ESS101- Modeling and Simulation
Lecture 13

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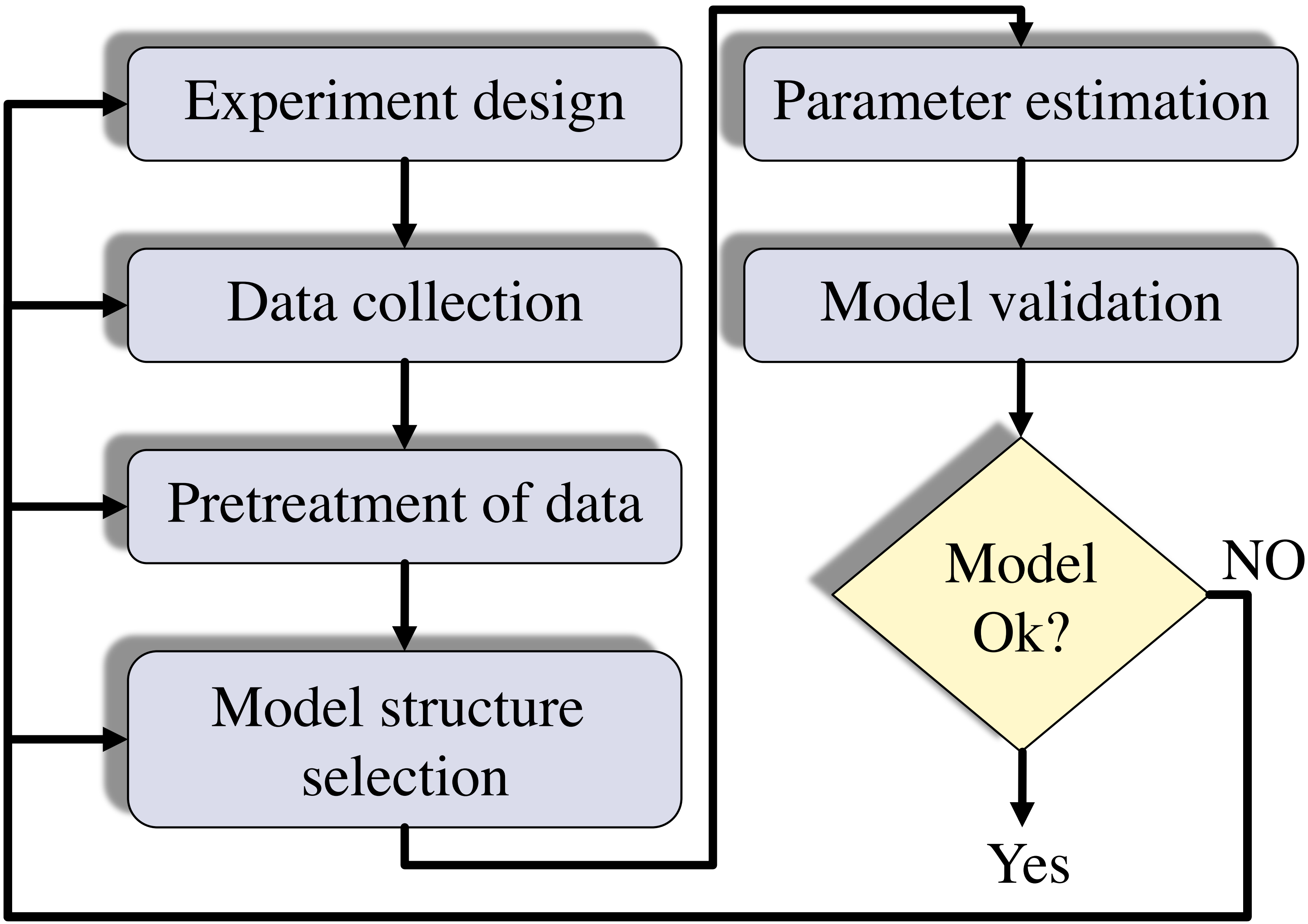
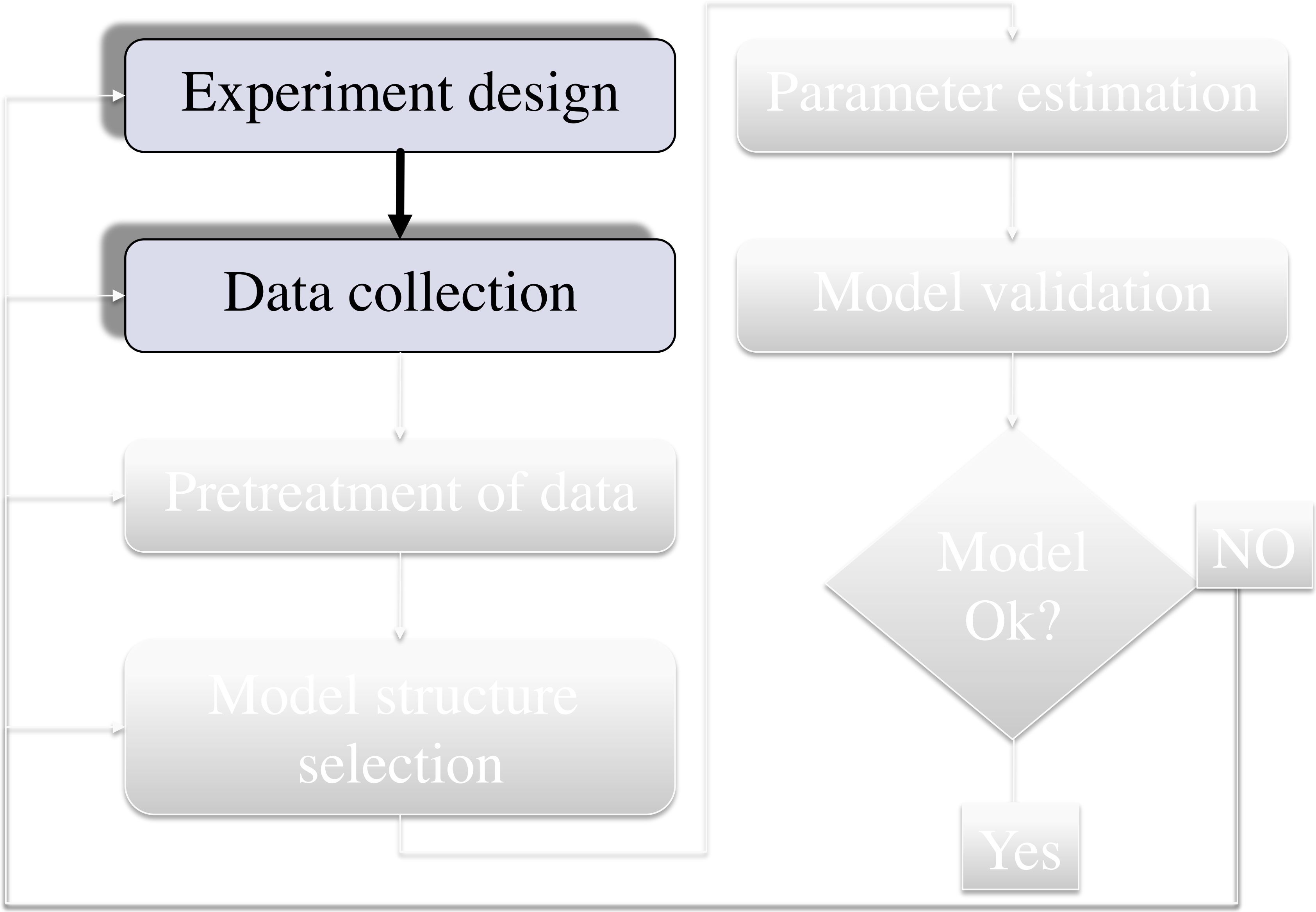
Today (Chapter 10)

