Part I

Sampling and Bandwidth

What is the relationship between B, f_c, f_{samp}, R_{s}, T_{s}, and T_{samp}?

- Define the baseband bandwidth B [Hz] of $V(f) = \mathcal{F}\{v(t)\}$ to be the lowest frequency $f > 0$, where $|V(f)| = 0$ (L_5).
- The passband bandwidth is then $2B$ [Hz] (L_5).
- The symbol rate is always $R_{\text{s}} = 1/T_{\text{s}}$ [symbols/s], where T_{s} [s] is the time between symbols (or samples at the MF output) (L_2).
- The received signal should be sampled every T_{samp} [s] with sampling frequency $f_{\text{samp}} = 1/T_{\text{samp}} > 2(f_c + B)$ [Hz], where f_c [Hz] is the carrier frequency (The Sampling Theorem in L_1).
- For Nyquist pulses (L_2 and L_4):

$$V_{\text{N}}(f) = \sum_{n=-\infty}^{\infty} V\left(f - \frac{n}{T_{\text{s}}}\right) = T_{\text{s}} v(0)$$

- For T_{s}-orthogonal pulses (L_4):

$$V_{\text{O}}(f) = \sum_{n=-\infty}^{\infty} \left|V\left(f - \frac{n}{T_{\text{s}}}\right)\right|^2 = T_{\text{s}} E_v$$

Rect and Sinc Pulses (L_2, L_4, and L_5)

$$\text{Rect: } v(t) = \begin{cases} 1 & \text{if } |t| \leq G \\ 0 & \text{otherwise} \end{cases}$$

$$\text{Rect: } V(f) = \operatorname{sinc}(2Gf) = \frac{\sin(\pi 2Gf)}{\pi 2Gf}$$

$$\text{Sinc: } v(t) = \operatorname{sinc}(t/G) = \frac{\sin(\pi t/G)}{\pi t/G}$$

$$\text{Sinc: } V(f) = \begin{cases} 1 & \text{if } |f| \leq 1/(2G) \\ 0 & \text{otherwise} \end{cases}$$

where $B = \frac{1}{2G}$ and G is just a parameter.

RC and RRC Pulses (L_2 and L_4)

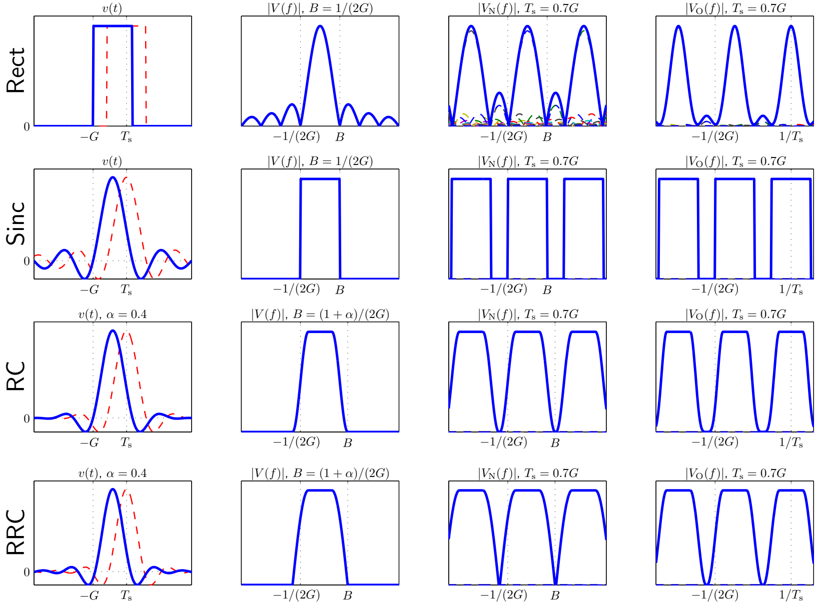
$$\text{RC: } v(t) = \operatorname{sinc}(t/G)\,\frac{\cos(\alpha\pi t/G)}{1-(2\alpha t/G)^2}$$

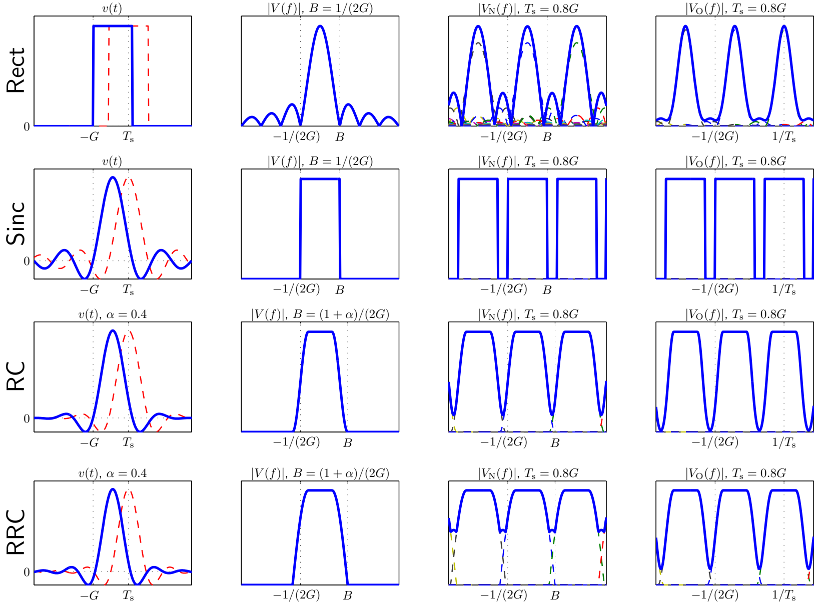
$$\text{RC: } V(f) = \begin{cases} 1 & \text{if } |f| \le \frac{1-\alpha}{2G} \\ \cos^2\left(\frac{\pi G}{2\alpha}\left(|f| - \frac{1-\alpha}{2G}\right)\right) & \text{if } \frac{1-\alpha}{2G} < |f| < \frac{1+\alpha}{2G} \\ 0 & \text{otherwise} \end{cases}$$

$$\text{RRC: } v(t) = \frac{\sin(\pi(1-\alpha)t/G) + (4\alpha t/G)\cos(\pi(1+\alpha)t/G)}{\sqrt{G}(\pi t/G)(1-(4\alpha t/G)^2)}$$

$$\text{RRC: } V(f) = \begin{cases} 1 & \text{if } |f| \le \frac{1-\alpha}{2G} \\ \cos\left(\frac{\pi G}{2\alpha}\left(|f| - \frac{1-\alpha}{2G}\right)\right) & \text{if } \frac{1-\alpha}{2G} < |f| < \frac{1+\alpha}{2G} \\ 0 & \text{otherwise} \end{cases}$$

where $B = (1+\alpha)\frac{1}{2G} > \frac{1}{2G}$ and G is just a parameter.


