Digital Communications
SSY125, Lecture 14

Coding beyond reliable communication

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CHALMERS

December 4, 2019

Coding beyond coding

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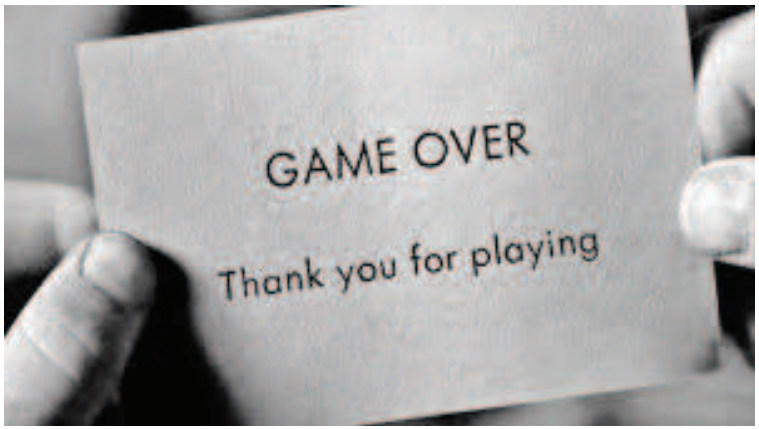

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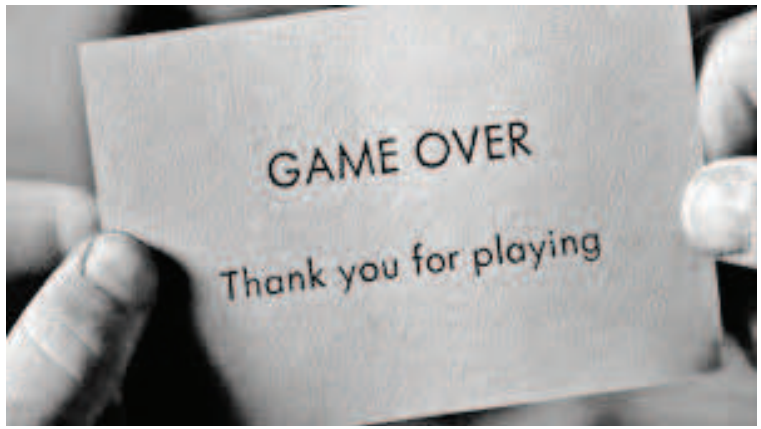

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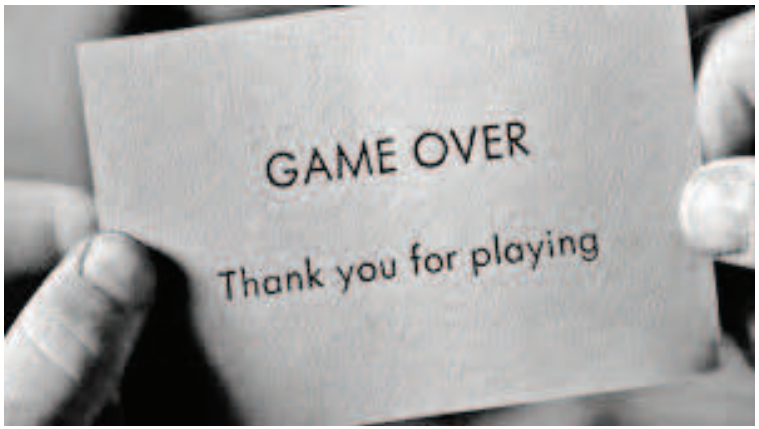
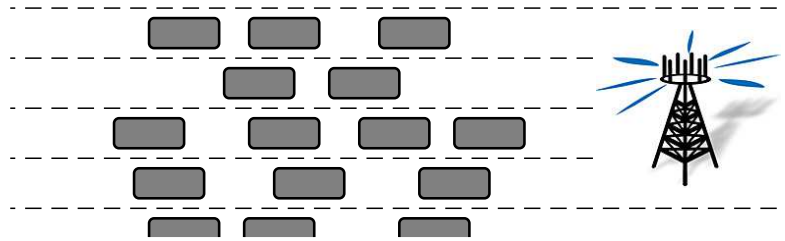


- A very powerful tool for many applications and research fields:
  - Uncoordinated multiple access
  - Post-quantum cryptography
  - DNA storage
  - Caching
  - Distributed computing

Uncoordinated multiple access
Slotted ALOHA



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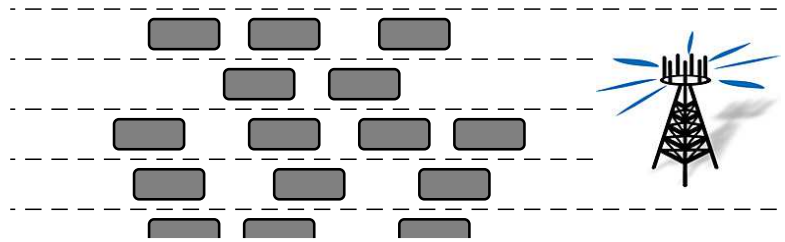

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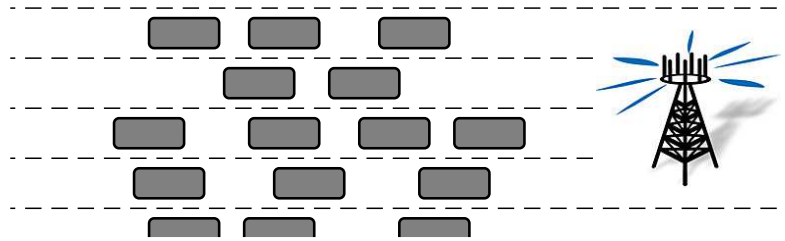

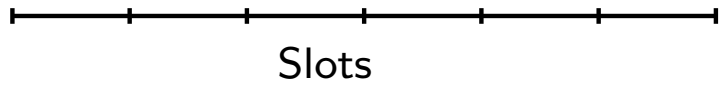

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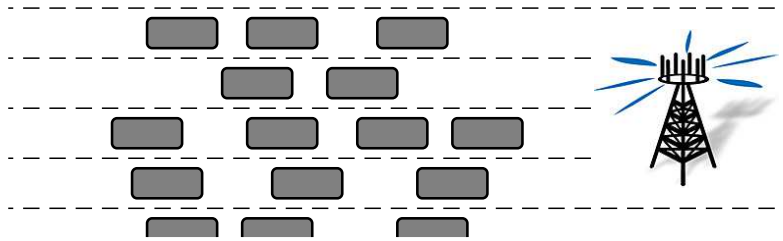

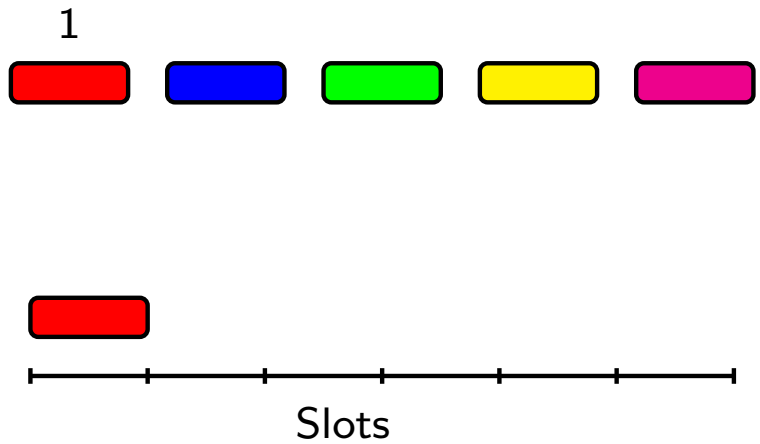

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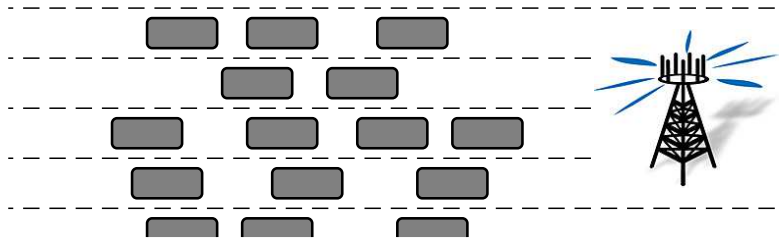

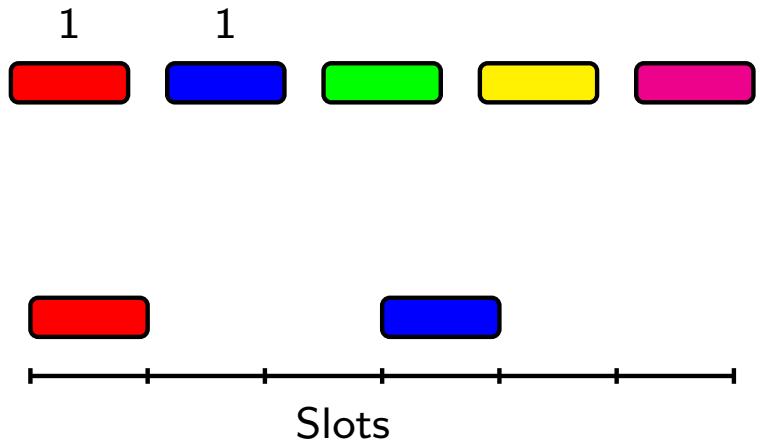

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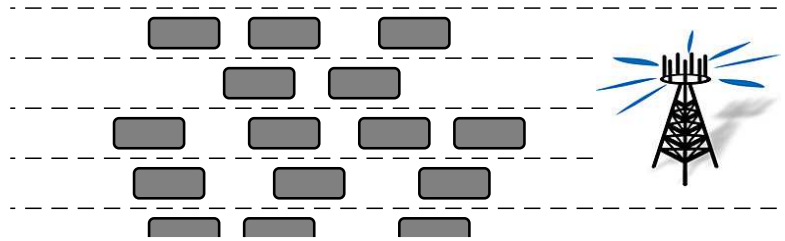

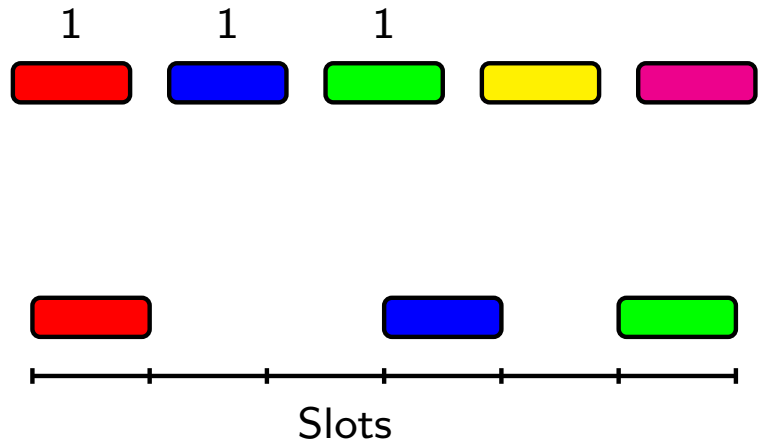

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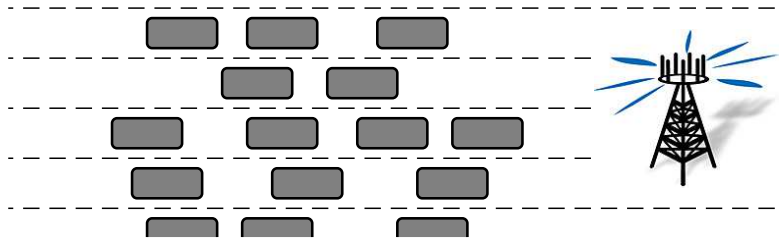

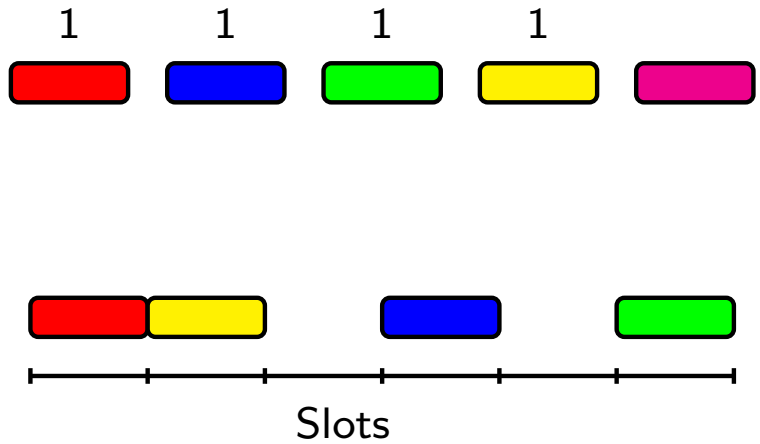

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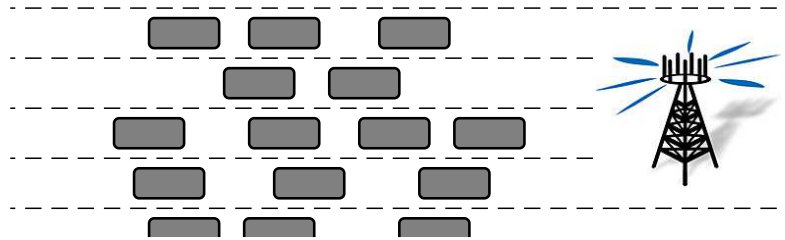

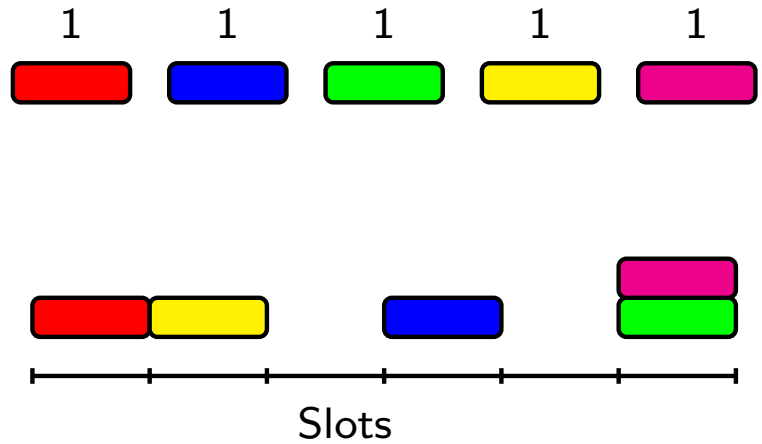

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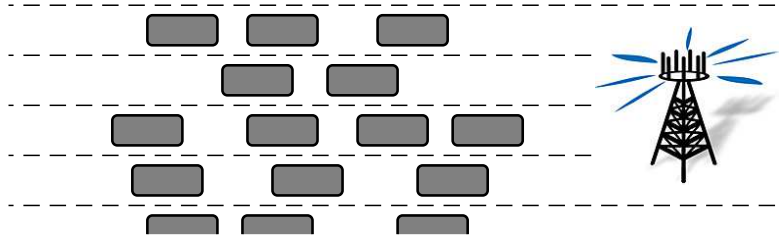

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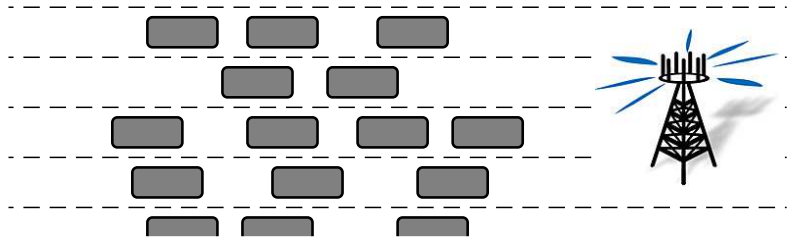

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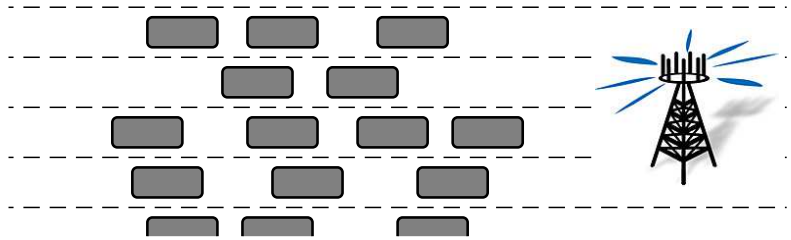

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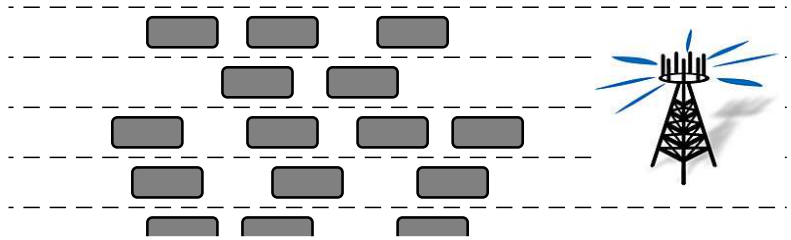

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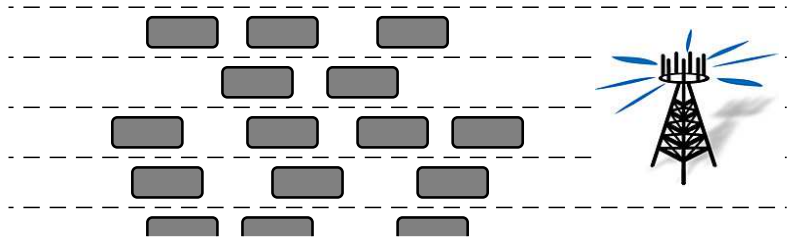

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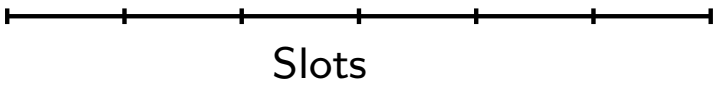

