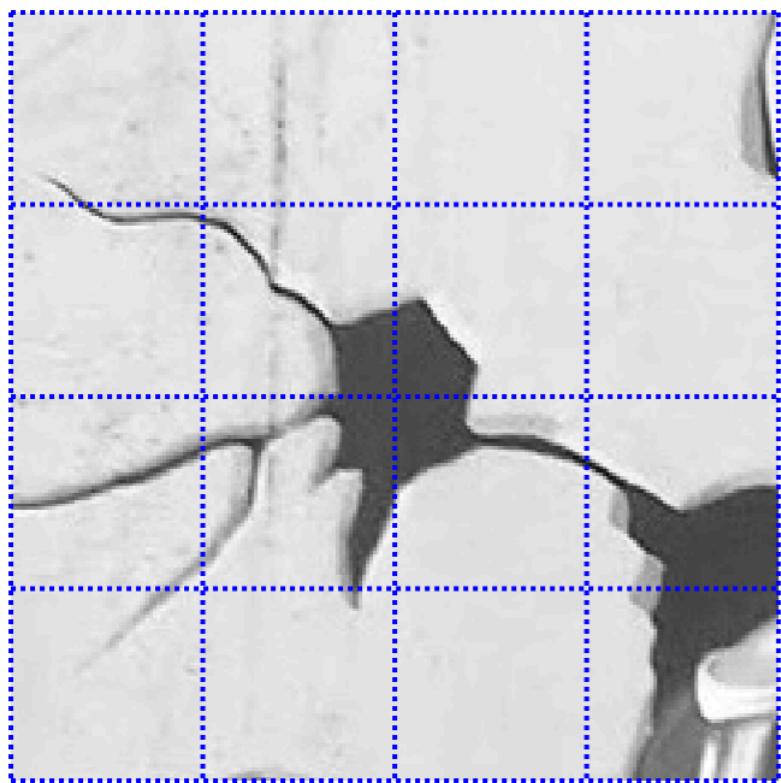
size=5

Map to canonical patch before descriptor extraction

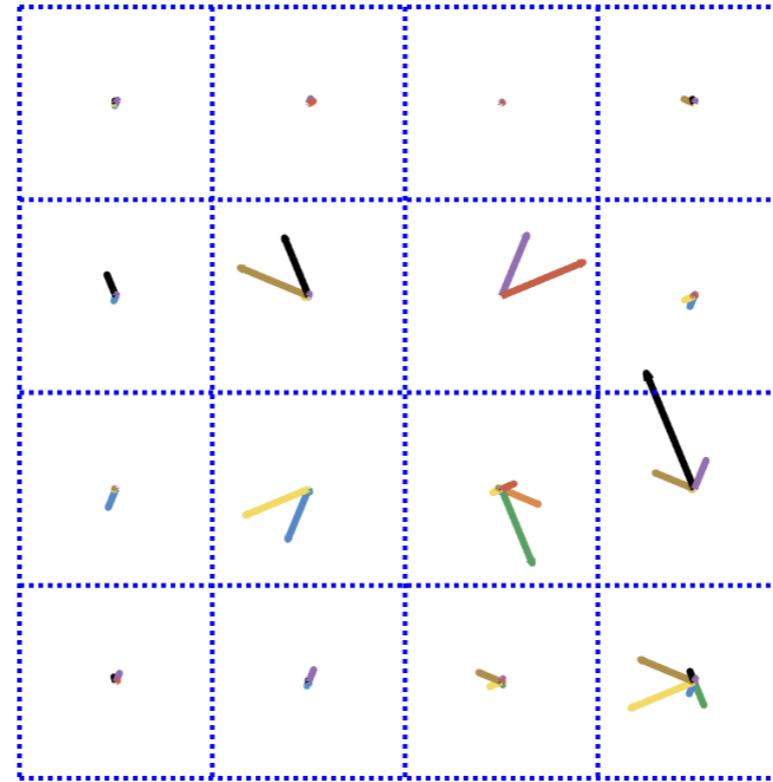
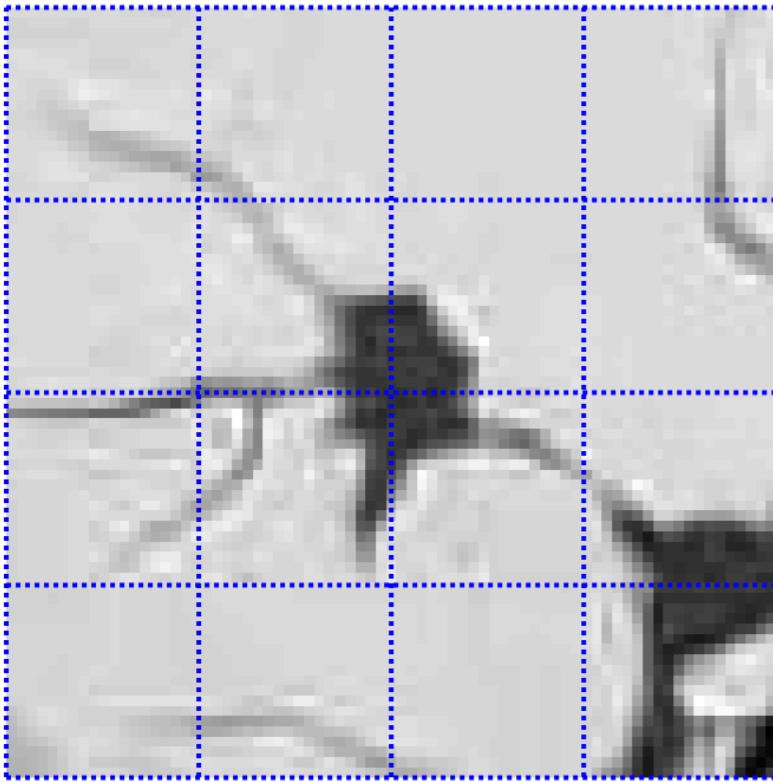
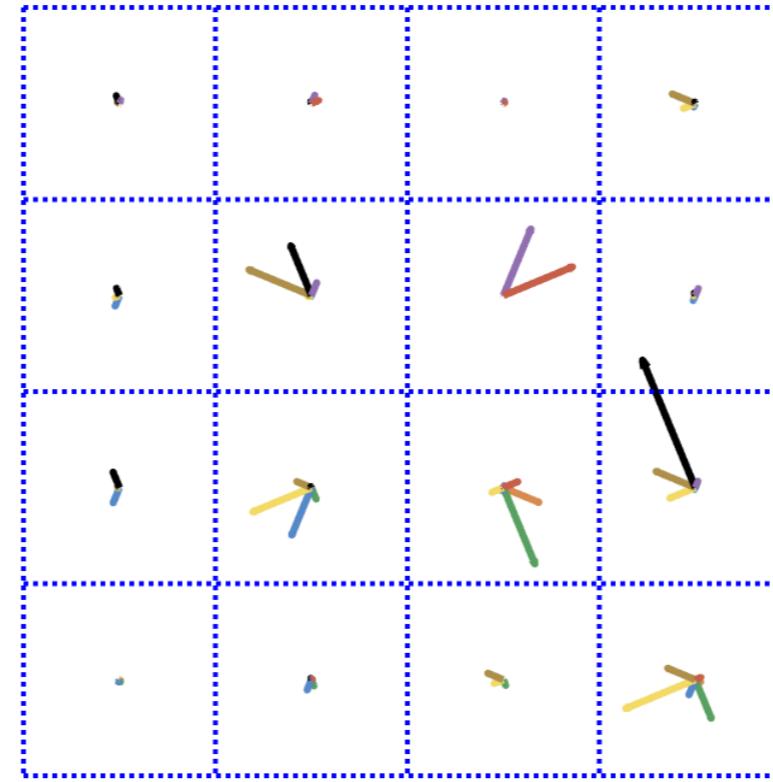
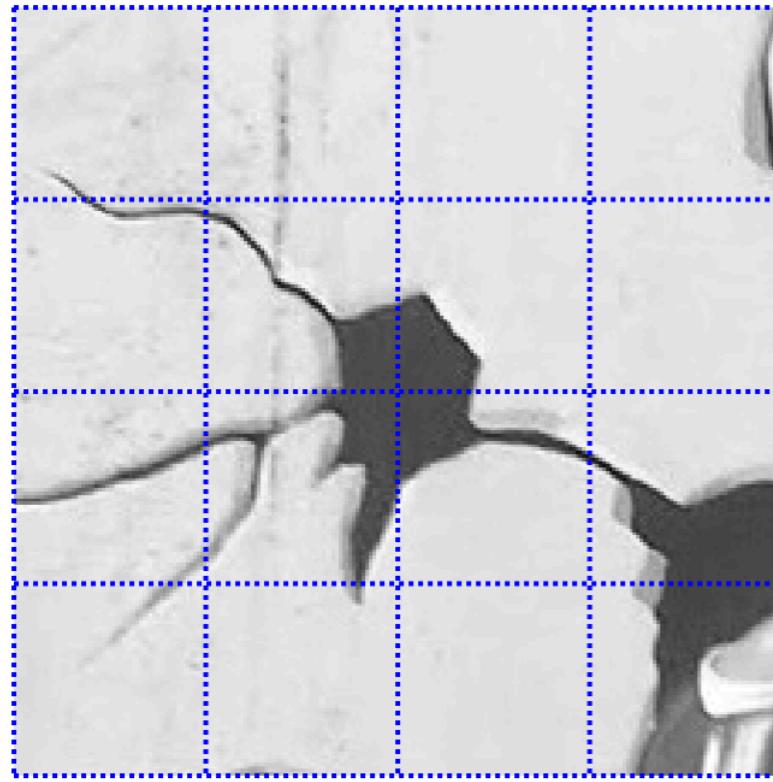


size=0.5

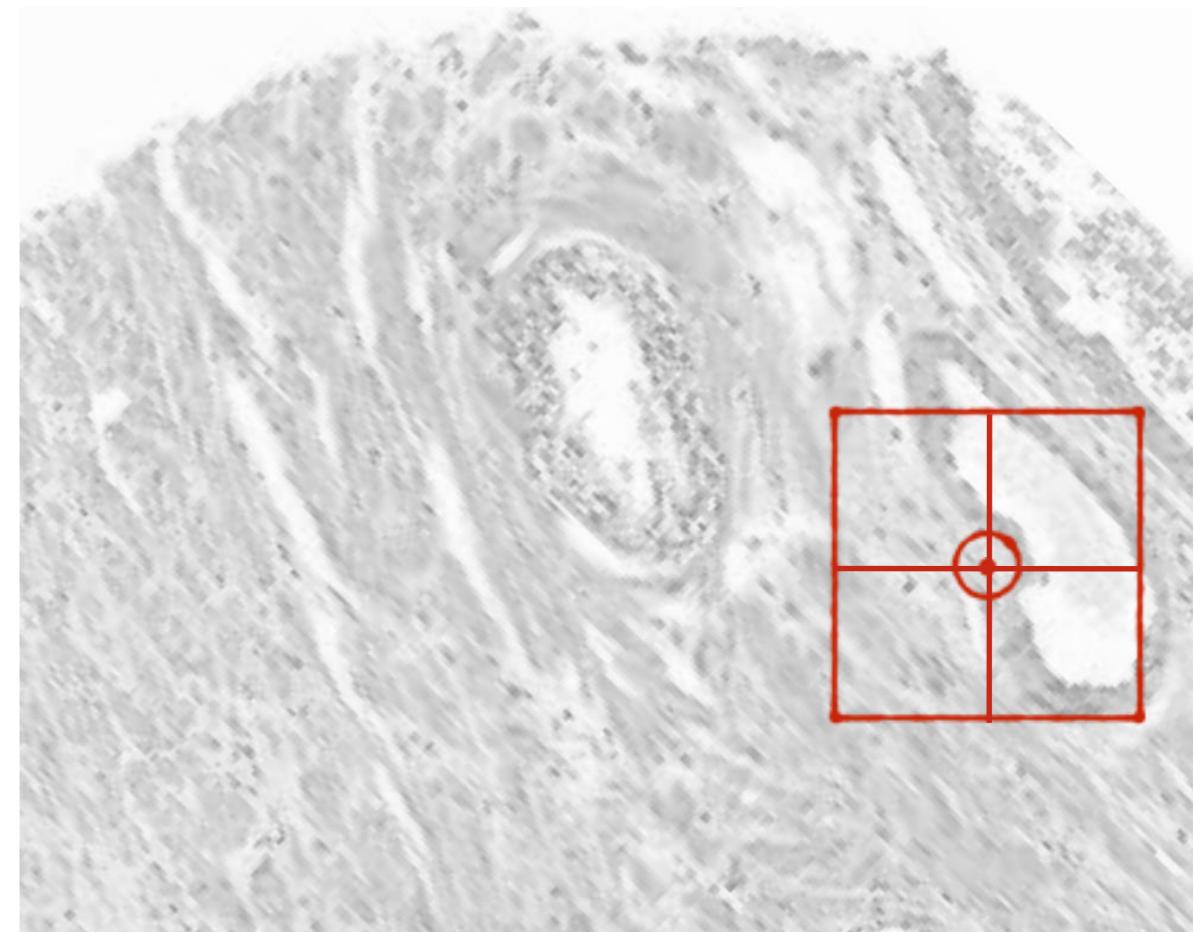
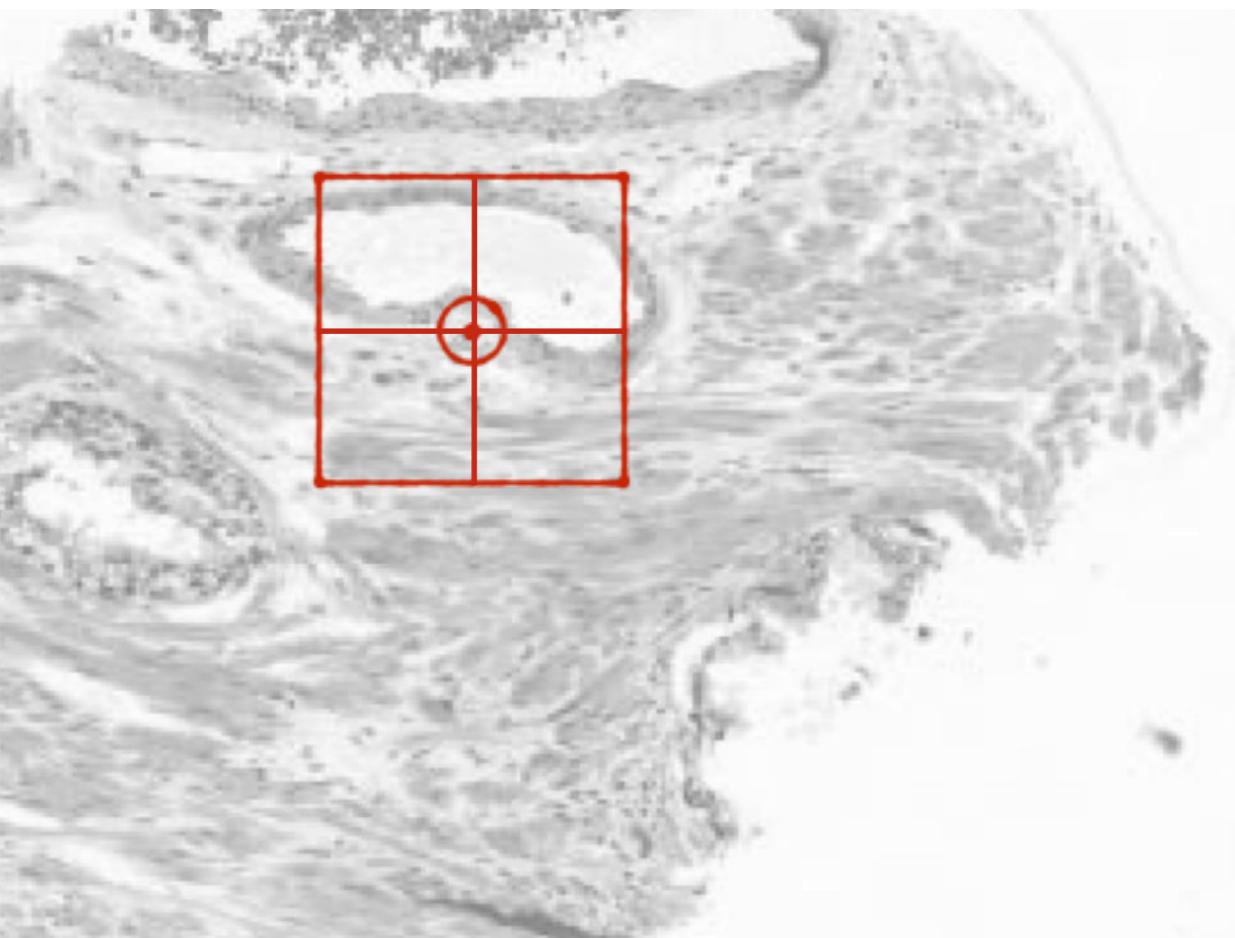
# Canonical Patches