Rect
Sinc
RC
RRC
$v(t)$
$v(t)$, $\alpha = 0.4$
$|V(f)|$, $B = 1/(2G)$
$|V(f)|$, $B = (1+\alpha)/(2G)$
$|V_\mathrm{N}(f)|$, $T_\mathrm{s} = 0.7G$
$|V_\mathrm{O}(f)|$, $T_\mathrm{s} = 0.7G$
0
$-G$
T_s
$-1/(2G)$
B
$1/T_\mathrm{s}$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RRC
$v(t)$
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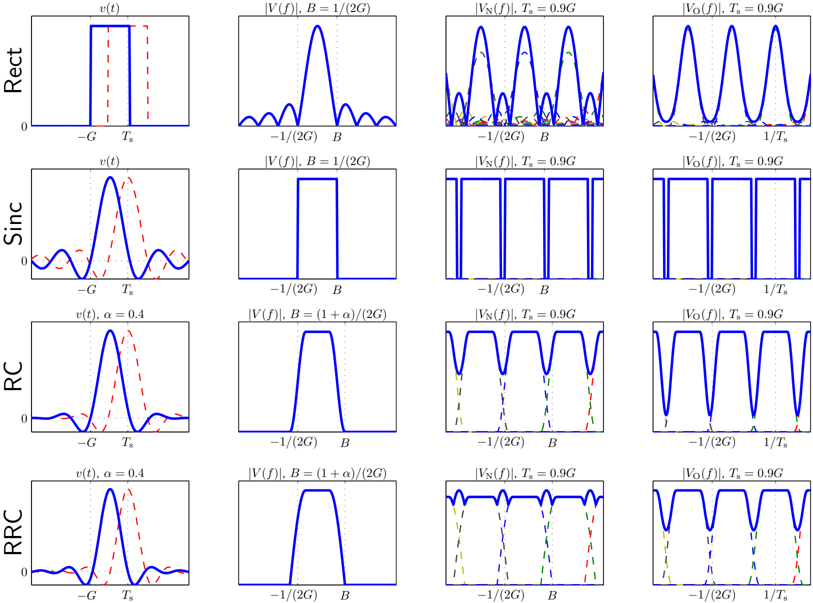
$|V_{\mathrm{O}}(f)|$, $T_{\mathrm{s}} = 0.8G$
$v(t)$, $\alpha = 0.4$
$|V(f)|$, $B = (1+\alpha)/(2G)$
0
$-G$
T_{s}
$-1/(2G)$
B
$1/T_{\mathrm{s}}$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$v(t)$, $\alpha = 0.4$
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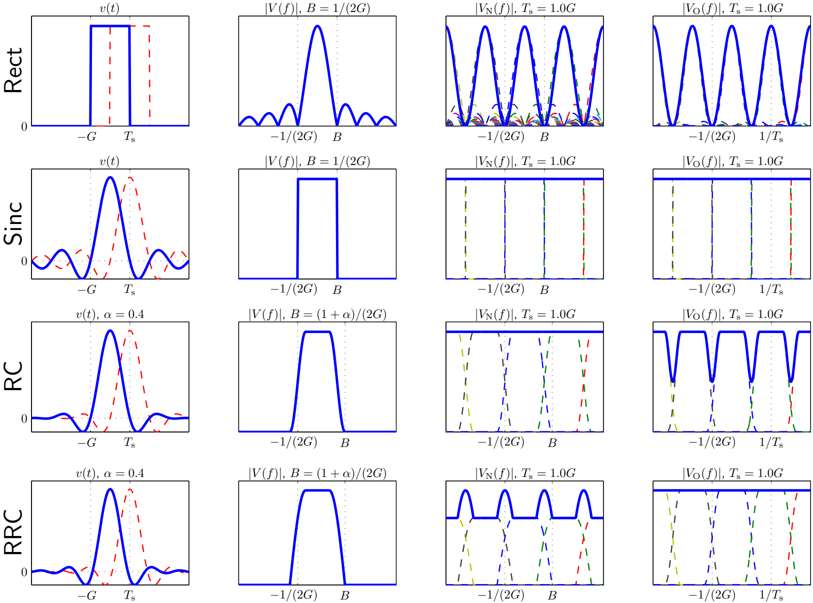
$|V(f)|$, $B = (1+\alpha)/(2G)$
$|V_{\mathrm{N}}(f)|$, $T_{\mathrm{s}} = 0.9G$
$|V_{\mathrm{O}}(f)|$, $T_{\mathrm{s}} = 0.9G$