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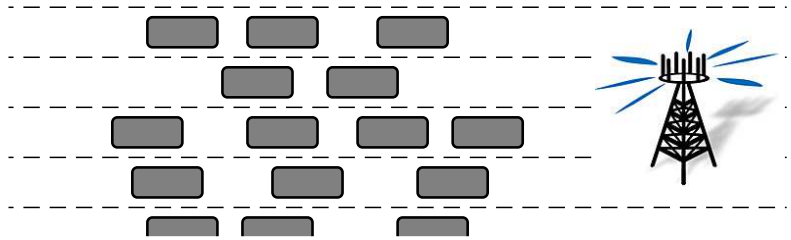

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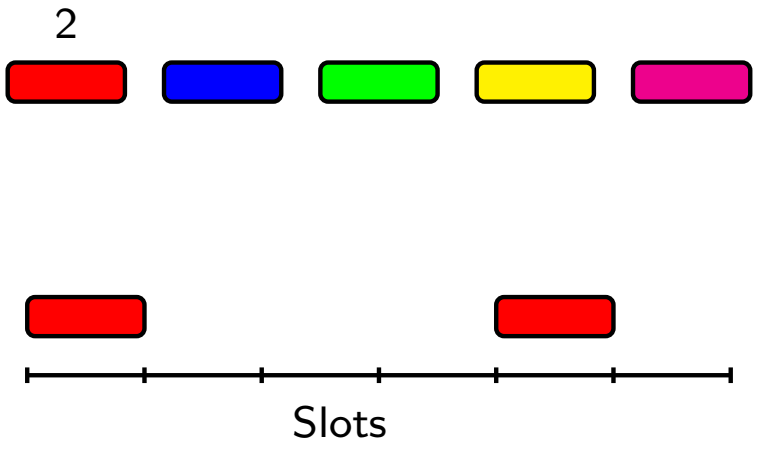

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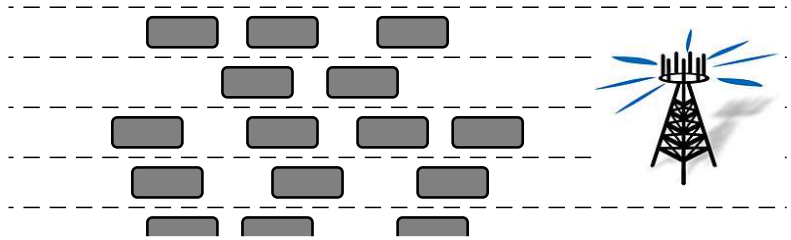

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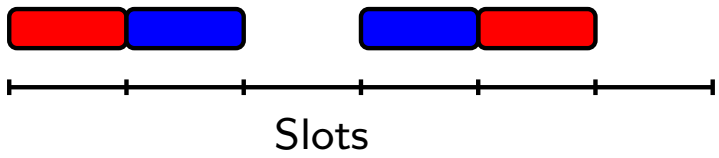

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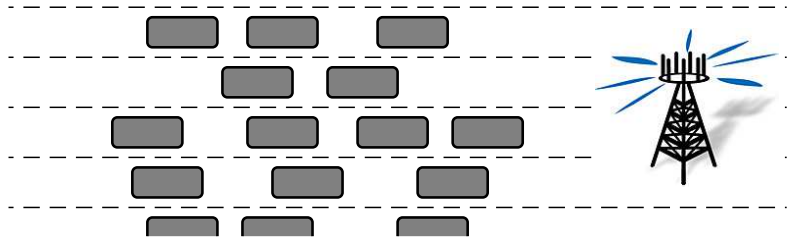

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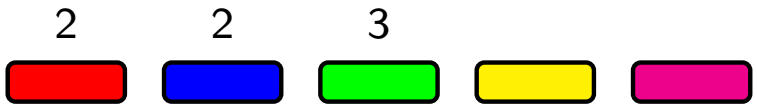

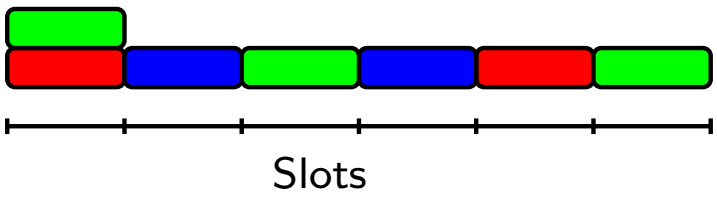

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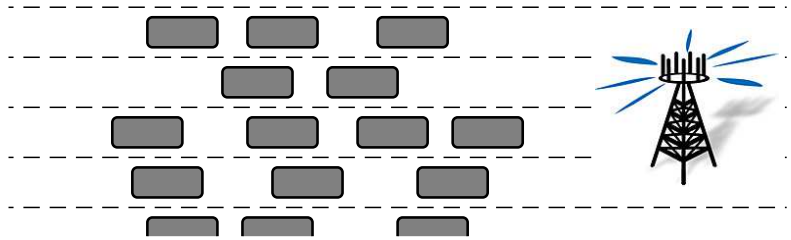

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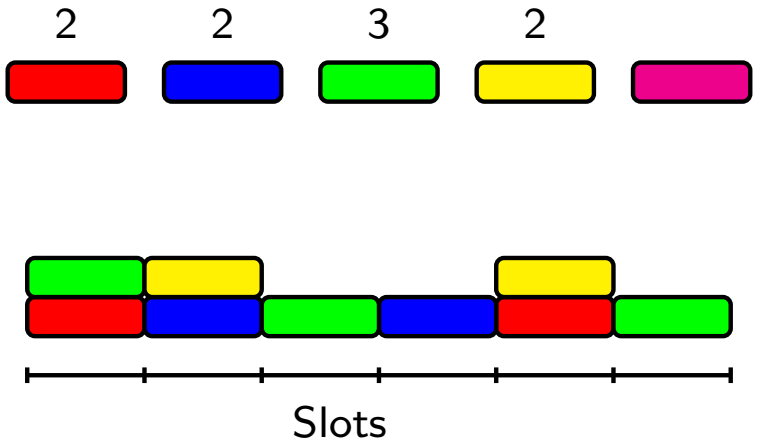

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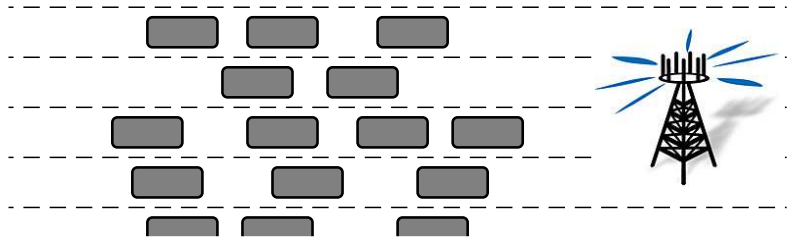

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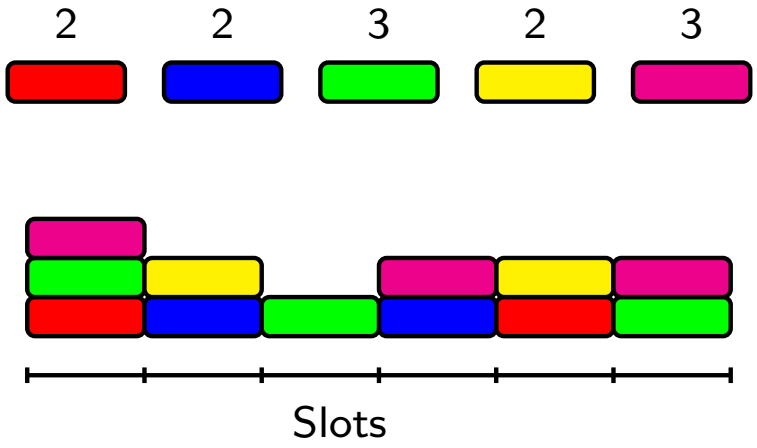

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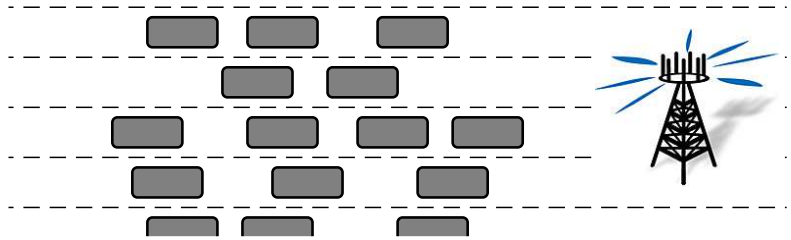

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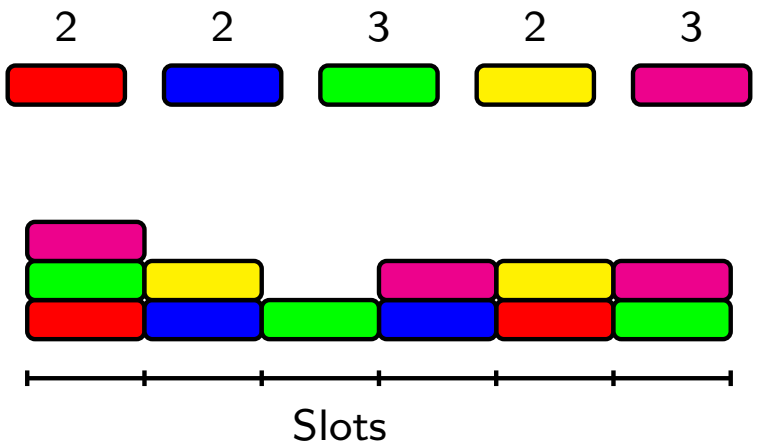

Alternative representation

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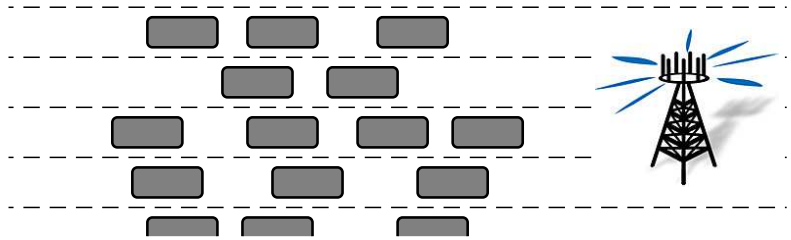

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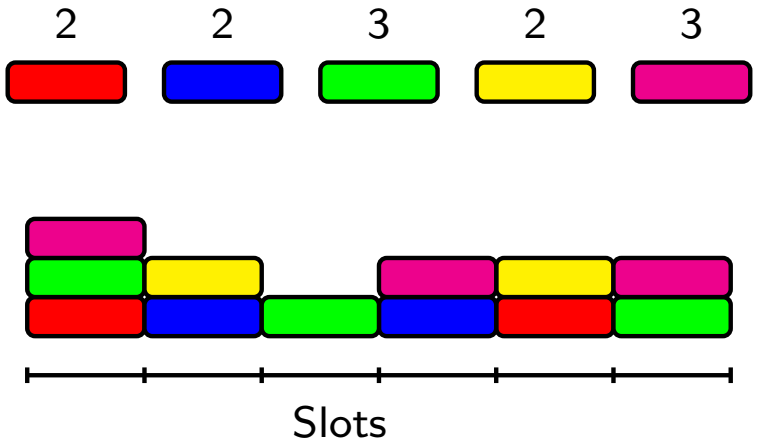

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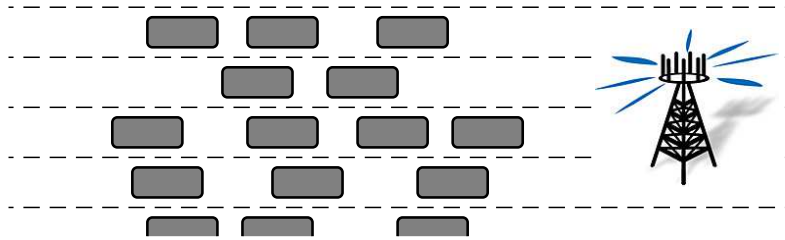

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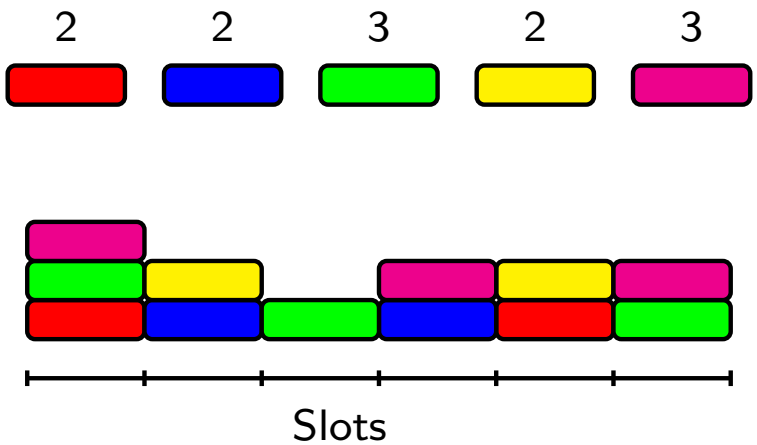

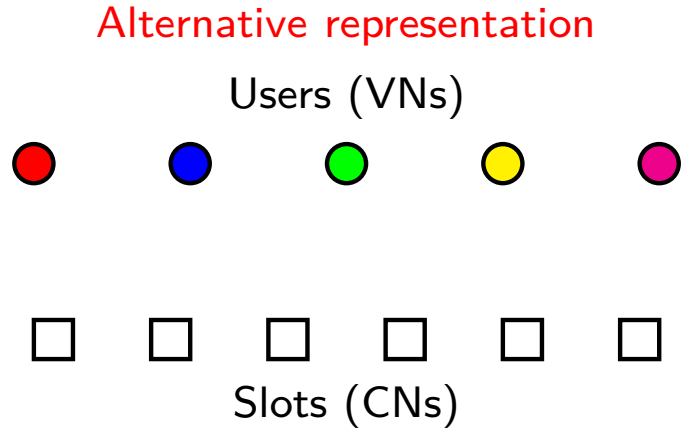

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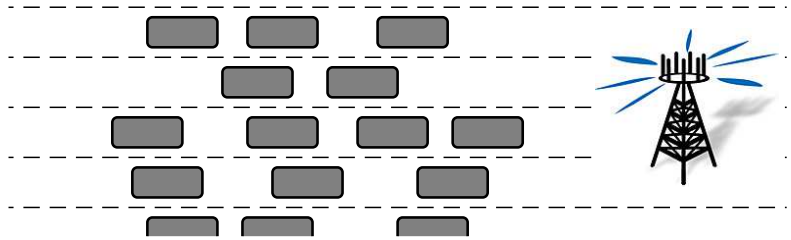

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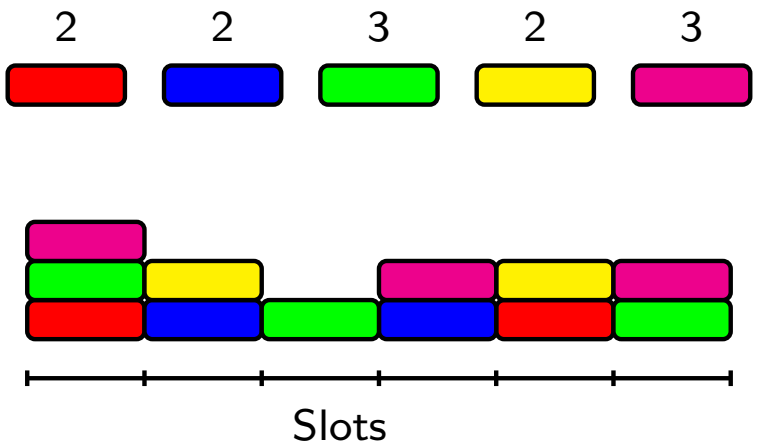

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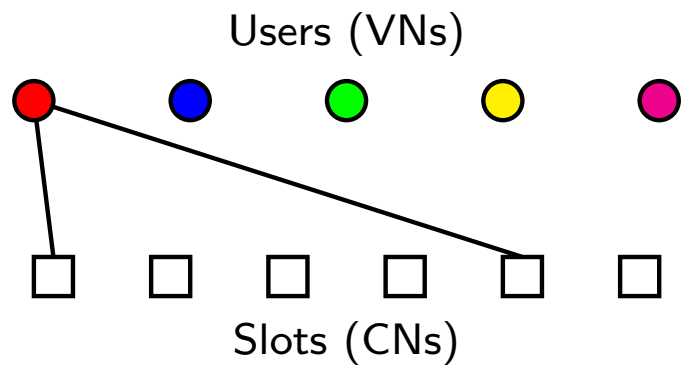

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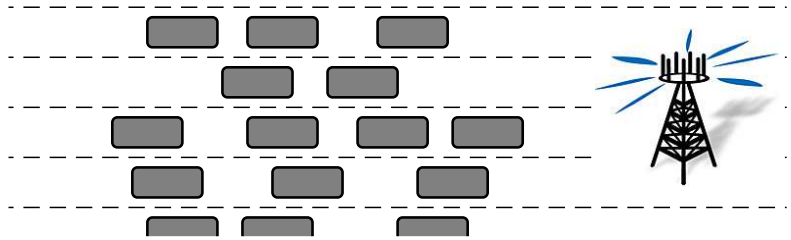