- ☞ Working principle in parametric identification
  - ☞ Experiment design
  - ☞ Pretreatment of data
  - ☞ Model structure selection
  - ☞ Model validation

Working principle



Working principle



Experiment design

- Choice of the input signal (if possible)*
- Detection of feedback*
- Sampling time

* Data can be collected only under normal operation of the plant

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Experiment design. The input signal

Criterion: excite all the features of the system (see Excitation on Lecture 9). E.g.,

- steady state behavior
- transient response
- response to different frequencies

Telegraph signal. A signal randomly switching between *two levels*.

Q1: how to select the two levels?

Q2: how to select the switching frequency?

Experiment design. The input signal

Q1: how to select the two levels?

- Usual input levels
- Maximum allowed signals levels
- Nonlinearities
  - Input signals levels within a range

Q2: how to select the switching frequency?

- Frequency within the system bandwidth
- Let the system get to steady state

Experiment design. The input signal. Procedure

1. (If possible) Get a step response

2. Calculate the system bandwidth through, e.g., the settling time

$$T_a^1 \cong \frac{3}{\omega_B} \qquad T_a^2 \cong \frac{3}{\xi\omega_B}$$

3. Set the maximum switching frequency within the system bandwidth

4. Hold the input constant for at least the settling time

Experiment design

✓ Choice of the input signal (if possible)*

- Detection of feedback*
- Sampling time

* Data can be collected only under normal operation of the plant

Experiment design. Feedback

Consider the system

$$y(t) + ay(t-1) = bu(t-1) + e(t)$$

under the feedback control law

$$u(t) = -fy(t)$$

The corresponding predictor is:

$$\hat{y}(t,\vartheta) = \underbrace{\left(-bf - a\right)}_{\vartheta} y(t-1)$$

$\exists \infty\ \hat{a}, \hat{b} : \ -\hat{b}f - \hat{a} = \hat{\vartheta}$ *identifiability issues*

Experiment design. Feedback

Define the input as

$$u(t) = f\left(r(t) - y(t)\right)$$

The corresponding predictor is:

$$\hat{y}(t \mid \vartheta) = \left(-bf - a\right)y(t-1) + br(t-1)$$

Observe that spectral analysis does not work with feedback

Experiment design

- ✓ Choice of the input signal (if possible)*
- ✓ Detection of feedback*
- ☝ Sampling time

