SSY130 – Hand in 3

Target Tracking Using the Kalman Filter

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Task 1.

Given the state in the target motion model is given by:

$$s(t) = \begin{bmatrix} s_1(t) \\ s_2(t) \\ s_3(t) \\ s_4(t) \end{bmatrix} = \begin{bmatrix} x(t) \\ \dot{x}(t) \\ y(t) \\ \dot{y}(t) \end{bmatrix}$$

Apply the finite-difference approximation

$$\dot{x}(t)|_{t=kT} \approx \frac{(kT+T)-x(kT)}{T}$$

T: is sample time = 0.01s.

Our goal to run equation like:

$$s(k+1) = As(k) + w(k)$$

$$z(x) = C\, s(k) + \, y(k)$$

We can drive the discrete space formulation by solving finite-difference approximation in stat target model equations, so we get

$x(k+1) - x(k) = \, T\, v_x(k)$, $x(kT+T) - x(kT) = Tv_x(k)$ (1).

$v_x(kT+1) - v_x(k) = \, 0$, $v_y(k+1) - v_y(k) = 0$ (2).

From (1) we can find

$$s(k+1) = As(k) + w(x)$$

Where. $A \, = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}$, w(k)= w ~ N(0,Q)

From (2) we can find

$$z(x) = C\, s(k) + \, v(k)$$

Where. $A \, = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, v(k)= w ~ N(0,R)

C matrix give us the position of x(t) & y(t) using state-space variables, that measured the normal distribution with zero mean and the uncorrelated noise v(k) with covariance matrix R. we need to know that the noise w(k) affects the states of vx(k) and vy(k). Where Q is the variance of w(k) and R is the variance of v(k).

Task 2.

The goal from this experiment is filter the noise from measured position by apply a Kalman-Filter.

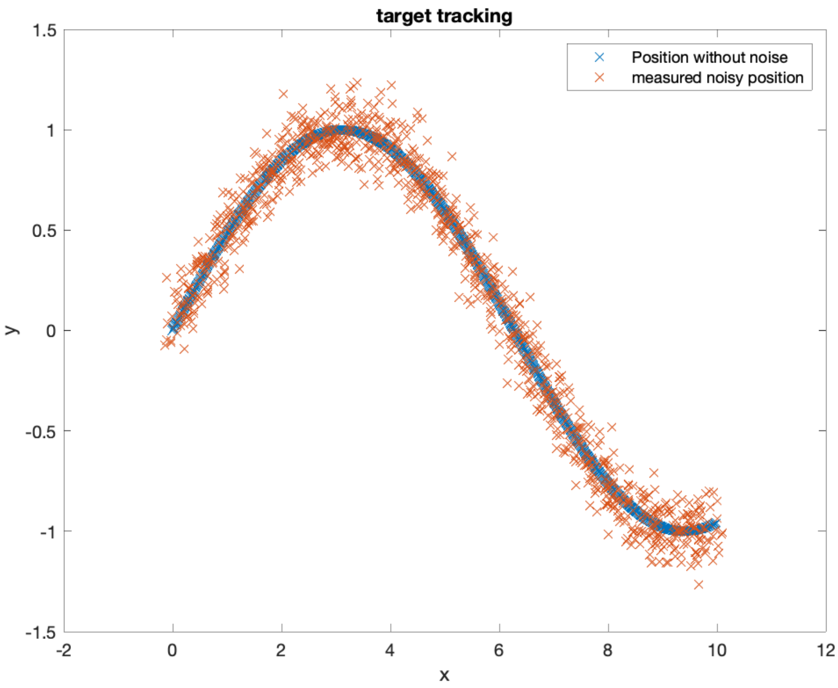


Figure 1. Measured vs Noise-free position

Task 3.

```
for t=1:N
            % Filter update based on measurement
            % Xfilt(:,t) = Xpred(:,t) + ...
            Xfilt(:,t) = Xpred(:,t) + P*C'*inv(C*P*C' + R)*(Y(:,t)
- C*Xpred(:,t));

            % Uncertainty update, from (11.17)
            Pplus = P - P*C'*inv(C*P*C' + R)*C*P;
            % Prediction, from (11.19)
            Xpred(:,t+1) = A * Xfilt(:,t);

            % From (11.20)
            P = A*Pplus*A' + Q;
        end
```

Task 4.

