Exercise session 1

Sept. 1, 2021

Problem 1 (LTI Systems)

Consider an LTI system whose response to the signal $x(t)$ in Figure 1(a) is the signal $y(t)$ in Figure 1(b). Sketch the response of the system to the input signal $z(t)$ shown in Figure 1(c).

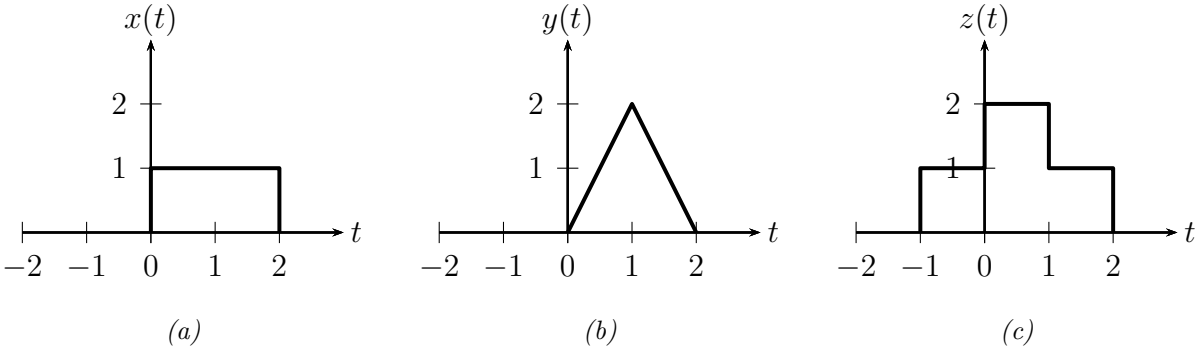

Figure 1: Problem 1.

Problem 2 (Fourier Transform Properties)

Find the Fourier transform of the signal $x(t)$ shown in Figure 2, in two ways as mentioned below.

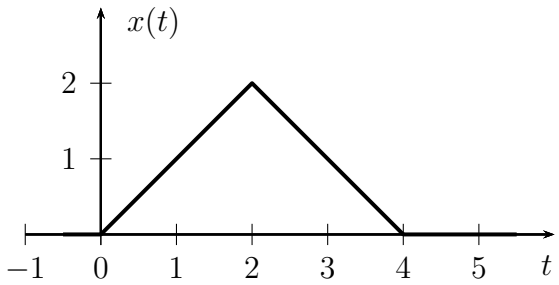

Figure 2: Problem 2.

1. Calculate the Fourier transform using the definition.
2. Use the Fourier transform properties.

Problem 3 (Fourier)

Let $x(t)$ be a signal that is band-limited to W.

1. Show that if $f > W$, then
$$\int_{-\infty}^{\infty} x(t)\cos(2\pi ft)\ \mathrm{d}t = \int_{-\infty}^{\infty} x(t)\sin(2\pi ft)\ \mathrm{d}t = 0.$$

2. Show that if $f > W/2$, then
$$\int_{-\infty}^{\infty} x(t)\cos^2(2\pi ft)\ \mathrm{d}t = \frac{1}{2}\int_{-\infty}^{\infty} x(t)\ \mathrm{d}t.$$

Problem 4 (Fourier)

Prove that
$$\mathrm{sinc}(2Wt)\cos(2\pi Wt) = \mathrm{sinc}(4Wt).$$
Illustrate this identity in the frequency domain.

Problem 5 (Nyquist pulse)

Let $v(t)$ be a continuous signal with limited energy, i.e., $\int_{-\infty}^{\infty} v^2(t)\ \mathrm{d}t < \infty$ and $v(0) = 1$. Define $g(t) = v(t)\mathrm{sinc}(t/T)$.

1. Show that $g(t)$ is a Nyquist pulse for the time interval T.
2. Argue that the raised-cosine pulse is a Nyquist pulse.
3. Find the Fourier transform $G(f)$ as a function of $V(f)$ and show that it satisfies the Nyquist criterion in the frequency domain.

Problem 6 (Nyquist Pulse)

The pulses are defined in frequency domain and their spectra are shown in the below figure (frequency is in MHz).

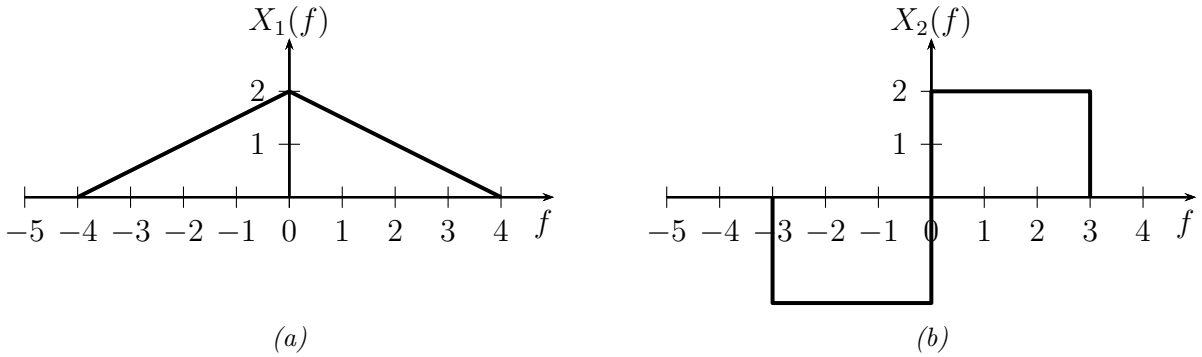

Figure 3: Problem 6.

$$X_1(f) = \begin{cases} 2 - 0.5|f| & \text{if } |f| \le 4 \\ 0 & \text{o.w.} \end{cases} \qquad X_2(f) = \begin{cases} 2 & \text{if } 0 \le f \le 3 \\ -2 & \text{if } -3 \le f < 0 \end{cases}$$

1. Which pulse(s) satisfy the Nyquist criterion and for which symbol rate?

2. Find the value at $t = 0$ and the energy for these signals.