Applied Signal Processing SSY130
Tutorial 1

Tomas McKelvey
Dept. of Signals and Systems
Chalmers University of Technology
Gothenburg, Sweden
mckelvey@chalmers.se

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Problems

1. Analyze the block diagrams of Figure 1 and derive the relations between $y(n)$ and $x(n)$. The z^{-1}-blocks are unit time delays. We assume $|d_1| < 1$ to ensure that the circuit is stable.

2. A periodic signal $x(n)$ with period N is applied to a causal linear time invariant (LTI) filter with impulse response $h(n)$ generating the output $y(n)$. Write out the expression (the convolution) for the output $y(n)$. Is the output a periodic signal? What is the period?

3. Show that the convolution between two signals $x_1(n)$ and $x_2(n)$ is commutative i.e.

$$x_1(n) * x_2(n) = x_2(n) * x_1(n) \tag{1}$$

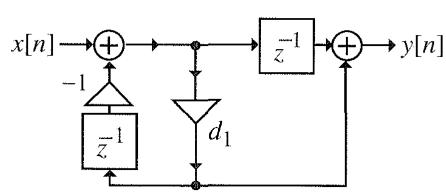

Figure 1: Signal diagram

4. Let the step response $s(n)$ be the output of a discrete time LTI system where the input is a unit step

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases} \tag{2}$$

Derive an expression for the output of this system given an arbitrary input $x(n)$ in terms of $s(n)$. (Hint: How can the impulse response of the system be calculated from the step response?)

5. The output of a system is defined recursively according to

$$y(n) = \alpha y(n-1) + x(n) \tag{3}$$

What is the impulse response of the system?

6. Consider the following two coupled recursive equations

$$\begin{aligned} y_r(n) &= y_r(n-1)\cos\omega - y_i(n-1)\sin\omega \\ y_i(n) &= y_r(n-1)\sin\omega + y_i(n-1)\cos\omega \end{aligned} \tag{4}$$

with the initial conditions $y_r(0) = 1$ and $y_i(0) = 0$. Show that the solution is $y_r(n) = \cos n\omega$ and $y_i(n) = \sin n\omega$. Hence the two equations forms an oscillator with frequency ω. (Hint: $\cos n\omega = \mathrm{Re}\{e^{j\omega n}\}$)

7. Which of the following impulse responses are causal?

(a)

$$h(n) = \begin{cases} 1/n^2 & n \neq 0 \\ 1 & n = 0 \end{cases}$$

(b)

$$h(n) = \begin{cases} e^{\alpha n} & n > 0 \\ 0 & n \leq 0 \end{cases}$$

(c)

$$h(n) = \begin{cases} 1/M & -M < n < M \\ 0 & \text{otherwise} \end{cases}$$

(d)

$$h(n) = \begin{cases} 1/M & 0 < n < M+1 \\ 0 & \text{otherwise} \end{cases}$$

8. The Discrete Time Fourier Transform is defined by the relations where Δt is the sampling period ans $\omega_s = 2\pi/\Delta t$ is the sampling frequency in radians/seconds.

$$\begin{aligned} X(\omega) &= \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n\Delta t} \\ x(n) &= \frac{1}{\omega_s}\int_0^{\omega_s} X(\omega)e^{j\omega n\Delta t}\,d\omega \end{aligned}$$

(a) Show that for any signal $x(n)$ (where the DTFT exists) we have

$$X(\omega) = X(\omega + k\omega_s),\ k = 0, \pm 1, \pm 2, \ldots$$

The following table show some properties of the DTFT

Property	Signal	DTFT
	$g(n)$	$G(\omega)$
	$x(n)$	$X(\omega)$
linearity	$\alpha g(n) + \beta x(n)$	$\alpha G(\omega) + \beta X(\omega)$
time shifting	$g(n - n_0)$	$e^{-j\omega n_0 \Delta t} G(\omega)$
frequency shifting/modulation	$e^{j\omega_0 n \Delta t} g(n)$	$G(\omega - \omega_0)$
convolution	$g(n) * x(n)$	$G(\omega) X(\omega)$
frequency convolution	$g(n) x(n)$	$\frac{1}{\omega_s} \int_0^{\omega_s} G(\theta) X(\omega - \theta)\, d\theta$

Prove the results in the table.

(b) The following table show some symmetry relations for complex signals

Signal	DTFT
$x(n)$	$X(\omega)$
$x(-n)$	$X(-\omega)$
$x^*(-n)$	$X^*(\omega)$
$\mathrm{Re}\{x(n)\}$	$\frac{1}{2}(X(\omega) + X^*(-\omega))$
$j\,\mathrm{Im}\{x(n)\}$	$\frac{1}{2}(X(\omega) - X^*(-\omega))$

Show the results in the table. Note that $x^*(n)$ denotes the complex conjugate of $x(n)$ and $x(-n)$ implies the time reversal of the sequence $x(n)$.

(c) If we now assume $x(n)$ is a real signal. What symmetries does then $X(\omega)$ have? Use the table in (b) to derive them and visualize them graphically.

9. An ideal low pass filter with cut off frequency ω_c has a DTFT given by

$$H_{\mathrm{LP}}(\omega) = \begin{cases} 1 & |\omega| \le \omega_c \\ 0 & \omega_c < |\omega| \le \omega_s/2 \end{cases} \tag{5}$$

Derive the impulse resonse $h_{\mathrm{LP}}(n)$ of the filter. Is the filter causal? How long is the impulse response?

10. Consider a filter described by equation (3). Assume the input is given by $x = \cos(\frac{\pi}{4} n)$. What is the output $y(n)$?