

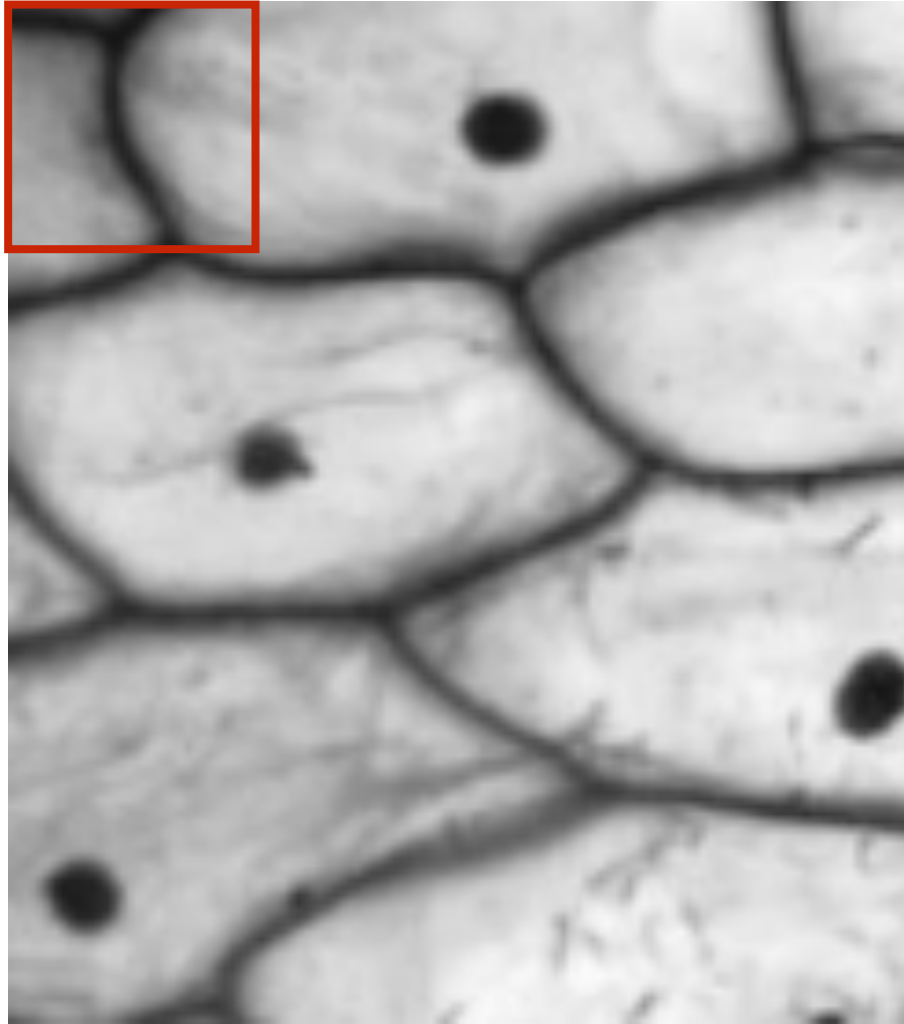
SSY097 - Image Analysis

Lecture 2 - Filtering, gradients and scale

Torsten Sattler

(slides adapted from Olof Enqvist)

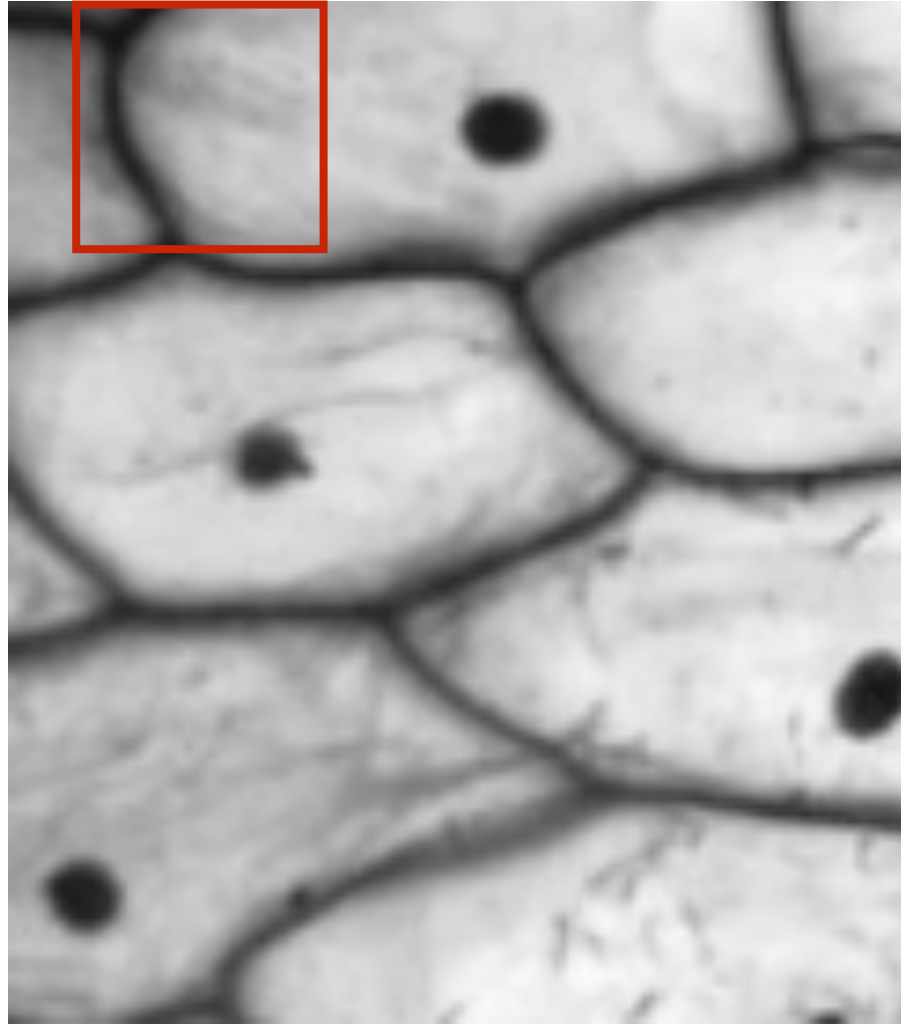
Last Lecture



$$\cdot w < \tau$$

Sliding window classification

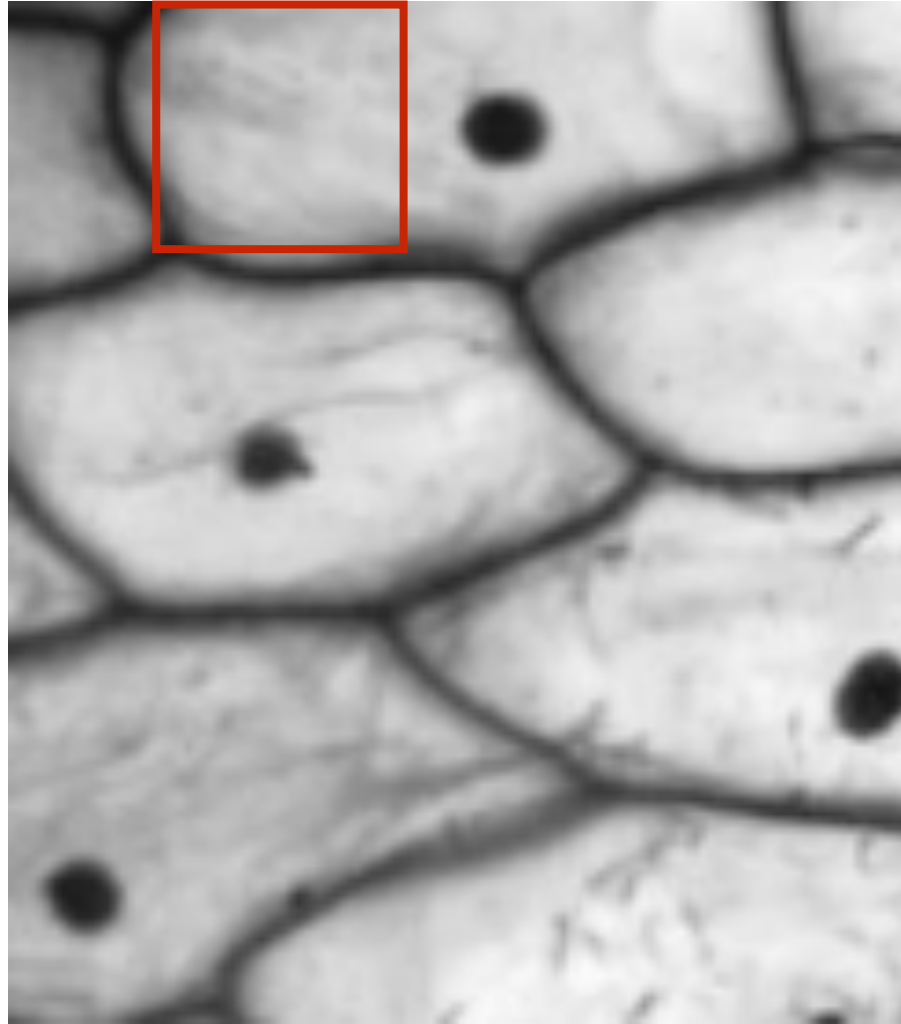
Last Lecture



$$\cdot w < \tau$$

Sliding window classification

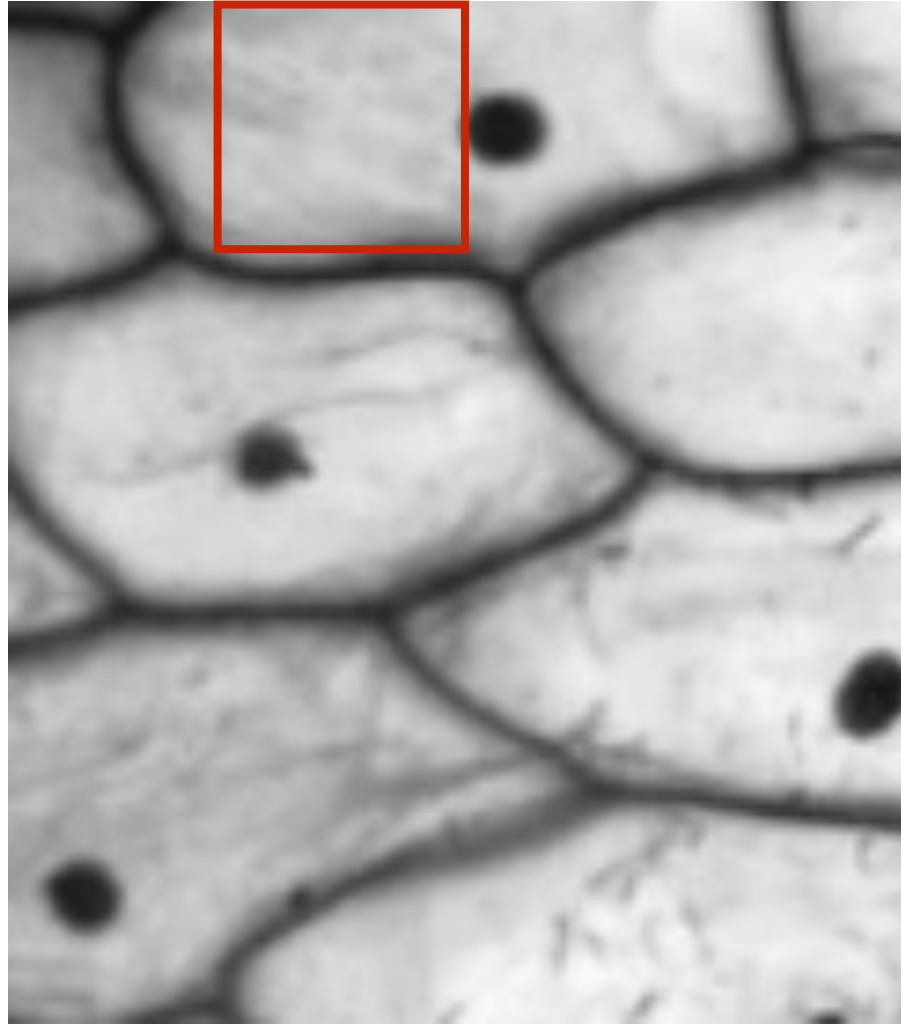
Last Lecture



$$\cdot w < \tau$$

Sliding window classification

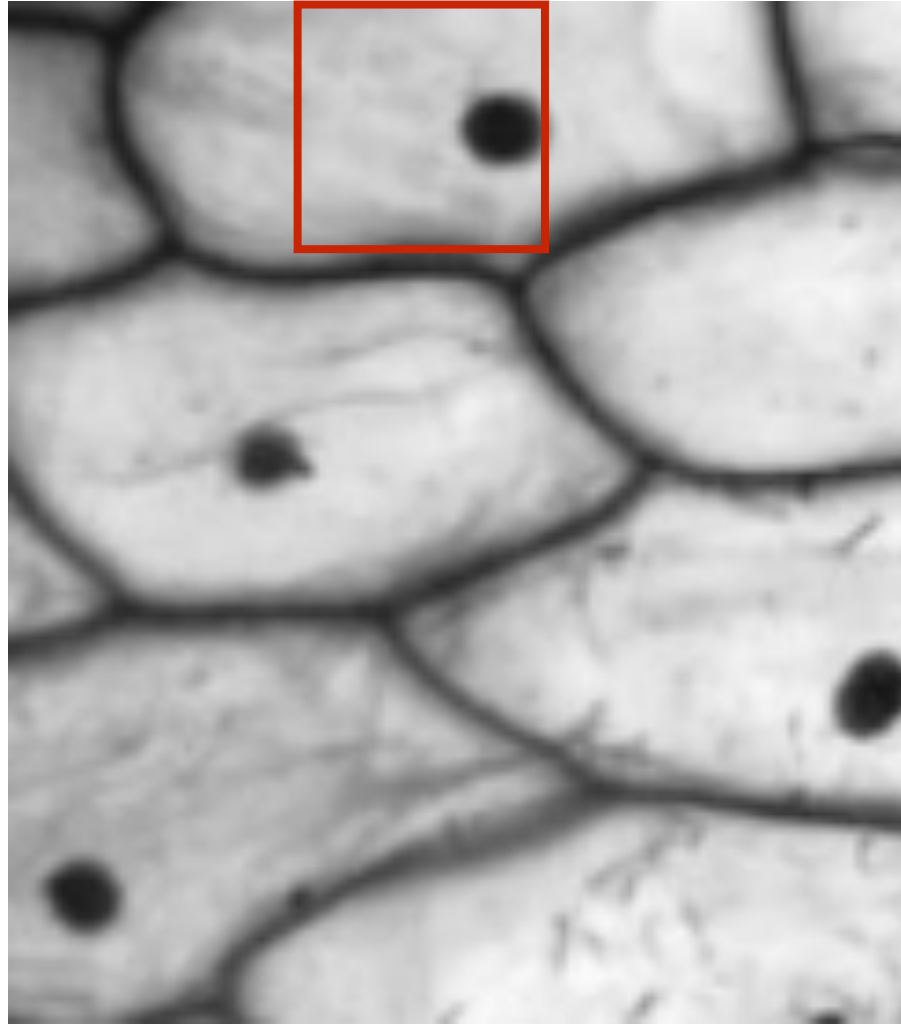
Last Lecture



$$\cdot w < \tau$$

Sliding window classification

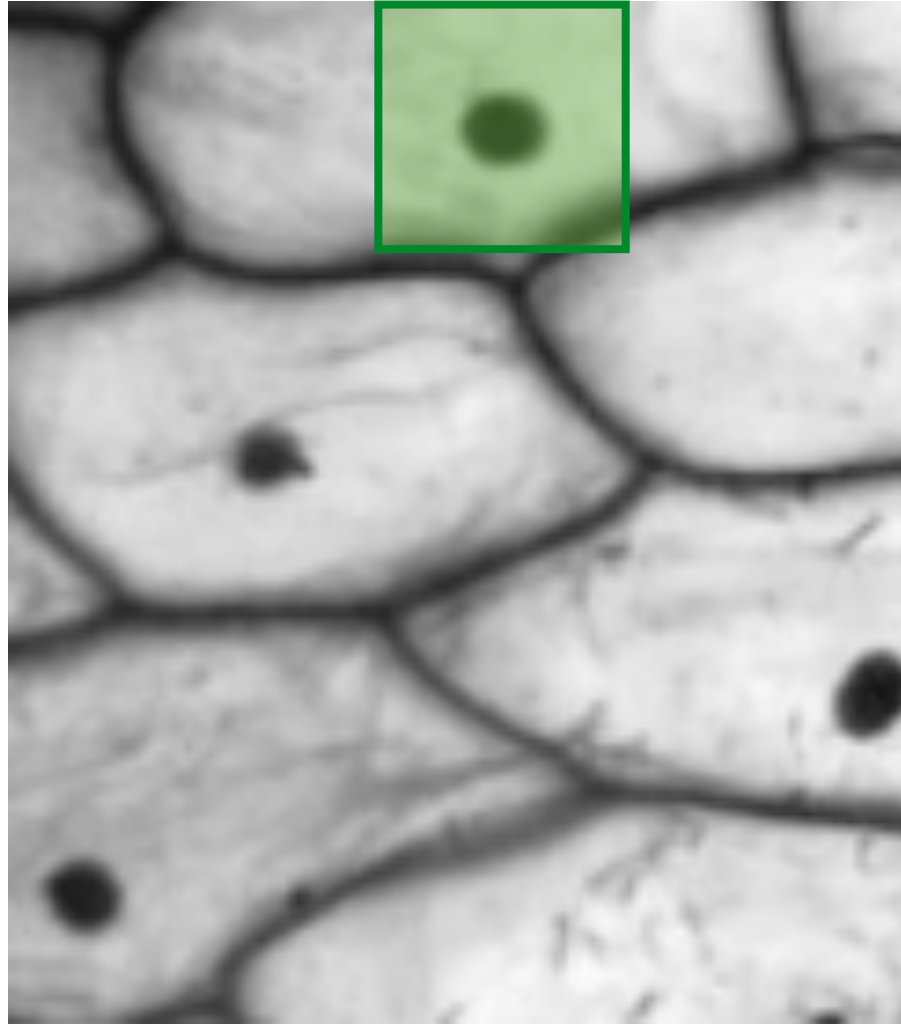
Last Lecture



$$\cdot w < \tau$$

Sliding window classification

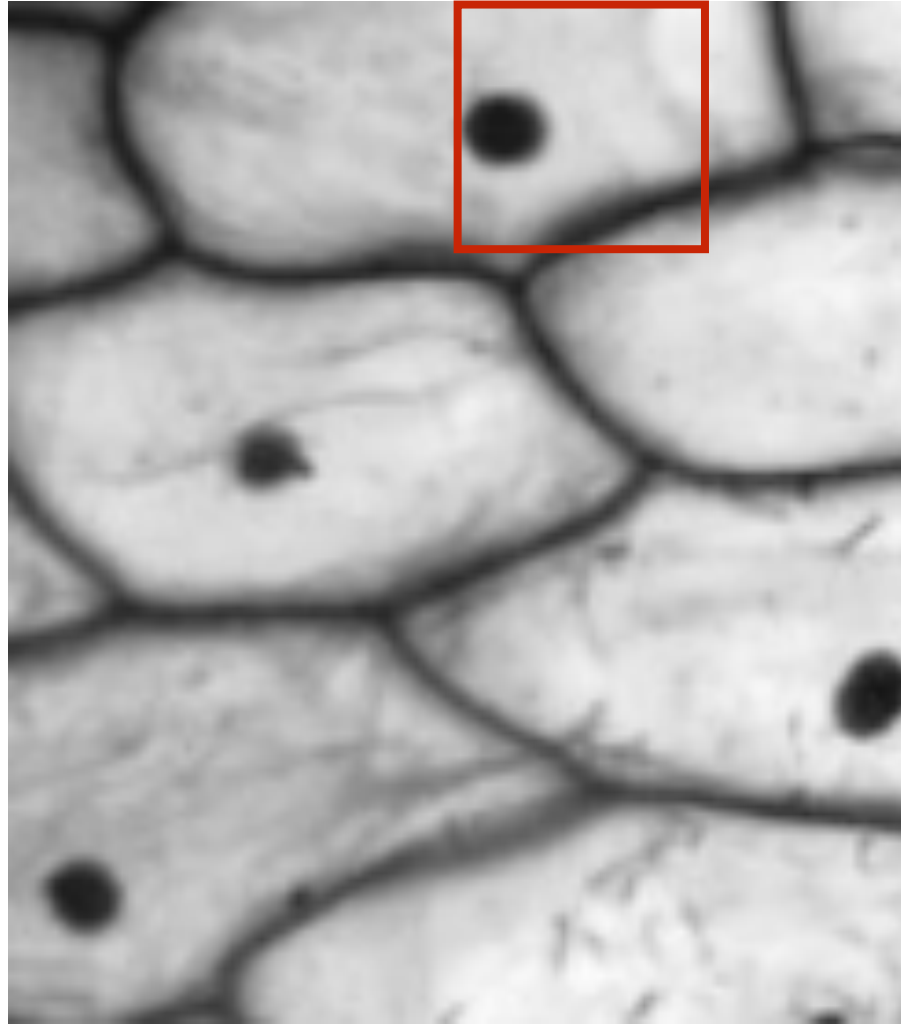
Last Lecture



$$\cdot w > \tau$$

Sliding window classification

Last Lecture

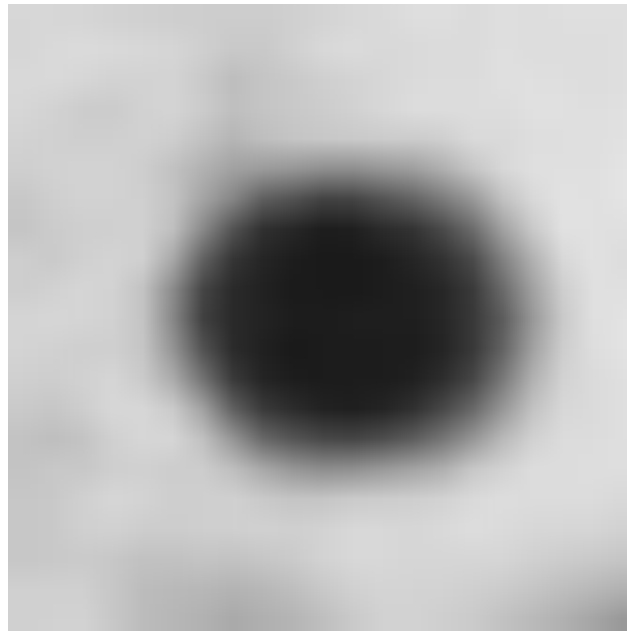


$$\cdot w < \tau$$

Sliding window classification

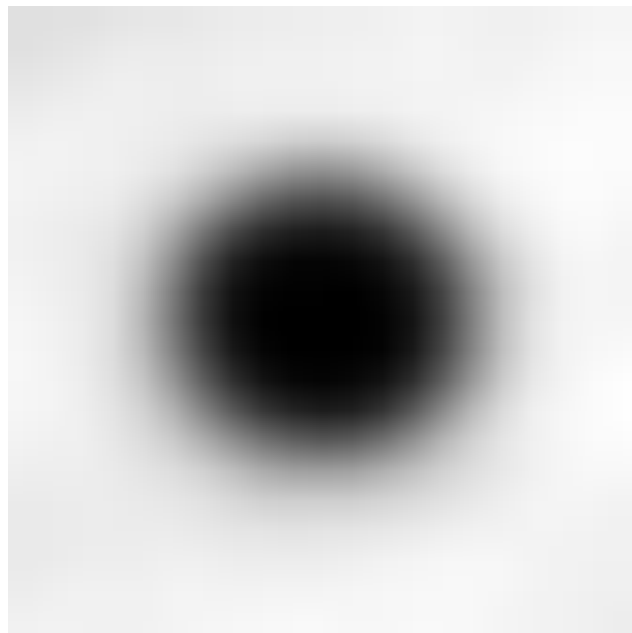
Last Lecture

$$w \cdot \text{img} > \tau$$

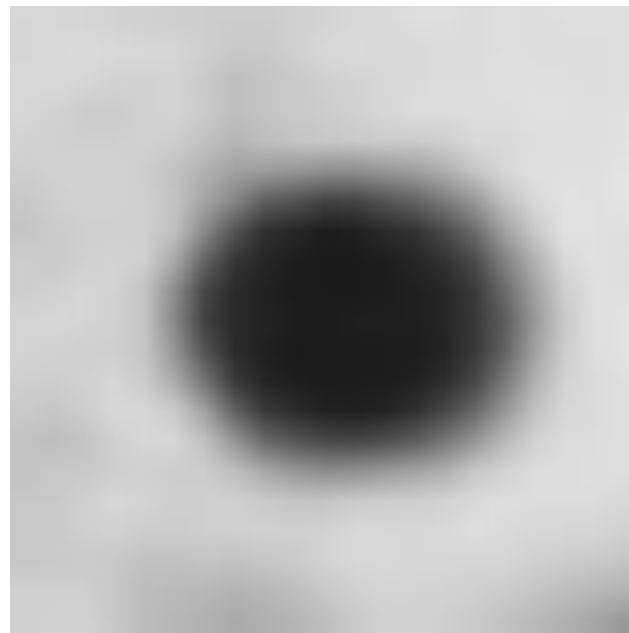


Linear filters

Last Lecture



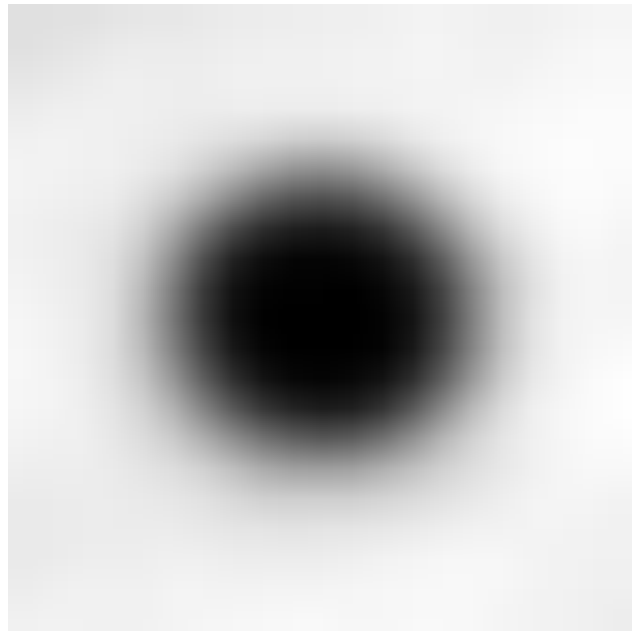
•



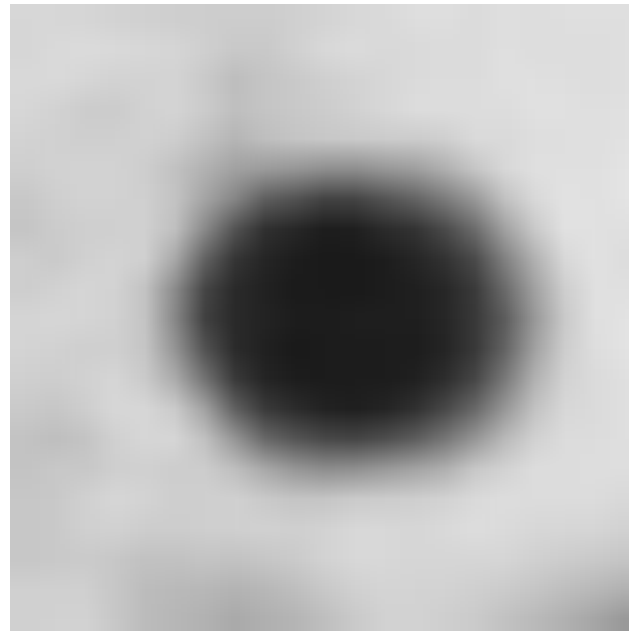
$> \tau$

Linear filters

Last Lecture



•



$> \tau$

$$\frac{1}{N_{\text{elements}}} (T - \mu_T \mathbf{1})$$

Linear filters

Last Lecture

1	2	1	2	3	4	2
11	21	22	21	14	6	7
12	20	45	32	21	12	11
11	12	11	16	21	12	21
21	22	23	25	35	22	20
12	11	16	17	16	6	0
0	7	0	21	12	11	0

input

	0	0	0	0	0	
	0	1				

result

Nonlinear filters: Non-maximum suppression

Today

- More Filters
- Similarity Measures
- Multi-Scale Processing

More Filters

Average Filter

$$\frac{1}{9}$$

1	1	1
1	1	1
1	1	1

Average Filter



$$\frac{1}{9}$$

1	1	1
1	1	1
1	1	1

image source: <http://graphics.stanford.edu/data/3Dscanrep/>

Average Filter



$$\frac{1}{9}$$

1	1	1
1	1	1
1	1	1

image source: <http://graphics.stanford.edu/data/3Dscanrep/>

Average Filter



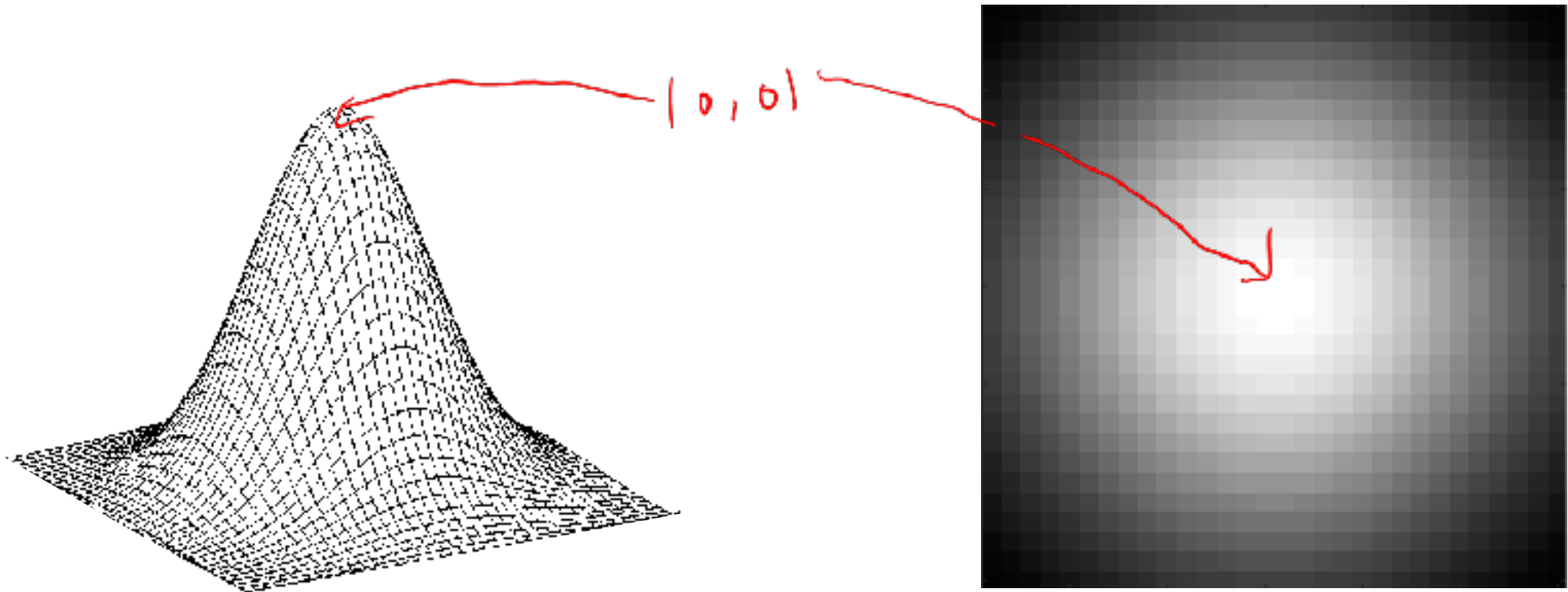
image source: <http://graphics.stanford.edu/data/3Dscanrep/>

