Convolutional Coding and the Viterbi Algorithm

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1 Hard vs. Soft Receiver

The first task regards the encoder ε_2. Results for comparing hard and soft receivers using QPSK-modulation is shown Figure 1.

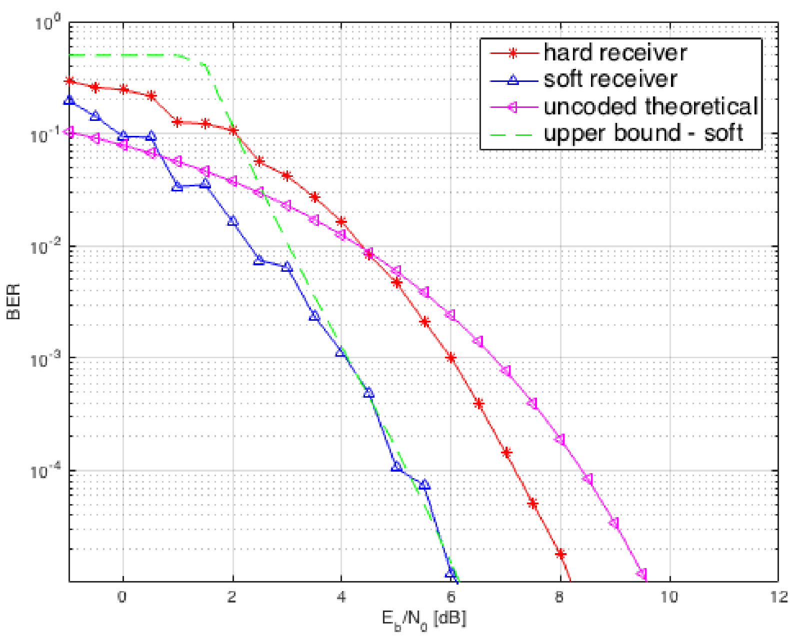

Figure 1: BER vs E_b/N_0 plot for the encoder ε_2 with both hard and soft receiver. The uncoded simulation is also shown as well as the upper bound error probability for the soft decoder

The coding gain at BER $= 10^{-5}$ can be seen approximately by looking at the difference in dB between the uncoded simulation and the different coded simulations. In the same way, by looking at where the graphs for uncoded and coded simulations cross, the minimum value for $\mathrm{E_b/N_0}$ can be seen for both the hard and soft case. The theoretical asymptotic coding gain for the soft receiver can be derived by knowing $\mathrm{d_{min}}$ of the encoder, and using equation 1. The data extracted can bee seen in table 1.

$$\mathrm{Gain_{lim}} = 10\log(\mathrm{R_c d_{min}}) \tag{1}$$

Table 1: Comparison between hard and soft receivers using encoder ε_2 and QPSK-modulation (BER $= 10^{-5}$)

	Hard Rx	Soft Rx
Coding gain [dB]	1.3	3.4
Asymptotic gain [dB]	-	3.9794
Minimum $\mathrm{E_b/N_0}$ [dB]	4.5	0.8

It can be seen in table 1 that the difference in coding gain between hard and soft receivers is 2.1 dB. This can be explained by the fact that accuracy of the decoder is partly lost in the process of quantization.

2 Encoder Comparison

Considering a QPSK constellation with Gray mapping, the BER of encoders ε_1, ε_2 and ε_3 using the soft receiver are shown in Figure 2 along with the upper bound of the theoretical BER for each encoder. In order to calculate the coding gain the BER for the uncoded QPSK simulation is shown in the same figure.