* Data can be collected only under normal operation of the plant

Experiment design. The sampling time

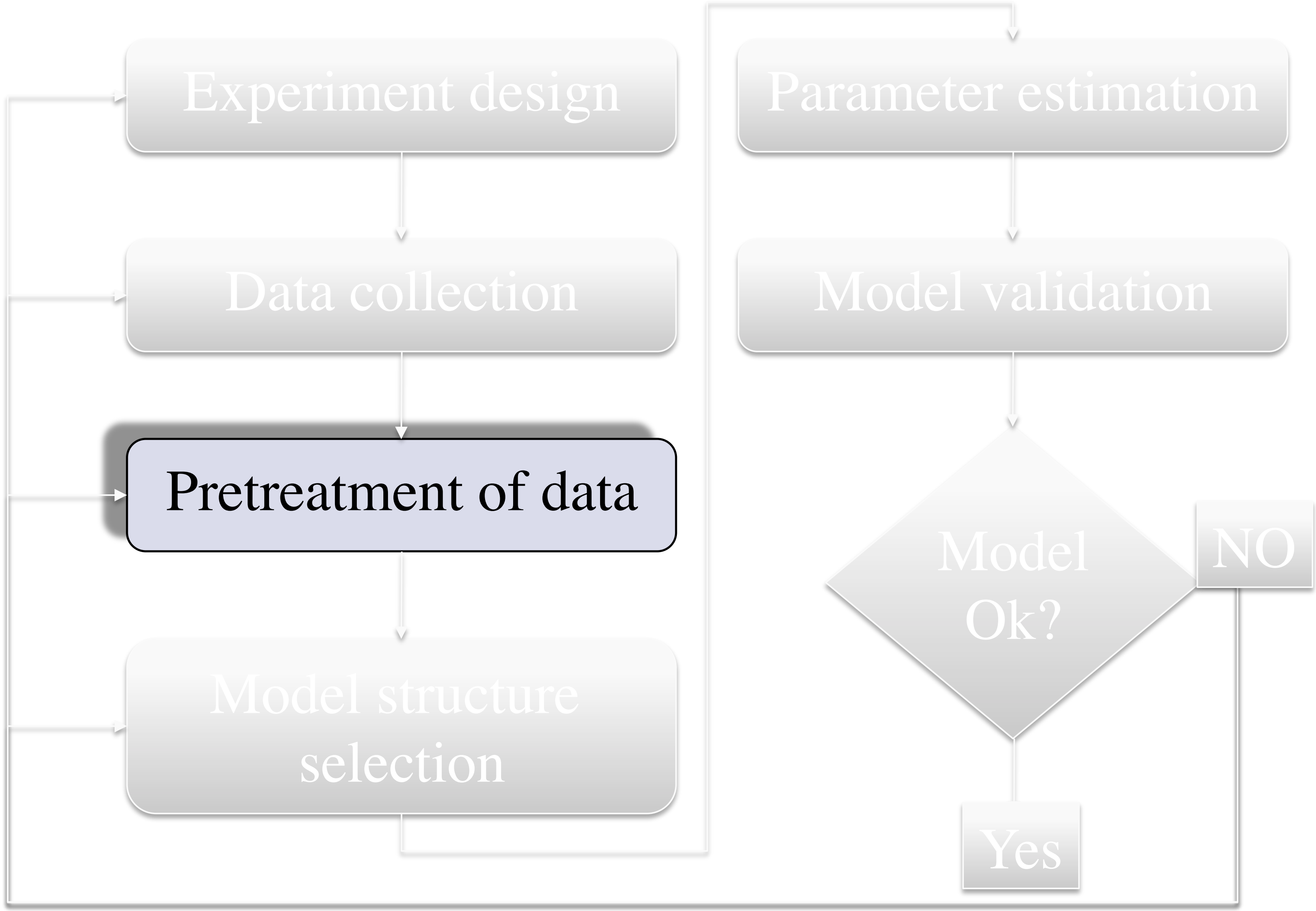
- **Evaluate the step response.** Chose the sampling time as 1/10 of the rise time.

- **Sampling frequency greater than the Nyquist frequency:**

$$\boxed{\omega_s > 2\omega_c}$$

- **Aliasing.** Before sampling, low pass filter the signal with cut-off frequency at the Nyquist frequency.

Working principle



Pretreatment of data

- Subtract the mean values and drift
  - If data is collected at steady state
- Outliers
  - Wrong data due to, e.g., spikes
- Low pass filter (anti-aliasing filter)
  - High frequency components due to not well designed anti-aliasing filters

Matlab command:
`detrend`

Model structure selection

☞ Tailor made models

Pro:

✌ Minimal set of parameters

✌ High quality model

Cons:

✗ Physical modeling efforts required

✗ Computationally expensive numerical optimization

☞ Ready made models

Pro:

