1A

This report investigates simulation of an audio channel for data transfer, encoding of mentioned bits as well as retrieval using equalization.

Task 1.a)

Introducing the ideal case:

$$h = h_1(n) = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise} \end{cases}, \quad \text{SNR} = \infty, \quad N_{\text{cp}} = 0 \tag{1}$$

and no sync error.

Simulation plots show:

$$\begin{cases} |H(k)| = 1 \\ \arg(H(k)) = 0 \end{cases} \quad \forall k \in [1, \text{length}(h)] \tag{2}$$

and indiscernible deviation between transmitted symbols for both post- and pre-equalization constellations. The error vector magnitude is $3.01 \cdot 10^{-16}$ and BER$= 0$. Given the channel is ideal (see (1)), we theoretically expect $y(n) = z(n) \iff r(k) = s(k)$ and this also coincides with the results obtained. The reason for a finite EVM is due to numerical computation on continuous value space $\mathbf{C}$, that is, matlab approximates/rounds off values after DFT/IDFT and equalization calculations and thus reciprocity is broken. Reason for BER to equal zero exactly is due to its discrete logical value space where decimal truncation pose as no source of error. Bits are either 1 or 0, nothing else.

Task 1.b)

A cyclic prefix of correct length N_{cp} enables us to achieve $y(n + N) = y(n)$ where $z(n + N) = z(n)$, that is, the received signal bear the same periodicity as our transmitted signal. The implemented scheme forming the channel transfer and equalization builds upon the assumption of y and z to be of the same periodicity and hence s, r and H to have the same length. The cyclic prefix need to compensate for the length M of h and so we have $N_{\text{cp}} = M - 1$. $N_{\text{cp}} \geq M - 1$ will assure minimum EVM with respect to finite impulse response induced errors. If too short cyclic prefix is appended the periodic length of y will be different from the transmitted signal length, thus their Fourier transforms will be sampled at different frequencies and no simple equalization relation can be employed. As currently implemented in MATLAB, appending a too short cyclic prefix will cause frequency sample mismatch and subsequent errors, see figure 1.

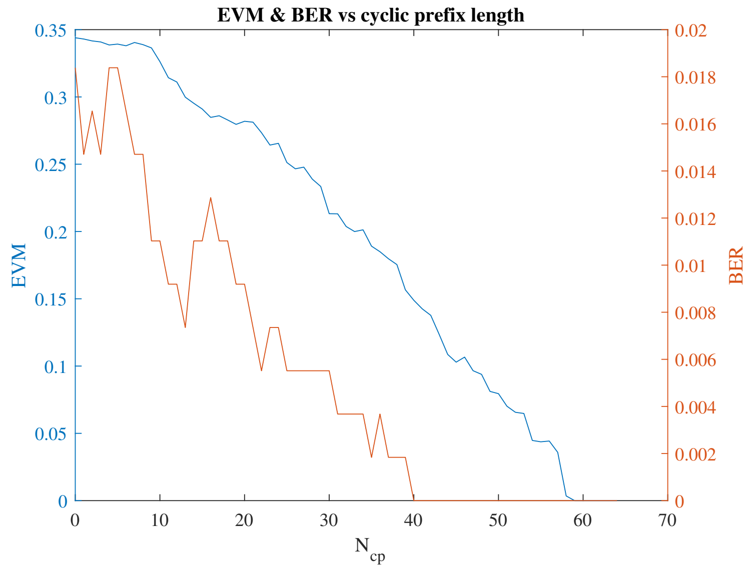


Figure 1: Simulation over noiseless and sync error free channel with $h(n) = \text{randn}(60,1)$.