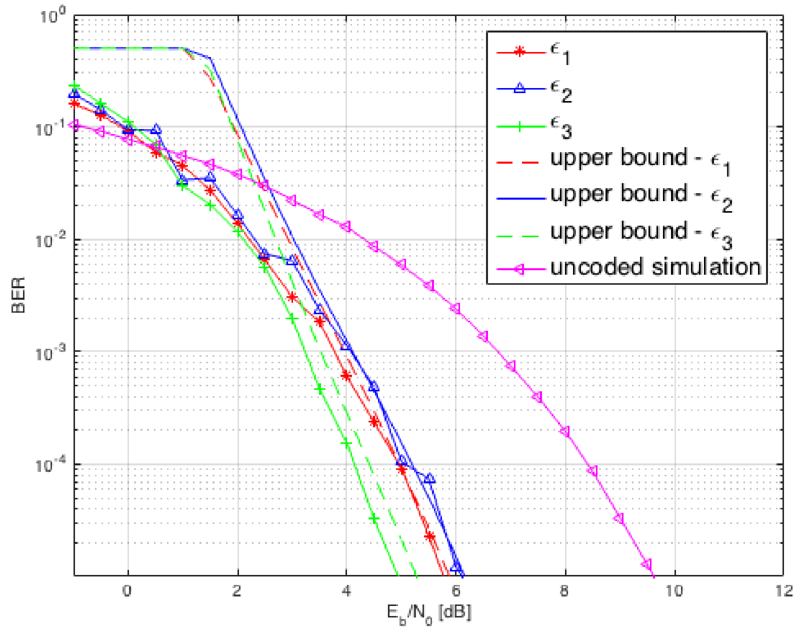

Figure 2: BER vs E_b/N_0 plot which compares the different encoders ε_1-ε_3. The upper bounds are also plotted for the different soft decoders and the uncoded simulation is plotted for reference.

The coding gain at BER $= 10^{-5}$ for the three codes was found in similar way as in section 1, or by looking at the difference in dB between the uncoded simulation and each of the encoders. The results for both coding gain and asymptotic coding gain are shown in table 2 where the asymptotic coding gain was found using equation 1.

Table 2: Comparison between encoders ε_1, ε_2 and ε_3 using QPSK-modulation (BER $= 10^{-5}$)

	ε_1	ε_2	ε_3
Coding gain [dB]	3.8	3.4	4.5
Asymptotic gain [dB]	3.9794	3.9794	4.77

From the table it is clear that the encoder ε_3 performs best at BER$= 10^{-5}$, followed by ε_1. The encoder ε_2 performs worst out of the three. It is also desirable to have low complexity in a system, and if the three encoders are analyzed from this point of view, it can be seen that the complexity of ε_2 and ε_3 are the same, where they require four registers and 5 `bitxor` for every information bit. ε_1 only requires two delay elements and still outperforms ε_2.

3 Coding can Increase Efficiency

In this section three different systems are compared, BPSK modulation using ε_3 encoder(system 1), QPSK also using ε_3(system 2), and AMPM using ε_4(system 3). The results of the BER for each systems are plotted in Figure 3 as well as the uncoded transmissions for all constellations.

