Introduction to Communication Engineering
SSY121, Lecture # 6

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Outline

Part I

A Short Summary of Last Lecture

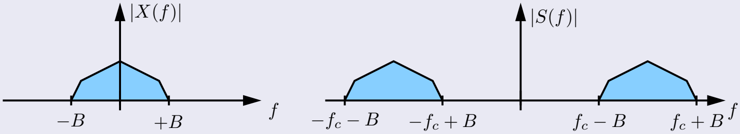
1D Passband Transmission

- The baseband signal $x(t)$ is multiplied by a sinusoid of frequency f_c:

$$s(t) = x(t)\cos(2\pi f_c t)$$

- In frequency domain:

$$\begin{aligned} S(f) &= \mathcal{F}\{x(t)\cos(2\pi f_c t)\} = \mathcal{F}\{x(t)\} * \mathcal{F}\{\cos(2\pi f_c t)\} \\ &= X(f) * \frac{1}{2}\Big[\delta(f-f_c)+\delta(f+f_c)\Big] = \frac{1}{2}\Big[X(f-f_c)+X(f+f_c)\Big] \end{aligned}$$



The spectrum of a train of pulses

Consider the signal

$$s(t) = \sum_{k=0}^{\infty} a_k v(t - kT_{\mathrm{s}}).$$

The linear modulation spectrum theorem (Theo. 2.3-1 in [p. 31, Anderson]) simply states that the spectrum of $s(t)$ has the same form of the spectrum of the pulse $v(t)$.

Consequences

The passband signal

$$s(t) = \sum_{k=0}^{\infty} a_k v(t - kT_{\mathrm{s}}) \cos(w_c t)$$

- has a spectrum that is simply the spectrum of $V(f)$ at $\pm f_c$,
- has a BW that depends on the symbol period (See “Symbol rate BW tradeoff” in L_2)

The spectrum of BPSK with sinc and RRC pulses (from [Anderson])

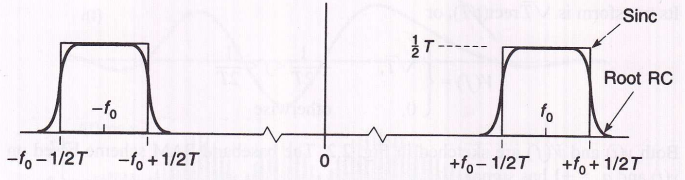


Figure 3.3 Idealized power spectral densities for BPSK for sinc and root RC pulses; T = symbol time.

2D Passband Transmission

- The transmitted signal will be of the form

$$s(t) = \sum_{k=0}^{\infty} a_k \phi_1(t - kT_\mathrm{s}) + b_k \phi_2(t - kT_\mathrm{s}),$$
$$\phi_1(t) = \sqrt{2} v(t) \cos(w_c t),$$
$$\phi_2(t) = \sqrt{2} v(t) \sin(w_c t),$$

  where $v(t)$ is a unit-energy baseband pulse
- Orthonormal basis: $||\phi_1(t)|| = ||\phi_2(t)|| = 1$ and $\langle \phi_1(t), \phi_2(t) \rangle = 0$
- ISI-free transmission if $v(t)$ is T_s-orthogonal:

$$\int_{-\infty}^{\infty} \phi_1(t)\phi_1(t - nT_\mathrm{s})dt = \int_{-\infty}^{\infty} \phi_2(t)\phi_2(t - nT_\mathrm{s})dt = 0,$$
$$\int_{-\infty}^{\infty} \phi_1(t)\phi_2(t - nT_\mathrm{s})dt = \int_{-\infty}^{\infty} \phi_2(t)\phi_1(t - nT_\mathrm{s})dt = 0,$$

  for $n = \pm 1, \pm 2, \ldots$

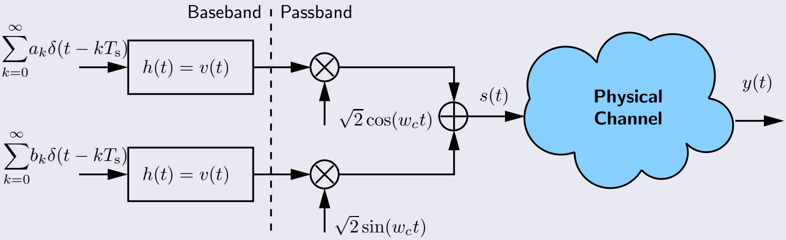
Passband signal for 2D modulations

- The passband signal for the 2D constellations can be written as

$$s(t) = \sqrt{2} \sum_{k=0}^{\infty} a_k v(t - kT_\mathrm{s}) \cos(w_c t) + b_k v(t - kT_\mathrm{s}) \sin(w_c t)$$

- Every point $(a_k, b_k) \in \mathbb{R}^2$ can be represented using an amplitude $A_k = \sqrt{a_k^2 + b_k^2}$ and an angle $\psi_k = \arctan(b_k/a_k)$
- The 1D signals can be obtained by using $b_k = 0$.

