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In depth analysis of noise effects in orthogonal frequency division multiplexing systems, utilising a large number of subcarriers

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Abstract. Orthogonal frequency division multiplexing (OFDM) is a multicarrier data transmission, where a single stream of information is divided over a large number of subcarriers. The primary purpose of this work was to find out the relationships connecting BER performance in noisy environments and the number of transmitted subcarriers. In order to simulate this kind of environment, various noise types where taken into consideration such as complex Rayleigh fading, complex rician noise, AWGN and phase noise.

Keywords: OFDM, AWGN, BER, Rayleigh fading, rician noise, phase noise, wireless channel estimation.
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INTRODUCTION

In our days a great amount of researchers all over the world study OFDM (orthogonal frequency division multiplexing) in order to implement it in wireless systems. Already, OFDM has been accepted in various network standards such as IEEE 802.11, high performance local area networks (HIPERLAN) and mobile multimedia access communication systems (MMAC). A very important reason for the wide acceptance of OFDM, is credited to utilisation of multicarrier production that has been found to reduce problems emerging from non-line-of-sight (NLOS) transmission [1, 2].

In order to go one step further in studying OFDM transmission, we simulated a system, approached in a relevant simple and flexible way of programming, which has proven to be very reliable even with the absence of encoding. This system is composed from an OFDM transmitter, OFDM receiver and different noise blocks for conducting various simulations. Their results helped us to create a large number of curves for the purpose of finding satisfactory equations for modelling channel noise characteristics. These noise characteristics emerged from the extensive simulation of Rayleigh fading, Rician noise, Additive White Gaussian Noise (AWGN) and Phase noise.

Our work was split into three stages. In the first stage we involved with system design and the good theory of operation in the absence of noise. In the second stage after implementing various blocks in the simulation environment for enhancing system reliability, we inserted AWGN block. Then, we conducted a complete set of noise simulations involved 64, 128, 256, 512, 1024, 2048 and 4096 carriers. These carriers were present in the input of Inverse Fourier Transform (IFFT) block. Finally, in the third stage we simulated our system while taking into consideration all noise types which are referred in the previous paragraph and for the same amount of produced OFDM sucarriers.

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THEORY OF SYSTEM OPERATION

OFDM combines modulation and multiplexing. We used one of the known modulation schemes like QPSK to produce a modulated signal. Then multiplexing was applied to independent signals, which were small parts of the produced QPSK, for creating various channels. This was accomplished by utilizing serial to parallel conversion which is called buffering. Particularly, modulated data pass through buffer and then these are separated to columns according to the number of subcarriers we wish to have in our OFDM transmission (See Table 1). Then, data representation is transformed into the time domain with the help of Inverse Fast Fourier Transform (IFFT) [3, 4].

TABLE 1. Example of buffering (serial-to-parallel) a data sequence of "a1, a2, a3, b1, b2, b3, c1, c2, c3" to 3 subcarriers.

Column1 (Subcarrier 1)	Column2 (Subcarrier 2)	Column3 (Subcarrier 3)
a1	a2	a3
b1	b2	b3
c1	c2	c3

A signal that passes through FFT is multiplied with complex exponentials over the range of frequencies and after each product is summed, the result is plotted as a coefficient of the corresponding frequency. The mechanism of FFT produces a frequency domain signal. The equation of FFT in sinusoids is presented below:

$$x(k)=\sum_{n=0}^{N-1}x(n)\cos\left(\frac{2\pi kn}{N}\right)+j\sum_{n=0}^{N-1}x(n)\sin\left(\frac{2\pi kn}{N}\right)=\sum_{n=0}^{N-1}x(n)\exp\left(j\frac{2\pi kn}{N}\right). \quad (1)$$

where n is the time index, k is the index of frequencies over N frequencies, x(n) is the n-time signal value and x(k) is the spectral value corresponding to k_{th} frequency.

