Appendix A - Matrix theory revision

The purpose of this appendix is to present the basic matrix conventions and operations which are used in this text. Dealing first with column matrices or vectors, we define a vector of order N as a column vector composed of N elements:

$$\mathbf{x} = \begin{bmatrix} x_o \\ x_1 \\ \cdot \\ \cdot \\ x_{N-1} \end{bmatrix} \tag{A.1}$$

The transpose of this vector is a row vector of order N:

$$\mathbf{x}^T = [x_o x_1 \cdots x_{N-1}] \tag{A.2}$$

The scalar, inner or dot product of two such vectors is given by:

$$\mathbf{x}^T\,\mathbf{y} = \sum_{i=0}^{N-1} x_i\, y_i = \mathbf{y}^T\,\mathbf{x} \tag{A.3}$$

The vector or outer product of these two vectors is given by:

$$\mathbf{x}\,\mathbf{y}^T = \mathbf{y}\,\mathbf{x}^T = \mathbf{A} \tag{A.4}$$

where the matrix **A** is an $(N \times N)$ square matrix. In general such an $(N \times N)$ square matrix is defined as follows:

$$\mathbf{A} = \begin{bmatrix} a_{00} & a_{01} & \cdot\,\cdot & a_{0,N-1} \\ a_{10} & a_{11} & \cdot\,\cdot & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ a_{0,N-1} & a_{N-1,1} & \cdot\,\cdot & a_{N-1,N-1} \end{bmatrix} \tag{A.5}$$

Transposition of the matrix **A** is achieved by reflection about the leading diagonal.

EXAMPLE A.1

Find the transpose of matrix **A** below:

$$\mathbf{A} = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \end{bmatrix}$$

$$\mathbf{B} = \mathbf{A}^T = \begin{bmatrix} a_{00} & a_{10} & a_{20} \\ a_{01} & a_{11} & a_{21} \\ a_{02} & a_{12} & a_{22} \end{bmatrix}$$

The matrix **A** is termed symmetric when $\mathbf{A}^T = \mathbf{A}$. The operation of transposition has the following properties:

$$(\mathbf{A}^T)^T = \mathbf{A}$$

$$(\mathbf{A}+\mathbf{B})^T = \mathbf{A}^T + \mathbf{B}^T$$

$$(\mathbf{AB})^T = \mathbf{B}^T\mathbf{A}^T \qquad \text{(A.6)}$$

Multiplication of two square ($N \times N$) matrices leads to a product matrix which is also square and ($N \times N$):

$$\mathbf{C} = \mathbf{A}\,\mathbf{B} = \sum_{k=0}^{N-1} \begin{bmatrix} (a_{0k} & b_{k0}) & (a_{0k}\, b_{k1}) & (a_{0k} & b_{k,N-1}) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ (a_{N-1,k}\, b_{k,N-1}) & . & . & (a_{N-1,k}\, b_{k,N-1}) \end{bmatrix} \qquad \text{(A.7)}$$

The properties of multiplication are:

$$\mathbf{A}\,\mathbf{B} \neq \mathbf{B}\,\mathbf{A}$$

$\mathbf{A}\,\mathbf{B} = \mathbf{A}\,\mathbf{C}$ *does not imply* $\mathbf{B} = \mathbf{C}$

$$\mathbf{A}\,(\mathbf{B}\,\mathbf{C}) = (\mathbf{A}\,\mathbf{B})\,\mathbf{C}$$

$$\mathbf{A}\,(\mathbf{B}+\mathbf{C}) = \mathbf{A}\,\mathbf{B} + \mathbf{A}\,\mathbf{C} \qquad \text{(A.8)}$$

Every square matrix **A** has associated with it a determinant which is a scalar defined as $|\mathbf{A}|$. It is defined as the sum of the products of the elements in any row or column of **A** with the respective cofactors c_{ij} of that element a_{ij}. The cofactor, c_{ij}, is defined as $(-1)^{i+j} m_{ij}$ where m_{ij} is the determinant of the $(N-1) \times (N-1)$ matrix formed by removing the ith row and jth column of **A**. The properties of the determinant of **A** are

$|\mathbf{A}| = 0$ if *one row is zero or two rows are identical*

$$|\mathbf{A}\,\mathbf{B}| = |\mathbf{A}|\,|\mathbf{B}|$$

$$|\mathbf{A}^{-1}| = |\mathbf{A}|^{-1} \qquad \text{(A.9)}$$

EXAMPLE A.2

Find the determinant of matrix **B** below.

$$\mathbf{B} = \begin{bmatrix} 3 & 1 & 0 \\ -1 & 2 & 4 \\ 5 & 6 & -2 \end{bmatrix}$$

$$|\mathbf{B}| = 3\,(-28) - 1\,(-18) + 0.0$$

$$= -66$$

The inverse of a matrix **A** is defined by:

$$\mathbf{A}^{-1}\,\mathbf{A} = \mathbf{A}\,\mathbf{A}^{-1} = \mathbf{I} \tag{A.10}$$

where:

$$\mathbf{I} = \begin{bmatrix} 1\,0\,0\,0 \\ 0\,1\,0\,0 \\ 0\,0\,1\,0 \\ 0\,0\,0\,1 \end{bmatrix}$$

is the identity matrix.

$$\mathbf{A}^{-1} = \frac{1}{|\mathbf{A}|}\,adj\,(\mathbf{A}) \tag{A.11}$$

where $adj(\mathbf{A})$ is the adjoint matrix of **A** and is the transpose of the matrix of the cofactors of **A**. The properties of the inverse are:

$$(\mathbf{A}^{-1})^{-1} = \mathbf{A}$$

$$(\mathbf{A}\,\mathbf{B})^{-1} = \mathbf{B}^{-1}\,\mathbf{A}^{-1}$$

$$(\mathbf{A}^{-1})^{T} = (\mathbf{A}^{T})^{-1} \tag{A.12}$$

For a square matrix **A**, a non-zero vector **a** is an eigenvector if a scalar λ exists such that:

$$\mathbf{A}\,\mathbf{a} = \lambda\,\mathbf{a}$$

$$(\mathbf{A} - \lambda\,\mathbf{I})\,\mathbf{a} = 0 \tag{A.13}$$

where λ is an eigenvalue of **A**. Equation (A.13) yields a non-trivial eigenvector only if:

$$|\mathbf{A} - \lambda\,\mathbf{I}| = 0 \tag{A.14}$$

This is the so-called characteristic equation for **A** and its roots are the eigenvalues of **A**. The properties of eigenvalues are:

$$|\mathbf{A}| = \prod_{i=0}^{N-1} \lambda_i$$

$$trace\,(\mathbf{A}) = \sum_{i=0}^{N-1} \lambda_i \tag{A.15}$$

More details on matrix theory can be found in texts such as [Broyden].

