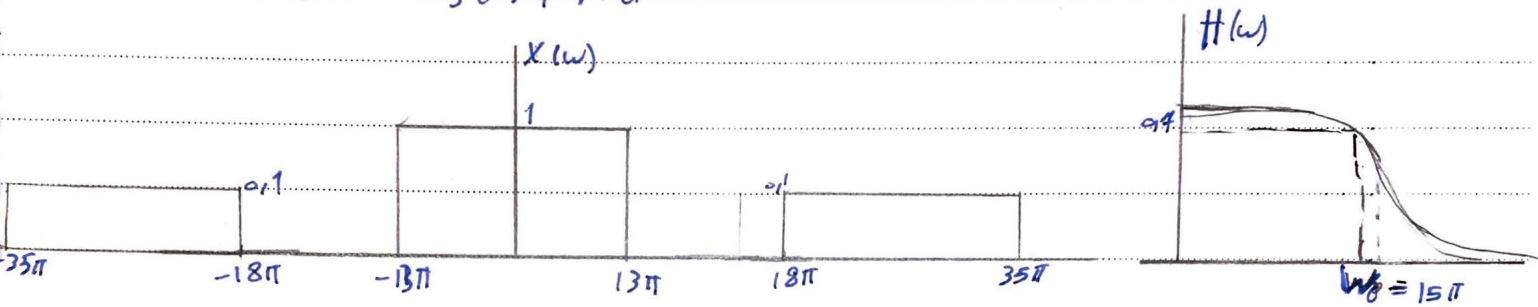


1] $H(\omega) = \frac{1}{1 + j\frac{\omega}{\omega_0}}$; $\omega_0 = 15\pi \times 10^3$

$x(t) = x_s(t) + n(t)$



The noise is separated from the useful signal $x_s(t)$ and the noise value is outside the filter range. Where the Nyquist frequency is

$\omega_s \geq 2\omega_0$

Minimum sampling frequency is

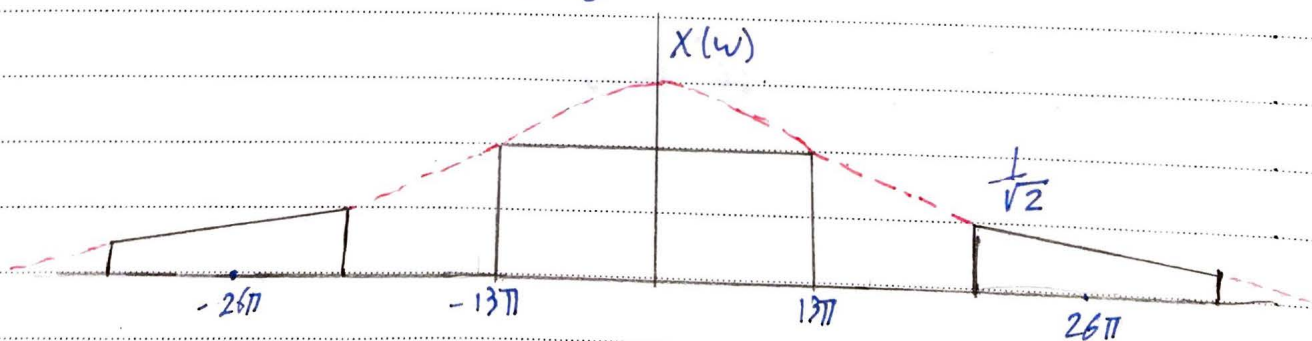
$\omega_s = 2 \times 15\pi \times 10^3 = 30\pi \times 10^3 \text{ rad/s}$

So the minimum sampling rate $R_s = f_s = 30\pi \times 10^3 \text{ rad/s}$

$x(\omega) = x_s(\omega) + N(\omega)$

$y(\omega) = H(\omega) \cdot x(\omega) = H(\omega) \cdot x_s(\omega) + H(\omega) \cdot N(\omega)$

$|H(\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}}$



$$\frac{1}{20} \text{DTFT}[H(\omega)N(\omega)] < \text{DTFT}[H(\omega)X_s(\omega)]$$

The Nyquist here will be.

$$\omega_s > 2\omega \quad ; \omega = 13\pi \times 10^3$$

Sampling here will be $2 \times 13\pi \times 10^3 = 26\pi \times 10^3 \text{ rad/s}$

$$H(\omega)N(\omega) = \frac{1}{20} ;$$

$$N(\omega) = 0.1, \quad H(\omega) = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}}$$

$$0.1 \cdot \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}} = \frac{1}{20} \Rightarrow \sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2} = 2$$

$$1 + \left(\frac{\omega}{\omega_0}\right)^2 = 4 \Rightarrow \left(\frac{\omega}{\omega_0}\right) = \sqrt{3}$$

$$\omega = \sqrt{3} \cdot \omega_0$$

$$\omega \approx 26\pi \times 10^3 \text{ rad/s}$$

New ω_s will be.

