Exercise session 4

October 14, 2019

Problem 1 (MSK)

Let T be a positive constant. Assume a binary frequency shift keying (FSK) modulation, i.e., the following signals are used to transmit 0 and 1:

$$s_0(t) = \begin{cases} \sqrt{\frac{2E}{T}}\cos(2\pi(f_c - \Delta)t) & \text{if } 0 \le t \le T, \\ 0 & \text{otherwise,} \end{cases}$$

$$s_1(t) = \begin{cases} \sqrt{\frac{2E}{T}}\cos(2\pi(f_c + \Delta)t) & \text{if } 0 \le t \le T, \\ 0 & \text{otherwise.} \end{cases}$$

Assume that $f_c T \gg E$.

1. Compute the energy of the signals.
2. Find the minimal frequency shift Δ such that the signals are orthogonal.
3. Assume that 0 and 1 are equally likely. For the found Δ, suggest the optimal detector to minimize the probability of error in the AWGN channel. Find the probability of error.
4. Find the optimal frequency shift to minimize the probability of error.

Problem 2 (Phase Error)

1. Give expressions for the BER of BPSK and QPSK with a Gray labeling in the AWGN channel in terms of E_b/N_0. Interpret the relation between these expressions.
   Note: In case of QPSK, the real part is used to decide on the first bit in the constellation, and the imaginary part is used to decide on the second bit.
2. Find expressions for the BER of BPSK and QPSK with a Gray labeling in the AWGN channel in the presence of the phase error ϕ. Does the same interpretation apply here? Why?
3. What are the asymptotic gains for the two systems when $\phi = \pi/6$. What is the probability of error for the two systems when $\phi = \pi/4$ in the absence of noise?

Problem 3 (Gain Control Error)

Consider a communication system that uses an equally spaced M-PAM constellation $\mathcal{S} = \{\pm(M-1)d, \pm(M-3)d, \pm d\}$ with equally likely symbols, where $M = 2^m$. The parameter d is chosen to normalize the average symbol energy to one. At the receiver side, the energy of the constellation is overestimated by a factor of $\alpha^2 > 1$, i.e., the detector thinks that the constellation has the energy of α^2. The receiver calculates the new observations $Y' = Y/\sqrt{\alpha^2}$ before detection.

1. Find d.
2. For $M = 4$, calculate the probability of symbol error for the detector with the energy estimate error if the transmitted symbol is $s_2 = -d$ over the AWGN channel with N_0. Did the probability of error increase because of the gain control error?
3. For the same setup, find the probability of symbol error if $s_1 = -3d$ was transmitted.