Appendix B - Signal transforms

Selected Laplace transforms

$x(t)$ $(t \geq 0)$	$X(s)$
$\delta(t)$	1
$\delta(t-\alpha)$	$\exp(-\alpha s)$
1 (unit step)	$\frac{1}{s}$
t (unit ramp)	$\frac{1}{s^2}$
$\exp(-\alpha t)$	$\frac{1}{s+\alpha}$
$t\exp(-\alpha t)$	$\frac{1}{(s+\alpha)^2}$
$\sin(\alpha t)$	$\frac{\alpha}{s^2+\alpha^2}$
$\cos(\alpha t)$	$\frac{s}{s^2+\alpha^2}$
$e^{-\alpha t}\sin(\omega t)$	$\frac{\omega}{(s+\alpha)^2+\omega^2}$
$e^{-\alpha t}\cos(\omega t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega^2}$

Selected z-transforms

$x(n) \quad (n \geq 0)$	$X(z)$
$\delta(n)$ (unit pulse)	1
$\delta(n-m)$	z^{-m}
1 (unit step)	$\frac{z}{z-1}$
n (unit ramp)	$\frac{z}{(z-1)^2}$
$\exp(-\alpha n)$	$\frac{z}{(z-e^{-\alpha})}$
$n\exp(-\alpha n)$	$\frac{e^{-\alpha}z}{(z-e^{-\alpha})^2}$
$\sin(\beta n)$	$\frac{z\sin(\beta)}{z^2-2z\cos(\beta)+1}$
$\cos(\beta n)$	$\frac{z^2-z\cos(\beta)}{z^2-2z\cos(\beta)+1}$
$e^{-\alpha n}\sin(\beta n)$	$\frac{z\,e^{-\alpha}\sin(\beta)}{z^2-2ze^{-\alpha}\cos(\beta)+e^{-2\alpha}}$
$e^{-\alpha n}\cos(\beta n)$	$\frac{z^2-ze^{-\alpha}\cos(\beta)}{z^2-2ze^{-\alpha}\cos(\beta)+e^{-2\alpha}}$

Four classes of Fourier transform

Fourier series – periodic and continuous in time, discrete in frequency	
$x(t)=\sum_{n=-\infty}^{+\infty} X_n \exp(jn\omega_0 t)$	$X_n=\frac{1}{T}\int_{-T/2}^{T/2} x(t)\exp(-jn\omega_0 t)\,dt$
Fourier transform – continuous in time, continuous in frequency	
$x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\exp(\,j\omega t\,)\,d\omega$	$X(\omega)=\int_{-\infty}^{\infty} x(t)\exp(-j\omega t)\,dt$
Discrete-time Fourier transform – discrete in time, continuous and periodic in frequency	
$x(n\Delta t)=\frac{\Delta t}{2\pi}\int_{-\pi/\Delta t}^{\pi/\Delta t} X(\omega)\exp(-jn\Delta t\omega)\,d\omega$	$X(\omega)=\sum_{n=-\infty}^{\infty} x(n\Delta t)\exp(-j\omega n\Delta t)$
Discrete Fourier transform – discrete and periodic in time and in frequency	
$x(n)=\frac{1}{N}\sum_{k=0}^{N-1} X(k)\exp\left(\frac{jnk2\pi}{N}\right)$	$X(k)=\sum_{n=0}^{N-1} x(n)\exp\left(\frac{-jnk2\pi}{N}\right)$

Solutions to self assessment questions

Chapter 1

SAQ 1.1 The energy in the signal is defined by equation (1.2):

$$E = \int_{-\infty}^{\infty} x^2(t)\, dt = \int_{0}^{1} (5t)^2 dt = 25/3$$

SAQ 1.2 Using equation (1.13), $T = 1$, $\omega_0 = 2\pi$, thus:

$$X_n = \int_{-1/2}^{1/2} 4t \exp(-jn2\pi t)\, dt$$

Integration by parts yields:

$$X_n = \left[\frac{2j\pi nt \exp(-2j\pi nt) + \exp(-2j\pi nt)}{\pi^2 n^2} \right]_{-1/2}^{1/2} = \frac{2}{\pi^2 n^2} (\sin(\pi n) - n\pi \cos(\pi n))$$

SAQ 1.3 Using equation (1.16):

$$X(\omega) = \int_{1}^{4} 4 \exp(-j\omega t)\, dt = \left[-\frac{4}{j\omega} \exp(-j\omega t) \right]_{1}^{4} = -\frac{4}{j\omega} (\exp(-j\omega) - \exp(-4j\omega))$$

This comprises two impulses at the above frequencies.