0
$-G$
T_{s}
$-1/(2G)$
B
$1/T_{\mathrm{s}}$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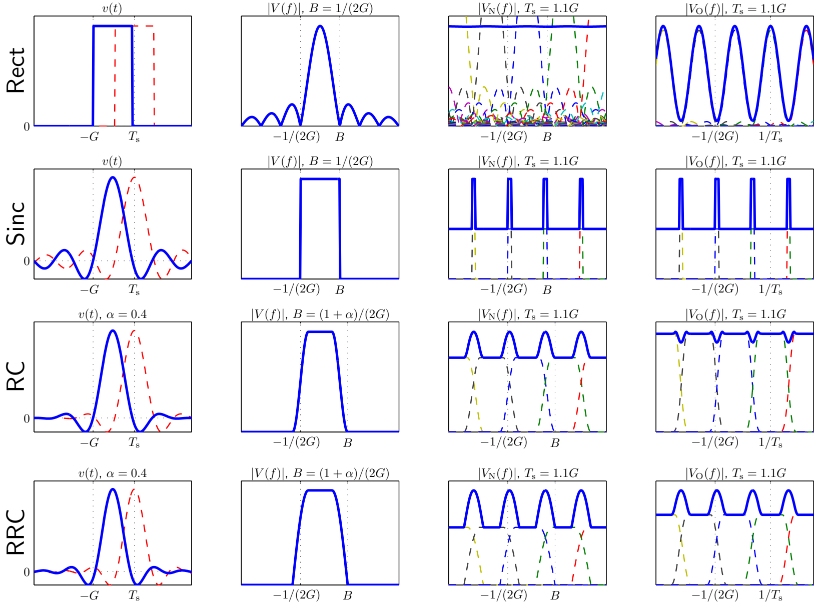
$v(t),\ \alpha = 0.4$
$|V(f)|,\ B = (1+\alpha)/(2G)$
0
$-G$
T_{s}
$-1/(2G)$
B
$1/T_{\mathrm{s}}$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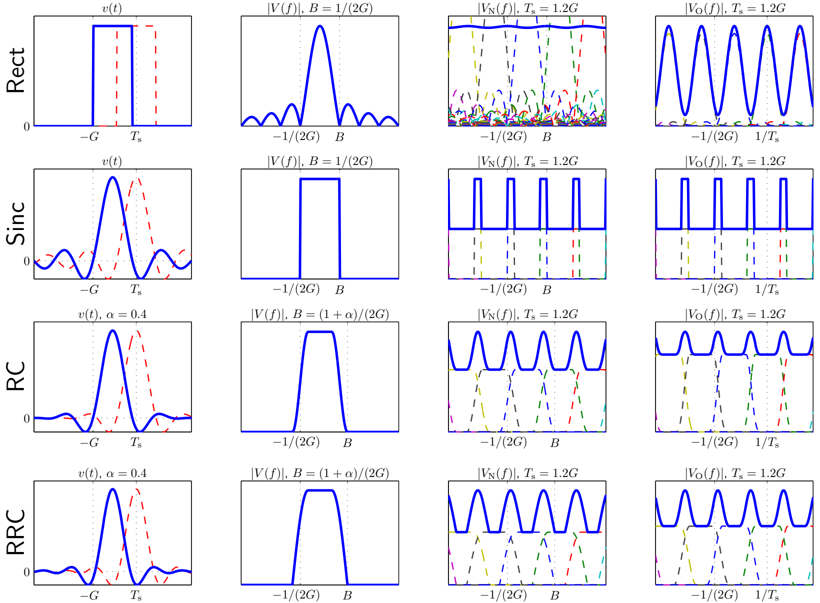
$v(t),\ \alpha = 0.4$
$|V(f)|,\ B = (1+\alpha)/(2G)$
0
$-G$
T_s
$-1/(2G)$
B
$1/T_s$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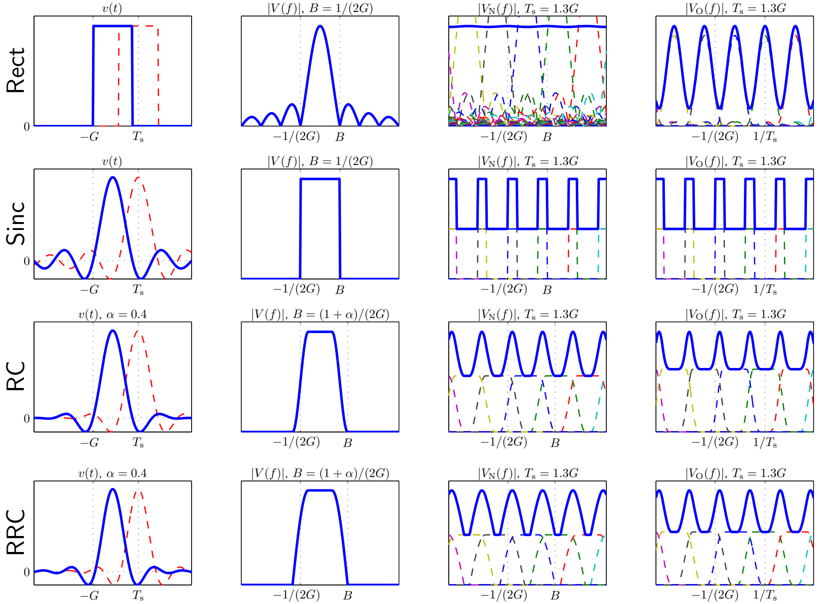
$|V_{\mathrm{O}}(f)|$, $T_{\mathrm{s}} = 1.2G$
$v(t)$, $\alpha = 0.4$
$|V(f)|$, $B = (1+\alpha)/(2G)$
0
$-G$
T_{s}
$-1/(2G)$
B
$1/T_{\mathrm{s}}$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RRC
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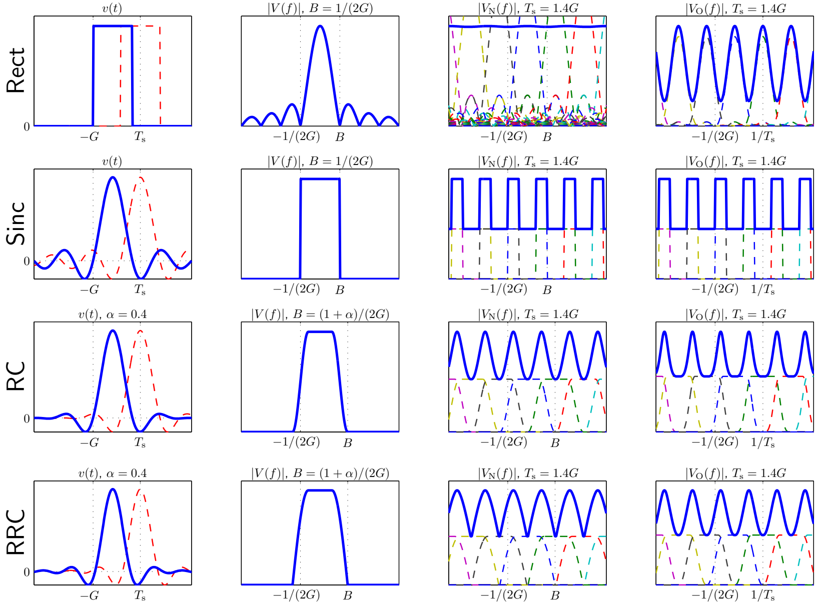
$|V_{\mathrm{O}}(f)|,\ T_{\mathrm{s}} = 1.3G$