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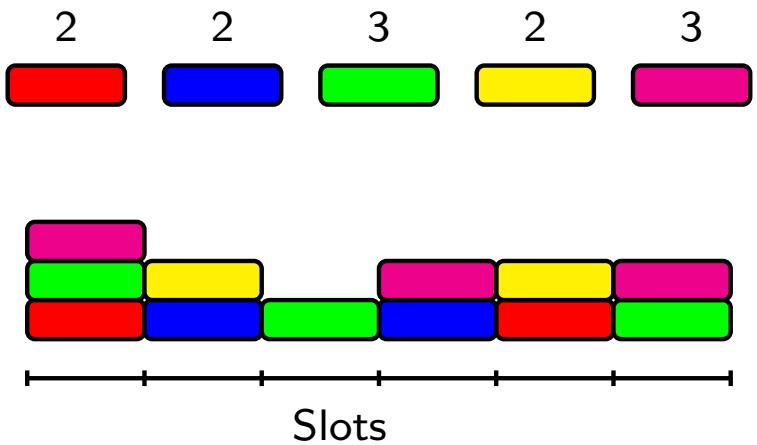
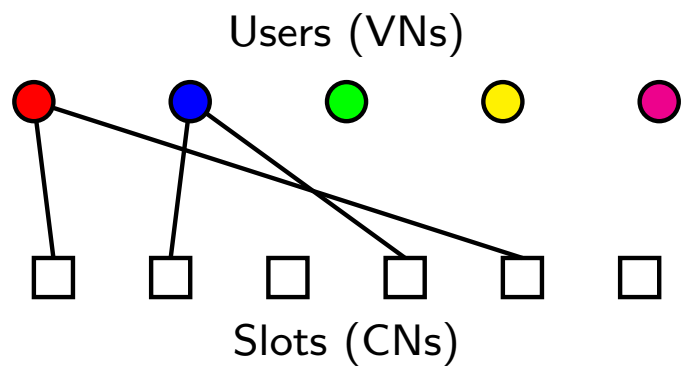


Alternative representation



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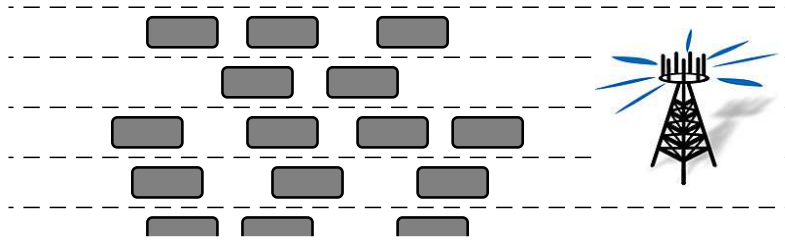

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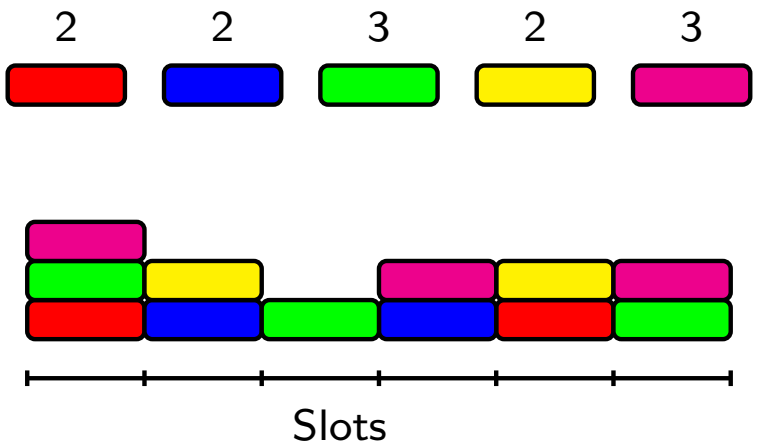
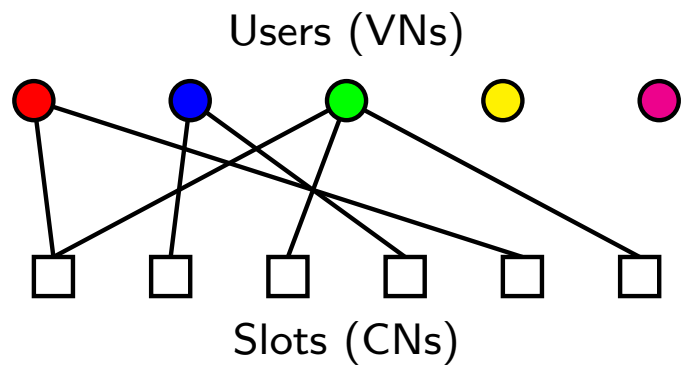


Alternative representation



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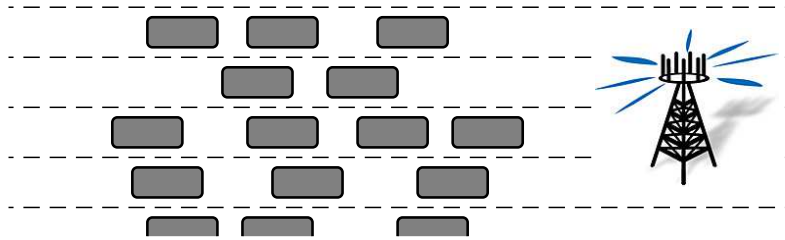

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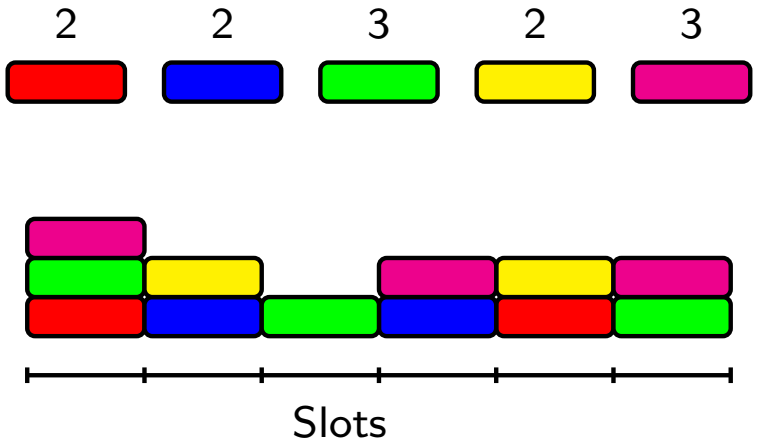
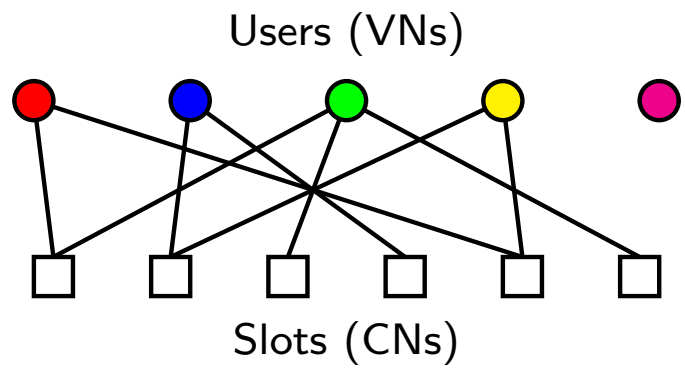


Alternative representation



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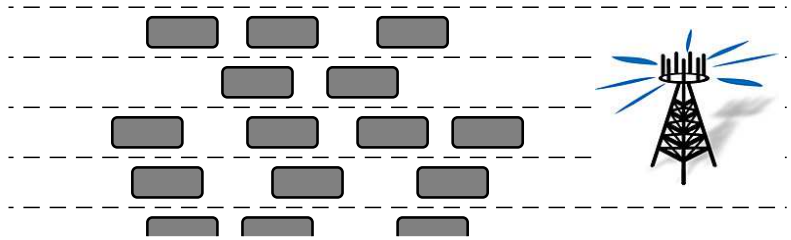

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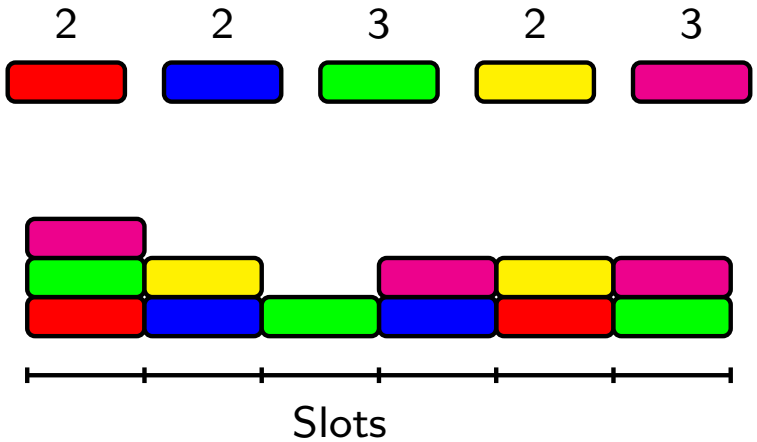
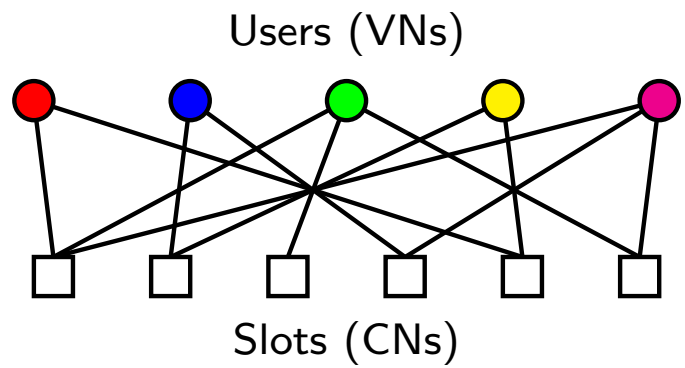


Alternative representation



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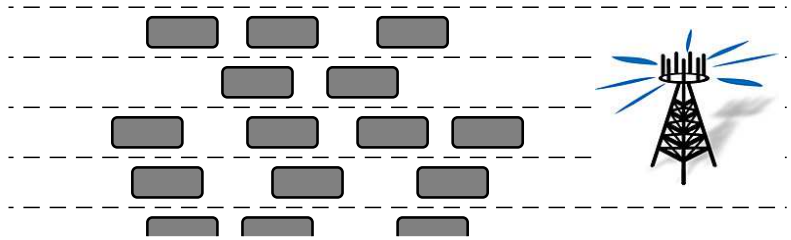

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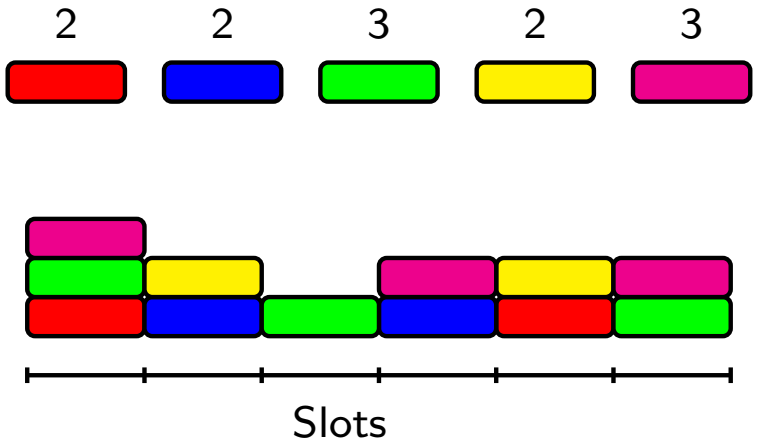
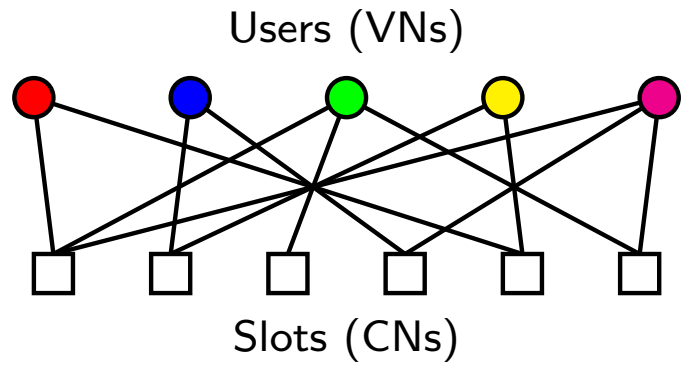
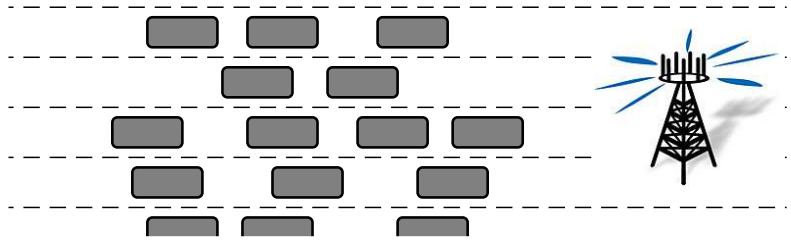


Alternative representation



- A coding problem!

Uncoordinated multiple access
Coded slotted ALOHA



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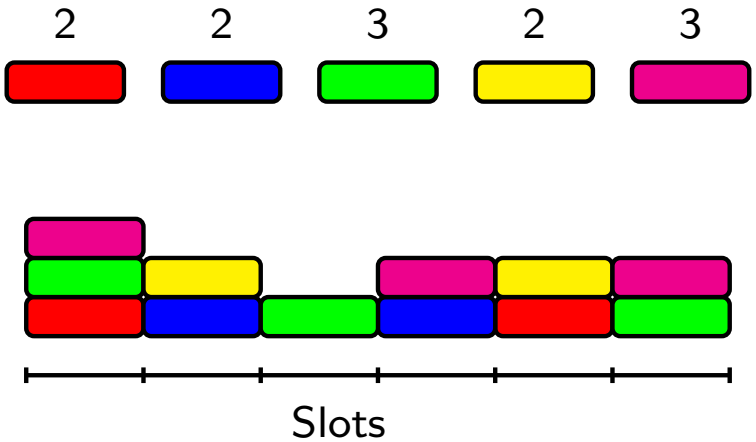
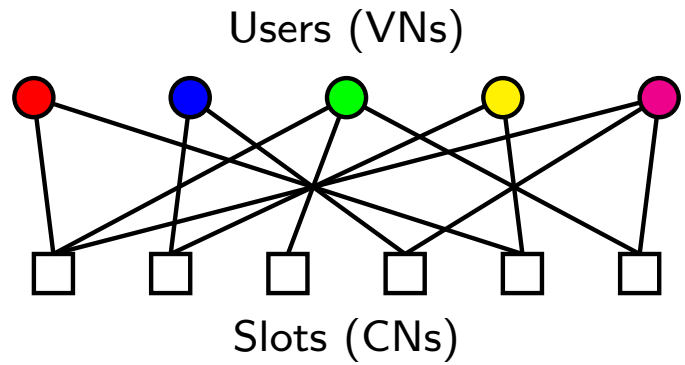
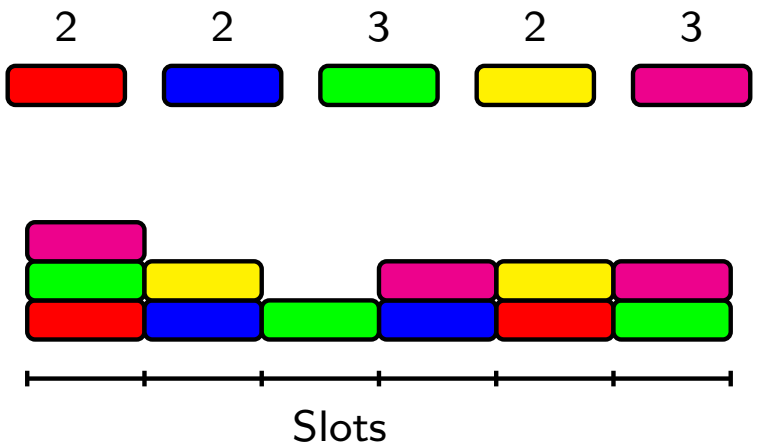


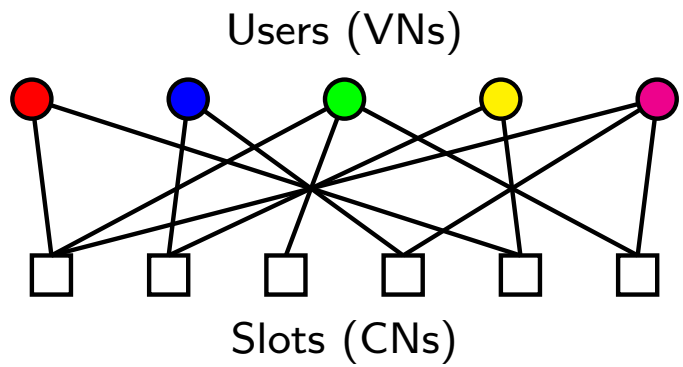
Alternative representation



- A coding problem! $\rightarrow$ Can be analyzed using the theory of codes on graphs