The inverse FFT converts the frequency domain signal to time domain signal by multiplying it with a successive range of sinusoids. These two processes constitute a linear pair. The equation is presented below:

$$X(n)=\frac{1}{N}\sum_{n=0}^{N-1}x(k)\cos\left(\frac{2\pi kn}{N}\right)-j\frac{1}{N}\sum_{n=0}^{N-1}x(k)\sin\left(\frac{2\pi kn}{N}\right). \quad (2)$$

In order to avoid delay spread of the transmitted signal and consequently inter-symbol-interference (ISI) and inter-carrier-interference (ICI), we used Cyclic Prefix (CP). This is a procedure where last n out of N samples coming from IFFT are replicated and they are placed at the beginning of OFDM symbol. Attention was given to find the proper value of CP. By using higher values of CP, data rate decreases by the factor R and in this way system efficiency is compromised accordingly [4, 5].

$$Data\ Rate\ Deterioration\ due\ to\ CP: R=\frac{N}{n+N} \quad (3)$$

In our transmitter model, between the two stages of serial to parallel conversion and IFFT we inserted the zero padding procedure. This procedure changes the dimensions of the input matrix from Mi-by-Ni to Mo-by-No by adding zeros along rows, or columns or both. Specifically, double zero padding was made. Zeros were appended at the end and at the beginning of the signal. The purpose of adding zeros was to maximize efficiency of the FFT algorithm as it dealt with signals contained 2^N data points and also for having a better resolution in the frequency spectrum. As soon as we inserted zero padding in our system, we also noticed, better error rate calculations (decrement of detected errors) in the presence of noise [6].

Finally we added a Blackman window function in the output of the transmitter model. This technique of windowing FIR filter shows less sideband leakage and slightly wider central lobe comparing to other Hanning and Hann windows. The equation for calculating the coefficients of a Blackman window is given below [7]:

$$w(k+1)=0.42-0.5\cos\left(2\pi\frac{k}{n-1}\right)+0.08\cos\left(4\pi\frac{k}{n-1}\right),\quad k=0,...,n-1 \quad (4)$$

An overview about the simulated system: In figure 1 is presented a simple design of our transmitter (shaded blocks) and receiver. A random generator produces integers which are modified to bit format and this signal is modulated using QPSK scheme. Then multiple carriers are generated with the help of the buffer and then zero padding is applied before the signal passes through IFFT. Finally a cyclic prefix is appended and all data are unbuffered before filtering. At receiver's part, the opposite procedure of modulation and multiplexing is conducted in order to take back the received signal and compare it with the one coming from random generator. Also, BER is found using Error Rate Calculation block.

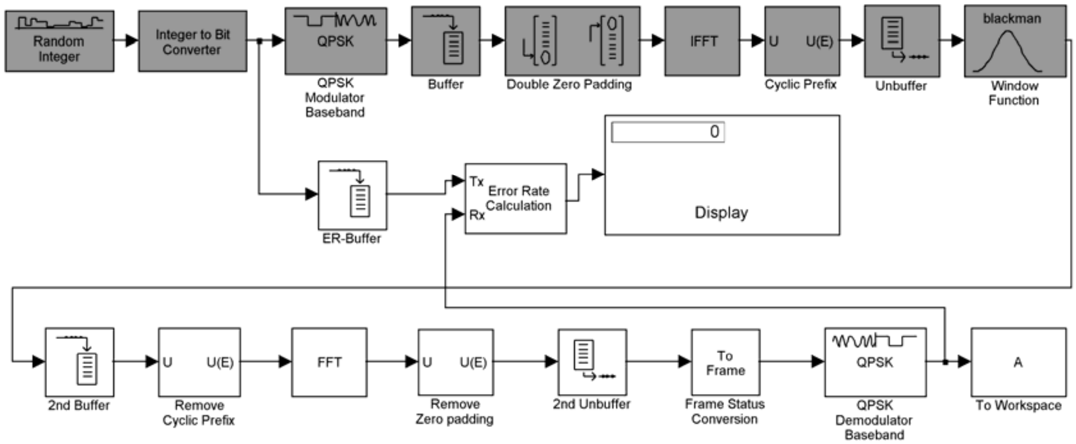

FIGURE 1. Simple representation of the designed OFDM system consisted of transmitter (shaded blocks) and receiver, without noise parameters.

