Lab 3: Bonus Questions

- There are two sets of bonus questions for this lab, each worth 3 points. In total, you can gain up to 18 points. Obtaining 12 out of the 18 points will improve your grade by one if you fulfill all requirements to pass the course.
- You have to work on the questions on your own and have to hand in your solutions individually.
- Your answers will be graded on a continuous scale, i.e., we try to reward partially correct answers.
- Please hand in your solutions as a PDF through Canvas.

1 RANSAC, 3 points

(a) Consider two minimal solvers for the same problem with the following properties:

- **Solver 1**: The solver requires $m = 3$ data points, takes 1 time unit to run, always returns 4 solutions[1], and evaluating one model on a single data point requires 0.01 time units.
- **Solver 2**: The solver requires $m = 4$ data points, takes 1 time unit to run, returns 1 solution if all data points are inliers and 0 solutions otherwise, and evaluating one model on a single data point requires 0.01 time units.

Derive equations for the expected run-time of RANSAC for each solver for a given inlier ratio ε, N data points, and the probability η for finding the best model. Assume that RANSAC requires exactly the maximum number of iterations for the given inlier ratio ε and probability η as predicted by the equation presented in the lecture.
Explain your solution.

(b) Let $\eta = 0.01$ and $N = 200$. Which of the two solvers would you prefer for inlier ratios of $\varepsilon = 0.1$, $\varepsilon = 0.2$, $\varepsilon = 0.5$, $\varepsilon = 0.8$, $\varepsilon = 0.9$?
Justify your answer.

(c) Consider the following scenario for 2D line fitting: We are given 10 sets of points $\mathcal{X}_i = \left(\mathbf{x} \in \mathbb{R}^2 \mid ||\mathbf{x} - (0, i \cdot 10)^T||_2 < \delta\right)$, each consisting of 2D points randomly distributed around a center point $(0, i \cdot 10)$, for $i = 1, \ldots, 10$. All center points are on a line and we assume we have 6 points per set, as well as an additional 40 points that are very far away ($> 100\delta$) from that line. For an inlier threshold of 3δ used by RANSAC, this results in an inlier ratio of $\varepsilon = 0.6$. For a probability $\eta = 0.01$ of missing the best line, the predicted number of RANSAC iterations for this scenario is thus 11 (rounding to the nearest larger integer).
Do you think the number of RANSAC iterations is a good estimate for the number of iterations it takes to find a line with 60 inliers through randomly sampling two points? Justify your answer.

[1] Out of the 4 solutions, at most one will be correct. The other solutions are introduced by the way the solver models the problem. However, you do not know which model is correct (i.e., the order in which the models are generated is arbitrary).

2 Model Fitting & Minimal Solvers, 3 points

(a) Consider a transformation of the form

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} a & 0 & -b \\ 0 & 1 & 0 \\ c & 0 & a \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} 0 \\ t_y \\ t_z \end{pmatrix} \tag{1}$$

that transforms one set of 3D points into another. Explain how to construct a solver that can be used inside RANSAC to estimate a transformation between two sets of 3D points. To this end, derive a linear system $\mathtt{M}\theta = \mathbf{v}$, where the vector θ contains the parameters of the transformation. What is the minimal number of correspondences that are needed by the solver to compute a transformation?

(b) For simple transformation such as the one from Eq. 1, computing the transformation via a minimal solver is typically orders of magnitude more efficient than evaluating the transformation on all matches during inlier counting in RANSAC.
The minimal number of correspondences for the minimal solver you derived in (a) provides more equations than unknowns. Describe a way how the additional equations can be used to significantly accelerate RANSAC.

(c) In the context of fitting a 3D line into a set of data points, considering the following vectors of residuals. Each vector corresponds to a line and each entry corresponds to the residual of a data point with respect to that line.

$$\begin{aligned}
\mathbf{r}_1 &= \begin{pmatrix} 0.16 & 0.97 & 0.0 & 0.49 & 0.0 & 0.14 & 0.42 & 0.92 & 0.79 & 0.96 \end{pmatrix} \\
\mathbf{r}_2 &= \begin{pmatrix} 0.71 & 0.03 & 0.28 & 0.05 & 0.09 & 0.82 & 0.69 & 0.32 & 0.95 & 0.03 \end{pmatrix} \\
\mathbf{r}_3 &= \begin{pmatrix} 0.0 & 0.38 & 0.77 & 0.80 & 0.18 & 0.48 & 0.45 & 0.0 & 0.70 & 0.0 \end{pmatrix} \\
\mathbf{r}_4 &= \begin{pmatrix} 0.28 & 0.67 & 0.66 & 0.16 & 0.12 & 0.49 & 0.96 & 0.34 & 0.0 & 0.22 \end{pmatrix}
\end{aligned}$$

Which of these lines were not generated by a minimal solver? Justify your answer!