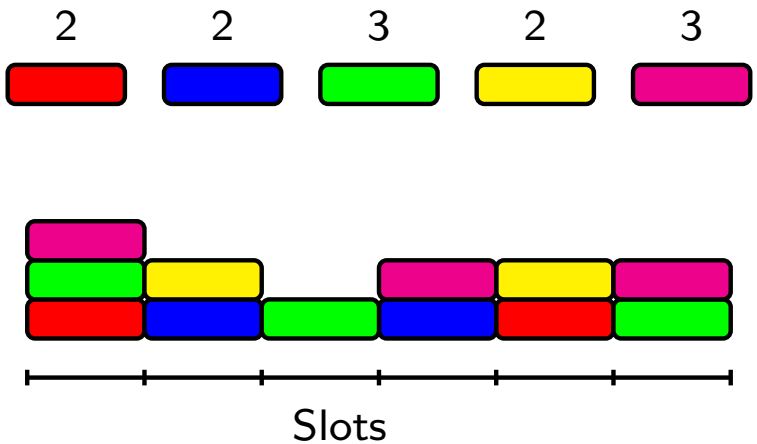
Coded slotted ALOHA: Decoding

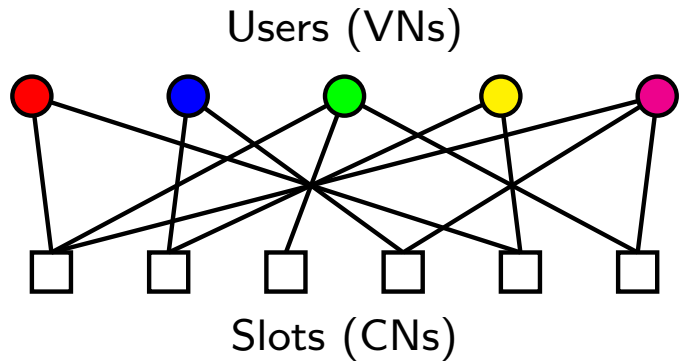




Coded slotted ALOHA: Decoding

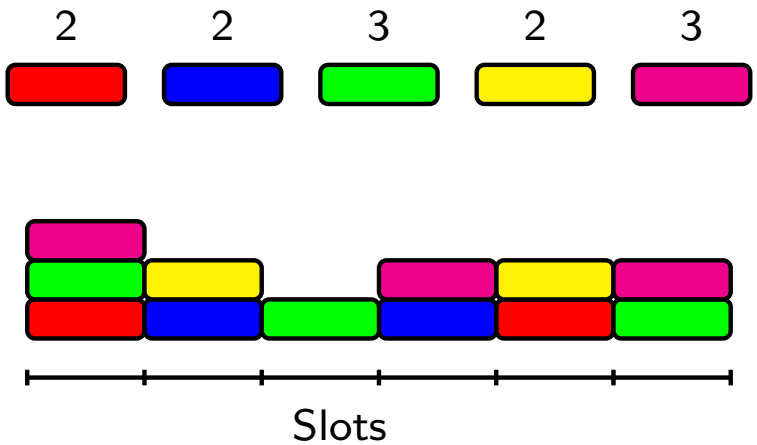
- A packet can be reliably decoded if no collision occurs

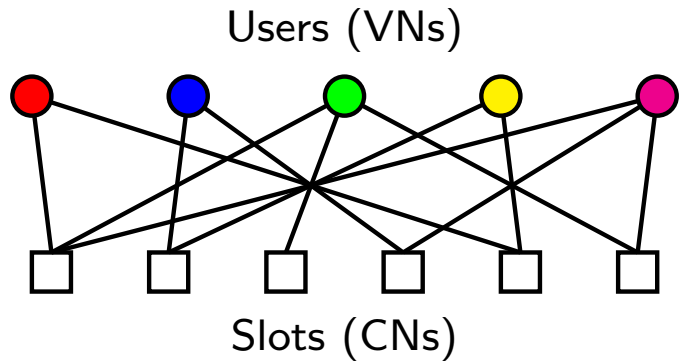




Coded slotted ALOHA: Decoding

- A packet can be reliably decoded if no collision occurs
- Each packet contains pointers to its replicas





Coded slotted ALOHA: Decoding

- A packet can be reliably decoded if no collision occurs
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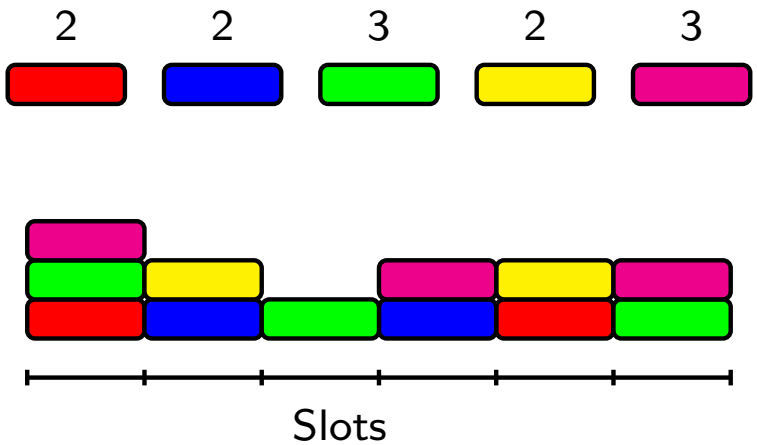

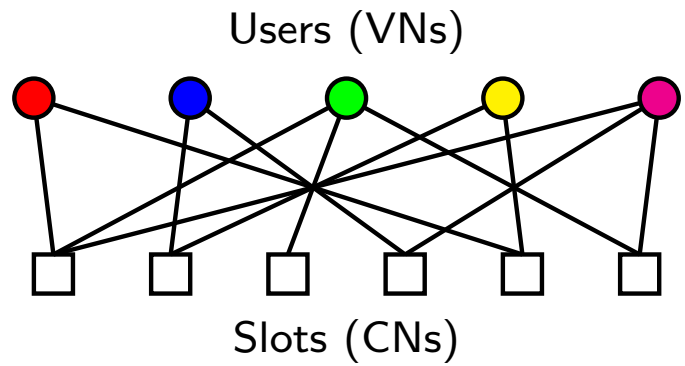

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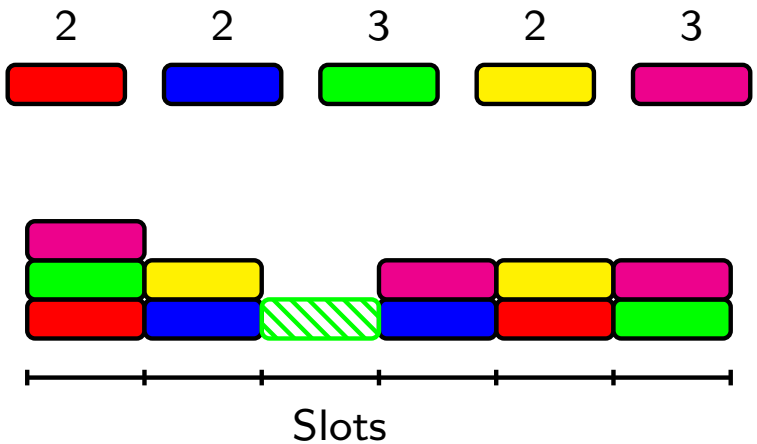

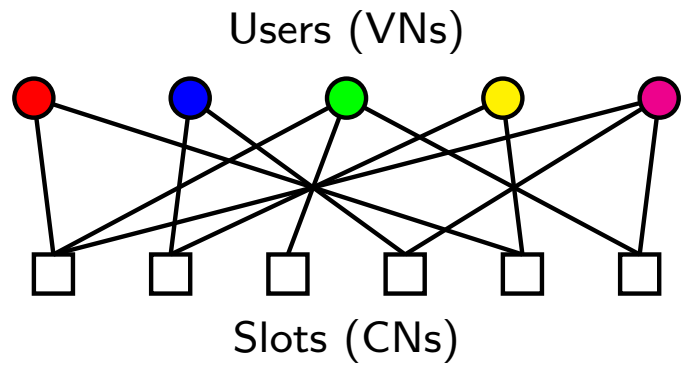

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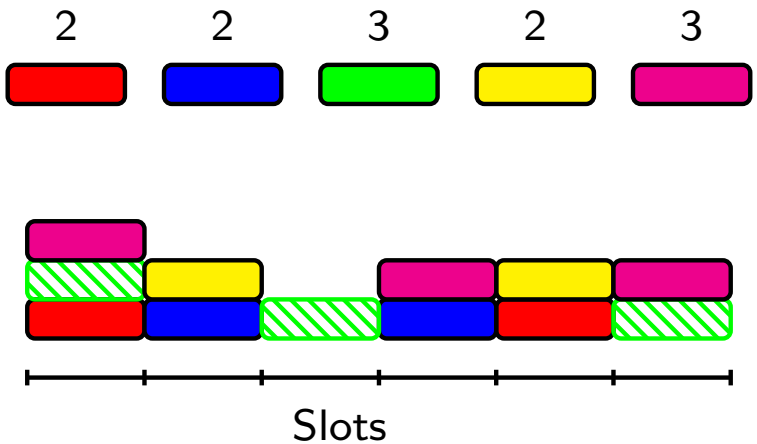

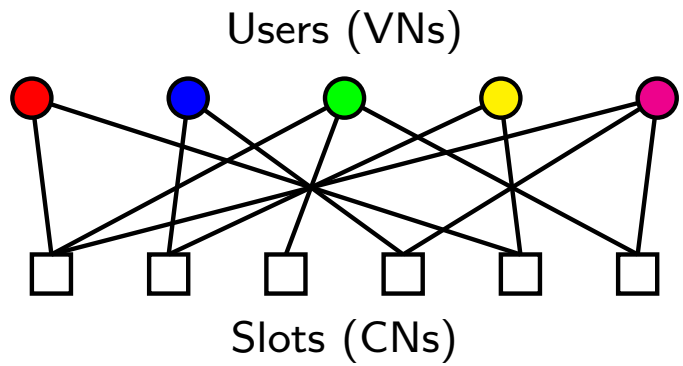

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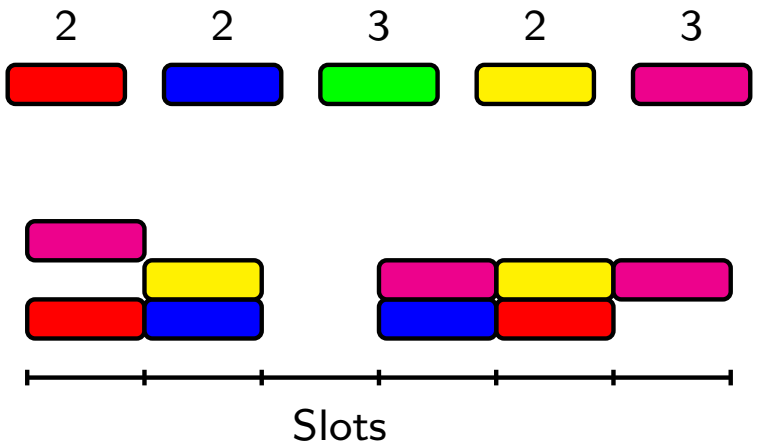

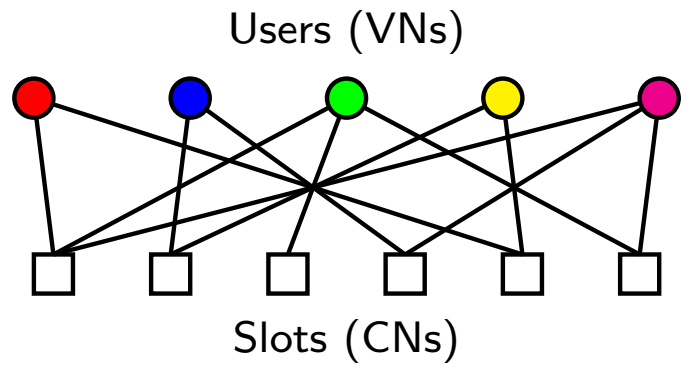

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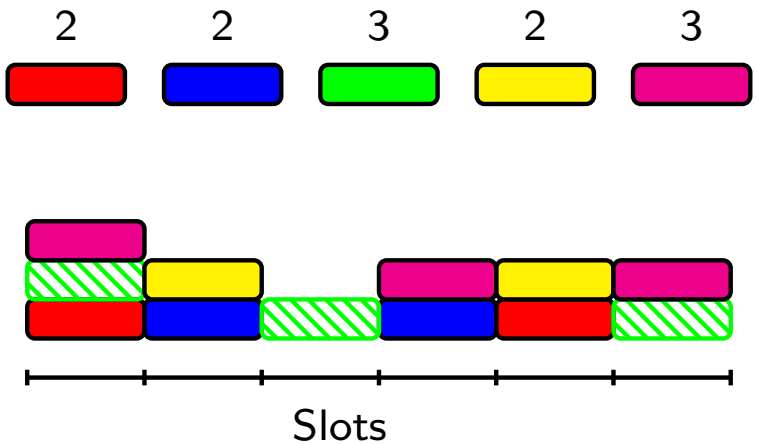

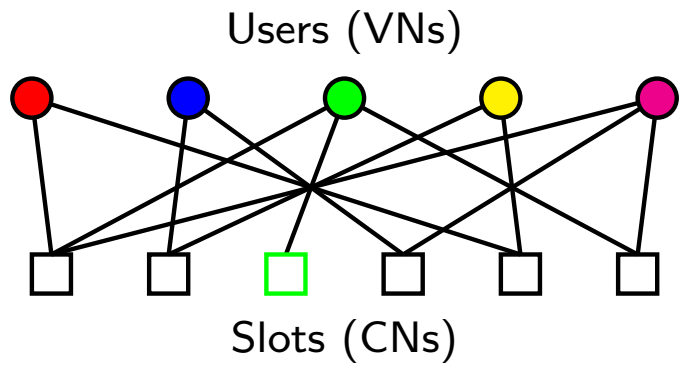

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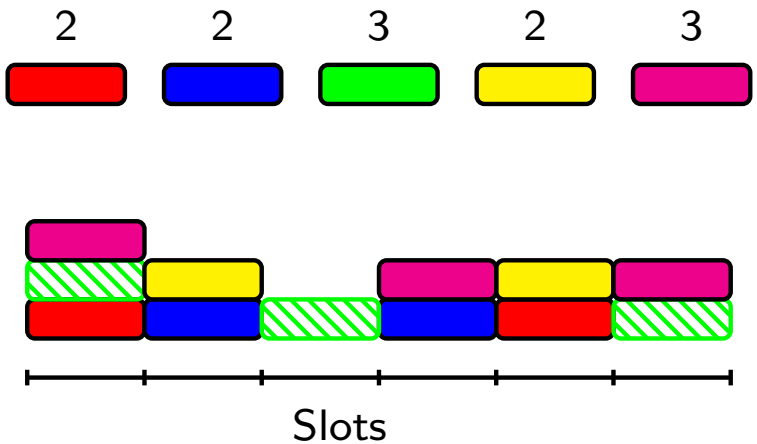

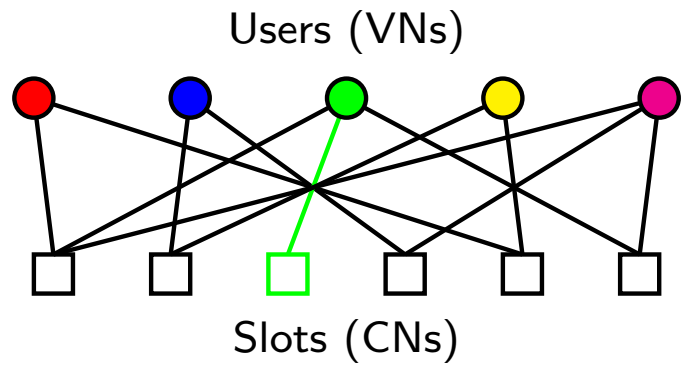

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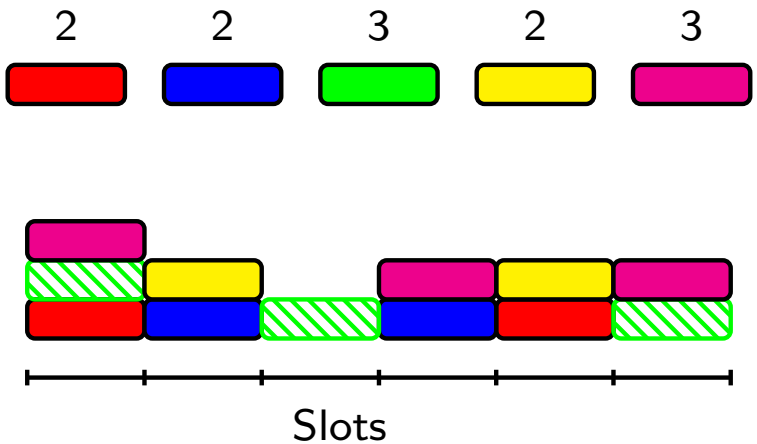

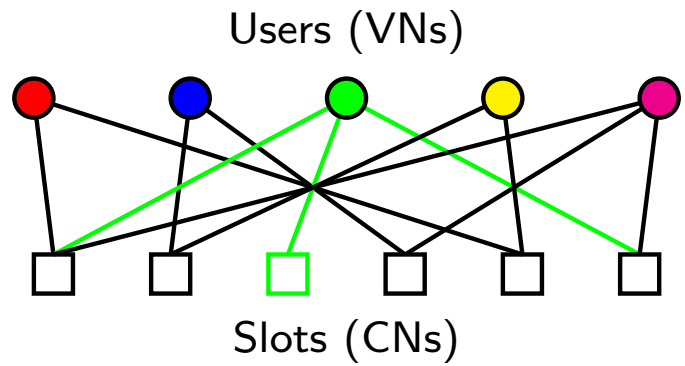

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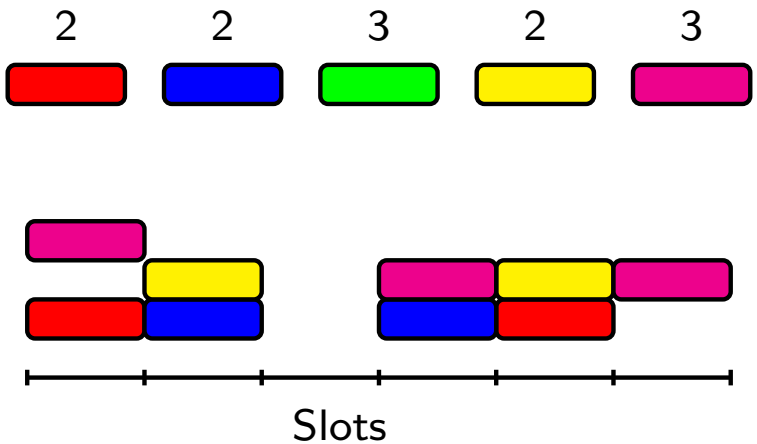