Gaussian Filter

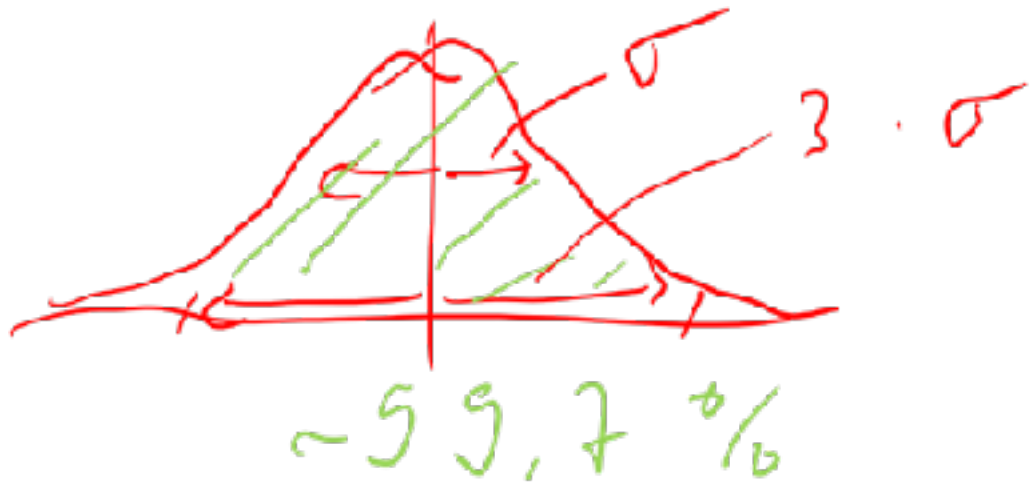
normalization factor

↓

$$a \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right)$$



Gaussian Filter



$$a \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right)$$

$$\frac{1}{273}$$

$$\sigma = 1$$

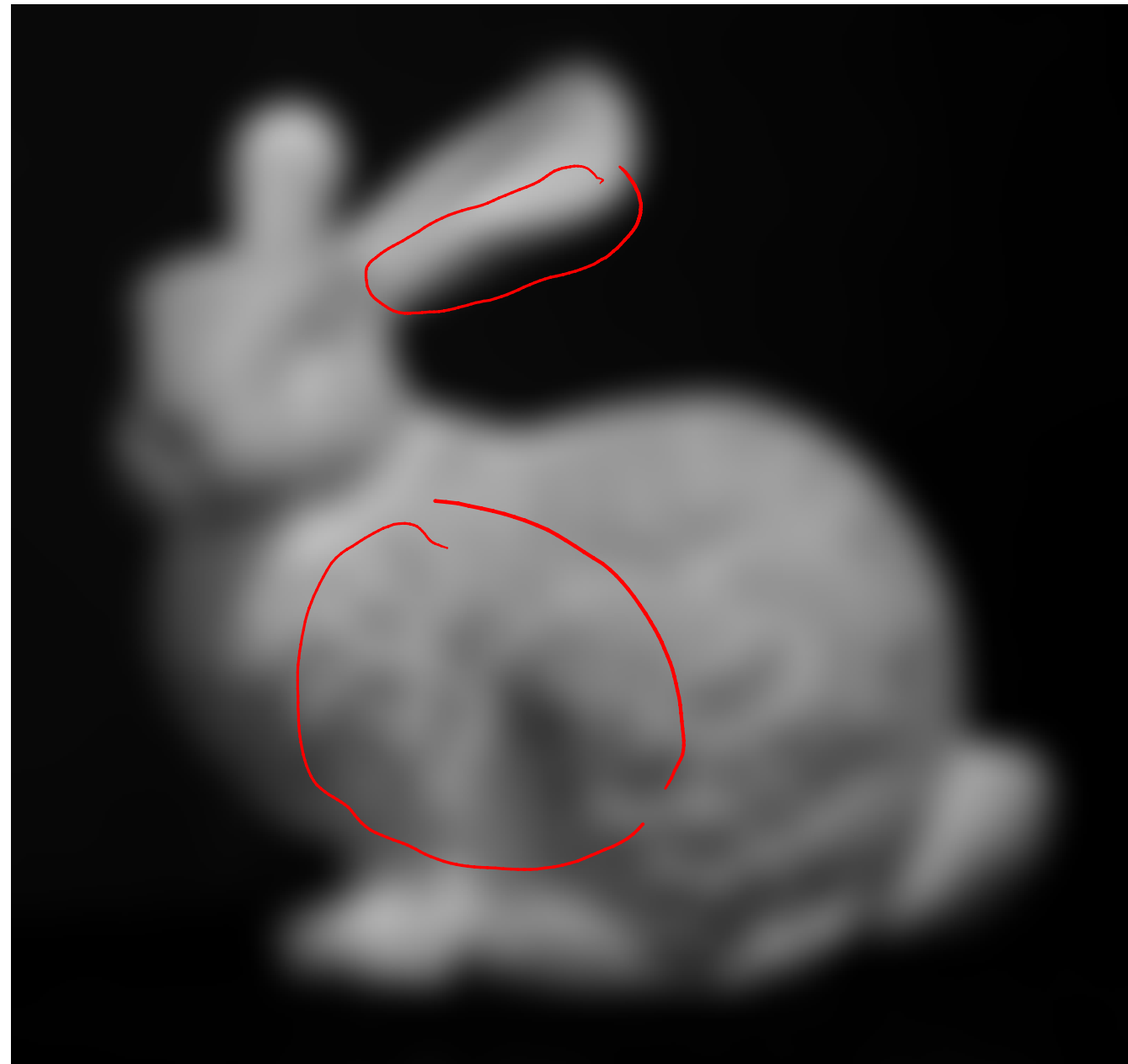
1	4	7	4	1
4	16	26	16	4
7	26	41	26	7
4	16	26	16	4
1	4	7	4	1

filter
size
 $\sim 6\sigma$

Gaussian Filter



Average filter



Gaussian filter

image source: <http://graphics.stanford.edu/data/3Dscanrep/>

Gaussian Filter

Separability

2D Gaussian

1D Filter

$$H \left[\begin{array}{|c|} \hline I \\ \hline \end{array} \right] \times G_{\sigma}(x, y) = \left[\begin{array}{|c|} \hline I \\ \hline \end{array} \right] * G_{\sigma}(y) * G_{\sigma}(x)$$

$w \times h$ $k \times k$ k k

$$w \cdot h \cdot k \cdot k$$

operations

$$k=5: 25$$

Example: $k=10$

>

$$2 \cdot w \cdot h \cdot k$$

operations

$$10 \text{ for } k=5$$

$$w \cdot h \cdot 100 \text{ vs. } w \cdot h \cdot 20$$

Gaussian Filter

Iterative application

$$I * G_{\sigma_1}(x, y) * G_{\sigma_2}(x, y) = I * G_{\sigma_3}(x, y)$$

$$\text{with: } \underline{\sigma_3^2 = \sigma_1^2 + \sigma_2^2}$$

Example:

$$2^2 + 2^2 = 8$$

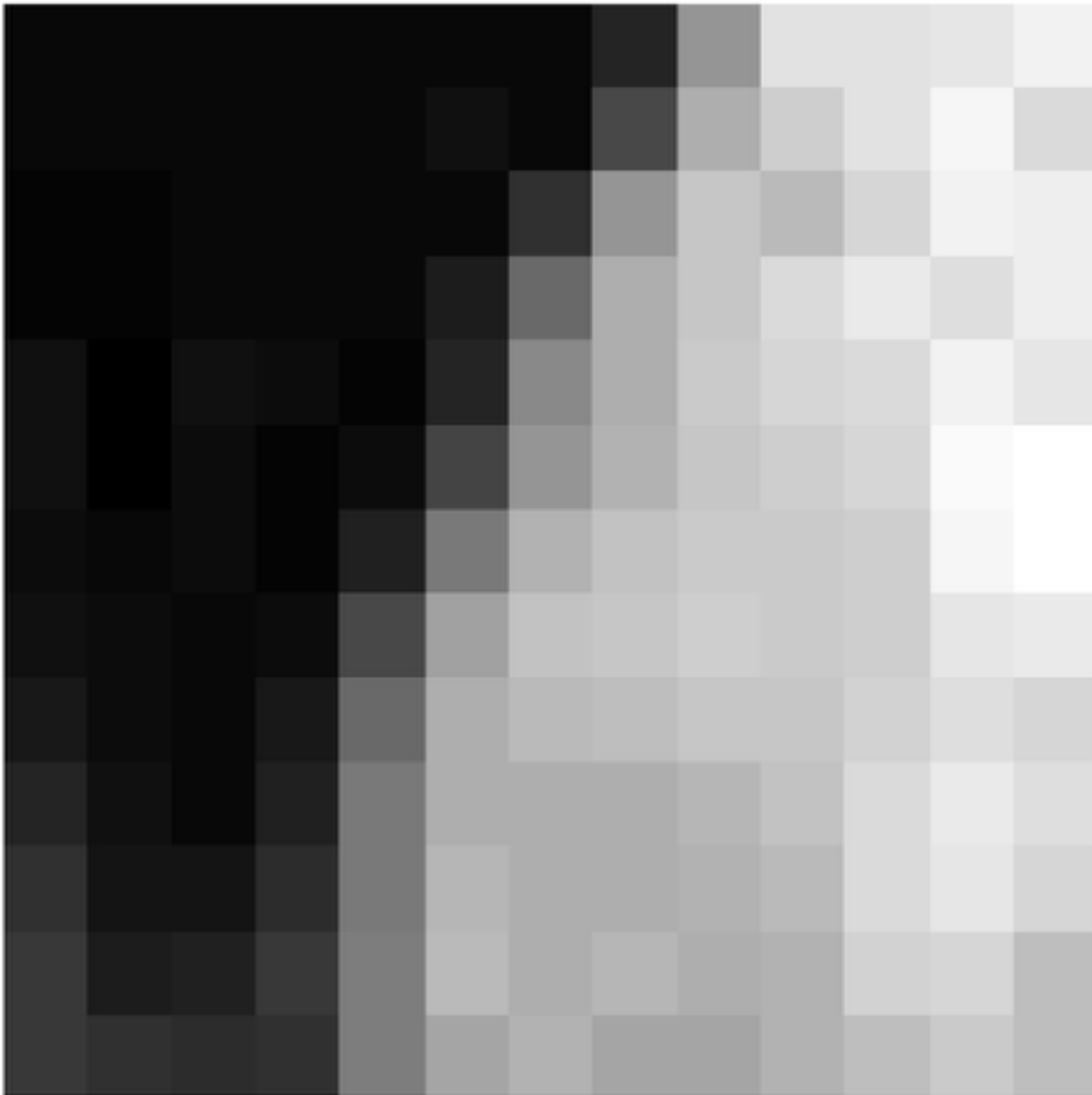
$$\boxed{I} * G_2(x, y) = \boxed{} * G_2(x, y) = \boxed{}$$

$\sigma \approx 2$ $\sigma = \sqrt{8}$

$$\dots \boxed{\sqrt{12}} \dots \boxed{\sigma = 4}$$

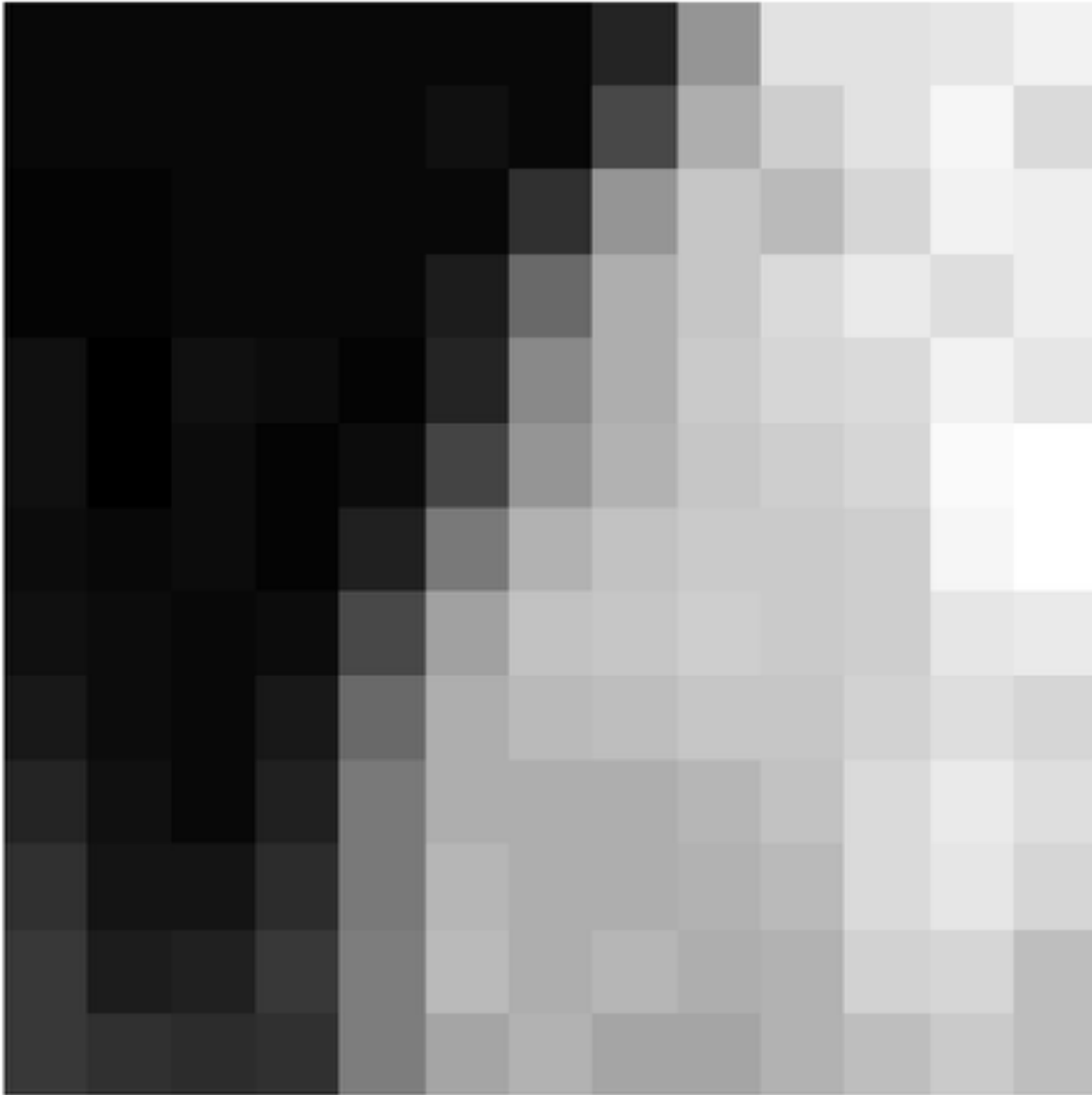
Image Gradients

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x-h)}{h}$$



$$\nabla I(x, y) = \begin{pmatrix} I'_x \\ I'_y \end{pmatrix}$$

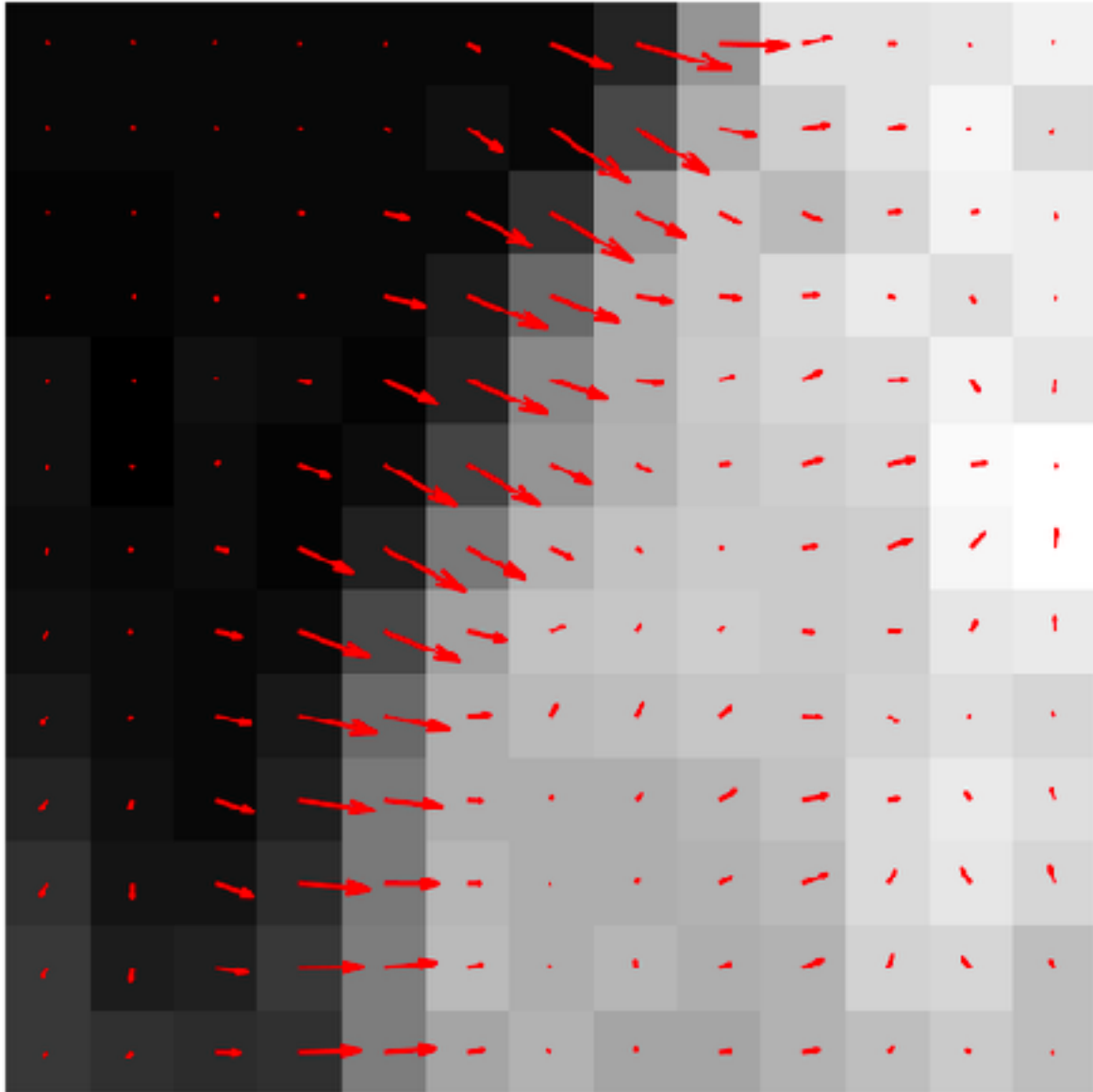
Image Gradients



$$\nabla I(x, y) = \begin{pmatrix} I'_x \\ I'_y \end{pmatrix}$$

$$I'_x(x, y) \approx \frac{I(x + 1, y) - I(x - 1, y)}{2}$$

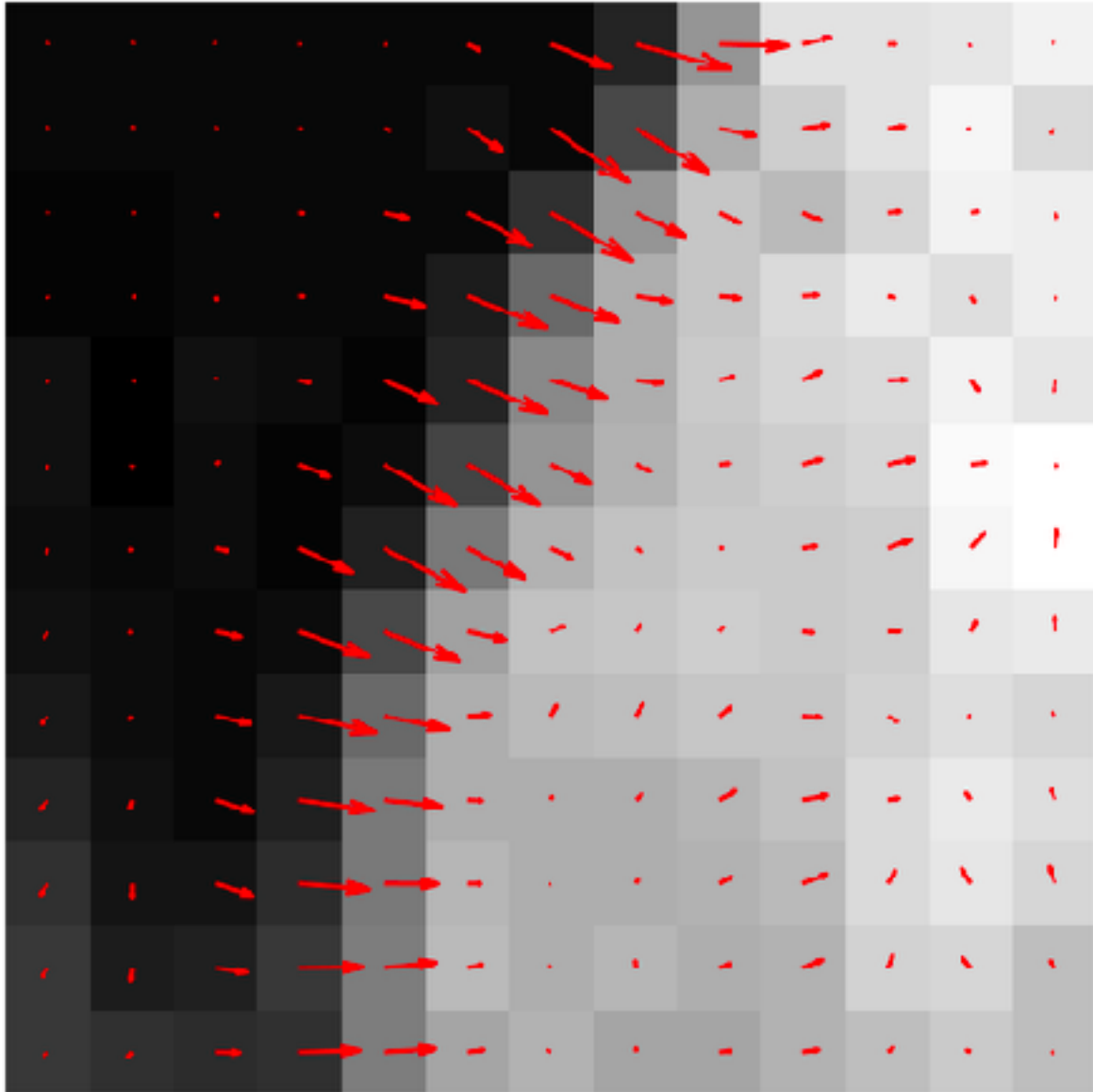
Image Gradients



$$\nabla I(x, y) = \begin{pmatrix} I'_x \\ I'_y \end{pmatrix}$$

$$I'_x(x, y) \approx \frac{I(x+1, y) - I(x-1, y)}{2}$$

Image Gradients

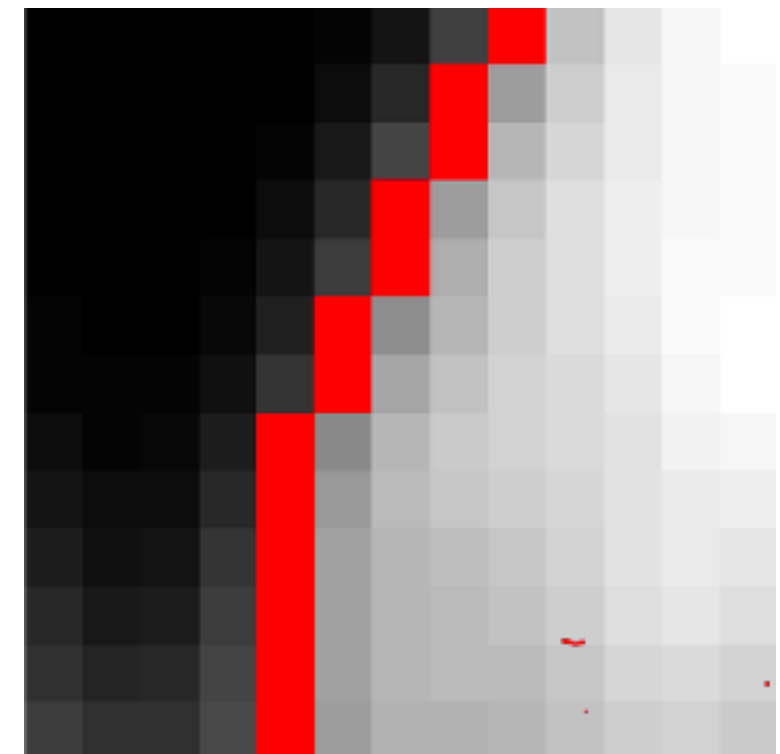
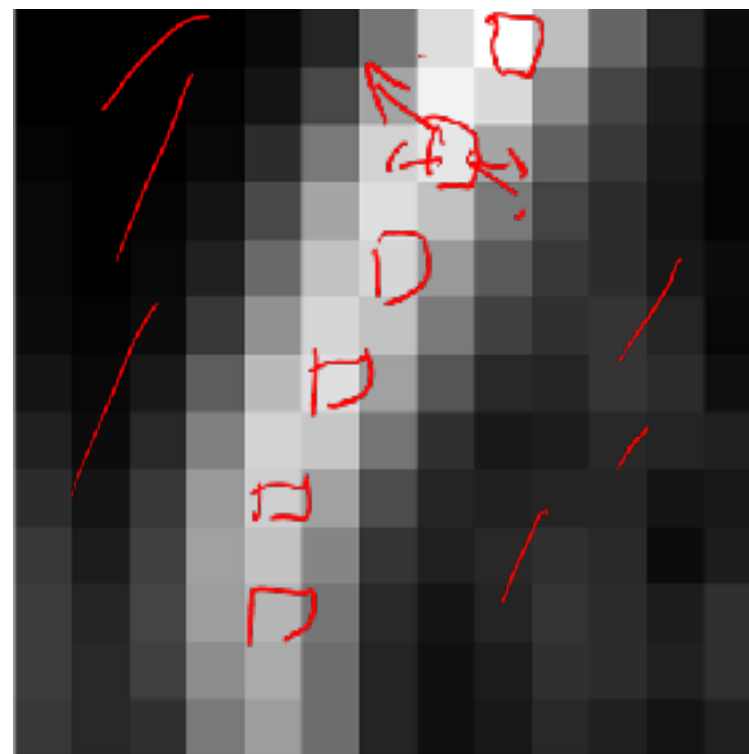
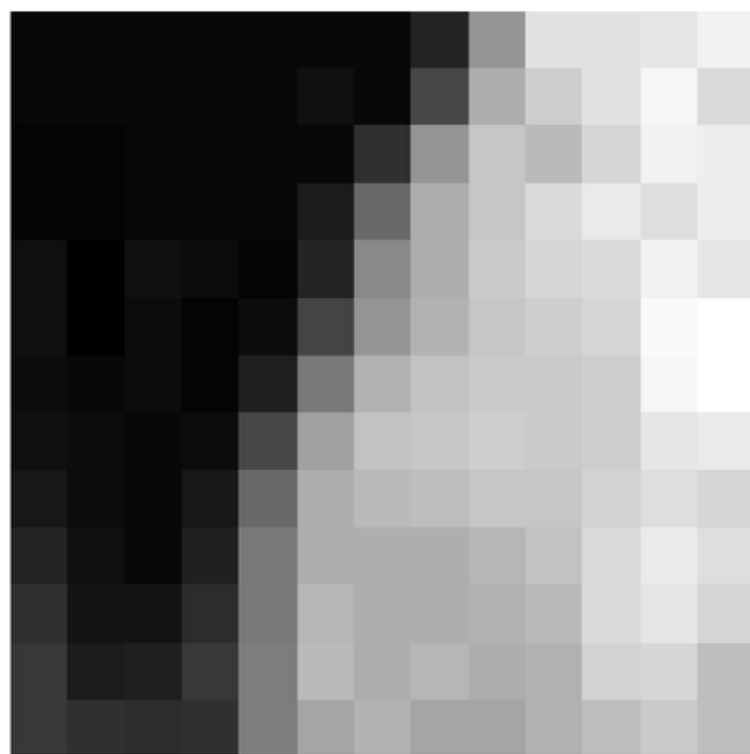


