Applied Signal Processing
Lecture 14

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Outline

- Signal Detection
  - Pulsed radar system
  - Signal Matched Filter
- Quantization errors
- Efficent implementation of decimation and intperpolation
  - Polyphase decomposition of FIR filters

Pulsed radar

Principle of operation: Measure *time difference* between transmitted and received pulse.

- Transmitter
  - Pulse generation (DAC)
  - Modulation
  - Amplification (PA)
- Receiver
  - Low Noise Amplification (LNA)
  - Demodulation
  - Sampling
  - **Pulse detection**

A hypothesis test problem:

$$\begin{aligned} \mathcal{H}_0 : &\quad \mathbf{x} = \mathbf{s} + \mathbf{v} \\ \mathcal{H}_1 : &\quad \mathbf{x} = \mathbf{v} \end{aligned}$$

where

$$\begin{aligned} \text{measured signal: } \mathbf{x} &= \begin{bmatrix} x(0) & x(1) & \cdots & x(N-1) \end{bmatrix}^T \\ \text{signal to detect: } \mathbf{s} &= \begin{bmatrix} s(0) & s(1) & \cdots & s(N-1) \end{bmatrix}^T \\ \text{measurement noise: } \mathbf{v} &= \begin{bmatrix} v(0) & v(1) & \cdots & v(N-1) \end{bmatrix}^T \end{aligned}$$

We assume $v(k)$ to be a white, zero mean stochastic process with variance σ_v and uncorrelated with $s(k)$. Shape of $\mathbf{s}$ is known.

A linear detector is

$$\mathbf{h}^*\mathbf{x} = \sum_{k=0}^{N-1} h^*(k)x(k) \begin{cases} > \alpha & \text{Select } \mathcal{H}_0 : \text{ Signal is detected} \\ \leq \alpha & \text{Select } \mathcal{H}_1 : \text{ No signal is detected} \end{cases}$$

How to select $\mathbf{h}$ and the threshold α.

Signal matched filter

To maximize the performance of the detector we can maximize the size of $|\mathbf{h}^*\mathbf{x}|$ when the signal is present compared to when the signal is not present:

$$\frac{\mathbf{E}\left|\mathbf{h}^*\mathbf{x}\right|^2\Big|_{\mathcal{H}_0}}{\mathbf{E}\left|\mathbf{h}^*\mathbf{x}\right|^2\Big|_{\mathcal{H}_1}} = \frac{|\mathbf{h}^*\mathbf{s}|^2 + \mathbf{E}\left|\mathbf{h}^*\mathbf{v}\right|^2}{\mathbf{E}\left|\mathbf{h}^*\mathbf{v}\right|^2} = \frac{|\mathbf{h}^*\mathbf{s}|^2}{\sigma_v^2|\mathbf{h}^*\mathbf{h}|} + 1$$

Since the Cauchy-Schwartz inequality for vectors state that

$$|\mathbf{h}^*\mathbf{s}|^2 \leq |\mathbf{h}^*\mathbf{h}||\mathbf{s}^*\mathbf{s}|$$

the fraction above is maximized when we select $\mathbf{h} = \mathbf{s}$.

The filter is *matched* to the pulse shape.

Tuning the detector

$$\mathbf{h}^*\mathbf{x} = \begin{cases} > \alpha & \text{Select } \mathcal{H}_0 : \text{ Signal is detected} \\ \leq \alpha & \text{Select } \mathcal{H}_1 : \text{ No signal is detected} \end{cases}$$

The performance of a detector is given as two metrics:

Probability of detection (PD) The probability that the detector detects a signal given that *the signal exists*.

Probability of False alarm (PFA) The probability that the detector detects a signal given that *there is no signal*.

The threshold α will give a balance between these two metrics.

The detector $\mathbf{h} = \mathbf{s}$ maximizes the PD given a fixed PFA.

Detection is FIR filtering

We have

$$\mathbf{h}^H \mathbf{x} = \sum_{k=0}^{N-1} h^*(k) x(k) = \sum_{k=0}^{N-1} h_{\text{det}}(k) x(N-1-k)$$

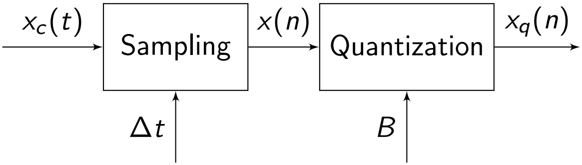
if

$$\mathbf{h}_{\text{det}} = \begin{bmatrix} h^*(N-1), & h^*(N-2), & \cdots, & h^*(0) \end{bmatrix}^T$$

The detector is a filter with the conjugate reverse signal as impulse response.

For streaming data the detector is used in a sliding window and can also be seen as the output of a FIR convolution. Also the name *correlator* is used.

In the sampling step that analog signal is converted to a finite representation encoded with B bits. This is called quantization.



The output of the ADC can be described by

$$x_q(n) = x_c(n\Delta t) + q(n) = x(n) + q(n)$$

where $x_q(n)$ is the quantized signal and $q(n)$ is the difference between the true sample $x(n)$ and the output $x_q(n)$.

Assume the quantizer is uniform with B bits. Then it has 2^B different levels between amplitudes $\pm A$.

Then the quantization step is $S_q = \frac{2A}{2^B - 1} \approx \frac{2A}{2^B}$ and the quantization error is bounded by $|q(n)| \leq \frac{S_q}{2} = \frac{A}{2^B}$.

Although completely determined by the value of the input signal, a good model of the error is to assume $q(n)$ is a zero mean random variable with a uniform distribution in the interval $[-\frac{A}{2^B}, \frac{A}{2^B}]$. The variance of the quantization error is

$$\sigma_q^2 = \int_{-A/2^B}^{A/2^B} x^2 \frac{1}{2A/2^B} \, dx = \frac{1}{3} \frac{A^2}{2^{2B}}$$

Quantization SNR

Relate the level of quantization to a sinusoidal signal with maximum amplitude A.

The power of the sinusoidal signal is $A^2/2$. The signal to noise ratio (SNR) or *dynamic range* is given by

$$R_D = 10\log_{10}\frac{\frac{A^2}{2}}{\frac{A^2}{3\times 2^{2B}}} = 10\log_{10}\frac{3}{2}+10\log_{10}4^B = 1.76+6.02B \quad \text{[dB]}. \tag{1}$$

- Every extra bit will provide an increased dynamic range with 6.02 dB.
- The quantization noise will have a constant power spectral density and will correspond to what is commonly referred to as a noise floor in the spectral domain.