To apply Kalman we need matrix of P_0, Q and R. We have given that, the zero vector as initial state vector, and $P0 = 10^6 . I$ as initial state covariance matrix.

Let start with assuming that the measurement noise so that led us to matrix of R will be diagonal and each element corresponds to the variance of the noise concerning each measurement. We can estimate the covariance matrix of R by analyzing the statistic of error measurement depending on our knowledge of real position.

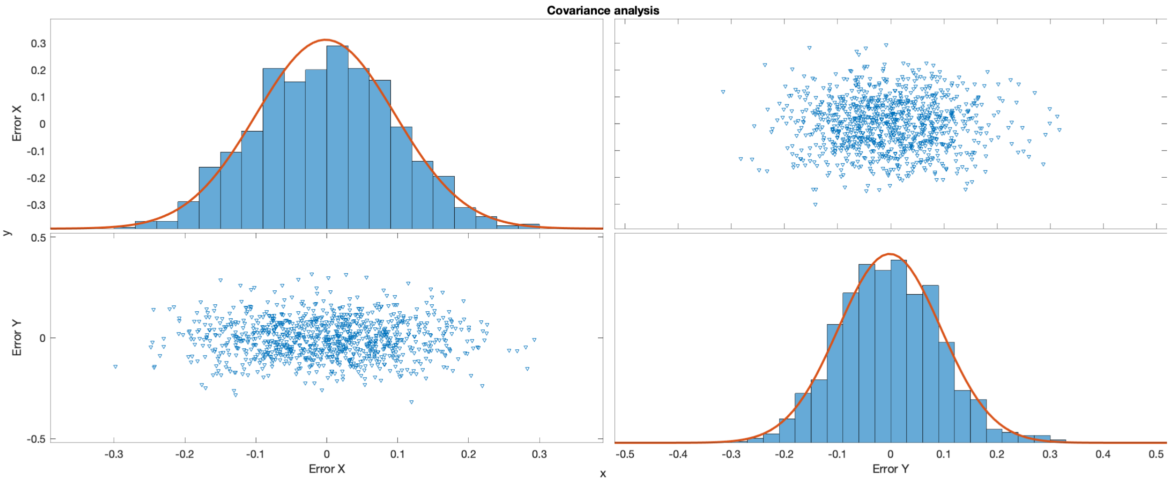


Figure 2. Covariance analysis to error of X&Y

Since N(0,R) we get:

$$R = \begin{bmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{bmatrix} = \begin{bmatrix} 9.7 * 10^{-3} & 0 \\ 0 & 9.7 * 10^{-3} \end{bmatrix}$$

However, Q matrix is relevant to the covariance of noise process. But in our case, we can't find any direct way to determine value of Q matrix. So, we have to do two assumption

1- The noise processes are affected the velocity.
2- The noise processes are uncorrelated.

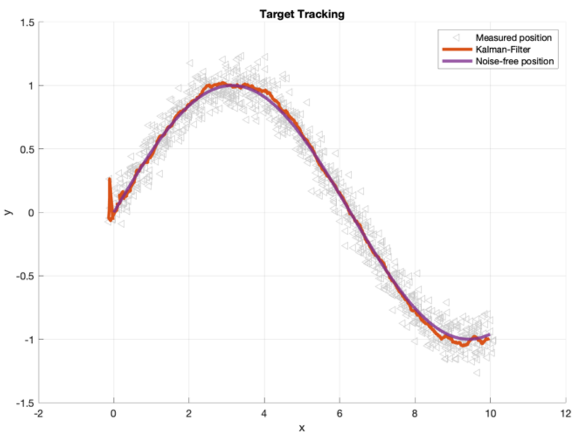
That will help us to define Q like:

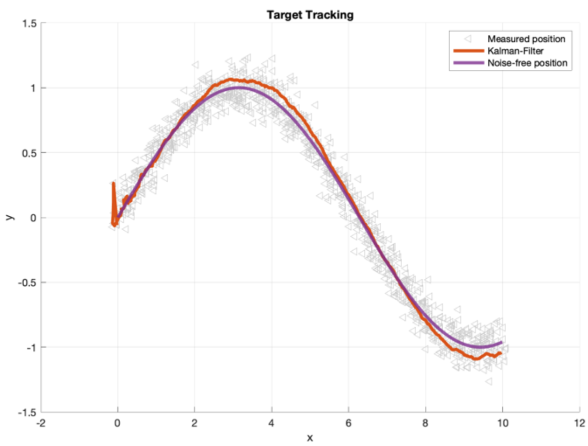
$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \sigma_{vx}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{vy}^2 \end{bmatrix} = \propto . \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where $\propto$ is changeable parameter which determine the performance of Kalman filter. We can see deferent plot of Kalman position and velocity tracking with deferent value of $\propto$.

1. $\propto = 10^{-2}$

2. $\propto = 10^{-3}$



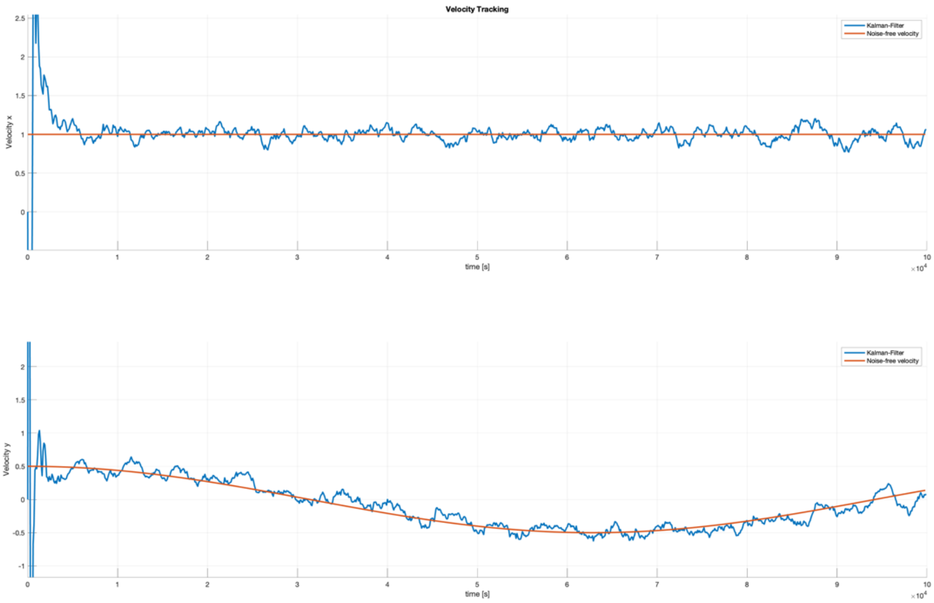


3. $\propto = 10^{-4}$ 4. $\propto = 10^{-5}$

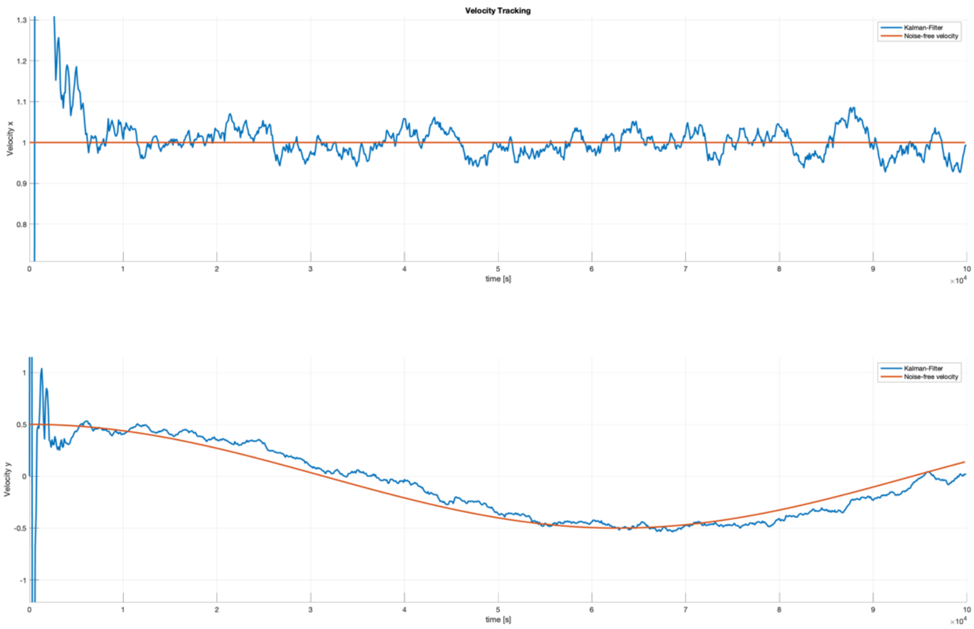
Figure. 3 position tracking for Kalman filter

