Digital Communications - SSY125

Convolutional Coding and the Viterbi Algorithm

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1 Hard vs. Soft Receiver

The first task is about the evaluation of the performance of the system for uncoded and coded transmission, using ϵ_2 and χ_{QPSK} with the Gray mapping. For the soft decoder also the upper bound error probability has been considered. The results are shown in Figure 1.

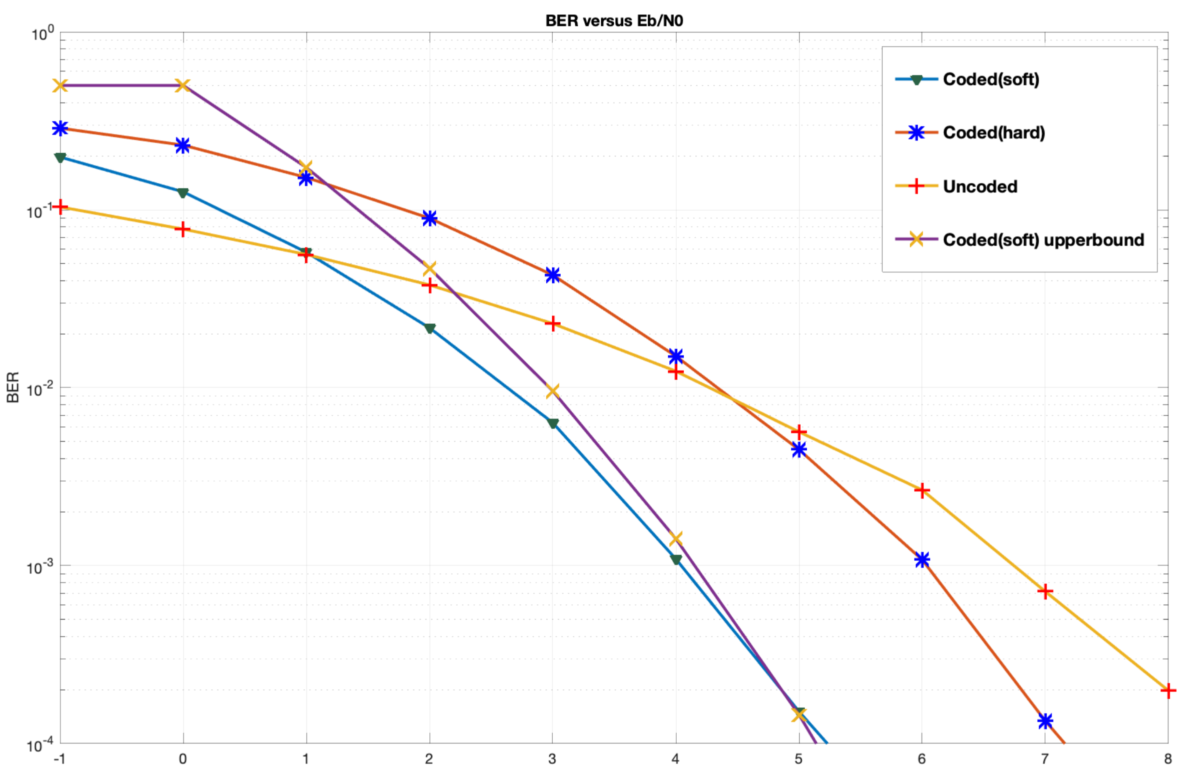

Figure 1: BER vs E_b/N_0 plot for the encoder ϵ_2 with both hard and soft receiver

In order to evaluate the coding gain at BER=10^{-4}, it is sufficient to look at the difference (in dB) between the uncoded simulation and the coded simulations.