The different number of the produced subcarriers in various stages of the transmitter is shown in Table 2. The symbol time is equal to $2 \cdot 10^{-6}$ sec. Taking into consideration that in QPSK modulation one symbol is consisted from two bits, then through a single carrier the maximum amount of information that can be transferred is 1 Mbps. Effective carriers denote information carriers.

TABLE 2. Various numbers of total carriers coming from a different number of effective carriers.

Effective Carriers	Zero padding	IFFT Input	Cyclic Prefix	Total Carriers
51	13	64	17	81
102	26	128	34	162
204	52	256	68	324
408	104	512	136	648
816	208	1024	272	1296
1632	416	2048	544	2592
3264	832	4096	1088	5184

AWGN CHANNEL SIMULATIONS

In this part of our work an Additive White Gaussian Noise (AWGN) Channel was implemented in the simulation. "Additive" means that noise is added based in the relation (5), "White" has to do with the flat power spectral density and "Gaussian" denotes the type of sample distribution. Our transmitter sent digital information through this channel. Since, the symbol duration equaled with T, the information could be transferred over the interval $0 \leq t \leq T$. This channel altered the transmitted signal by adding a large quantity of noise samples which their production based on White Gaussian Noise theory. The final signal which reached the receiver can be expressed by the following equation [8]:

$$r(t) = s_m(t) + n(t) \quad 0 \leq t \leq T \tag{5}$$

where $s_m(t)$ is the transmitted signal and $n(t)$ is expressed in terms of AWGN samples with the following power spectral density:

$$\Phi_m(f) = \frac{1}{2} N_o W / Hz \tag{6}$$

We adjusted our system according to Table 2 and we conducted many simulations for finding OFDM spectrum response. Simulation curves which are presented in Figure 2 represent the spectrum of OFDM signal with and without AWGN noise for 5184 total subcarriers.

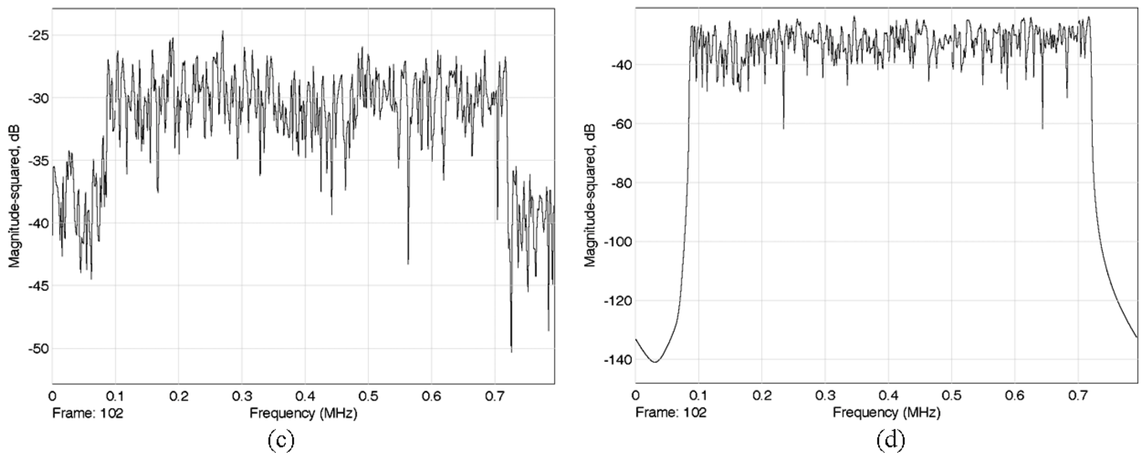

FIGURE 2. (c) Spectrum plot - 5184 OFDM subcarriers, 25 dB SNR, AWGN channel with extra phase noise and (d) Spectrum plot - 5184 OFDM subcarriers, without noise.