$$\nabla I(x, y) = \begin{pmatrix} I'_x \\ I'_y \end{pmatrix}$$

$$I'_x(x, y) \approx \frac{I(x+1, y) - I(x-1, y)}{2}$$

$$I'_x = I \star (-0.5 \quad 0 \quad 0.5)$$

Application: Edge Detection

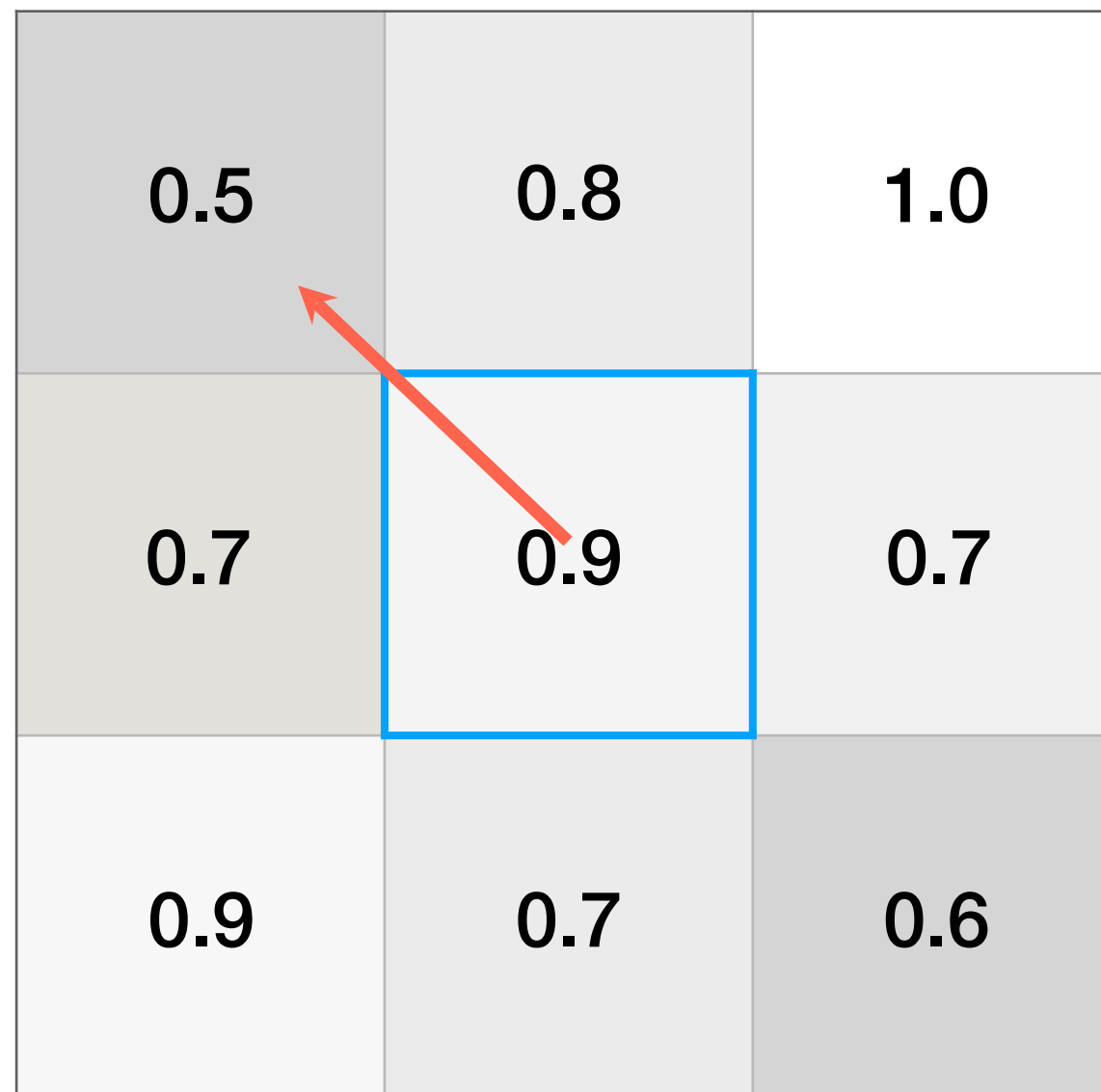


Magnitude of
gradients

$$\|\nabla I(x,y)\|$$

Thresholding and
non-maximum
suppression

Non-Maximum Suppression

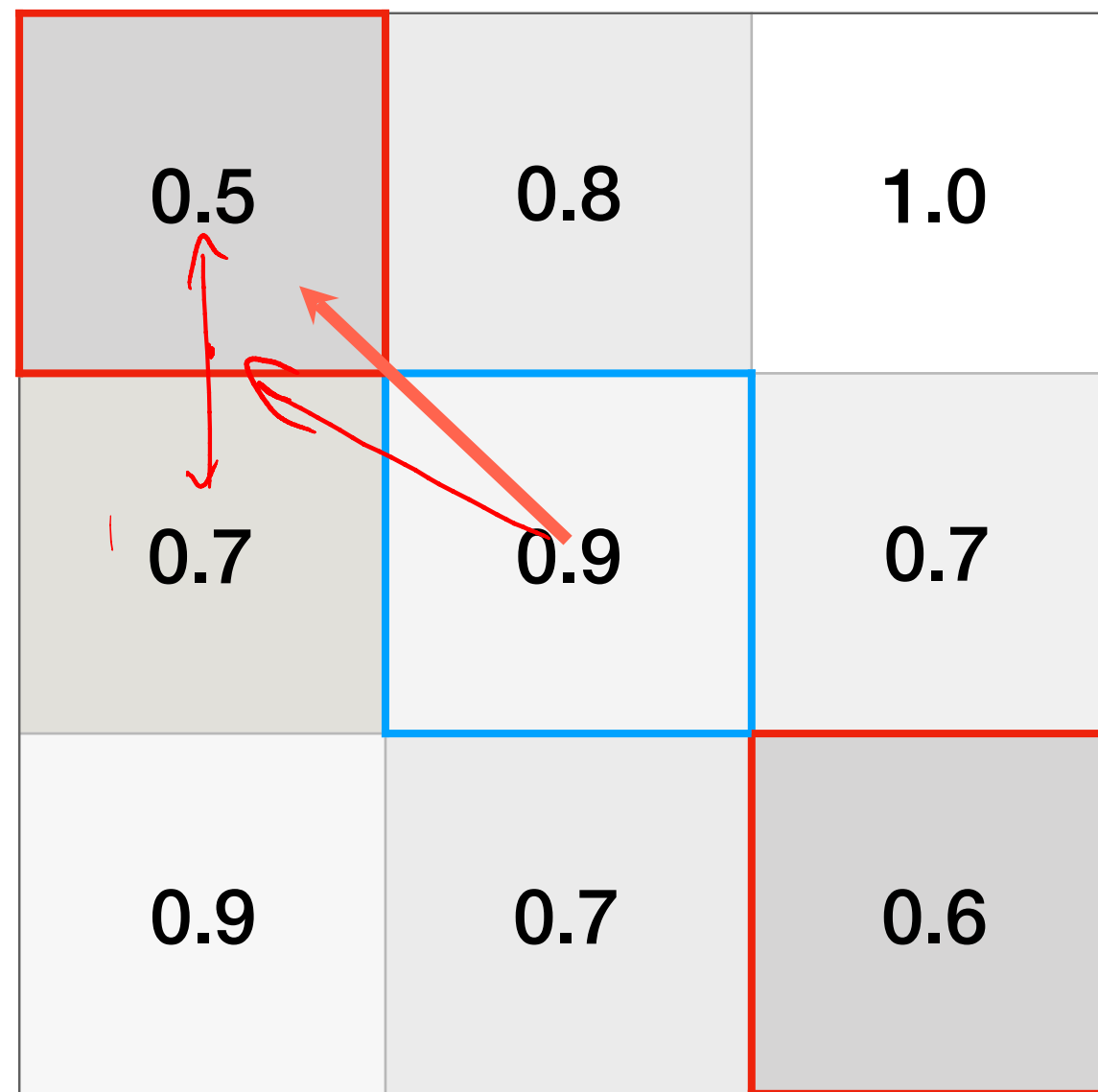


Non-Maximum Suppression



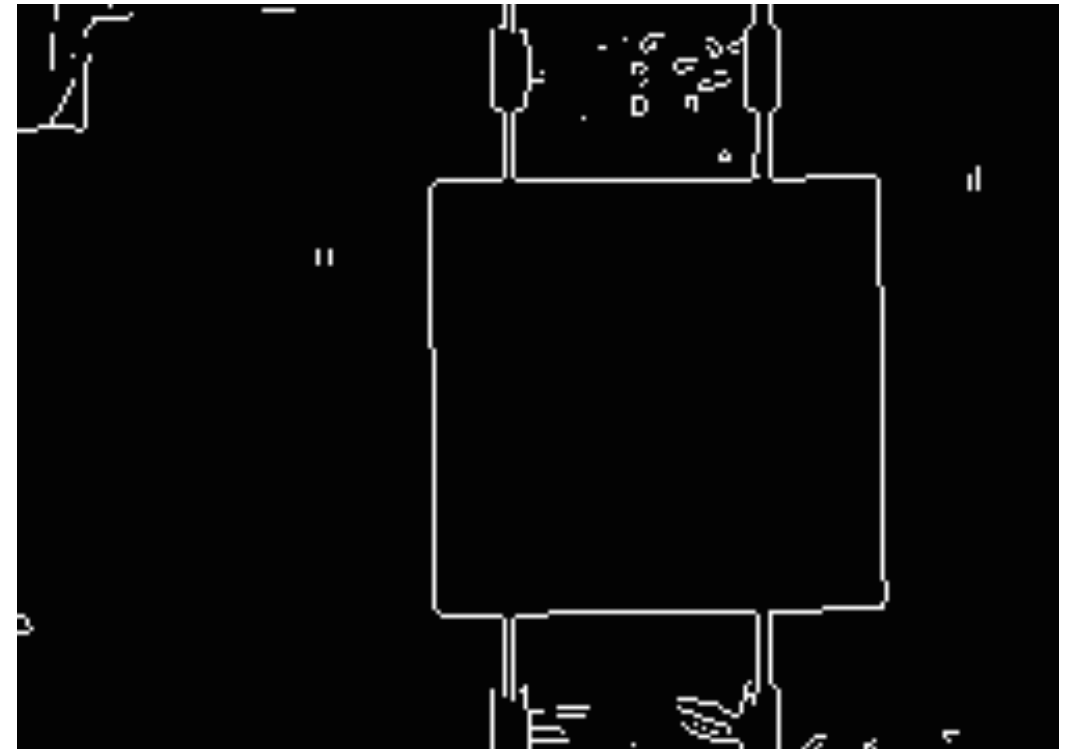
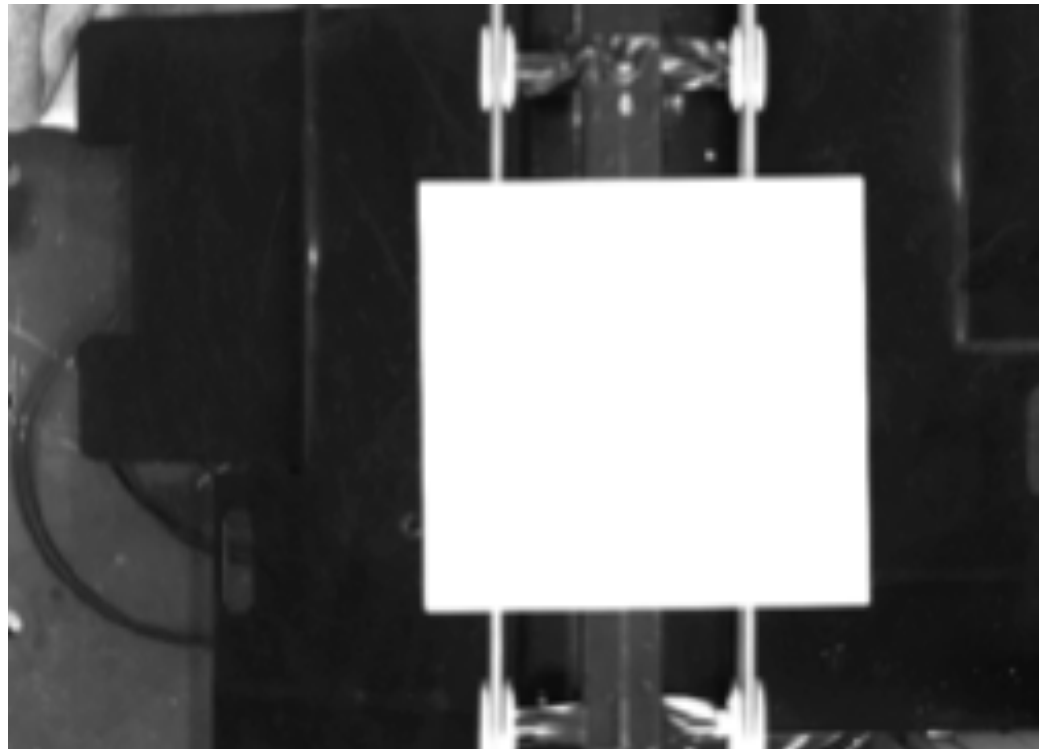
Only compare to neighbors along the gradient.

Non-Maximum Suppression



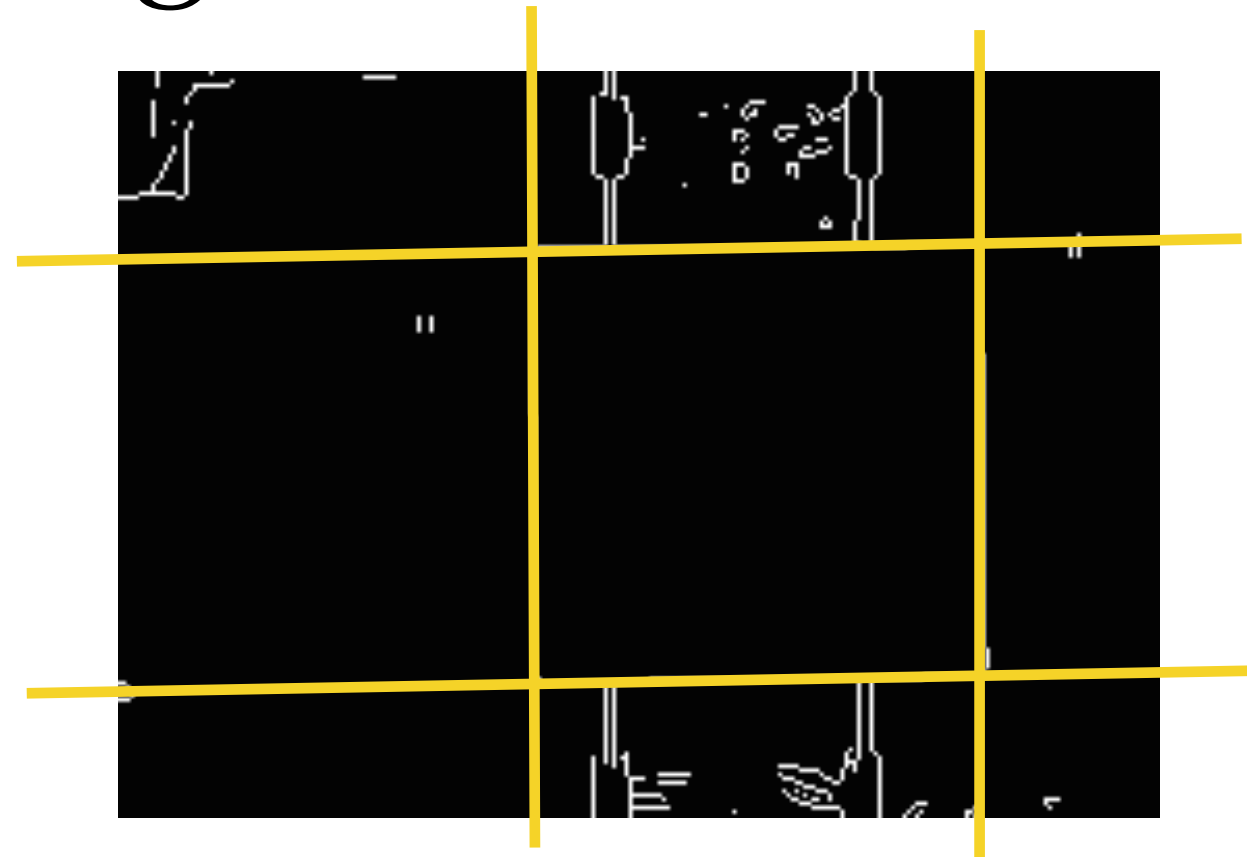
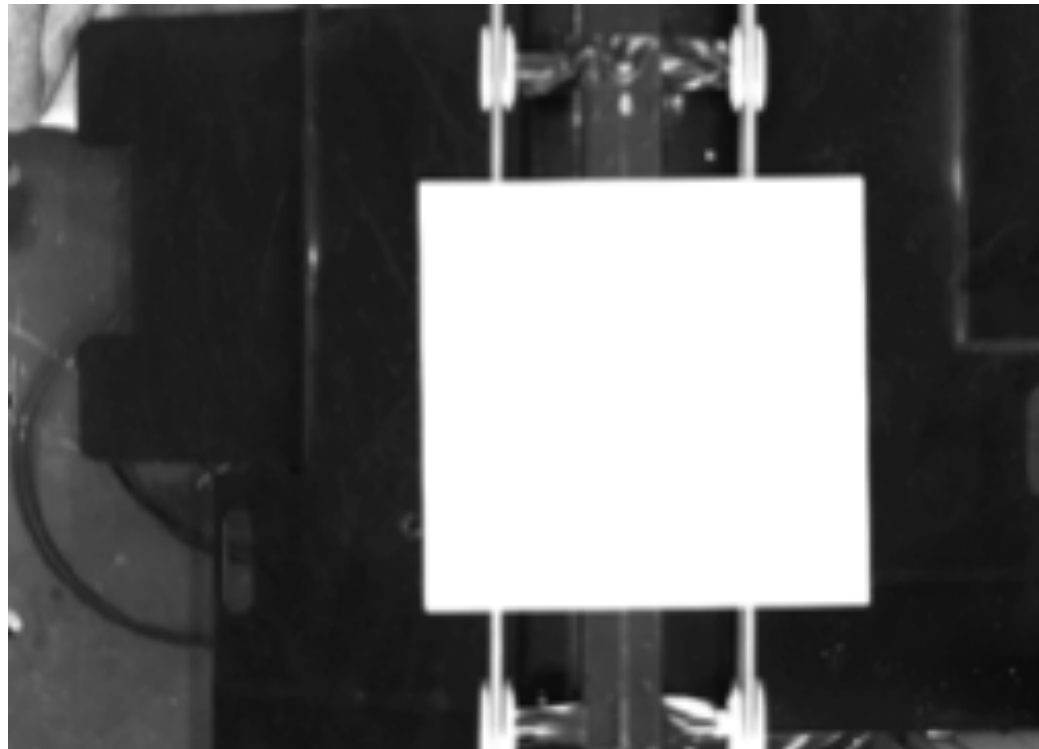
Only compare to neighbors along the gradient.
Keep if larger response than these.

Edge Detection



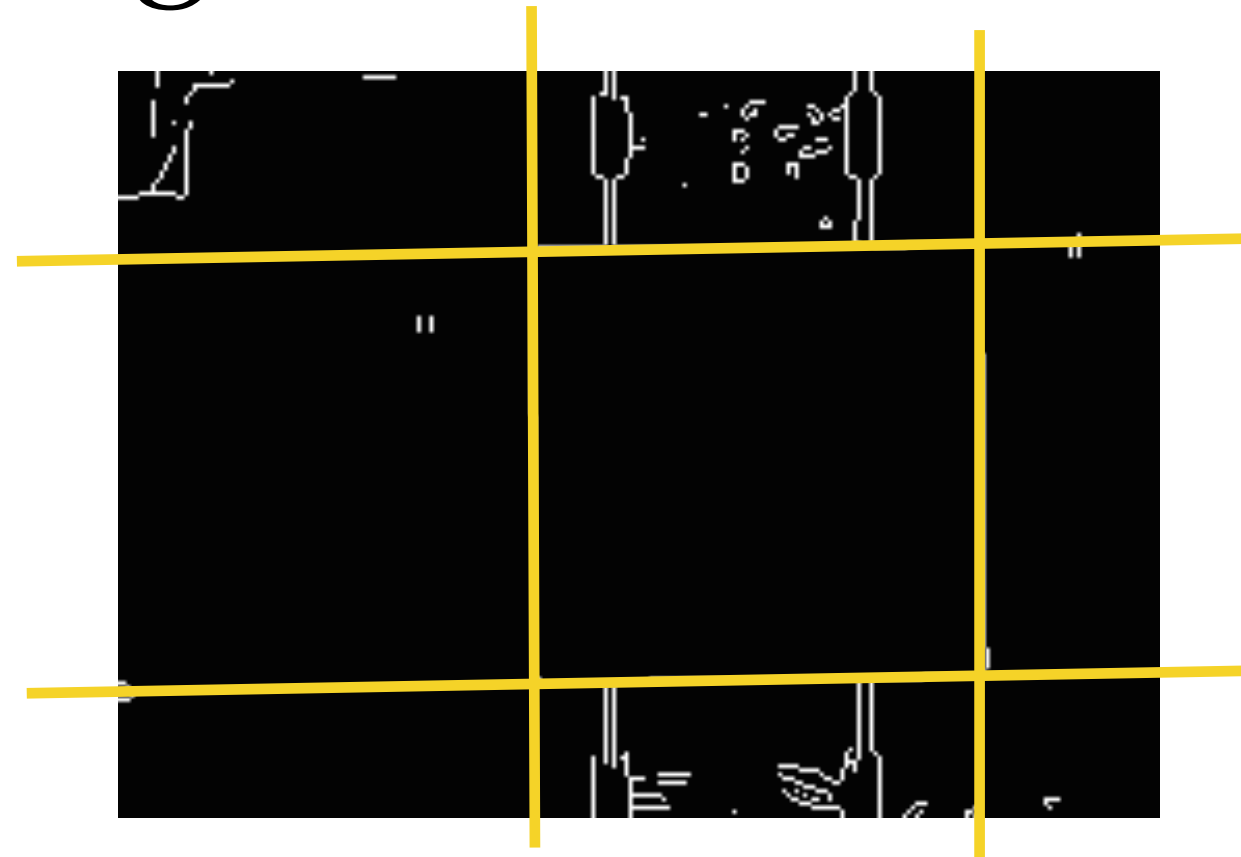
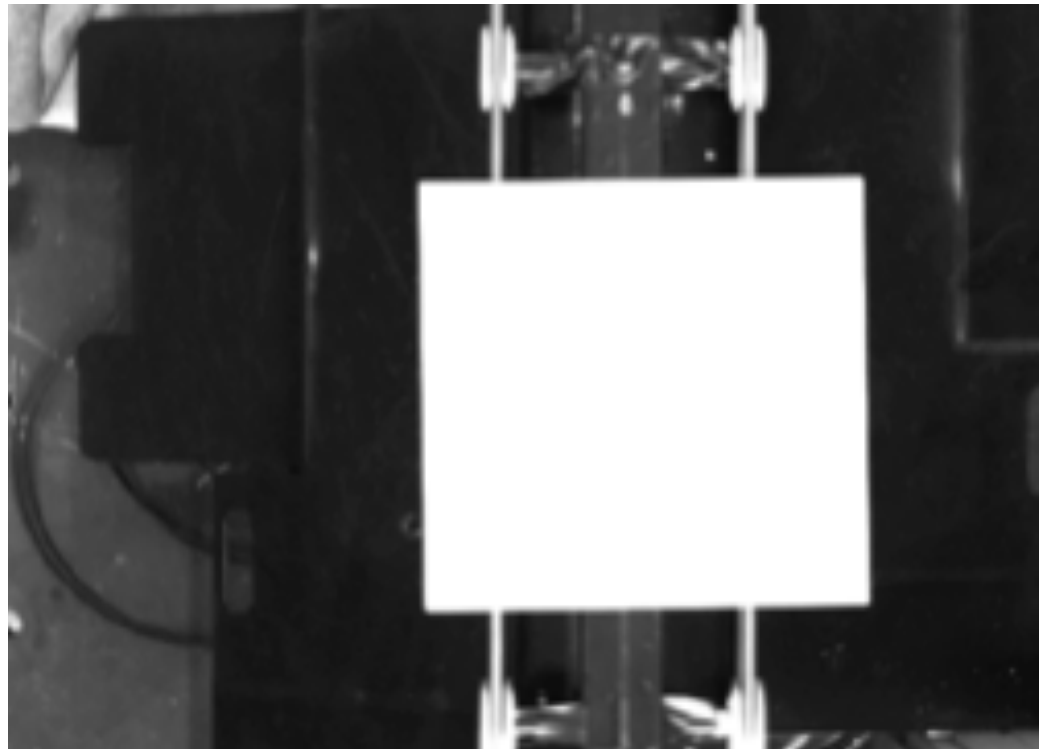
Using Matlab with default thresholds

Edge Detection

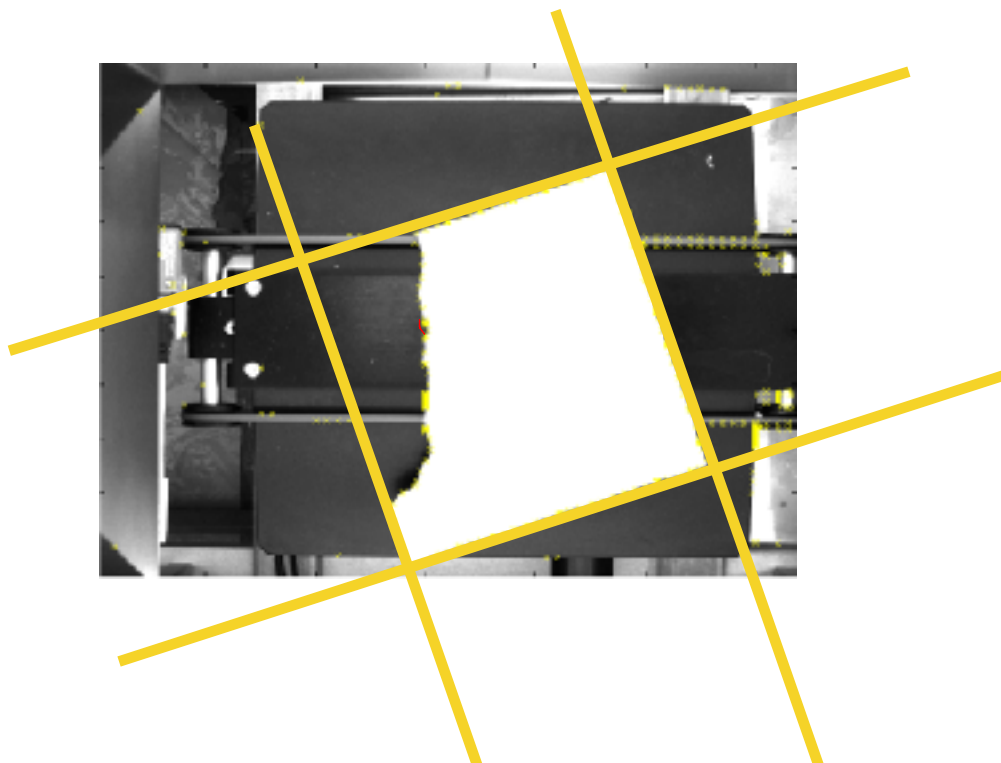


Using Matlab with default thresholds

Edge Detection



Using Matlab with default thresholds



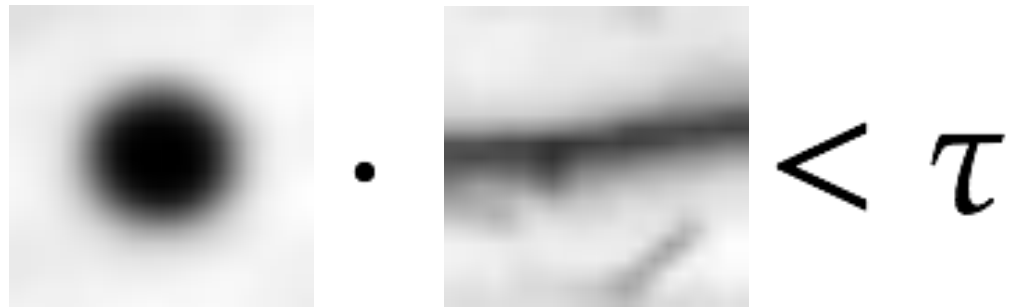
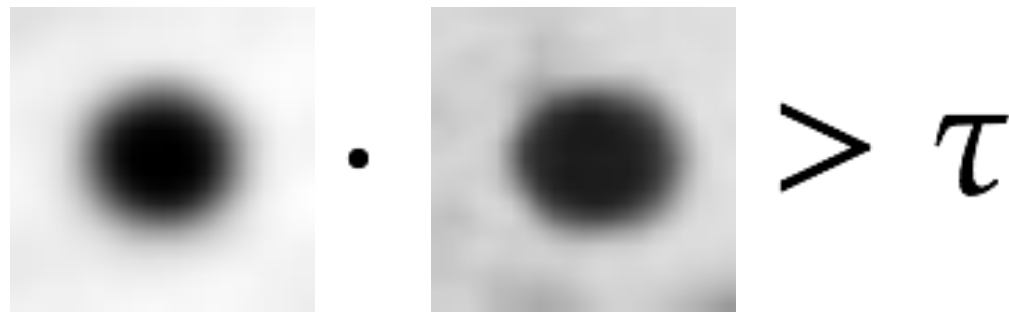
Similarity Measures

