SSY125 - Digital Communications
Project 2

Group 9
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1. Hard vs. Soft Receiver

From figure 1 the coding gain of encoder $\mathcal{E}_2$ using $\mathcal{X}_{\text{QPSK}}$ with gray mapping is shown. The values at a bit error rate (BER) of 10^{-4} are represented in table 1 together with the calculated values for the asymptotic coding gain.

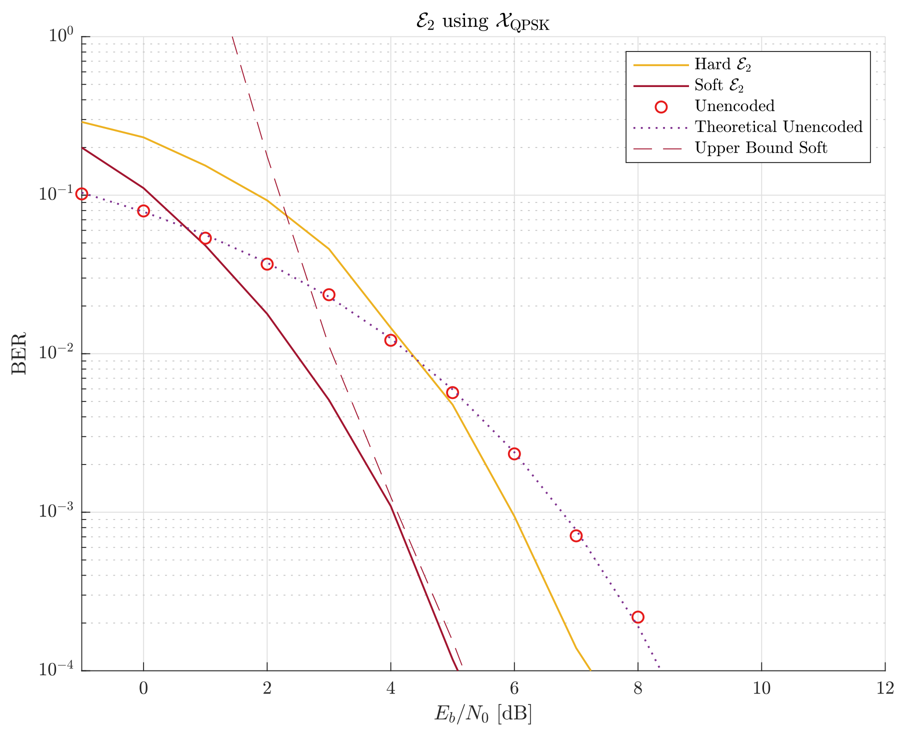

Figure 1: Encoder $\mathcal{E}_2$ and $\mathcal{X}_{\text{QPSK}}$ with gray mapping.

The asymptotic coding gain is calculated according to equation 1 where the coding rate, R_c, is 1/2 and the free distance, d_{min}, is 5. The free distance is the minimal Hamming distance between different encoded sequences and is calculated in `MATLAB` using the function `distspec`. Equation 1 is only true for soft decoding, hence the asymptotic gain is not presented for hard decoding.

$$G_{\text{C},\infty} = 10\log_{10}(R_c d_{min}) \tag{1}$$

Table 1: Coding gain at a BER of 10^{-4} and asymptotic gain.

Decision	Coding gain [dB]	Asymptotic gain [dB]
Hard	1.2	-
Soft	3.3	3.98

For hard-decision maximum likelihood (ML) decoding, the minimum hamming distance is used for every received bit in order to find the correct corresponding bit and accordingly code word. In other words part of the information is discarded for an efficient decoding to take place. For soft-decision ML decoding the euclidean distance is used, i.e. information is not discarded, on every received symbol to see which code word has the least euclidean distance and thus is the most likely one.

From figure 1 one can see that using soft-decision decoding instead of hard-decision decoding, the coding gain difference between the two at a BER of 10^{-4} is in this case ≈ 2.1 [dB]. It is true that soft-decision decoder performs better in aspect to BER for a specific signal-to-noise ratio (SNR) since the hard-decision decoder does not exploit all the information at the output of the channel. The downside when using soft-decision decoding is the much more complex decoding, causing it not to be realizable in many applications.