SIMULATIONS INCLUDING RAYLEIGH FADING AND RICIAN NOISE

In radio systems a severe but expected problem that can be emerged, is the reflection of the transmitted signal from various obstacles. This causes receiving problems due to the existence of a large number of transmitted paths. Interference is imminent as the relative phase of reflected signals leads to constructive or deconstructive procedures at the receiver's part. Rayleigh fading can be used to analyze propagation of radio signals on a statistical approach. It can be used better under conditions where there is no dominant signal (Non-line-of-sight). The Rayleigh probability distribution is defined as [8]:

$$P_R(r) = \frac{r}{\sigma^2} e^{-r^2/2\sigma^2}. \quad r \geq 0 \tag{7}$$

where "σ^2" is the variance of each of the components (in-phase and quadrature) that consist received signal and they are Gaussian random variables , and "r" is the Rayleigh amplitude.

The model of Rician fading has many similarities with Rayleigh fading, except the fact that it includes in its calculations the presence of deterministic LOS component. This dominant component is treated as fully predictable process. The Rician fading is characterized by the K factor. This factor is the power ratio of the deterministic LOS component to the multipath components [8, 9]:

$$K = \frac{s^2}{2\sigma^2} \ , \begin{pmatrix} K = 0, \ being \ the \ most \ severe \ fading - Rayleigh \\ K \to \infty, AWGN \ channel \\ K > 0 \end{pmatrix} \tag{8}$$

The next step in our study was to investigate Rayleigh fading in the presence of AWGN and phase noise. SNR (in AWGN channel) was equal to 30 and 40 dB. Phase noise level had a nominal value of -50 dBc/Hz with a frequency offset of 100 Hz and Rayleigh fading envelope was equal to 1 dB. In figure 4 are presented the diagrams of BER in relation with the total number of OFDM carriers that were produced through simulation procedures. Also, in figure 5 are shown the scattering plots for the produced Rayleigh channel shift of 15 and 30 degrees in reference to 0 degrees of channel shift.