Measuring Similarity

Measuring similarity between images / patches central problem

Measuring Similarity

Measuring similarity between images / patches central problem



Measuring Similarity

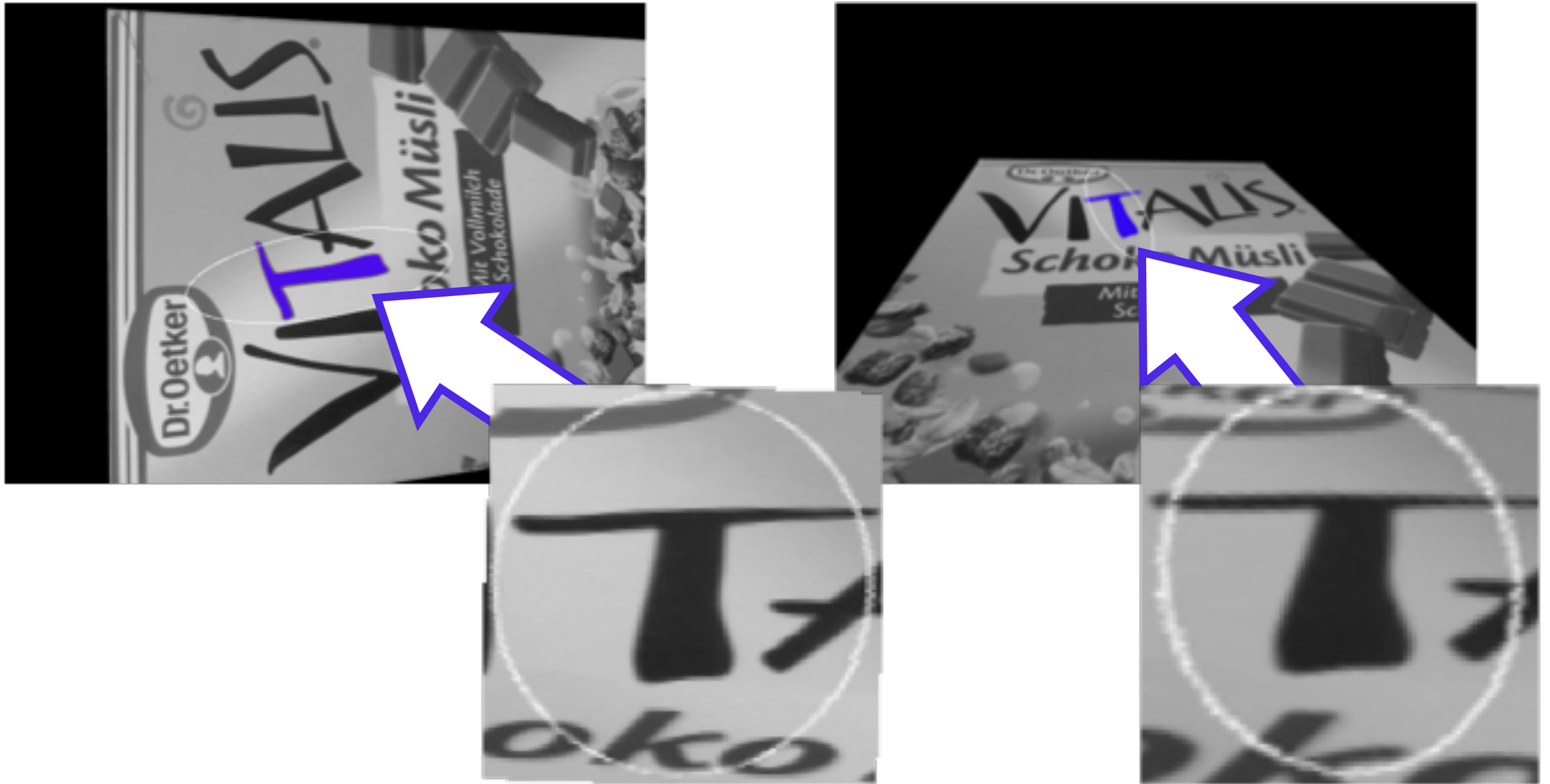
Measuring similarity between images / patches central problem



slide credit: Marc Pollefeys, Kevin Köser

Measuring Similarity

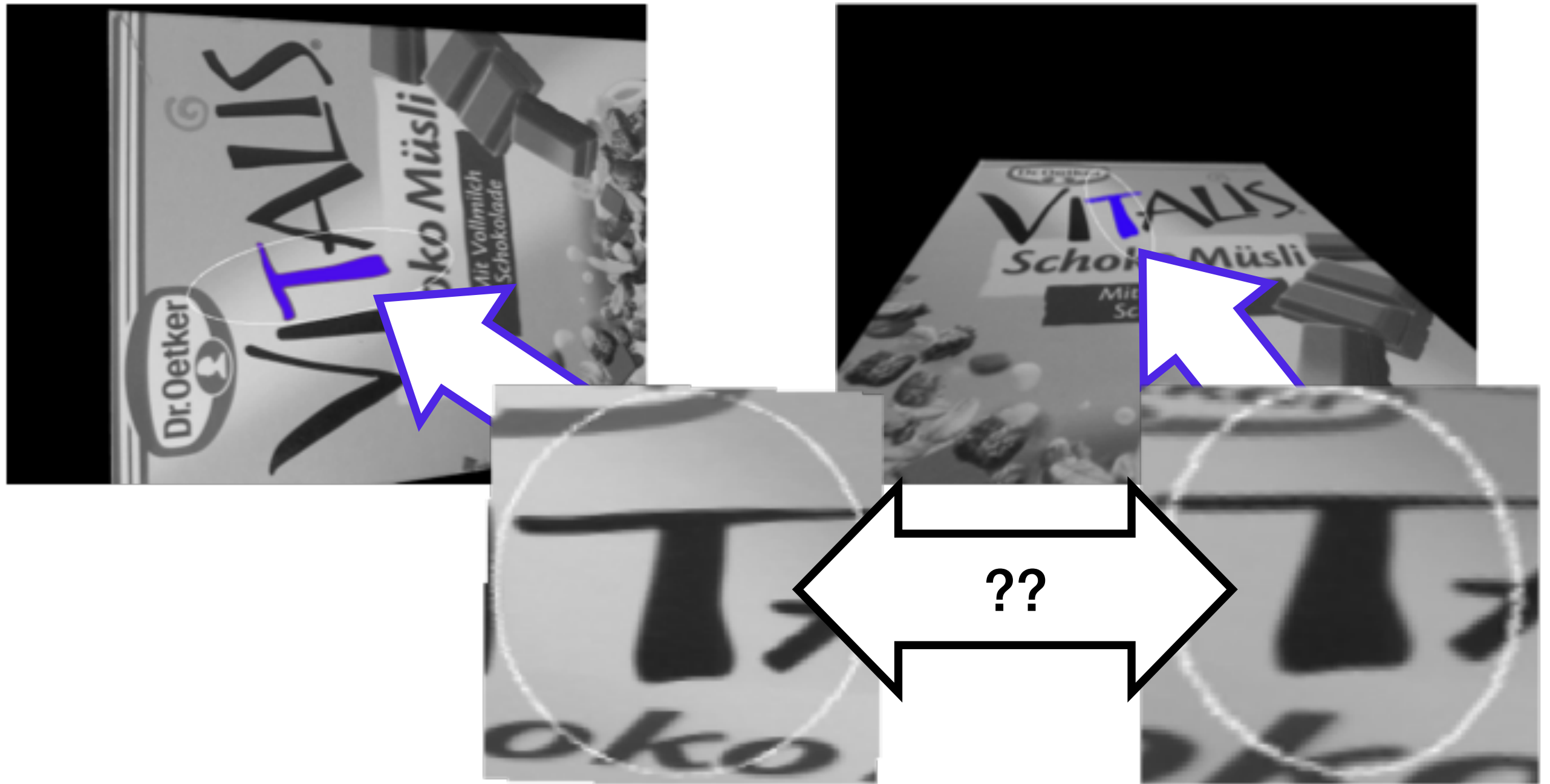
Measuring similarity between images / patches central problem



slide credit: Marc Pollefeys, Kevin Köser

Measuring Similarity

Measuring similarity between images / patches central problem



slide credit: Marc Pollefeys, Kevin Köser

Measuring Similarity

Covariance:



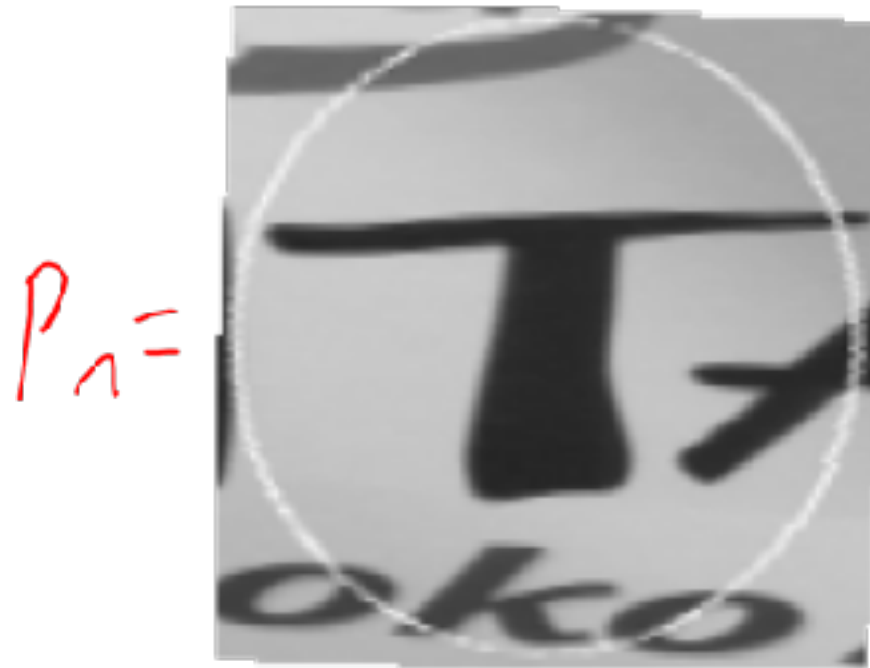
$$\begin{aligned}
 \text{Cov}(P_1, P_2) &= \frac{1}{N} \underbrace{(P_1 - \mu_1 \mathbf{1})}_X (P_2 - \mu_2 \mathbf{1}) \\
 &= \frac{1}{N} \left[X \cdot P_2 - \underbrace{\mu_1}_{N\mu_2} \underbrace{(\mathbf{1} \cdot P_2)}_N - \mu_2 \underbrace{\mathbf{1} \cdot \mathbf{1}}_N \right]
 \end{aligned}$$

Measuring Similarity

Is Covariance a good similarity measure?

Measuring Similarity

Is Covariance a good similarity measure?



$$\begin{aligned}
 \text{Cov}(P_1, P_2) &= \frac{1}{N} (P_1 - \mu_1 \mathbf{1}) (\alpha P_1 - \alpha \mu_1 \mathbf{1}) \\
 &= \alpha \frac{1}{N} (P_1 - \mu_1 \mathbf{1}) \cdot (P_1 - \mu_1 \mathbf{1}) \\
 &= \alpha \text{Cov}(P_1, P_1) > \text{Cov}(P_1, P_1)
 \end{aligned}$$

Measuring Similarity

Correlation:



$$\text{Corr}(P_1, P_2) = \frac{\text{Cov}(P_1, P_2)}{\sqrt{\text{Cov}(P_1, P_1)} \sqrt{\text{Cov}(P_2, P_2)}}$$

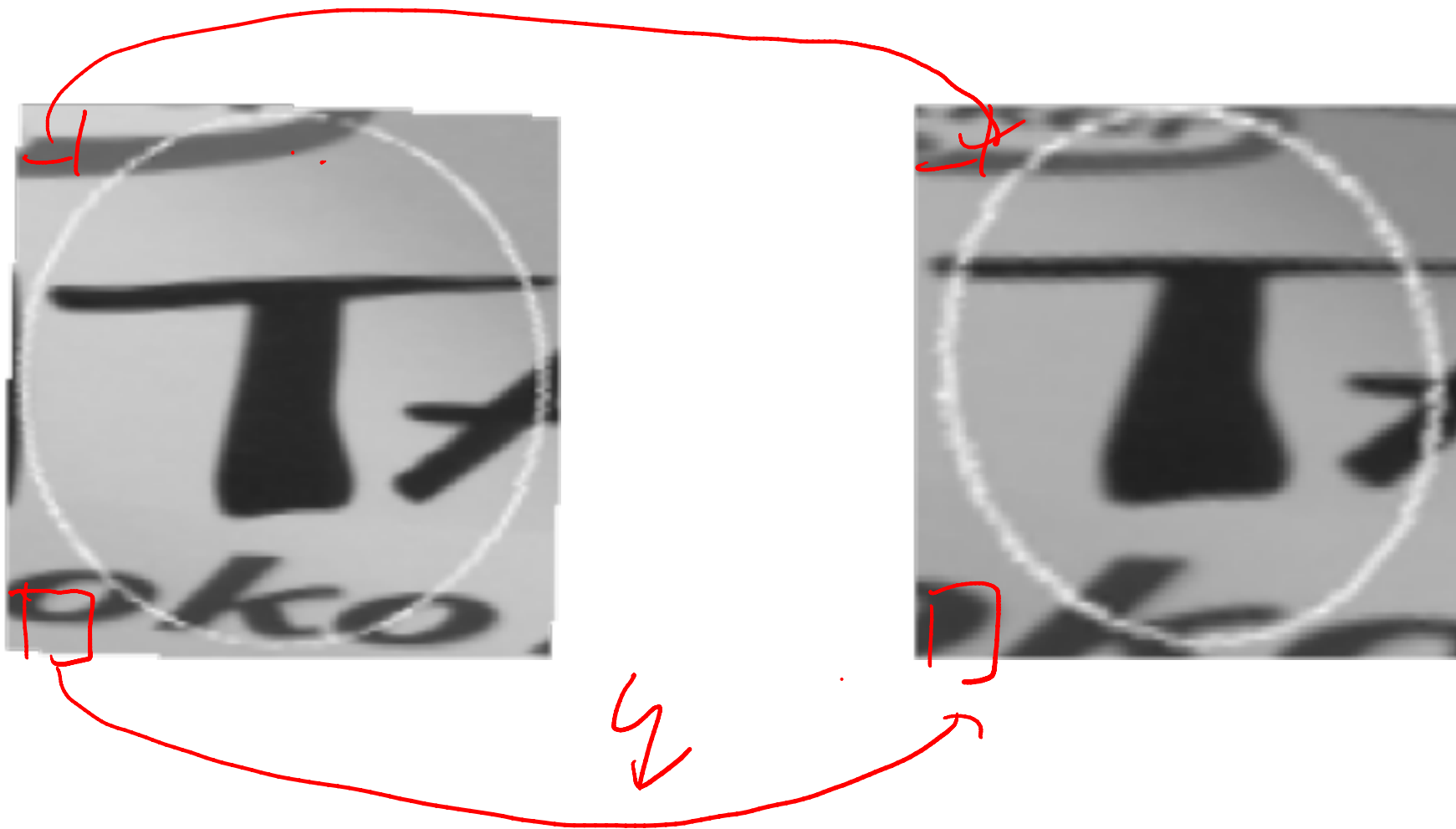
Also known as **Zero-Mean Normalized Cross-Correlation** (ZNCC)

Measuring Similarity

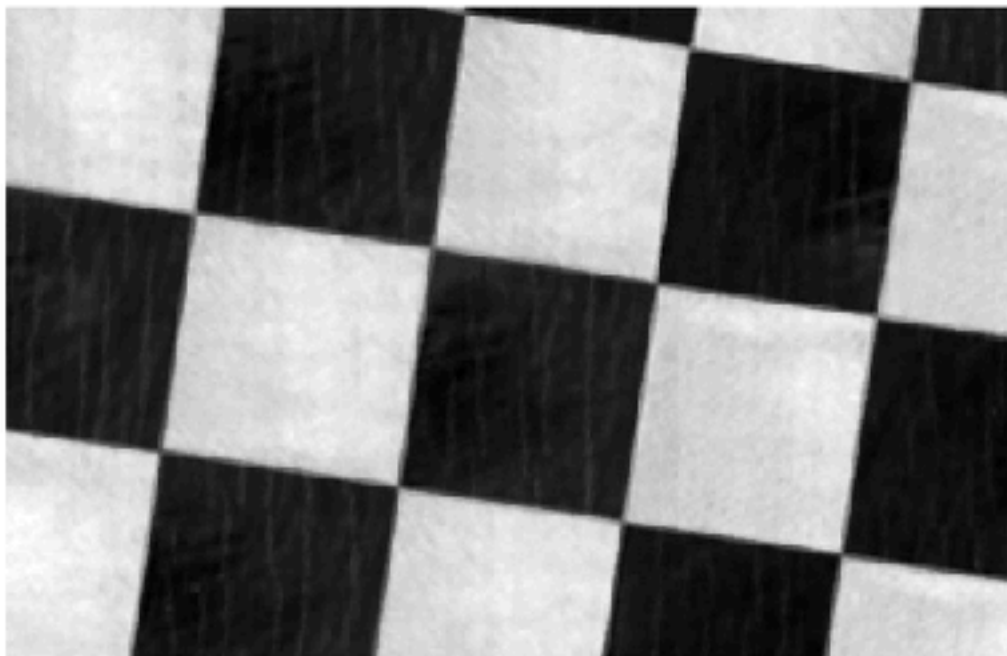
When doesZNCC work?

Measuring Similarity

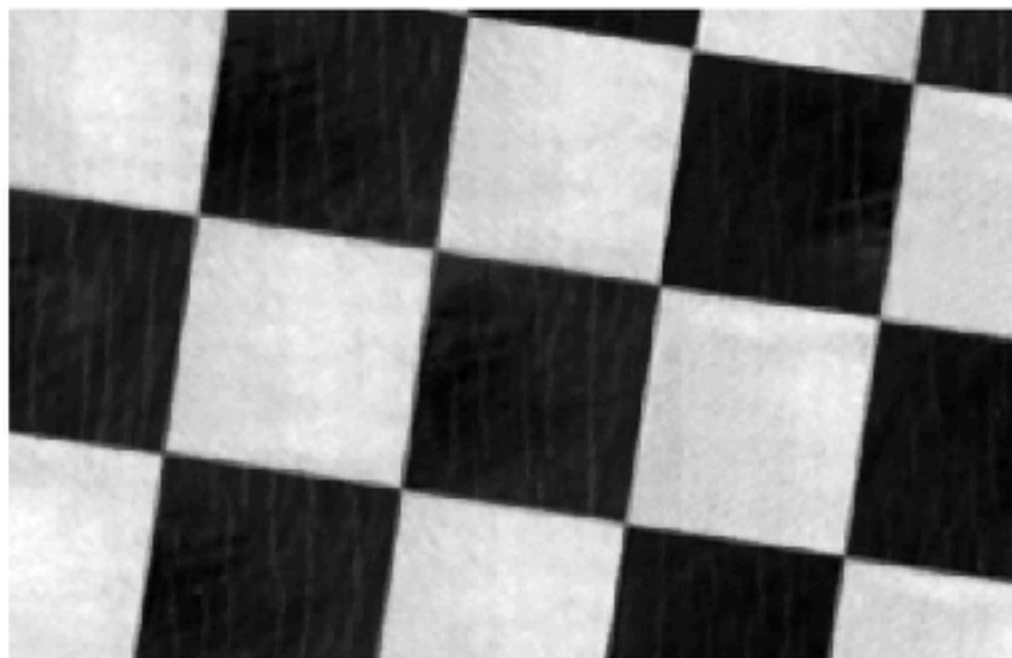
When does ZNCC work?



Measuring Similarity

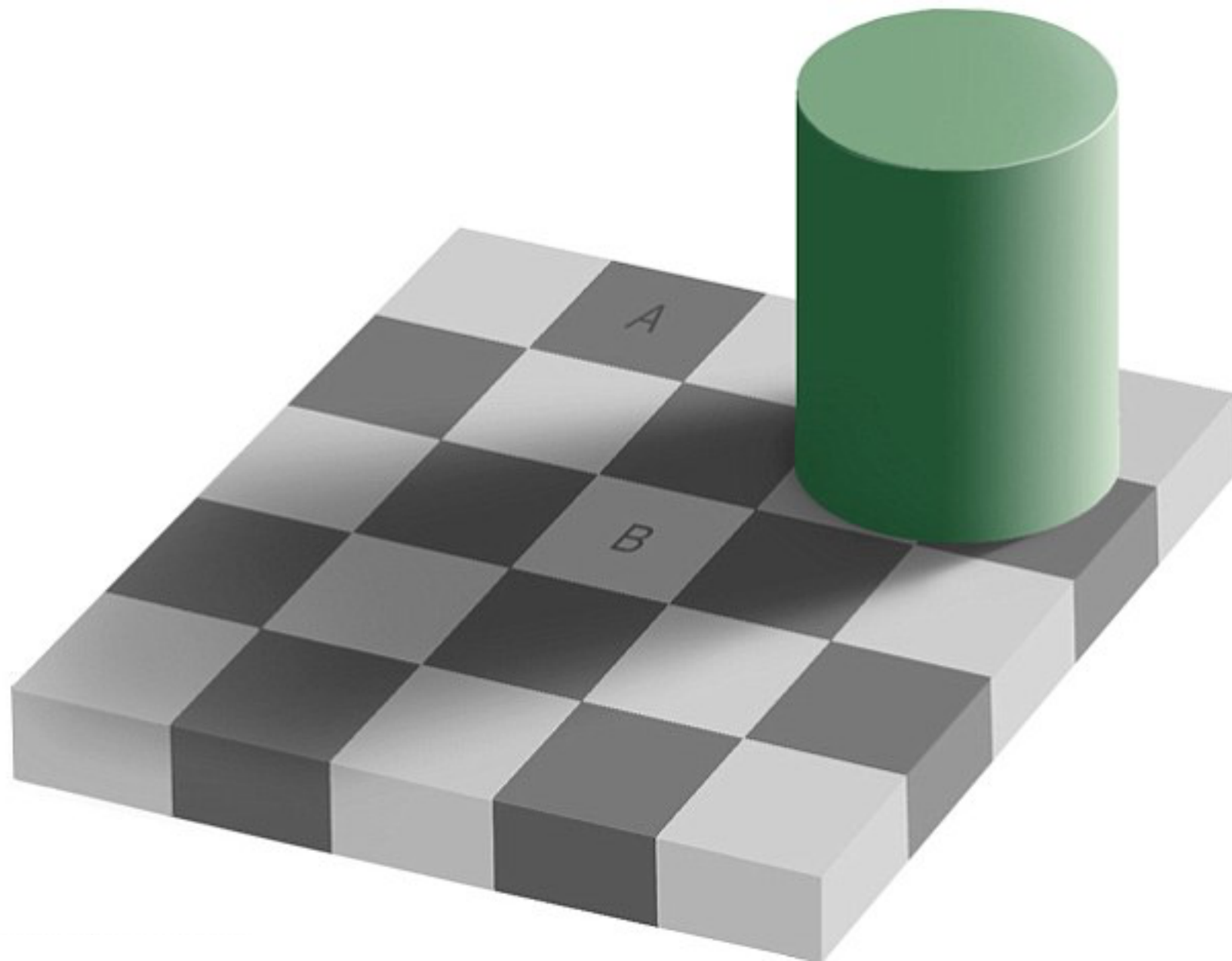


Measuring Similarity

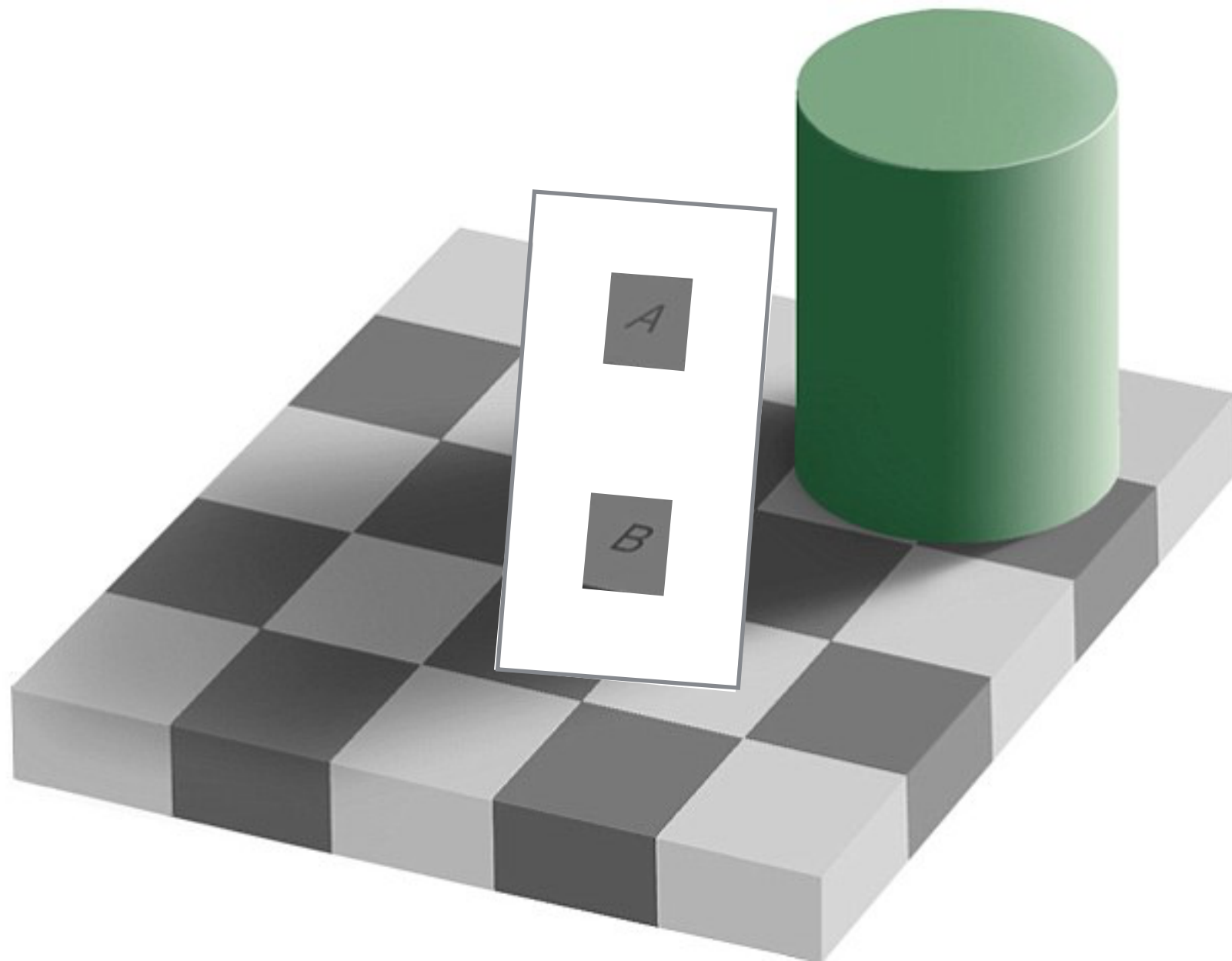


Difference image
black = no difference
white = difference

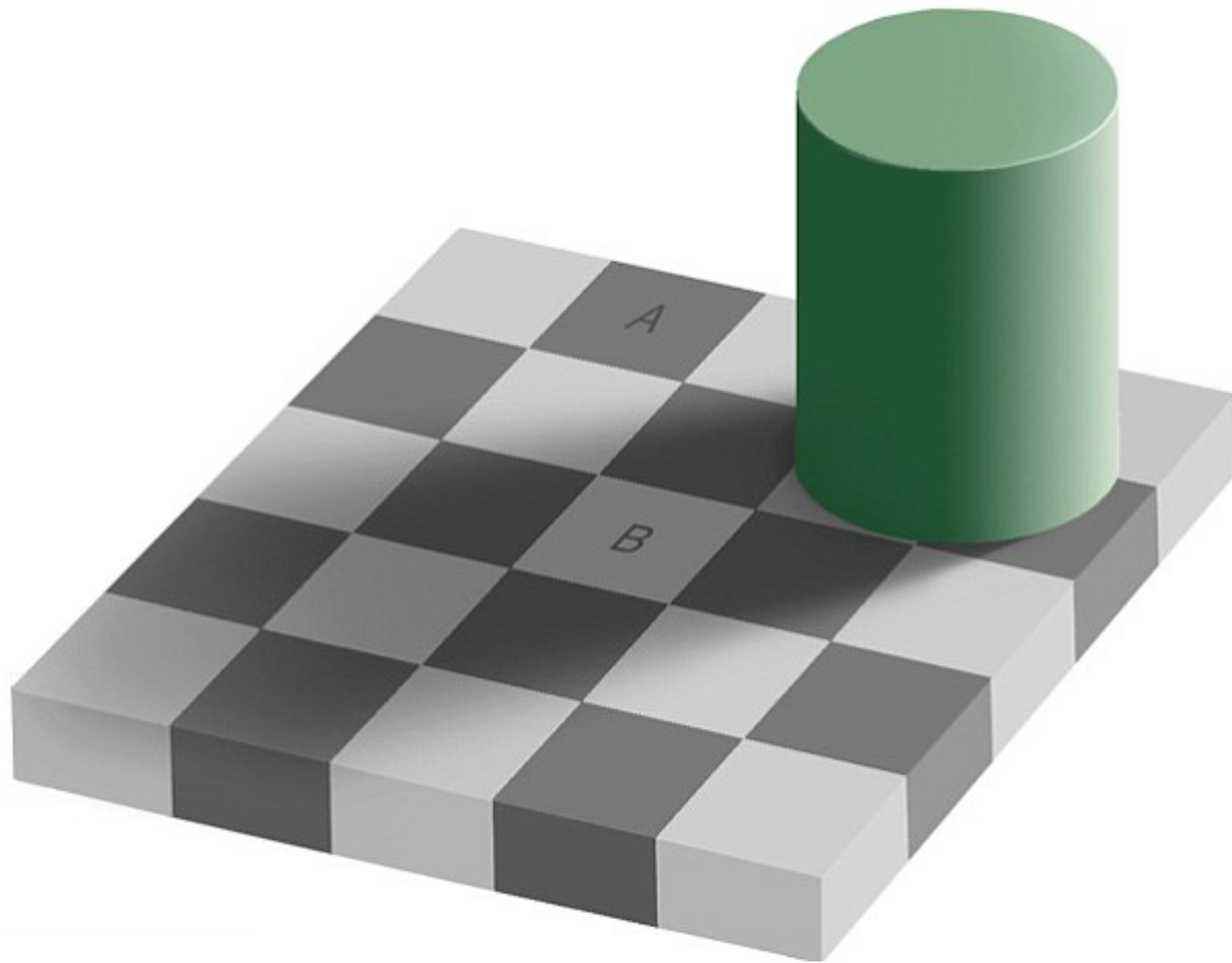
A Hint?



