Part B

1

Since the sampling frequency, after 8 times upsampling, is $16kHz$, before upsampling, the sampling frequency has been $\frac{16kHz}{8} = 2kHz$. Therefore, in baseband, the signal should occupy the maximum frequency of $-1kHz$ to $1kHz$ and the bandwidth would be $2kHz$.

Modulating the signal with $4kHz$ carrier frequency will move the center baseband frequency to $4kHz$, therefore, the signal will occupy the spectrum of $3kHz$ to $5kHz$.

2

According to the answer to the first question, the baseband signal should be band-limited between $-1kHz$ to $1kHz$ in order to avoid aliasing during downsampling.

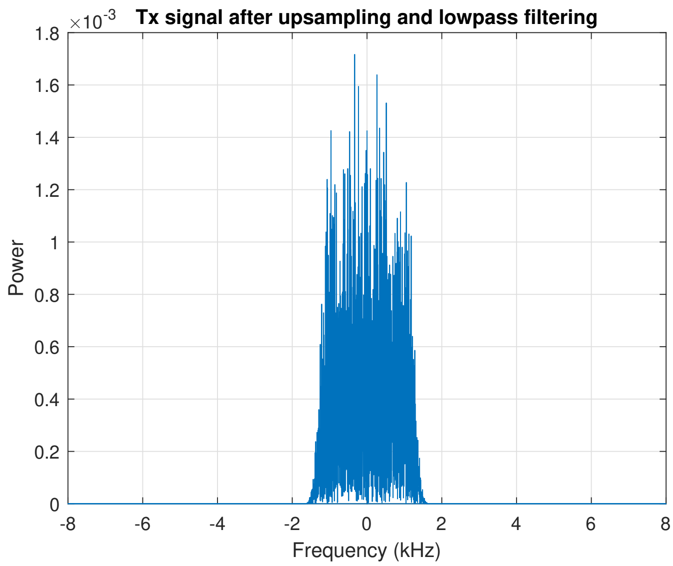

Figure 1: Tx signal after 8 times upsamling and lowpass filtering.

On the other hand, it can be seen in figure 1 that the signal is not completely limited in the desired spectrum to avoid aliasing. Hence the nonzero EVM. The reason for this over-spill in frequency content is accredited to the non-ideal lowpass filtering performed in the interpolation process. After upsampling, we have $L - 1$ extra copies of the signal which lie adjacent to the original and when a filter with finite transition-band and non-zero stop-band is applied, a portion of the power present in these copies will remain on the signal. The lowpass filtering in the decimation operation apply yet another non-ideal filter and thus, at the subsequent downsampling we suffer inevitable aliasing. This means there is a significant difference between the constellation points of the transmitted symbols and the received symbols. Therefore, EVM cannot be zero.

3

Let us first define what a channel (H) represents. Here, we think that a channel can be defined as any part of the system which introduces distortion to our transmitted data.

- Propagation over physical channel

  The physical channel is definitely a part of H ,and can add random noise to our transmitted signal, which cannot be removed completely from the signal. Also, the response of the physical channel can vary for different frequency components and can change the amplitude/phase of the frequency components of the signal.

- Modulation/demodulation

  These two parts in the transmitter and receiver do not cause any distortion to our data because their functionality is reversible with respect to their input and output. Therefore, based on our definition of channel, these parts cannot be part of H.

- Interpolation/decimation

  If the bandwidth of the signal satisfies the Nyquist criteria, then interpolation and decimation will not add any distortion to our signal but if this Nyquist criteria is not satisfied then aliasing will occur and the signal will be distorted, and a filter is used complimentary in each process, therefore they should be part of the channel in this case.

- Taking the real part of the signal

  Assuming that z is the transmitted signal, the real part of signal is $z_r = \frac{1}{2}(z + \overline{z})$. In the frequency domain, we are sure that $Z_r(w) = \frac{1}{2}(Z(w) + \overline{Z}(-w))$. We notice that Fourier transform (FT) of the real signal includes FT of original complex signal ($\frac{1}{2}Z(w)$). After demodulation, we can filter out the information related to $FT[z]$ in the base band, which is $\frac{1}{2}Z(w)$. So it contributes a constant factor of $\frac{1}{2}$ to H. More detailed explanation are in our response to question 4.