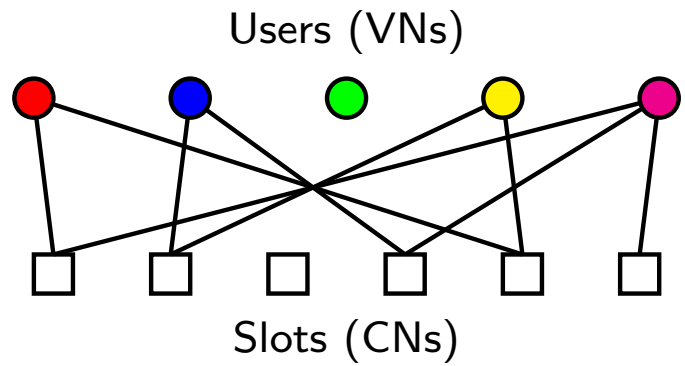

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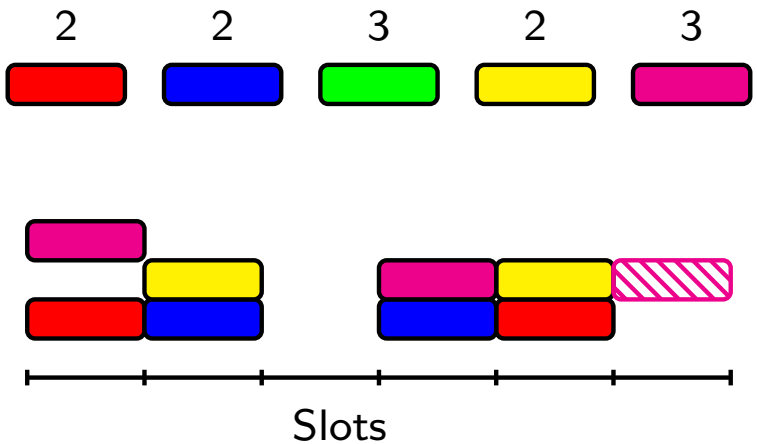

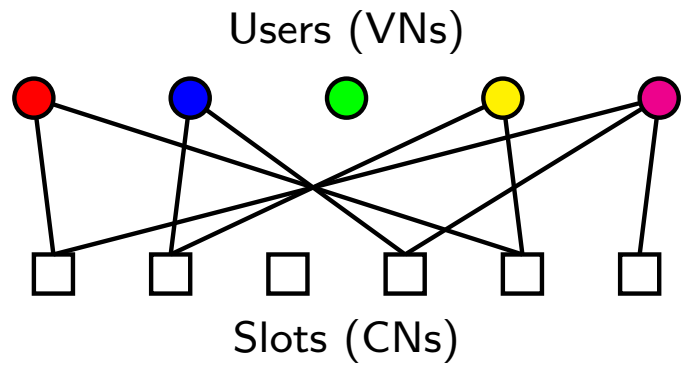

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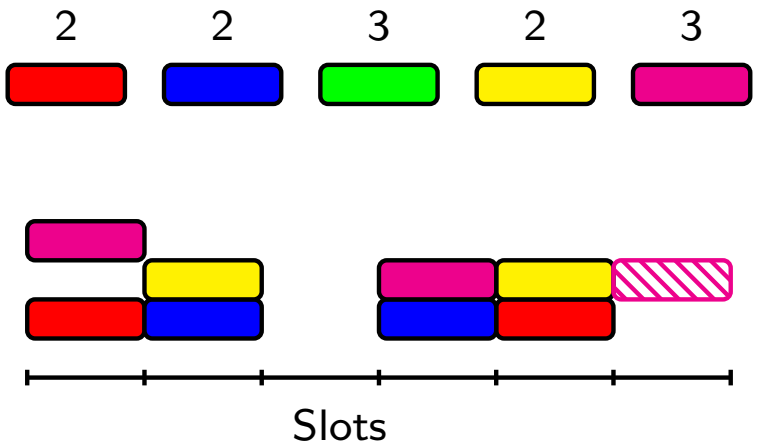

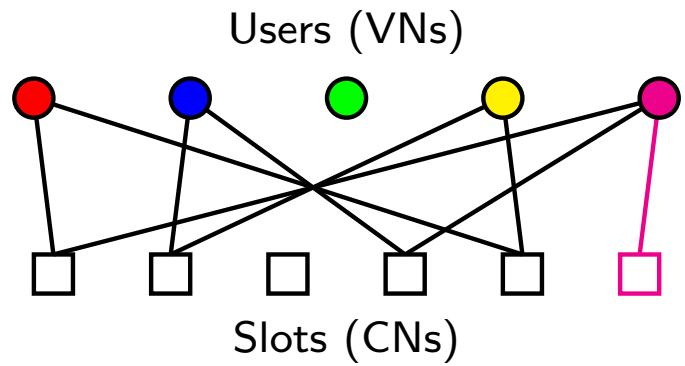

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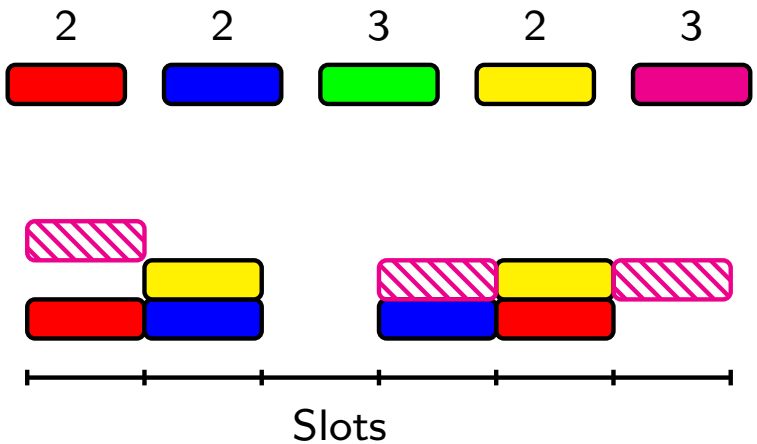

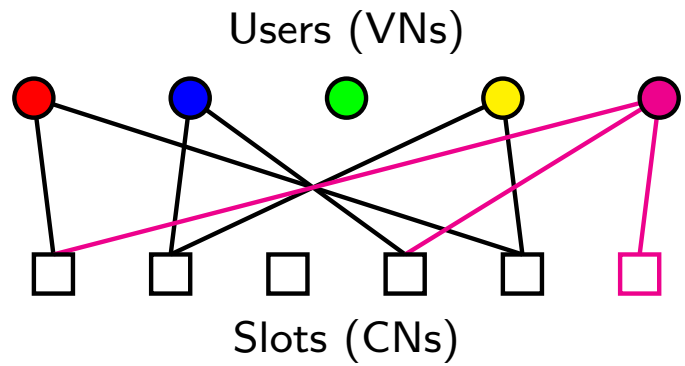

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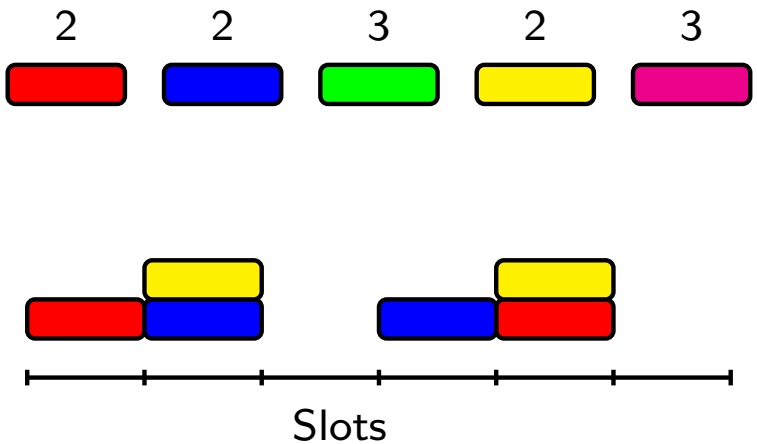

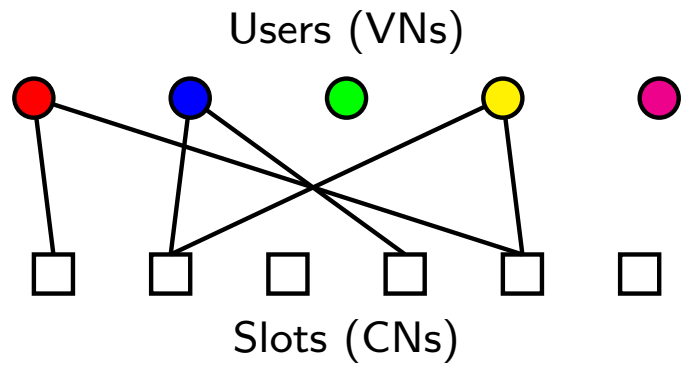

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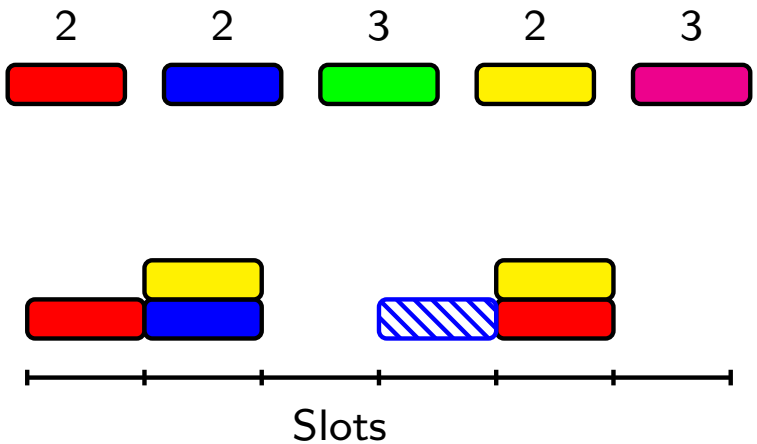

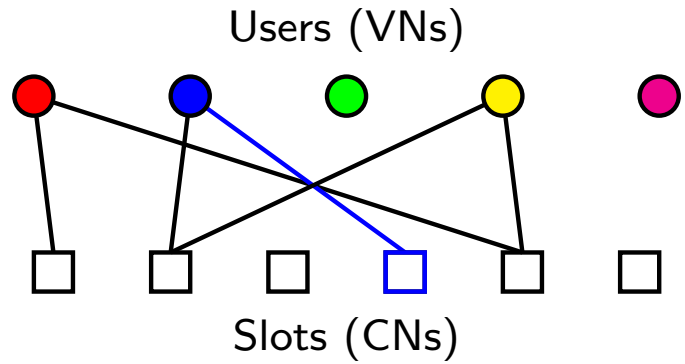

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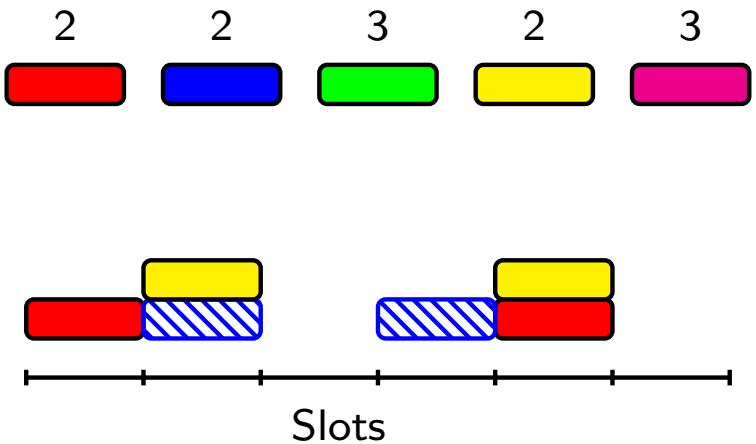

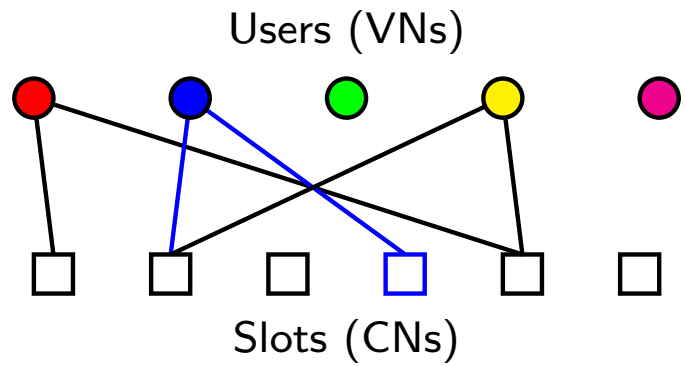

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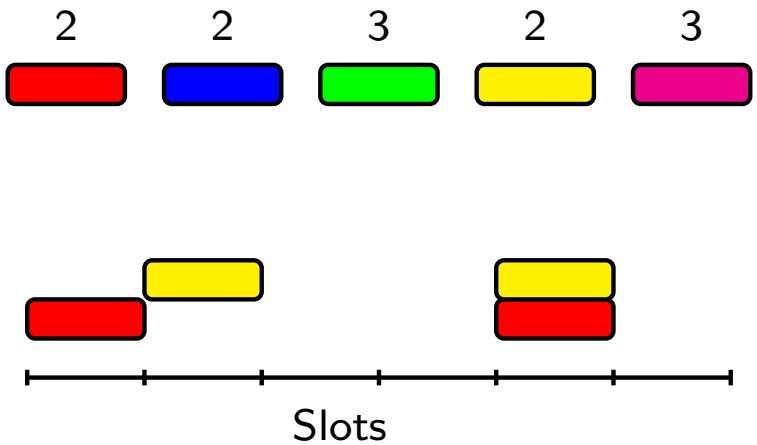

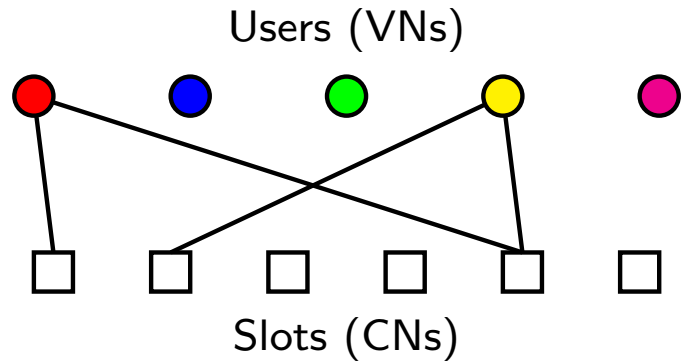

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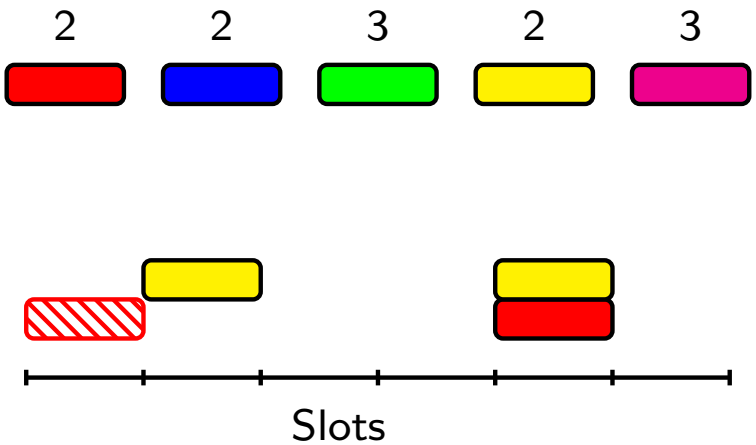

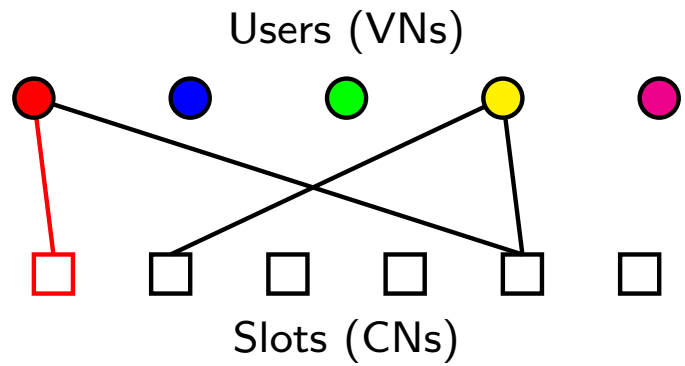

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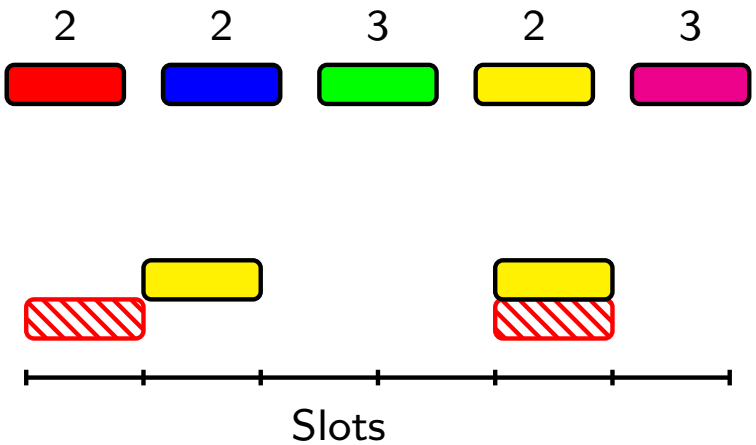