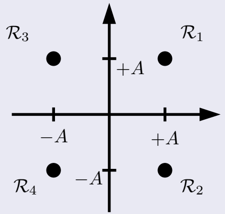
2D Tx



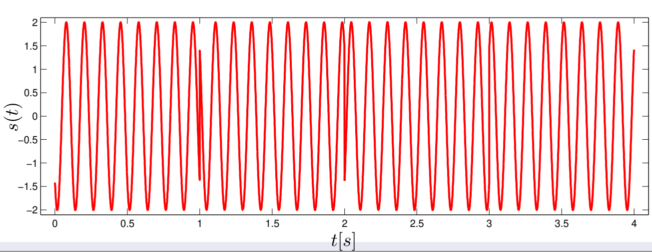
Quaternary phase shift keying (QPSK)

$$s_1(t) = +A\phi_1(t) + A\phi_2(t), \quad \mathsf{s}_1 = [+A, +A]$$
$$s_2(t) = +A\phi_1(t) - A\phi_2(t), \quad \mathsf{s}_2 = [+A, -A]$$
$$s_3(t) = -A\phi_1(t) + A\phi_2(t), \quad \mathsf{s}_3 = [-A, +A]$$
$$s_4(t) = -A\phi_1(t) - A\phi_2(t), \quad \mathsf{s}_4 = [-A, -A]$$

Constellation



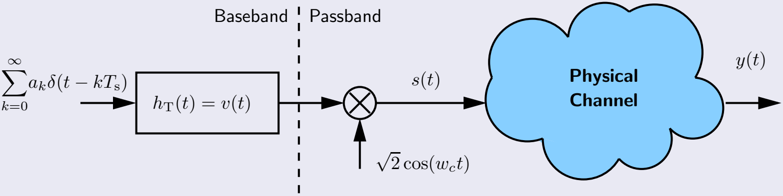
QPSK signal using square pulses



Part II

MFR for 1D and 2D Modulations

1D Tx



The transmitted signal

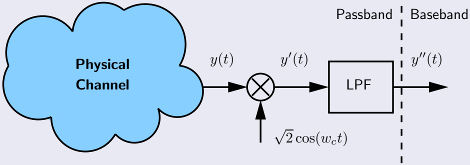
The passband signal for the 1D modulations can be written as

$$s(t) = \sqrt{2} \sum_{k=0}^{\infty} a_k v(t - kT_\mathrm{s}) \cos(w_c t).$$

And the 1D Rx?

How do we design the 1D MFRx?

1D Rx



Why?

If the channel is good, we can assume $y(t) \approx s(t)$, and thus

$$
\begin{aligned}
y'(t) &= \sqrt{2}y(t)\cos(w_c t) \approx \sqrt{2}s(t)\cos(w_c t) \\
&= 2\sum_{k=0}^{\infty} a_k v(t-kT_\mathrm{s})\cos^2(w_c t) \\
&= \sum_{k=0}^{\infty} a_k v(t-kT_\mathrm{s})\left[1+\cos(2w_c t)\right] \\
&= \sum_{k=0}^{\infty} a_k v(t-kT_\mathrm{s}) + {\color{red}\sum_{k=0}^{\infty} a_k v(t-kT_\mathrm{s})\cos(2w_c t)}
\end{aligned}
$$

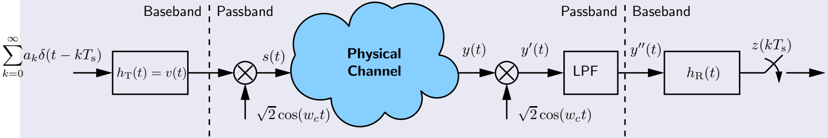
After the LPF

After the LPF, we obtain

$$y''(t) = \sum_{k=0}^{\infty} a_k v(t - kT_s),$$

which is an expression we have seen before, haven't we?