The theoretical asymptotic coding gain can be calculated from the following formula:

$$Gain_{\infty} = 10\log_{10}(R_c d_{min}) \tag{1}$$

Where R_c is the coding rate (on our case 1/2) and d_{min} is the free distance (minimum Hamming distance between different codes). Equation 1 is valid only for soft decoding, and for this reason the asymptotic gain for hard decoding has not been calculated. In order to calculate the minimum value of E_b/N_0 for both the hard and the soft case, it is sufficient to look at where the graphs for uncoded and coded simulations cross. However, equation 2 has been used for our case:

$$\frac{E_b}{N_0} = \frac{2^R - 1}{R} \tag{2}$$

Where R is the rate, that in our case is 1 since the code rate of ε_2 is 1/2 and the number of bits per symbol is 2 (QPSK). From equation 2 we can notice that the minimum E_b/N_0 is 0 dB, that is the capacity of the channel. In order to achieve reliable communication, the value of E_b/N_0 must be greater than 0dB. Table 1 summarizes all the values mentioned so far.

Table 1: Comparison between hard and soft receivers using encoder ϵ_2 and QPSK modulation (BER=10^{-4})

	Hard Receiver [dB]	Soft Receiver [dB]
Coding gain	-	1.25
Asymptotic gain	3.40	3.98
Minimum E_b/N_0	4.50	0.8

From Figure 1, we can see that the difference in coding gain between hard and soft receivers is 1.9 dB. The soft-decoding is much more complex than the hard-decoding. Even if it performs better in terms of BER for a specific SNR, it is not so easy to apply in many applications.

2 Encoder Comparison

In this section, we will compare the performance of the considered three encoders ϵ_1, ϵ_2 and ϵ_3 for χ_{QPSK} with the Gray mapping, by only considering the soft receiver. For each encoder, the upper bound of the theoretical BER has also been calculated. Figure 2 shows the obtained results .

