1

a

$$H(\omega) = \sum_{n=-\infty}^{+\infty} h(n) e^{-jn\omega} \rightarrow H(0) = \sum_{n=-\infty}^{+\infty} h(n)$$

As, $\forall n \notin \{0, 1, \dots, M\}$; $h(n) = 0$, we have that

$$H(0) = \sum_{n=0}^{M} h(n)$$

b Note that defining $k = M-n$, we have that

$$H(0) = \sum_{n=0}^{M} h(n) = \sum_{k=0}^{M} h(M-k)$$

Using the fact that $h(M-k) = -h(k)$, we get that

$$H(0) = \sum_{k=0}^{M} -h(k) = -H(0)$$

Thus,

$$2H(0) = 0 \longrightarrow H(0) = 0$$

d $\omega = 0$ corresponds to $z = e^{j0} = 1$. Thus

$$H(\omega = 0) = H(z = 1) = \frac{b_0 + b_1 + b_2}{1 + a_1 + a_2}$$

(2)

a) K samples:

$$h_{shift}(n) = h(n-K)$$

b) Using the properties of the Fourier Transform, we get ~~that~~

$$H_{shift}(\omega) = H(\omega)e^{-jK\omega}$$

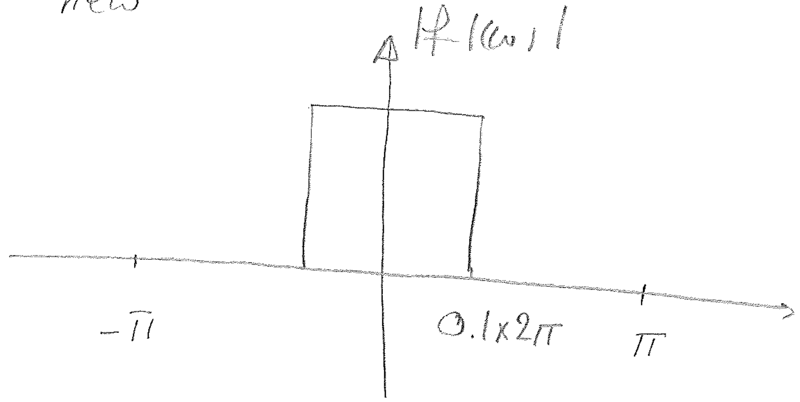
Thus, the amplitude is not changed but a linear phase with ω is added

(3) We have that

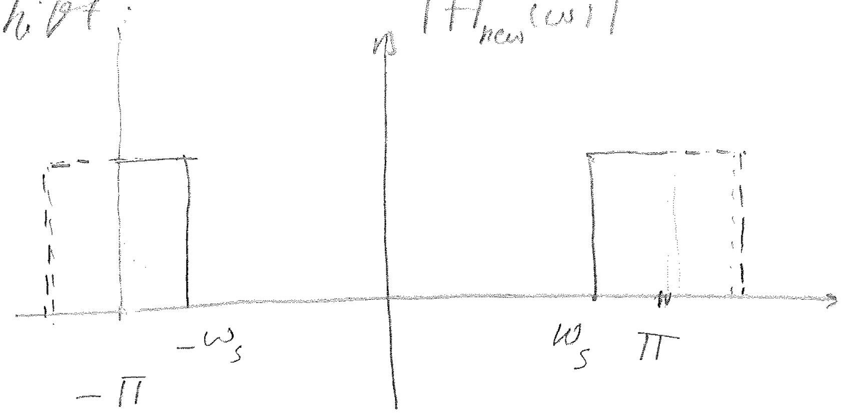
$$h_{new}(n) = (-1)^n h(n) = e^{jn\pi} h(n)$$

Thus,

$$H_{new}(\omega) = H(\omega - \pi)$$



after shift:



$$\omega_s = \pi - 0.1 \times 2\pi = 0.8\pi$$

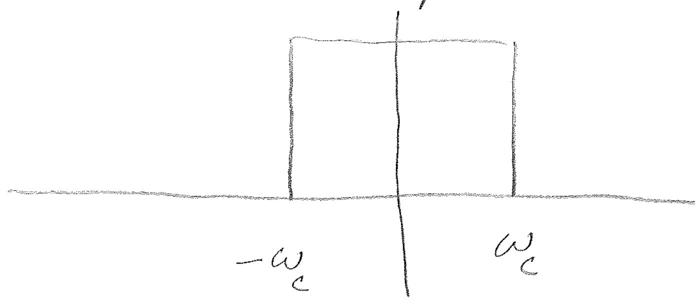
The filter is high-pass for real-valued signals as π $(= \frac{\omega_s}{2})$ is the highest possible frequency