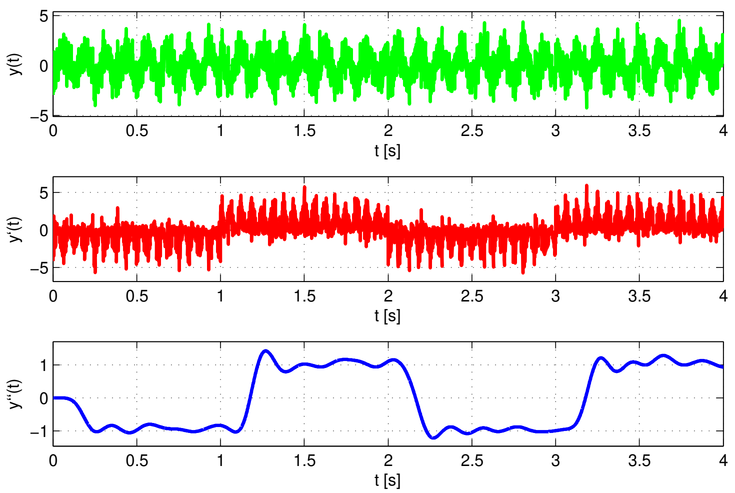
1D Tx and Rx



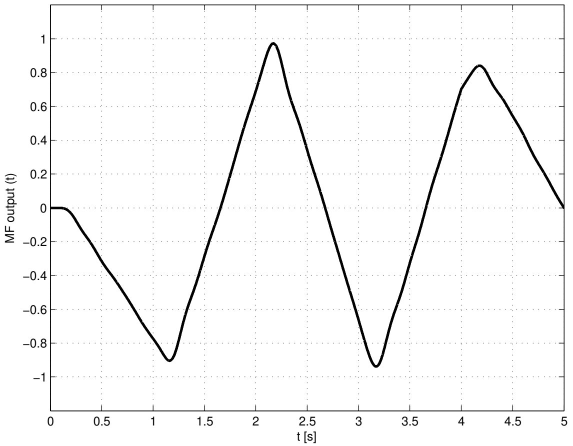
The selection of $h_R(t)$?

How do we choose the impulse response of the Rx filter?

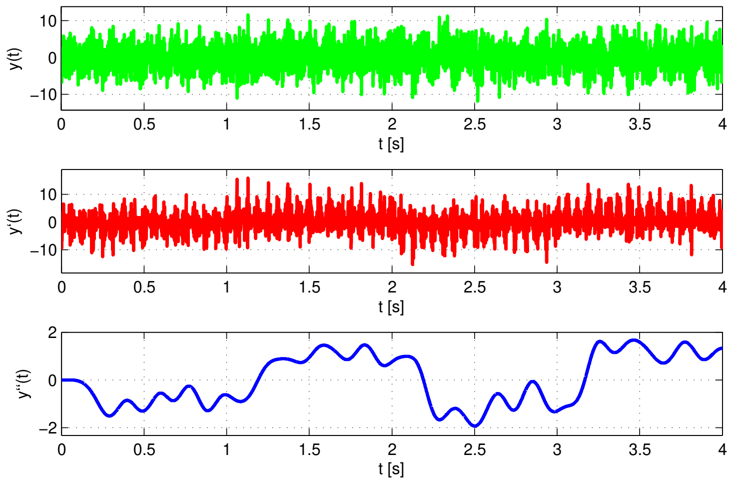
BPSK Receiver and a good channel



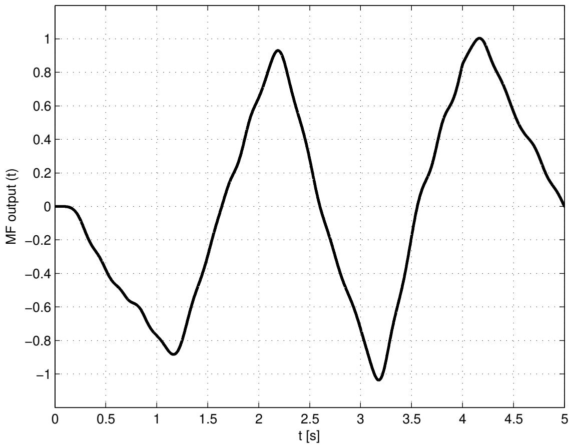
Matched filter's output



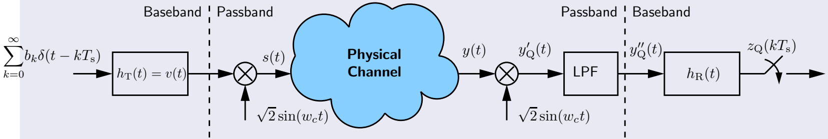
BPSK Receiver and a bad channel



Matched filter's output



Quadrature component in 2D Rx



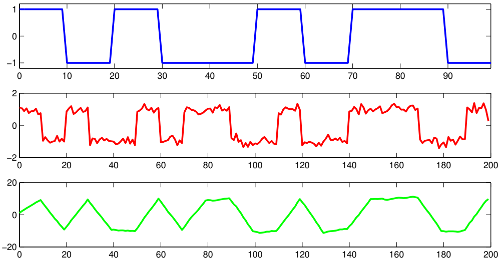
If the channel is good, we can assume $y(t) \approx s(t)$, and thus