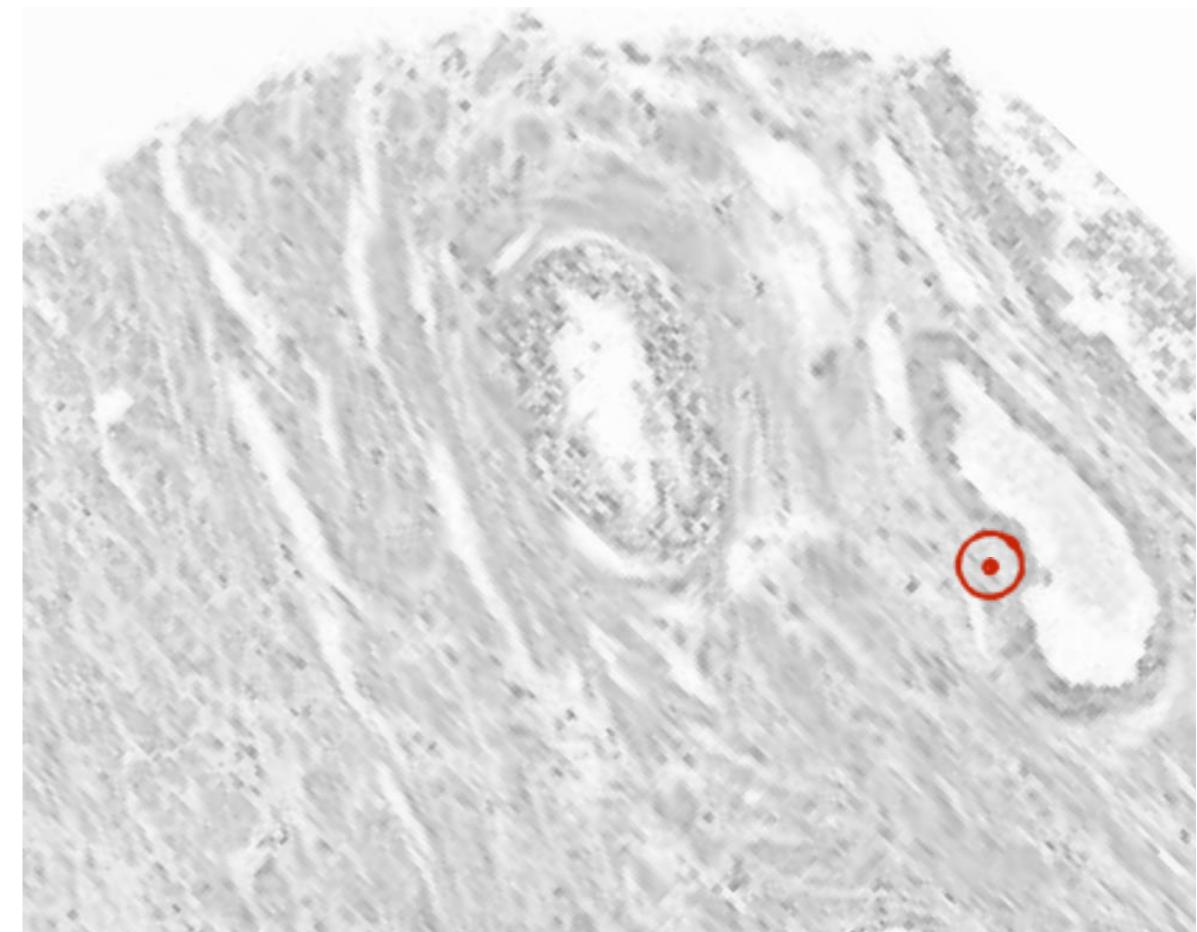
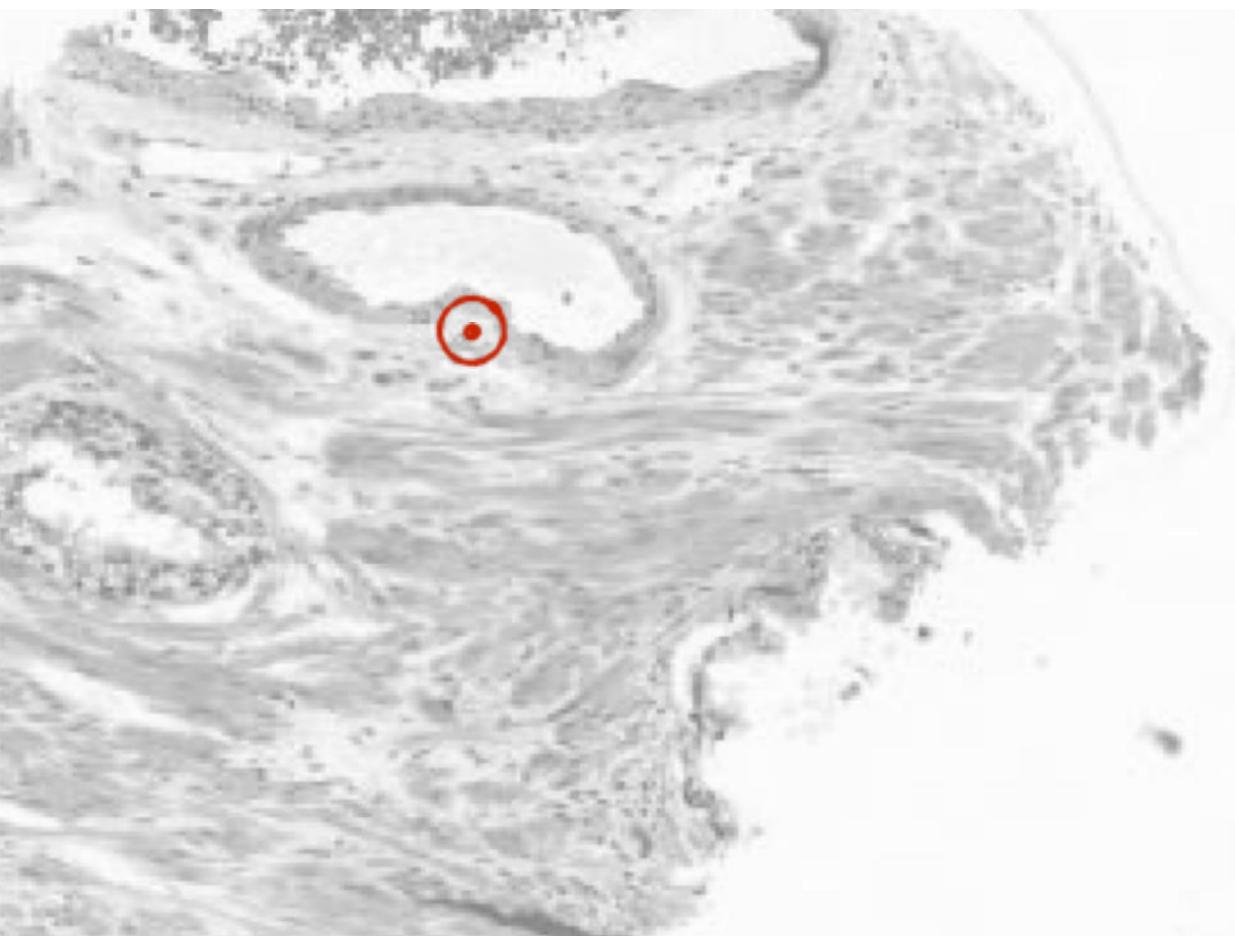
# Canonical Patches



# Challenge: Rotations

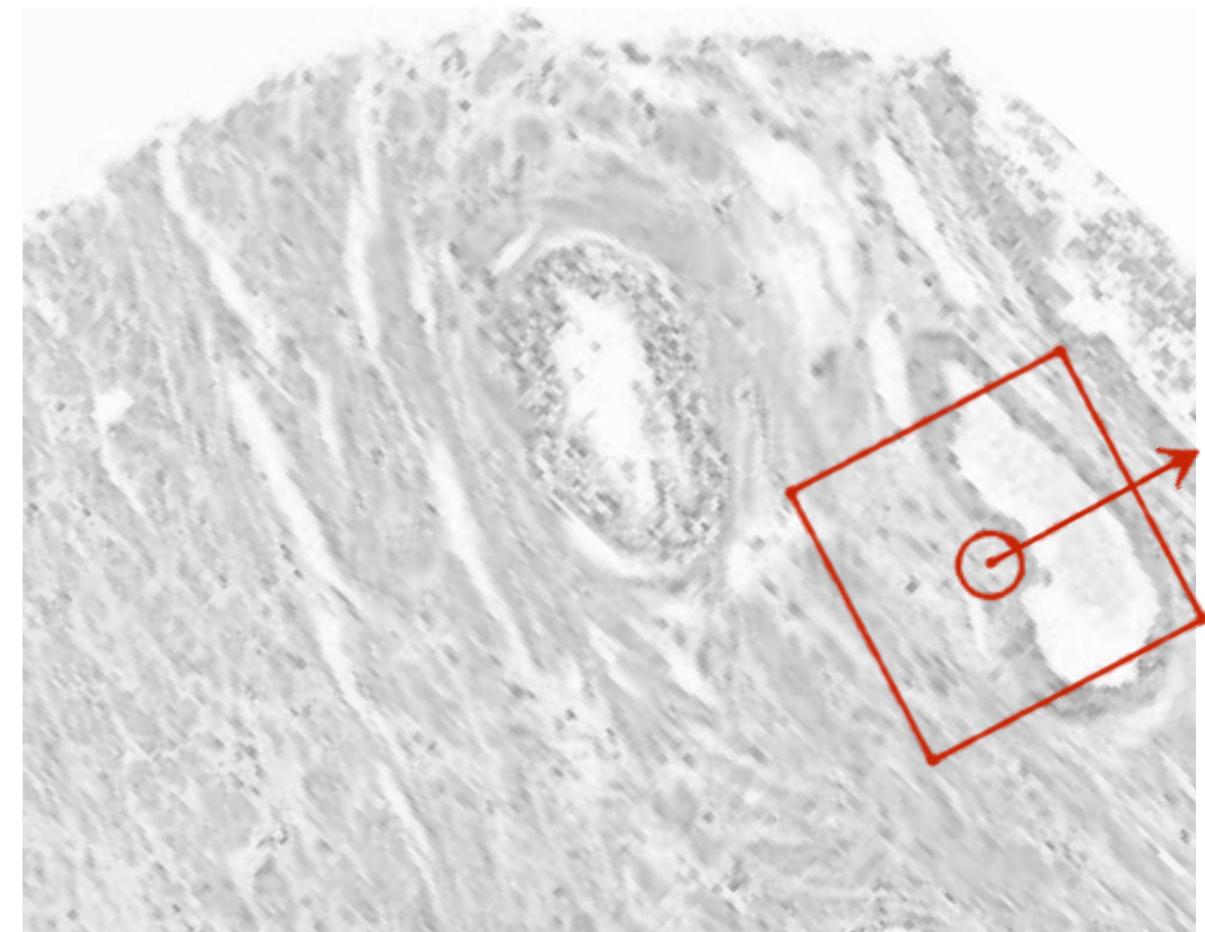
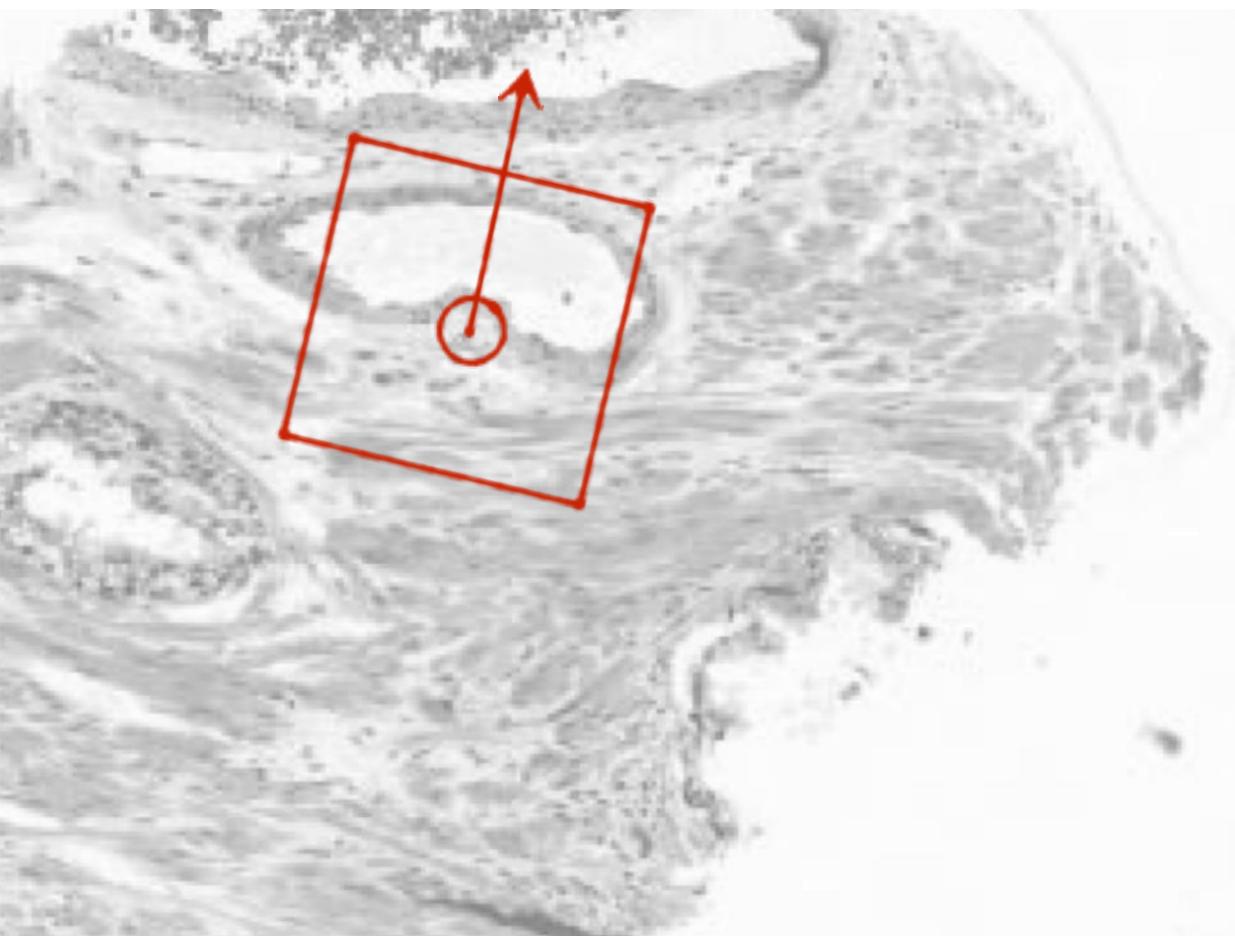


# Challenge: Rotations



Estimate a dominant direction

# Challenge: Rotations



Estimate a dominant direction

Align patches to this direction prior to descriptor extraction

# Estimating the Dominant Orientation



Estimated gradients

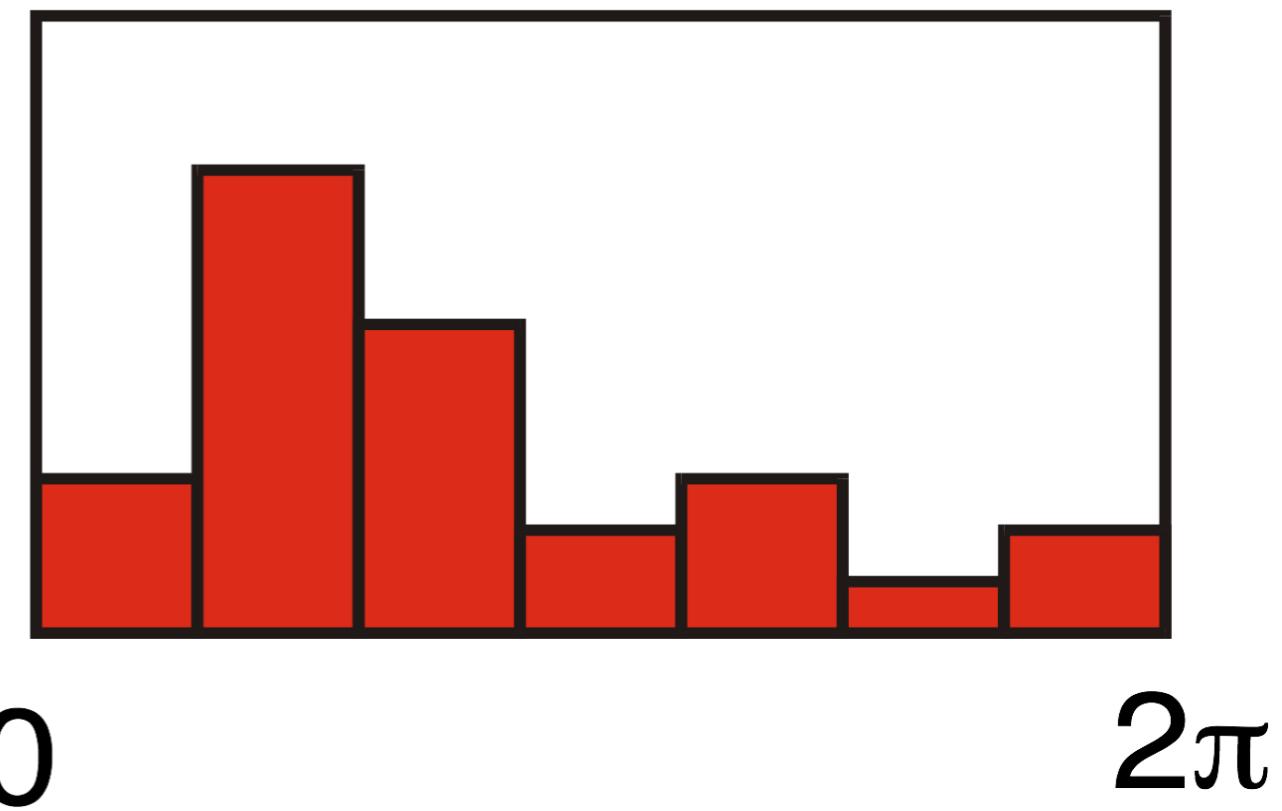
illustrations from  
[Lowe, Distinctive Image Features from Scale-Invariant Keypoints, IJCV 2004]