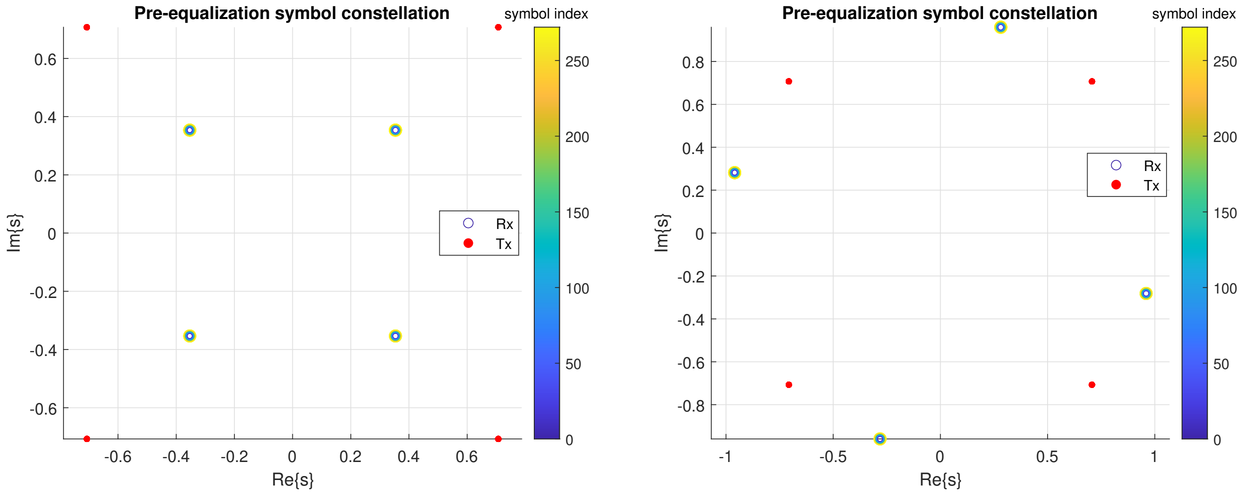
Task 1.c)

In this task we have

$$h(n) = \alpha h_1(n), \text{ where } \alpha = \begin{cases} 0.5, & h = h_2 \\ e^{\frac{j}{2}}, & h = h_3 \end{cases}, \quad \text{SNR} = \infty, \quad N_{\text{cp}} = 0 \tag{3}$$

When evaluating the convolution with h we can extract the constant α out of the integral and, since $h = h_1$ is ideal, we simply obtain $y(n) = \alpha z(n) \iff r(k) = \alpha s(k)$. Simple equalization: $\hat{s}(k) = r(k)/H_{2,3}(k) = \alpha s(k)/\alpha H_1(k) = s(k)$ results in the same as for the ideal case in task 1.a). Hence we understand that $\alpha = 0.5$ leaves the pre-equalization symbol magnitude scaled with 0.5 (see figure 2a) and $\alpha = e^{\frac{j}{2}}$ leaves each pre-equalization symbol rotated by 1/2 radians counterclockwise in symbol space (see figure 2b).

$h(n) = h_2(n)$ $\qquad\qquad$ $h(n) = h_3(n)$



(a) $\qquad\qquad$ (b)

Figure 2

Task 1.d)

As it is stated in the question, the synchronization error corresponds to a delay/early reception of the received time domain signal which in frequency domain will cause a phase shift in the frequency components. The phase shift depends on two parameters: the amount of delay and the frequency component. Since the transmitted symbols are QPSK, a symbol error occurs when the symbol is rotated more than $\frac{\pi}{4}$. With the synchronization error, the amount of phase shift for frequency components is calculated as $e^{-j2\pi n_{se}\frac{f}{f_s}}$ which $\frac{f}{f_s} \in \{\frac{0}{N}, \frac{1}{N}, \frac{2}{N}, ..., \frac{N-1}{N}\}$. In order to see which frequency components (x) get a phase shift less than $\frac{\pi}{4}$, we have to solve the below equations and find the valid values:

$$2\pi n_{se}\frac{x}{N} < 2m\pi + \frac{\pi}{4} \tag{4}$$

$$2\pi n_{se}\frac{x}{N} > 2m\pi + 2\pi - \frac{\pi}{4} \tag{5}$$

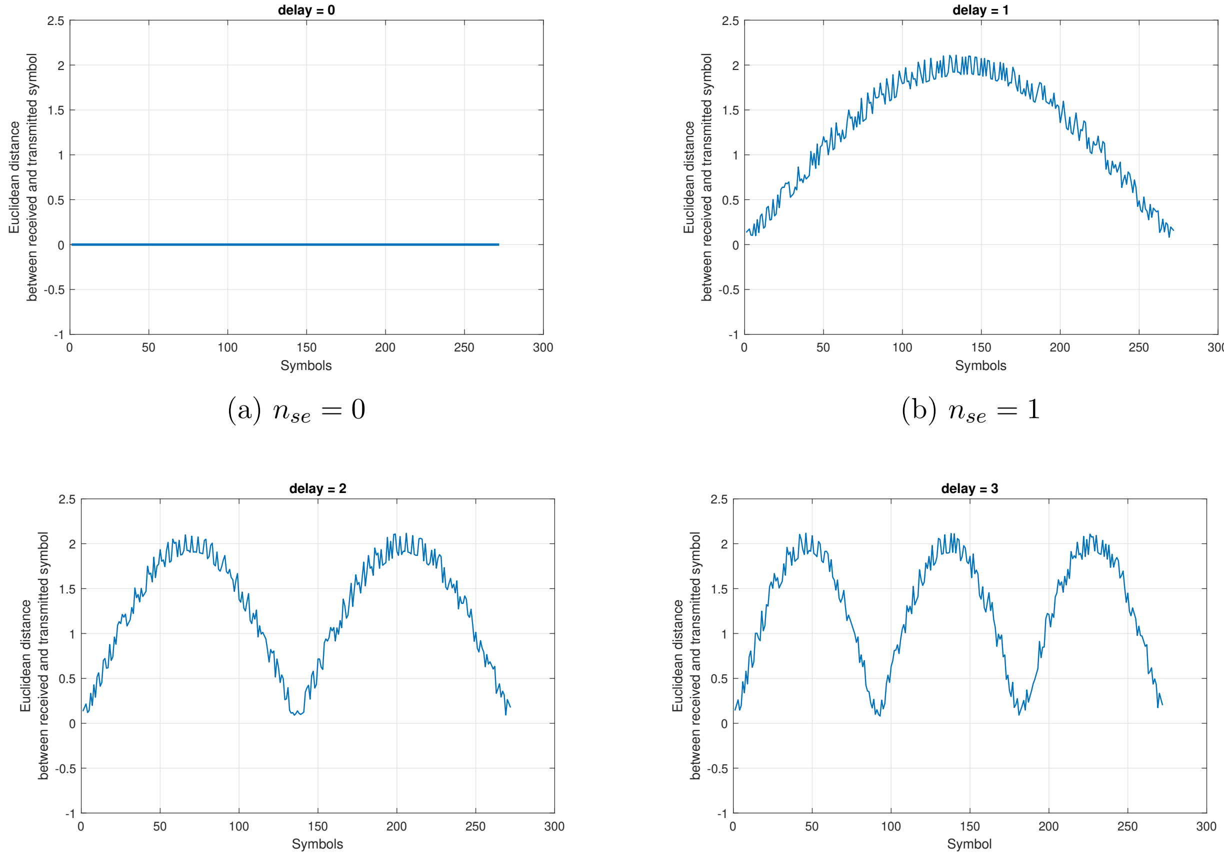
$$x \in \{0, 1, 2, ..., N-1\}, \quad m \in \{0, 1, 2, ...\}$$

For instance, in the case that $n_{se} = 1$, the frequency components which have a phase change less than $\frac{\pi}{4}$ are: $x < \frac{N}{8}$ and $x > \frac{7N}{8}$.

For $n_{se} = 2$, the components are $x < \frac{N}{16}$, $\frac{7N}{16} < x < \frac{9N}{16}$, and $x > \frac{15N}{16}$. For $n_{se} \geq 2$ we generally obtain

$$x \in \left\{0 \leq x < \frac{N}{8n_{se}}\right\} \cup \bigcup_{k=1}^{n_{se}-1} \left\{\frac{N}{n_{se}}\left(k - \frac{1}{8}\right) < x < \frac{N}{n_{se}}\left(k + \frac{1}{8}\right)\right\} \cup \left\{\frac{N}{n_{se}}\left(n_{se} - \frac{1}{8}\right) < x \leq N-1\right\} = \mathbf{X} \tag{6}$$

This behavior is also shown in figure 3 where the Euclidean distance between the received and transmitted symbols is plotted for different delays.



(a) $n_{se} = 0$

(b) $n_{se} = 1$

(c) $n_{se} = 2$

(d) $n_{se} = 3$

Figure 3: Euclidean distance between the transmitted and received symbols for different synchronization error values.