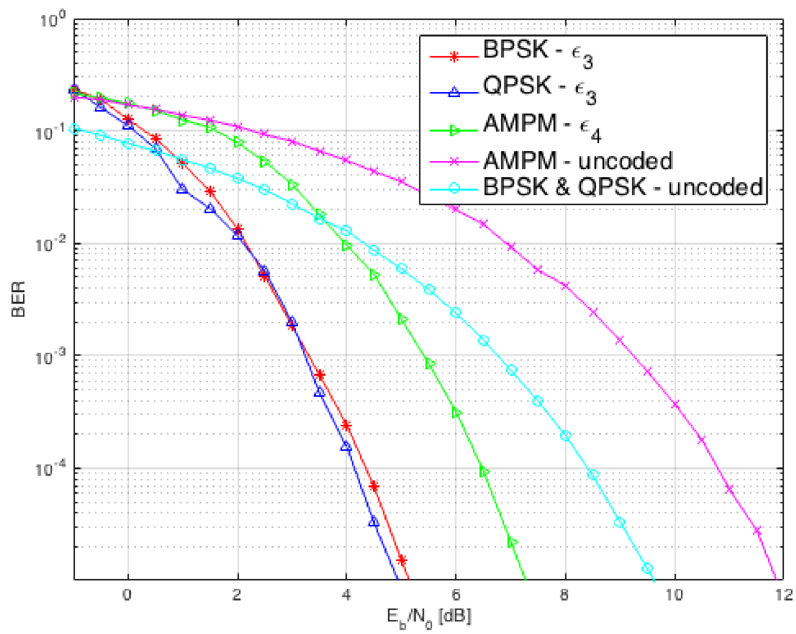

Figure 3: BER vs E_b/N_0 plot comparing 3 different systems, BPSK - ε_3, QPSK - ε_3 and AMPM - ε_4, for both coded and uncoded transmissions.

In terms of the power efficiency the performance of system 1 and 2 are very similar since they require similar E_b/N_0 in order to get the same BER. On the other hand, system 3 requires more E_b/N_0, for example about 2 dB at BER = 10^{-5}. When it comes to the spectral efficiency, system 3 performs best because the AMPM constellation uses more bits/symbol than both BPSK and QPSK. For the uncoded case, system 1 uses 1 bit/symbol, system 2 uses 2 bits/symbol and system 3 uses 3 bits/symbol. After coding, system 3 has 2 bits/symbol whereas system 2 has 1 bit/symbol and system 1 has 0.5 bits/symbol.

In order to look at how far the different systems performs from capacity, measurement data for what E_b/N_0 was required for BER=10^{-5} was extracted and since the the spectral efficiency in terms of bits/channel is known, points could be plotted out for the different systems, this is shown in Figure 4.

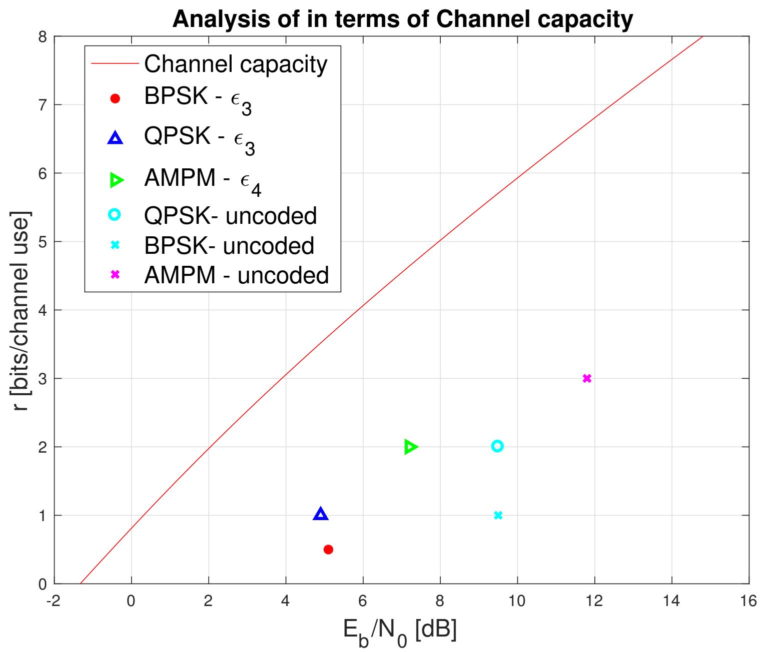

Figure 4: Comparison of systems 1 - 3 to channel capacity at BER=10^{-5}

From the figure, it can be concluded that the coded systems 2 and 3 perform closest to channel capacity at their respective given r. From analysing the systems in terms of different aspects, the conclusion is that no system is strictly better than another. System 3 is superior in terms of spectral efficiency but is not as power efficient.

When comparing system 1 and 2, it can be seen that they perform similarly in terms of bit error rate. System two uses a more advanced modulation scheme and is therefore better (twice as good) in terms of spectral efficiency. Why they still have the same behaviour in figure 3 is because QPSK can be considered as two independent BPSK modulations in the in-phase and quadrature planes.