For reliable transmission the minimum E_b/N_0 to support a rate R is calculated by equation (2). The rate of $\mathcal{E}_2$, R equals 1, since the code rate equals 1/2 and the number of bits per symbol equals 2. Hence according to equation 2 the minimum required E_b/N_0 for reliable transmission is 0 dB. Mind that this is the capacity of the channel, not what our code can achieve. Assuming that for our purpose a reliable transmission requires an BER of 10^{-4} our code would work reliably for an power efficiency of 7.2 [dB] and 5.1 [dB] using a hard- respectively soft decoder.

$$\frac{E_b}{N_0} > \frac{2^R - 1}{R} \tag{2}$$

2. Encoder Comparison

The coding gains for encoders $\mathcal{E}_1$, $\mathcal{E}_2$ and $\mathcal{E}_3$ using $\mathcal{X}_{\text{QPSK}}$ with Gray mapping can be seen in figure 2. The corresponding values for each encoder, including minimum free distance and asymptotic coding gain, are presented in table 2. The free distance is calculated in `MATLAB` using the function `distspec`. The asymptotic coding gain is calculated using equation 1.

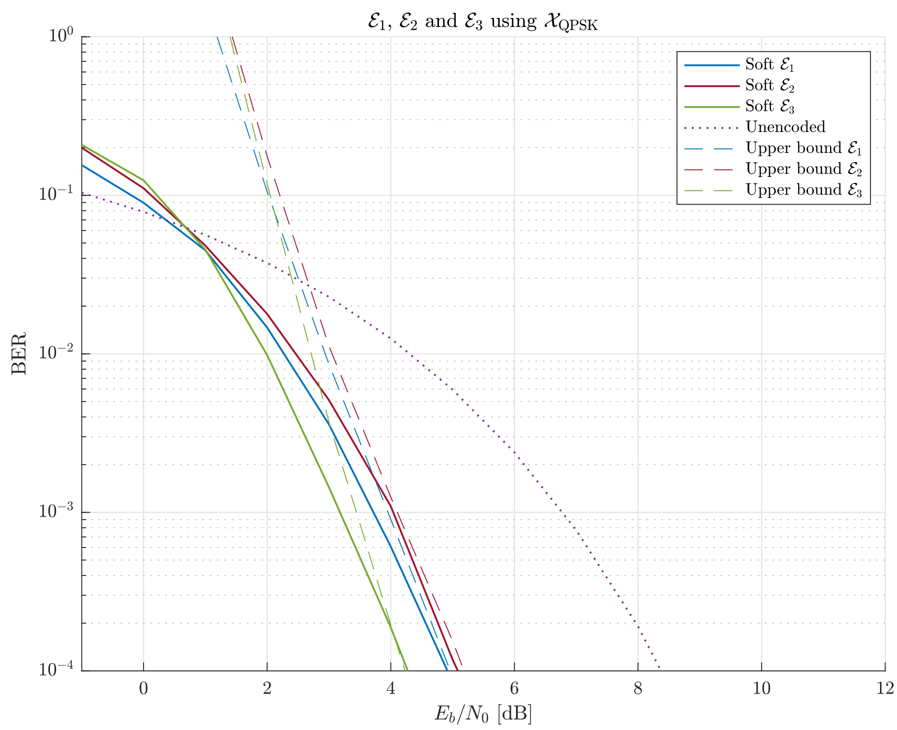

Figure 2: Performance of encoders $\mathcal{E}_1$, $\mathcal{E}_2$ and $\mathcal{E}_3$ for $\mathcal{X}_{\text{QPSK}}$ with Gray mapping along with their respective theoretical upper bound BER.

Table 2: Respective encoders gain at a BER of 10^{-4}, free distance and calculated asymptotic coding gain.

