Lecture 1b

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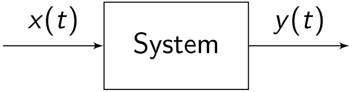
Signals and Systems



$x(t)$ - input, $y(t)$ - output

The system is an operator which given x produces y.

Signals $y(t)$ and $x(t)$ could be real-valued or complex valued.

time (or space) variable t is always real-valued.

We often analyze signals and systems based on spectral properties.

The complex exponential $e^{j\alpha t}$ forms a basis for all solutions to linear differential equations.

Fourier Transform (FT) the right tool for analysis

$$X(\omega) \triangleq \mathrm{FT}[x(t)] \triangleq \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\, dt$$

$$x(t) = \mathrm{FT}^{-1}[X(\omega)] \triangleq \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\, d\omega$$

Signal Energy (Parseval's relation)

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 \, d\omega \tag{1}$$

The delta function $\delta(t)$

$$\int_{-\infty}^{\infty} \delta(t - t_0) f(t) \, dt = f(t_0)$$

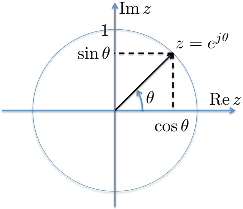
for "well behaved" $f(t)$.

Euler's relations:

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\cos\theta = \frac{1}{2}(e^{j\theta} + e^{-j\theta}) = \text{Re}(e^{j\theta})$$

$$\sin\theta = \frac{1}{2j}(e^{j\theta} - e^{-j\theta}) = \text{Im}(e^{j\theta})$$



FT pairs

$$x(t) \quad \Leftrightarrow \quad X(\omega)$$

Time delay

$$x(t - t_0) \quad \Leftrightarrow \quad e^{-j\omega t_0} X(\omega)$$

Modulation

$$e^{j\omega_0 t} x(t) \quad \Leftrightarrow \quad X(\omega - \omega_0)$$

Constant

$$1 \quad \Leftrightarrow \quad 2\pi\delta(\omega)$$

Delta function

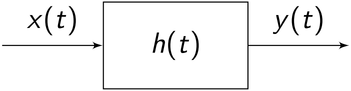
$$\delta(t) \quad \Leftrightarrow \quad 1$$

Convolution

$$\int_{-\infty}^{\infty} h(\tau)x(t - \tau)\, d\tau \quad \Leftrightarrow \quad H(\omega)X(\omega)$$

Linear Systems

Convolution is linear filtering



$x(t)$ - input,
$y(t)$ - output,
$h(n)$ - impulse response

$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)\,d\tau$$

$$\Leftrightarrow$$

$$Y(\omega) = H(\omega)X(\omega)$$

Causal system if	$h(t) = 0$ for $t < 0$
Anti-causal if	$h(t) = 0$ for $t > 0$
Non-causal	otherwise

Linear and Time Invariant (LTI):
input $\alpha x(t - t_0)$ yields output $\alpha y(t - t_0)$

Complex exponential input

Assume $x(t) = e^{j\omega_0 t}$, then $X(\omega) = \mathrm{FT}[x(t)] = 2\pi\delta(\omega - \omega_0)$ and the system output is

$$Y(\omega) = H(\omega)X(\omega) = H(\omega)2\pi\delta(\omega - \omega_0) \quad \Rightarrow$$

$$y(t) = \mathrm{FT}^{-1}[Y(\omega)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)2\pi\delta(\omega - \omega_0)e^{j\omega t}\, d\omega =$$

$$= H(\omega_0)e^{j\omega_0 t}$$

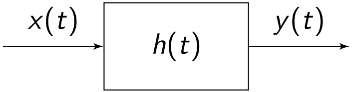
We can define

Amplitude function:	$A(\omega) \triangleq \lvert H(\omega)\rvert$
Phase function:	$\phi(\omega) \triangleq \angle H(\omega)$

which gives

$$y(t) = A(\omega_0)e^{j(\omega_0 t + \phi(\omega_0))}$$

Exponential impulse response



Assume $x(t) = e^{j\omega_0 t}$ and

$$h(t) = \begin{cases} e^{-\alpha t} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Then

$$\begin{aligned} H(\omega) &= \int_{-\infty}^{\infty} h(t) e^{-j\omega t}\, dt \\ &= \int_{0}^{\infty} e^{-(\alpha + j\omega)t}\, dt = \left[\frac{-e^{-(\alpha+j\omega)t}}{\alpha + j\omega} \right]_0^{\infty} = \frac{1}{\alpha + j\omega} \end{aligned}$$

and hence

$$y(t) = H(\omega_0) e^{j\omega_0 t} = \frac{1}{\alpha + j\omega_0} e^{j\omega_0 t}$$