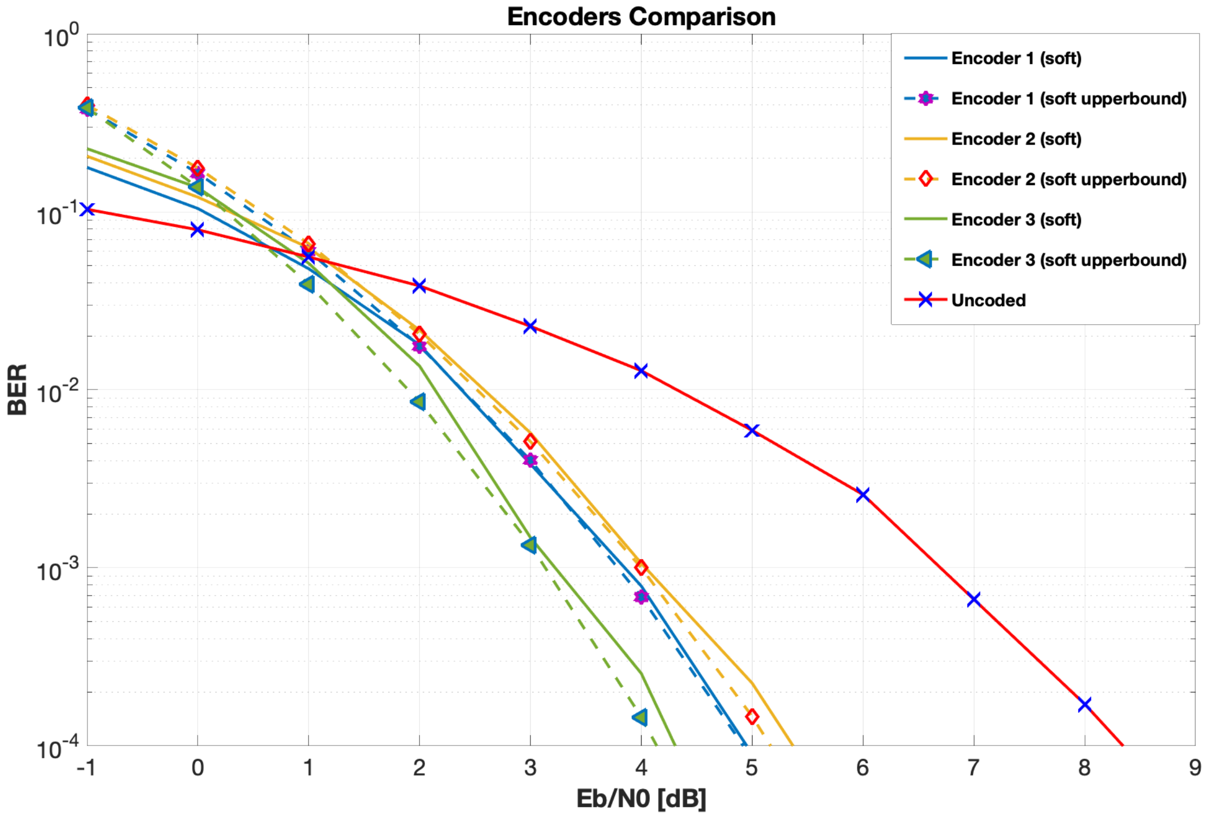

Figure 2: BER vs E_b/N_0 plot where the encoders (ϵ_1 , ϵ_2 and ϵ_3 are compared. The upper bounds are plotted as well.

The coding gain and the asymptotic gain at BER=10^{-4} has been calculated in the same way as in the previous task. The results are shown in Table 2.

Table 2: Comparison between ϵ_1, ϵ_2 and ϵ_3 using QPSK modulation (BER = 10^{-4})

	ϵ_1	ϵ_2	ϵ_3
Coding gain	3.58	3.44	4.11
Asymptotic gain	3.98	3.98	5.53

Table 2 shows that the encoder ϵ_3 performs best at BER=10^{-4}, followed by ϵ_1. The worst encoder among the three is ϵ_2 because of low free hamming distance. Since it is useful to keep the complexity in the system as low as possible, it can be seen that the complexity of ϵ_1 and ϵ_2 are the same, where ϵ_2 require four registers and 5 bitxor for every information bit. ϵ_1 instead requires two delay elements and its performance is still better than ϵ_2. However ϵ_3 perform better than both of the two encoders because it has larger hamming distance =7 where the other's have = 5.