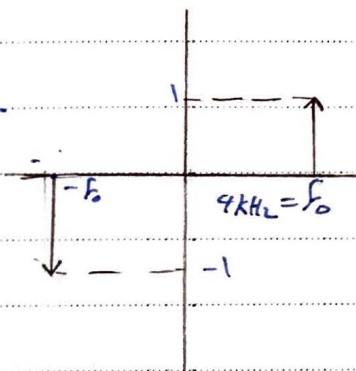
$$\omega_s \geq 26\pi \times 10^3 - (-13\pi \times 10^3)$$

$$\omega_s \geq 39\pi \times 10^3 \text{ rad/s}$$

(2)

2 $X_d(n) = \sin\left(2\pi n \frac{f_0}{f_s}\right)$; $f_0 = 4 \text{ kHz}$
 $f_s = 15 \text{ kHz}$

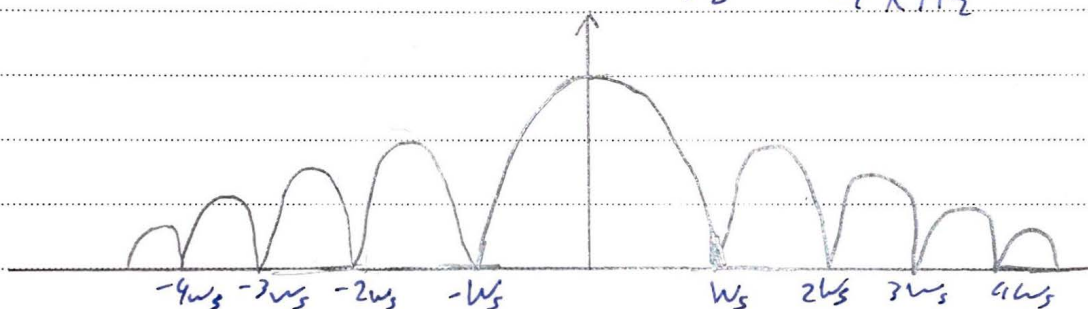
$$X(t) = \sin(2\pi f_0 t) = \frac{e^{j2\pi f_0 t} - e^{-j2\pi f_0 t}}{2j}$$



$$H_{ZOH}(\omega) = \Delta t e^{-j\pi \frac{\omega}{\omega_s}} \frac{\sin(\pi \omega / \omega_s)}{\pi \frac{\omega}{\omega_s}}$$

$$H_{ZOH}(0) = 1$$

number of Harmonics = $\frac{f_s}{f_0} = \frac{15 \text{ kHz}}{4 \text{ kHz}} = 3.75 \approx 4$.



at Δt , $|\omega| < \frac{\omega_s}{2}$

$$\omega_0 = 2\pi f_0 \Rightarrow 2\pi \cdot 4 \times 10^3 = 8\pi \times 10^3 \text{ Hz}$$

\Rightarrow main frequency 8 kHz .

$$H_{ZOH} = \Delta t e^{-j\pi \frac{4}{15}} \frac{\sin(\pi \frac{4}{15})}{\pi \frac{4}{15}} =$$

Amplitude = $|H_{ZOH}(\omega)| \cdot X_d(\omega)$

$$X_d(\omega) = \frac{\pi}{j} [\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$$

$$\omega = \omega_s = 15 \text{ kHz}, \omega_0 = 4 \text{ kHz}$$

(3)

$$\text{b) } X(\omega) = \frac{\pi}{j} [\delta(\omega) - \delta(\omega_0)]$$

$$\star \omega = 2\omega_s = 30 \text{ KHz}, \omega_0 = 4 \text{ KHz}$$

$$X(\omega) = \frac{\pi}{j} [\delta(30-4) - \delta(30+4)]$$

our harmonics at 11 KHz, 19 KHz, 26 KHz, 34 KHz.

$$|Y(\omega)| = |H_{20K}(\omega)| \cdot X(\omega)$$

$$|Y(\omega)|_{\omega=\omega_0} = 2.7868 \quad \text{main frequency.}$$

$$\text{1st harmonic } |Y(\omega)|_{\omega=\omega_s-\omega_0} = 1.0134$$

$$\text{2nd: harmonic } |Y(\omega)|_{\omega=\omega_s+\omega_0} = 0.5867$$

$$\text{3rd harmonic } |Y(\omega)|_{\omega=2\omega_s-\omega_0} = 0.4287$$

$$\text{4th harmonic } |Y(\omega)|_{\omega=2\omega_s+\omega_0} = 0.3279$$

(4)