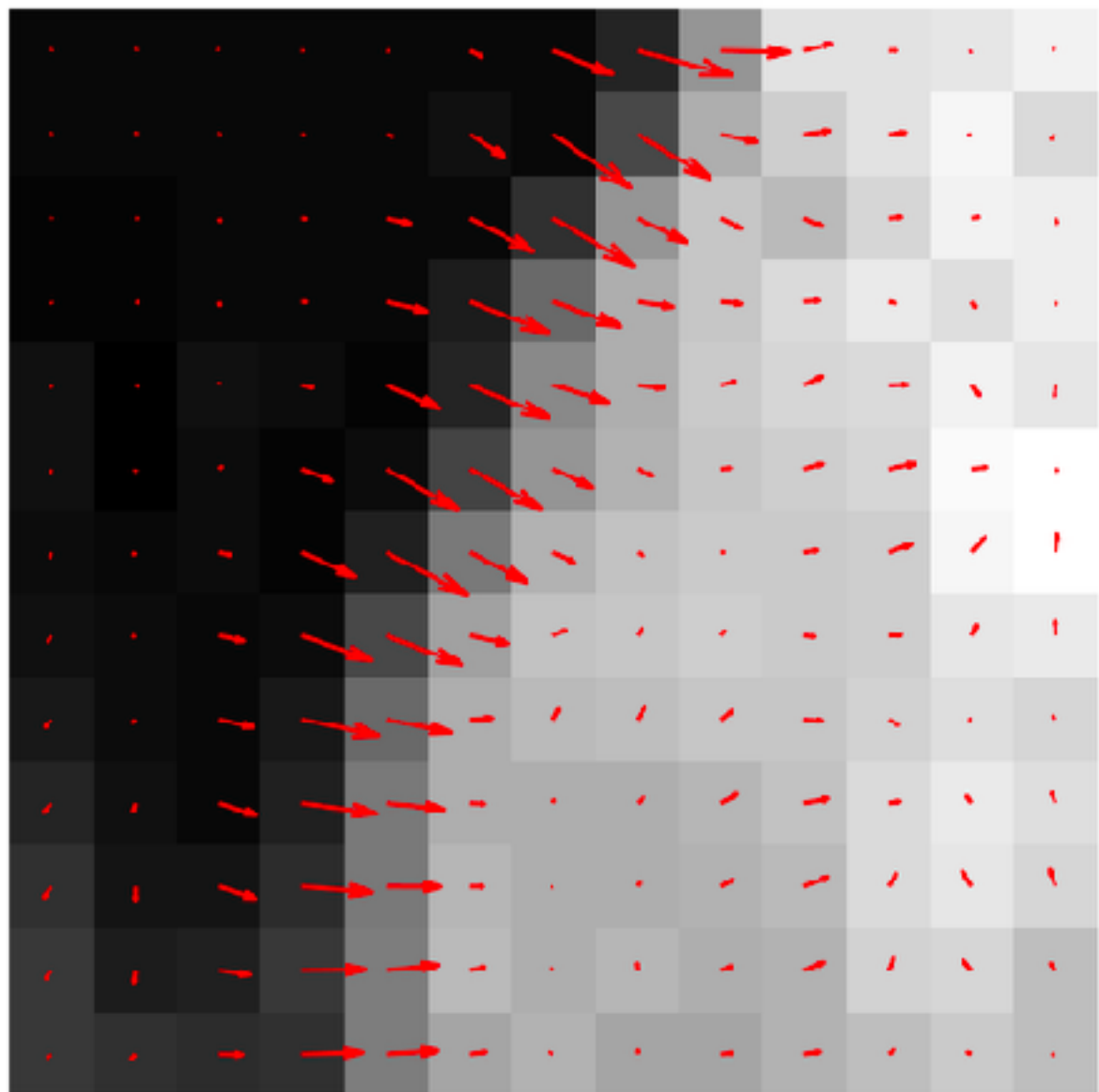
A Hint?



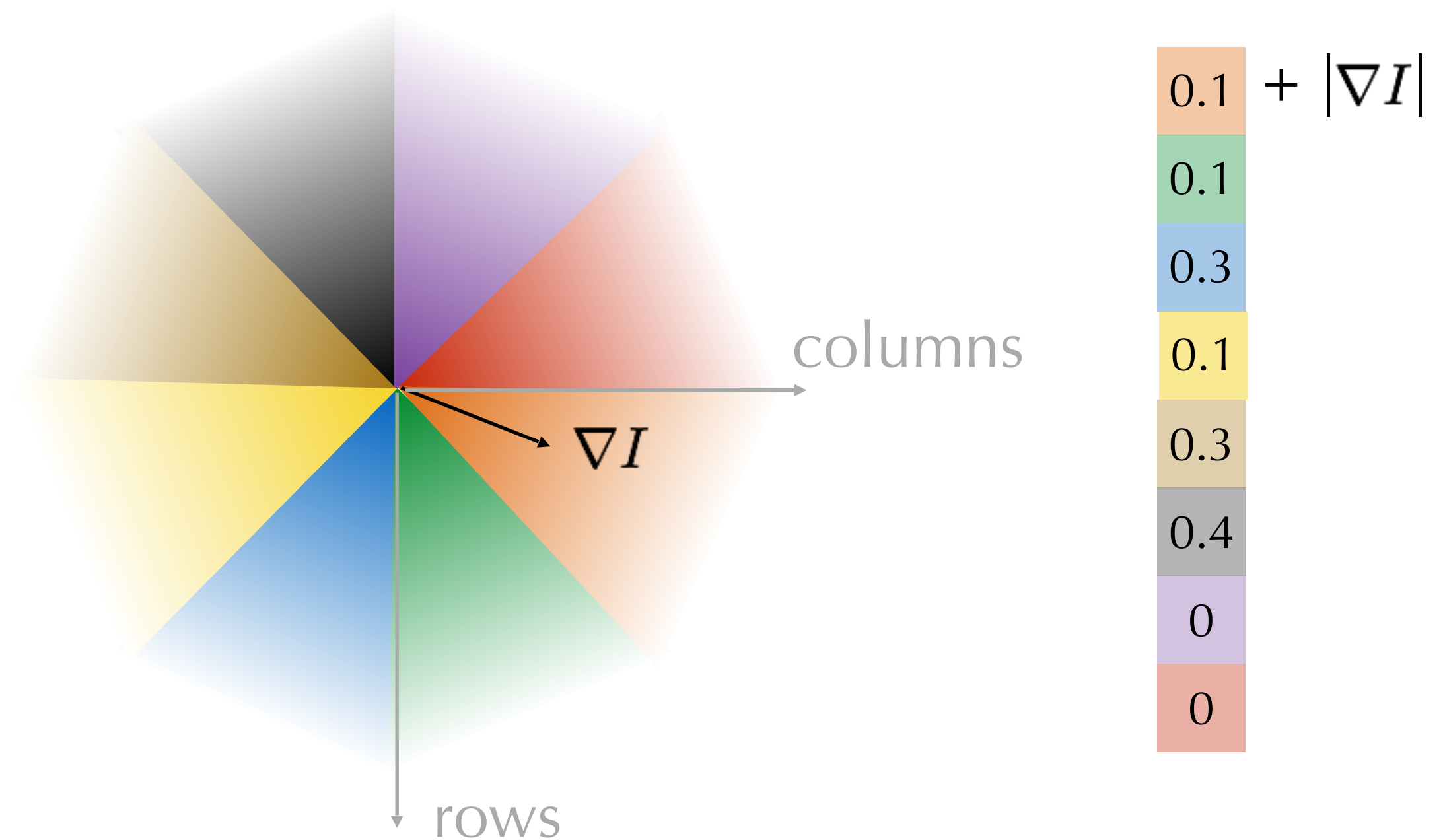
A Hint?



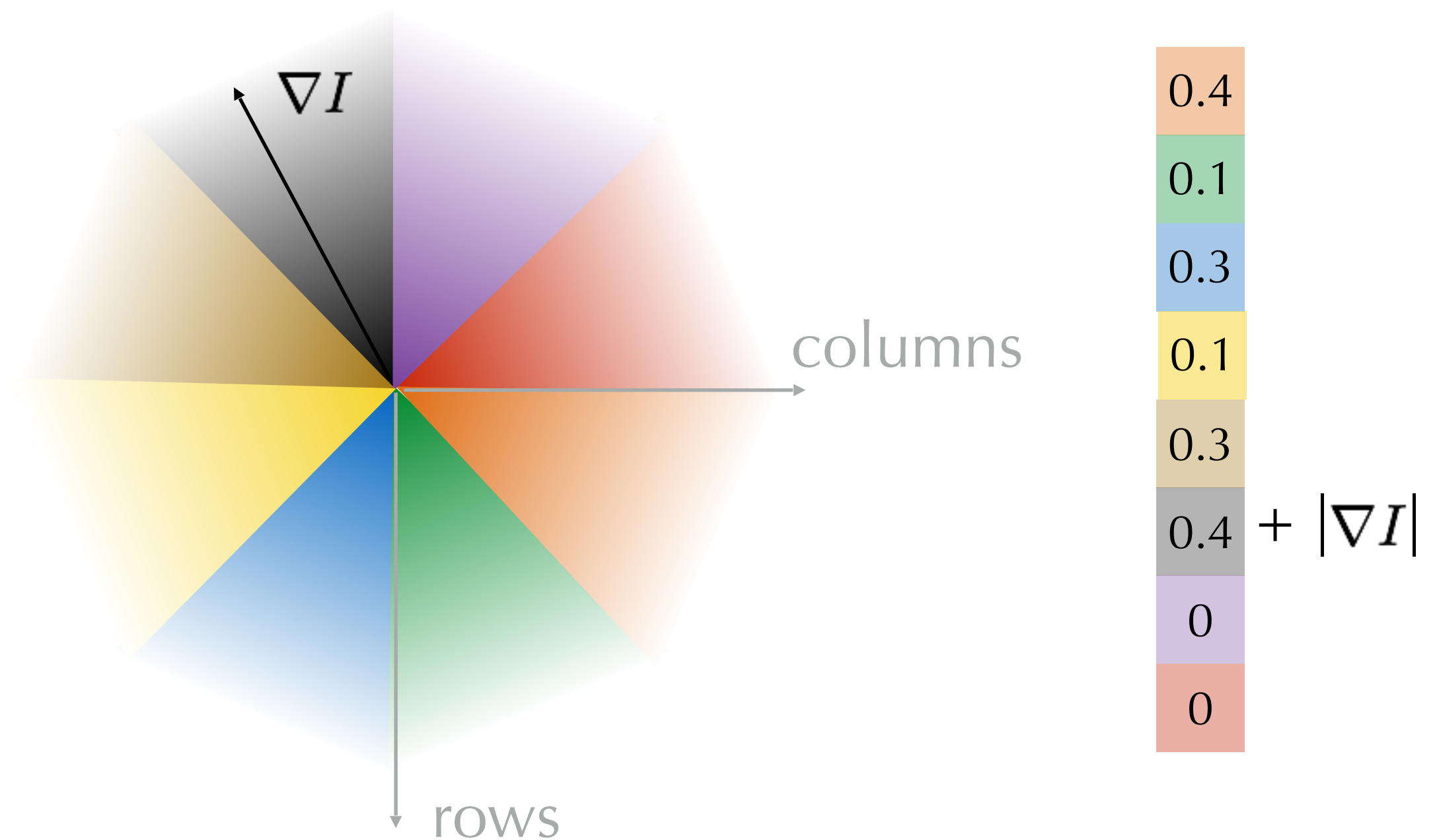
Gradient Histogram



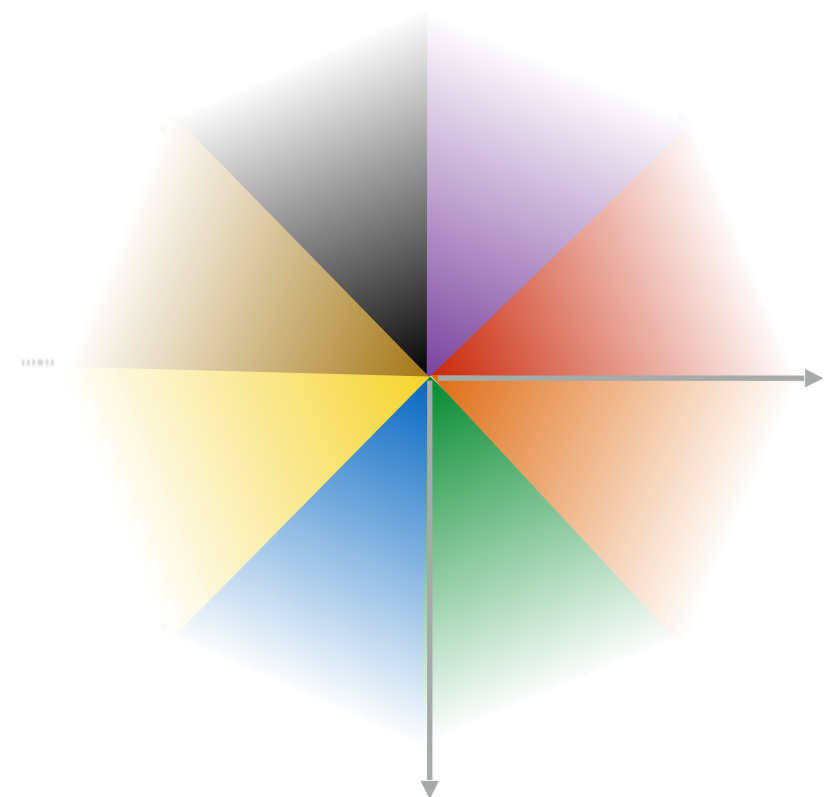
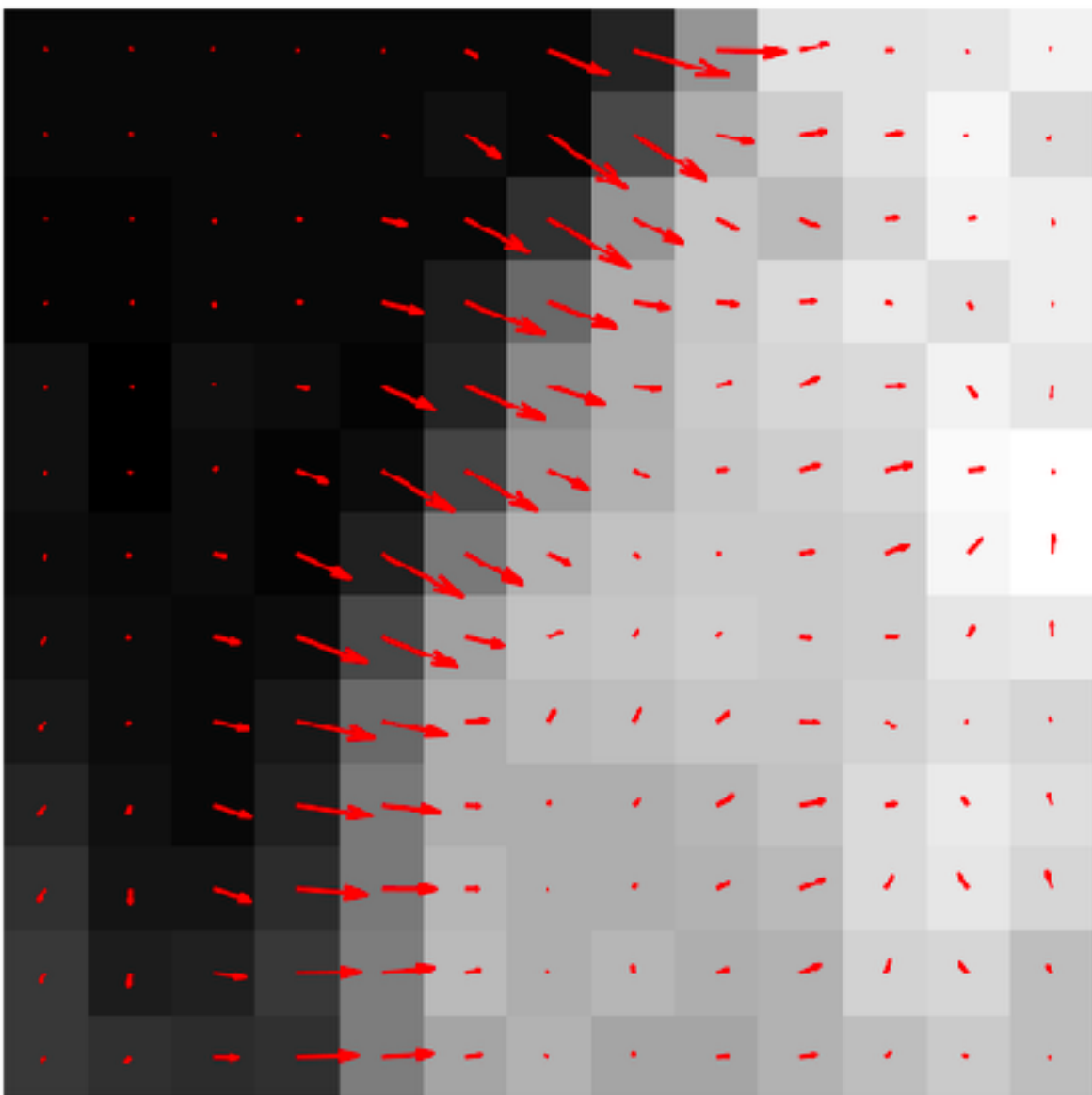
Gradient Histogram



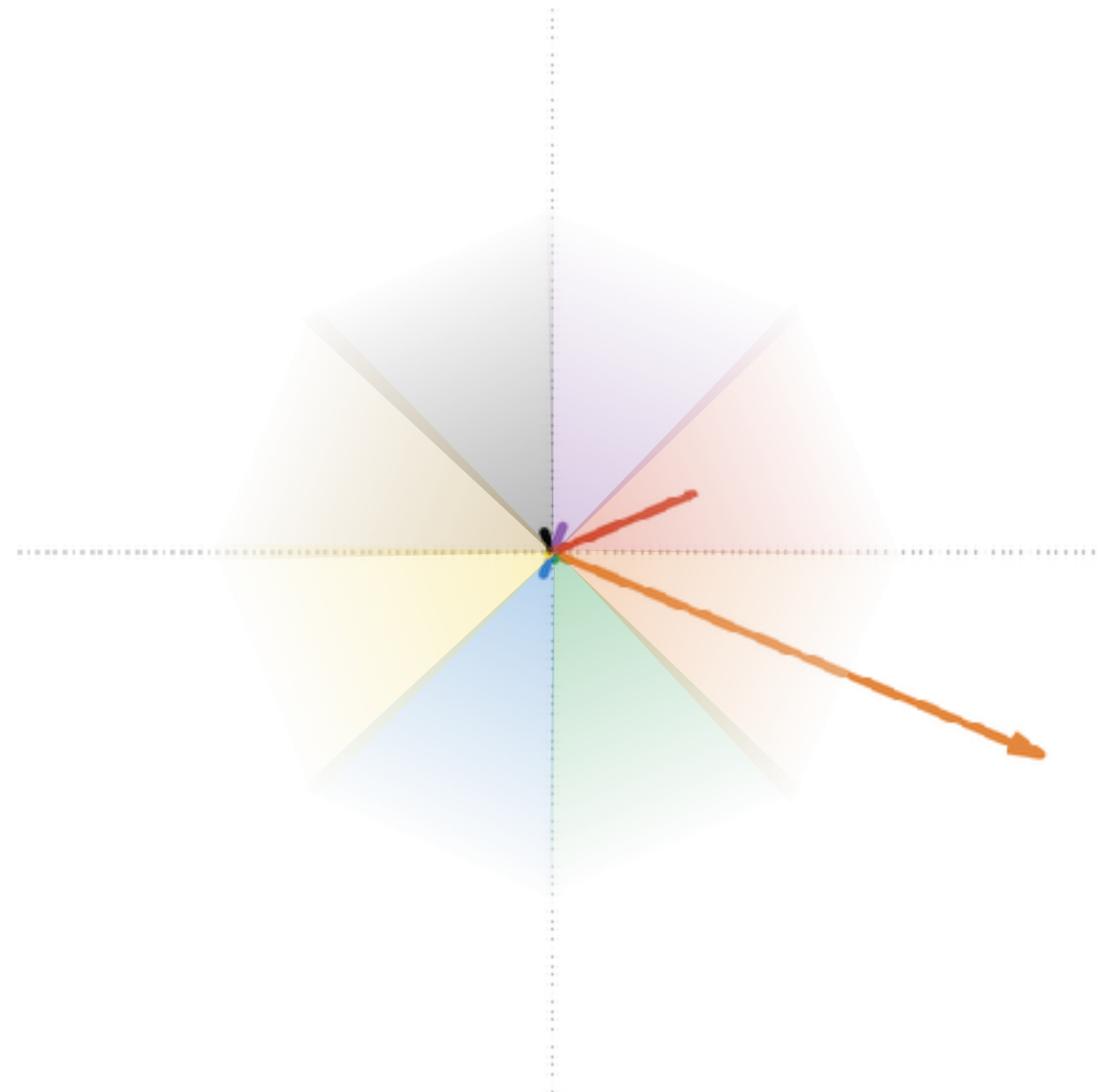
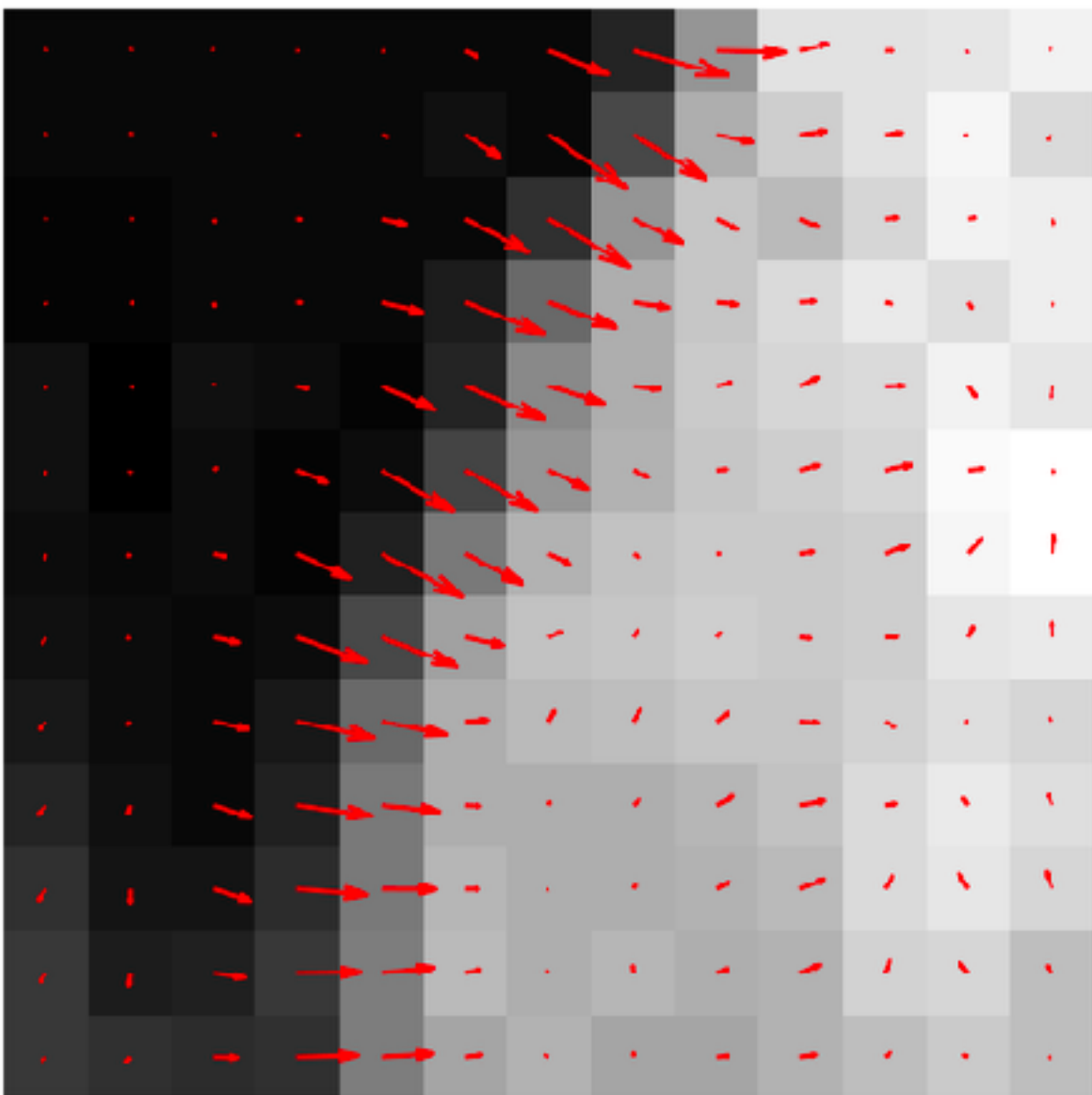
Gradient Histogram