SAQ 1.4 Using equation (1.20):

$$X(s) = \int_{0^-}^{\infty} \exp(-t/5) \exp(-st) dt = \left[-5 \frac{\exp(-st) \exp(-t/5)}{1 + 5s} \right]_{0^-}^{\infty} = \frac{5}{5s + 1}$$

SAQ 1.5 Using KCL:

$$\text{current} = C \frac{d}{dt} (\exp(jn\omega_0 t) - y) = \frac{y}{R}$$

Hence:

$$C\exp(jn\omega_0 t)jn\omega_0 - C\frac{dy}{dt} = \frac{y}{R}$$

thus:

$$y + RC\frac{dy}{dt} = RCjn\omega_0\exp(jn\omega_0 t)$$

Adopt an assumed solution, i.e.: $y_n(t) = K\exp(jn\omega_0 t)$. Substitute back into equation:

$$K\exp(jn\omega_0 t) + RCKjn\omega_0\exp(jn\omega_0 t) = RCjn\omega_0\exp(jn\omega_0 t)$$

Solving for K:

$$K + RCKjn\omega_0 = RCjn\omega_0 \Rightarrow K = \frac{RCjn\omega_0}{1 + RCjn\omega_0}$$

SAQ 1.6 Using method (i):

$$C\frac{d}{dt}(x(t) - y(t)) = \frac{y}{R} \Rightarrow RC\frac{dx}{dt} = RC\frac{dy}{dt} + y$$

Take Laplace transforms and assume initial conditions are zero:

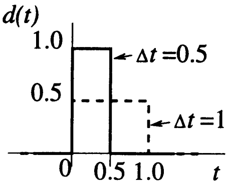
$$RCsX(s) = RCsY(s) + Y(s) \Rightarrow H(s) = \frac{RCs}{RCs + 1}$$

Method (ii), transform circuit:

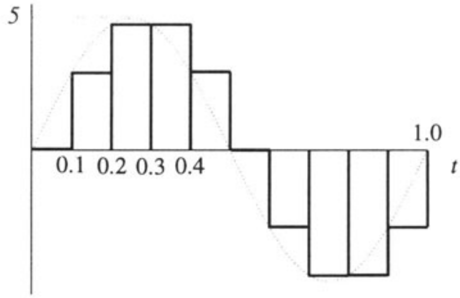
$$\frac{X(s) - Y(s)}{1/(sC)} = \frac{Y(s)}{R} \Rightarrow H(s) = \frac{RCs}{RCs + 1}$$

Chapter 2

SAQ 2.1



SAQ 2.2



First pulse: $5\sin(2\pi 0) \times d(t-0) \times 0.1 = 0$
Second pulse: $5\sin(2\pi \times 0.1) \times d(t-0.1) \times 0.1 = 0.294d(t-0.1)$
Third pulse: $5\sin(2\pi \times 0.2) \times d(t-0.2) \times 0.1 = 0.476d(t-0.2)$

SAQ 2.3 Transfer function:

$$H(s) = \frac{sL}{sL+R} = \frac{s}{s+R/L}$$

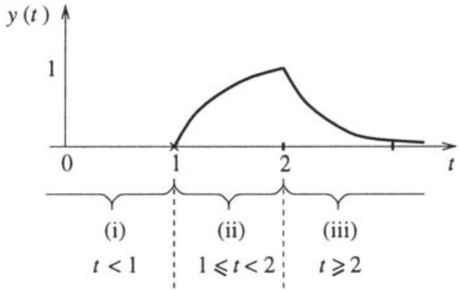
This is not a proper fraction so divide the numerator by the denominator to give:

$$H(s) = 1 - \frac{R/L}{s+R/L}$$

Taking inverse Laplace transforms yields:

$$h(t) = \delta(t) - \frac{R}{L}\exp\left(\frac{-t}{R/L}\right)$$

SAQ 2.4 Solution:



Region (i): $y(t) = 0$
Region (ii):

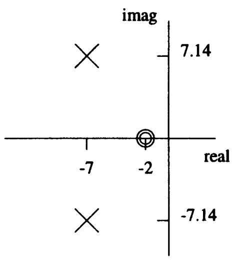
$$y(t) = 1 - \exp\left(-\frac{t-1}{RC}\right)$$

Region (iii):

$$y(t) = \exp\left(-\frac{t-1}{RC}\right)\left[\exp\left(\frac{1}{RC}\right) - 1\right]$$

Chapter 3

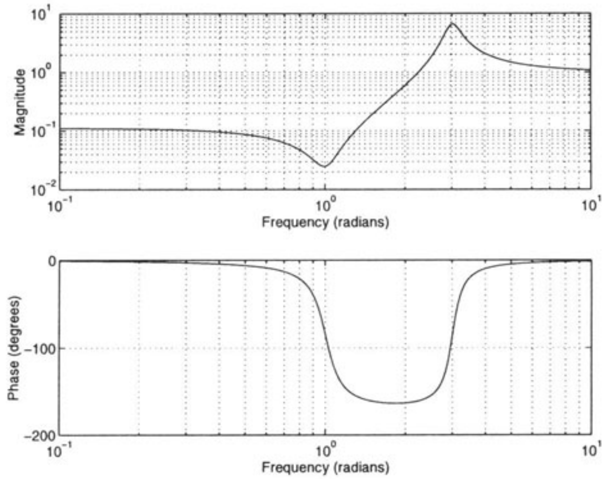
SAQ 3.1 Two zeros at $s = -2$ and poles at $s = -7 \pm j7.14$:



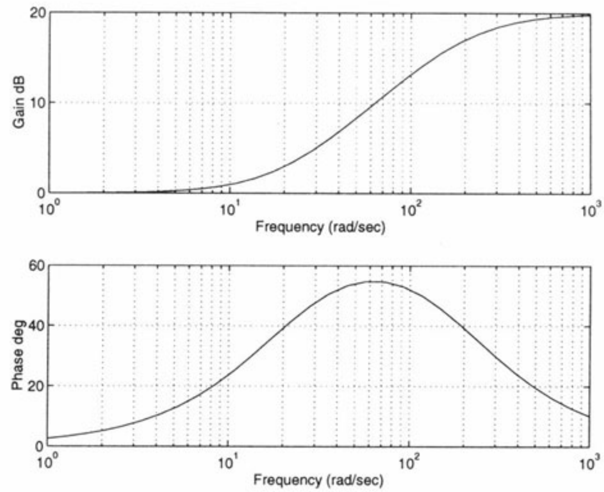
SAQ 3.2

$$H(s) = 1/(s^2 + 4s + 4) \Rightarrow H(j\omega) = 1/((j\omega)^2 + 4(j\omega) + 4)$$

SAQ 3.3 Zeros at $s = 0.1 \pm j$, poles at $s = 0.2 \pm 3j$ and 0.