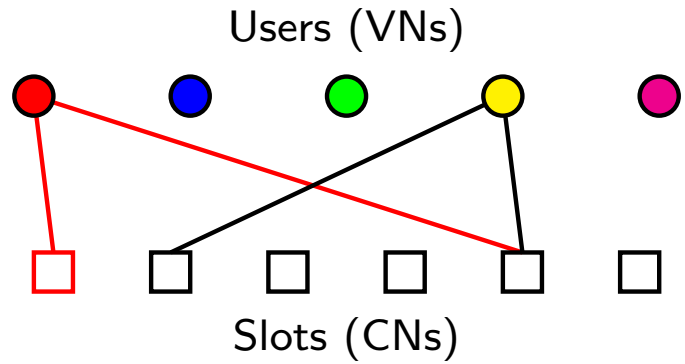

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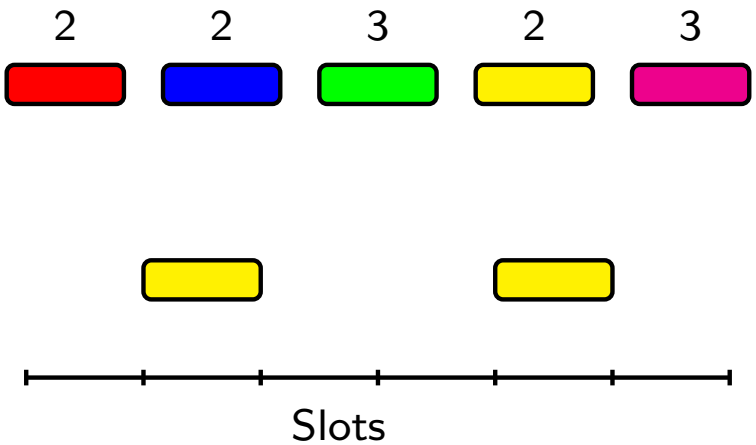

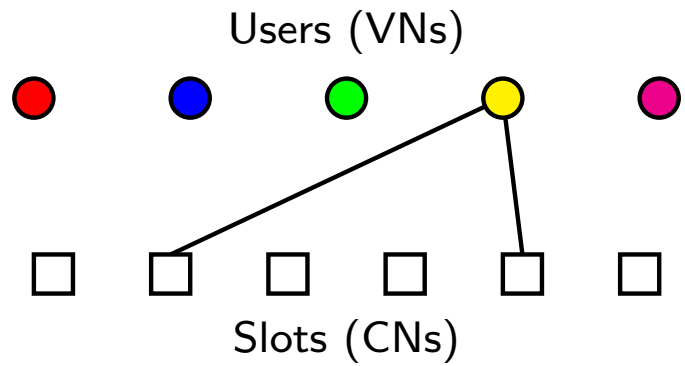

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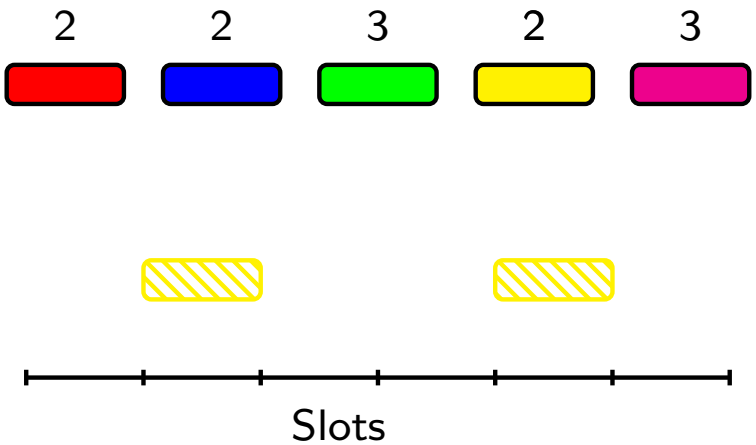

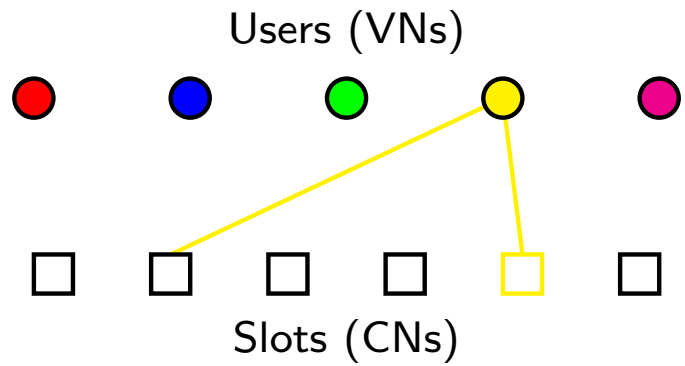

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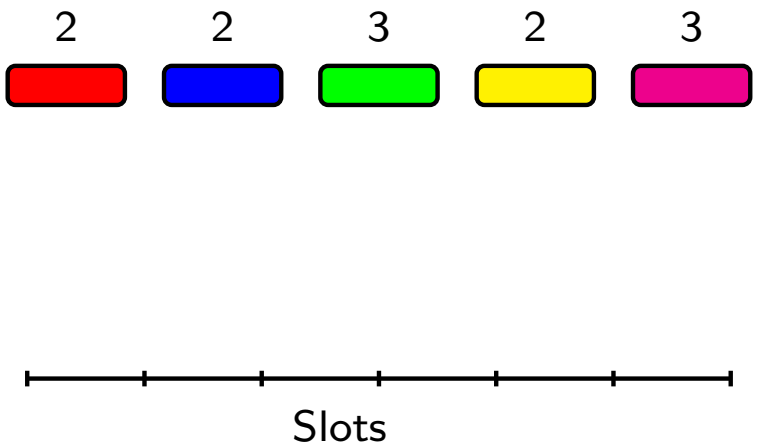

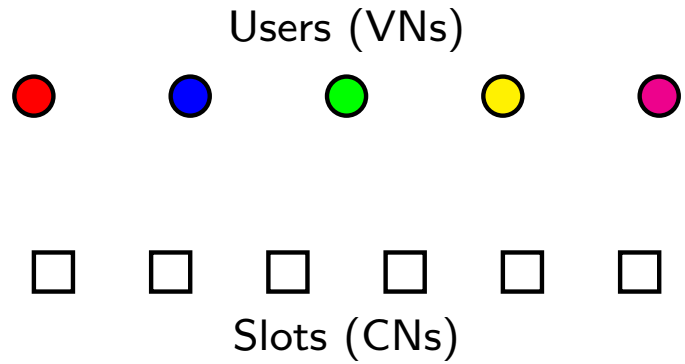

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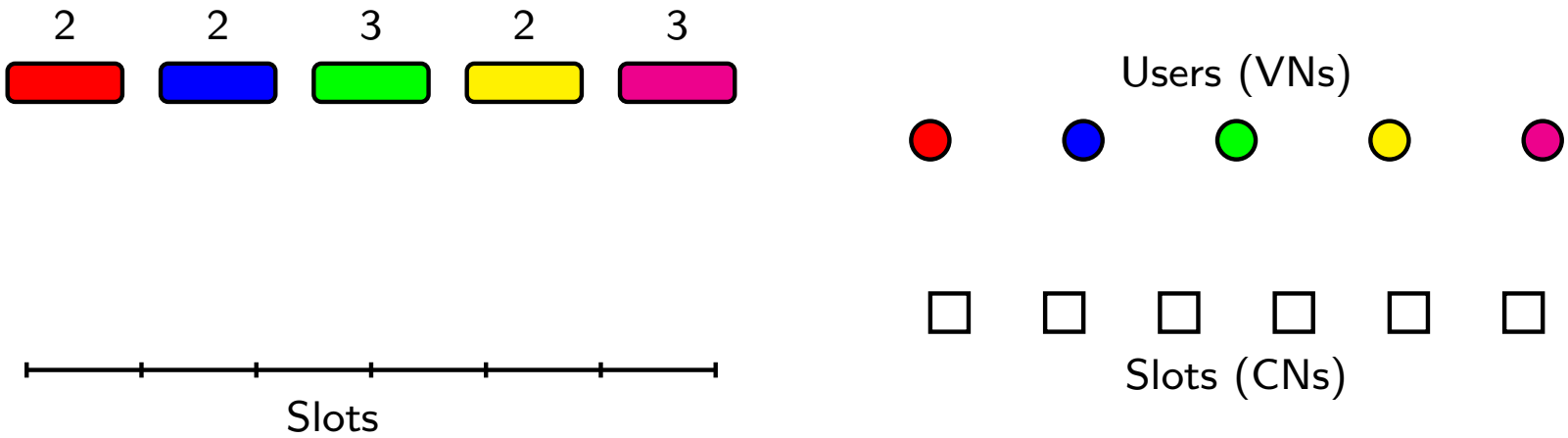

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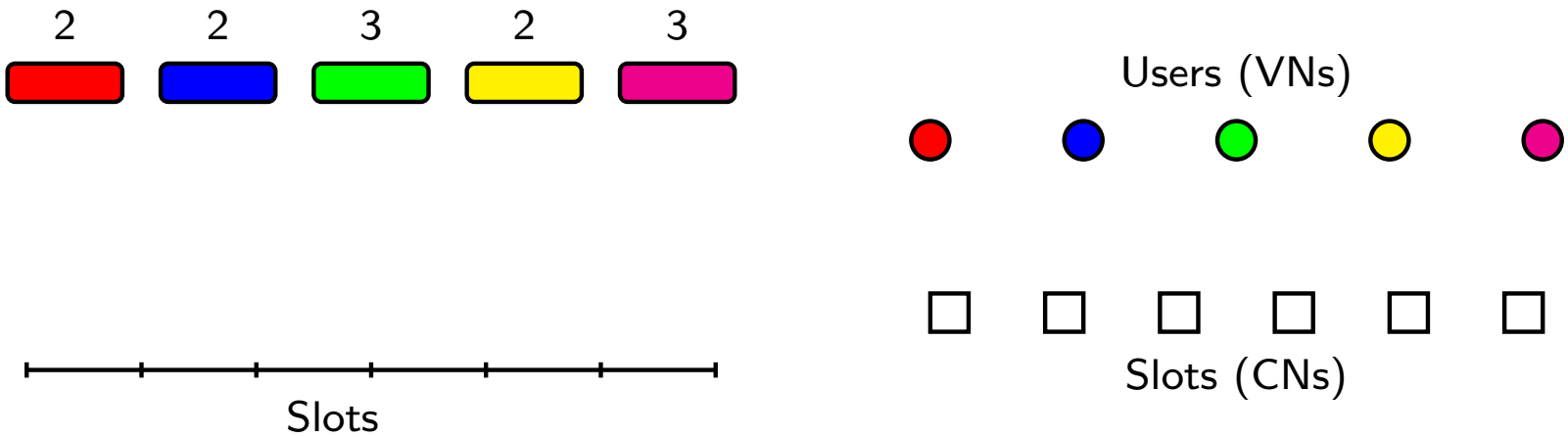

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Problem

Very big key sizes (hundreds of thousands of bits, compared to few thousands for RSA)

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- Once desired information is stored in DNA, it may be rewritten using DNA editing techniques.

DNA storage

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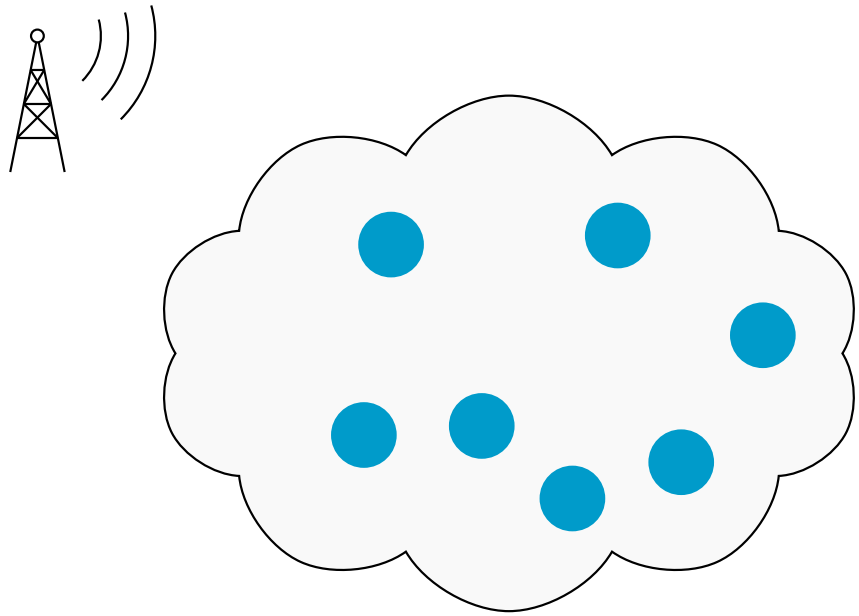
- Coding to correct insertions, deletions and substitutions.
- Constrained coding to avoid some DNA patterns.

Coding for content delivery: Distributed caching



video on demand

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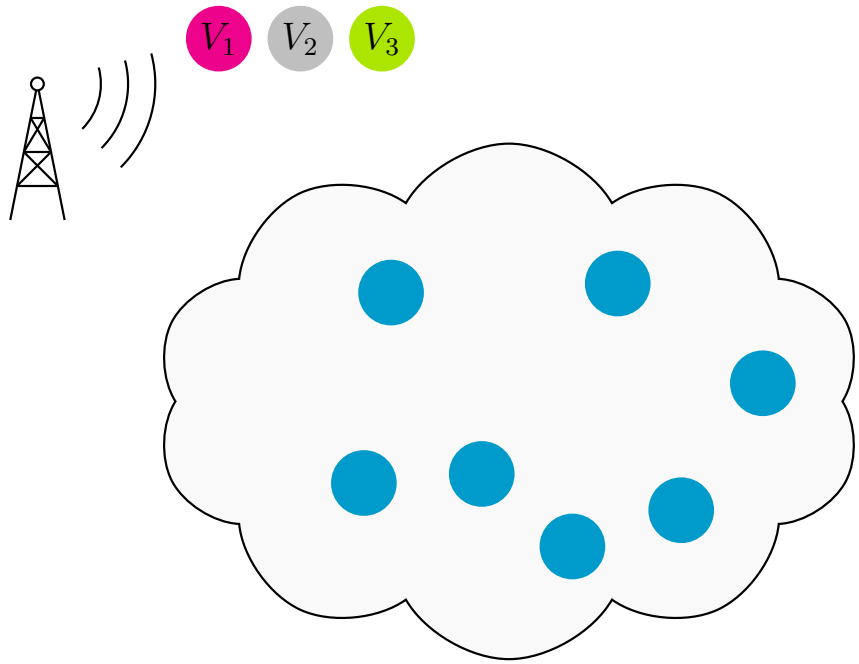

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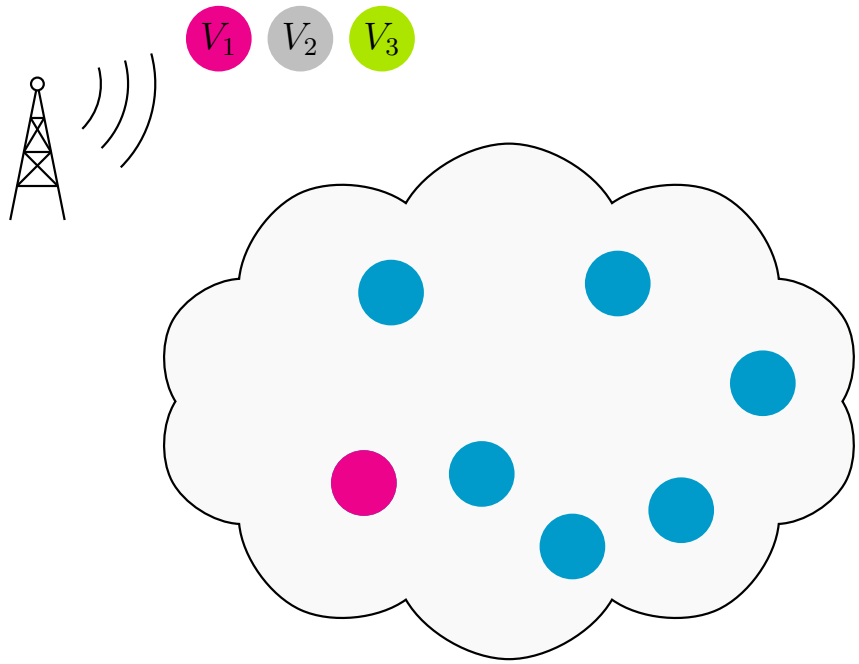

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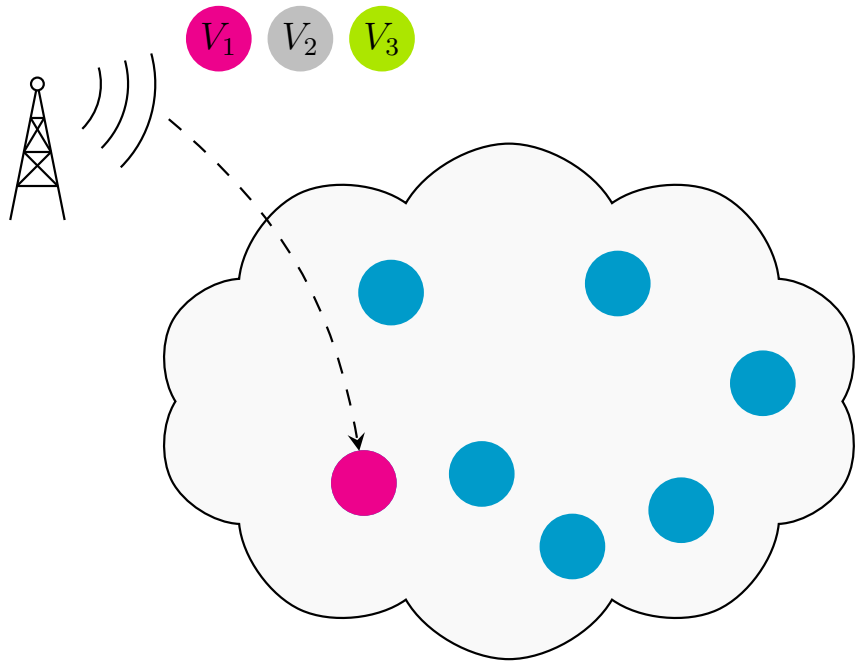