From figure.3 we can see that, changing alpha has no such a huge effect on velocity variance. Conclusion is filter position has slower effect on velocity variance and little noise effect. We need to find balance between noise and speed response for the filter by choosing as much as possible good value to alpha.

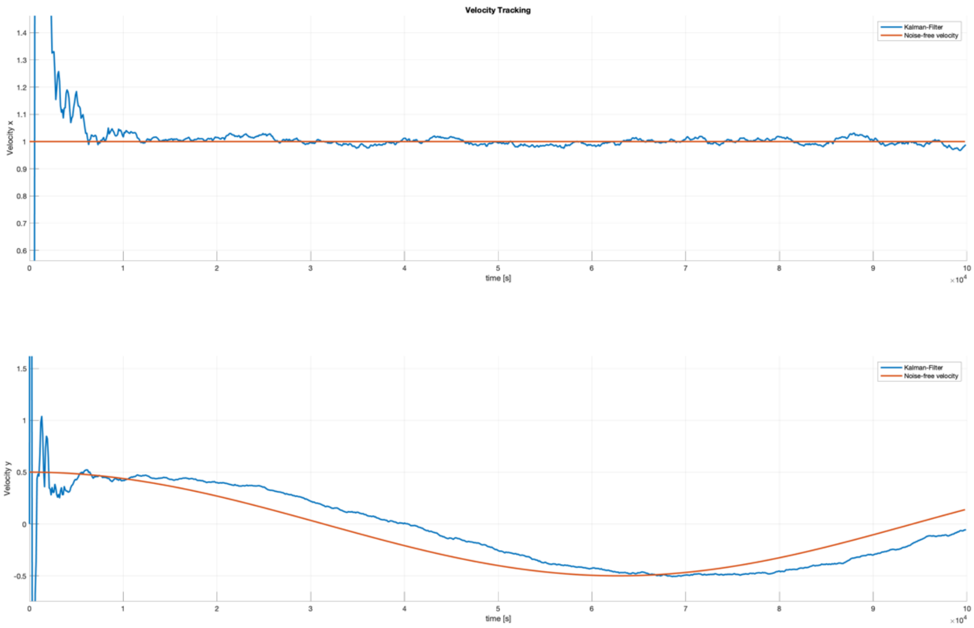
After many tests in our case alpha is 10^{-3}. The estimated velocity at alpha = 10^{-3} can be seen in figure below.



a) $\propto = 10^{-3}$



b) $\propto = 10^{-4}$



c) $\propto = 10^{-5}$

Figure 4. variance of velocity tracking with changing of $\propto$

If we tack the derivative of the noisy signal without using Kalman filter the result will be very bad.