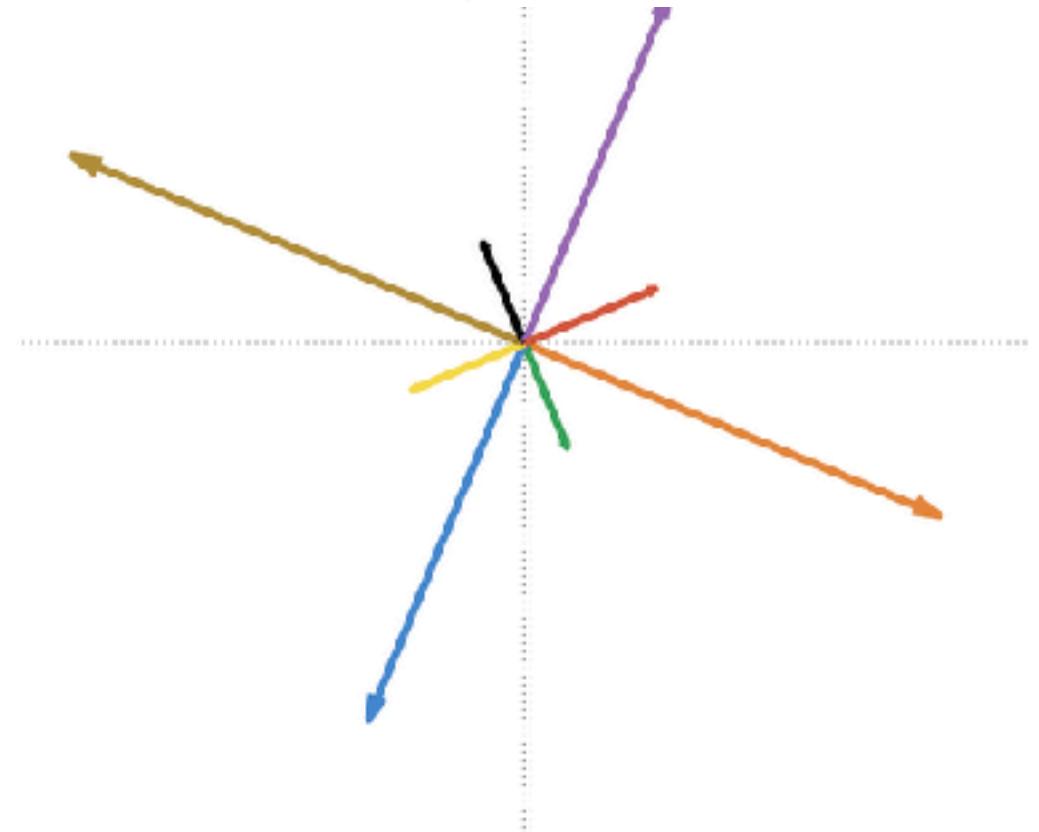
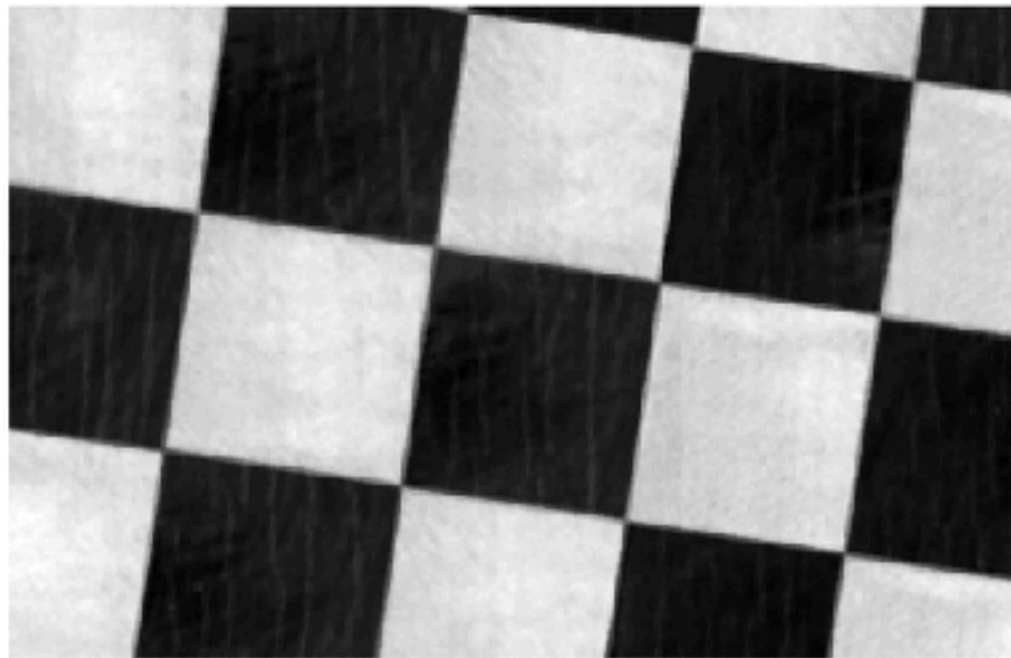
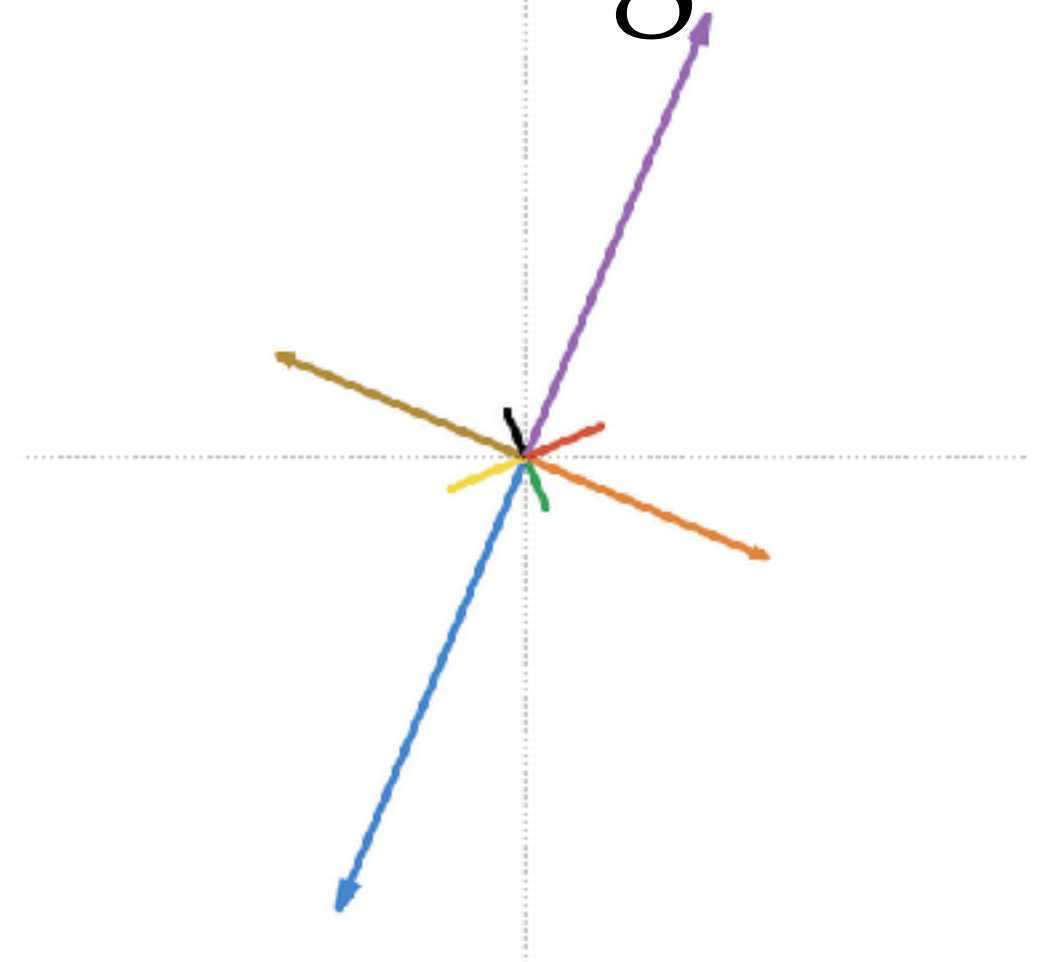
Gradient Histogram



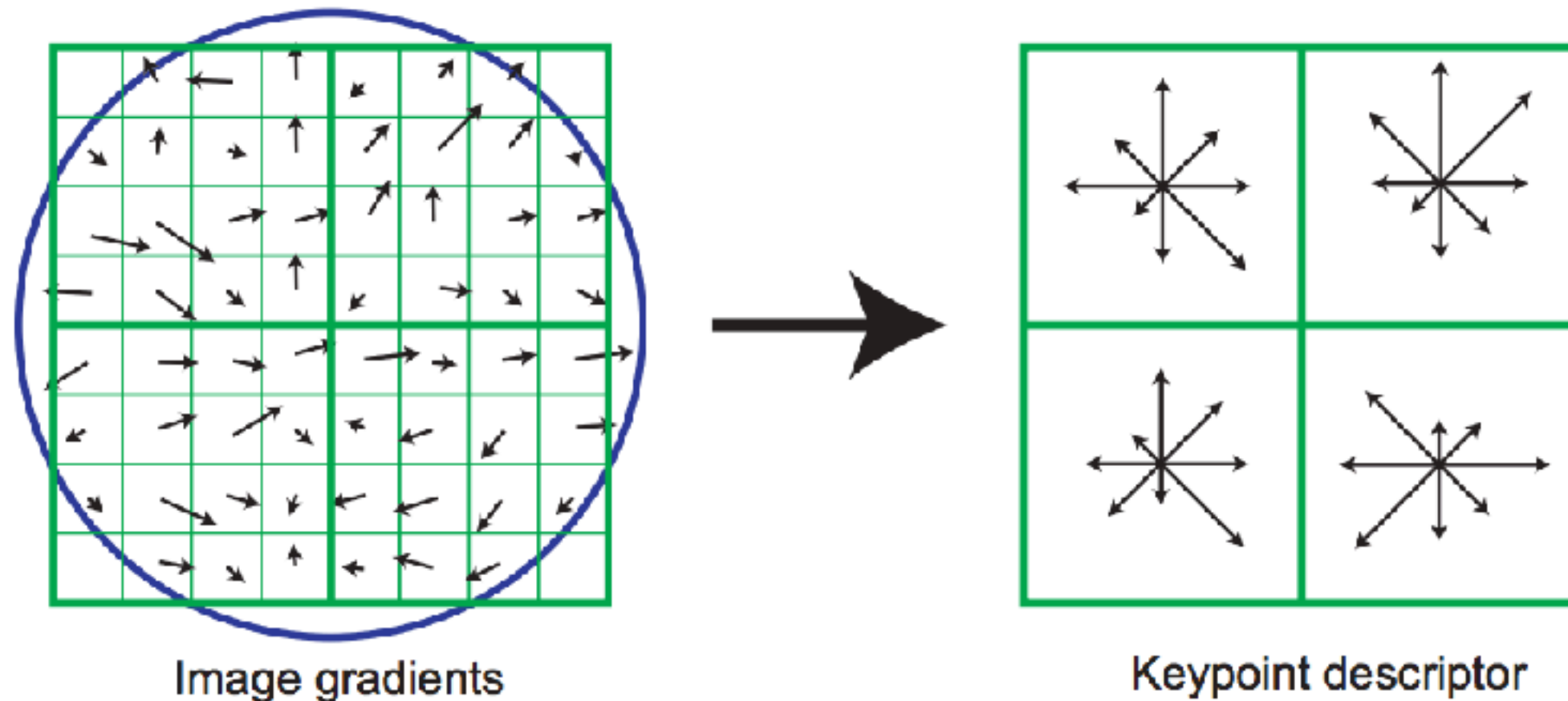
Gradient Histogram



Gradient Histogram



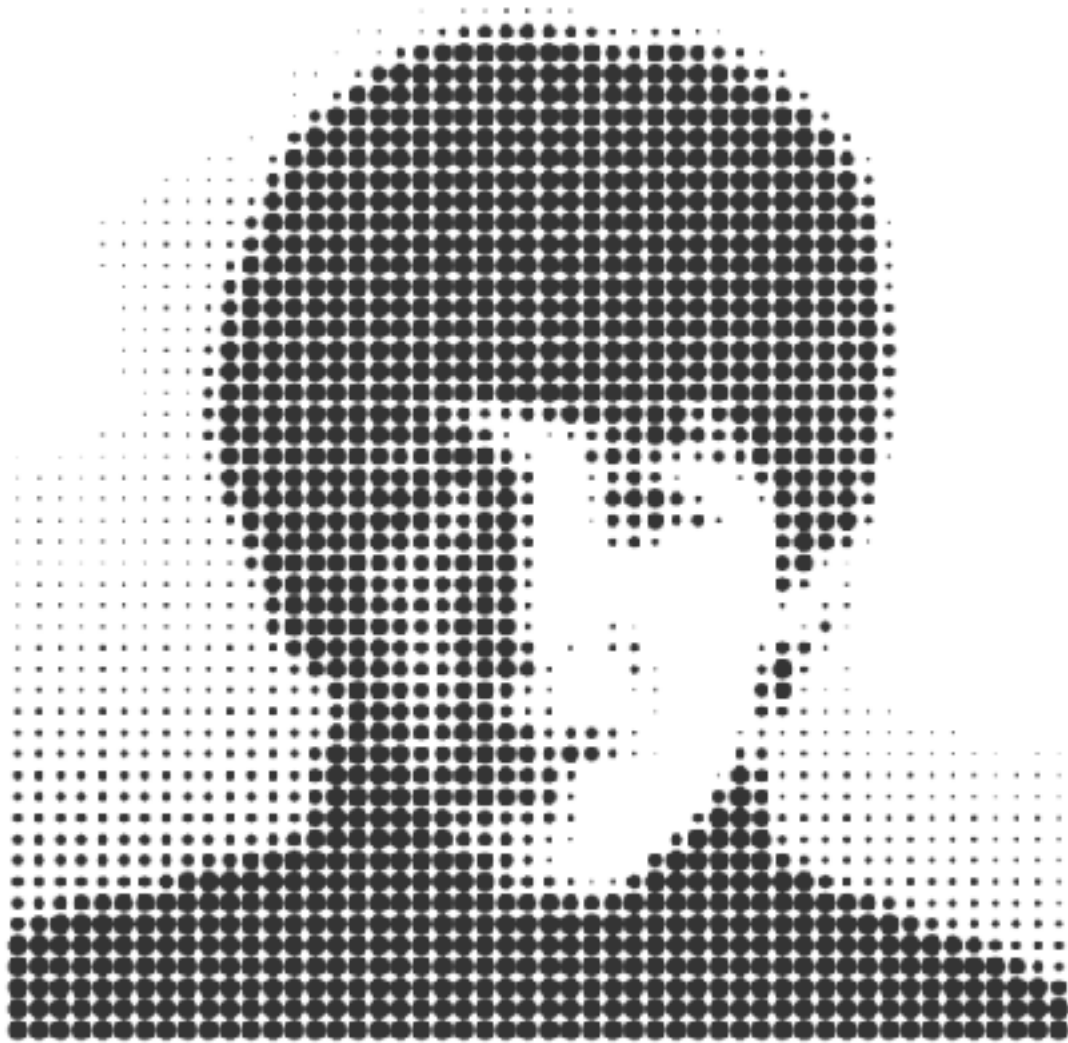
SIFT / HOG



Known as **SIFT** or Histogram of Oriented Gradients (**HOG**)
More details in next lecture

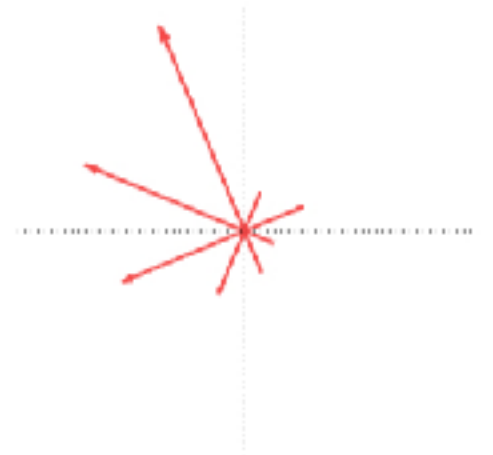
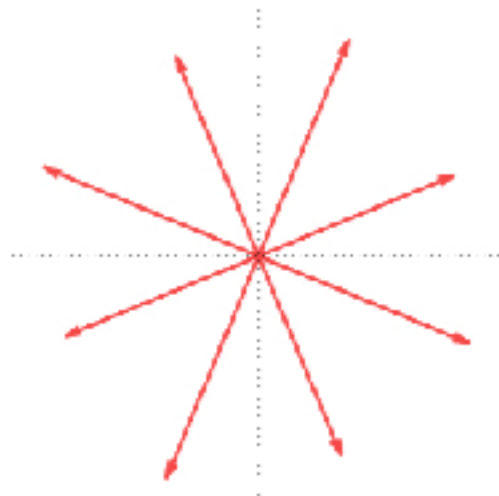
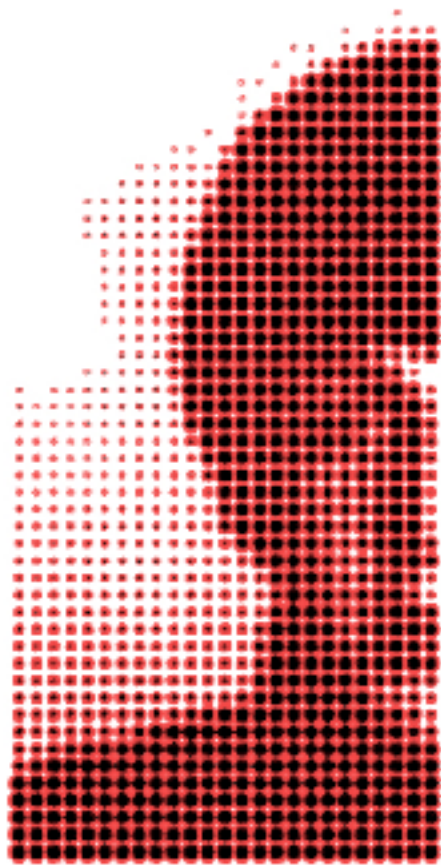
Multi-Scale Processing

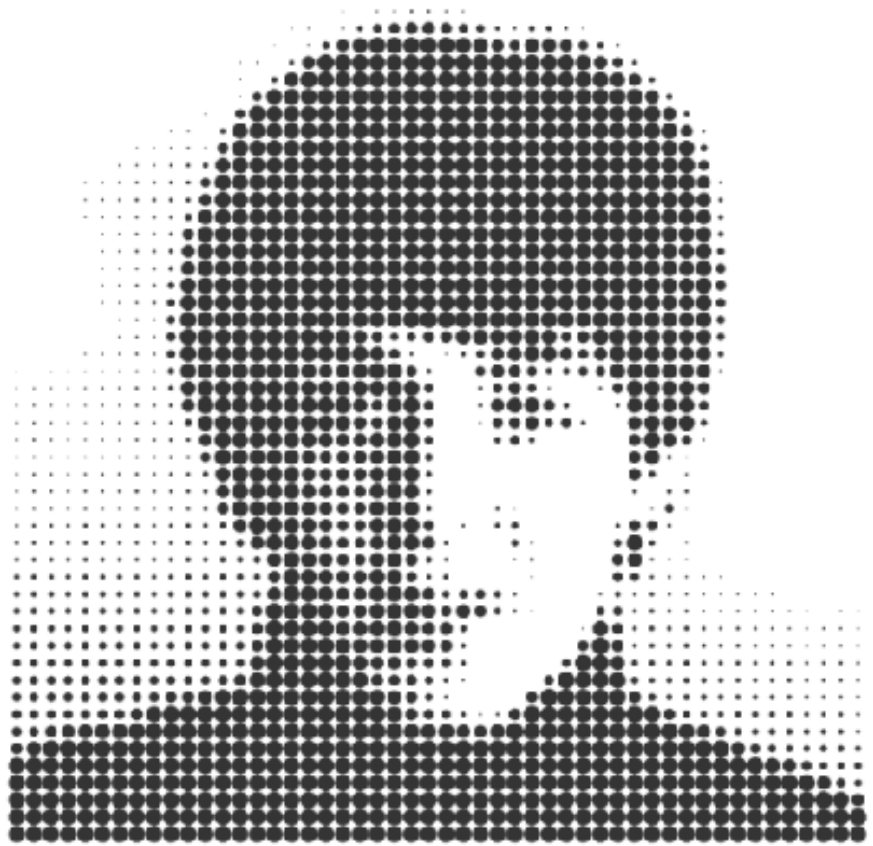
Scale

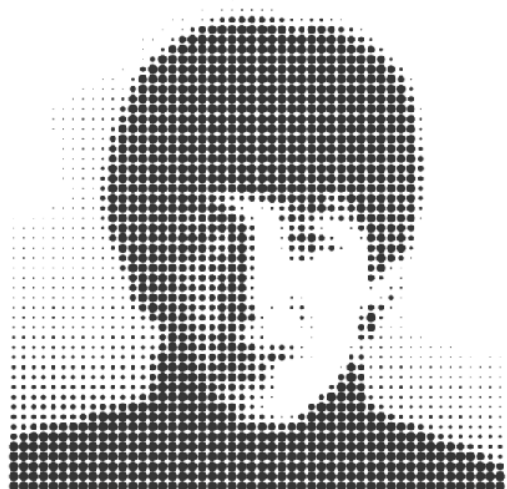
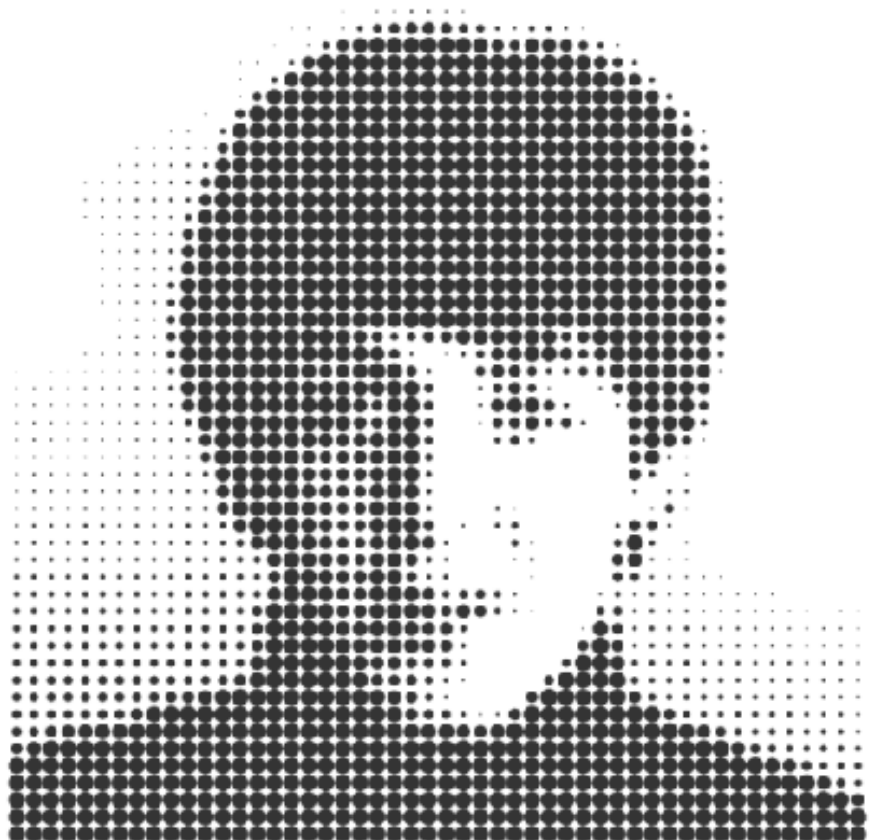


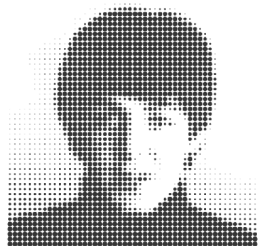
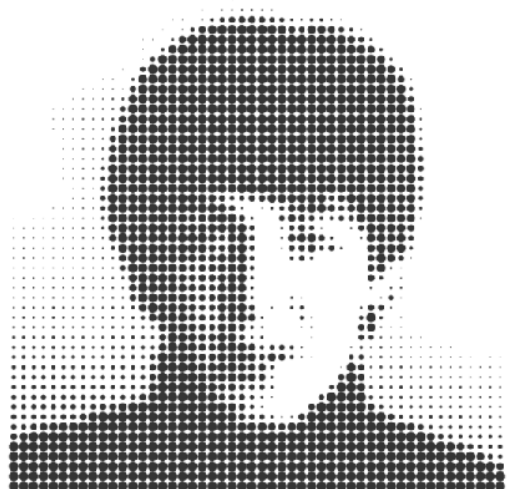
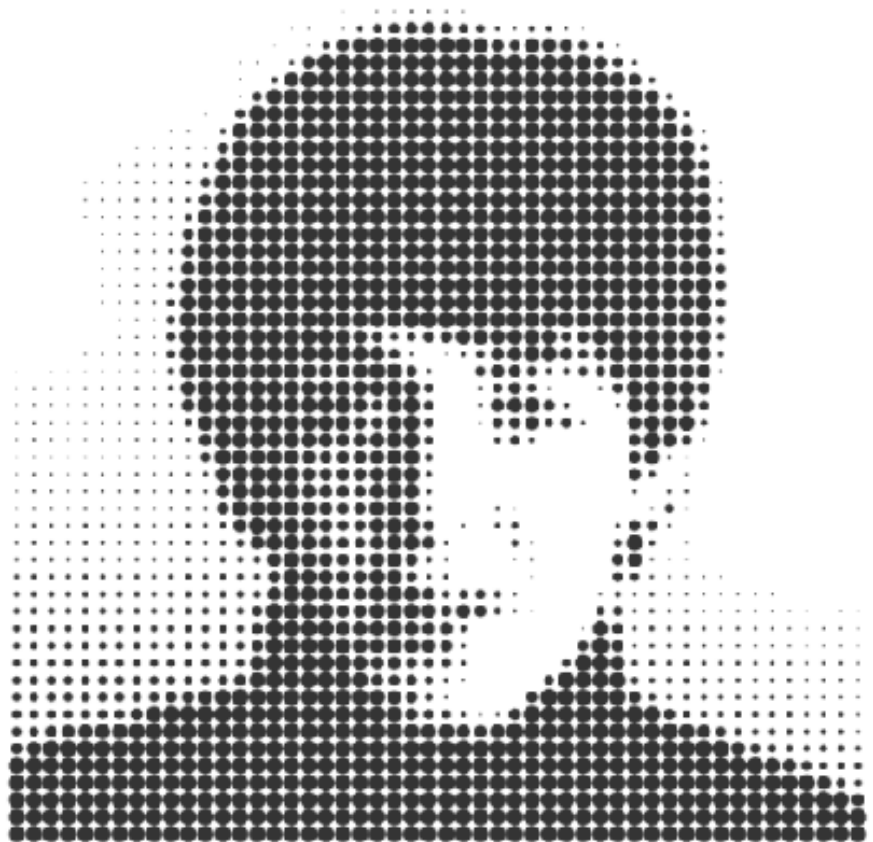
How to recognize objects at different distances?

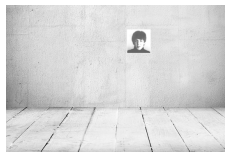
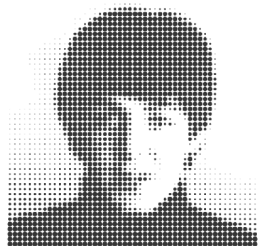
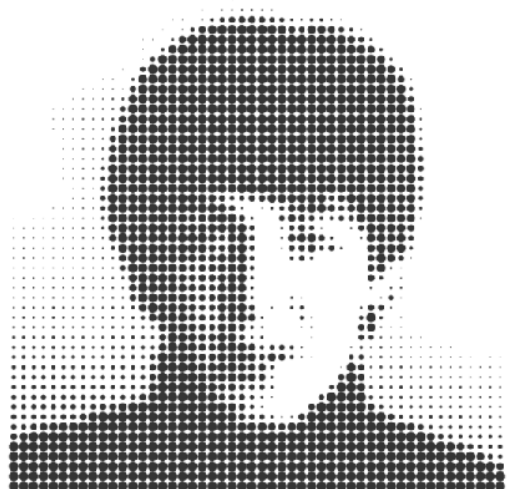
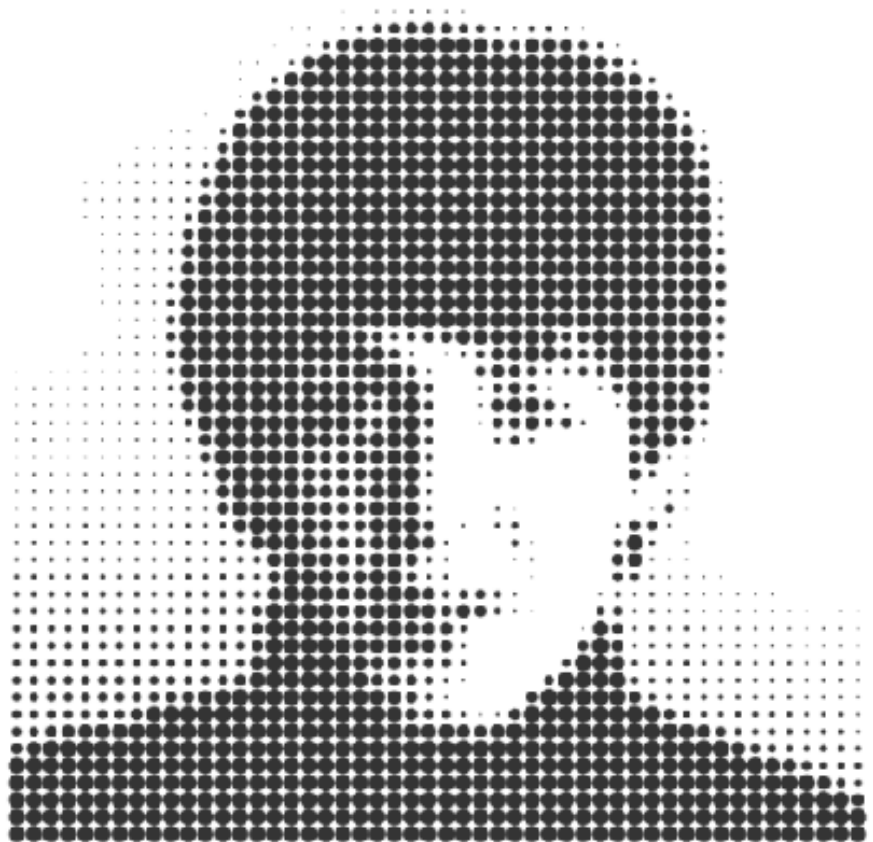
Gradient histogram

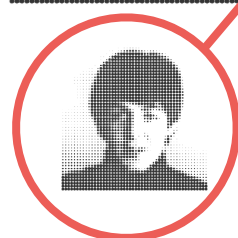
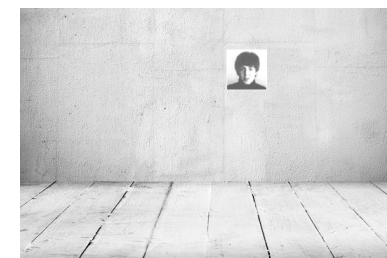
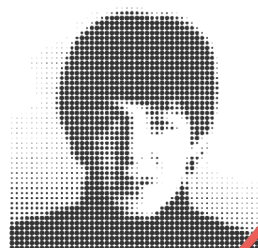
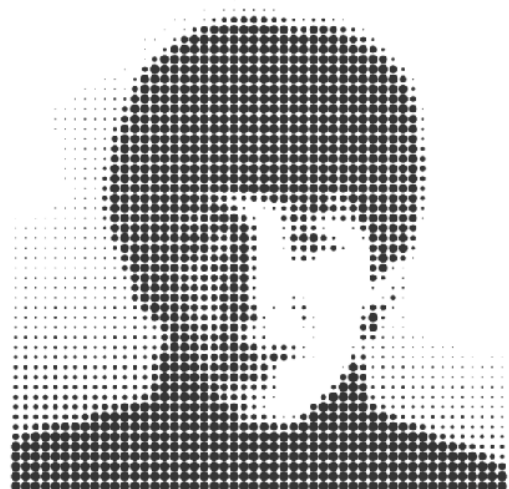
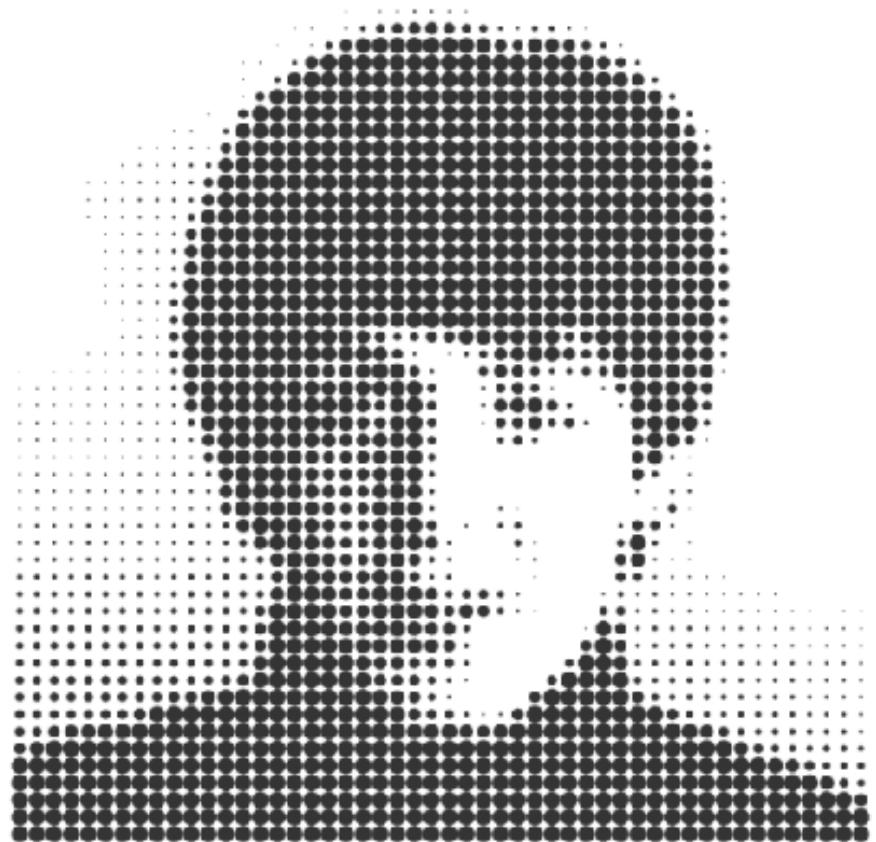