✌ No additional modeling effort

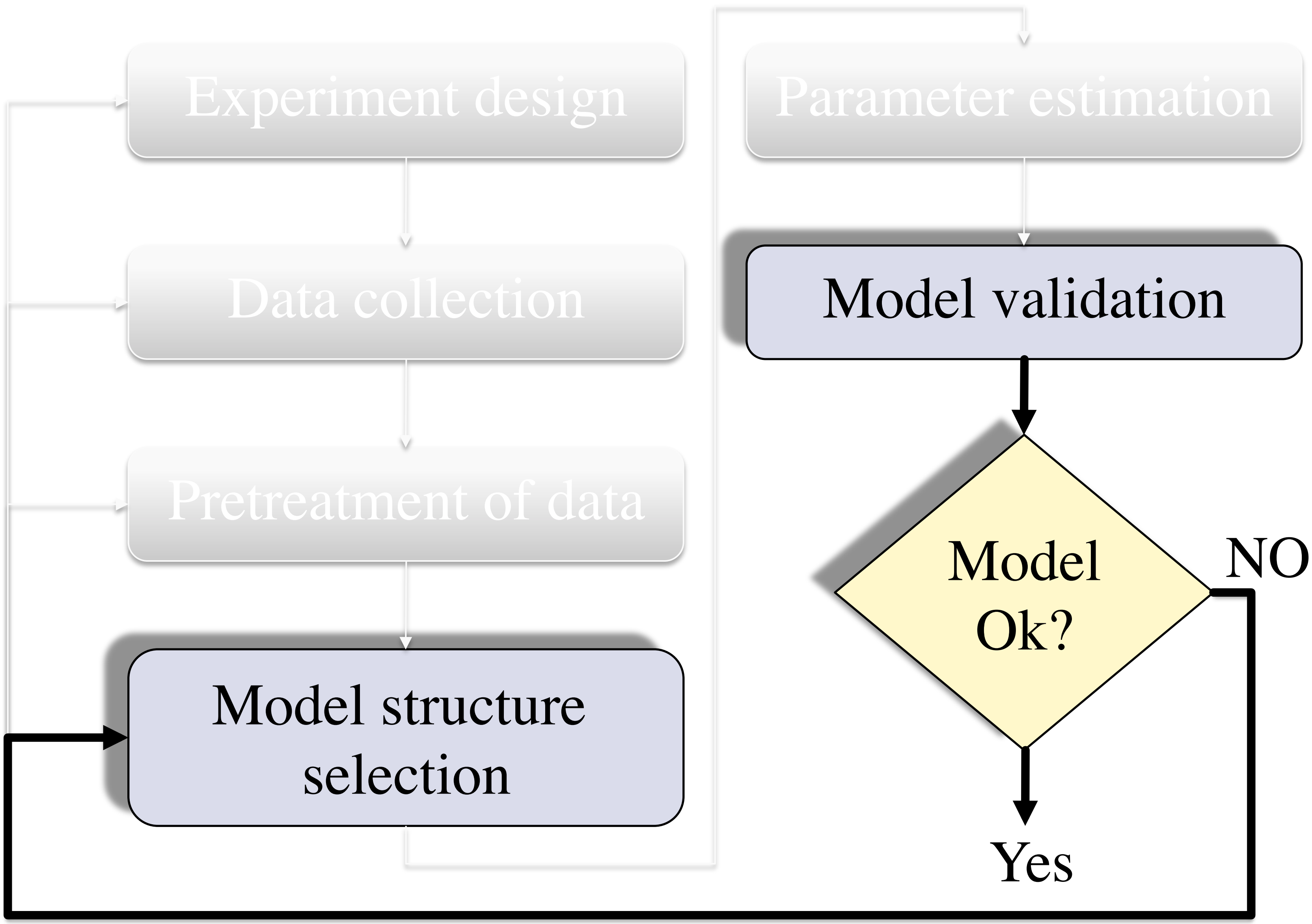
✌ Computationally efficient (ARX)

Cons:

✗ Bias and variance

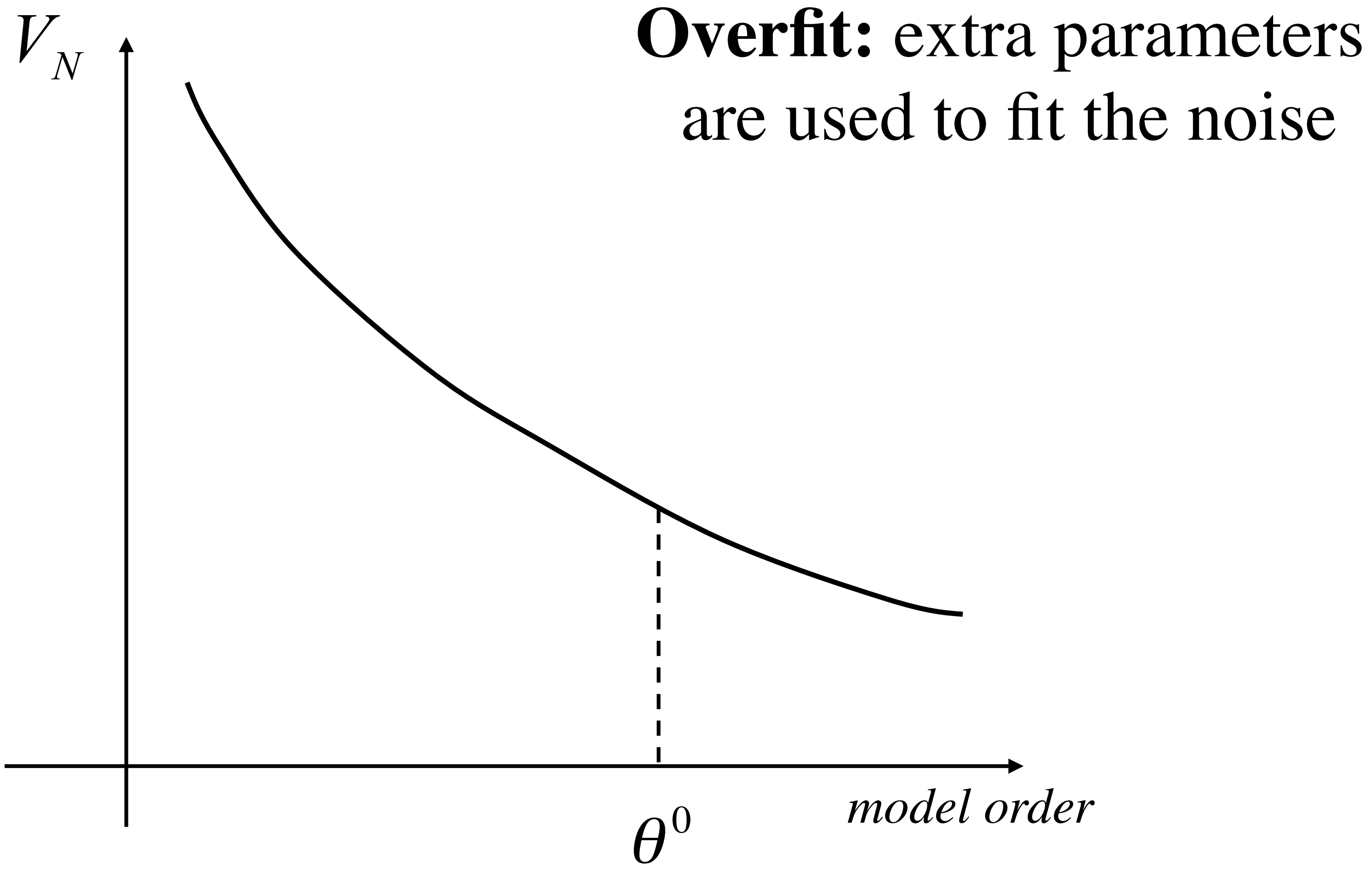
Conclusion. Start with the simplest (ARX) and complicate if needed.

Working principle



Model structure selection

Cross validation. Test the model by using fresh data



Model structure selection

Reformulate the identification problem as:

$$\min_{d,\theta} f(d,\theta)\sum_{t=1}^{N}\varepsilon^2(t,\theta) \quad \text{with} \quad d=\dim(\theta) \quad \text{and}$$

Akaike's Information Criterion (AIC):

$$f(d,\theta)=1+\frac{2d}{N}$$

Final Prediction Error (FPE):

$$f(d,\theta)=\frac{1+d/N}{N(1-d/N)}$$

Moreover compare the frequency responses

Model evaluation. Residual analysis

Evaluate the residuals:

$$\varepsilon(t) = y(t) - \hat{y}\left(t \mid \hat{\vartheta}_N\right)$$

$$\hat{R}_{\varepsilon u}(\tau) = \frac{1}{N}\sum_{t=1}^{N} \varepsilon(t+\tau)u(t)$$

- Feedback.

$$\hat{R}_{\varepsilon u}(\tau) \neq 0, \ \tau < 0$$

- Delay and estimate of n_b $\quad \hat{R}_{\varepsilon u}(\tau_0) \neq 0$