$v(t),\ \alpha = 0.4$
$|V(f)|,\ B = (1+\alpha)/(2G)$
0
$-G$
T_{s}
$-1/(2G)$
B
$1/T_{\mathrm{s}}$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$v(t)$, $\alpha = 0.4$
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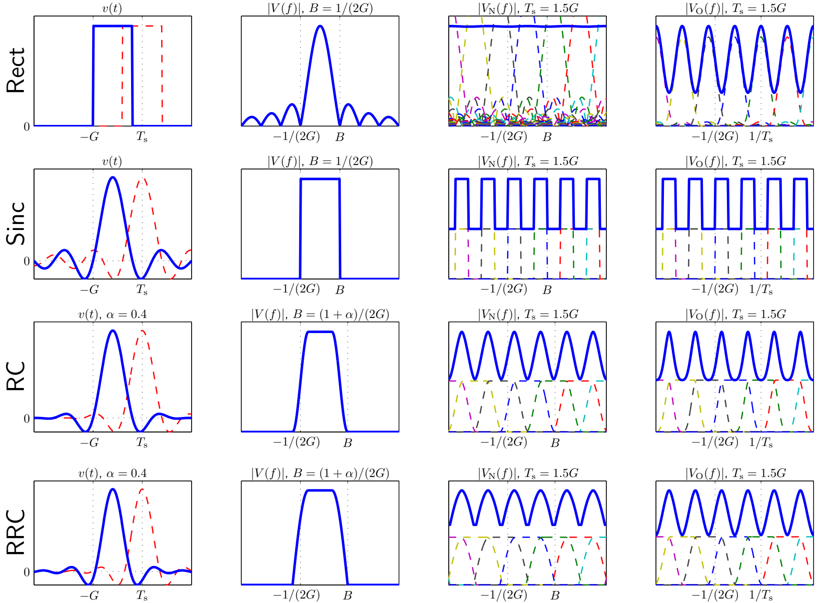
0
$-G$
T_{s}
$-1/(2G)$
B
$1/T_{\mathrm{s}}$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$|V_{\mathrm{O}}(f)|$, $T_{\mathrm{s}}=1.5G$
$v(t)$, $\alpha=0.4$
$|V(f)|$, $B=(1+\alpha)/(2G)$
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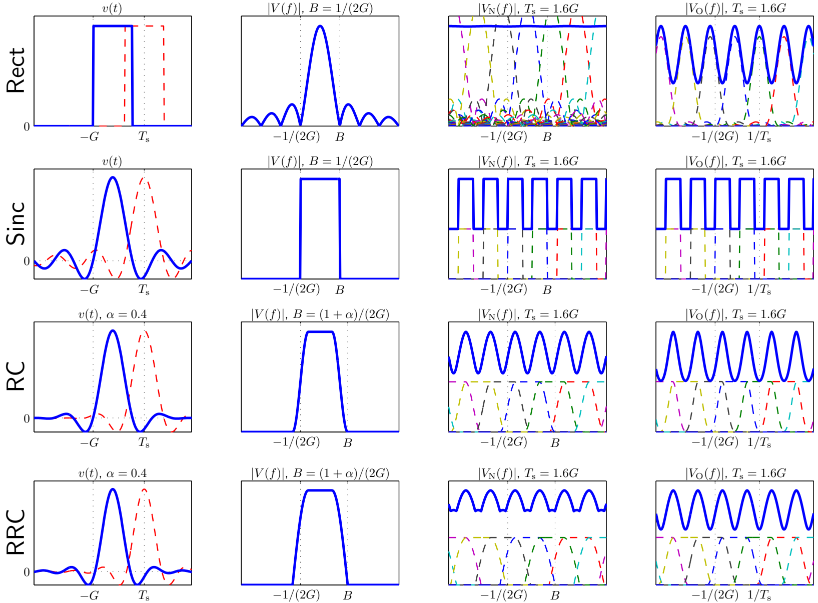
$-G$
T_{s}
$-1/(2G)$
B
$1/T_{\mathrm{s}}$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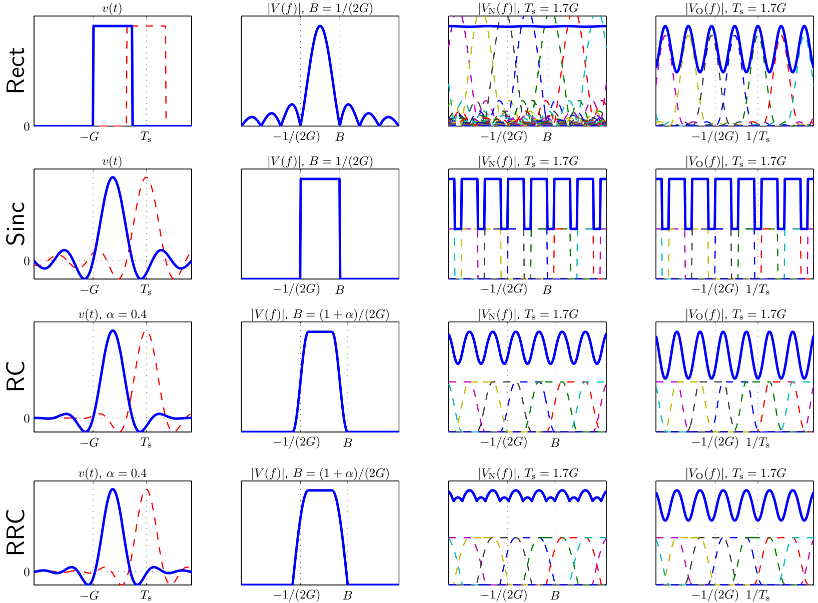
RC
RRC
v(t)
v(t), α = 0.4
|V(f)|, B = 1/(2G)
|V(f)|, B = (1 + α)/(2G)
|V_N(f)|, T_s = 1.6G
|V_O(f)|, T_s = 1.6G
0
−G
T_s