SAQ 3.4 Exact Bode plot:



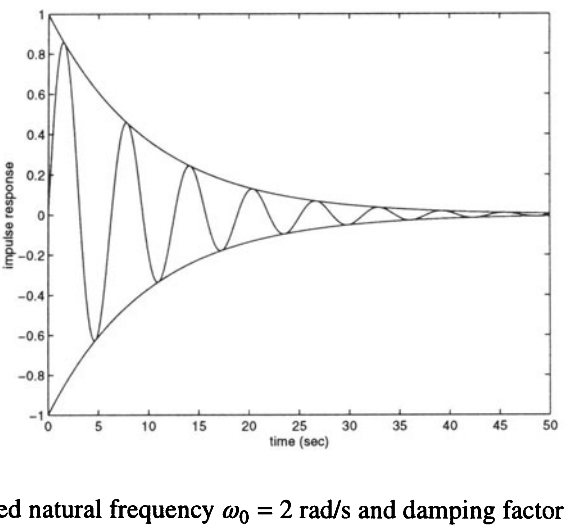
SAQ 3.5 Transfer function:

$$H(s) = 1/(s^2 + 0.2s + 1.01)$$

Poles at $s = -0.1 \pm j$ and hence system is stable. Partial fraction expansion:

$$H(s) = \frac{0.5j}{s + 0.1 + j} - \frac{0.5j}{s + 0.1 - j}$$

Impulse response: $h(t) = \exp(-0.1\ t)\sin(t)$; $t \geq 0$; period of oscillation is 2π seconds and time constant of decay is 10 seconds. Impulse response with envelope of delay shown below:



SAQ 3.6 Undamped natural frequency $\omega_0 = 2$ rad/s and damping factor $\zeta = 1/2$.

Chapter 4

SAQ 4.1 The frequencies which are present at the output of the D/A are: $3 + 10n$ kHz where n 1, 2, $\cdots$, etc. and $7 + 10n$ kHz where n 1, 2, $\cdots$, etc.

SAQ 4.2 Using (4.6):

$$X(z) = \sum_{n=0}^{\infty} \alpha^n z^{-n} = \sum_{n=0}^{\infty} (\alpha z^{-1})^n = \frac{1}{1 - \alpha z^{-1}} = \frac{z}{z - \alpha}$$

SAQ 4.3 Not a proper fraction, thus:

$$\frac{X(z)}{z} = \frac{(z-1)}{(z^2 - 5z + 6)} = \frac{2}{z-3} - \frac{1}{z-2}$$

and:

$$X(z) = \frac{2z}{z-3} - \frac{z}{z-2}$$

From tables in Appendix B:

$$x(n) = 2(3^n) - 2^n \quad n \geq 0$$

SAQ 4.4 At $n = 2$ the convolution equation becomes:

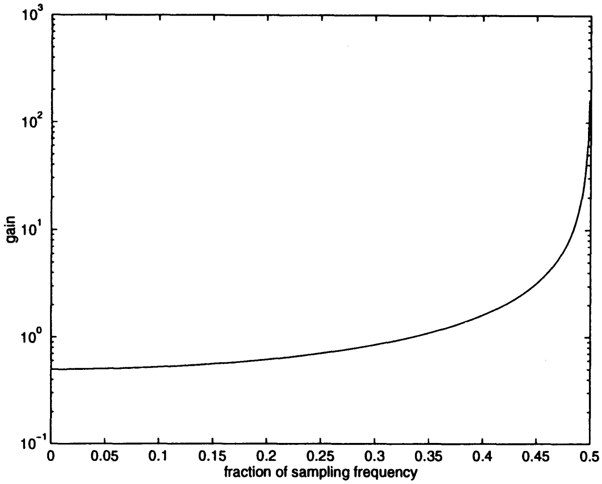
$$y(2) = \sum_{m=0}^{2} h(m)\, x(2-m) = 3(1) + 2(0.5) + 1(0.25) = 4.25$$

At $n = 3$ the convolution equation becomes:

$$y(3) = \sum_{m=0}^{2} h(m)\, x(3-m) = 0(1) + 3(0.5) + 2(0.25) = 2$$

SAQ 4.5 At a quarter of the sampling frequency, $\omega = (2\pi/\Delta t)/4$ and hence $\omega\Delta t = \pi/2$. Frequency response: $H(\omega) = 2 + \exp(j\omega\Delta t)$. Thus the gain is: $|2 + \exp(j\pi/2)| = 2.236$.

SAQ 4.6 Zero at the origin and pole at −1. Frequency response:



Chapter 5

SAQ 5.1 From the 'brick wall' frequency response by Fourier transform theory, the impulse response must have a sinc (x) shape.

SAQ 5.2 The number of poles is independent of the cut-off frequency and depends only on the filter order. It is 2 poles per order, thus for this question the answer is 8 poles.

SAQ 5.3 For a 4th order filter, the roll-off rate is $4 \times 20 = 80$ db/decade or $4 \times 6 = 24$ dB/octave. Thus at 2 kHz there is 24 dB attenuation, at 10 kHz there is 80 dB and at 100 kHz there is 160 dB.

SAQ 5.4 Stopband is specified at 12/3 or 4 times the cut-off frequency. Thus for a single stage filter the attenuation is $4 \times 6 = 24$ dB. For two stages the attenuation is 48 dB and for 3 stages it is 72 dB. Thus 3 stages will be required to meet the design specification of −60 dB.

SAQ 5.5 Chebyshev provides fastest roll-off or best rejection for a given number of stages of filter order and thus it is very efficient from a computation standpoint. The disadvantage is that there is ripple in the passband (Figure 5.4(a)), and also the phase response is highly non-linear.

