PROBLEM 1 Olalekan Peter Adare : Haitham Babbili .

Given

$X = \{x_0, x_1, \dots, x_m\}$ with probabilities $P_0, P_1, \dots, P_m$

$Y = \{y_0, y_1, \dots, y_m\}$ with probabilities $q_0, q_1, \dots, q_m$

where $i = 0, 1, \dots, j-2, j+1, \dots, M$, with $0 < j < M$

$$q_i = P_i$$
$$q_j = q_{j-1} = \frac{P_j + P_{j-1}}{2}$$

Using Basic formula, represent the entropies with $H(x)$ and $H(y)$

$$H(x) = \sum_{i=1}^{M} P_i \log_2\left(\frac{1}{P_i}\right) \text{ and } H(y) = \sum_{i=1}^{M} q_i \log_2\left(\frac{1}{q_i}\right)$$

Measure of Entropy associated with each possible data value is

$$H_s = -\sum_{i=1}^{M} P_i \log P_i \text{ , negative Logarithm}$$

Exploring; that $q_i \doteq P_i$

$$H_{q_i} = -\sum_{i=1}^{M} q_i \log q_i$$

$$H_{pi} = -\sum_{i=1}^{M} P_i \log P_i$$

Therefore, $H_{q_i} - H_{pi} = -\sum_{i=1}^{M} q_i \log q_i - \left(-\sum_{i=1}^{M} P_i \log P_i\right)$

$$= -\sum_{i=1}^{M} q_i \log q_i + \sum_{i=1}^{M} P_i \log P_i$$

$$= -(q_{j-1} \log_2 q_{j-1} + q_j \log_2 q_j) + (P_{j-1} \log_2 P_{j-1} + P_j \log_2 P_j)$$

$$= -q_{j-1} \log_2 q_{j-1} - q_j \log_2 q_j + P_{j-1} \log_2 P_{j-1} + P_j \log_2 P_j$$

There is a given function that

$$f_a(x) = -x\log x - (a-x)\log(a-x) \quad \text{——— (1)}$$

from the above, the highest value of $f_a(x)$ will be at $x = a/2$

Already given that $q_i = P_i$ and

$$q_j = q_{j-1} = \frac{P_j + P_{j-1}}{2}.$$

let $P_j + P_{j-1} = A$, $\Rightarrow P_{j-1} = A - P_j$

Therefore, $q_j = q_{j-1} = \frac{A}{2}$, and put them into our equation

$$H_{q_i} - H_{p_i} = -\frac{A}{2}\log_2\frac{A}{2} - \frac{A}{2}\log_2\frac{A}{2} + (A-P_j)\log_2(A-P_j) + P_j\log_2 P_j$$

comparing with equation (1), of $f_a(x)$

$$H_{q_i} - H_{p_i} = \left(-\frac{A}{2}\log_2\frac{A}{2} - \frac{A}{2}\log_2\frac{A}{2}\right) - \left(-P_j\log_2 P_j - (A-P_j)\log_2(A-P_j)\right)$$

$$= f_A\left(\frac{A}{2}\right) - f_A(P_j).$$

$$H_{q_i} - H_{p_i} = f_A\left(\frac{A}{2}\right) - f_A(P_j)$$

This is possible, if and only if, $H_{q_i} = H_{p_i}$ or

$H_{q_i} > H_{p_i}$

Finally, $H(Y) \geq H(x)$ (It can be greater or equal)