−1/(2G)
B
1/T_s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RC
RRC
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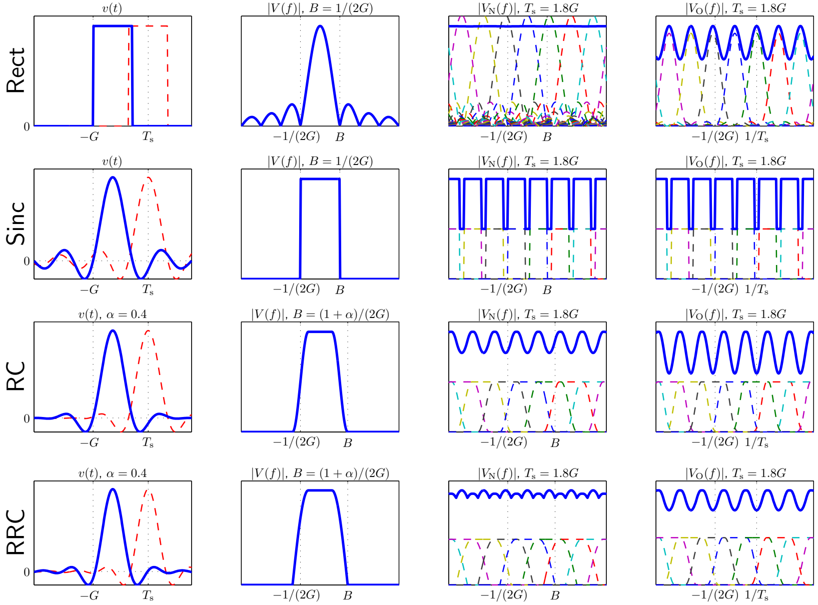
$|V(f)|$, $B = (1+\alpha)/(2G)$
$|V_\mathrm{N}(f)|$, $T_\mathrm{s} = 1.7G$
$|V_\mathrm{O}(f)|$, $T_\mathrm{s} = 1.7G$
0
$-G$
T_s
$-1/(2G)$
B
$1/T_\mathrm{s}$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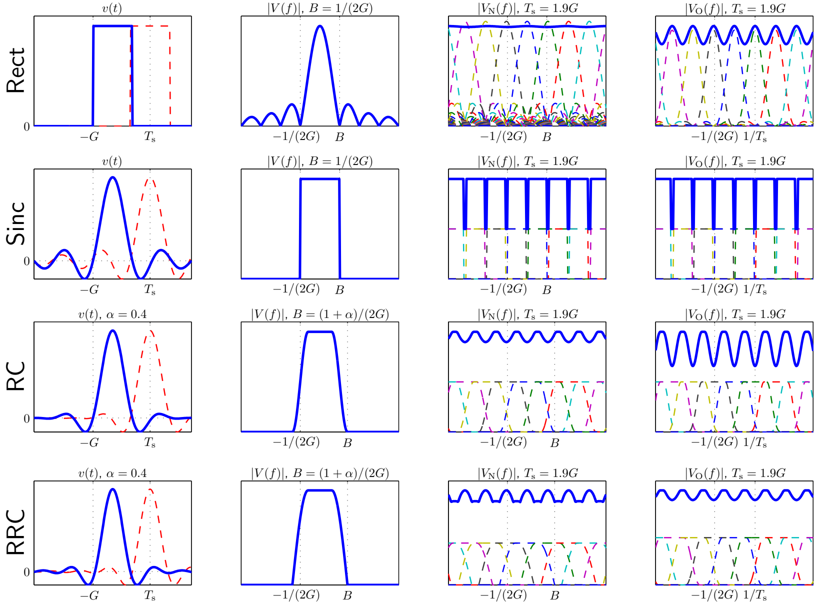
$v(t),\ \alpha = 0.4$
$|V(f)|,\ B = (1+\alpha)/(2G)$
0
$-G$
$T_{\rm s}$
$-1/(2G)$
B
$1/T_{\rm s}$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$v(t)$, $\alpha = 0.4$
$|V(f)|$, $B = 1/(2G)$
$|V(f)|$, $B = (1+\alpha)/(2G)$
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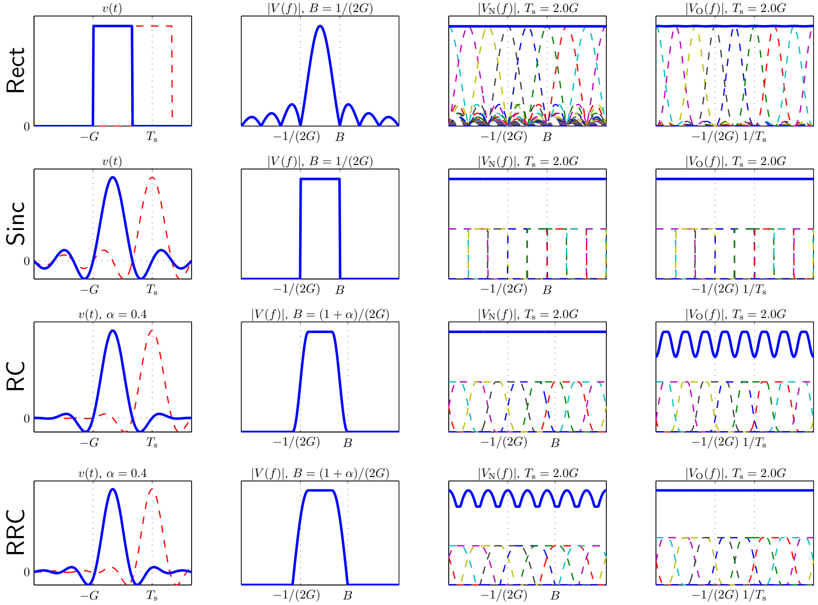
$|V_\mathrm{O}(f)|$, $T_\mathrm{s} = 1.9G$
0
$-G$
T_s
$-1/(2G)$
B
$1/T_\mathrm{s}$