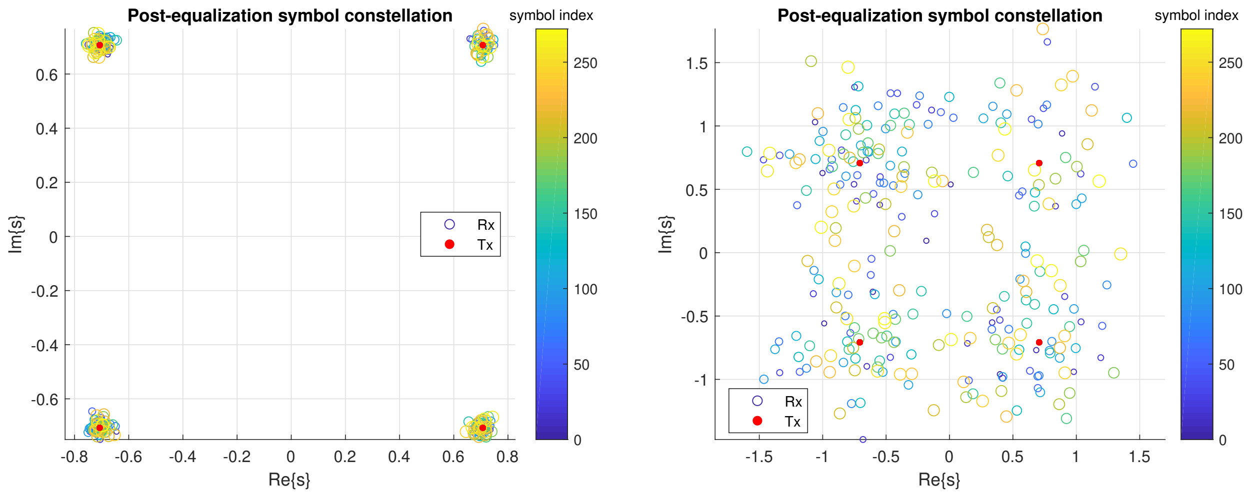
When $n_{se} = \pm 1$, the values for EVM and BER are approximately 1.4 and 0.5. By increasing the synchronization error, these values do not change considerably and stay the same. The reason is because of the periodic structure that can also be seen in figure 3. E.g, if we sum $\mathbf{X}$ we have from (6):

$$\sum \mathbf{X} = \sum_{k=1}^{n_{se}} \frac{N}{n_{se}} \Big(k - k + \frac{1}{4} \Big) = \frac{N}{4}, \quad \forall n_{se} \geq 1 \tag{7}$$

Synchronization error also causes a large increase in the EVM and BER because most of frequency components will have a phase shift which is more than $\frac{\pi}{4}$.

Task 1.e)

By adding Gaussian noise to the signal, the symbols start to move in random directions from their original point. Increasing the power of the noise will result in reducing the SNR and makes the received symbols spreading more in the complex constellation plane. The effect of Gaussian noise and increasing its power can be seen in figure 8.