SAQ 5.6 Solution is as in Figure 5.8 but a_1 and b_1 are zero so these paths do not exist. $a_1 = 1$, $a_2 = 10$ and $b_2 = 5$.

SAQ 5.7 The theoretical duration is infinity.

SAQ 5.8 All bilinear z-transform filters have infinite attenuation at half the sampling frequency, i.e. 25 kHz in this case.

SAQ 5.9

$$H(z) = \frac{z^2 + 2z + 1}{10.2z^2 - 9.62z + 3.37} = \frac{(z+1)^2}{z + \frac{9.6 \pm \sqrt{92.16 - 137.5}}{20.4}}$$

$$= \frac{(z+1)^2}{(z + 0.5 + 0.33)\,(z + 0.5 - j\,0.33)}$$

This has two zeros at $z = -1$ and poles at $-0.5 \pm j0.33$.

SAQ 5.10 Replace $\{z\}$ by $\{-z\}$ for LP to HP transform:

$$H(z) = \frac{z^2 - 2z + 1}{10.2z^2 + 9.62z + 3.37} = \frac{(z-1)^2}{(z - 0.5 + j0.33)\,(z - 0.5 - j0.33)}$$

Thus the poles and zeros reflect about the vertical axis:

$$H(z) = \frac{0.098 - 0.195\, z^{-1} + 0.098\, z^{-2}}{1 + 0.942\, z^{-1} + 0.333\, z^{-2}}$$

In comparison with the LPF of Figure 5.11, the signs of the a_1 and b_1 weights alter but the numerical values are all unchanged.

SAQ 5.11 Compared with SAQ 5.7, the finite precision causes the impulse response to be of finite duration. Because of the limit cycle effects in 5.5.2, there can also be an oscillating output for no input signal.

Chapter 6

SAQ 6.1 In this FIR filter $H(z) = 1 + z^{-1} = (z + 1)/z$. This has one zero at $z = -1$ and a pole at $z = 0$. The frequency response, which can be obtained by applying the Figure 4.24(a) constructions, peaks at DC and f_s and has a minimum value of zero at $f_s/2$.

SAQ 6.2 For (a): $H(z) = 1 + \frac{1}{2} z^{-1} = (z + \frac{1}{2})/z$. This has a zero at $z = -\frac{1}{2}$ and a pole at $z = 0$. The frequency response peaks at DC and f_s with a magnitude of 1.5 and at $f_s/2$ the minimum value is 0.5. The phase response is double humped with zero values at $f = 0, f_s/2, f_s$. For (b): $H(z) = \frac{1}{2} + z^{-1} = (\frac{1}{2}z + 1)/z$. This has its zero at $z = -2$. The frequency response is as in (a) with maximum value of 3 and a minimum value (at $f_s/2$) of 1. The phase response is 0° at DC, −180° at $f_s/2$ and 0° again at f_s.

SAQ 6.3 Linear phase filtering is essential in radar and communications to enhance pulsed signal returns and measure accurately the timing of the received signals.

SAQ 6.4 This introduces a symmetrical impulse response.

SAQ 6.5 The coefficient values would each be scaled to $10 \times$ their values.

SAQ 6.6 For this increased duration impulse response, the –3 dB bandwidth is reduced to one tenth of the sample frequency.

SAQ 6.7 For half the original bandwidth, by Fourier theory, the impulse response main-lobe width will be doubled in duration. Now if the gain of both filters is identical then the values will have to be scaled by ½ as there are twice as many of them.

SAQ 6.8 For the Hamming filter the transition bandwidth is:

$$\frac{3.3}{N\Delta t} = \frac{3.33}{21 \times 10^{-3}} = 0.157 \text{ kHz}$$

SAQ 6.9 For the Hanning filter design, the transition bandwidth must be only 50 Hz. Thus for filter order N:

$$50 = \frac{3.1}{N\Delta t} = \frac{3.1}{N \times 10^{-3}} \quad \text{i.e.} \quad N = \frac{3.1}{0.05} = 62$$

The filter order is thus 62. Note that as the transition bandwidth is one third that of SAQ 6.7 then the filter order is three times larger!

SAQ 6.10 From Figure 6.10 find δ_1 for a passband ripple of ½ dB. Substitution into the equation yields $\delta_1 = 0.06$. Also for –40 dB stopband attenuation, $\delta_2 = 0.01$. Now using equation (6.17):

$$\hat{N} = \frac{2}{3}\frac{1}{\Delta t \Delta f}\log_{10}\left(\frac{1}{10 \times 0.06 \times 0.01}\right) = \frac{2}{3}\frac{10^3}{50}\log_{10}\frac{100}{0.6}$$

This yields $\hat{N} = 30$.

SAQ 6.11 For an input variance of 1/12, the output variance will be 1/36 in this 3-stage FIR averager, see example 7.3.

SAQ 6.12 The frequency response of the matched filter is the complex conjugate of the expected signal.

SAQ 6.13 The optimum filter in white Gaussian noise is a matched filter.

SAQ 6.14 Clearly this has peak values of ± 3 every 3 samples as the signal propagates through the FIR receiver.

