Exercise session 2

September 18, 2019

Problem 1 (Inner Product)

Consider the signals $s_1(t) = \frac{1}{\sqrt{T}} e^{j2\pi t/T} \mathrm{I}\{0 \le t \le T\}$ and $s_2(t) = \frac{1}{\sqrt{T}} e^{j4\pi t/T} \mathrm{I}\{0 \le t \le T\}$. Compute the following inner products

- $\langle s_1, s_1 \rangle$,
- $\langle s_1, s_2 \rangle$.

Problem 2 (Orthonormal Basis)

Consider the signals $s_1(t) = \mathrm{I}\{0 \le t \le 1\} - \mathrm{I}\{1 < t \le 2\}$; $s_2(t) = \mathrm{I}\{0 \le t \le 2\}$; $s_3(t) = \mathrm{I}\{0 \le t \le 0.5\} + \mathrm{I}\{1.5 \le t \le 2\}$; and $s_4(t) = \mathrm{I}\{0 \le t \le 0.5\} - \mathrm{I}\{1.5 \le t \le 2\}$.

1. Plot the signals.
2. Find a **minimal set** of orthonormal basis for $\mathrm{span}(s_1, s_2, s_3, s_4)$.
3. Express each of the signals as a linear combination of the basis functions.
4. Find energy of the signals using the results of the previous item.

Problem 3 (Simplex Constellations)

A communication system uses three signals shown in Figure 3 for transmission. The signals are transmitted equiprobably. Answer the following questions.

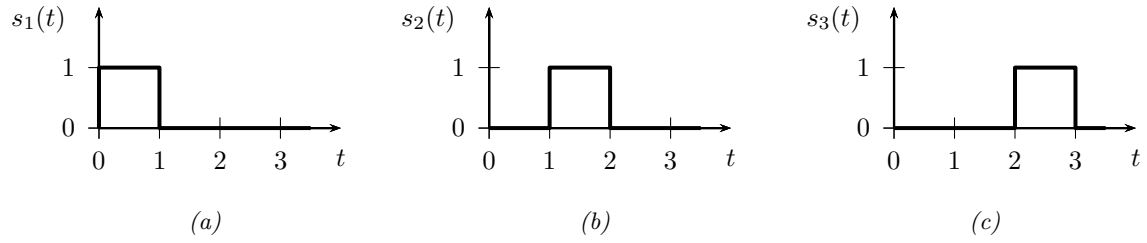

Figure 1: Signal alternatives.

1. Find an orthonormal basis $\{\phi_i(t)\}$ for these signals. Sketch a constellation diagram.
2. Find a new set of signals $\{\tilde{s}_j(t)\}$, $j = 1, 2, 3$ by subtracting the mean signal from the original signals, i.e.,
$$\tilde{s}_j(t) = s_j(t) - \frac{1}{3}\sum_{k=1}^{3} s_k(t).$$
Plot the new signals.
3. Find an orthonormal basis $\{\tilde{\phi}_i(t)\}$ for the new set of signals $\{\tilde{s}_j(t)\}$ and sketch them. Express the signals as a superposition of the basis functions.
4. Sketch a constellation diagram for the new signals and show the decision boundaries for the ML detector.

Problem 4 (Energy)

Of all unit energy real signals that are bandlimited to W Hz, which one has the largest value at $t = 0$? What is its value at $t = 0$? Repeat for $t = 17$. Hint: find a good orthonormal basis for bandlimited signals and express signal energy in terms of its coordinates.