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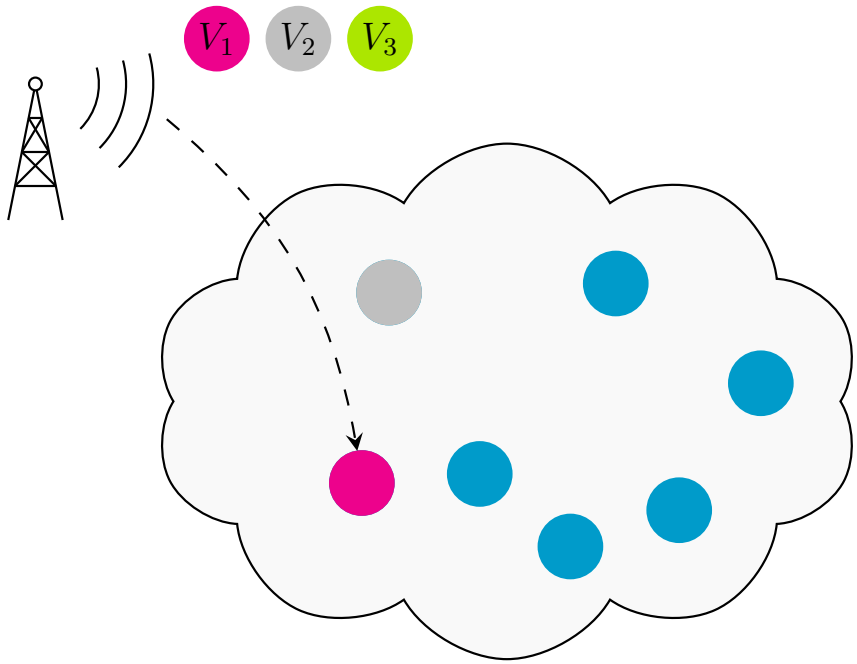

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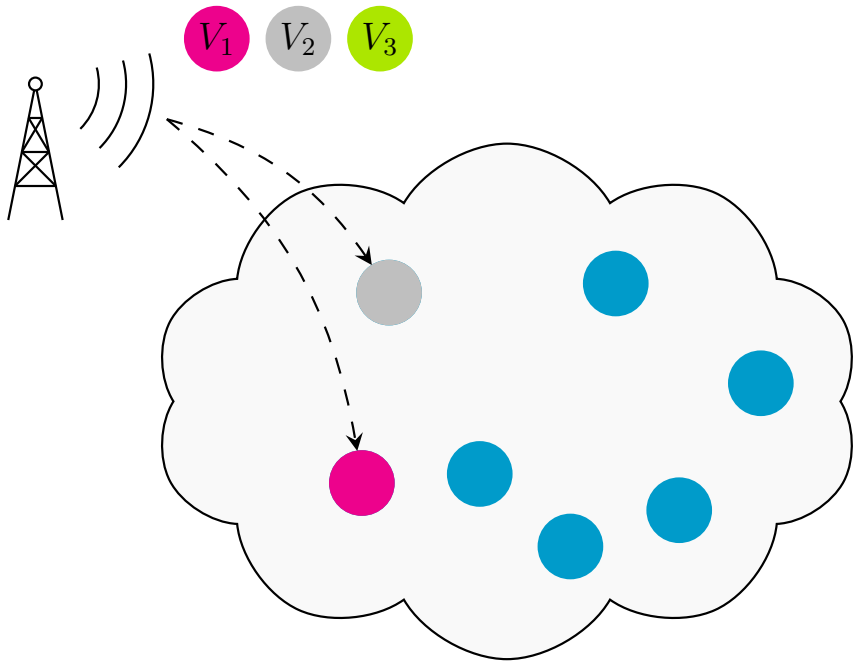

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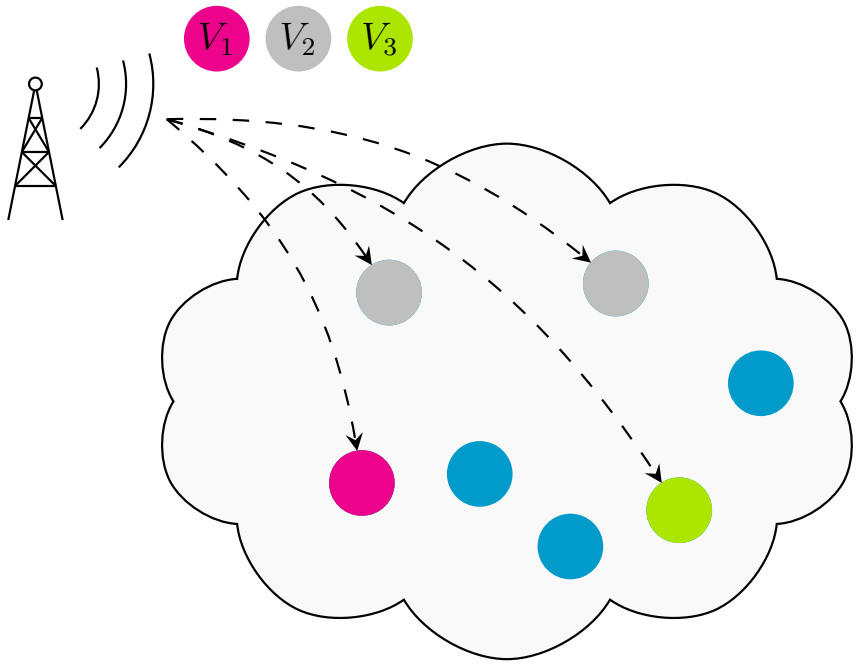

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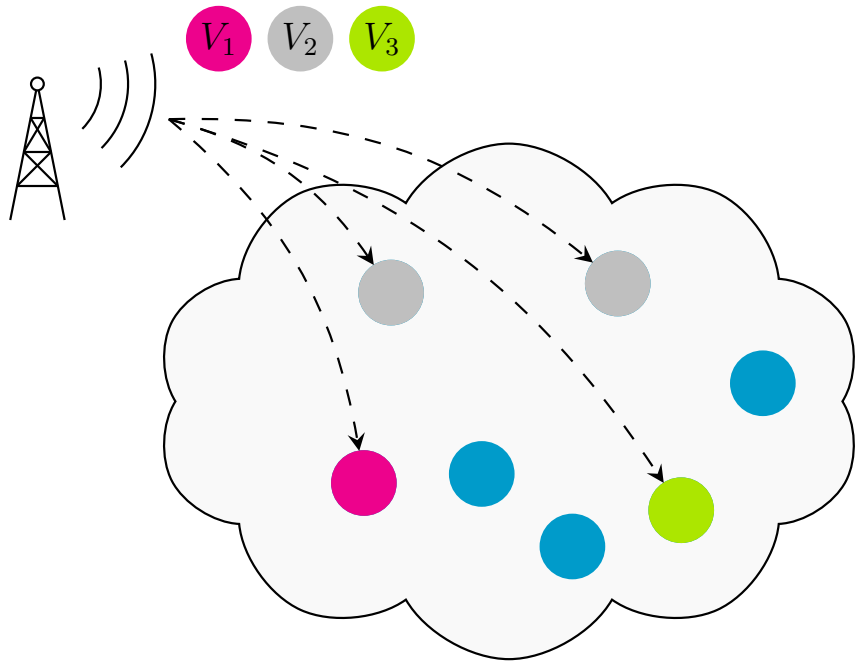

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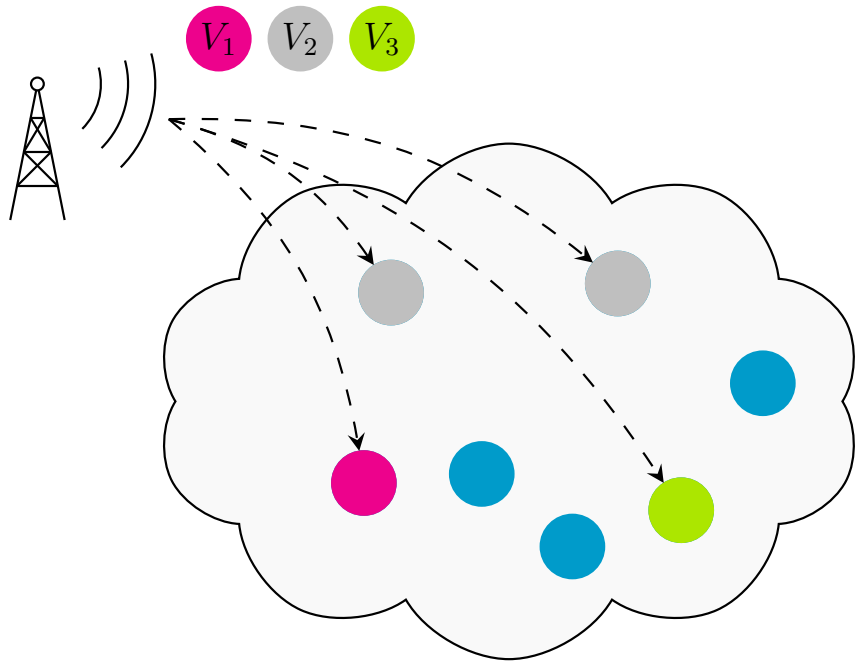

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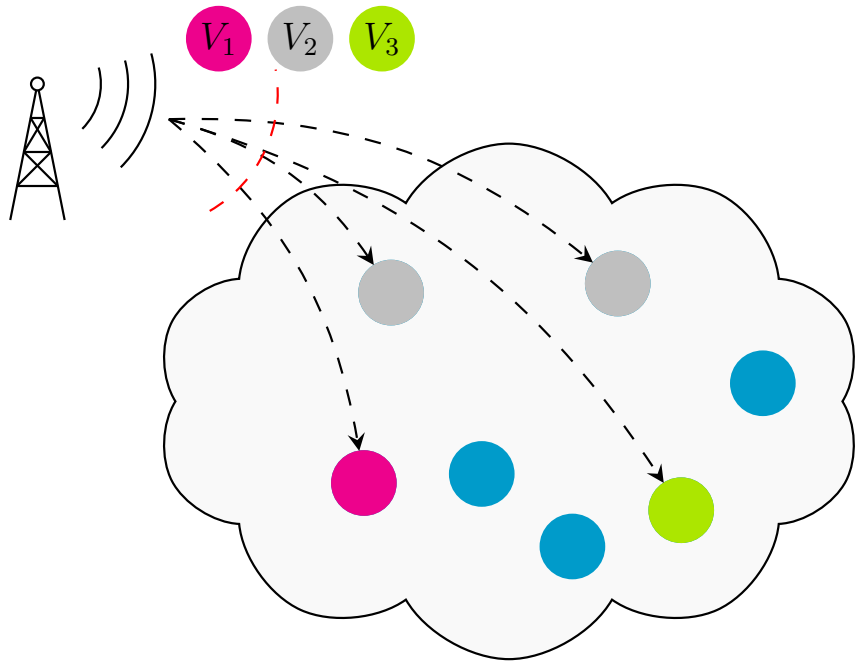

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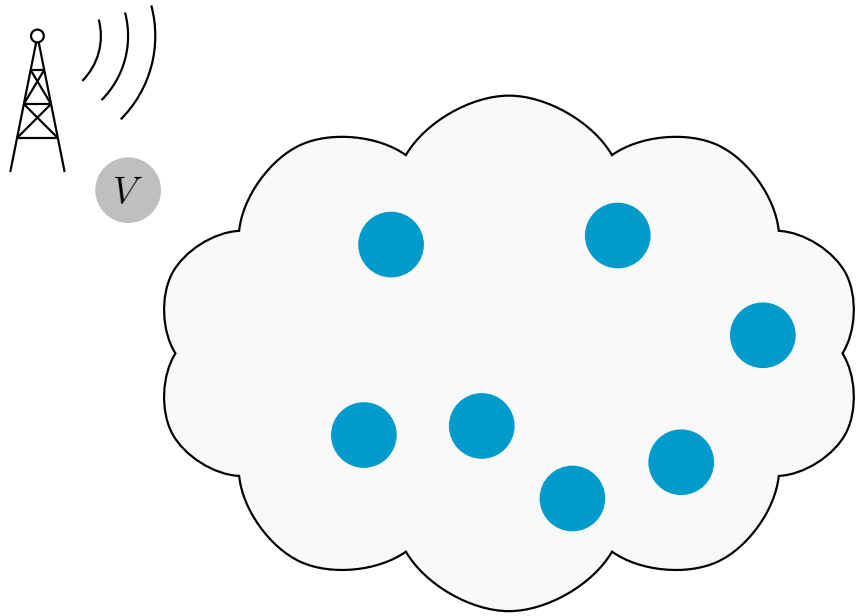

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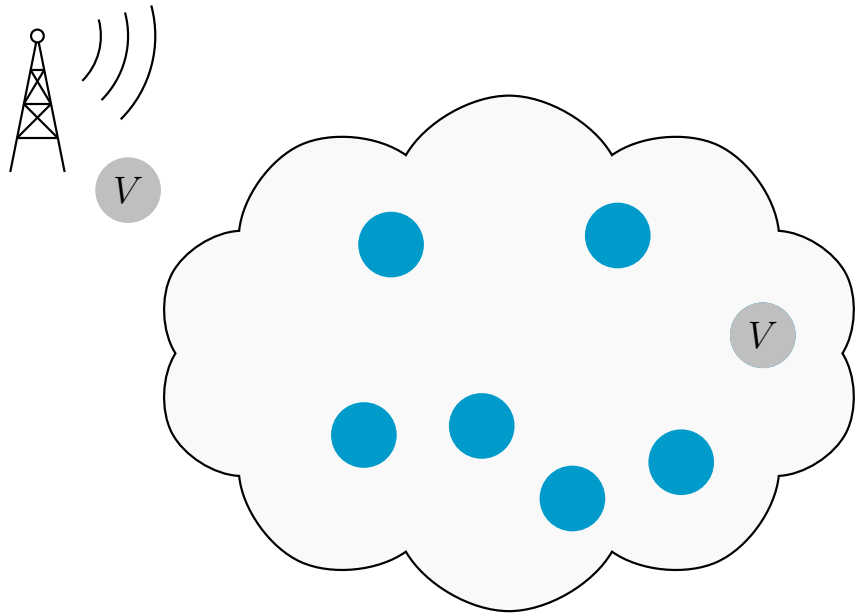

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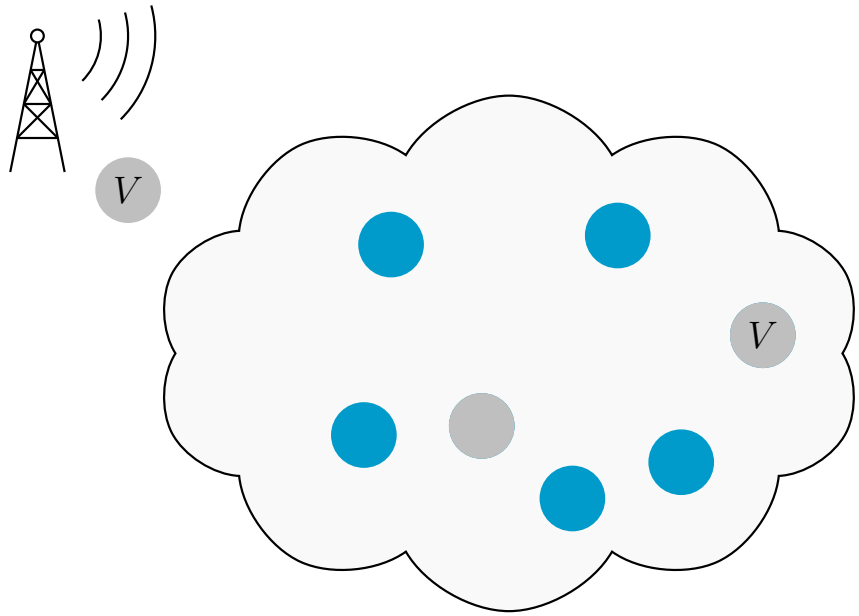

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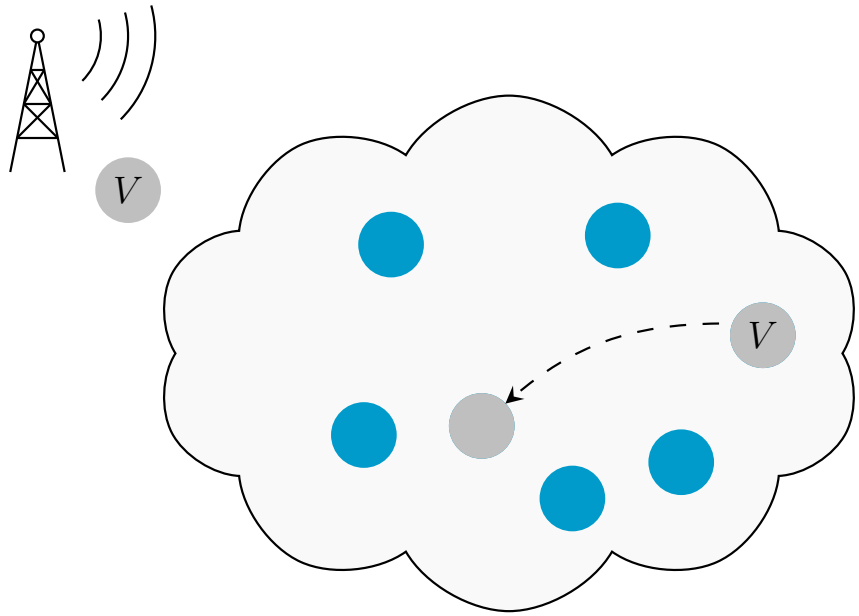