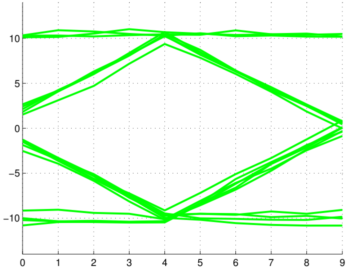
$$
\begin{aligned}
y'_Q(t) &\approx \left[\sqrt{2}\sum_{k=0}^{\infty} a_k v(t-kT_s)\cos(w_c t) + b_k v(t-kT_s)\sin(w_c t)\right]\sqrt{2}\sin(w_c t) \\
&= 2\sum_{k=0}^{\infty} a_k v(t-kT_s)\cos(w_c t)\sin(w_c t) + 2\sum_{k=0}^{\infty} b_k v(t-kT_s)\sin^2(w_c t) \\
&= \sum_{k=0}^{\infty} a_k v(t-kT_s)\sin(2w_c t) + \sum_{k=0}^{\infty} b_k v(t-kT_s)\left[1-\sin(2w_c t)\right] \\
&= \sum_{k=0}^{\infty} b_k v(t-kT_s) + \sum_{k=0}^{\infty}(a_k-b_k)v(t-kT_s)\sin(2w_c t)
\end{aligned}
$$

HW: The inphase component for 2D Rx: $y'_I(t) = y(t)\sqrt{2}\cos(w_c t)$?

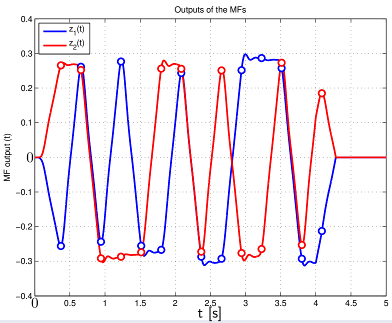
Eye Diagram

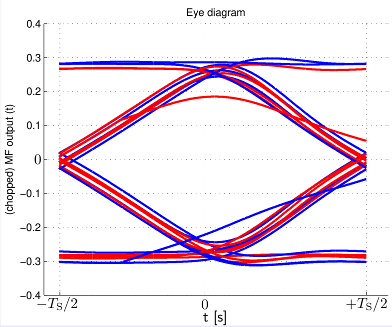
- Study the output of the MF (before sampling)
- Chop it in segments of length T_s
- Overlay the segments





MF's output and Eye Diagram



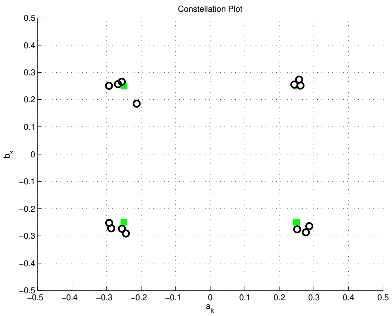


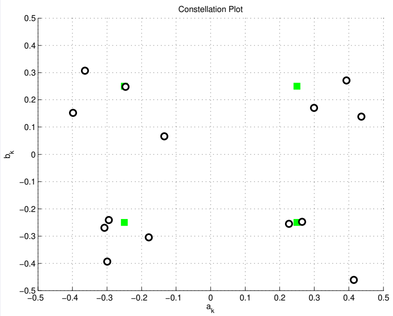
Why are eye diagrams important?

Because it illustrates

- The best sampling instant
- The ability to separate the signal alternatives
- The sensitivity to synchronization errors

Constellation Plot QPSK (Good and Bad Channel)

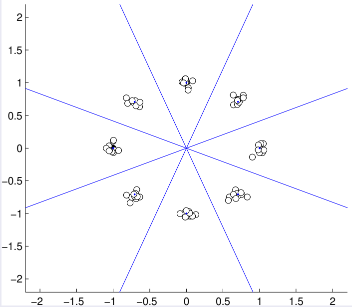


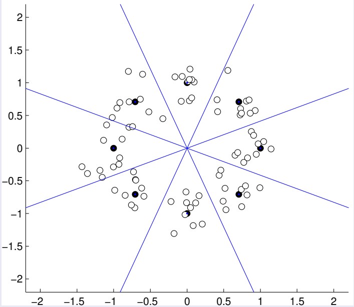


Constellation Plots

- Study the samples of the MFs
- Let the sampled values at a given time be the coordinates of a point
- Plot those points for a large number of samples

Constellation Plot 8PSK (Good and Bad Channel)





Why are constellation plots important?

Because it illustrates

- The energy of the signal alternatives
- How reliable the outputs of the MFs are (distances between the constellation points)
- It shows how much the channel affects the decisions

Today's Summary

- 1D and 2D Passband Transmission
- Spectrum of Pulses
- 1D and 2D MFRx
- Eye Diagrams
- Constellation Plots
- Constant Envelope Modulations (FSK or CPFSK) and PA