# Estimating the Dominant Orientation



Estimated gradients

Gradient histogram



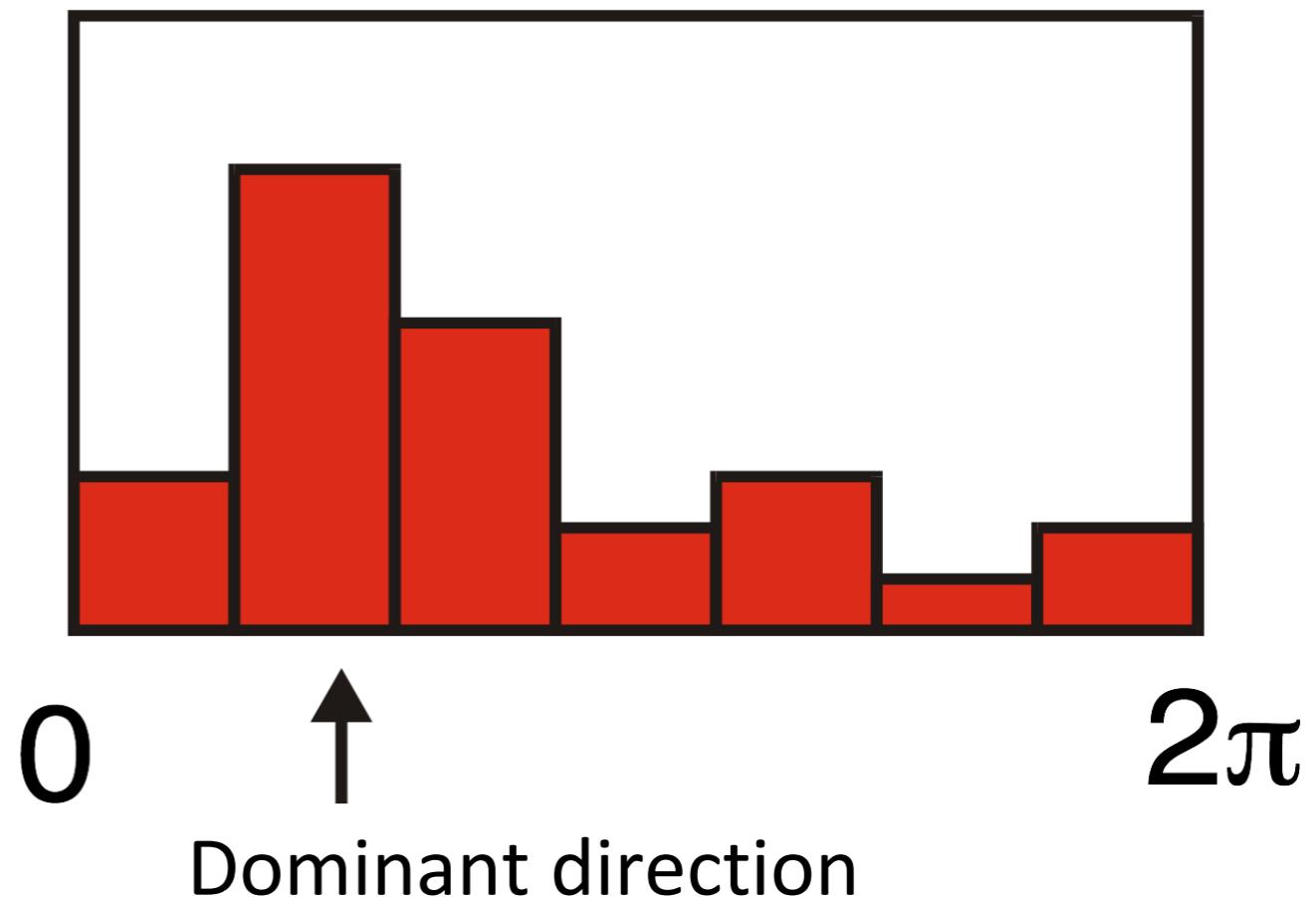
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# Estimating the Dominant Orientation



Estimated gradients

Gradient histogram



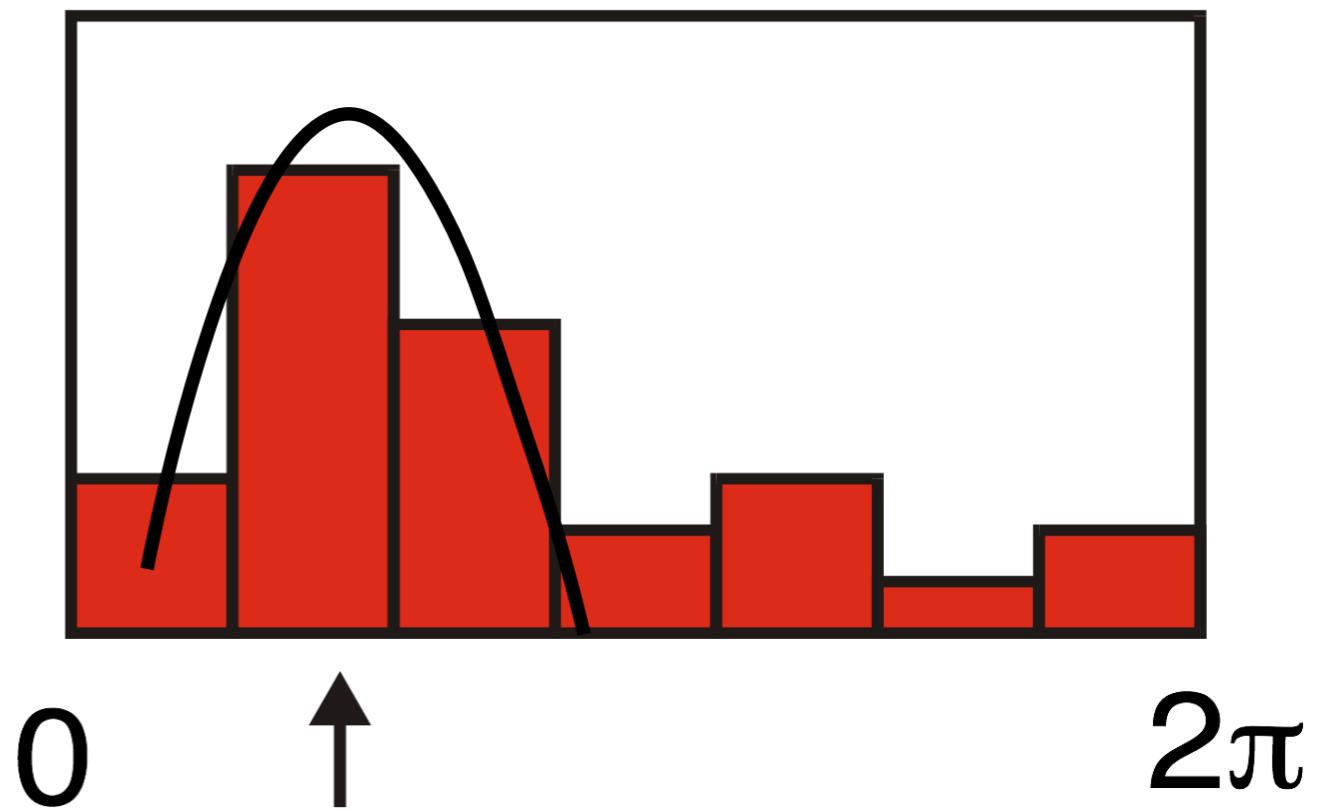
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# Estimating the Dominant Orientation



Estimated gradients

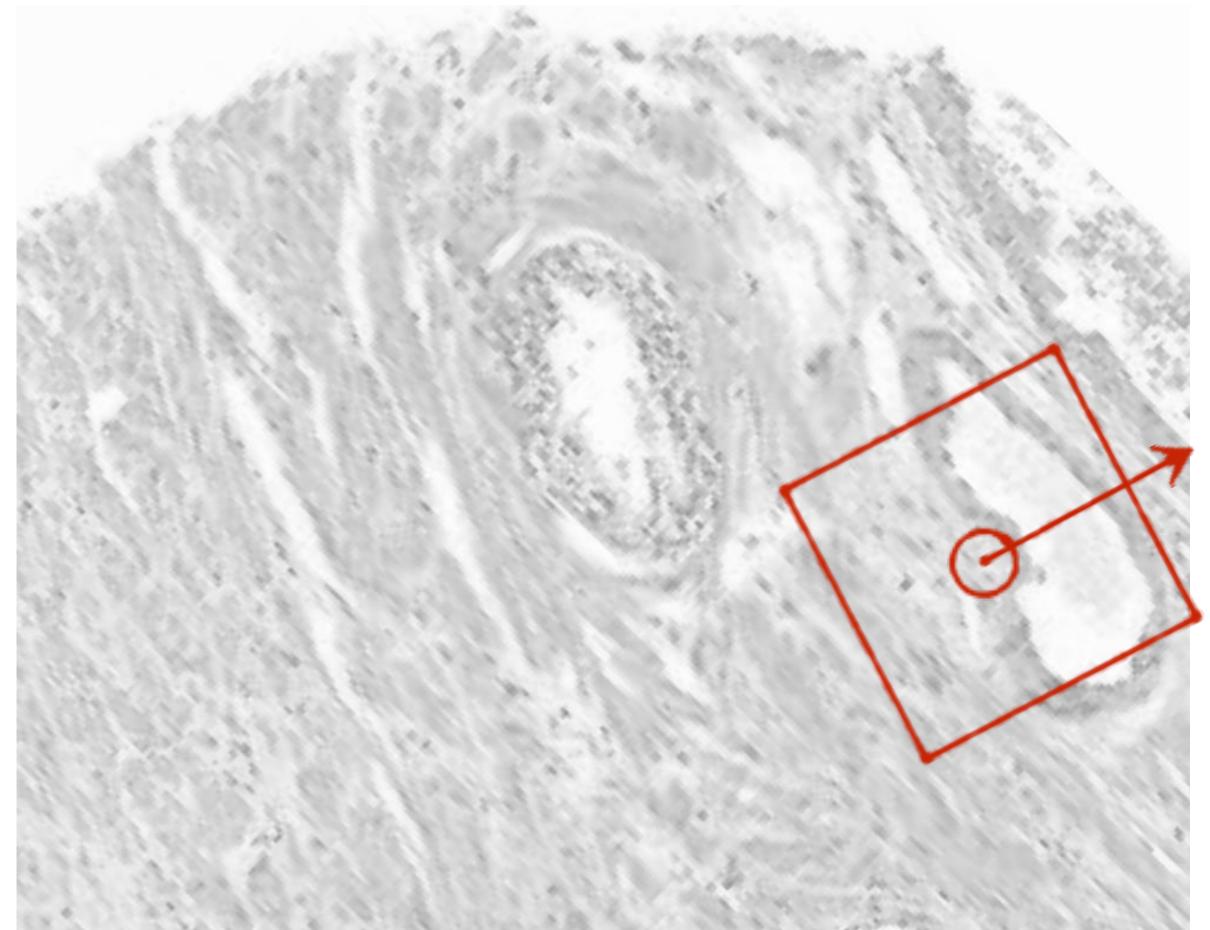
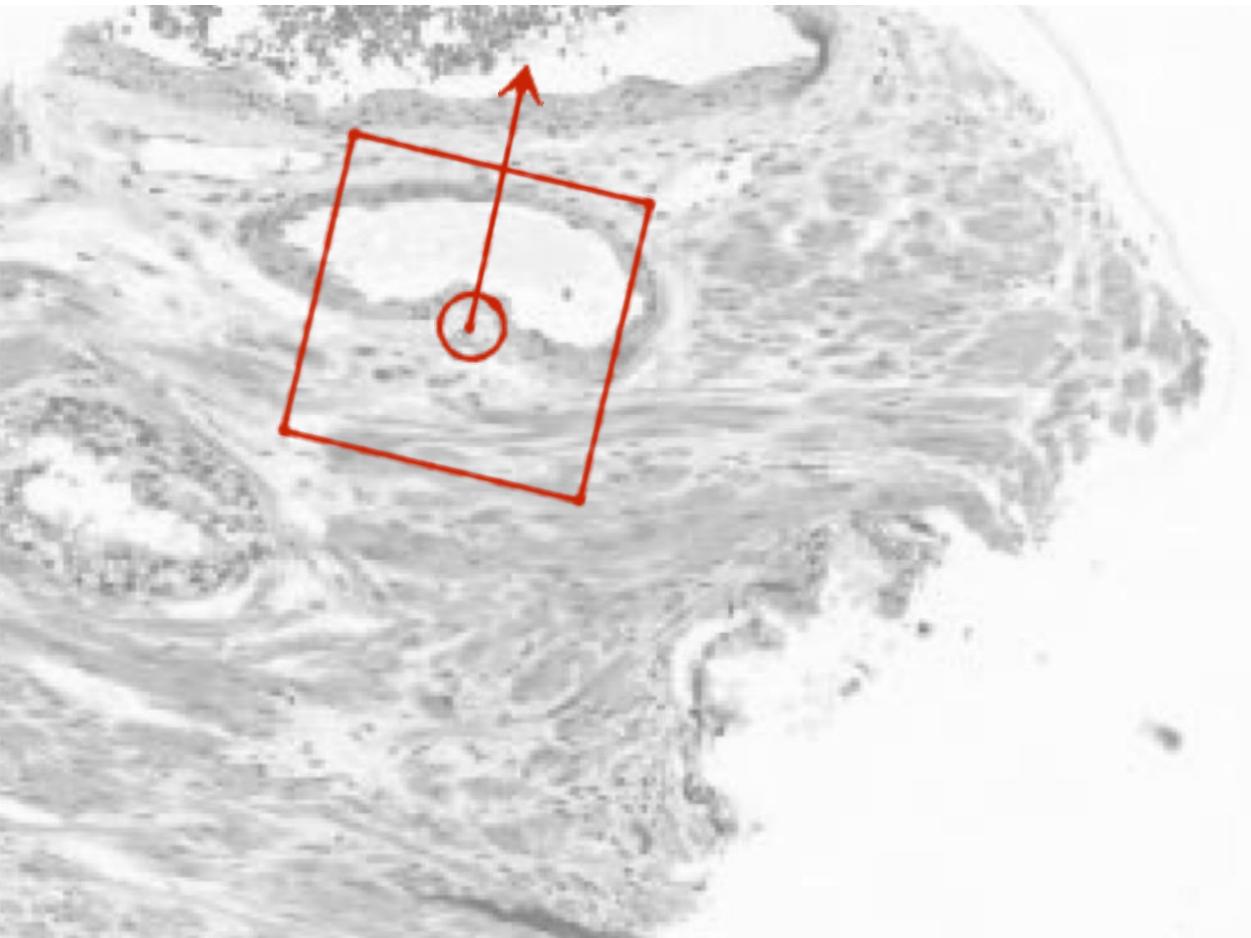
Gradient histogram



↑  
Dominant direction  
refine by curve fitting

illustrations from  
[Lowe, Distinctive Image Features from Scale-Invariant Keypoints, IJCV 2004]

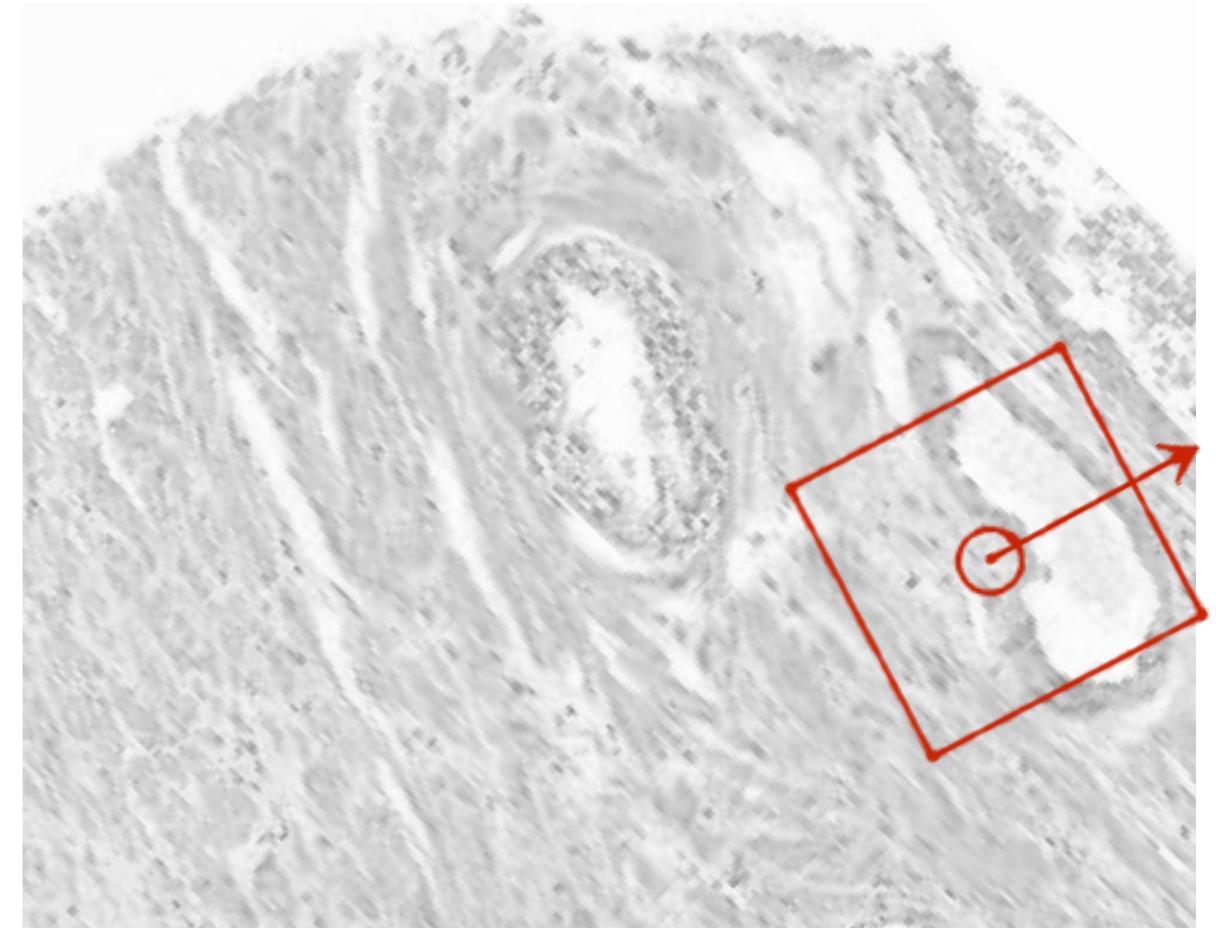
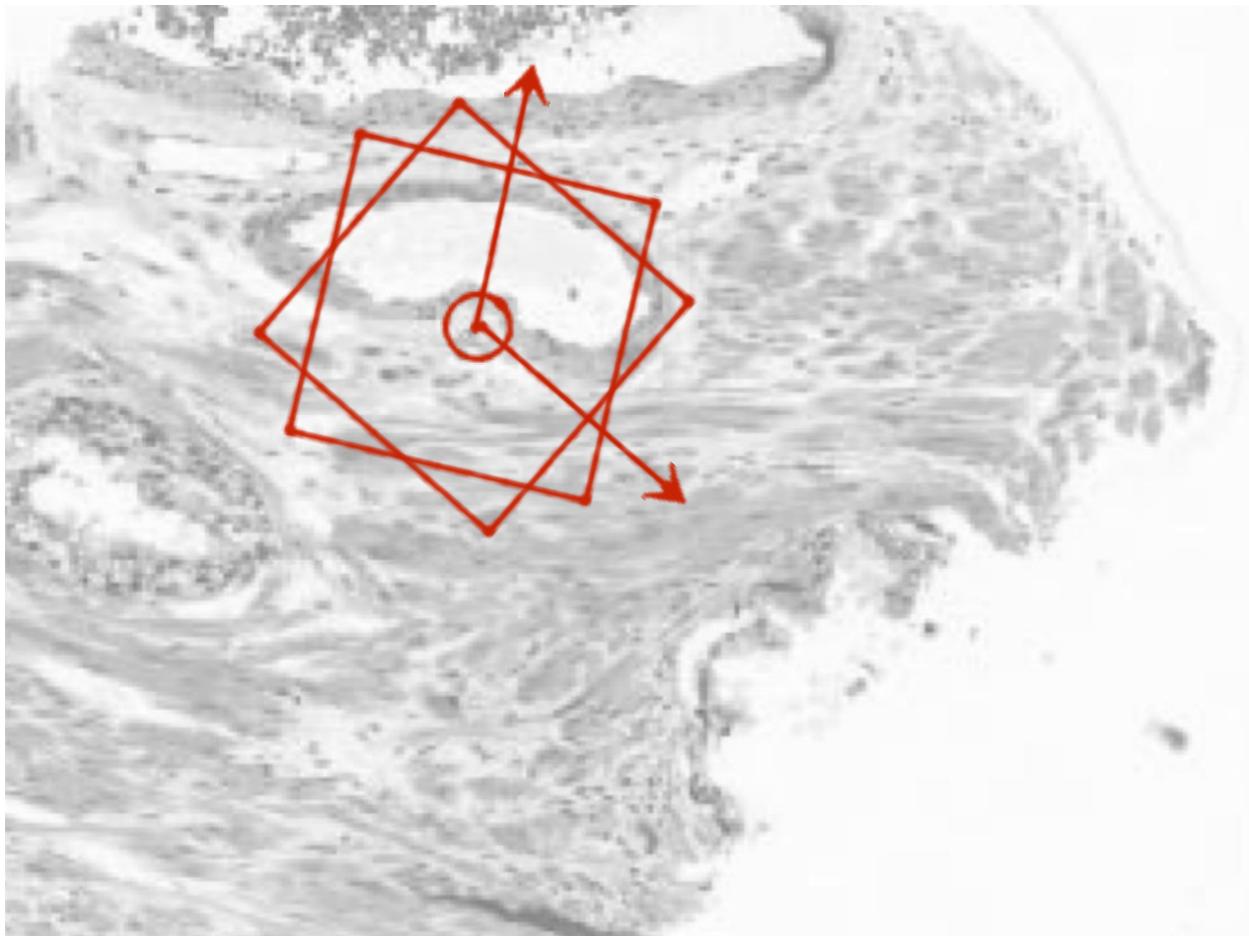
# Estimating the Dominant Orientation