Chapter 7

SAQ 7.1 By definition:

$$\sigma_x^2(n) = E[(x(n) - m_x(n))^2]$$

Start by multiplying out all the term inside the expectation operator:

$$\sigma_x^2(n) = E[x^2(n) - 2x(n)m_x(n) + m_x^2(n)]$$

As in equations (7.1) and (7.3), expectation is an integration operation and hence we can re-order the expectation and addition operations:

$$\sigma_x^2(n) = E[x^2(n)] - 2E[x(n)m_x(n)] + E[m_x^2(n)]$$

The mean is a constant and hence can be taken outside the expectation operator:

$$\sigma_x^2(n) = E[x^2(n)] - 2E[x(n)]m_x(n) + m_x^2(n)$$

From the definition of the mean in equation (7.1):

$$\sigma_x^2(n) = E[x^2(n)] - m_x^2(n)$$

SAQ 7.2 Let $\{x(n)\} = \{1.3946, 1.6394, 1.8742, 2.7524, 0.6799\}$. Using equation (7.2) an estimate of the mean is:

$$\hat{m}_x = \frac{1}{5}\sum_n x(n) = 1.6681$$

An estimate of the variance would then be:

$$\hat{\sigma}_x^2 = \frac{1}{5}\sum_n (x(n) - \hat{m}_x)^2 = 0.4541$$

SAQ 7.3 The power at the input is given by the product of the PSD and the bandwidth of the signal – PB_s. The power at the output is given by the product of the PSD, the filter gain and the bandwidth of the filter PGB_f. Since the gain is unity, the power at the output of the filter will be less than at the input by a factor of 10.

SAQ 7.4 The PSD describes how the average power in a signal is distributed in the frequency domain. The pdf describes how the instantaneous value of a signal is distributed.

SAQ 7.5

$$S_{yy}(z) = H(z)\,H(z^{-1})\,S_{xx}(z) = (0.5 + 0.75\,z^{-1})\,(0.5 + 0.75\,z)\,\sigma_x^2$$

$$= (0.375\,z + 0.8125 + 0.375\,z^{-1})\,\sigma_x^2$$

The autocorrelation at the output is the inverse z-transform of this:
$\phi_{yy}(-1) = 0.375\sigma_x^2$ $\phi_{yy}(0) = 0.8125\sigma_x^2$ $\phi_{yy}(1) = 0.375\sigma_x^2$. The variance of the output is $0.8125\sigma_x^2$ with a corresponding RMS value of $\sqrt{0.8125}\sigma_x$. The PSD:

$$S_{yy}(\omega) = (0.375\,e^{-j\omega\Delta t} + 0.8125 + 0.375\,e^{j\omega\Delta t})\,\sigma_x^2$$

$$= (0.8125 + 0.75\cos(\omega\Delta t))\,\sigma_x^2$$

SAQ 7.6 At output of filter:

$$S_{yy}(z) = (0.1 - 0.8z^{-1})(0.1 - 0.8z)$$

with zeros at $z = 8$ and 1/8. The term $(0.1 - 0.8z)$ is minimum since it has a root at 1/8 but it is non-causal because of the positive power of z. Rewriting we have: $(0.1 - 0.8z) = z(0.1z^{-1} - 0.8)$ and $(0.1 - 0.8z^{-1}) = (0.1z - 0.8)z^{-1}$. Thus:

$$S_{yy}(z) = (0.1z - 0.8)(0.1z^{-1} - 0.8)$$

The minimum phase causal filter is $(0.1z^{-1} - 0.8)$. The whitening filter is the inverse of this, i.e.: $1/(0.1z^{-1} - 0.8)$. This is NOT the inverse of $H(z)$.

SAQ 7.7 In section 7.6.2 the dynamic range was calculated on the basis of a sine wave signal occuppying the whole dynamic range. In this example the signal amplitude is a factor of 2 smaller than the max. voltage – hence signal power is reduced by a factor of 4. Thus signal-to-quantisation noise is:

$$1.76 + 6M - 10\log_{10}(4) = -88.3 \text{ dB}$$

Chapter 8

SAQ 8.1 Using equation (8.4):

$$\xi = E[x^2(n)] - 2\,\mathbf{h}^T\,\Phi_{yx} + \mathbf{h}^T\,\Phi_{yy}\,\mathbf{h}$$

Then using equation (8.8):

$$\xi = E[x^2(n)] - 2\,\mathbf{h}^T\,\Phi_{yx} + \mathbf{h}^T\,(\Phi_{yy}\,\mathbf{h}) = E[x^2(n)] - 2\,\mathbf{h}^T\,\Phi_{yx} + \mathbf{h}^T\,\Phi_{yx}$$
$$= E[x^2(n)] - \mathbf{h}^T\,\Phi_{yx}$$

SAQ 8.2 Thus:

$$\Phi_{yy} = \begin{bmatrix} 1.35 & 0.5 & 0 \\ 0.5 & 1.35 & 0.5 \\ 0 & 0.5 & 1.35 \end{bmatrix}$$

Since the lag d is 2, the z-transform of the cross-correlation sequence is:

$$S_{yx'}(z) = z^{-2}\,(z + 0.5) = z^{-1} + 0.5z^{-2}$$

Hence the cross-correlation terms are: $\phi_{yx'}(0) = 0$, $\phi_{yx'}(1) = 1$ and $\phi_{yx'}(2) = 0.5$. The cross-correlation vector is: $\Phi_{yx'} = [0.0\ \ 1.0\ \ 0.5]^T$. Solving the three simultaneous equations yields the impulse response vector of the equaliser: $\mathbf{h} = [-0.3081\ \ 0.8318\ \ 0.0623]^T$. The minimum MSE can then be obtained using equation (8.9):

$$E[e^2] = \sigma_x^2 - \mathbf{h}^T\,\Phi_{yx'} = 0.14$$

SAQ 8.3 Using equation (8.14):

$$\mathbf{R}_{yy}(n) = \sum_{k=0}^{n} \mathbf{y}(k)\, \mathbf{y}^T(k) = \sum_{k=0}^{n-1} \mathbf{y}(k)\, \mathbf{y}^T(k) + \mathbf{y}(n)\mathbf{y}^T(n)$$

$$= \mathbf{R}_{yy}(n-1) + \mathbf{y}(n)\mathbf{y}^T(n)$$

SAQ 8.4 Using equation (8.15):

$$\mathbf{r}_{yx}(n) = \sum_{k=0}^{n} \mathbf{y}(k)\; x(k) = \sum_{k=0}^{n-1} \mathbf{y}(k)\; x(k) + \mathbf{y}(n)x(n)$$

$$= \mathbf{r}_{yx}(n-1) + \mathbf{y}(n)x(n)$$

SAQ 8.5 A 2-tap filter with a white noise input, hence: $\Phi_{yy} = 2I_2$ where I_2 is a (2×2) identity matrix. Using equation (8.23):

$$\nabla = 4\begin{bmatrix} 2.9 \\ 4.0 \end{bmatrix} - 2\begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5.6 \\ 14.0 \end{bmatrix}$$

SAQ 8.6 i) LMS: $N = 4$, 8 multiples and 8 additions; $N = 128$, 256 multiplies and 256 additions. ii) BLMS: $N = 4$, 28 multiples and 60 additions; $N = 128$, 78 multiplies and 135 additions. The BLMS is more expensive than the LMS for short filters but more computationally efficient for long filters.