Encoder	Coding gain [dB]	d_{min}	Asymptotic coding gain [dB]
$\mathcal{E}_1$	3.5	5	3.98
$\mathcal{E}_2$	3.3	5	3.98
$\mathcal{E}_3$	4.0	7	5.44

The three encoders $\mathcal{E}_1$, $\mathcal{E}_2$ and $\mathcal{E}_3$ all have equal code rate, $R_c = 1/2$, that is one information bit gives two coded bits. The number of memory elements differs encoder $\mathcal{E}_1$ from the other two though. While encoder $\mathcal{E}_1$ simply have 2 memory elements, encoder $\mathcal{E}_2$ and $\mathcal{E}_3$ both have 4 each. Because of this, the encoder $\mathcal{E}_1$ only have 4 different states, while encoder $\mathcal{E}_2$ and $\mathcal{E}_3$ will have 16 different states. More states means a more complex coding algorithm and thus longer computational times.

As can be seen in figure 2, encoder $\mathcal{E}_3$ performs best at BER 10^{-4}. Interestingly, both encoder $\mathcal{E}_1$ and $\mathcal{E}_2$ performs much alike even though encoder $\mathcal{E}_2$ has twice the amount of memory elements compared to encoder $\mathcal{E}_1$. The reason for this, and why encoder $\mathcal{E}_3$ performs best, depends on the free distance. Both encoder $\mathcal{E}_1$ and $\mathcal{E}_2$ has d_{min} 5 while encoder $\mathcal{E}_3$ has d_{min} 7. As one could expect it is desired to have a large free distance.

3. Coding can Increase Efficiency

In figure 3 a fair comparison is made between system 1 ($S1$): $\mathcal{X}_{\text{BPSK}}$ and encoder $\mathcal{E}_3$, system 2 ($S2$): $\mathcal{X}_{\text{QPSK}}$ with Grey mapping and encoder $\mathcal{E}_3$ and system 3 ($S3$): $\mathcal{X}_{\text{AMPM}}$ and encoder $\mathcal{E}_4$. The power efficiency at BER 10^{-4} is compared between the three systems in table 3.

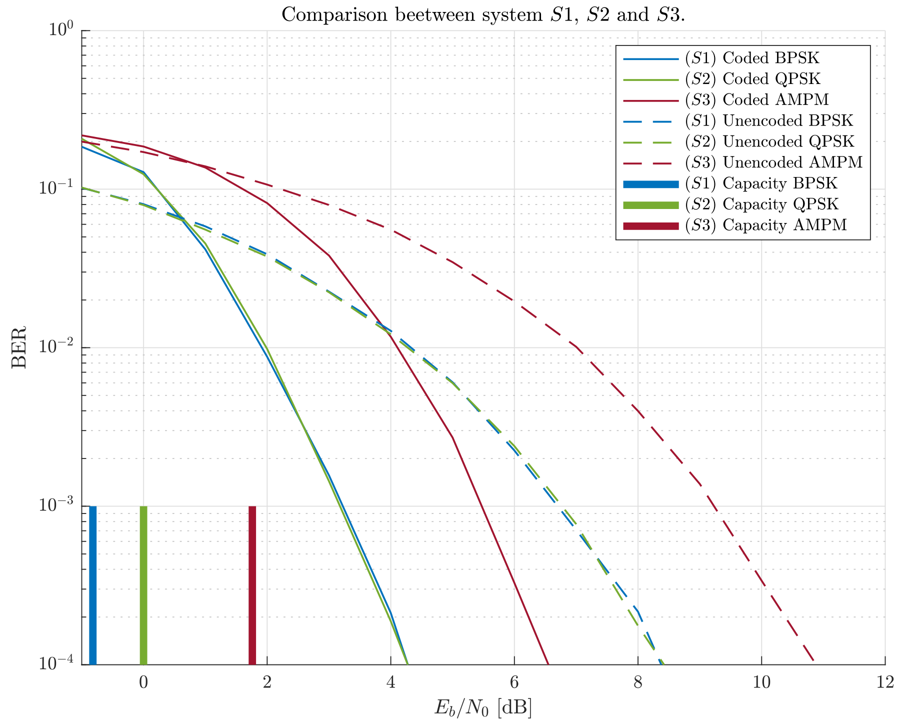

Figure 3: Comparison of the three systems: $\mathcal{E}_3$ and $\mathcal{X}_{\text{BPSK}}$, $\mathcal{E}_3$ and $\mathcal{X}_{\text{QPSK}}$ with Grey mapping, and $\mathcal{E}_4$ and $\mathcal{X}_{\text{AMPM}}$ and their capacities.