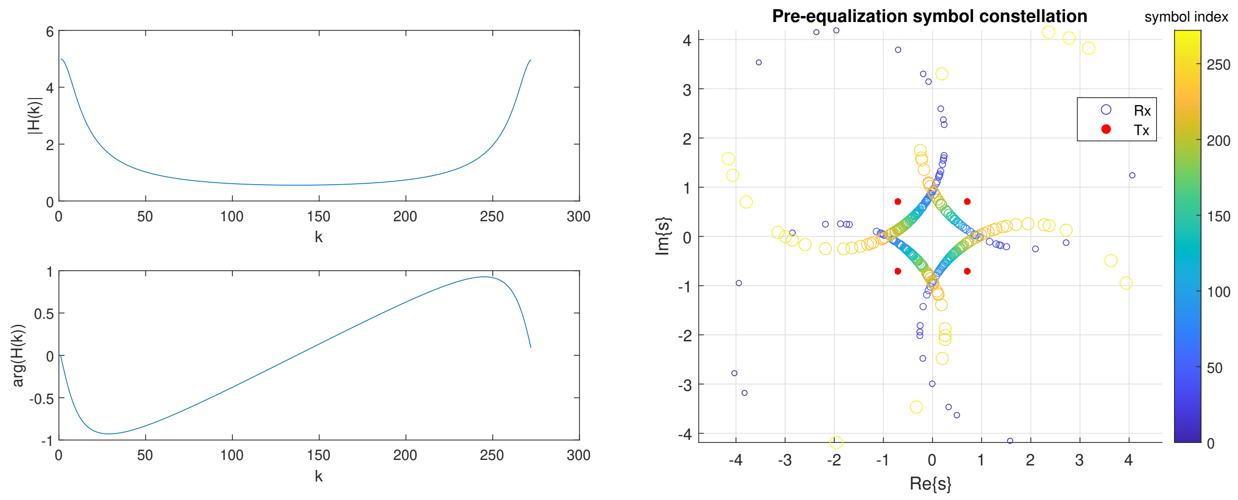


(a) SNR = 30dB (b) SNR = 5dB

Figure 4: Received symbols after passing in a complex additive white Gaussian noise channel

Task 2.a)

When channel model is changed to $h_4(n)$, the system does have dynamics property, which means that each symbol has individual channel gain, $r(k) = H(k) \cdot s(k)$. As $H(1) = 5$, one symbol is found at $5 \cdot \sqrt{\frac{1}{2}}(\pm 1 \pm j)$ (In figure 5b at $5 \cdot \sqrt{\frac{1}{2}}(-1 + j)$). The symbol $r(k)$ then rotates in clockwise direction with decreasing amplitude until k reaches 136. After that, the amplitude increase again while rotation direction do not change. $H(k)$ and pre-equalization symbol constellation can be seen in figure 5a and 5b, separately.

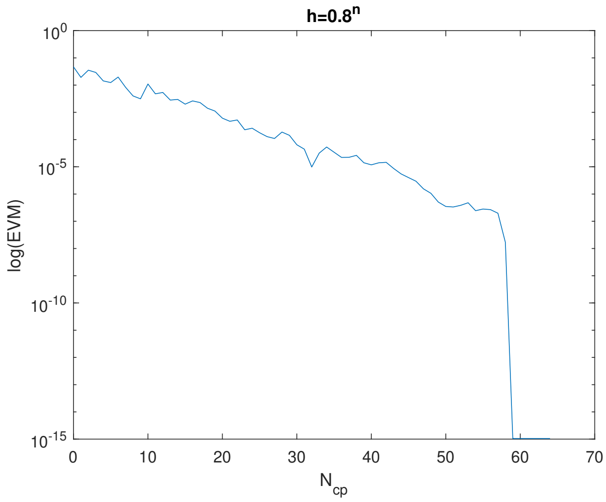


(a) H(k) (b)

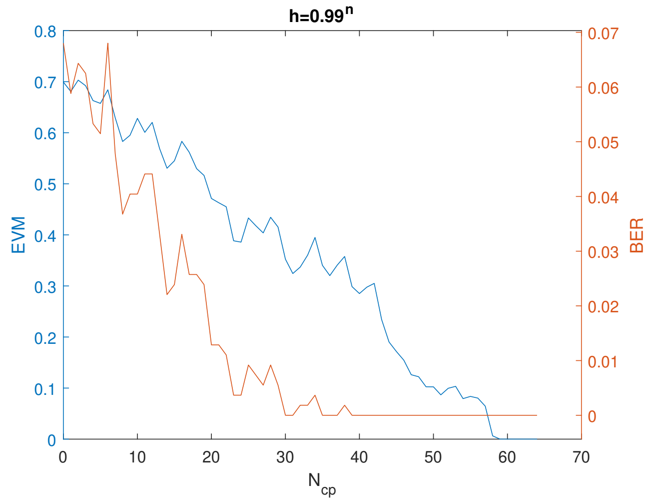
Figure 5

Task 2.b)

EVM$= 1.05 \cdot 10^{-15}$ and BER$= 0$ for this setup. Varying the cyclic prefix length for this channel yields the result in figure 6a with regards to EVM. Clearly, for $N_{cp} \geq 59$ the EVM vanishes. An explanation for 'why' was presented with regards to task 1.b). BER however is 0 independent of N_{cp} for this channel.