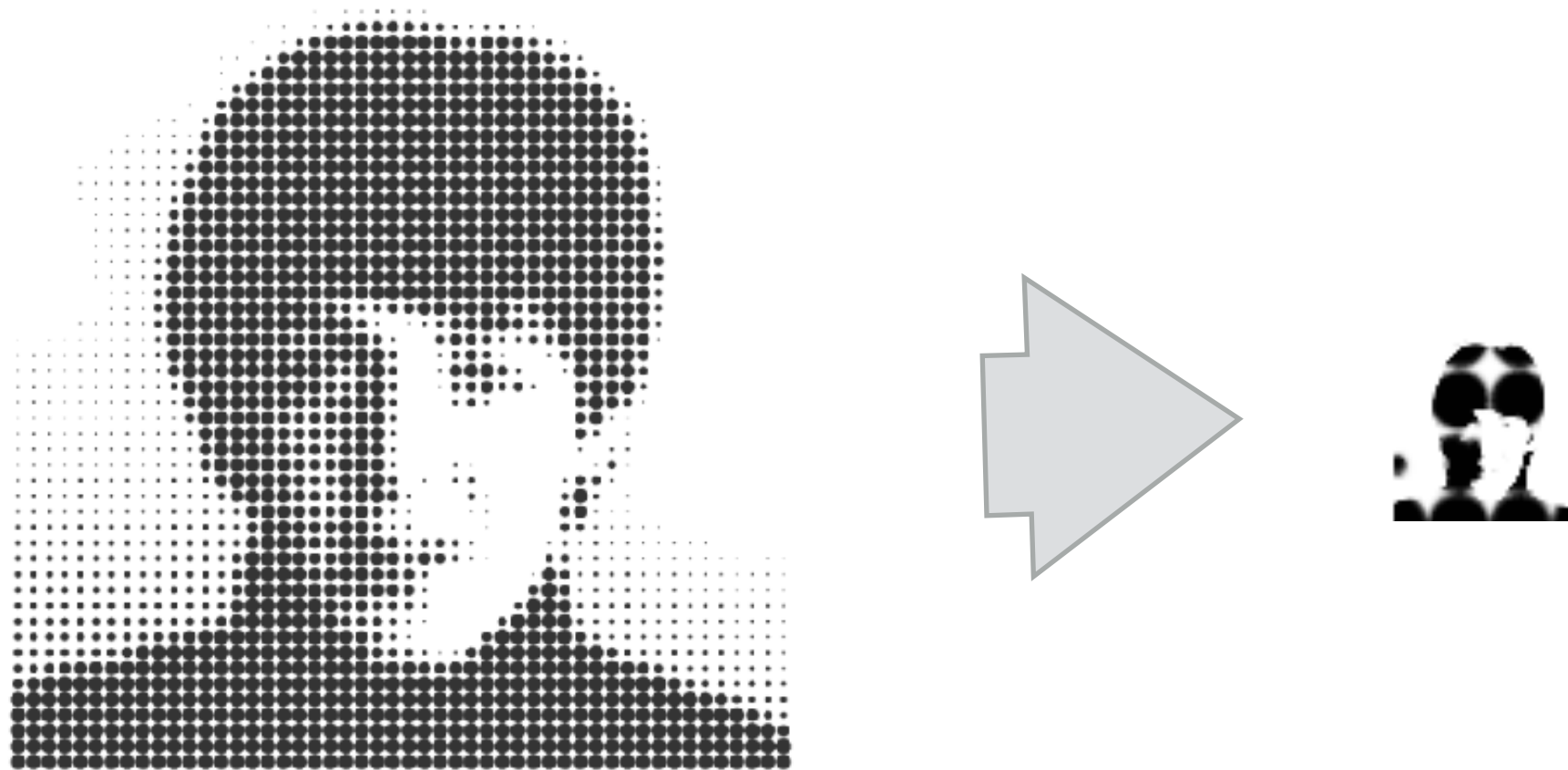




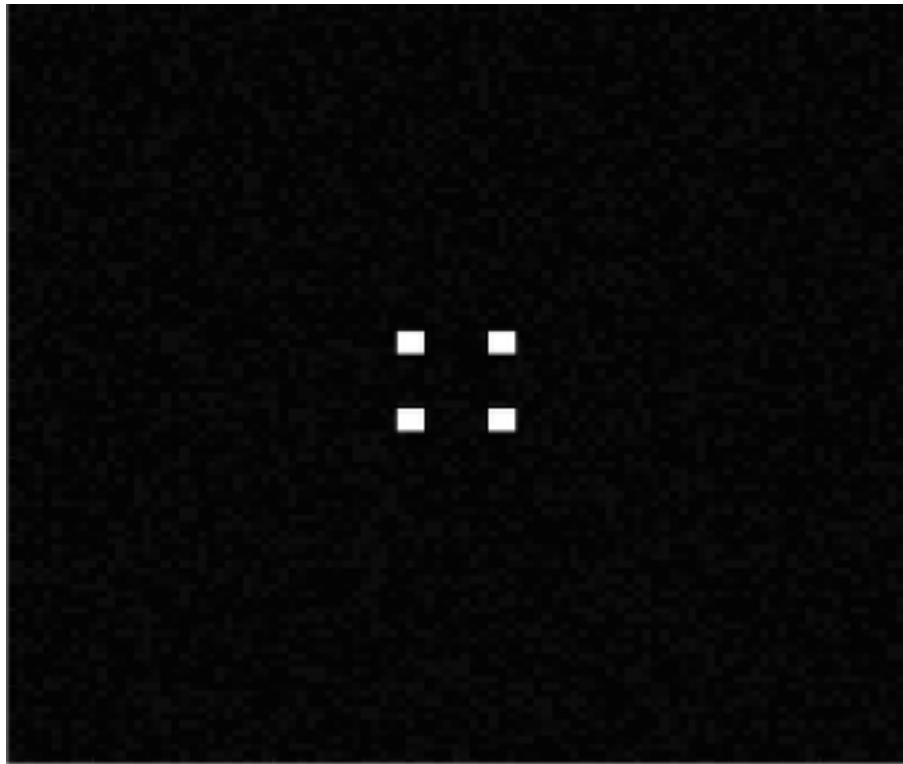




Downsampling?



Averaging

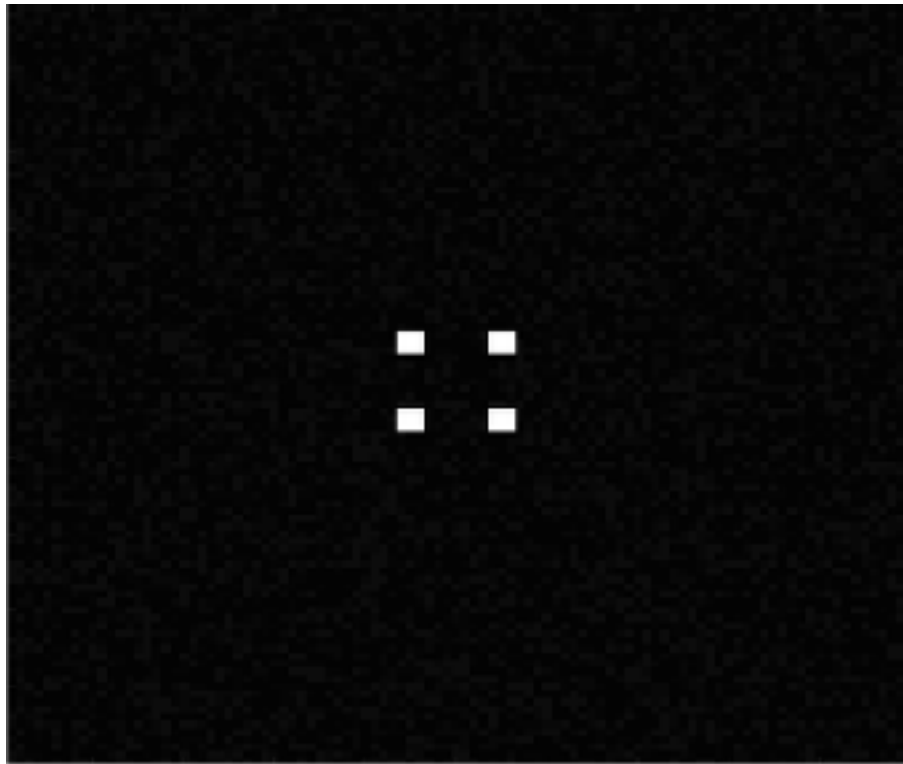


*

$$\frac{1}{25}$$

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

Averaging

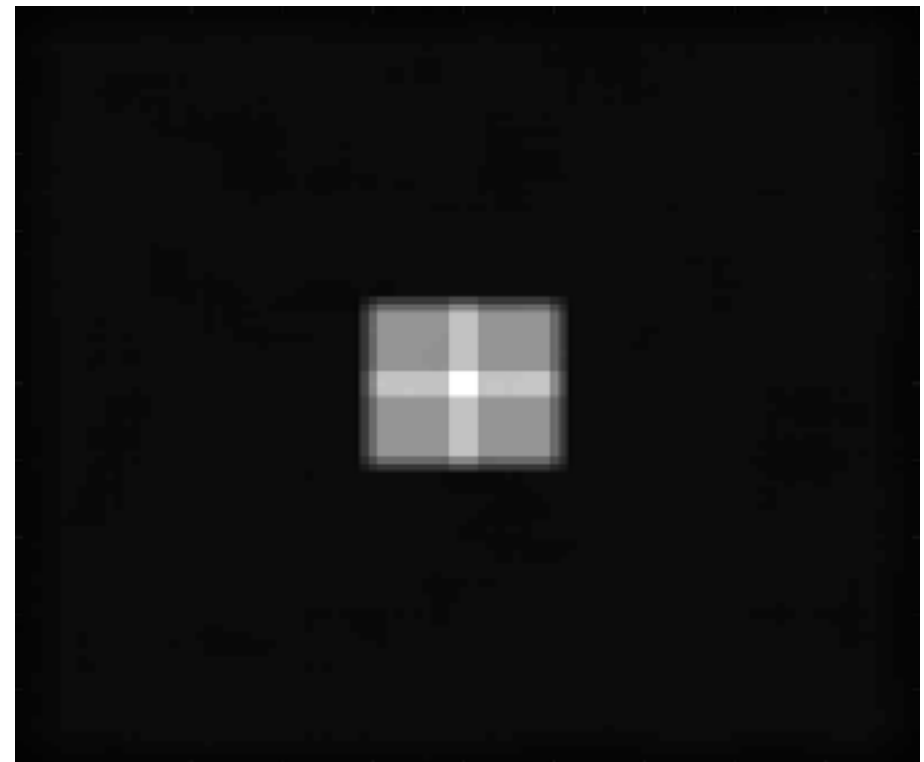


*

$$\frac{1}{25}$$

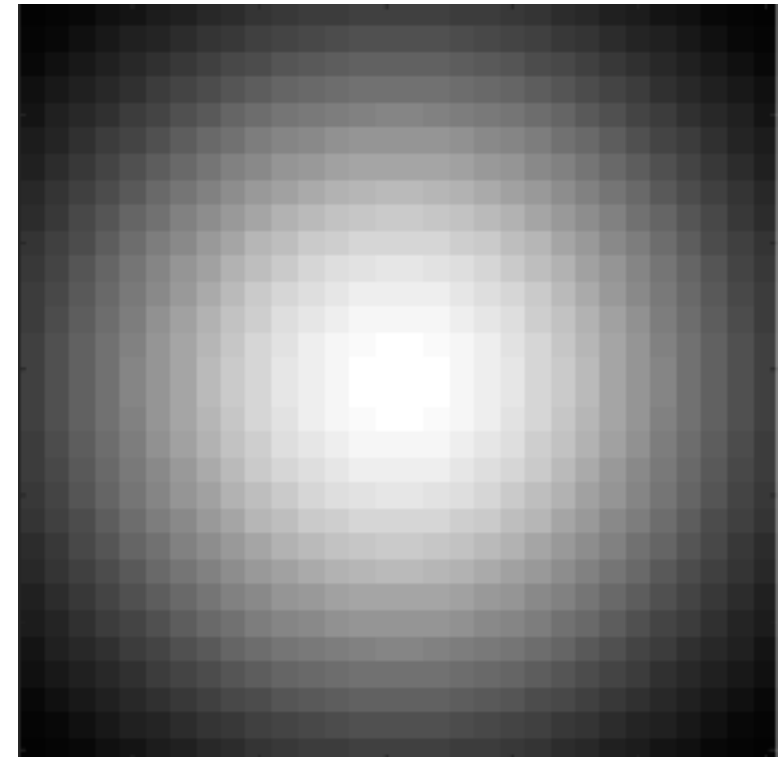
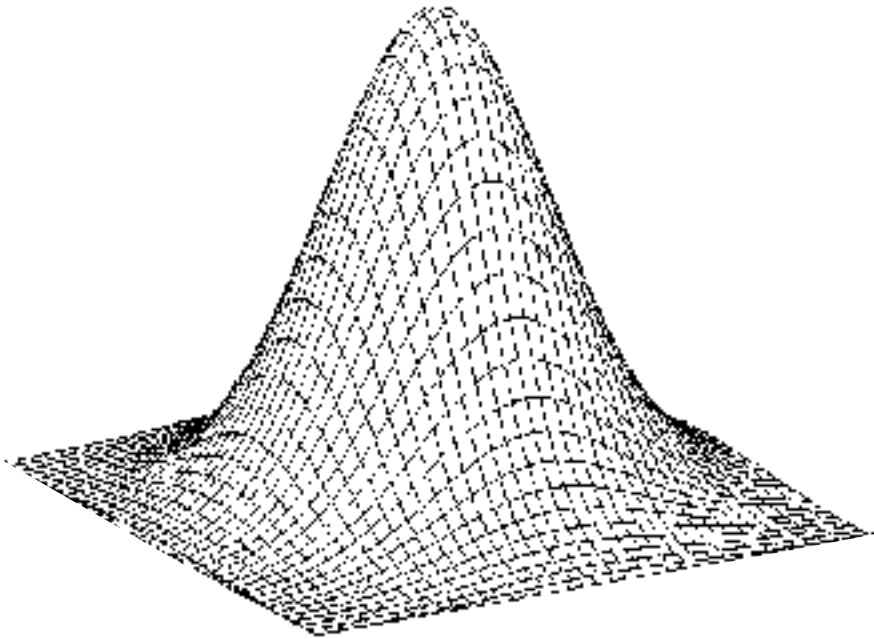
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

=

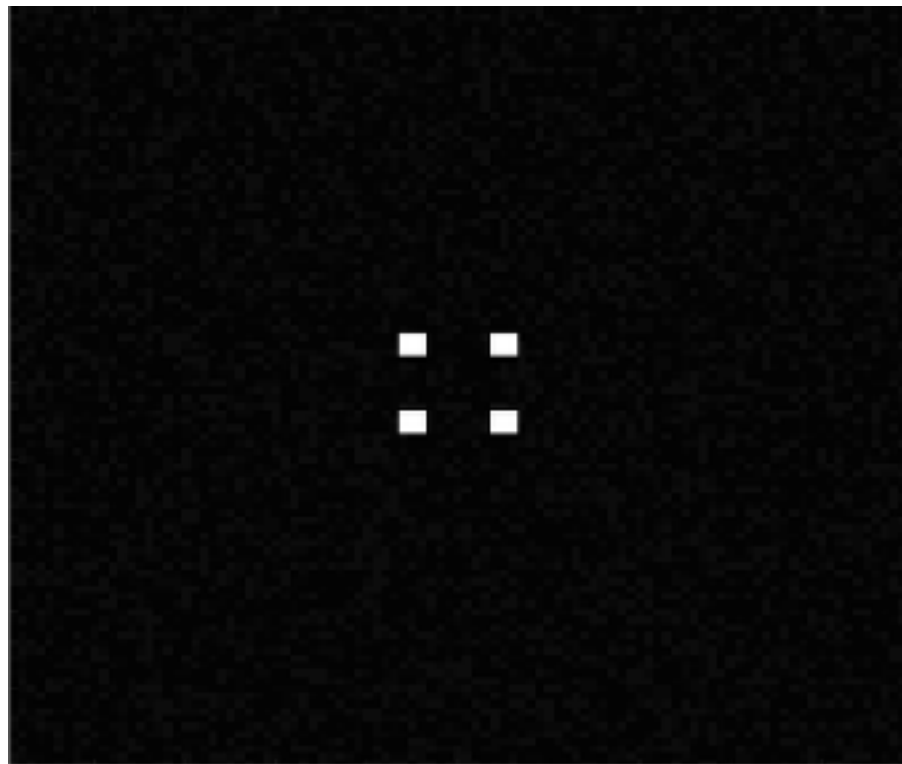


Gaussian Filter

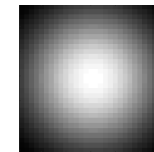
$$a \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right)$$



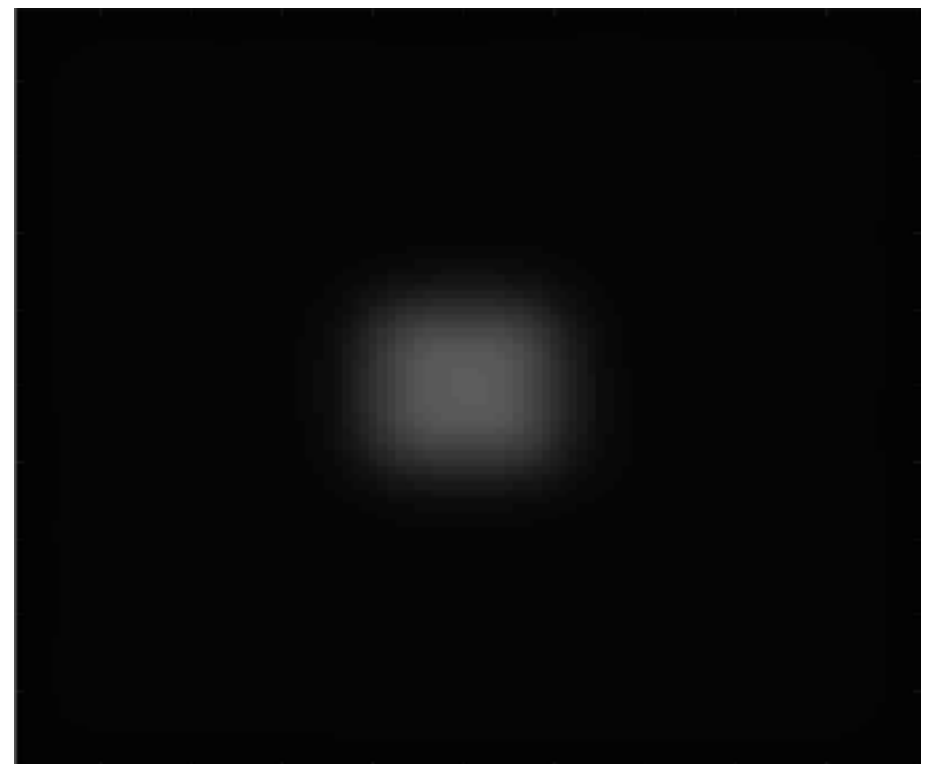
Gaussian Filter



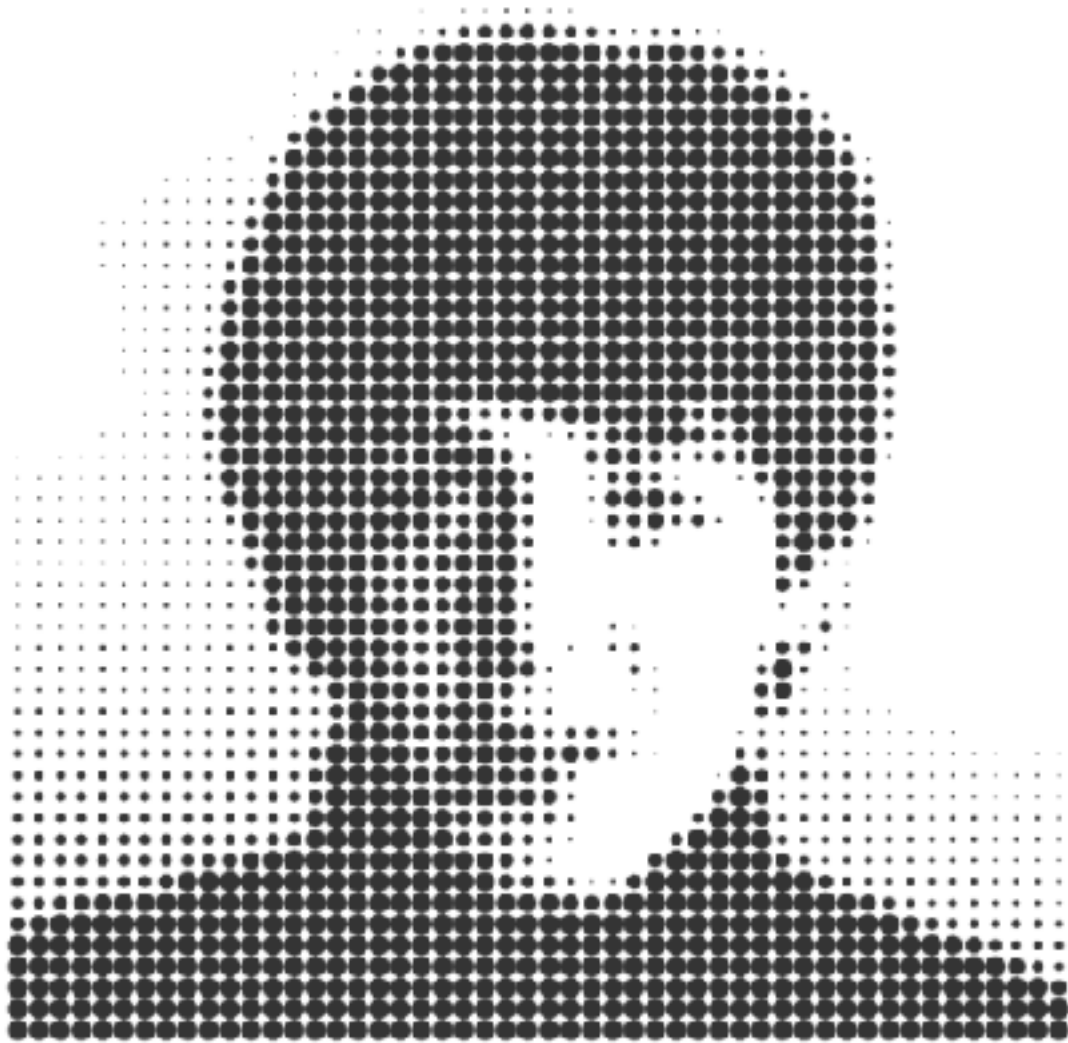
*



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Scale



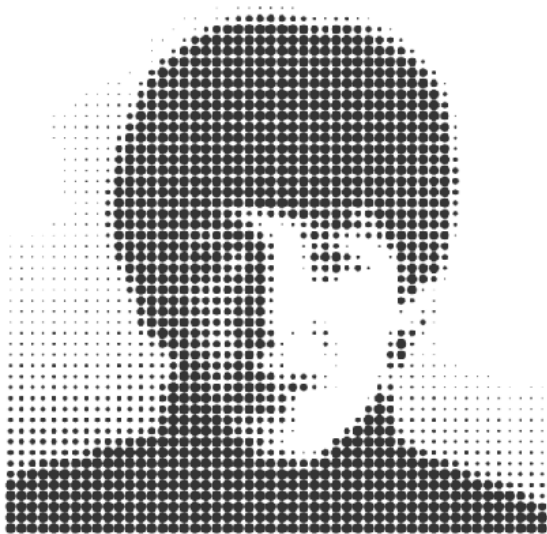
Scale



Scale Space Representation

$$L(x, y, \sigma^2)$$

$$L(x, y, 0.5^2)$$



$$L(x, y, 8^2)$$



$$L(x, y, 16^2)$$

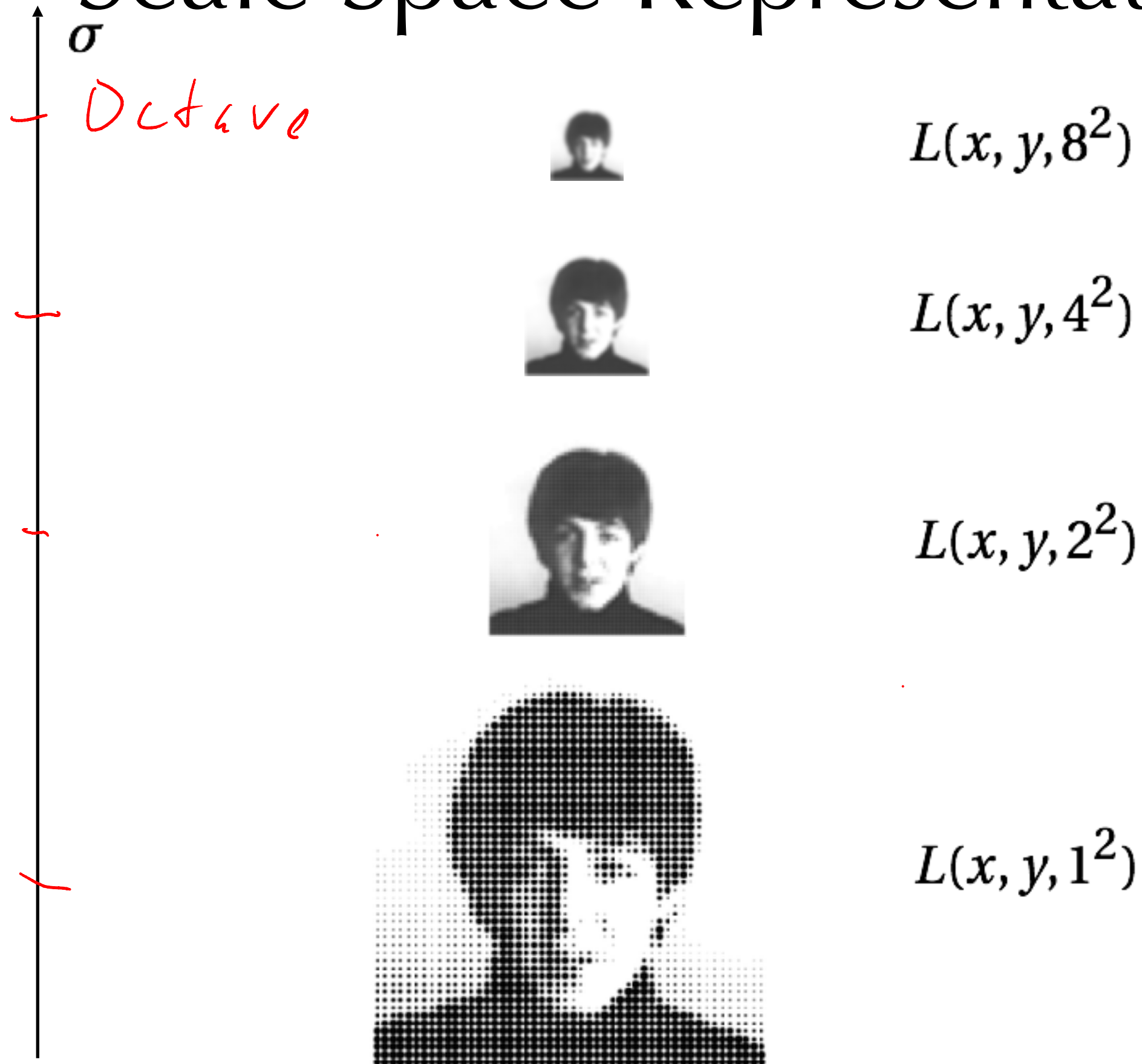


$$L(x, y, 4^2)$$

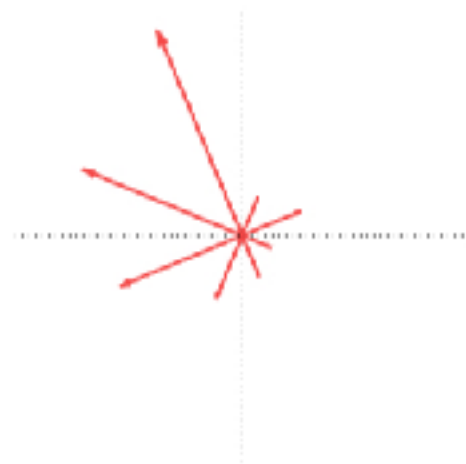
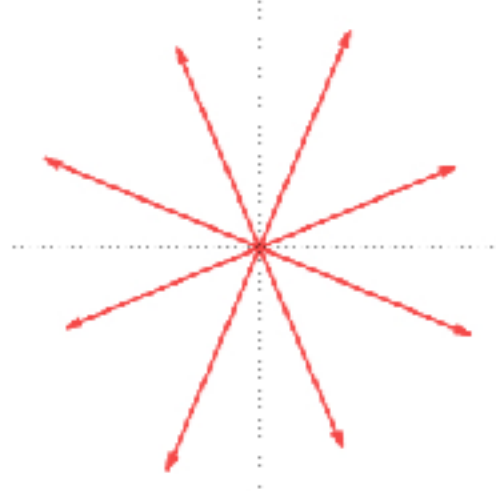


$$L(x, y, 12^2)$$

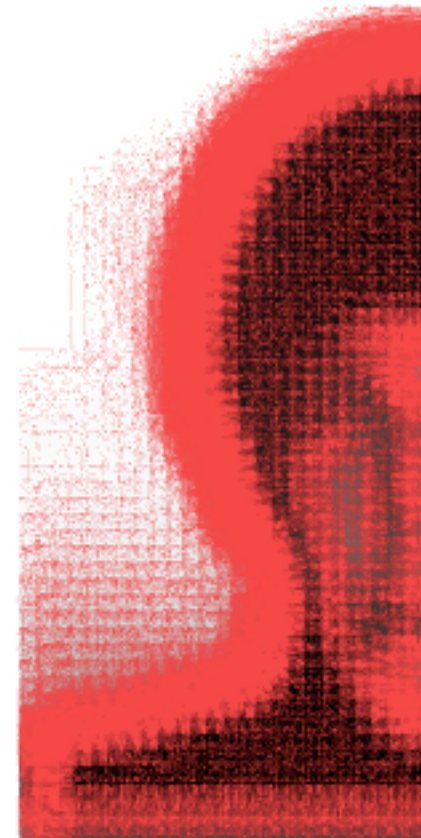
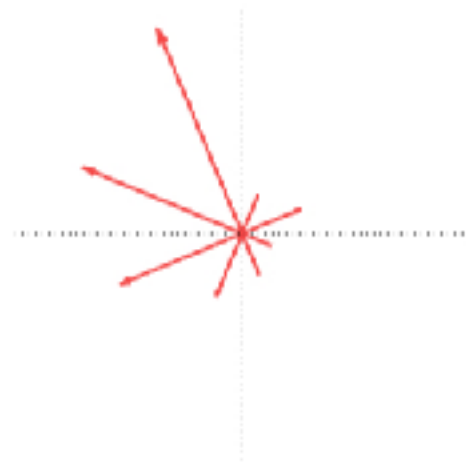
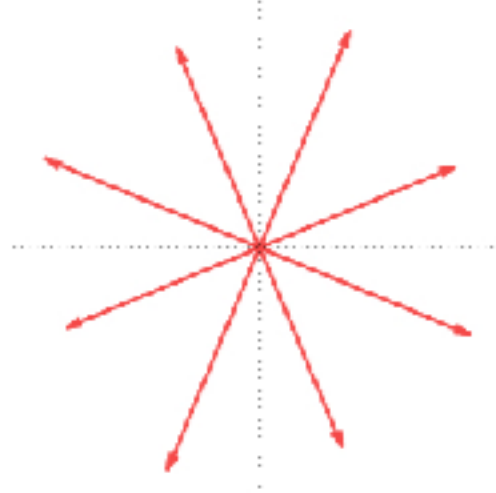
Scale Space Representation



