Bonus Assignment 4 – Linear Regression

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1. Estimation of Parameters

In this problem, the equation hypothesis is assumed as below, in which *y* is the logarithm of pressure, *x* is the reciprocal of temperature. *A* and *B* are parameters to be estimated.

$$y = A + Bx \qquad \text{Eq. 1}$$

At first, some statistics are computed, such as standard deviations of *x* and *y* respectively as well as the covariance between *x* and *y*. The coefficient of determination r can be obtained as Equation 2.

S_x = 1.5662e-04 S_y = 3.3317 S_{xy} = -5.2150e-04

$$r = \frac{S_{xy}}{S_x S_y} = -0.9994 \qquad \text{Eq. 2}$$

Now, two parameters *A* and *B* can be estimated as Equation 3 and 4, in which $\bar{x}$ and $\bar{y}$ are sample mean that calculated from data.

$$\hat{B} = \frac{r \cdot S_y}{S_x} = \mathbf{-21259.69} \qquad \text{Eq. 3}$$

$$\hat{A} = \bar{y} - \hat{B}\bar{x} = \mathbf{18.18} \qquad \text{Eq. 4}$$

Before compute the 95% confident intervals for two estimated parameters, the standard error of residuals should be got first. Equation 5 shows the method to estimate this standard error. *n* is the number of elements in dataset, which is 32 in this case.

$$S = \sqrt{\frac{n-1}{n-2} S_y^2 (1 - r^2)} = 0.1164 \qquad \text{Eq.5}$$

We have everything in Equation 6 and 7 to compute the standard errors for two estimated parameters.

$$S_{\hat{A}} = \frac{S\sqrt{\sum x_i^2}}{S_x\sqrt{n(n-1)}} = \mathbf{0.14} \qquad \text{Eq.6}$$

$$S_{\hat{B}} = \frac{S}{S_x\sqrt{n-1}} = \mathbf{133.45} \qquad \text{Eq.7}$$

95% confident intervals for two parameters can be generated by checking the table of t distribution at the degree of freedom (*n* - 2).

95% CI for $\hat{A}$: **18.18 ± 0.29**
95% CI for $\hat{B}$: **-21259.69 ± 272.55**

2. Plot Estimation Result

The estimated regression line is shown in Figure 1 with original measurements. Figure 2 displays the residuals of all elements in dataset. Residuals are around the horizontal zero line. It is obvious that the performance of estimation is quite good.

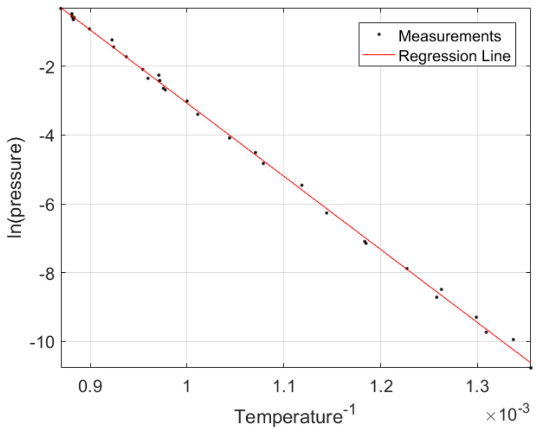

Figure 1. Regression Line of Estimated Parameters

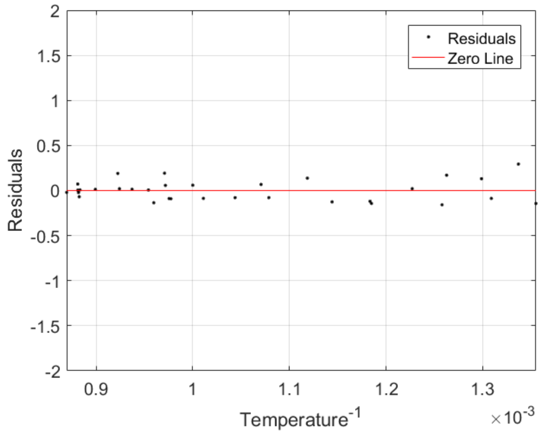

Figure 2. Residuals between Measurements and Estimations

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