Estimate a dominant direction

Align patches to this direction prior to descriptor extraction

# Estimating the Dominant Orientation



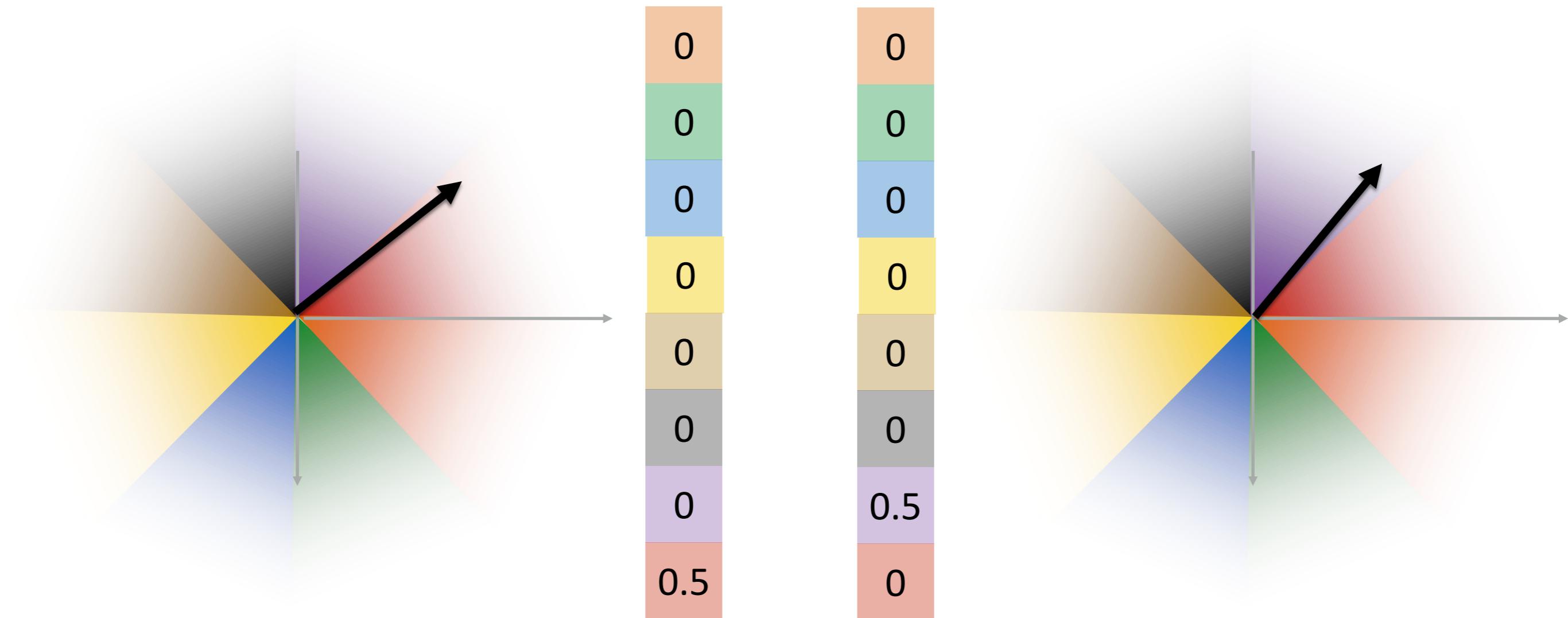
Estimate a dominant direction

Align patches to this direction prior to descriptor extraction

Can create multiple patches per keypoint (multiple dominant directions)

# Estimating the Dominant Orientation

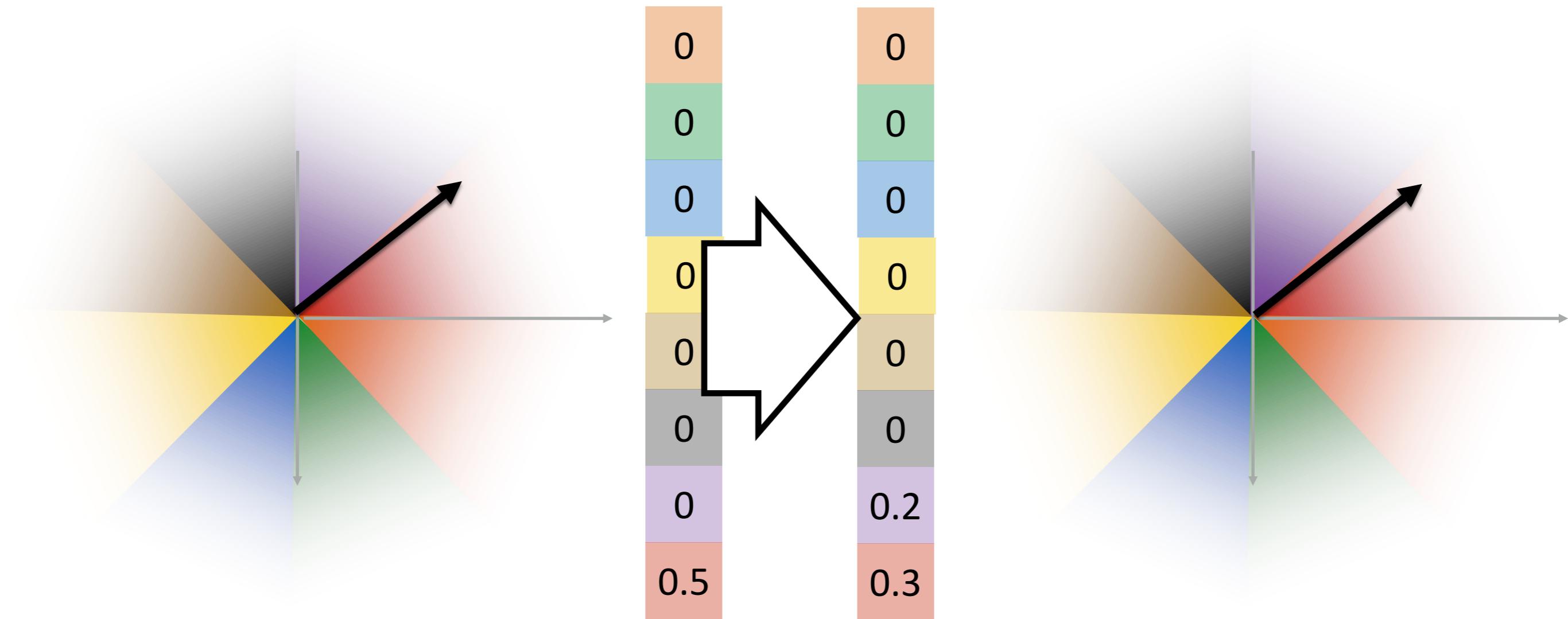
Boundary effects in descriptor due to small errors in orientation



slide credit: Marc Pollefeys, Kevin Köser

# Soft-Binning

Distribute weights to multiple bins via linear interpolation



slide credit: Marc Pollefeys, Kevin Köser

# Invariances?



Changes in scale



in-plane rotation



Changes in brightness

# Invariances?



Changes in scale



in-plane rotation



Changes in brightness



Changes in viewpoint



# Invariances?



Changes in scale



in-plane rotation



Changes in brightness

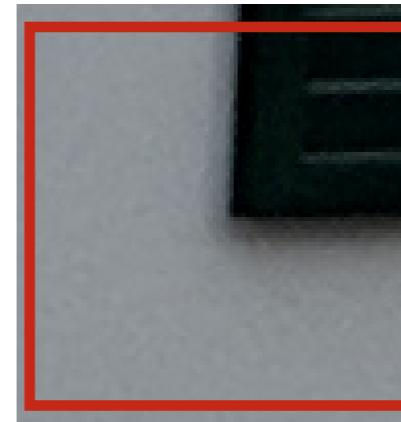


Not invariant, but reasonably robust



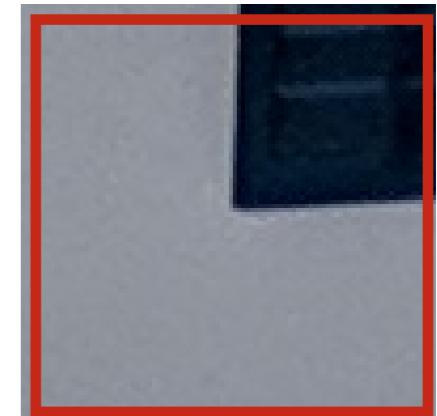
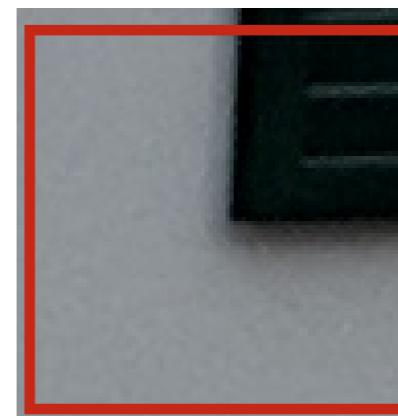
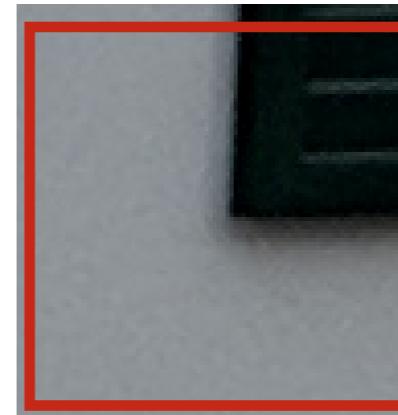
Changes in viewpoint