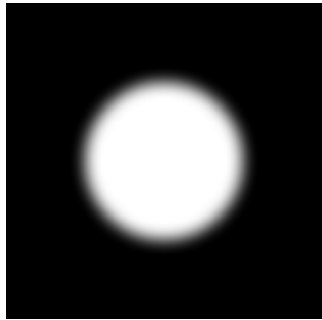
Gradient Histograms



Gradient Histograms

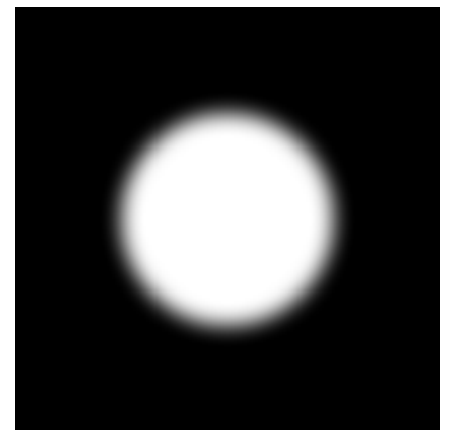
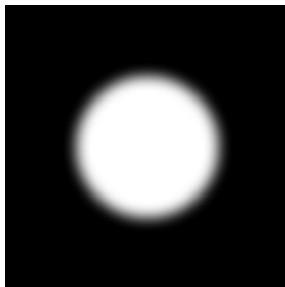
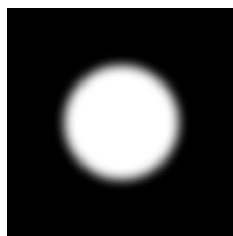


Estimating Scale



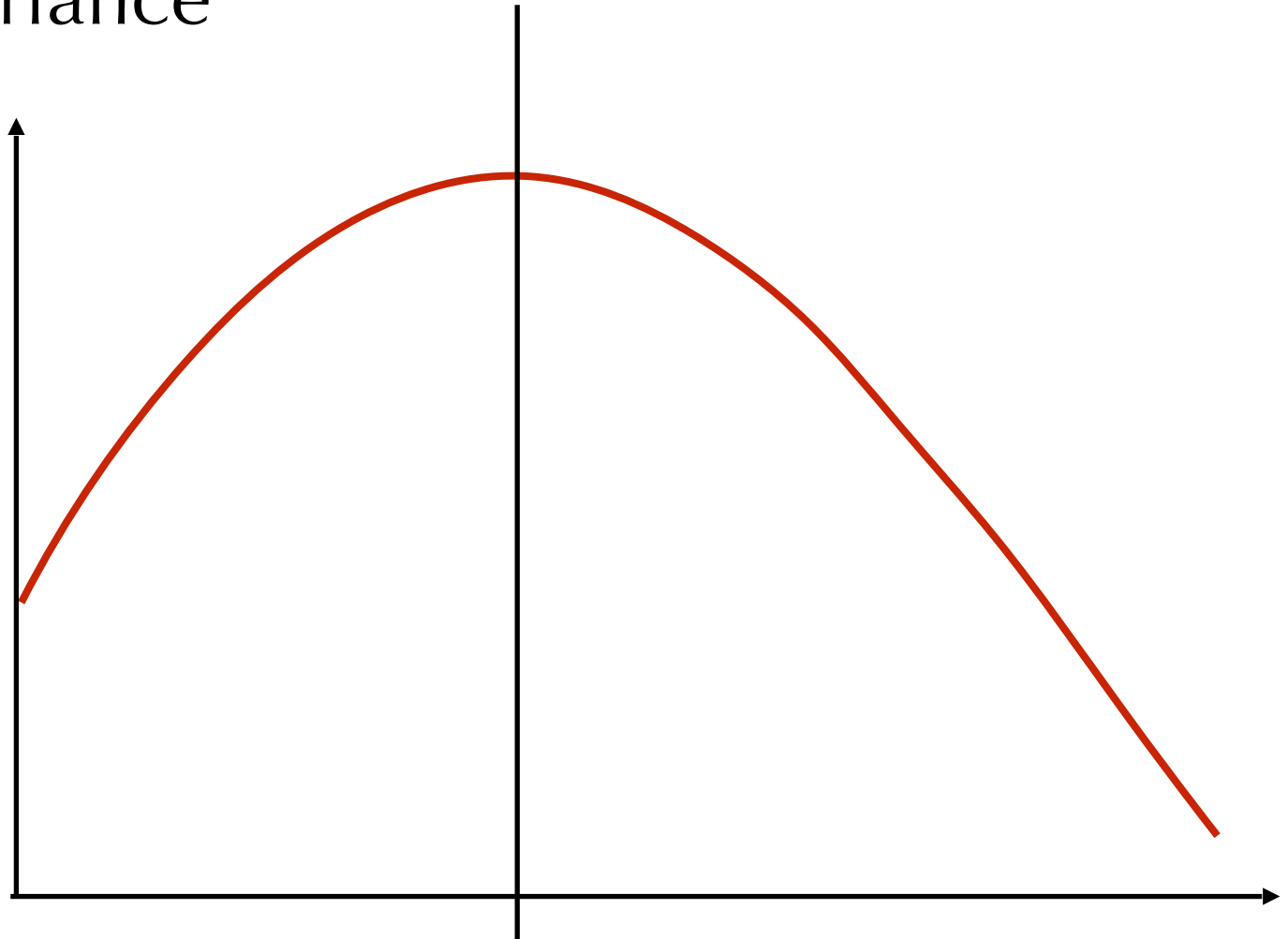
Query. $r = 67$

Templates:



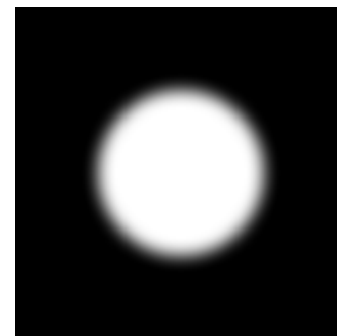
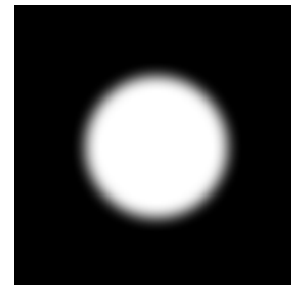
Estimating Scale

Covariance

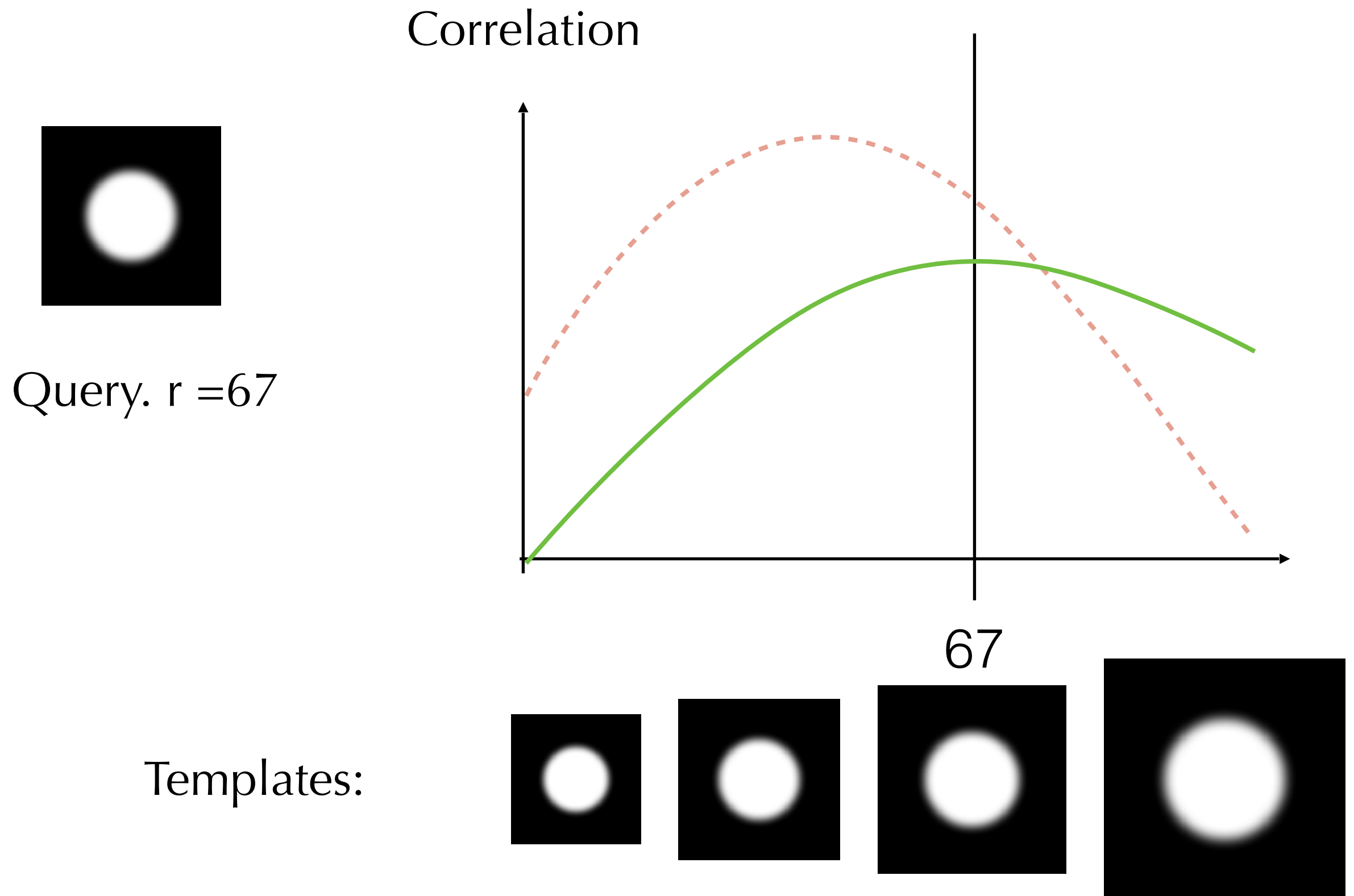


Query. $r = 67$

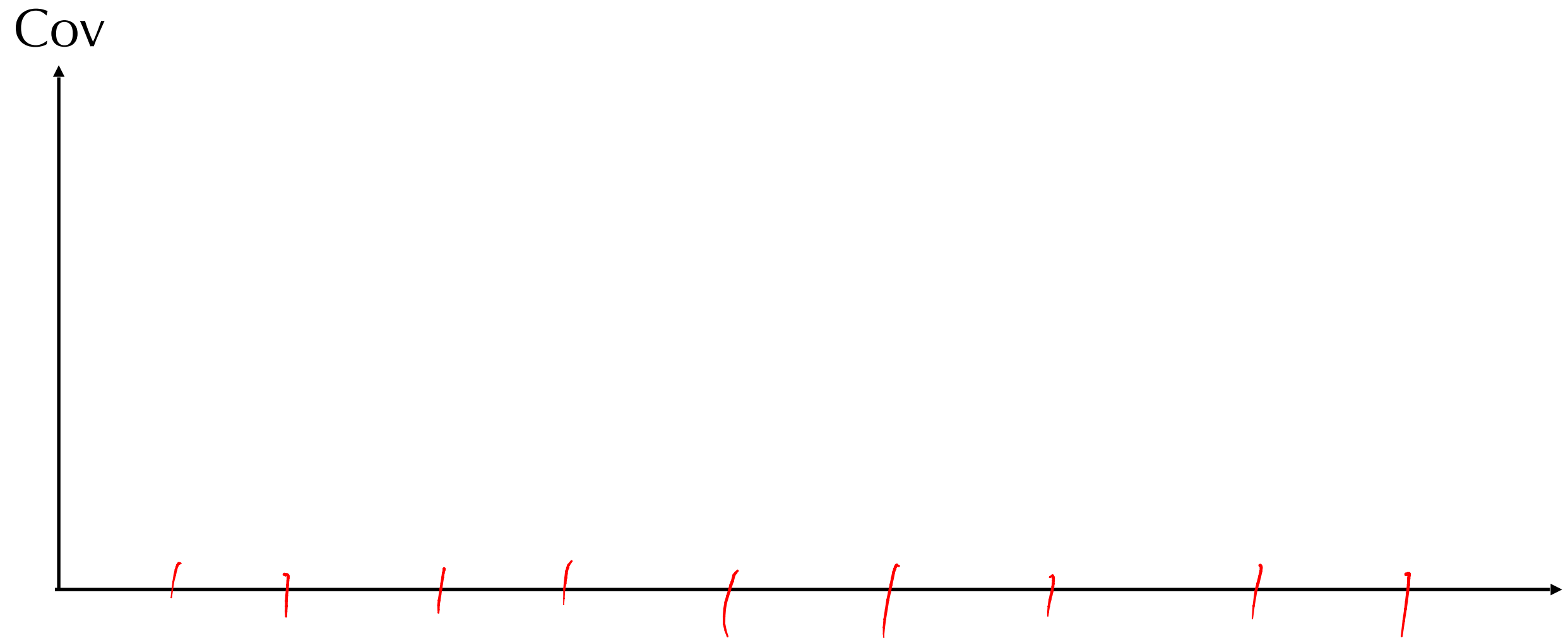
Templates:



Estimating Scale



Estimating Relative Scale

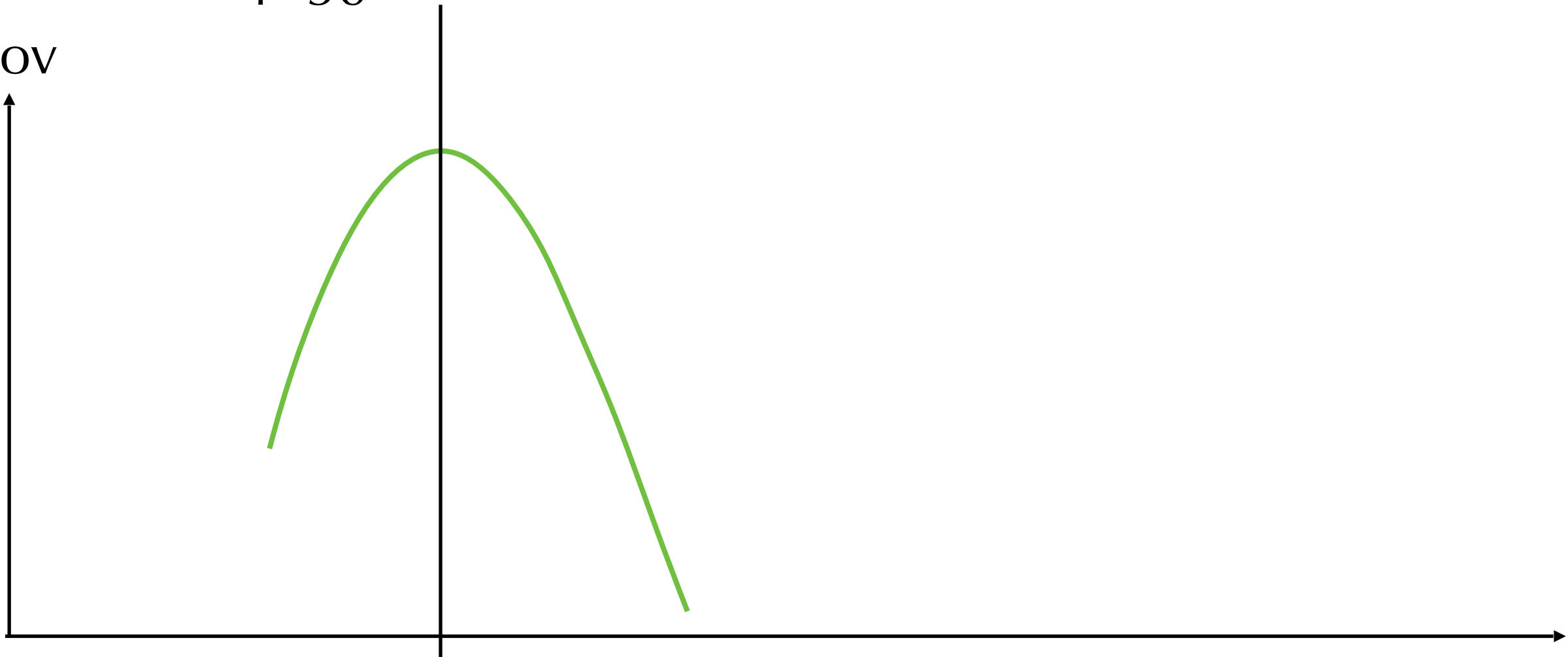


Estimating Relative Scale



$r=50$

Cov



45

Estimating Relative Scale

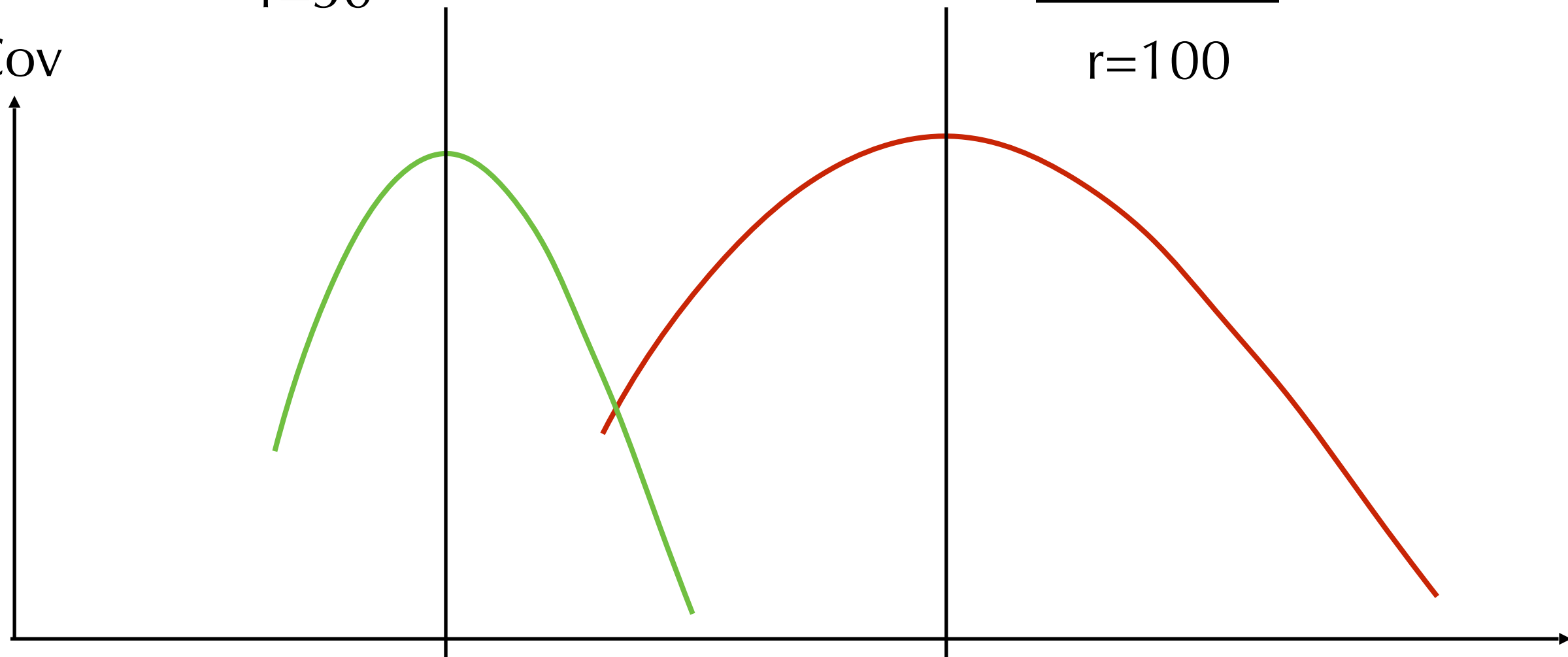


$r=50$



$r=100$

Cov

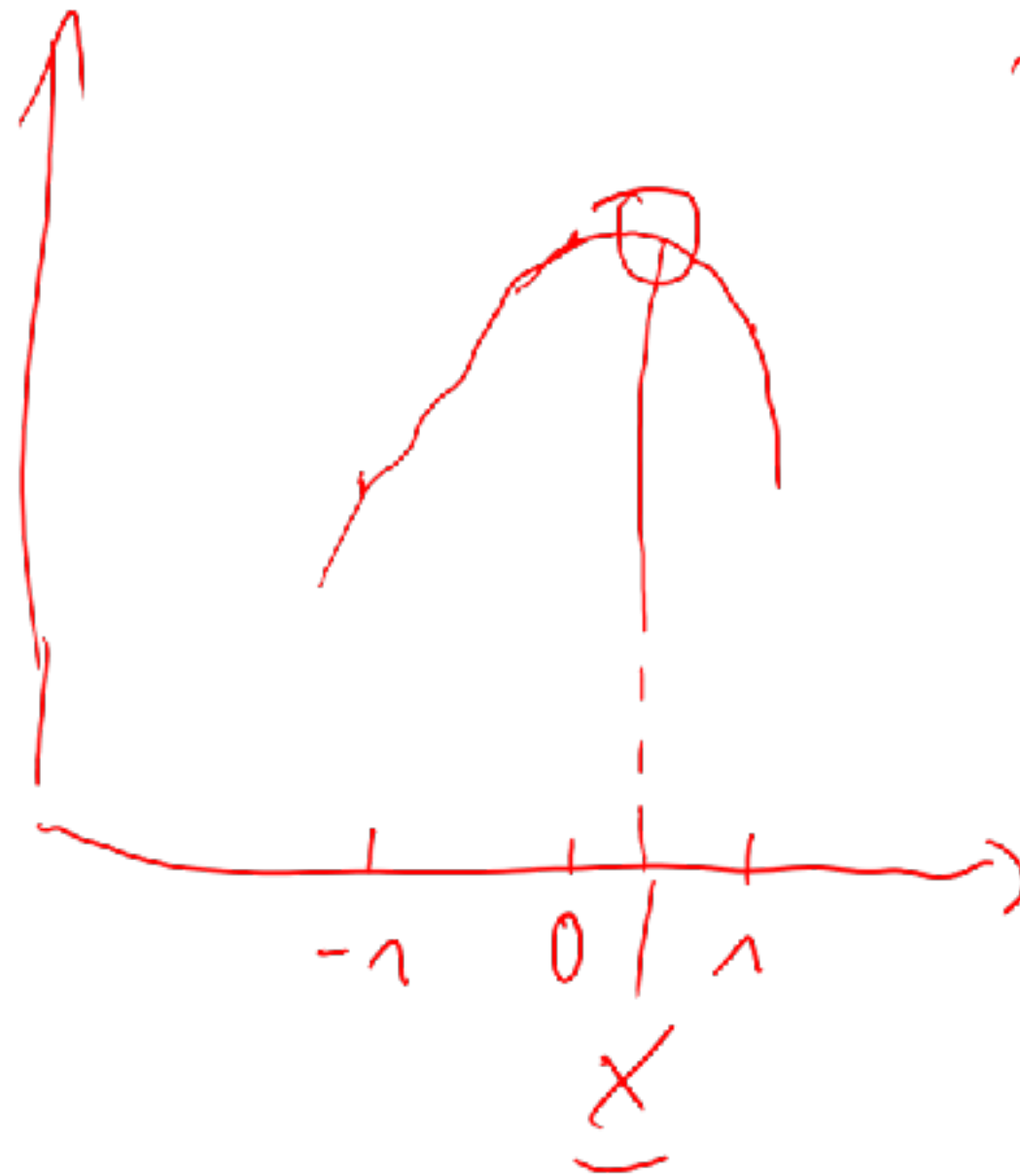


45

90

θ

“Sub-Pixel” Precision



2nd order
curve
fitting

``Sub-Pixel'' Precision

2nd order Taylor expansion

$$f(x) \approx f(0) + x f'(0) + \frac{x^2}{2} f''(0)$$

$$f'(0) \approx \frac{f(1) - f(-1)}{2} = a$$

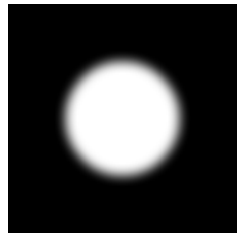
$$f''(0) \approx \underbrace{f(1) - 2f(0) + f(-1)} = b$$

discrete Laplacian

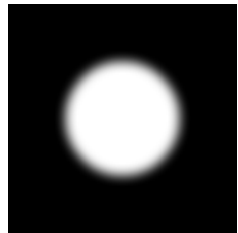
$$\max \quad f(x) \approx f(0) + ax + b \frac{x^2}{2}$$

$$0 \stackrel{!}{=} f'(x) = a + bx \Rightarrow x = -\frac{a}{b}$$

Detect Scale and Position



Detect Scale and Position



1	0	1	2	3	4	2
2	7	8	2	3	3	3
2	3	1	4	5	6	2
2	7	12	15	20	7	2
2	8	12	25	19	11	3
3	9	11	18	16	7	1
1	2	3	2	1	3	1

2	1	2	2	2	3	3
1	2	3	2	2	3	4
5	11	9	8	9	8	1
1	11	15	20	22	9	5
2	12	16	35	22	11	4
3	9	15	18	21	7	1
2	2	9	8	7	9	1

3	3	3	3	3	3	3
3	4	4	4	4	4	3
3	5	6	6	6	5	2
2	7	12	12	12	7	2
2	5	12	20	13	11	3
3	9	12	13	13	9	1
1	2	5	5	5	3	1

Detect Scale and Position



1	0	1	2	3	4	2
2	7	8	2	3	3	3
2	3	1	4	5	6	2
2	7	12	15	20	7	2
2	8	12	25	19	11	3
3	9	11	18	16	7	1
1	2	3	2	1	3	1

2	1	2	2	2	3	3
1	2	3	2	2	3	4
5	11	9	8	9	8	1
1	11	15	20	22	9	5
2	12	16	35	22	11	4
3	9	15	18	21	7	1
2	2	9	8	7	9	1

3	3	3	3	3	3	3
3	4	4	4	4	4	3
3	5	6	6	6	5	2
2	7	12	12	12	7	2
2	5	12	20	13	11	3
3	9	12	13	13	9	1
1	2	5	5	5	3	1

Larger than neighbors in image **and scale!**

Sub-Pixel Refinement

Sub-Pixel Refinement

Sub-Pixel Refinement

Lessons Learned

- Main lessons from this lecture
 - Image filters: Gaussian, gradients (edge detection)
 - Similarity measures: ZNCC, histogram of gradients
 - Scale space: Gaussian pyramid by iterative Gaussian Filtering
 - Sub-pixel refinement: Fitting 2nd order surfaces

Lessons Learned

- Main lessons from this lecture
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 - Similarity measures: ZNCC, histogram of gradients
 - Scale space: Gaussian pyramid by iterative Gaussian Filtering
 - Sub-pixel refinement: Fitting 2nd order surfaces
- Next lecture: Local features