Table 3: Fair comparison between the three systems ($S1$, $S2$ and $S3$) at a BER of 10^{-4}.

	Decoder	System	Power efficiency [dB] (at BER 10^{-4})	Distance to capacity [dB]
Coded	Soft	$S1$	4.28	5.10
		$S2$	4.28	4.28
		$S3$	6.55	4.79
Uncoded	Hard	$S1$	8.38	9.20
		$S2$	8.42	8.42
		$S3$	10.88	9.12

The spectral efficiency is the rate, R, of the system and it is calculated by multiplying the code rate of the system with $\log_2(M)$, where M is the number of symbols in the constellation. In other words, the spectral efficiency does not change when changing between coded or uncoded and hard- or soft decoding. The spectral efficiencies for the different systems are presented in table 4 together with the free distances.

Table 4: Spectral efficiencies and free distance of systems $S1$, $S2$ and $S3$.

System	Spectral efficiency	d_{min}
S1	0.5	7
S2	1	7
S3	2	2

All systems have different strengths and weaknesses, but for all systems a coded transmission is better than transmitting uncoded. The data from table 3 and table 4 shows that $S1$ and $S2$ have equal power efficiency, while $S2$ also have twice the spectral efficiency as $S1$. Both of these systems perform approximately 1 dB better than $S3$ at a BER equal to 10^{-4}, even though $S3$ has the highest spectral efficiency. One of the main reasons for this is because of the free distance, which is lower for $S3$ compared to the other two systems.

For low SNR, $S1$ or $S2$ is preferable since the power efficiency is higher. For higher SNR where the BER of $S3$ is acceptable it is better. This is due to the spectral efficiency being the highest and thus it allows a higher bit rate. Over all non of them are strictly better, since they all have an area where they excel in comparison to the other codes.

Looking closer at $S1$ and $S2$ in figure 3 shows that they almost look identical. This is due to them using the same encoder and decoder. The only difference between them is that they are BPSK and QPSK, hence what separates them is the spectral efficiency, $R = R_c \cdot log_2(M)$, where the M is 2 for BPSK and 4 for QPSK.

Detta skall inte vara med i rapporten sen. Bara för studerandets skull

1 System overview

What are the lengths of each of the vectors shown in the block diagram?

Are the elements discrete, continuous, complex, . . . ?

How does one measure the performance of such a system?

1.1 Convolutional Encoder

Are the encoders systematic?

Can you draw a trellis diagram corresponding to a given encoder?

How long is the vector c for the three encoders?

1.2 Symbol Mapper

What is the average energy per symbol or per information bit?

What is the minimum distance normalized by the average energy for each constellation?

Can you draw the optimal symbol decision regions assuming additive white Gaussian noise?

1.3 Additive Gauissian Noise Channel

Which components usually follow after the symbol mapper in a real communication system?

Why can we assume that they are not there?

Does that have any implications for the obtained performance results with respect to a real communication system?

What is the relationship between σ^2 and N_0 (the one-sided power spectral density of an additive white Gaussian noise (AWGN) channel)?

1.4 Hard receiver

Why is this receiver structure not optimal?

Can you think of another way to pass information about the symbols/coded bits from the symbol detector to the decoder?

1.5 Soft receiver

What is the cost function that the soft-input Viterbi decoder minimizes?

Is this receiver structure optimal?

What is the difference between the hard-input Viterbi and soft-input Viterbi decoder from an implementation point-of-view?