4C7 – Digital Communications – Tutorial 1 Solutions

1. Consider a systematic block code whose parity check equations are:

$$b_0 = m_0 + m_1 + m_3$$
$$b_1 = m_0 + m_2 + m_3$$
$$b_2 = m_0 + m_1 + m_2$$
$$b_3 = m_1 + m_2 + m_3$$

where m_i are the message bits, i = 0, 1, 2, 3, and b_i are the check bits, i = 0, 1, 2, 3.

(a) What are the parameters n and k? Find the generator matrix for the code.
(b) What is the minimum Hamming distance? How many errors can the code correct?
(c) Is the vector [1 0 1 0 1 0 1 0] a valid codeword?
(d) Is the vector [0 1 0 1 1 1 0 0] a valid codeword?

Solutions:

(a) n = 8, k = 4

$$G = \begin{pmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Note: I assumed that the parity bits come at the beginning of the codeword. It would also have been fine to swap the matrices I and P, making the parity bits come at the end of the codeword.

(b) All possible messages and corresponding codewords are tabulated below:

m	**c**		**m**	**c**
0000	00000000		1000	11101000
0001	11010001		1001	00111001
0010	01110010		1010	10011010
0011	10100011		1011	01001011
0100	10110100		1100	01011100
0101	01100101		1101	10001101
0110	11000110		1110	00101110
0111	00010111		1111	11111111

d_{min} = 4, so we can correct all single errors.

(c) $H = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$, and $H^T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{pmatrix}$.

$(1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0)H^T = (0 \quad 0 \quad 1 \quad 1)$, so not a codeword.

(d) $(0 \quad 1 \quad 0 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0)H^T = (0 \quad 0 \quad 0 \quad 0)$, so a valid codeword.

2. Consider a (7,4) linear block code whose generator matrix is:

$$G = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

(a) Find all the valid codewords.
(b) Find H, the parity check matrix of the code.
(c) Compute the syndrome for the received vector [1 1 0 1 1 0 1]. Is this a valid codeword?
(d) What is the error correcting capability of the code?
(e) What is the error detecting capability of the code?

Solutions:

(a) All messages and corresponding codewords are listed below:

m	c		m	c
0000	0000000		1000	1111000
0001	1100001		1001	0011001
0010	0110010		1010	1001010
0011	1010011		1011	0101011
0100	1010100		1100	0101100
0101	0110101		1101	1001101
0110	1100110		1110	0011110
0111	0000111		1111	1111111

(b) $H = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$

(c) $s = (1 \quad 1 \quad 0 \quad 1 \quad 1 \quad 0 \quad 1)H^T = (0 \quad 1 \quad 0)$, so not a codeword.

(d) d_{min} = 3, so can correct all single errors.

(e) Can detect all single and double errors.

3. The generator matrix for a linear binary code is

$$G = \begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}.$$

(a) Express G in systematic [P | I] form.
(b) Determine the parity check matrix H for the code.
(c) Construct the table of syndromes for the code.
(d) Determine the minimum distance for the code.
(e) Demonstrate that the codeword corresponding to information sequence 101 is orthogonal to H.

Solutions:

(a) $G = \begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$, by swapping columns 1 and 7, columns 2 and 6, and columns 3 and 5.

(b) $H = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$

(c) The syndrome table is below:

Syndrome	Coset leader		Syndrome	Coset leader
0000	0000000		1000	1000000
0001	0001000		1001	1001000,

				0010100, 0000011
0010	0010000		1010	1010000, 0100010, 0001100
0011	1000100, 0100001, 0011000		1011	0000100
0100	0100000		1100	1100000, 0010010, 0000101
0101	0101000, 0010001, 0000110		1101	1101000, 0110100, etc.
0110	1000010, 0110000, 0001001		1110	0000010
0111	0000001		1111	1000001, 0100100, 0001010

(d) $d_{min} = 4$.

(e) $m = (1 \quad 0 \quad 1)$, $c = mG = (1 \quad 1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 1)$, $cH^T = (0 \quad 0 \quad 0 \quad 0)$, so orthogonal.