(a) EVM plotted vs N_{cp} for $h = 0.99^n$.

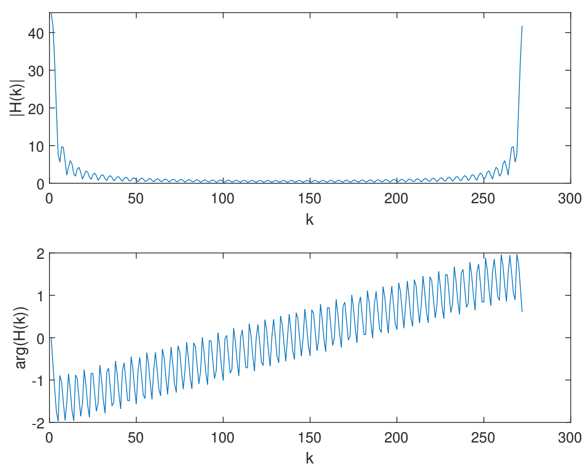


(b) EVM and BER plotted vs N_{cp} for $h = 0.99^n$.

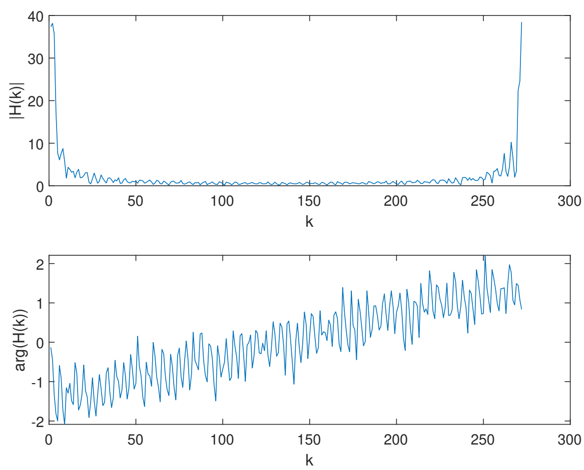
Figure 6

Task 2.c)

EVM and BER simulated for a range of different cyclic prefix lengths are plotted in figure 6b.

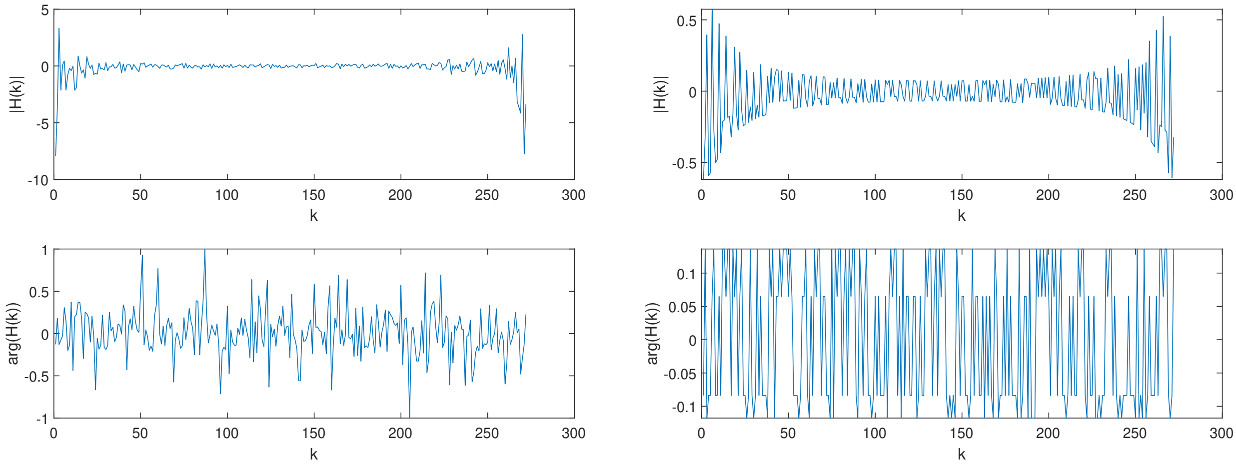


(a) $H(k)$ for $N_{cp} = 60$



(b) Pilot-generated $H(k)$ for $N_{cp} = 0$. Real $h(n) = 0.99^n$.