# How Much Invariance Do We Want?



Invariance can also reduce performance

# How Much Invariance Do We Want?



Invariance can also reduce performance

# Is Your Visual System Rotation Invariant?

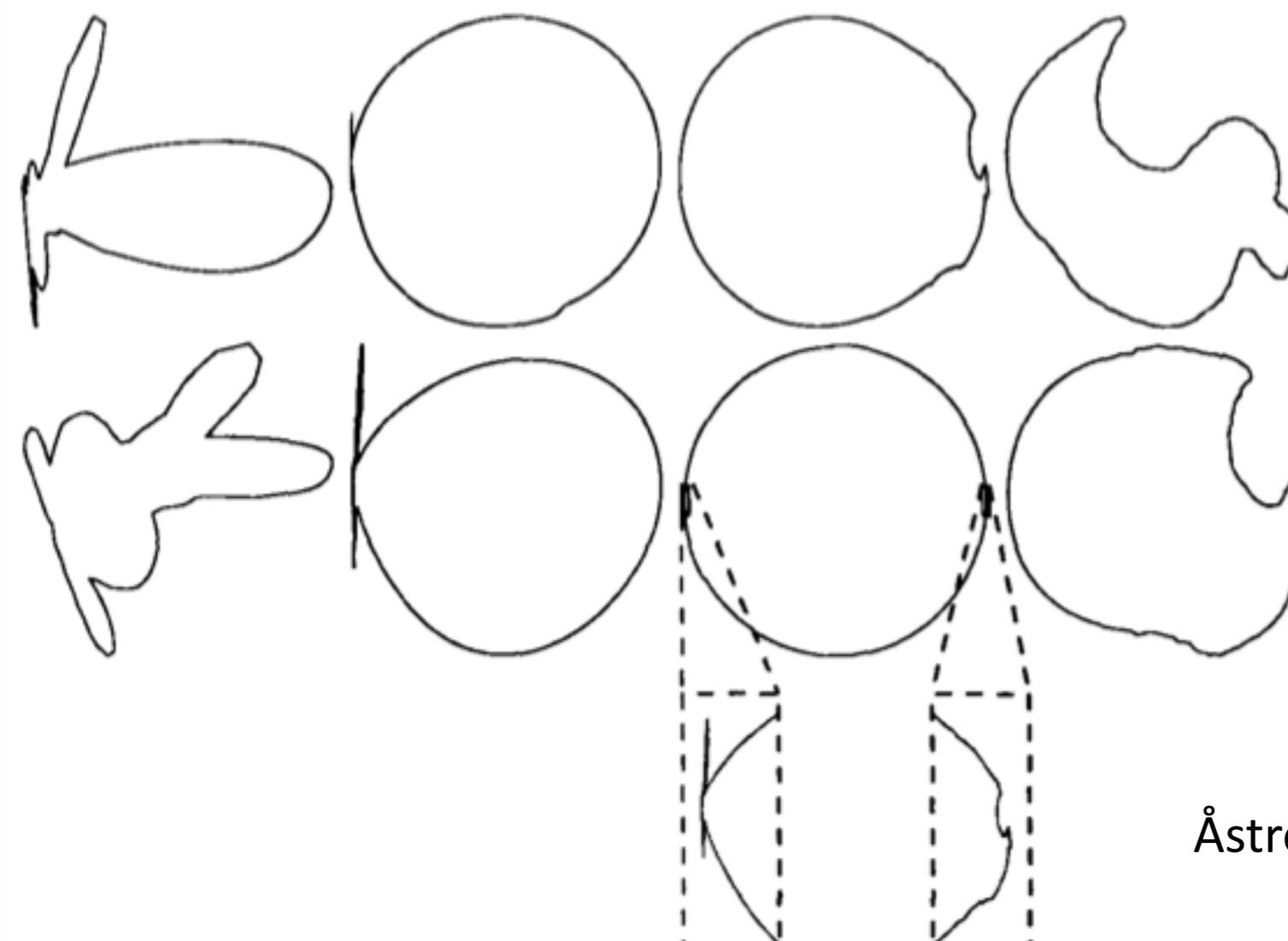


# Is Your Visual System Rotation Invariant?



Thatcher illusion

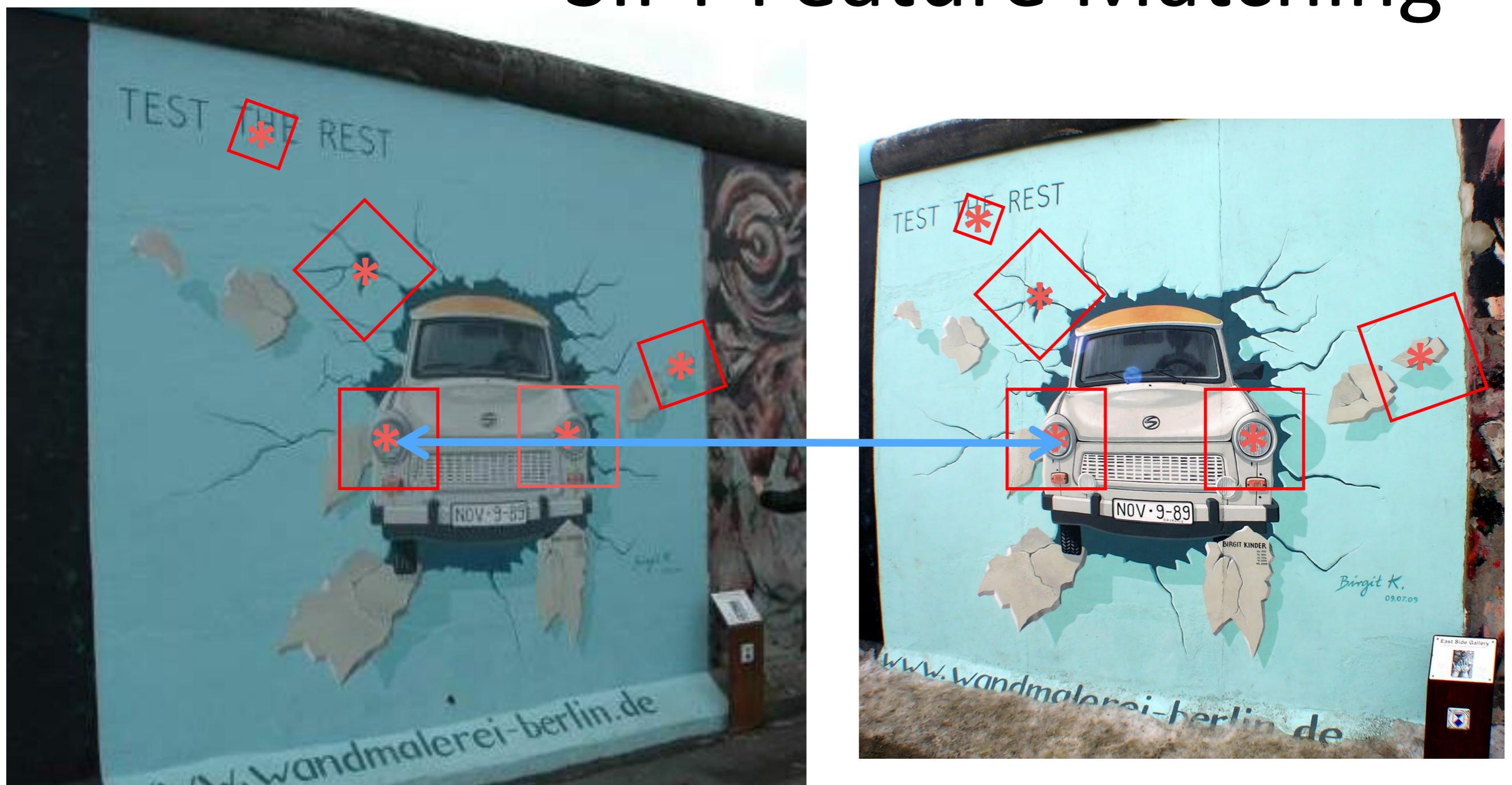
# Invariance vs. Robustness



Invariance = no influence on result

Robustness = we can deal with a moderate amount

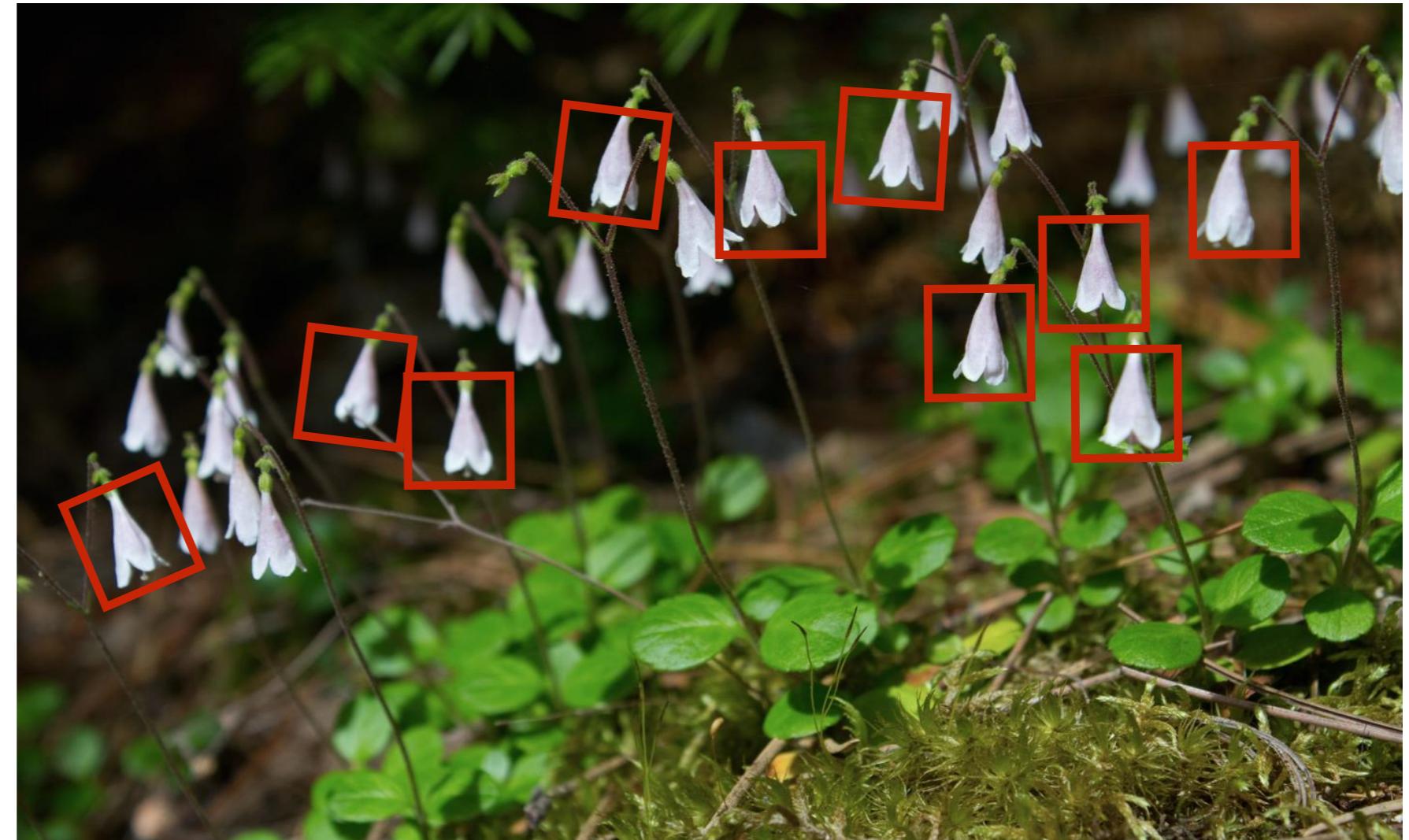
# SIFT Feature Matching



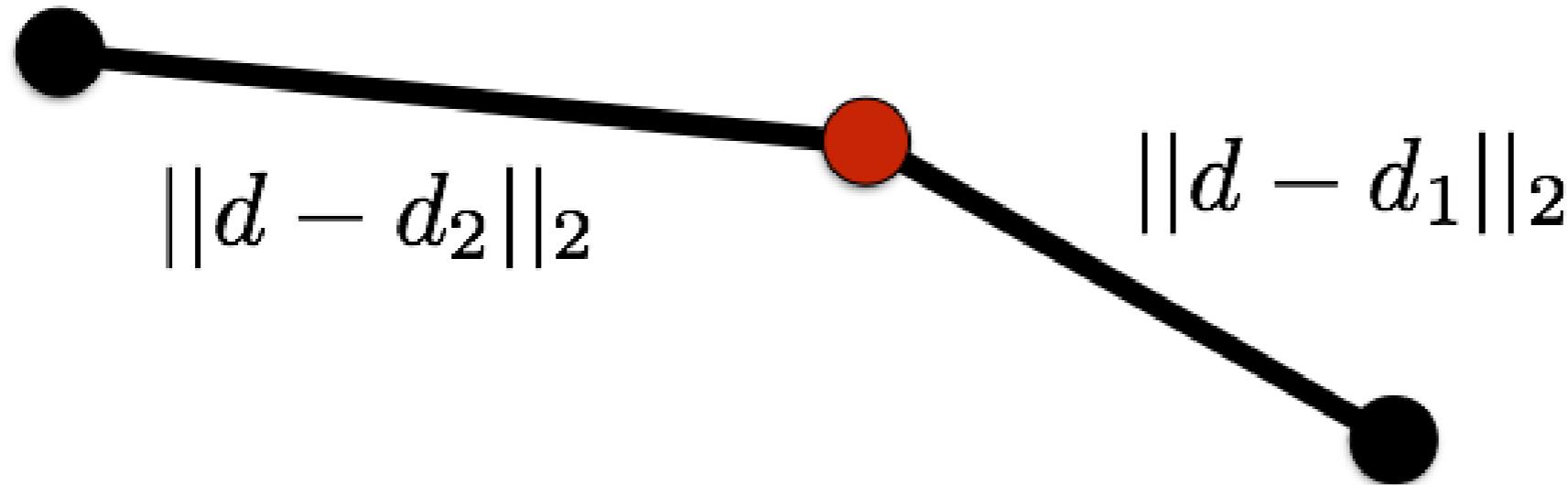
Use descriptor similarity ( $L_2$  distance) to find corresponding points in different images

# $L_2$ Similar = Correct?

query:



# Lowe's Ratio Test



Accept match with 1<sup>st</sup> neighbor if

$$\frac{\|d - d_1\|_2}{\|d - d_2\|_2} < 0.8$$

# Faster Matching

- Exhaustively comparing all descriptors is expensive

# Faster Matching

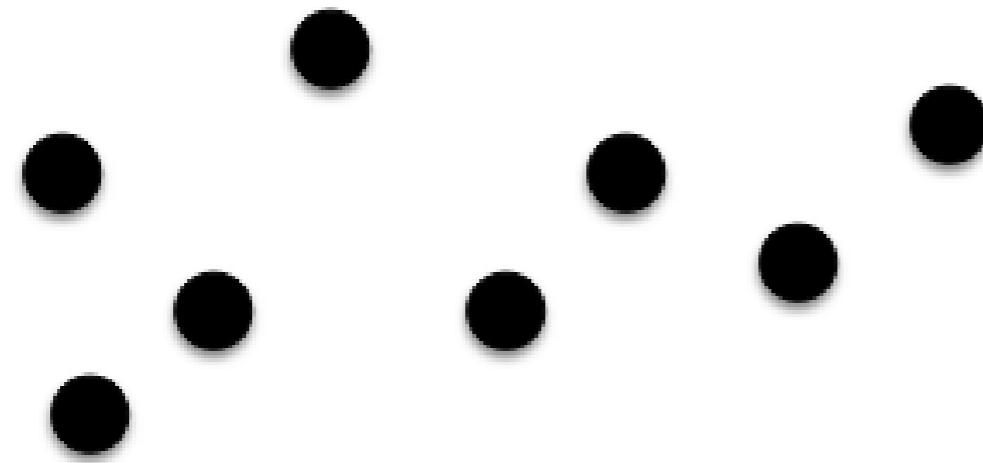
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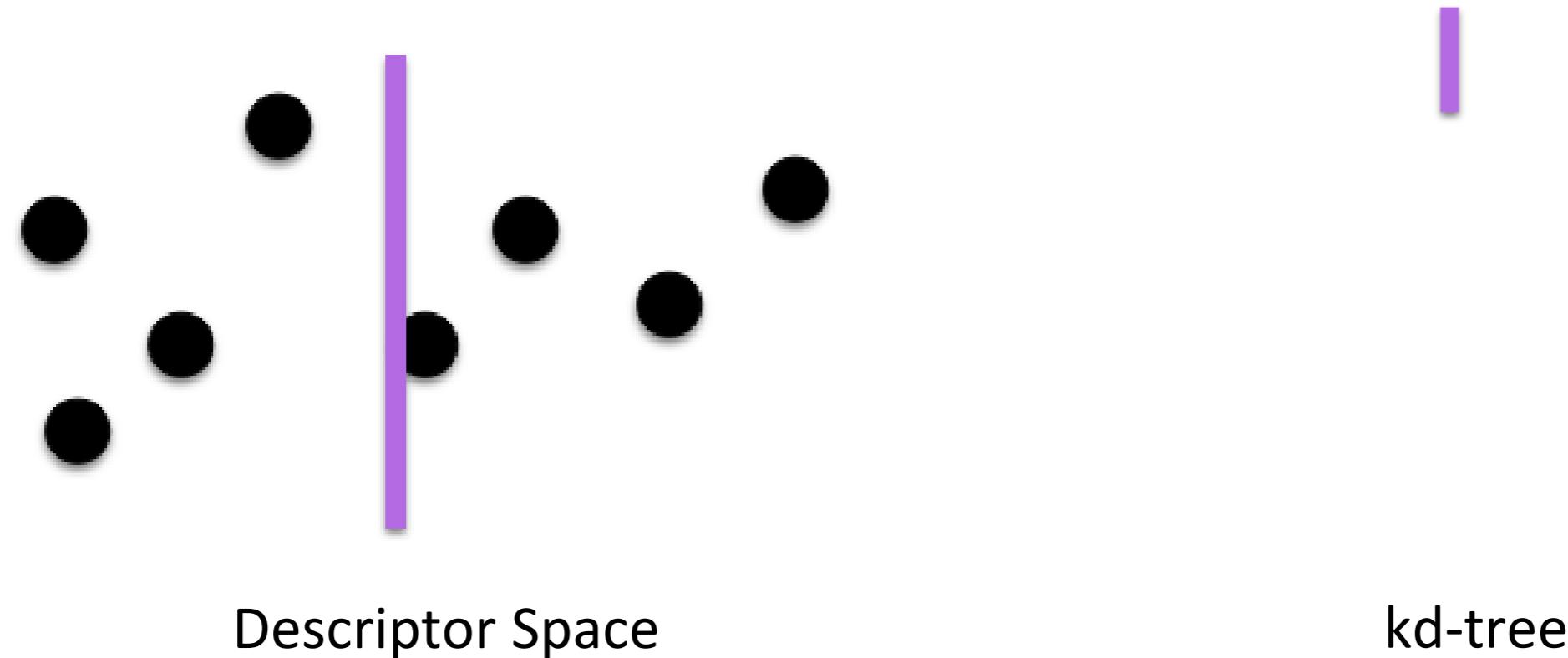