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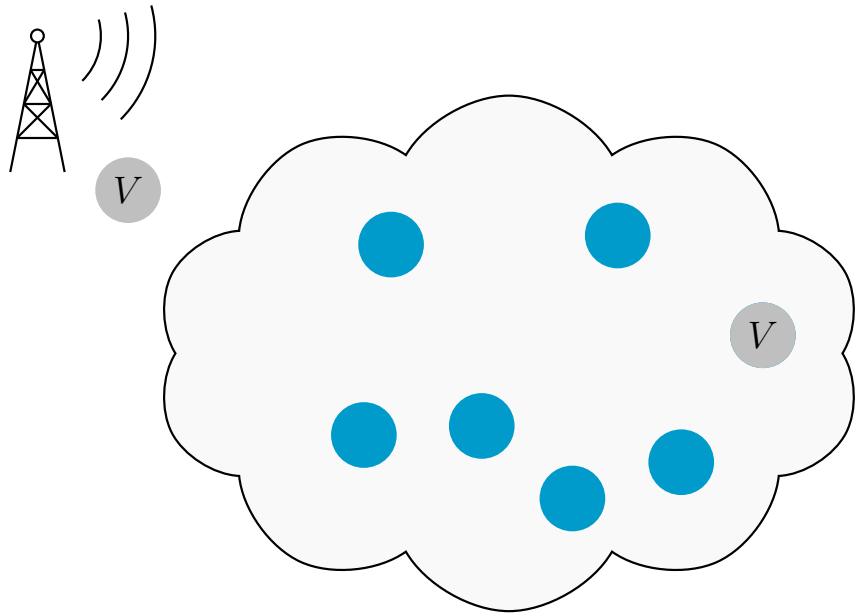

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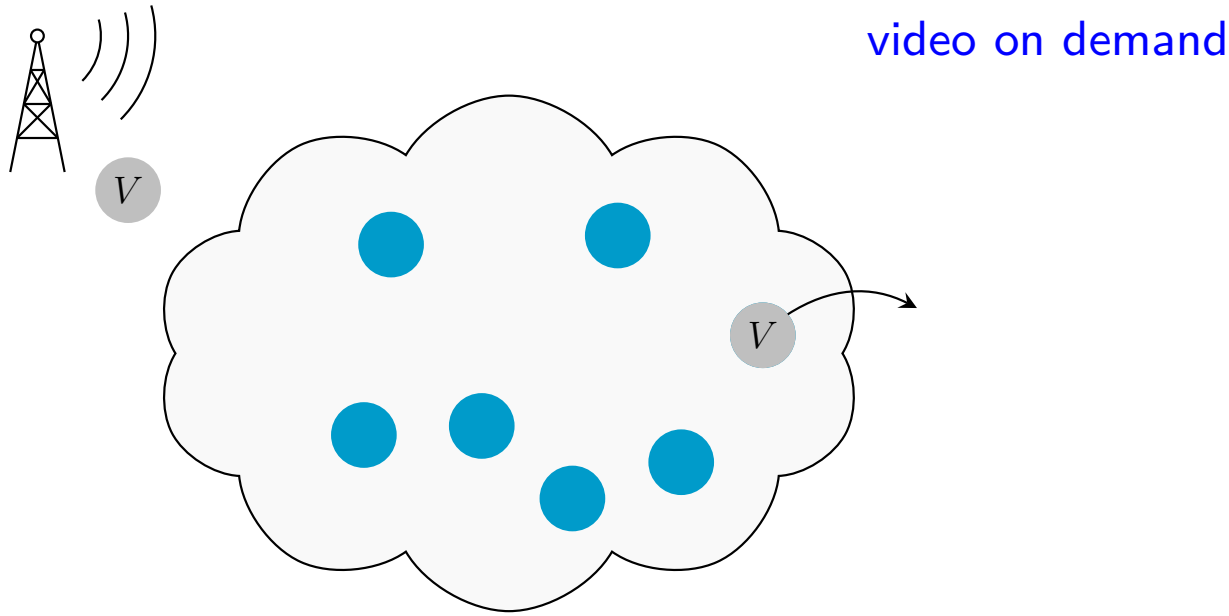

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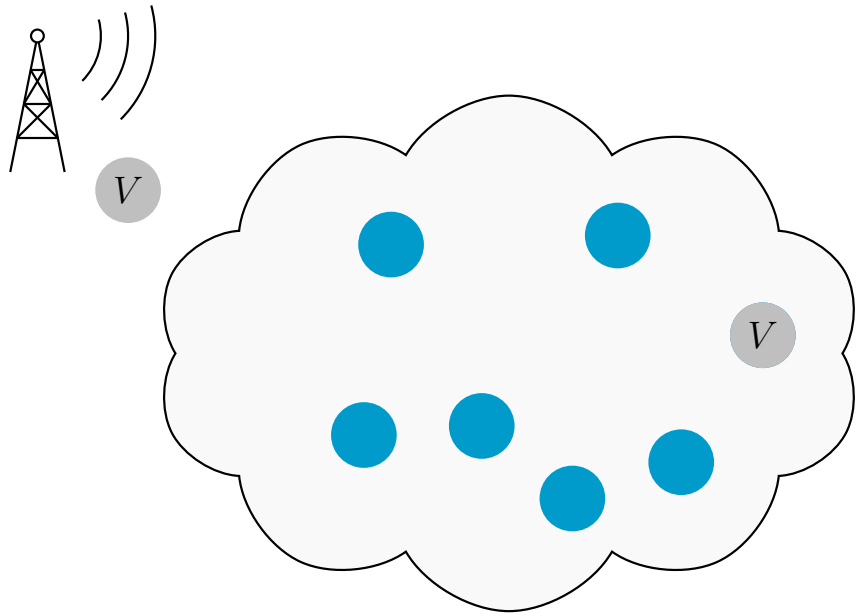

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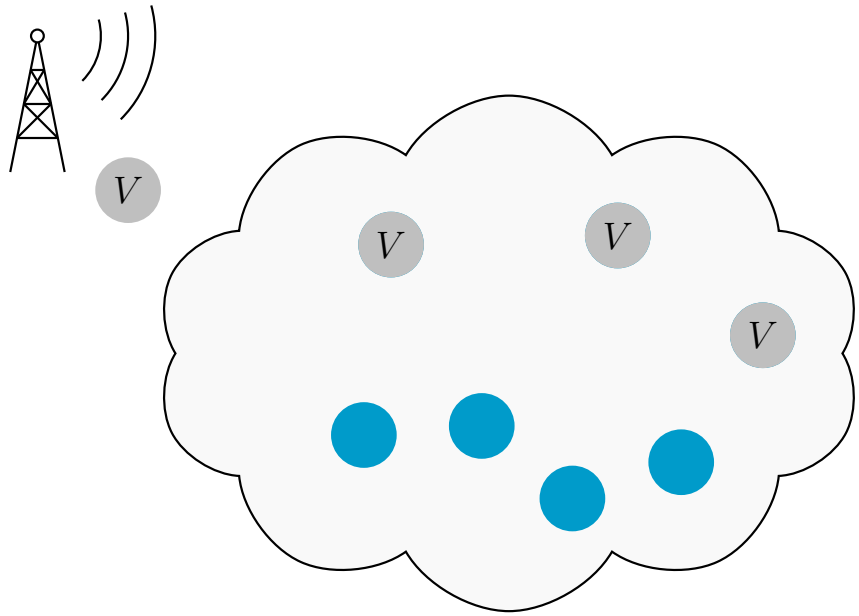

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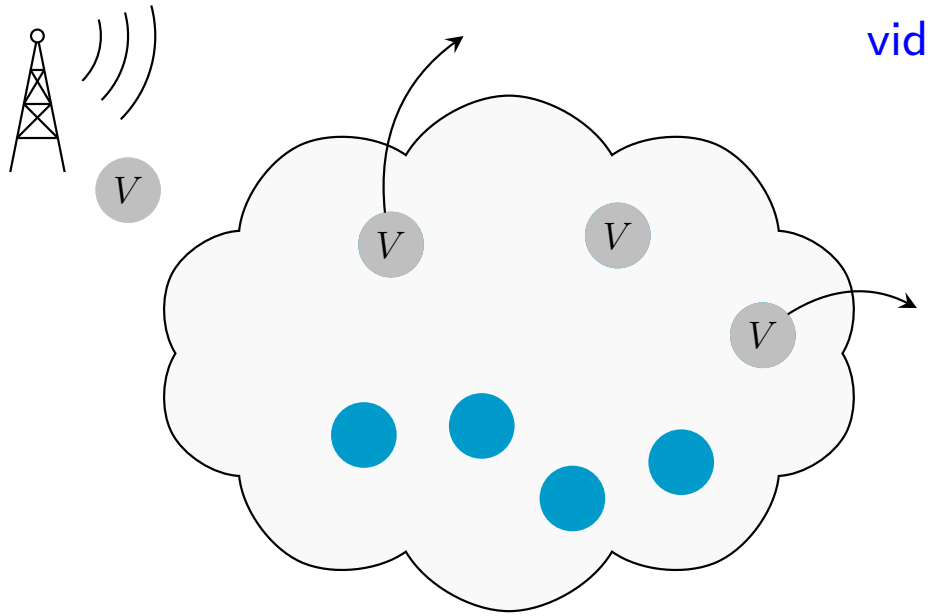

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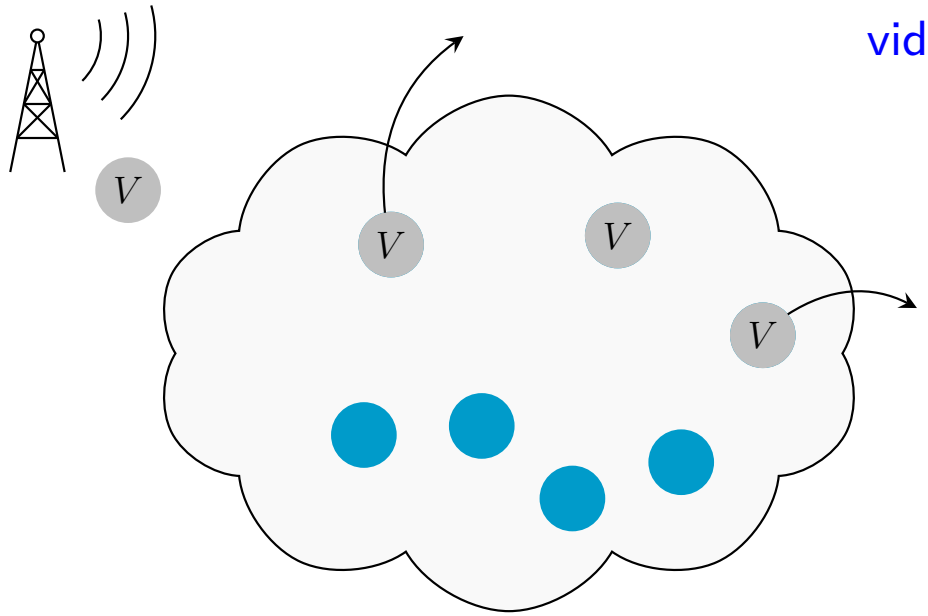

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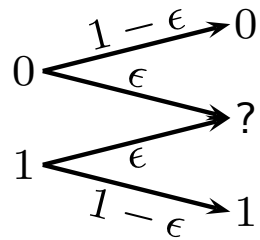

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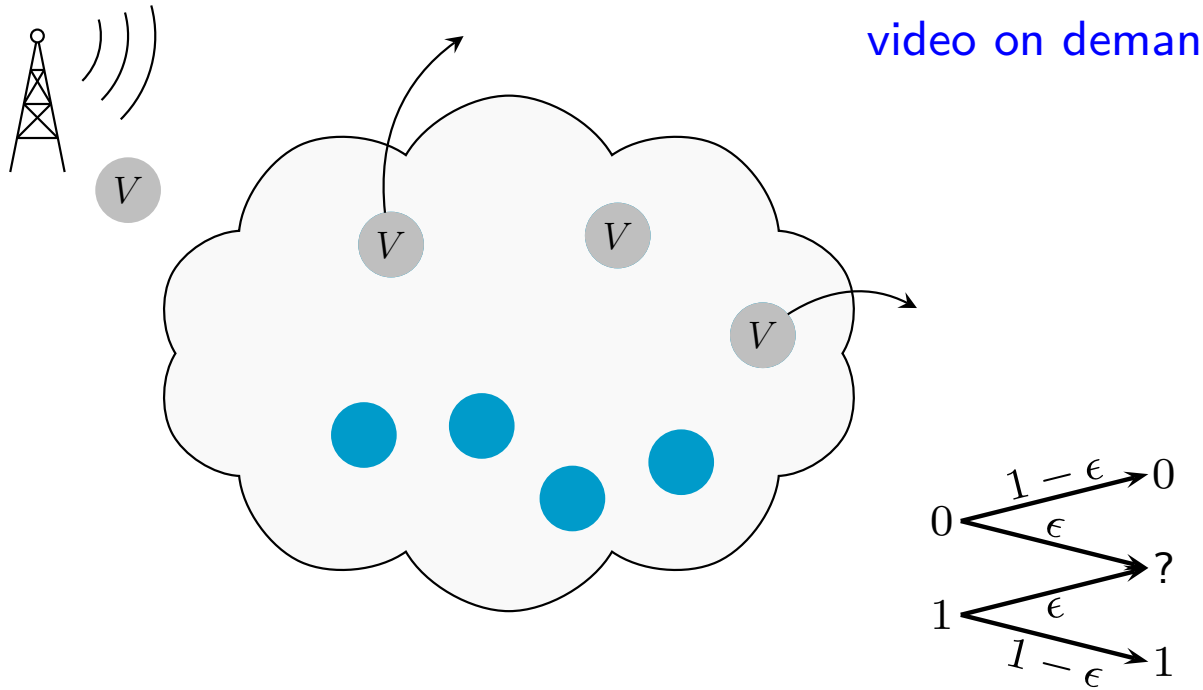

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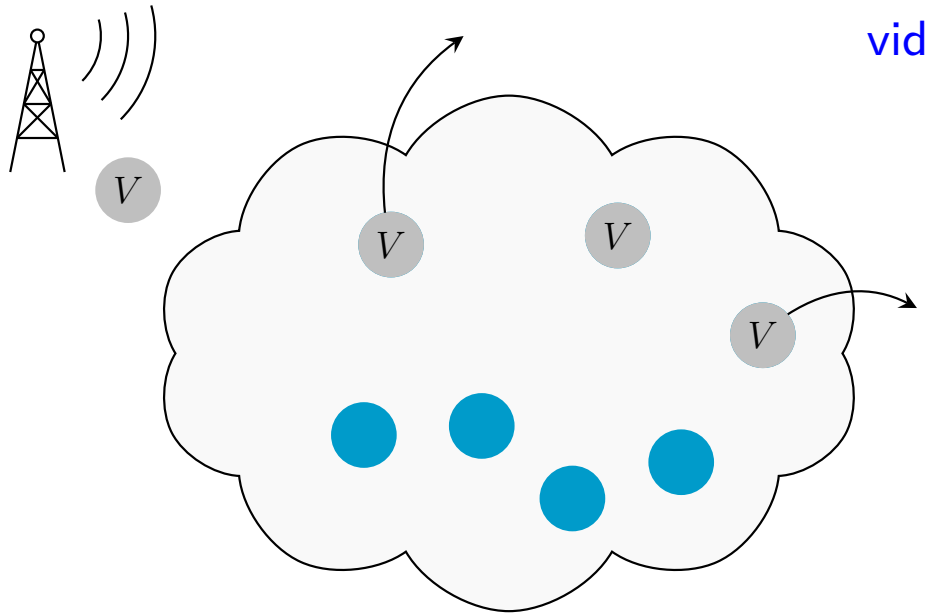

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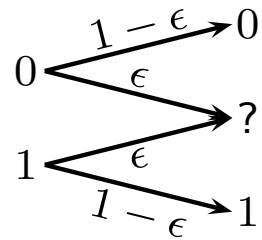

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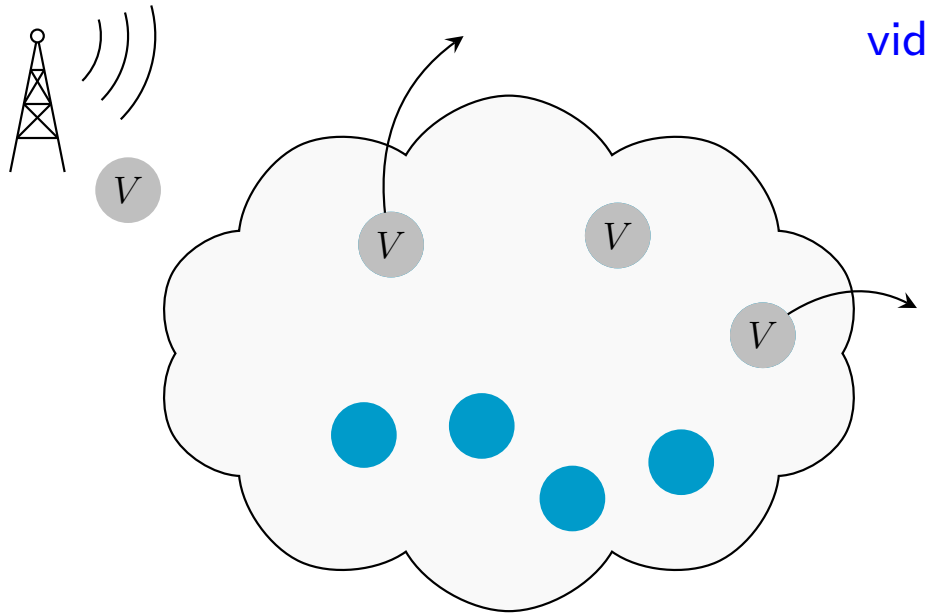
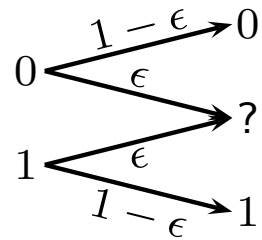
Coding for content delivery: Distributed caching





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Idea

Use erasure correcting codes to store content!

And many more applications...

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- Distributed computing
- Distributed storage
- Security and privacy
- Quantum key distribution
- Quantum error correction
- ...