Figure 7



(a) $H(k)$ for $N_{cp} = 60$ (b) Difference between real $H(k)$ a

Figure 8

Task 3.a)

After equalization, received symbols can be expressed as $\hat{s}(k) = r(k)/H_1(k) = r(k)$ for known-channel $h_1(n)$. The synchronization error corresponds to a delay/early reception of the received time domain signal which in frequency domain will cause a phase shift in the frequency components. We obtain shifted symbols $\hat{s}(k) = r(k) \cdot e^{-j2\pi n_{se}\frac{f}{f_s}}$ which $\frac{f}{f_s} \in \{\frac{0}{N}, \frac{1}{N}, \frac{2}{N}, ..., \frac{N-1}{N}\}$, so errors exist if phase shift is larger than $\frac{\pi}{4}$. However, in the unknown channel, the equalized symbol is given by $\hat{s}(k) = r(k)/(r_p(k)/x_p(k)) = x_p(k) \cdot r(k)/r_p(k)$ which $r_p(k)$ and $x_p(k)$ are received and transmitted pilot symbol. As $r(k)$ and $r_p(k)$ have same phase shift component $e^{-j2\pi n_{se}\frac{f}{f_s}}$, the equalized symbol $\hat{s}(k)$ is not affected by the synchronization error.

Task 3.c)

The cyclic prefix need to compensate for the length M of h and so we have $N_{\text{cp}} = M - 1$. $N_{\text{cp}} \geq M - 1$ will assure minimum EVM with respect to finite impulse response induced errors. As channel h_5 is selected, e.g.$h_5(0) = 0.5$ and $h_5(8) = 0.5$, we set $N_{cp} = 8$ for this length $M = 9$.

Impulse response of this multi path channel can be expressed as $h_5(n) = 0.5 \cdot h_1(n) + 0.5 \cdot h_1(n-8)$ which $h_1(n)$ is ideal channel. We can also simply obtain that:

$$H_5(k) = 0.5H_1(k) + 0.5H_1(k) \cdot e^{-j2\pi k\frac{8}{N}} \quad \forall k \in [0, N-1] \tag{8}$$

We can notice that $H_5(k) = 0$ when $2\pi k\frac{8}{N} = m\pi$ which $m \in \{1, 3, 5, ...\}$ and $N = 272$. By solving the above equation, we obtain $k = 17(2a - 1), \forall a \in [1, 8]$. We can also observe the same result in figure 9. Since the received symbol is $r(k) = H(k) \cdot z(k)$, we lose the symbol when $\mid H(k) \mid= 0$ and can not recover it after equalization. This is the reason that we always have errors even in noiseless and sync error free channel.

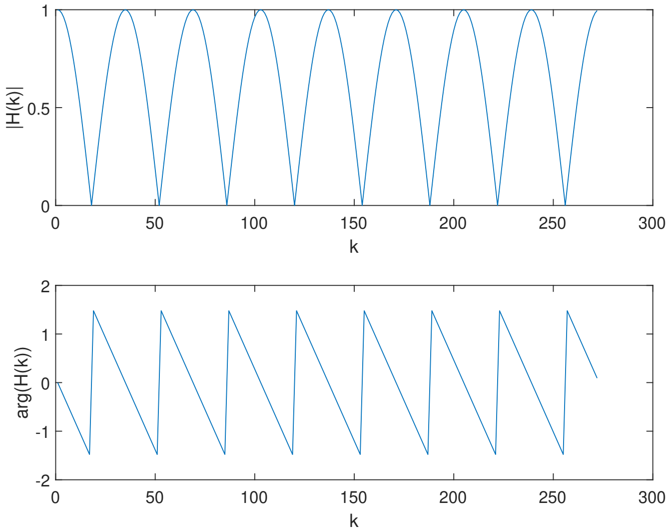


Figure 9

When the second nonzero coefficient of h_5 is set to 0.4, the channel gain is changed to:

$$H_5'(k) = 0.5H_1(k) + 0.4H_1(k) \cdot e^{-j2\pi k \frac{8}{N}} \quad \forall k \in [0, N-1] \tag{9}$$

We notice that there are no zeros value for $\mid H_5'(k) \mid$ because of the strength difference in these two paths. So BER becomes zero when no noise and sync error exist in this channel.