Rect
Sinc
RC
RRC
v(t)
v(t), α = 0.4
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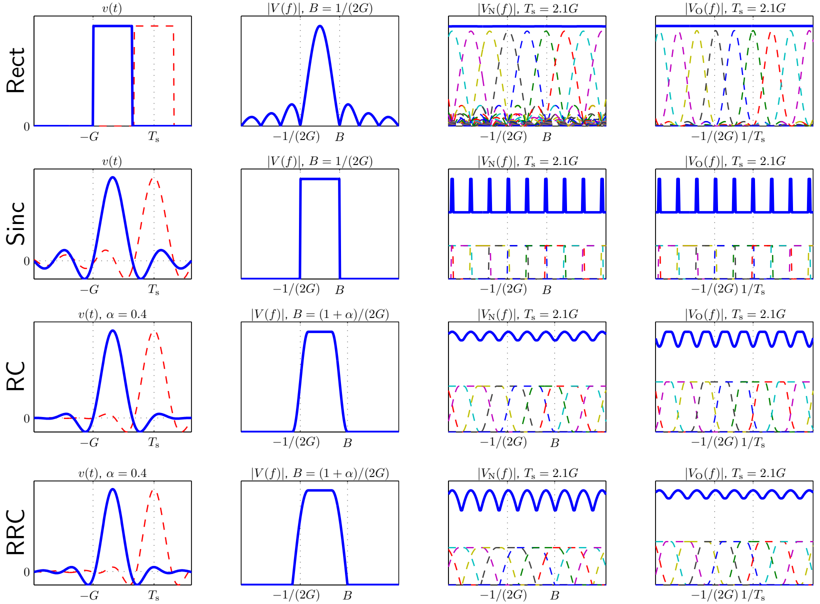
|V(f)|, B = (1 + α)/(2G)
|V_N(f)|, T_s = 2.0G
|V_O(f)|, T_s = 2.0G
0
−G
T_s
−1/(2G)
B
1/T_s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RC
RRC
v(t)
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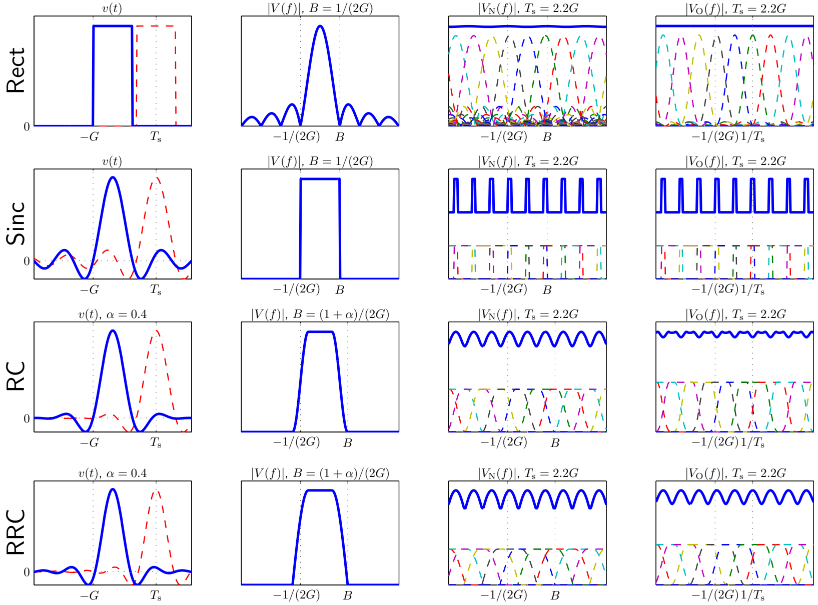
|V_N(f)|, T_s = 2.1G
|V_O(f)|, T_s = 2.1G
v(t), α = 0.4
|V(f)|, B = (1 + α)/(2G)
0
−G
T_s
−1/(2G)
B
1/T_s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RRC
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$|V(f)|,\ B = 1/(2G)$
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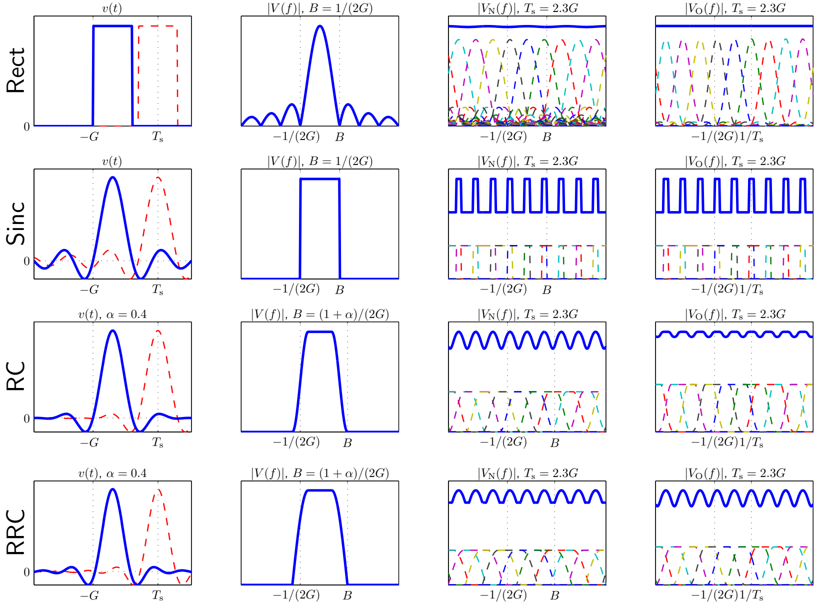
$|V_\mathrm{O}(f)|,\ T_\mathrm{s} = 2.2G$
$v(t),\ \alpha = 0.4$
$|V(f)|,\ B = (1+\alpha)/(2G)$
0
$-G$
T_s
$-1/(2G)$
B
$1/T_\mathrm{s}$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RC
RRC
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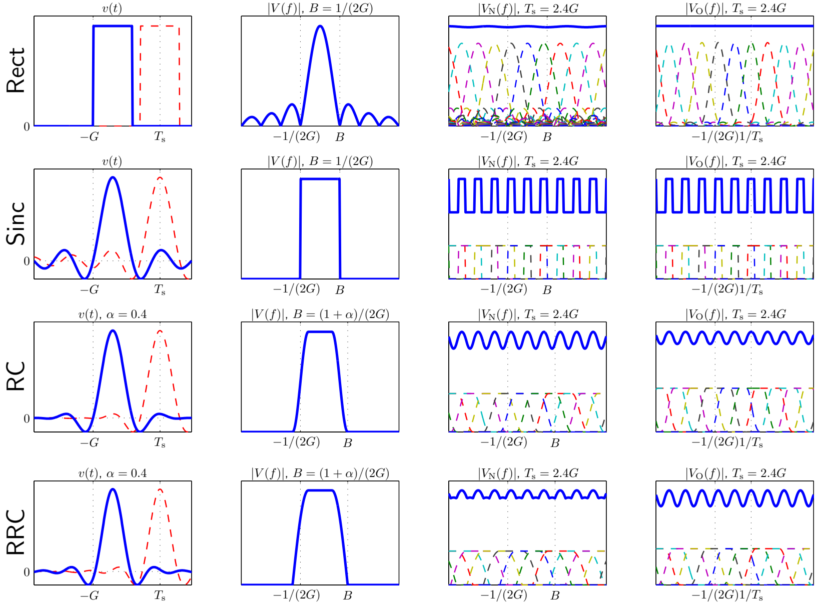
$|V_{\mathrm{O}}(f)|$, $T_{\mathrm{s}} = 2.3G$
$v(t)$, $\alpha = 0.4$
$|V(f)|$, $B = (1+\alpha)/(2G)$
0
$-G$
T_{s}
$-1/(2G)$
B
$-1/(2G)1/T_{\mathrm{s}}$