4

The Fourier transform (FT) of original signal $X(w)$ is shown in Figure 2(a). We are sure that FT of the upsampled and filtered signal $Y(w)$ is shown in Figure 2(b). After modulation, the FT of the signal is given by $Z(w) = Y(w + w_0)$, which includes a positive frequency shift w_0 and is show in Figure 2(c). If we take the real part of the modulated signal $z(t)$, we have $z_r(t) = \frac{1}{2}[z(t)+\overline{z}(t)]$ in time domain and $Z_r(w) = \frac{1}{2}[Z(w)+\overline{Z}(-w)]$ in frequency domain. $Z(w)$ and $\overline{Z}(-w)$ is shown in Figure 2(c) and 2(d), separately. One important issue is that $Z(w)$ and $\overline{Z}(-w)$ can not be overlapped, otherwise taking the real part of the signal will distort original signal. We should avoid this condition to protect original signal. So we have requirement to safely discard imaginary part of the signal: (1) $w_{s_2} - w_0 - B/2 \geq w_0 + B/2 \Rightarrow w_{s_2} \geq 2w_0 + B$ or (2) $w_{s_2} - w_0 + B/2 \leq w_0 - B/2 \Rightarrow w_{s_2} \leq 2w_0 - B$. Once (1) or (2) is achieved, there will not be any distortion due to taking the real part of the signal. It means that we can recover the signal in the receiver side based on demodulating and filtering out the signal shown in Figure 2(c). Since $Z_i(w) = \frac{1}{2}[Z(w) - \overline{Z}(-w)]$, We have the same analysis for taking the imaginary part of the signal. In our system setup, $w_{s_2} = 16KHz, w_0 = 4KHz,\ B = 2KHz \Rightarrow w_{s_2} \geq 2w_0 + B$, which meets requirement (1). Thus we can safely have chosen to simply discard the imaginary or real part of the signal.

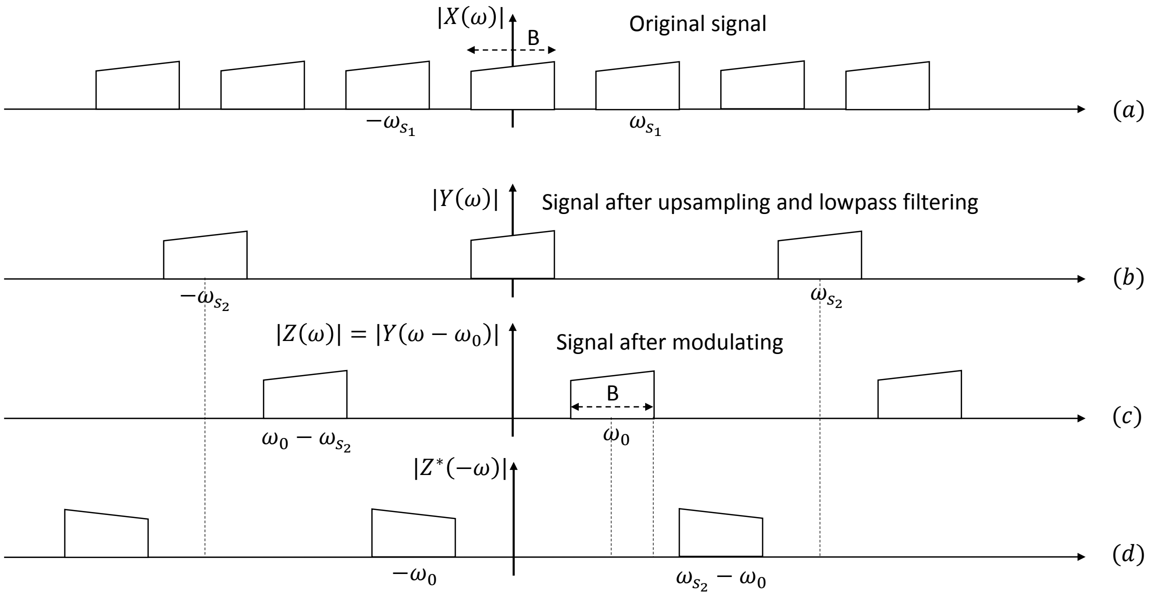

Figure 2: Signal spectrum

5

Let's give separate analysis about the importance of the lowpass filter properties in interpolation and decimation stages.