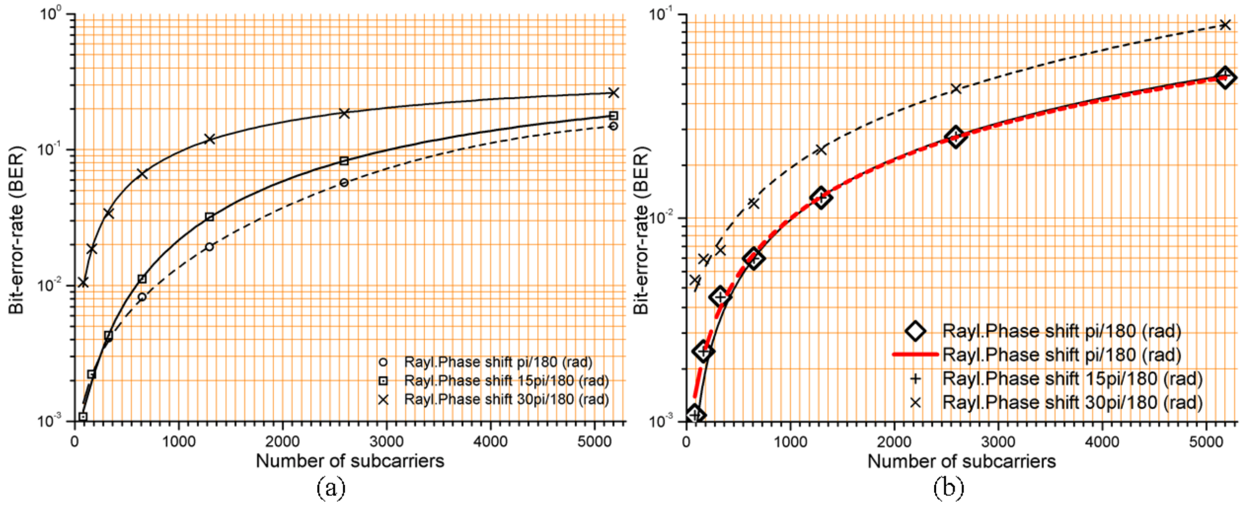

FIGURE 4. BER in relation with the number of the total produced carriers for given Rayleigh phase channel shifts. (a) SNR was equal to 30 dB and (b) SNR was equal to 40 dB.

The best sets of simulation results in Figures 4a and 4b are corresponding to lower BER curves. Through proper fitting, we extracted the equations that characterized the system behavior for different number of carriers (NC):

$$BER = -0.0112 + \frac{0.19615}{1+10^{(3360{,}8315-NC)\ \ 3{,}5514\ \ 10^{-4}}}, \qquad Phase\ shift = 1\ \deg, \quad SNR = 30\ dB \tag{9}$$

$$BER = -2.6249\, e^{\frac{NC}{-313741.3}} - 0.5518\, e^{\frac{NC}{-526808.9}} + 3.1772, \quad Phase\ shift = 1\ \deg, \quad SNR = 40\ dB \tag{10}$$

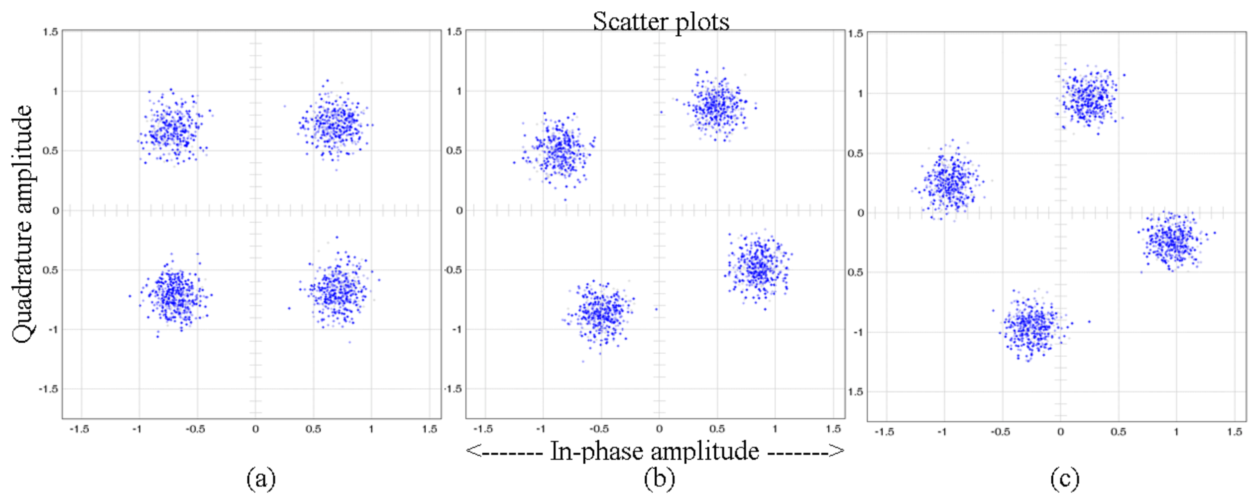

FIGURE 5. (a) Scatter plot for OFDM with 5184 total carriers with AWGN noise and no phase shift (b) Scatter plot for OFDM with 5184 total carriers and 15 deg Rayleigh shift, (c) Scatter plot for OFDM with 5184 total carriers and 30 deg Rayleigh shift.

