SSY130 - Applied signal processing
Hand in Problem 1

Qixun Qu (901001-5551)

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1. (a) The magnitudes of filter and two signal components are shown in Figure 1.1.

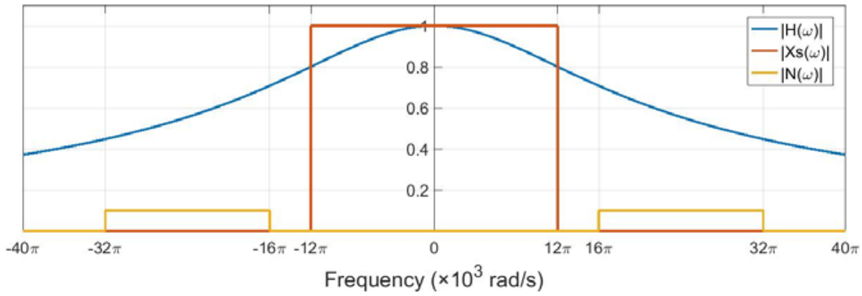

Figure 1.1 Magnitudes of Filter and Two Components

On the basis of Nyquist sampling theorem, sampling rate should be greater than twice of the highest frequency of the signal. Thus, in this problem, the minimal sampling rate is $24\pi \times 10^3$ rad/s.

1. (b) The magnitudes of two filtered signal components are displayed in Figure 1.2.

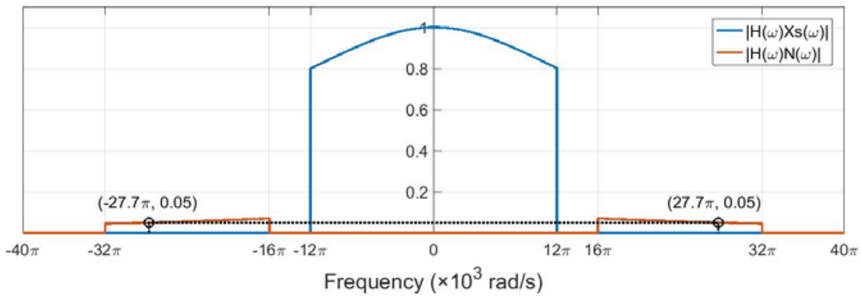

Figure 1.2 Magnitudes of Filtered Components

It is can be seen that, at $\omega = 0$, the magnitude of filtered desired signal is 1. First, calculate the positions whose magnitudes are 20 times lower than 1 in filtered noise signal part. In the interval of $16\pi \times 10^3 \leq |\omega| \leq 32\pi \times 10^3$, the magnitude of filtered noise signal should satisfy

$$|\mathrm{H}(\omega)\mathrm{N}(\omega)| = \left| \frac{0.1}{1 + \frac{j\omega}{\omega_0}} \right| \leq 0.05$$

where $\omega_0 = 16\pi \times 10^3\ rad/s$, the result is $27.7\pi \times 10^3 \leq |\omega_r| \leq 32\pi \times 10^3$, which is shown in Figure 1.2.

To insure that the magnitude of sampled and filtered noise signal is 20 times lower than sampled desired signal in the interval of $|\omega| \leq 12\pi \times 10^3$, the critical condition of sampled noise signal is shown in Figure 1.3. Thus, the minimal interval can be determined as follows.

$$12\pi \times 10^3 - (-27.7\pi \times 10^3) = 39.7\pi \times 10^3\ rad/s$$

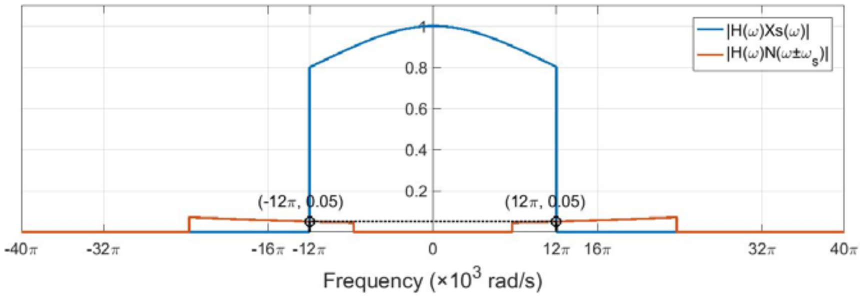

Figure 1.3 Magnitudes of Filtered Components

2. The discrete time sinusoidal signal can be written as 2.1.

$$x_d(n) = \sin\left(2\pi n \frac{f_0}{f_s}\right) = \begin{bmatrix} \omega_0 = 2\pi f_0 \\ \Delta t = 1/f_s \end{bmatrix} = \sin(\omega_0 n \Delta t) \tag{2.1}$$

So, the continuous time signal *x(t)* can be presented as 2.2.

$$x(t) = \sin(\omega_0 t) \tag{2.2}$$

Thus, the Fourier transform of *x(t)* can be calculated as $X(\omega)$ in 2.3.

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt = \int_{-\infty}^{\infty} \sin(\omega_0 t)\, e^{-j\omega t} dt = \frac{\pi}{j}[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)] \tag{2.3}$$

Then, DTFT of discrete time sinusoidal signal $x_d(n)$ is shown as 2.4.

$$X_d(\omega) = \frac{1}{\Delta t} \sum_{k=-\infty}^{\infty} X(\omega + \omega_s k) = \frac{\pi}{j\Delta t} \sum_{k=-\infty}^{\infty} [\delta(\omega + \omega_s k + \omega_0) - \delta(\omega + \omega_s k - \omega_0)] \tag{2.4}$$

The ZOH reconstruction is $Y_{ZOH}(\omega) = H_{ZOH}(\omega) X_d(\omega)$. The $H_{ZOH}(\omega)$ is

$$H_{ZOH} = \Delta t e^{-j\pi\omega/\omega_s} \cdot \frac{\sin(\pi\omega/\omega_s)}{\pi\omega/\omega_s} \tag{2.5}$$

Thus, $Y_{ZOH}(\omega)$ is shown as follows,

$$Y_{ZOH}(\omega) = \frac{\pi}{j} e^{-j\pi\omega/\omega_s} \cdot \frac{\sin(\pi\omega/\omega_s)}{\pi\omega/\omega_s} \sum_{k=-\infty}^{\infty} [\delta(\omega + \omega_s k + \omega_0) - \delta(\omega + \omega_s k - \omega_0)] \tag{2.6}$$

where $\omega_0 = 2\pi f_0 = 6\pi kHz, \omega_s = 2\pi f_s = 20\pi kHz$. Because $Y_{ZOH}(\omega)$ is symmetric, so the following calculation only takes $k \leq 0$ into account.

k	ω	$\lvert Y_{ZOH}(\omega)\rvert$
0	$\omega_1 = \omega_0$ $\omega_2 = -\omega_0$	2.7
-1	$\omega_1 = -\omega_0 + \omega_s = 14\pi\text{kHz}$	1.16
	$\omega_2 = \omega_0 + \omega_s = 26\pi\text{kHz}$	0.62
-2	$\omega_1 = -\omega_0 + 2\omega_s = 34\pi kHz$	0.48
	$\omega_2 = \omega_0 + 2\omega_s$	0.35

Table 2.1 Frequencies and Magnitudes of Harmonic Components

Thus, the frequencies and magnitudes of first three harmonic components are show in the grey area of Table 2.1.