- Interpolation stage

  Passband ripple and phase linearity are less important in this process, because we can equalize these frequency responses using pilot symbols. However, both stopband attenuation and transition bandwidth are important. We should have large attenuation and narrow transition bandwidth to ensure that most of the signal energy is confined in frequency $|w| \leq w_{s_1}/2$, where w_{s_1} is the sampling frequency before upsampling. Therefore, it will not cause any distortion if we take the real or imaginary part of the signal after modulation. Assuming that we have signal energy leaking outside signal bandwidth B, signal spectrum $Z(w)$ and $\overline{Z}(-w)$ shown in Figure 2, it will overlap and induce distortions to the signal, which we should avoid by selecting proper stopband attenuation and transition bandwidth.

- Decimation stage

  Just same as in the interpolation stage, passband ripple and phase linearity are less important. In the decimation process, we also require a low-pass filter to remove signal energy with frequency $|w| \geq w_{s_1}/2$. If we take the real part of the signal, the Fourier transform of this signal becomes $Z_r(w) = \frac{1}{2}[Z(w) + \overline{Z}(-w)]$. After demodulation, we can recover the signal at the receiver side by filtering out $\frac{1}{2}Z(w)$ and have the inverse Fourier transform. We notice that there is a "guard band" between $Z(w)$ and $\overline{Z}(-w)$. In this condition, the transition band width can be wider, but stop band attenuation should definitely be large enough to avoid distortion.

7

To avoid using the division operation that requires complex computing, we can estimate the channel as $\hat{H} = \overline{T} \cdot R$. In an ideal system, we assume that frequency response of the channel is given by $H = \frac{R}{T} = \frac{\overline{T} \cdot R}{\overline{T} \cdot T} = \frac{\overline{T} \cdot R}{|T|^2}$. Assuming that the generated QPSK signal is $T \in \{\sqrt{1/2}(1 + 1j), \sqrt{1/2}(1 - 1j), \sqrt{1/2}(-1 + 1j), \sqrt{1/2}(-1 - 1j)\}$, then we have $|T|^2 = 1$. Therefore, $H = \overline{T} \cdot R = \hat{H}$, which means that H and $\hat{H}$ have the same magnitude and phase.

8

The equalized symbol can be expressed as $R_{eq} = R \cdot \overline{\hat{H}} = R \cdot \overline{\overline{T} \cdot R} = R \cdot \overline{R} \cdot T = |R|^2 \cdot T$. It shows that the equalized signal R_{eq} has exactly same phase as transmitted signal, T, because $|R|^2$ is a real value that only contributes to magnitude response. Therefore, we can conclude that the equalization process is correct for the phase of R_{eq}.

9

We checked the transmission of high and low amplitude of the signal by increasing (+) and decreasing (-) the volume of the speakers. The result for high amplitude signal is shown in figure 3 and low amplitude signal in figure 4, respectively.

As it can be seen in these figures, when the amplitude is high, the effect of noise is less on the signal and the EVM value is small. On the other hand, when the signal amplitude is small, the effect of noise will dominate the system and EVM value will increase significantly. Therefore we can say, the lower the amplitude, the lower the SNR, the higher the noise and the larger the EVM values, and vice-versa. That is, equalization using low SNR pilot allows risk of large error and symbol spread (EVM)