SAQ 8.7 The RLS has faster initial convergence than the LMS and its convergence rate is much less dependent on the statistics of the input signal. The RLS is computationally more expensive than the LMS.

SAQ 8.8 As the bandwidth of the filter is decreased, the signal at its output becomes more correlated from sample to sample and hence more predictable. At the end of the day the signal is more predictable than it was at the beginning of the day.

Chapter 9

SAQ 9.1 The eight roots of unity are:

$W_8^0 = 1$
$W_8^1 = 0.7 - j\,0.7$
$W_8^2 = -j$
$W_8^3 = -0.7 - j\,0.7$
$W_8^4 = -1$
$W_8^5 = -0.7 + j\,0.7$
$W_8^6 = +j$
$W_8^7 = 0.7 + j\,0.7$

SAQ 9.2 This involves the following 8 multiply and sum operations:
$(1 \times 1) + (0.7 + j0.7) \times -j + (j \times -1) + (-0.7 + j0.7) \times j - (1 \times 1) +$
$(-0.7 - j0.7) \times -j\,(-j \times -1) + (0.7 - j0.7) \times j = 0$

SAQ 9.3 In the first channel we have an N-tap FIR where all the weights are unity. This is a low-pass filter, centred on 0 Hz, whose impulse response is N samples long. Fourier theory tells us that such a signal has sinc x response with a mainlobe width of $1/N\Delta t$ Hz to the first null. sinc x responses can then be overlapped by arranging the next filter in the bank to have a sinc x response, centred on $1/N\Delta t$ Hz, and the filterbank then overlaps at the –3 dB points, as shown in Figure 9.8.
Thus the first low-pass channel has a narrow bandwidth and, as in Figure 4.10(c) the output sample rate can be set at $1/N\Delta t$ Hz. As the input sample rate is $1/\Delta t$ Hz, the downsampling is by the factor N.

SAQ 9.4 As in SAQ 9.3, the sinc x responses with $\Delta t = 10^{-3}$ and an $N = 8$ transform have zeros at 1/8, 1/4, 3/8, 1/2, 5/8, 3/4, 7/8, 1 kHz. These are the bin frequencies in the 8-point DFT.

SAQ 9.5 Resolution $\Delta f = 1/N\Delta t$ Hz. For a 1024-point DFT sampled at 512 kHz:

$$\Delta f = \frac{1}{1024\left(\dfrac{1}{512 \times 10^3}\right)} = \tfrac{1}{2} \times 10^3 = 500 \text{ Hz.}$$

SAQ 9.6 For 16-bit arithmetic, dynamic range is 6 dB per bit. Some allowance must be given for the peak-to-mean ratio of the signal as the quantiser must not clip the large signal transients. Thus typically the SNR is 6 dB per bit minus 6–10 dB to allow for this. Thus for 16-bit arithmetic, the dynamic range is 85–90 dB.

SAQ 9.7 The level of leakage can be calculated by noting that the signal at the bin 16 position is given by the sinc$x$ value where x is equal to $11\pi/2$, i.e. it falls on to the top of the fifth sidelobe in the sinc$x$ response in Figure 10.11(a). This magnitude is $1/(5.5\pi)$ which equals 0.0579 or –24.8 dB, as shown in Figure 10.11(b).

SAQ 9.8 This can only be achieved by using *a priori* knowledge to change either the sample rate or the transform length to accommodate the signal. For the 128-point DFT sampled at 128 kHz, the resolution is 100 Hz and the signal is at 1.05 kHz. Changing the sampling frequency to $12.8 \times 10/10.5$, i.e. to 121,904.76 Hz will put the frequency on to bin 10. Note that this is signal specific and it is not a GENERAL solution to the problem.

SAQ 9.9

$$\Delta f = \frac{1}{256}\left(\frac{1}{51.2 \times 10^3}\right) = \frac{2 \times 10^3}{10} = 200 \text{ Hz.}$$

Signal is at 63 Hz, the zeros on the sinc x are every 200 Hz and bin 7 occurs at 1400 Hz.

Thus on the sinc x response, where zeros occur every π rad, bin 7 is $\pi(1400 - 63)/200$ rad from the peak, i.e. 6.685π. Evaluating $\text{sinc}(x) = (\sin(\pi x))/(\pi x)$ where $x = 6.685$ gives the value of $0.835/(6.685\pi) = 0.0398$, or –28 dB.

SAQ 9.10 For the 64-sample Hanning window $w_0 = 0.5 - 0.5 = 0$, $w_{32} = 0.5 - 0.5\cos(2\pi\ 32/64) = 0.5 + 0.5\cos\pi = 1$, $w_{64} = 0.5 - 0.5 = 0$.

SAQ 9.11 For the 64-sample Hamming window $w_0 = 0.54 - 0.46 = 0.08$, $w_{32} = 0.54 + 0.46 = 1$, $w_{64} = 0.54 - 0.46 = 0.08$.

SAQ 9.12 For –40 dB sidelobe level, the Hamming window is the most appropriate function. For –55 dB there are several choices but the Dolph–Chebychev has the narrowest width for the main peak and so gives the best spectrum analyser capabilities.

