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Problem 3

(1) Explain why a prefix-free code is always uniquely decodable

A code is a prefix-free code if no code word is a prefix of another one.
A code is uniquely decodable if the symbols generated by the source can be uniquely decoded from the encoded string of bits
Therefore, a prefix-free code is always uniquely decodable because we can at once, or instantaneously, decode code words at the receiver, without waiting for additional received bits. So a prefix-free code is always uniquely decodable. Whereas a uniquely decodable code is not always prefix-free

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	a	b	c	d	e
x_1	00	0	0	1	0
x_2	10	10	01	101	1
x_3	01	11	11		01
x_4	11				

Code a: It is uniquely decodable but not prefix-free.
We can improve on it by using Variable-length code
Example {0, 10, 110, 111}

Code b: This is uniquely decodable and prefix-free

Code c: It uniquely decodable, but not prefix-free, because if receive 011, we will need more received bits to decide
A better code { 0, 10, 11}

Code d: It is uniquely decodable and prefix-free. However, we can achieve same with a lower number of bits. Example $\{0, 1\}$

Code e: This is not uniquely decodable, and so not prefix-free as well. A better example $\{0, 10, 11\}$ or $\{0, 10, 110\}$, with variable length code.

3 Using kraft's inequality, with codeword length, l

$$\sum_{i=1}^{M} 2^{-l} \leq 1$$

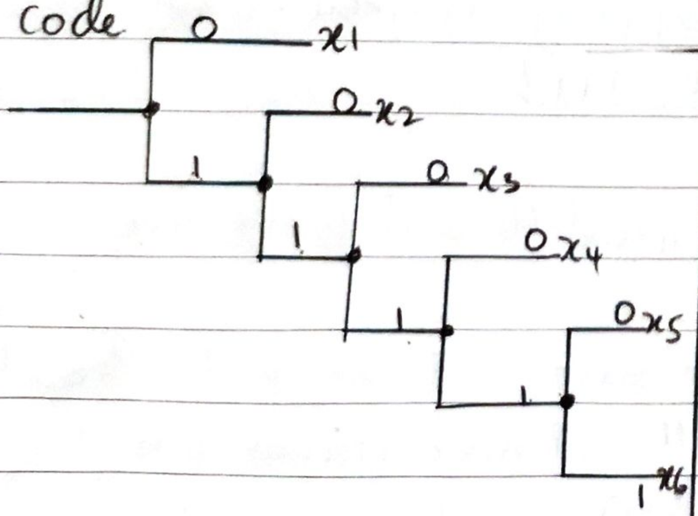
(5) $(1, 2, 3, 4, 5, 5)$

$$\sum_{i=1}^{6} (2^{-1} + 2^{-2} + 2^{-3} + 2^{-4} + 2^{-5} + 2^{-5}) \leq 1$$

$$= \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{32}$$

$$= \frac{32}{32} = 1$$

Since kraff's inequality is satisfied, it is prefix-free code



Symbol	Length	codeword
x_1	1	0
x_2	2	10
x_3	3	110
x_4	4	1110
x_5	5	11110
x_6	5	11111

(b) (1, 2, 3, 4, 4, 5)

$$\sum_{i=1}^{6} (2^{-1} + 2^{-2} + 2^{-3} + 2^{-4} + 2^{-4} + 2^{-5}) \leq 1$$

$$= \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} + \frac{1}{32} = \frac{33}{32}, = 1.03125$$

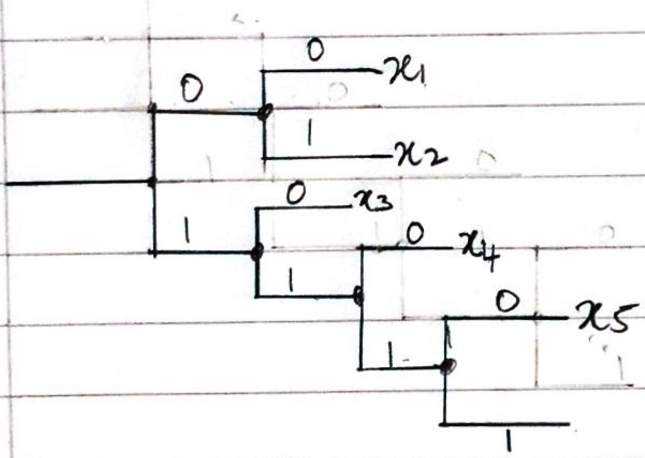
This is more than 1 and so Kraff's inequality is not obeyed Therefore, it is not prefix-free

(c) (2, 2, 2, 3, 4)

$$\sum_{i=1}^{5} (2^{-2} + 2^{-2} + 2^{-2} + 2^{-3} + 2^{-4}) \leq 1$$

$$= \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = \frac{15}{16}, = 0.9375$$

Kraff's inequality is satisfied and so it is prefix-free



Symbol	Length	Codeword
x_1	2	00
x_2	2	01
x_3	2	10
x_4	3	110
x_5	4	1110

This is uniquely decodable and prefix-free