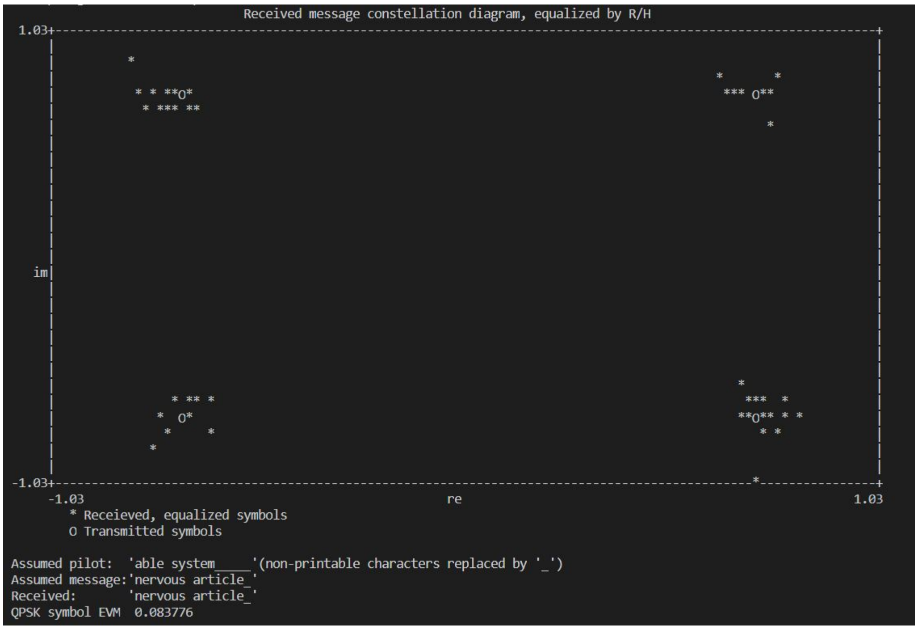

Figure 3: High amlitude signal

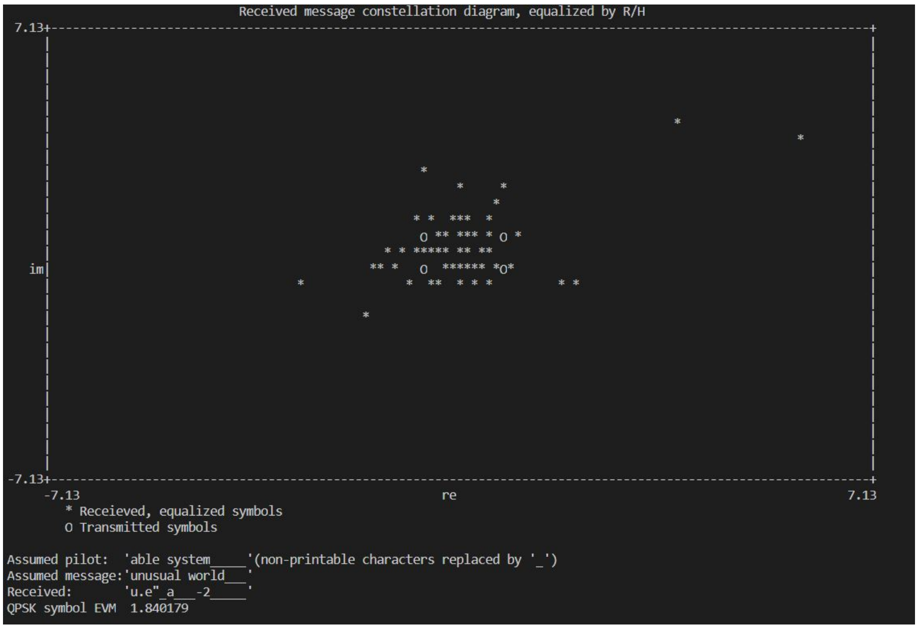

Figure 4: Low amplitude signal

10

Changing the position of the speaker with respect to the microphone will cause a delay for the channel that are seen by the pilot signal, or training symbols, and the payload. This effect of delay cannot be removed by equalizing the payload by the channel estimate that has been calculated for the pilot. Therefore, since the delay in time domain corresponds to a rotation in the frequency domain, we expect that the complex constellation will rotate counter clockwise or clockwise depending on the sign of the relative radial velocity. This effect can be seen in figures 5 and 6.