(4) Not that

$$f_s = \frac{1}{50\mu s}$$

and

$$\omega_c = 2\pi \times \frac{6\ KHz}{\frac{1}{50\mu s}} = 2\pi \times 0.3 = 0.6\pi$$



$$h(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(\omega) e^{jn\omega} d\omega = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{jn\omega} d\omega = \frac{e^{jn\omega_c} - e^{-jn\omega_c}}{2\pi \times jn}$$

$$= \frac{\sin(n\omega_c)}{\pi n} = \frac{\sin(0.6\pi n)}{\pi n}$$

We only keep the largest components corresponding to $n \in \{-2, -1, 0, 1, 2\}$:

n	0	± 1	± 2
$h(n)$	0.6	0.3027	-0.0935

The final filter is given by

$$h_f(n) = h(n) \times w(n)$$

	0	± 1	± 2
$w(n)$	1	0.54	0.08
$h_f(n)$	0.6	0.1635	-0.0075

5

(a) Take the z-transform:

$$Y(z) = a_1 z^{-1} Y(z) + (b_0 + b_1 z^{-1}) X(z)$$

$$\rightarrow Y(z) = \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}} X(z)$$

Thus,

$$H(\omega) = \frac{b_0 + b_1 e^{-j\omega}}{1 - a_1 e^{-j\omega}}$$

(b) from above the pole is:

$$1 - a_1 z^{-1} = 0 \rightarrow z = a_1$$

when $b_0 + b_1 a_1 = 0$. This is not a pole anymore!

Thus the system is stable if

$$|a_1| \leq 1 \quad \text{or} \quad b_0 + b_1 a_1 = 0$$

6

(a) $$|H_{BW}(\Omega)|^2 = \frac{1}{(1-\Omega^2)^2 + 2\Omega^2} = \frac{1}{2}$$

Thus,

$$(1-\Omega^2)^2 + 2\Omega^2 = 2 \rightarrow 1 + \Omega^4 = 2 \rightarrow \Omega = 1$$

(b) Let us denote the new filter by H'_{BW}.

Then,

$$H'_{BW}(\Omega) = H_{BW}\left(\frac{\Omega}{\omega_c}\right)$$

$$|H'_{BW}(\Omega)| = \frac{1}{\sqrt{2}} \rightarrow \left|H_{BW}\left(\frac{\Omega}{\omega_c}\right)\right| = \frac{1}{\sqrt{2}} \rightarrow \frac{\Omega}{\omega_c} = 1 \rightarrow \Omega = \omega_c$$

(c)

i)

$$\omega_c = 2\pi \times \frac{5\,kHz}{20\,kHz} = \frac{\pi}{2} \quad rad/sec$$

ii)

$$\Omega_c = 2\tan\left(\frac{\pi}{4}\right) = 2 \quad Hz$$

iii.

$$H_{BW}(s) = \frac{1}{\left(\frac{s}{2}\right)^2 + \sqrt{2}\,\frac{s}{2} + 1}$$

iv.

$$H(z) = \frac{1}{\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2 + \sqrt{2}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 1}$$

$$= \frac{1+2z^{-1}+z^{-2}}{2+\sqrt{2}+(2-\sqrt{2})z^{-2}}$$

The system can be written as

$$(2+\sqrt{2})y(n) + (2-\sqrt{2})y(n-2) = x(n) + 2x(n-1) + x(n-2)$$

$$y(n) = \frac{1}{2+\sqrt{2}}\left[-(2-\sqrt{2})y(n-2) + x(n) + 2x(n-1) + x(n-2)\right]$$

It is of order 2.

The poles are at

$$z = \pm\sqrt{\frac{2-\sqrt{2}}{2+\sqrt{2}}} = \pm 0.41$$

$$|z| \leq 1$$

Thus, the system is stable