Applied Signal Processing SSY130
Tutorial 2

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Problems

1. The static gain of a filter, also called the DC-gain, is the amplitude of the frequency response at frequency 0, i.e. $H(0)$. What is the static gain for the following types of filters:
   (a) A general FIR filter $h(n)$ which is non-zero for $n = 0, 1, \ldots, M$?
   (b) An anti-symmetric filter where $h(n) = -h(M - n)$ and M is an odd integer.
   (c) An anti-symmetric filter where $h(n) = -h(M - n)$ and M is an even integer.
   (d) An IIR filter with the Z-transform
   $$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-1}}$$

2. A non-causal FIR filter has a non-zero impulse response for time indices $n = -K, -K + 1 \ldots, 0, \ldots, K$.
   (a) How many samples should the impulse response be delayed in order for the new filter to become causal?
   (b) The original filter has a frequency function $H(\omega)$. What is the frequency response of the resulting causal filter?

3. A FIR filter $h(n)$ of low pass type has a cut-off frequency of $0.1\omega_s$. A new filter is created by changing the sign of every second filter coefficient
   $$h_{\text{new}}(n) = -1^n h(n), \quad n = 0, \ldots, M$$
   What type of filter is the new filter? Sketch the magnitude. What is the cut-off frequency? (Hint: Investigate $H(\omega + \omega_s/2)$ and note that $e^{j\pi k} = (-1)^k$.)

4. Use the window design method to design a causal low-pass FIR filter with 5 filter coefficients ($M = 4$) and a cut off frequency at 6 kHz. The sample interval is 50 μs. Use a Hamming window:
   $$w_n = 0.54 + 0.46 \cos(n2\pi/M)$$

5. A first order IIR filter can be described by the following equation

$$y(n) = a_1 y(n-1) + b_0 x(n) + b_1 x(n-1)$$

   (a) What is the frequency function $H(\omega)$ for this filter?

   (b) For what values of a_0, b_0, b_1 is this filter asymptotically stable?

6. A second order Butterworth filter has the Laplace transform

$$H_{BW}(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$$

and the frequency function

$$H_{BW}(\Omega) = \frac{1}{1 - \Omega^2 + \sqrt{2}j\Omega}$$

   (a) The magnitude of the frequency function is a monotonically decreasing function with increasing frequency. At what frequency (the cut-off frequency) has the magnitude reached $1/\sqrt{2} \approx -3$dB?

   (b) What is the cut-off frequency for the following frequency scaled filter

$$H_{BW}(s) = \frac{1}{\left(\frac{s}{\omega_c}\right)^2 + \sqrt{2}\left(\frac{s}{\omega_c}\right) + 1}$$

   (c) Design a second order discrete time IIR filter of Butterworth type with - 3dB cut off frequency at 5 kHz and the sample rate of the filter is 20 kHz. Follow the bilinear-transform design method:

      i. Derive the cut-off frequency in unit radians/sample, i.e. normalized with respect to the sampling frequency.

      ii. Translate the discrete time cut-off frequency specifications to continuous time, $\Omega = 2\tan(\omega/2)$ where ω where ω has unit rad/sample.

      iii. Design a continuous time filter with the correct cut off frequency.

      iv. Use the bilinear transform to convert the filter to a discrete time filter.

$$s \rightarrow \frac{2(1 - z^{-1})}{1 + z^{-1}}$$

      What are the coefficients of the filter? What order does the filter have? Is the filter stable?