Descriptor Space

kd-tree

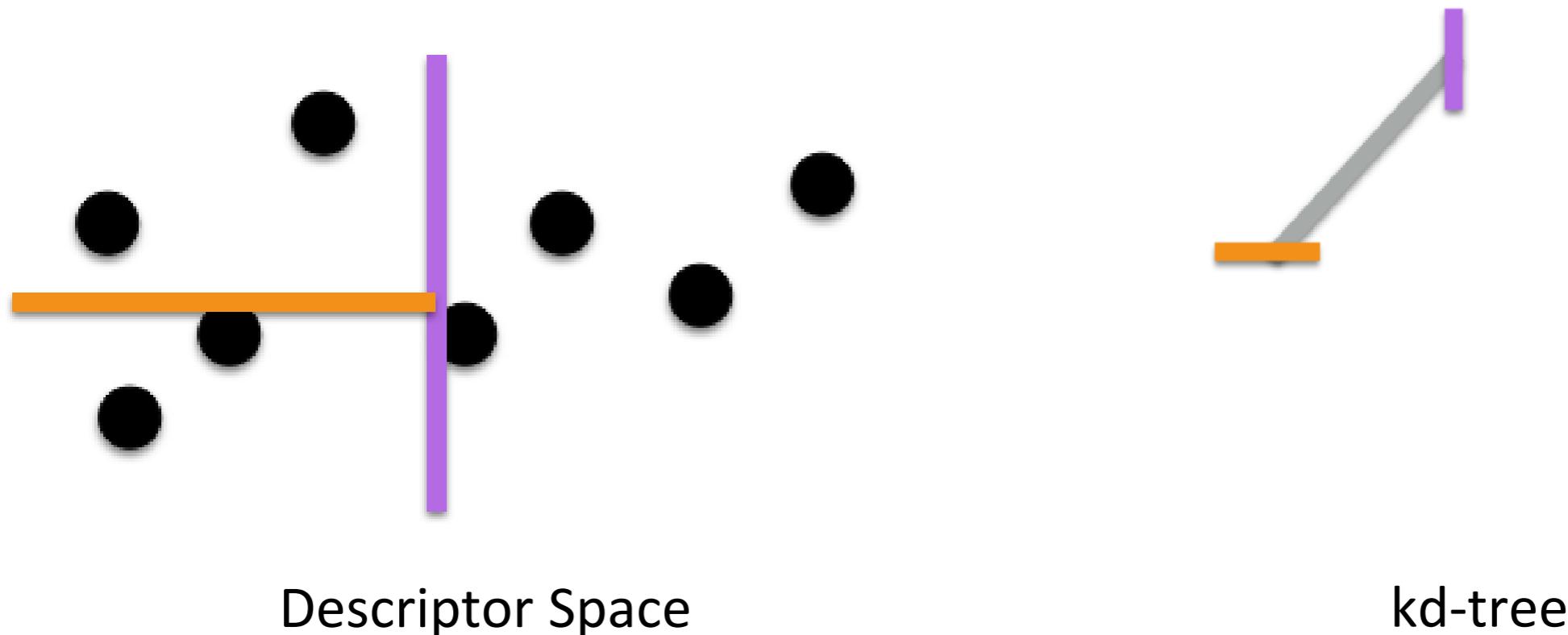
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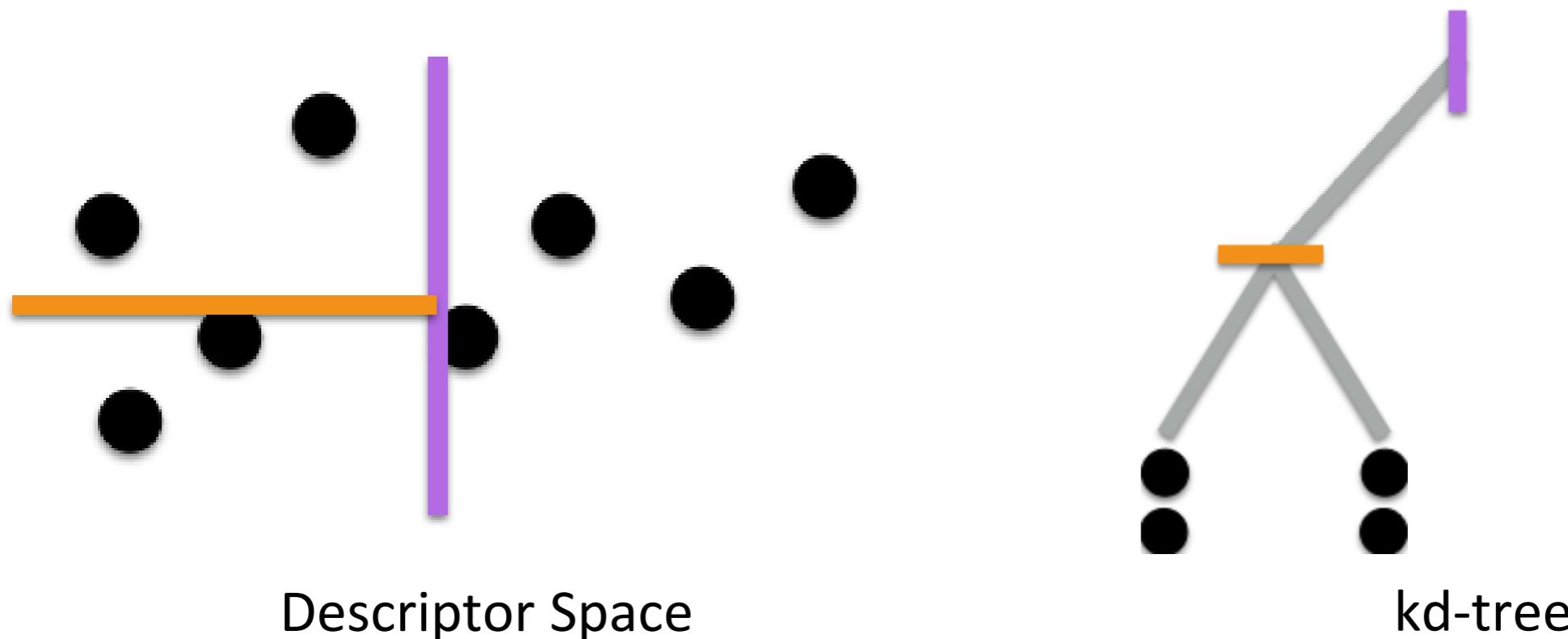
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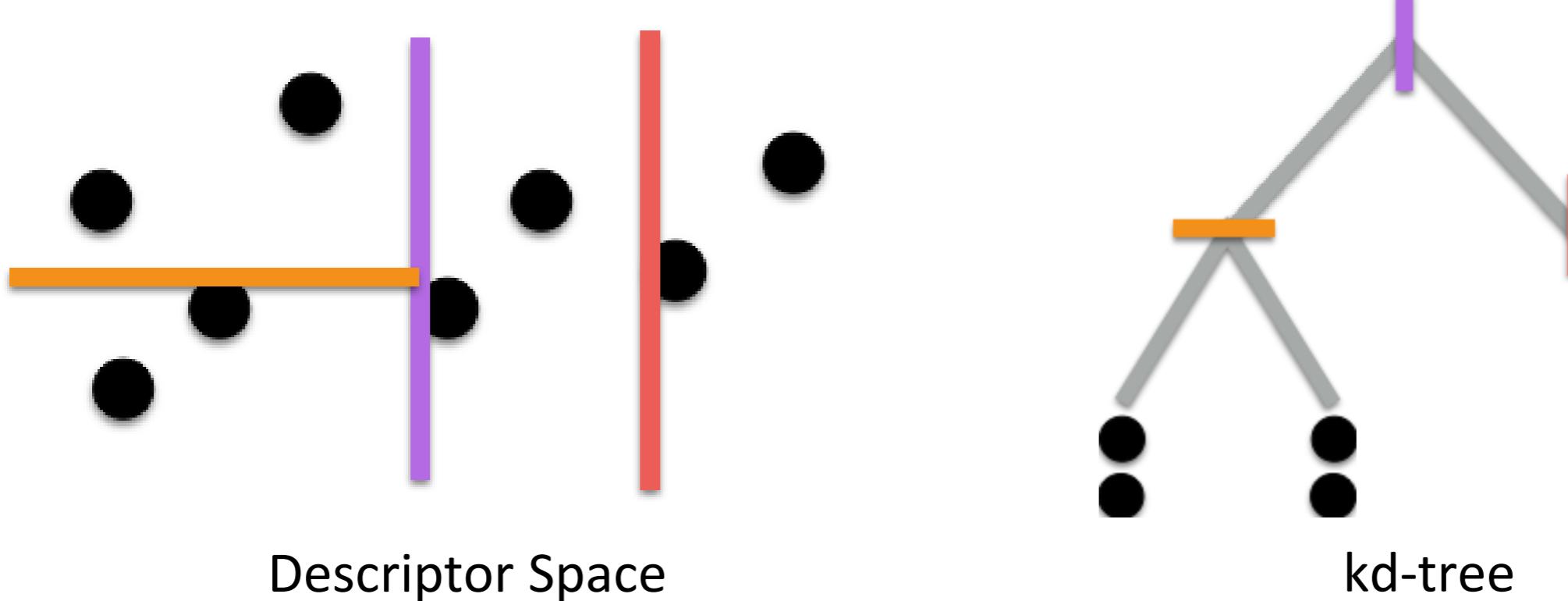
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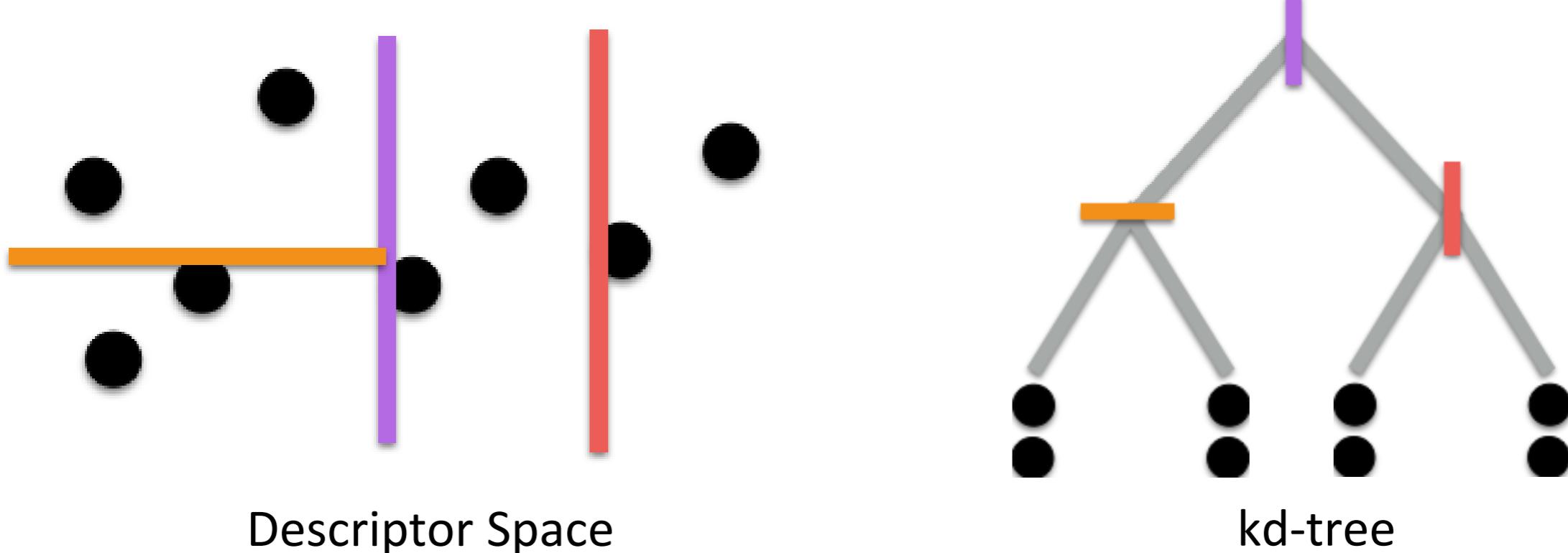
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# Faster Matching

- Curse of dimensionality: No acceleration scheme (asymptotically) faster than exhaustive search

# Faster Matching

- Curse of dimensionality: No acceleration scheme (asymptotically) faster than exhaustive search
- Intuition: Consider SIFT (128 dimensions)
  - Around 16,000 features lead to kd-tree of depth 14
  - By the time you reach a leaf node, you have only considered 14 out of the 128 dimensions
  - High chance that the other dimensions are very different
  - High chance that the best leaf node does not contain nearest neighbor
  - Need to essentially search through all leaves

# Faster Matching

- Curse of dimensionality: No acceleration scheme (asymptotically) faster than exhaustive search
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  - High chance that the other dimensions are very different
  - High chance that the best leaf node does not contain nearest neighbor
  - Need to essentially search through all leaves
- **Approximate search:** Search only through fixed number of leaves

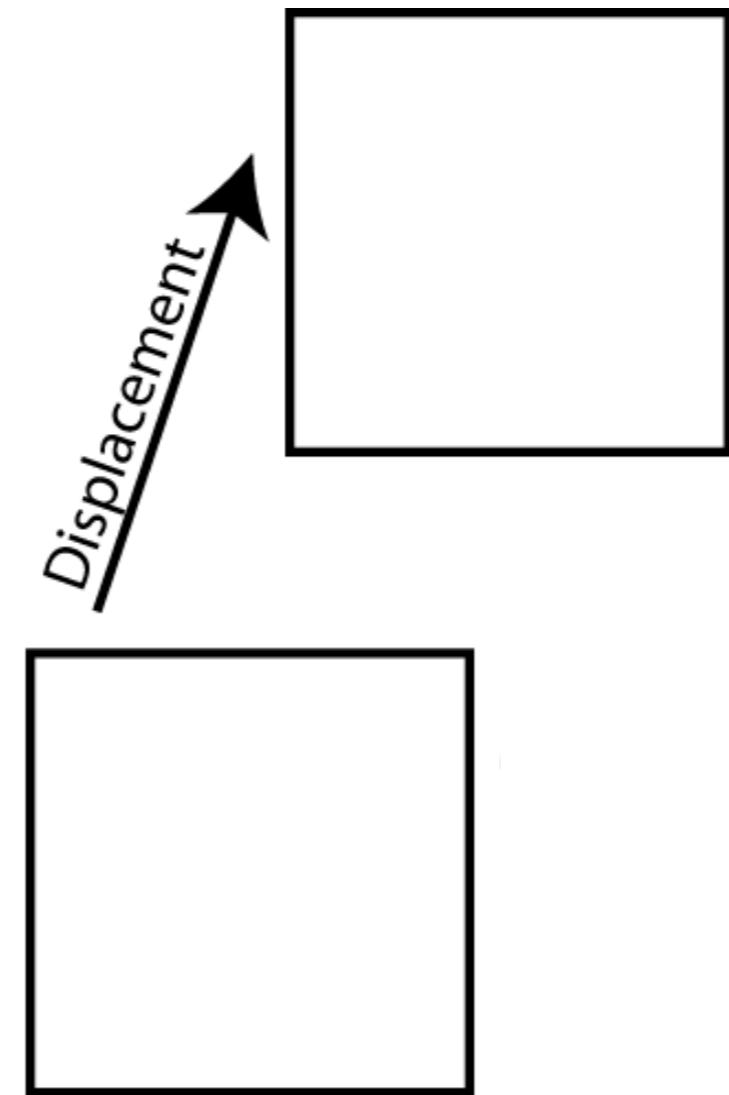
# SIFT Features

- Skipped some details:
  - Down-weighting of pixels far away from center of spatial bin
  - Bilinear interpolation between spatial bins
  - For further details, read Lowe's 2004 IJCV paper

# Today

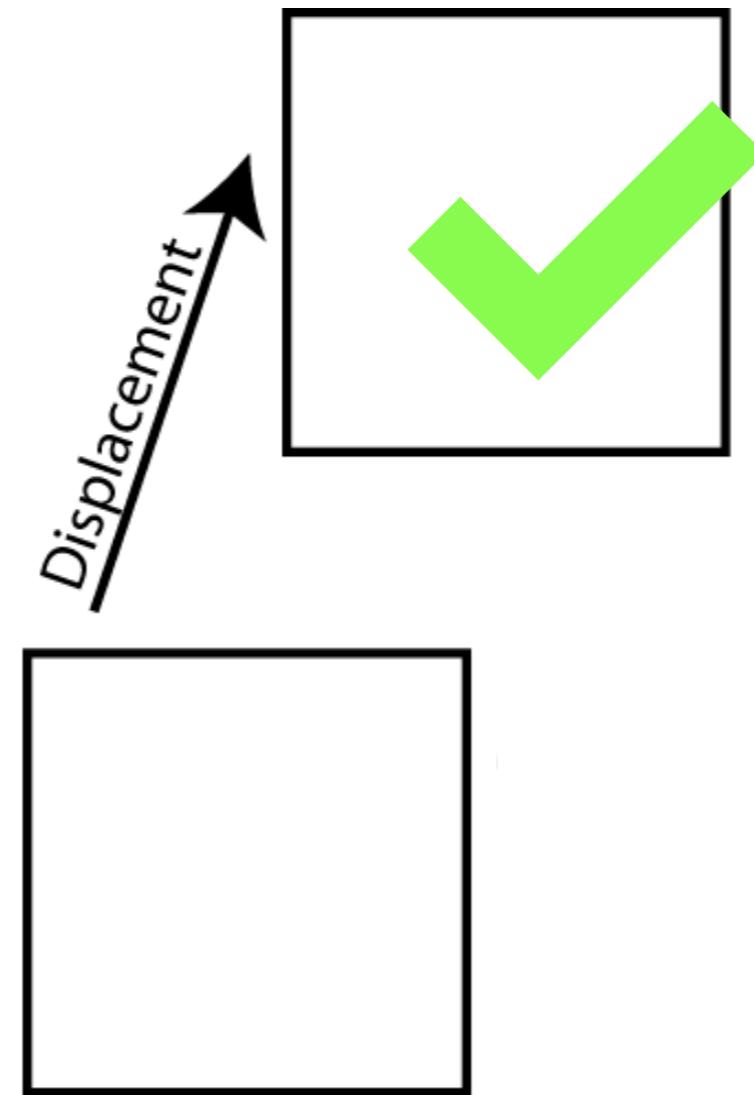
- Scale-Invariant Local Features (SIFT)
- Geometric Invariances