SAQ 9.13 Power spectral density is given by equation (9.24) as the Fourier transform of the autocorrelation function. The power spectrum measure is simply the squared modulus of the amplitude spectrum or DFT of the signal and it does not use correlation measures – see later equation (9.26) and Figure 9.16.

SAQ 9.14 Classical spectrum analysers are general processors that are not signal dependent. They offer poor resolution. Modern analysers offer superior resolution but they assume a signal type and this is the basis of the model employed.

Chapter 10

SAQ 10.1 The number of passes is $\log_2 N$. For $N = 256$ then $\log_2 512 = 8$ and so there are eight passes.

SAQ 10.2 If the binary addresses of the input locations 0 to 7 are reflected 'mirror' fashion with the MSB being regarded as the LSB etc., then the resulting number indicates which data sample should be stored in any given location. For example location 1 (address 001_2) would hold data sample 4 (sample 100_2). Thus data re-ordering for a DIT FFT processor is easily achieved by simply adjusting the address decoder.

SAQ 10.3 The number of DFT complex multipliers is N^2 and for $N = 512$ this is 512^2 or 262,144. The number of FFT complex multipliers is $(N/2)\log_2 N$ which is $256\log_2 512$ or 2304.

SAQ 10.4 Extending SAQ 10.3, we can make the following table for the number of complex multiply operations:

N	DFT	FFT	% saving
32	1024	80	92.19
256	65,536	1024	98.44
512	262,144	2304	99.12
1024	1,048,576	5120	99.51
8192	67,174,416	49,152	99.93

SAQ 10.5 In the radix-4 case, multiplication is by $+1$, -1 or $+j$, $-j$. Thus from equation (10.16):

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} a+jb \\ c+jd \\ e+jf \\ h+ji \end{bmatrix}$$

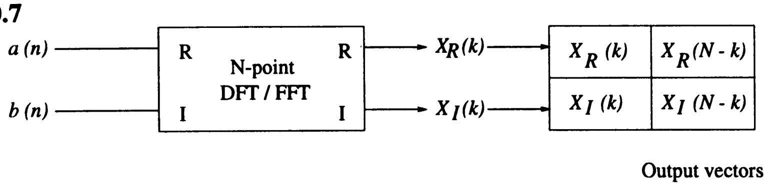
Multiplication by the complex number $\pm j$ may be simply achieved by interchanging real and imaginary parts and changing the sign of the real or imaginary parts of the resultant respectively. Thus:

$$\begin{aligned} X(0) &= a+c+e+h \ +j(b+d+f+i) \\ X(1) &= a+d-e-i \ +j(b-c-f+h) \\ X(2) &= a-c+e-h \ +j(b-d+f-i) \\ X(3) &= a-d-e+i \ +j(b+c-f-h) \end{aligned}$$

and no actual multiply operations are required!

SAQ 10.6 The number of passes is $\log_4 N$ and, for $N = 256$, there are four passes.

SAQ 10.7



$$A(k) = 1/2 \times \begin{array}{|c|c|} \hline X_R(k) & +X_R(N-k) \\ +jX_I(k) & -jX_I(N-k) \\ \hline \end{array}$$

$$B(k) = 1/2 \times \begin{array}{|c|c|} \hline X_I(k) & X_I(N-k) \\ -jX_R(k) & +jX_R(N-k) \\ \hline \end{array}$$

SAQ 10.8 For a 4096-point convolution, Figure 11.9 thus involves $8192 \times 13 \times 2$ complex FFT MAC operations plus a further 8192 complex multiplications for the $X(k)H(k)$ operation. The total number of MAC operations is thus 8193×27 which is a considerable saving over the $4096^2 \approx 16$M operations of Figure 6.1.

Chapter 11

SAQ 11.1 For this signal the normal minimum (Nyquist) sample rate would be 3 kHz. At input sample rate 10 kHz, decimation by 3 would then give 3.33 kHz output.

SAQ 11.2 For $n = Q = 8$ we get: $2500 < f_s < 2571$. The sample rate is thus 2535.5 Hz with an accuracy of 1.4%.

SAQ 11.3 The quotient $Q = 10.5$ is now no longer an integer. The lowest allowed sampling rate is therefore given by $n = int(Q) = 10.0$. The sampling rate is now bound by:

$$2.100 \leq f_s \leq 2.111 \text{ (kHz)}$$

This gives a required sampling rate of 2.106 ± 0.006 kHz or 2.106 kHz $\pm 0.26\%$.

SAQ 11.4 Now there is no difference between filterbank and DFT.

SAQ 11.5 Because in the N-channel multirate filterbank the filter length or impulse duration is not constrained to be N samples long, as in the DFT. Thus making the length longer than N gives the improved filterbank performance shown in Figure 11.15.

SAQ 11.6 If the signal bandwidth is 102.4 kHz then the sample rate is 204.8 kHz. Resolution is $f_s/N = 200$ Hz.

SAQ 11.7 For Figure 11.17(b), f_c in the FIR filter is now 11.15 kHz but the signal bandwidth is still 2.5 kHz so the solution is as before. If the Figure 11.9 bandpass sampling criterion had been used, then:

$$Q = \frac{f_H}{B} = \frac{12.4}{2.5} = 4.96$$

Thus the n value must be 4 to avoid the spectral aliases. Now:

$$5 \times 10^3 \left(\frac{4.96}{4} \right) \leq f_s \leq 5 \times 10^3 \left(\frac{4.96 - 1}{4 - 1} \right)$$

This gives a range for the f_s value of 6.2 to 6.6 kHz. Note that as the input FIR filter of Figure 11.17(b) is not present, a higher sample rate is required to stop the straddling of Figure 11.9 occurring.

SAQ 11.8 For 4 equal width sub-bands covering the 0 kHz wide spectrum, i.e. 2.5 kHz wide bands, the output sample rate in each band is 5.0 kHz. The total bit rate for the output signal is thus $5(8 + 6 + 4 + 2) = 5 \times 20 = 100$ kbit/s. The compression is thus $160/100 = 1.6$ times.

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