3 Coding can Increase Efficiency

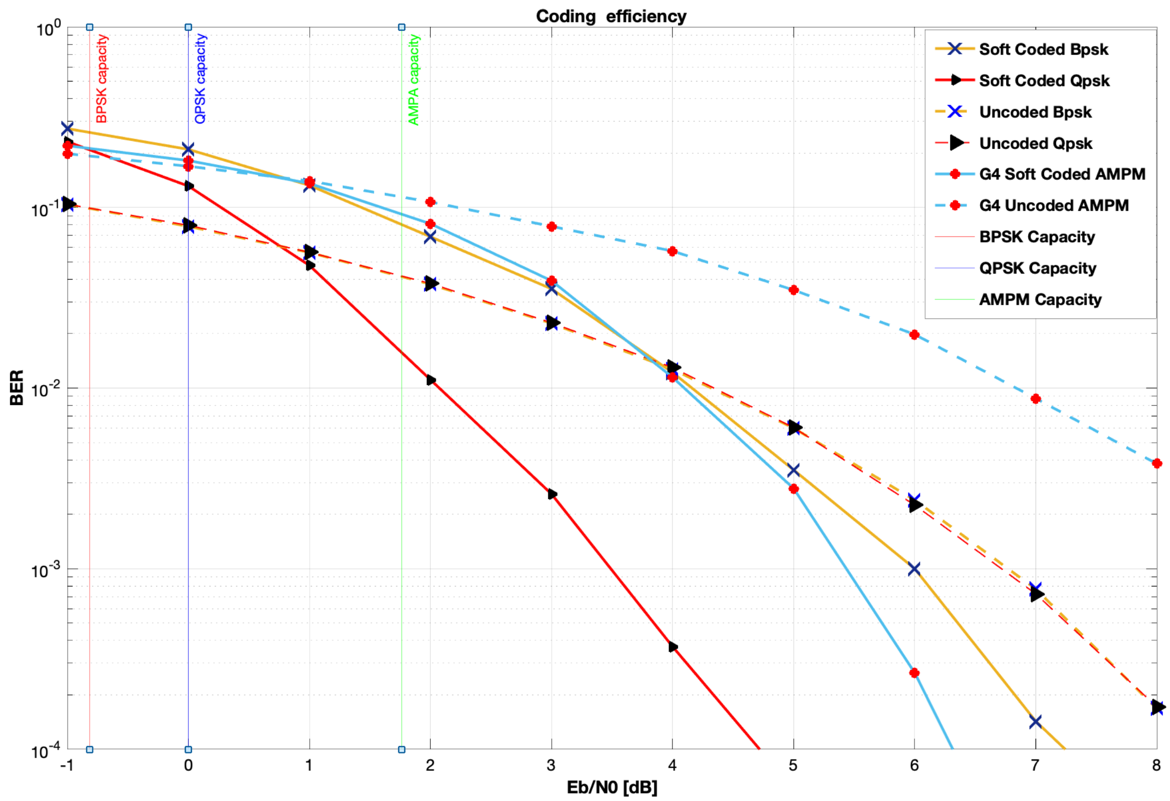

Figure 3: BER vs E_b/N_0 plot where the encoders ϵ_1 , ϵ_2 and ϵ_3 are compared. The upper bounds are plotted as well.

In this section, we will compare System 1 (ϵ_3 with BPSK modulation), System 2 (ϵ_3 with QPSK modulation),and System 3 (ϵ_4 with AMPM modulation) for different performance parameters like power efficiency at BER=10^{-4} and spectral efficiency for coded and uncoded scenarios. The results are shown in Table 3.

The spectral efficiency can be written as product of encoder rate R_c and the modulation order of the system (number of bits per symbol= $\log_2 *M$ where M is the number of symbols in the constellation). However, R_c=1 for uncoded transmission. Therefore, we can say that coding is modifying our spectral efficiency. In fact it decreases it as depicted in the Table

3 for all considered systems. Whereas, S3 is better than S1 and S2 in term of spectral efficiency (because of higher free hamming distance) but it needs almost 2dB more power than S1 and S2 in terms of power efficiency. In addition to that, S1 and S2 BERs are almost similar (because the BERs for QPSK and BPSK are the same) because we used same encoder and decoder. The difference is the spectral efficiency since BPSK sends one bit per symbol whereas QPSK sends 2 bits per symbol.

The system performance in term of capacity distance can be seen from Figure 3, which tells us that the difference is less for coded than uncoded which means its achieving higher capacity as compared to the uncoded part for different systems. The System 2 for coded part is outperforming all the other systems since it's value is 4.29 as compared to 4.31 (S1), 6.58 (S3), and uncoded values are 8.37 (S1), 8.41 (S2) and 10.87(S3) respectively.

All the coded systems are performing worst than uncoded systems in term of spectral efficiency as shown in Table 3. In the other way, coded power efficiency in all systems is almost double than uncoded. But coding decreases the BER with respect to uncoding. QPSK soft is outperforming all other scenarios in term of BER and AMPM is also performing better then rest of the others except QPSK (soft). In general coding enhances BER performance more than uncoded system. Overall, coding increases efficiency in terms of BER and power efficiency but at the loss in spectral efficiency .

Table 1: Table 3: Comparison between ϵ_3 using BPSK QPSK modulation and ϵ_4 using AMPM modulation (BER = 10^{-4})

Decoder	**System type**	**Power efficiency**	**Spectral efficiency**
Coded (soft)	S1	4.31	0.5
	S2	4.29	1
	S3	6.58	2
Uncoded	S1	8.37	1
	S2	8.41	2
	S3	10.87	3