# Patch Transformations



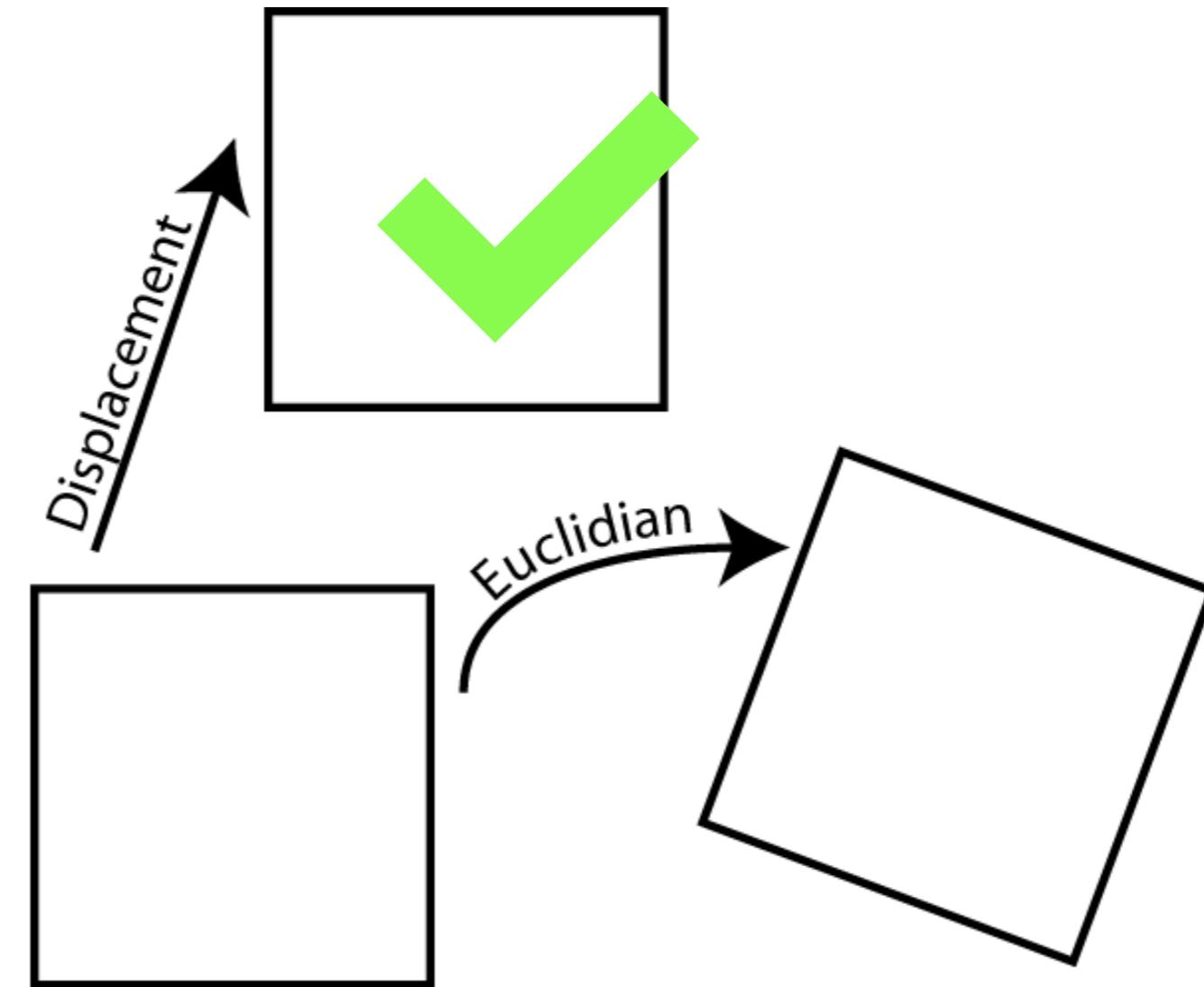
slide credit: Marc Pollefeys

# Patch Transformations



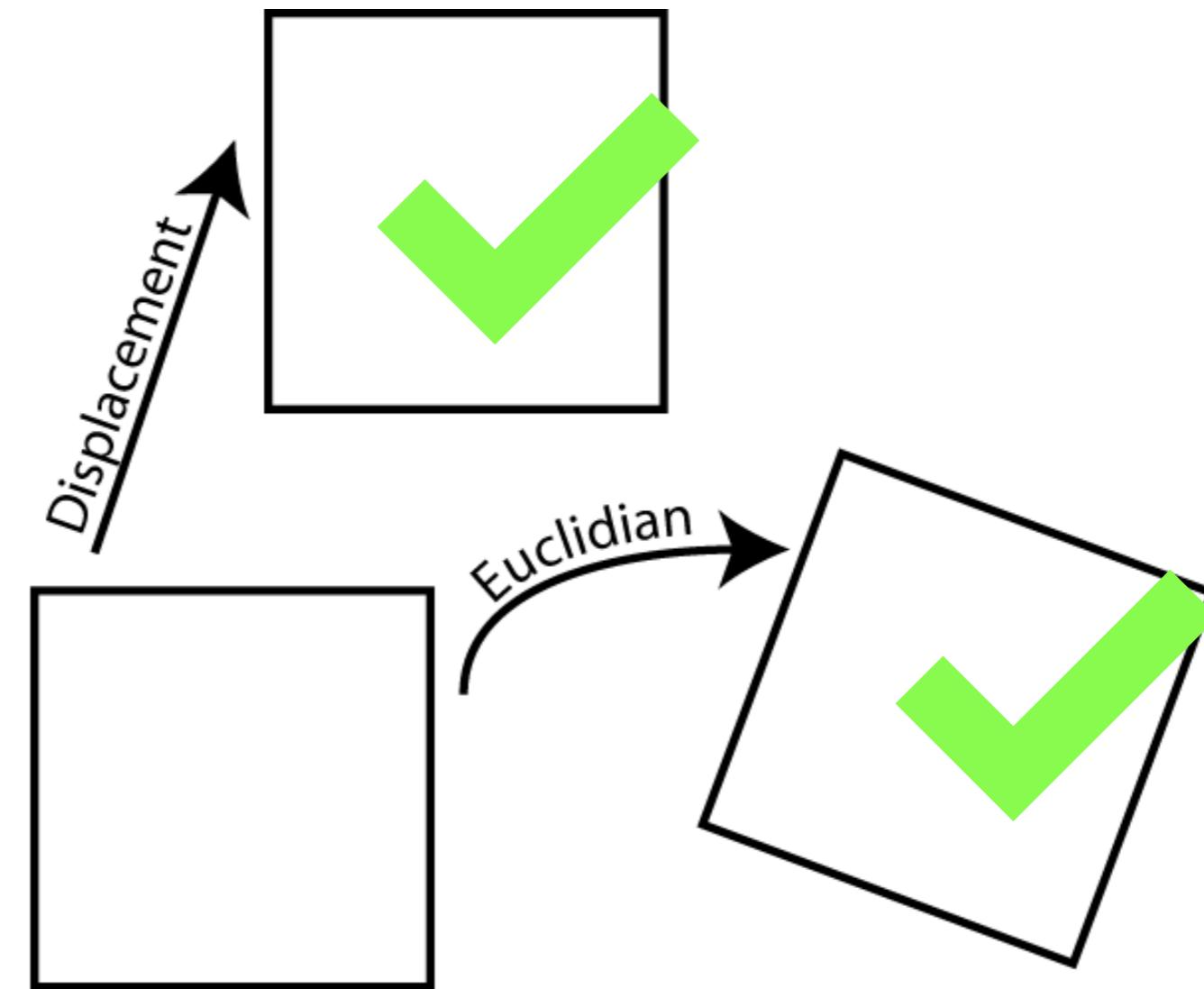
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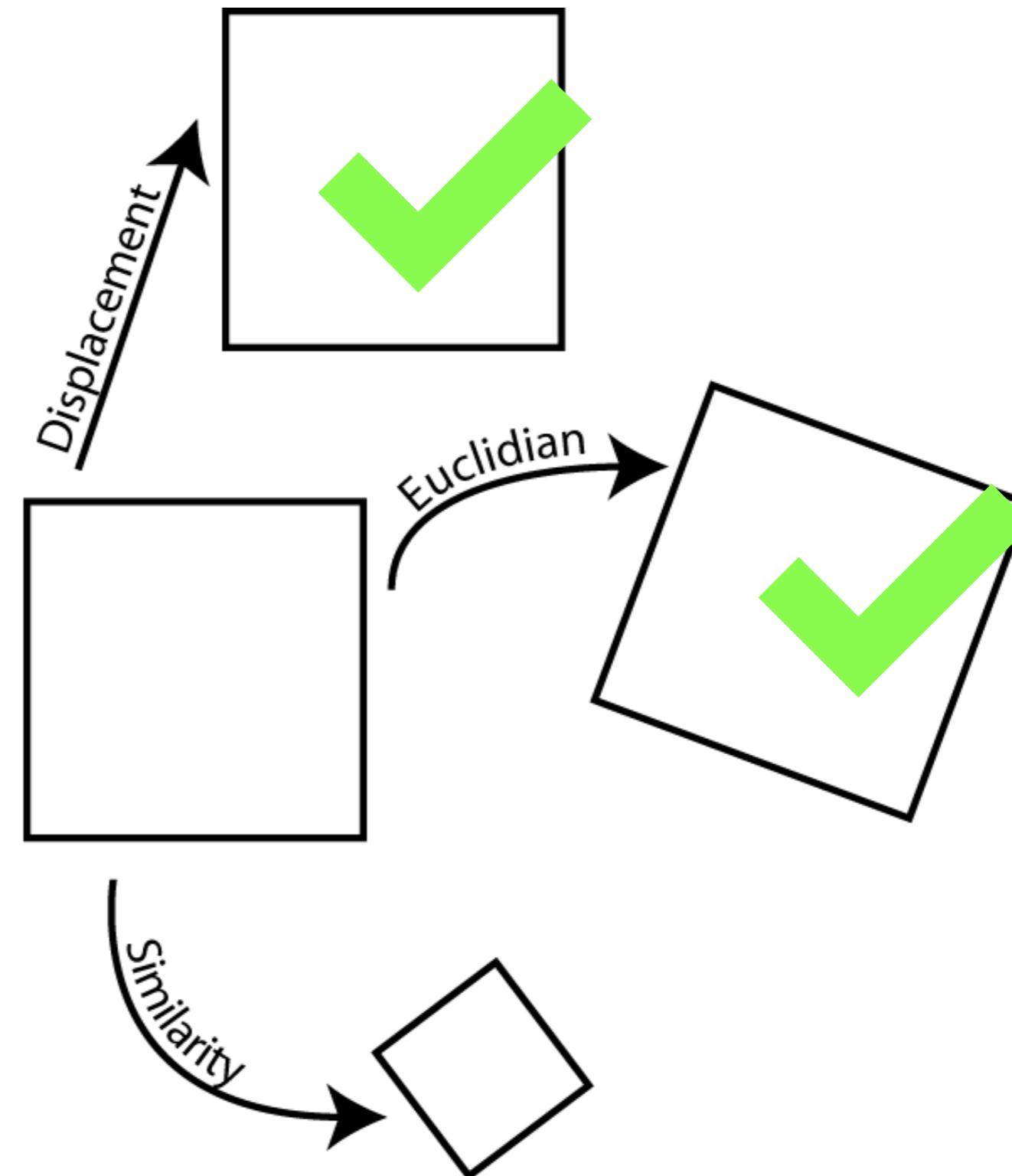
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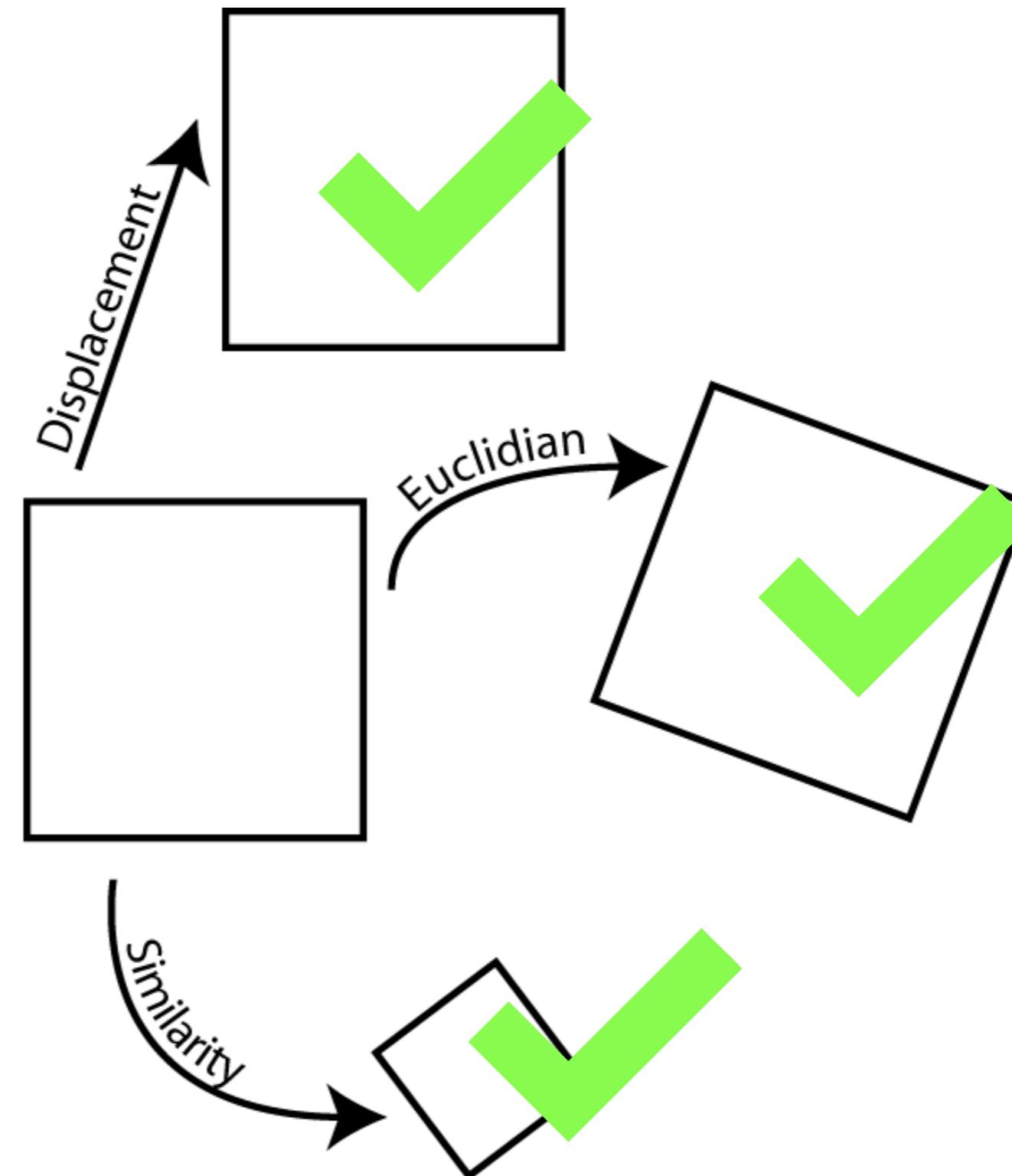
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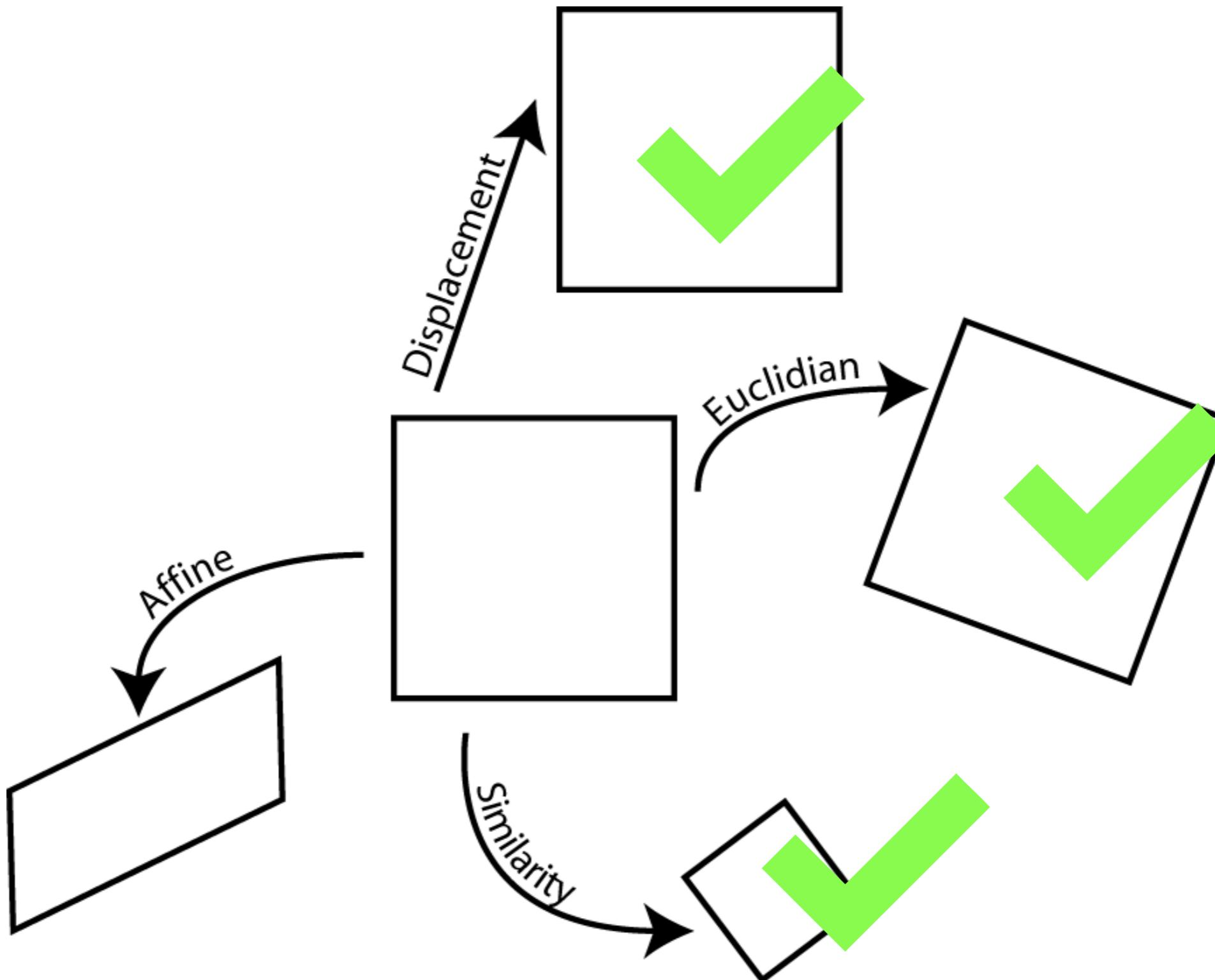
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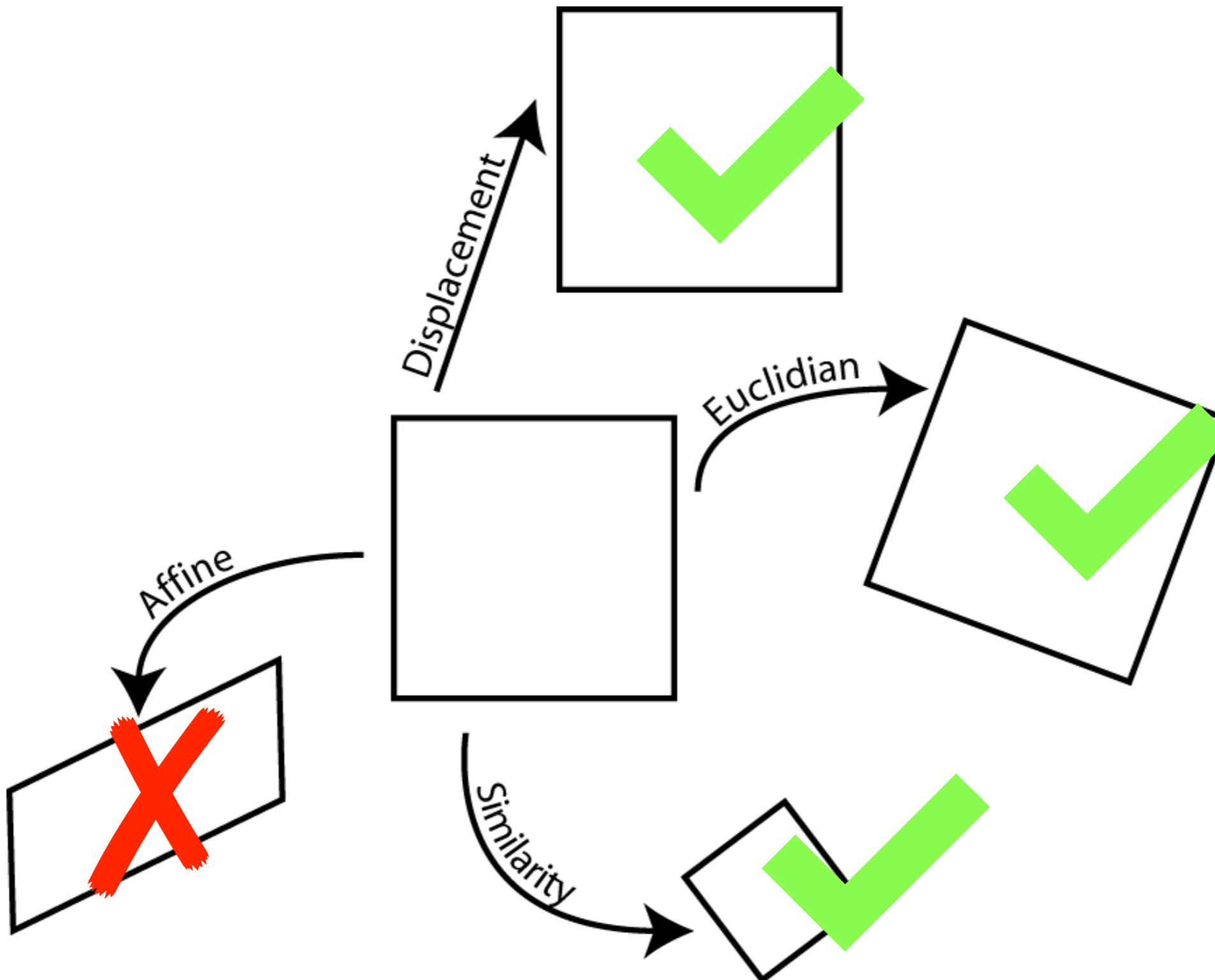
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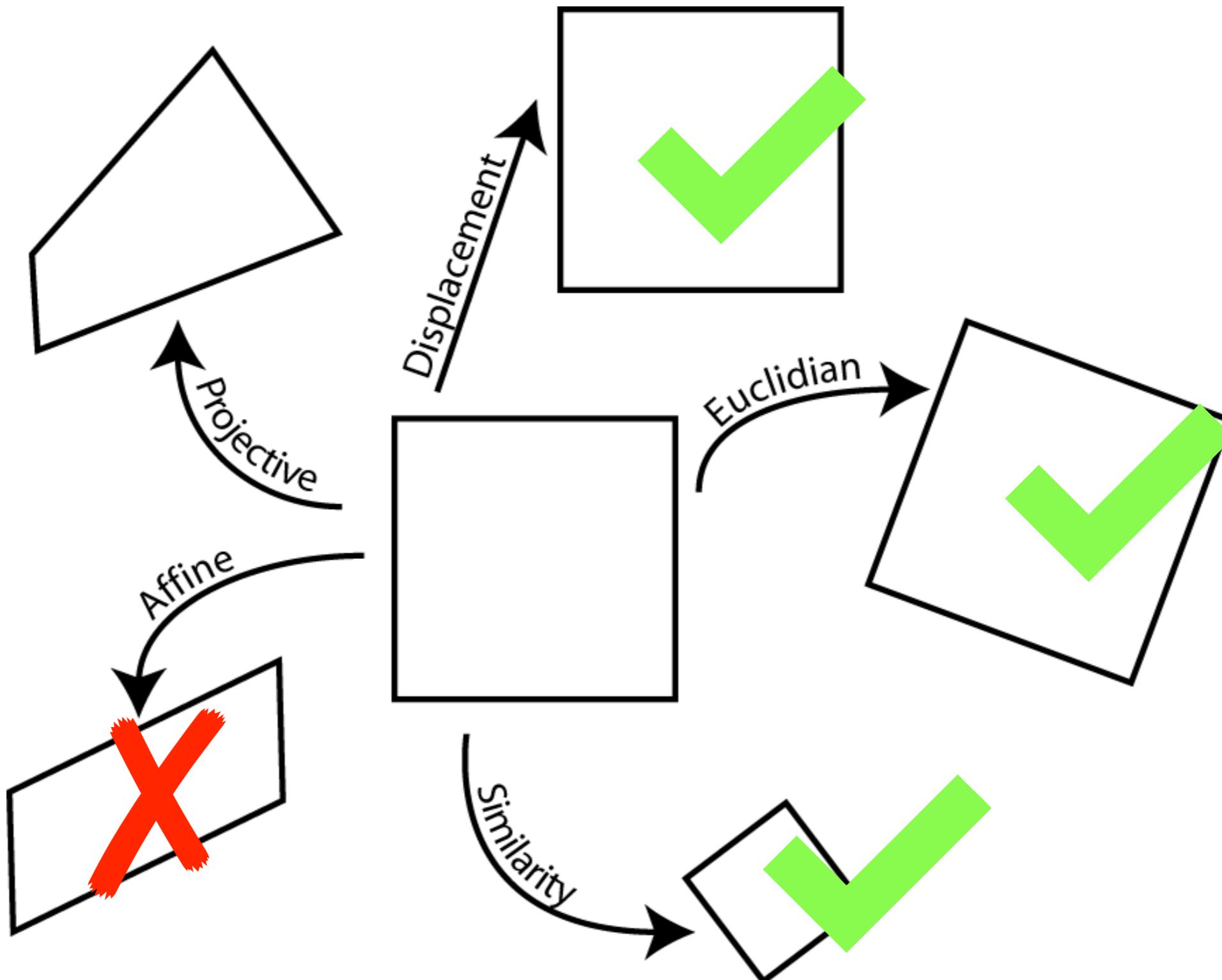
slide credit: Marc Pollefeys