Rect
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RRC
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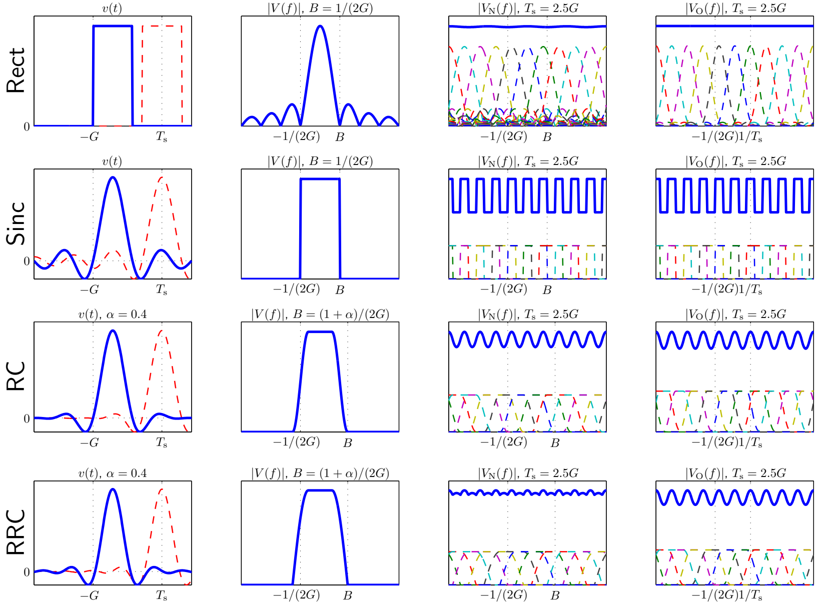
$|V_\mathrm{N}(f)|$, $T_\mathrm{s} = 2.4G$
$|V_\mathrm{O}(f)|$, $T_\mathrm{s} = 2.4G$
$v(t)$, $\alpha = 0.4$
$|V(f)|$, $B = (1+\alpha)/(2G)$
0
$-G$
T_s
$-1/(2G)$
B
$-1/(2G) 1/T_\mathrm{s}$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RC
RRC
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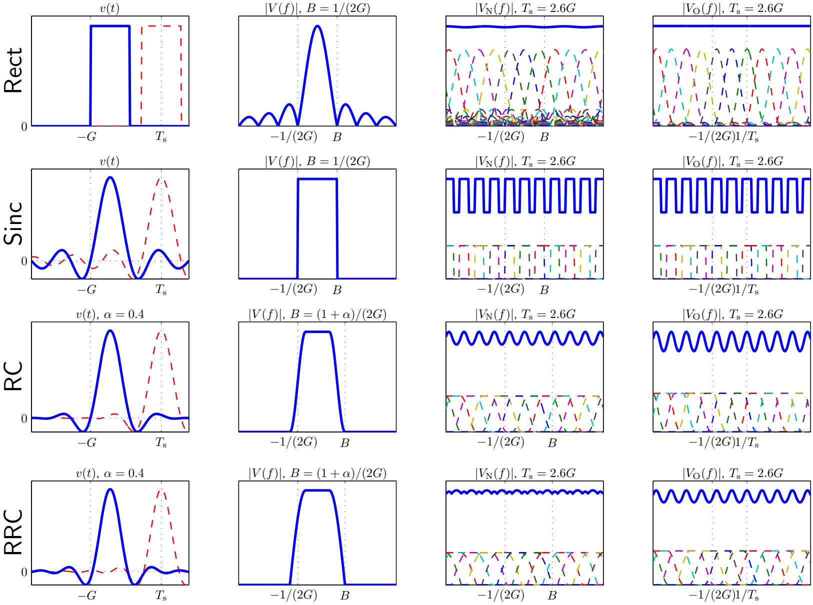
$|V_{\mathrm{N}}(f)|,\ T_{\mathrm{s}}=2.5G$
$|V_{\mathrm{O}}(f)|,\ T_{\mathrm{s}}=2.5G$
$v(t),\ \alpha=0.4$
$|V(f)|,\ B=(1+\alpha)/(2G)$
0
$-G$
T_{s}
$-1/(2G)$
B
$-1/(2G)1/T_{\mathrm{s}}$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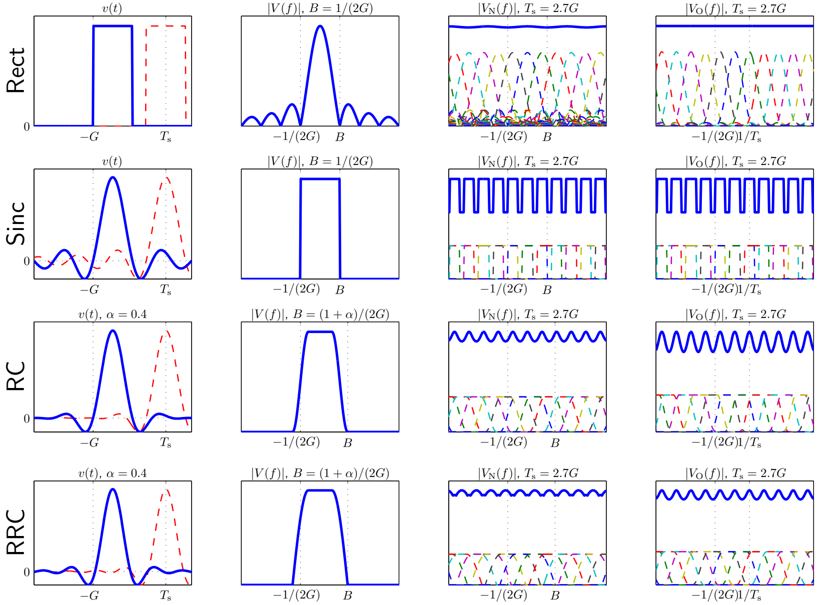
$v(t)$
$v(t)$, $\alpha = 0.4$
$|V(f)|$, $B = 1/(2G)$
$|V(f)|$, $B = (1+\alpha)/(2G)$
$|V_\mathrm{N}(f)|$, $T_\mathrm{s} = 2.6G$
$|V_\mathrm{O}(f)|$, $T_\mathrm{s} = 2.6G$
0
$-G$
T_s
$-1/(2G)$
B
$-1/(2G)1/T_\mathrm{s}$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$v(t),\ \alpha = 0.4$
