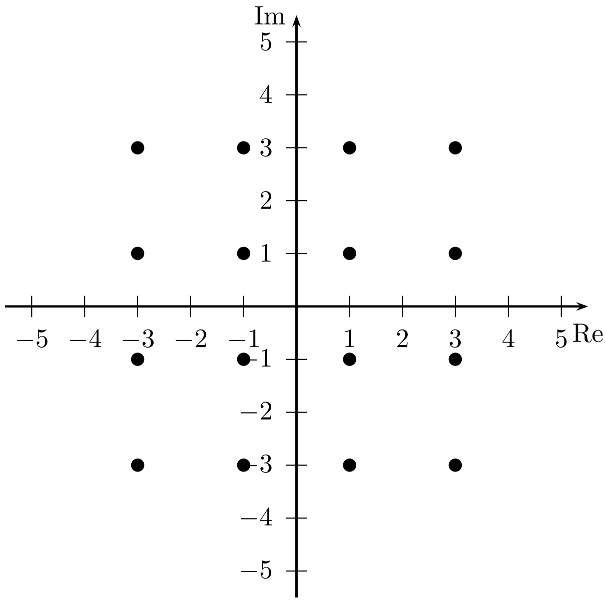
Exercise session 3

October 2, 2019

Problem 1 (Constellations)

Two 16-point QAM signal sets are shown in Figure 2. The first one is a standard square 4×4 constellation; the second one is the V.29 constellation. These constellations have a minimum squared distance of $D^2 = 4$.



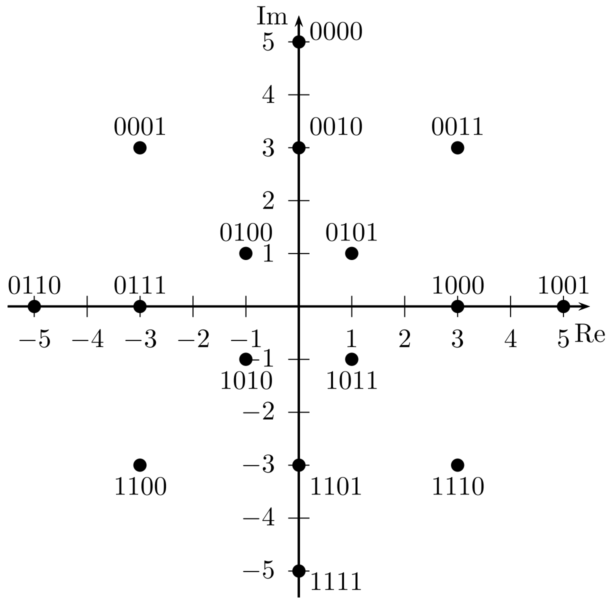


Figure 1: Problem 1.

1. Compute the average energy of each constellation if all points are equiprobable.
2. Find a high-snr approximation for the SER. Compare the power efficiencies of the two constellations in dB.
3. Find a high-snr approximation for the BER for the constellation in Fig.1(b).
4. Sketch the decision regions of a minimum-distance detector for the two constellations.

Problem 2 (Sandwiching the BER)

Consider a message $\mathbf{D}$ of k bits, i.e., $\mathbf{D} = [D_1, \ldots, D_k]$. The bits are not necessarily independent and equally likely. Show that the probability of bit error rate is bounded as

$$\frac{1}{k}\Pr[\hat{\mathbf{D}} \neq \mathbf{D}] \leq \frac{1}{k}\sum_{j=1}^{k}\Pr[\hat{D}_j \neq D_j] \leq \Pr[\hat{\mathbf{D}} \neq \mathbf{D}],$$

where $\Pr[\hat{\mathbf{D}} \neq \mathbf{D}]$ is the probability that a message error occurs.

Problem 3 (MAP for BPSK)

Assume a discrete-time Gaussian channel. The received symbol is given by $Y = X + Z$, where X takes on a value $-d$ with probability 0.75 and d with probability 0.25 and $Z \sim \mathcal{N}(0, \sigma^2)$.

- Find the BER expression if the MAP demodulation is used.
- Find the BER expression if the ML demodulation is used.

Problem 4 (MAP for QPSK)

A QPSK constellation is used for transmission. The constellation points $s_i,\ i = 1, \ldots 4$ are defined as $s_i = \mathrm{e}^{j(i-1)\pi/2 + j\pi/4}$. Let $\Pr[s_1] = 0.4$ and $\Pr[s_2] = \Pr[s_3] = \Pr[s_4] = 0.2$. The SNR $E_s/N_0 = 0$ dB.

1. Sketch carefully the Maximum-Likelihood decision regions for the constellation points.
2. Sketch carefully the Maximum a posteriori decision regions for the constellation points.