# Patch Transformations



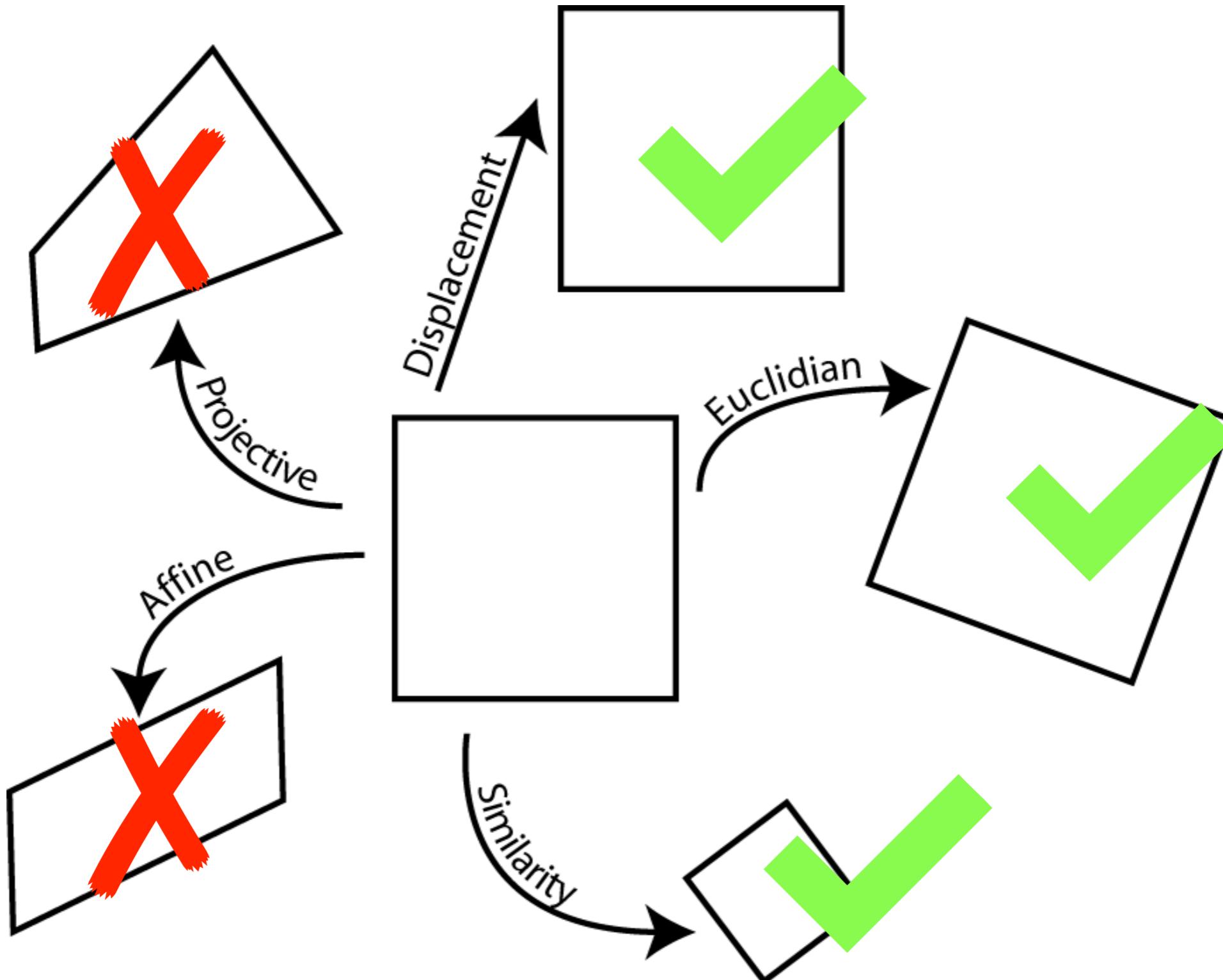
slide credit: Marc Pollefeys

# Patch Transformations



slide credit: Marc Pollefeys

# Patch Transformations



slide credit: Marc Pollefeys

# Projective Transformations



Affine transformations can locally approximate perspective transformations quite well

slide credit: Marc Pollefeys, Kevin Köser

# Affine Invariant Features

- Detect regions rather than points
- Fit ellipse into regions
- Warp ellipse to circle (= remove affine distortion)
- Compute SIFT descriptor (including orientation assignment)
- Example: MSER detector [Matas et al., Robust Wide Baseline Stereo from Maximally Stable Extremal Regions, BMVC 2002]

# Maximally Stable Extremal Regions (MSER)

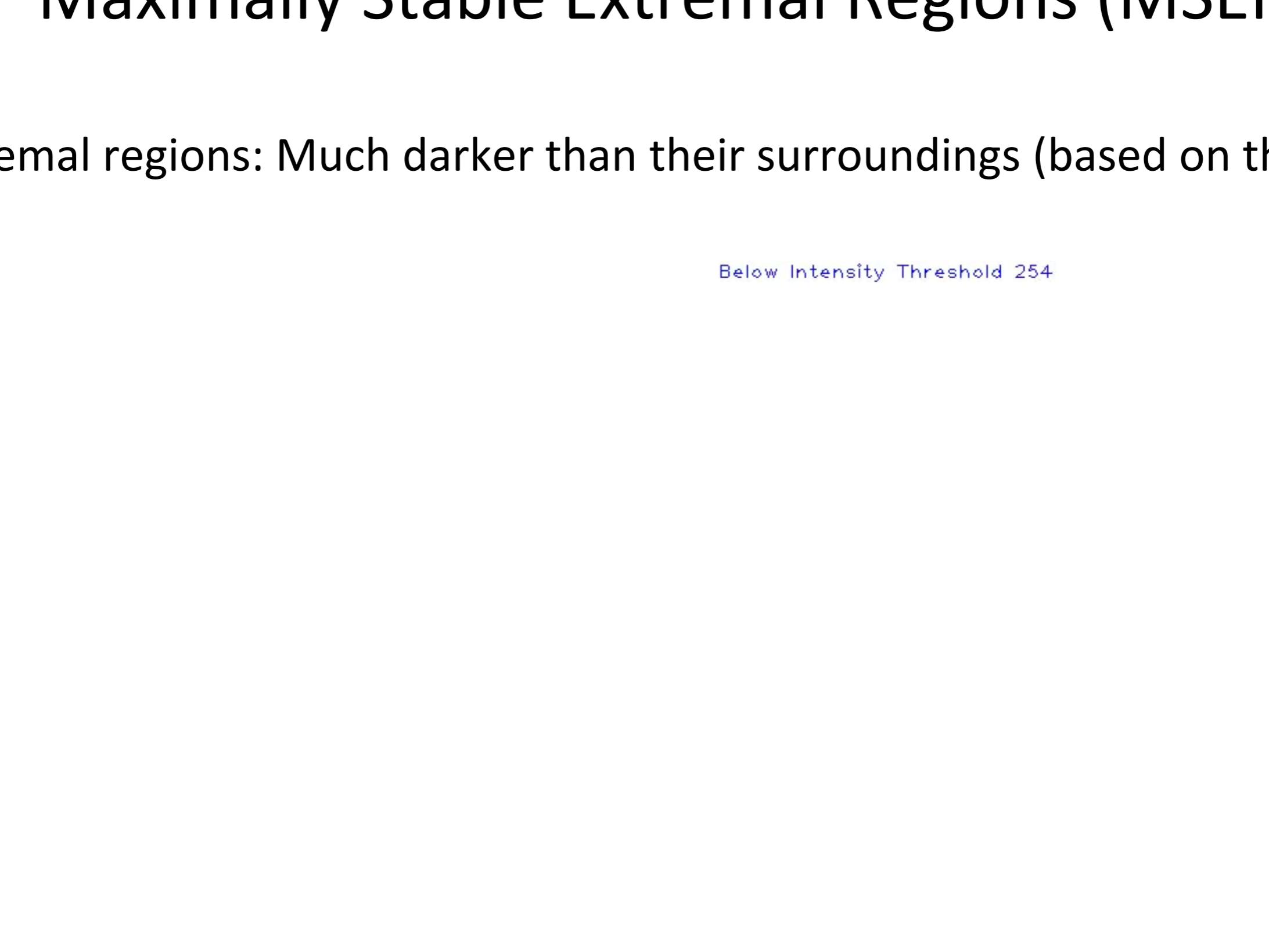
Extremal regions: Much brighter than their surroundings (based on thresholding)

Above Intensity Threshold 0.00

# Maximally Stable Extremal Regions (MSER)

Extremal regions: Much darker than their surroundings (based on threshold)

Below Intensity Threshold 254



# Maximally Stable Extremal Regions (MSER)

Maximally stable regions: Connected components whose size does not change around a threshold



# From Regions to Normalized Patches



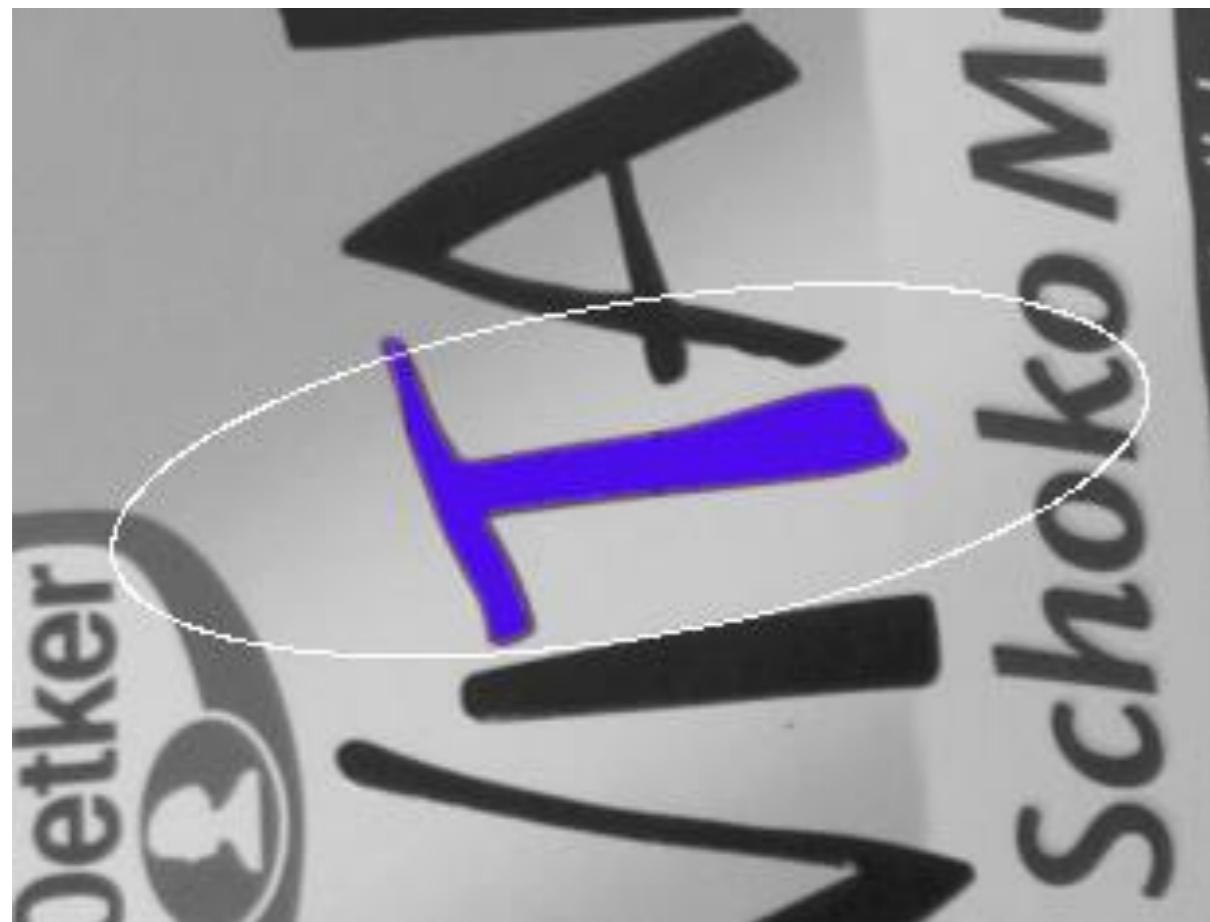
Fit ellipse into region (via PCA)

# From Regions to Normalized Patches



Fit ellipse into region (via PCA)

# From Regions to Normalized Patches



Warp ellipse to unit circle

# From Regions to Normalized Patches



Rotate into canonical coordinate system  
based on dominant orientation

# Lessons Learned

- Main lessons from this lecture
  - SIFT feature detector: Difference of Gaussians over scale space
  - Extraction of oriented and scale-dependent patches
  - SIFT feature descriptor: Concatenation of gradient histograms
- Next lecture: Learning classifiers

# Last Lecture

Jan. 20	Introduction, Linear classifiers and filtering	
Jan. 23	Filtering, gradients, scale	Lab 1
Jan. 27	Local features	
Jan. 30	<b>Learning a classifier</b>	
Feb. 3	Convolutional neural networks	Lab 2
Feb. 6	More convolutional neural networks	
Feb. 10	Robust model fitting and RANSAC	
Feb. 13	Image registration	Lab 3
Feb. 17	Camera Geometry	
Feb. 20	More camera geometry	
Feb. 24	Generative neural networks	
Feb. 27	Generative neural networks	
Mar. 2	TBA	
Mar. 9	TBA	