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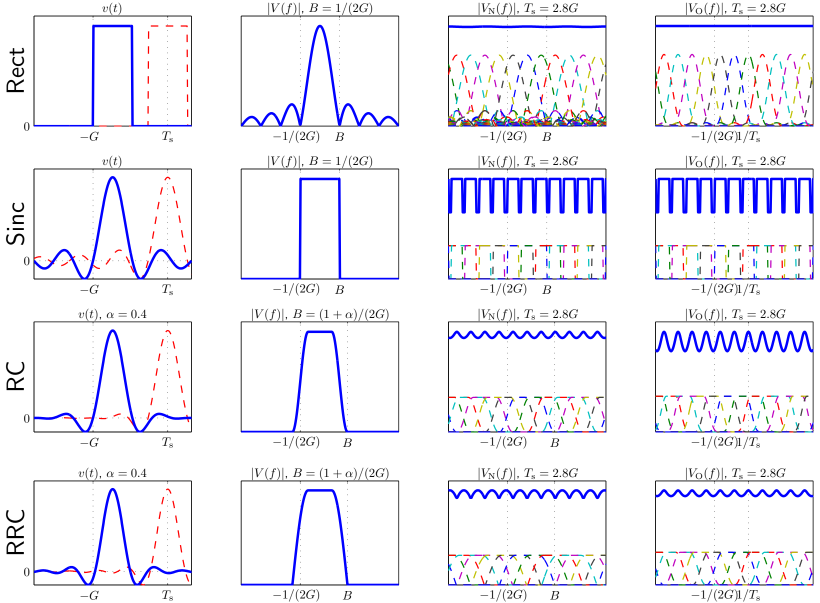
0
$-G$
T_s
$-1/(2G)$
B
$1/T_\mathrm{s}$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$v(t)$, $\alpha = 0.4$
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0
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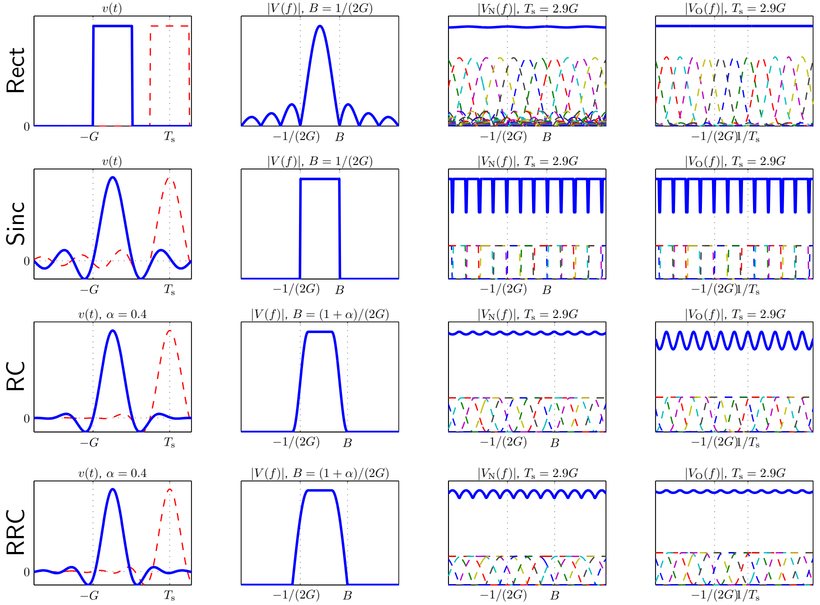
T_s
$-1/(2G)$
B
$1/T_\mathrm{s}$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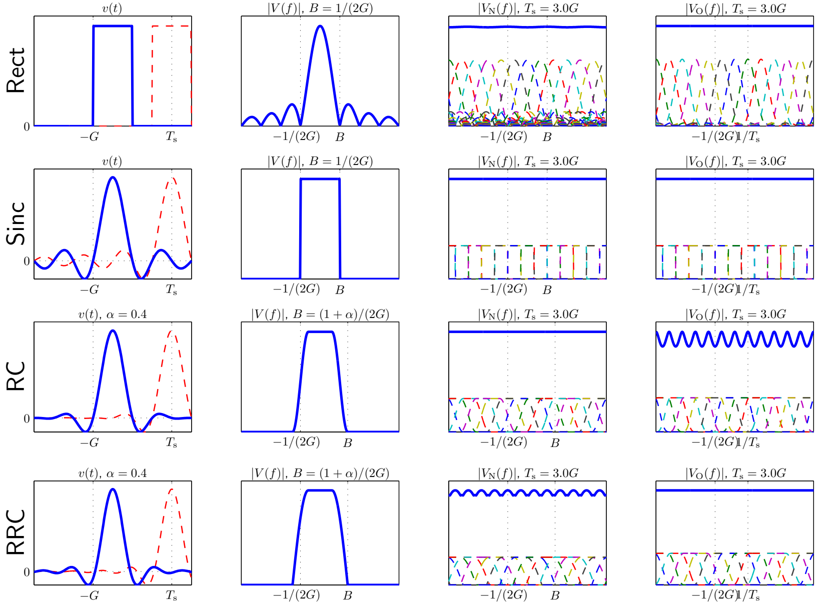
$|V_\mathrm{N}(f)|$, $T_\mathrm{s} = 2.9G$
$|V_\mathrm{O}(f)|$, $T_\mathrm{s} = 2.9G$
0
$-G$
T_s
$-1/(2G)$
B
$1/T_\mathrm{s}$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$v(t),\ \alpha = 0.4$
$|V(f)|,\ B = (1+\alpha)/(2G)$
0
$-G$
T_{s}
$-1/(2G)$
B
$1/T_{\mathrm{s}}$

Summary and Conclusions

Pulse	Nyquist	T_s-orthogonal	B
Rect	$T_s > G$	$T_s \geq 2G$	$1/(2G)$
Sinc	$T_s = kG$	$T_s = kG$	$1/(2G)$
RC	$T_s = kG$	–	$(1+\alpha)/(2G)$
RRC	–	$T_s = kG$	$(1+\alpha)/(2G)$

where $k = 1, 2, 3, \ldots$