Also, simulations were conducted in order to find the BER dependence with K-factor (Figure 7a) which characterizes rician channels. We placed in the position of Complex Rayleigh fading the Additive Rician Noise with a signal total mean power (LOS component with scattered components) of -72 dBm, SNR (AWGN Channel) was equal to 40 dB, and Phase noise had a nominal value of -50 dBc/Hz with a frequency offset of 100 Hz.

The final stage with our work (Figure 7b) had to do with the use of all the types of noises that we implemented in turns. The simulated channel that included Rayleigh fading and additive Rician noise is shown in Figure 6. Phase Noise had a level of -50dBc/Hz with a frequency offset of 100 Hz, SNR was equal to 50 dB, and Additive Rician noise had a K-factor equal to 5.

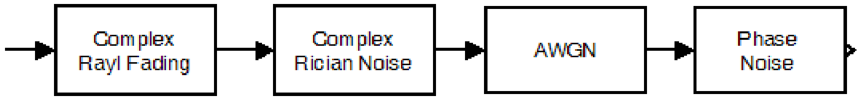

FIGURE 6. Noise Channel (Rayleigh fading with additive Rician noise, AWGN and Phase noise)

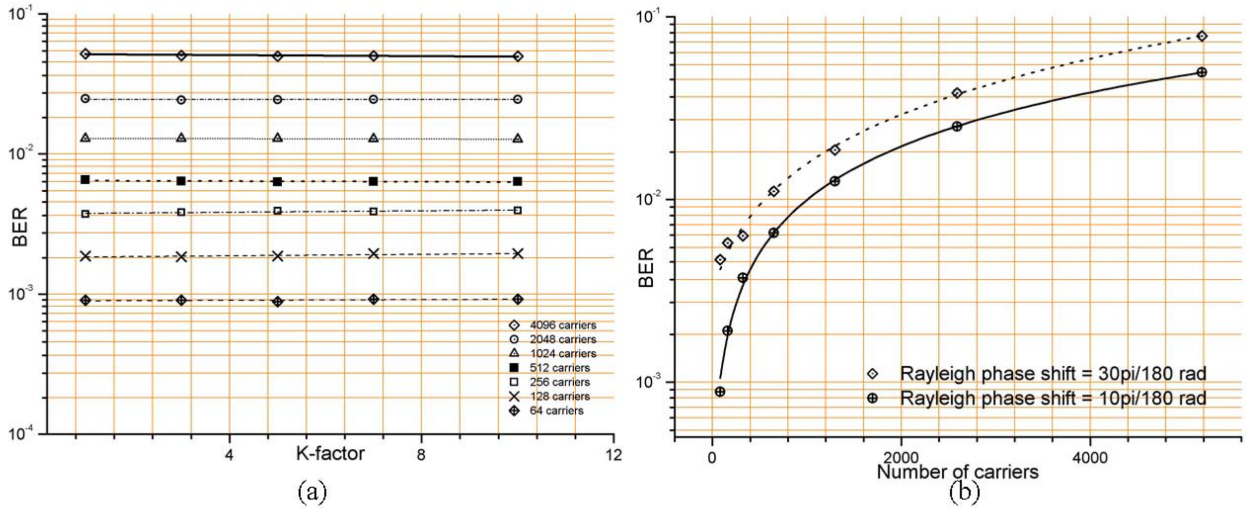

FIGURE 7. (a) BER in relation with K-factor in presence of Rician noise, AWGN and phase noise, (b) BER in relation with total number of carriers in presence of Rayleigh fading, Additive Rician noise, AWGN and phase noise.

CONCLUSION

In this paper we presented an OFDM system which doesn't use encoding. The purpose was to study the raw effects of noise to a large number of produced orthogonal carriers through simulation. The result indicates that the technique of stricting an even bigger amount of spectrum for holding only products of zero in the beginning and in the end of the signal, along with the appropriate percentage of cycling prefixing, is very helpful and even in the absence of encoding, a radio link can be established under conditions. Given these results, a future goal will be to use encoding and interleaving in our system for implementing it in a DSP.

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