Figure 5: Moving the speaker toward the microphone

Figure 6: Moving the speaker away from the microphone

We calculate the speed of our movement for the case in figure 6. Since this is an acoustic set-up, based on sound waves, we will have the Doppler effect on our signals. The Doppler effect creates a shift in time and phase. Usually, this can be corrected by resampling the received signal and performing pitch conversion, as well. The speed of sound is taken to be approximately 340m/s. The movement of the speaker at about 2-5cm/s, helps us to monitor the effect gradually. Usually, allowable distance using QPSK modulation in an acoustic set-up should not exceed 3m range, else EVM and bit errors start getting larger in value. First, we considered an approximate average point for a group of received symbols in the bottom right of figure 6. Then we calculated the phase rotation for that average point. In this case, the received angle for the average point was -53 degree. This will cause the phase rotation of $-53 - (-45) = -8$ degree.

The phase rotation in this case can be calculated based on the average of the carrier frequencies (f_c) and the delay which happens because of the movement (ΔT).

$$\Delta\phi = 2\pi f_c \Delta T$$
$$\Delta T = \frac{-v}{v_{\text{sound}}} N \Delta t$$

where Δt is the sampling intervals ($\frac{1}{16kHz}$) and N is the number of upsampled samples for the pilot including its cyclic prefix ($8 * (64 + 32) = 768$). This implies that the message symbols are separated by N samples from the respective pilot symbols they are equalized against.

Putting these numbers in the above equations will give us the movement speed of the speaker close to $4cm/sec$.

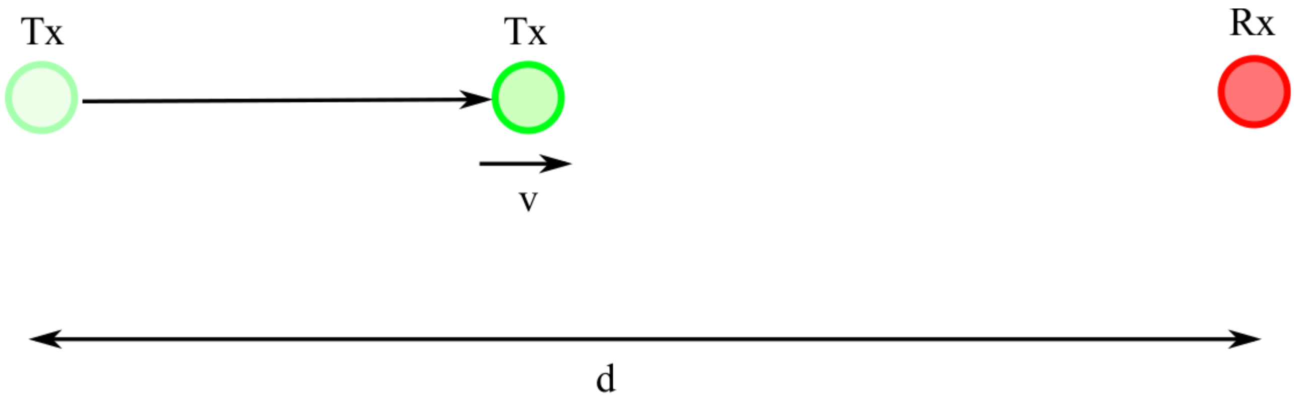

Figure 7: The time transmitted symbol s reaches the receiver is: $T(s, v) = t_0 + s\Delta t + (d - vs\Delta t)/v_{\text{sound}}$ where t_0 is the time at which the transmitter is turned on, Δt is the symbol time spacing (time between emitted symbols), v is the radial relative velocity between transmitter and receiver and d the initial distance in between at time $t = t_0$. The doppler shift due to the movement of the Transmitter contributes to an unanticipated time delay that varies with symbol according to the above equation. The difference between the actual reception time of the symbol s and the expected time is $T(s, v) - T(s, v = 0) = \frac{-v}{v_{\text{sound}}} s\Delta t = \Delta T$.

In order to calculate the highest possible speed, would restrict ourselves to the maximum frequency component in the spectrum of the signal which in case we assume this is $5kHz$. The speed of the movement can be calculated, using the constellation diagram by extracting the time delay, ΔT, by doing a cross correlation between transmitted and received symbols. The maximum possible phase rotation for this frequency component should be less than $\frac{\pi}{4}$. Using these numbers again in our equations will given to us $35cm/sec$.