Wireless Communications SSY135 – Lecture 3

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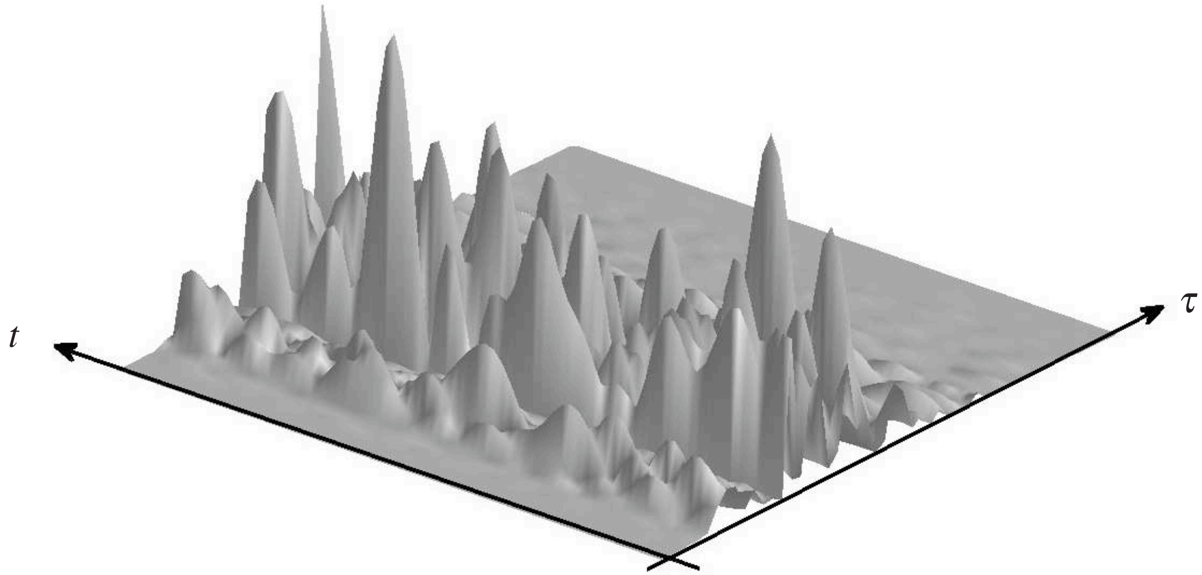

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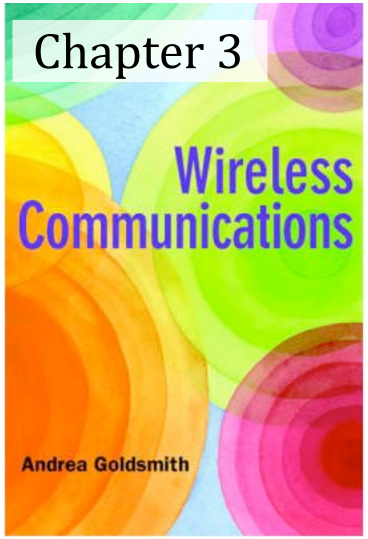
Topics for today

- Lecture learning outcomes
- Doppler shift
- Time-varying channel impulse response
- Narrowband fading models
  - Rayleigh, Rice
  - Jakes model
- Level crossing rate, average fade duration



Suggested reading:
- Every section from Chapter 3, except 3.2.4
- No derivations from sections 3.2.3
- Not derivations (3.18)-(3.27)

Today's learning outcomes

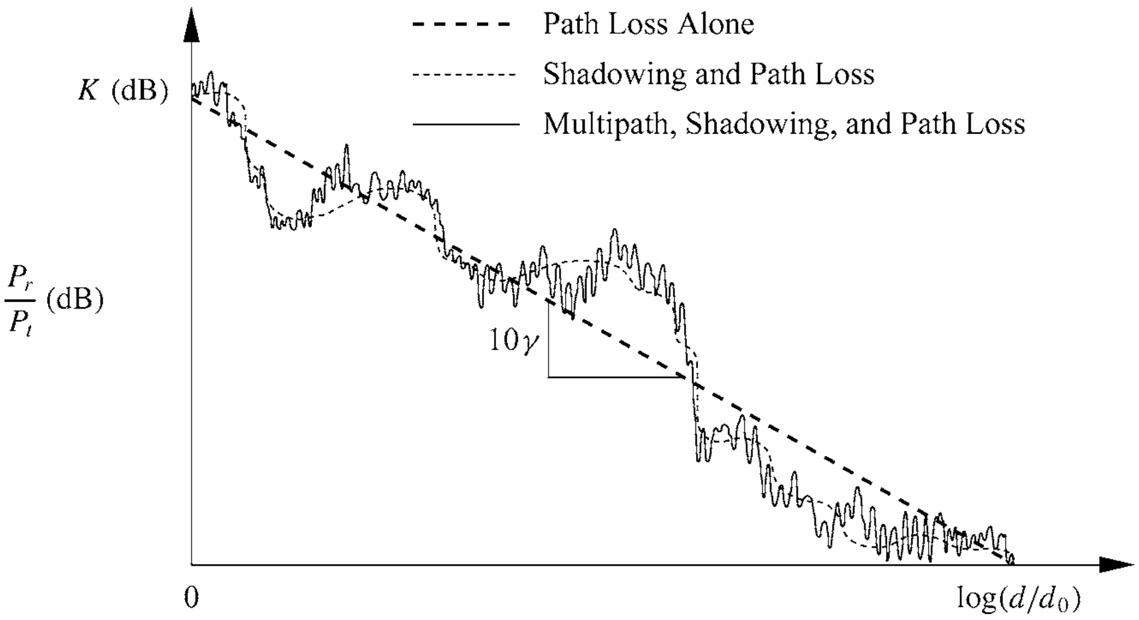
At the end of this lecture, you must be able to

- Distinguish between wideband and narrowband communication
- Derive Doppler shift due to mobility
- Define what resolvable paths are
- Express the general formula for time-varying channels
- Identify Rayleigh and Rician fading
- Define the autocorrelation and PSD of a WSS fading process

Last time

$$P_r[\mathrm{dBm}] = P_t[\mathrm{dBm}] + \mathrm{pathloss}[\mathrm{dB}] + \mathrm{shadowing}[\mathrm{dB}] + \mathrm{multipath}[\mathrm{dB}]$$

- Propagation in complex environments

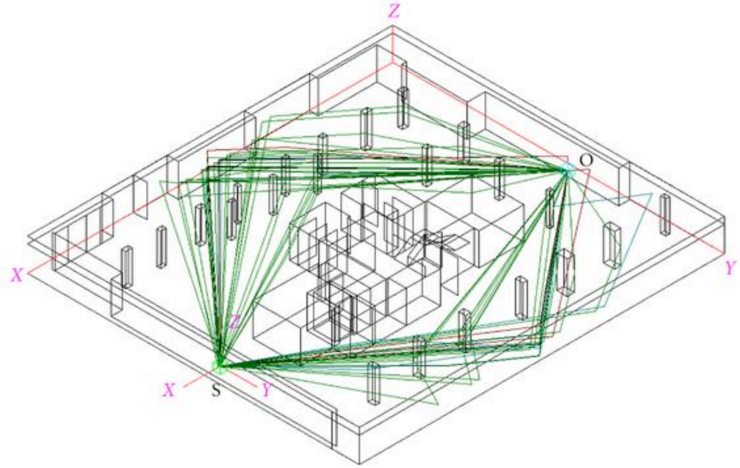


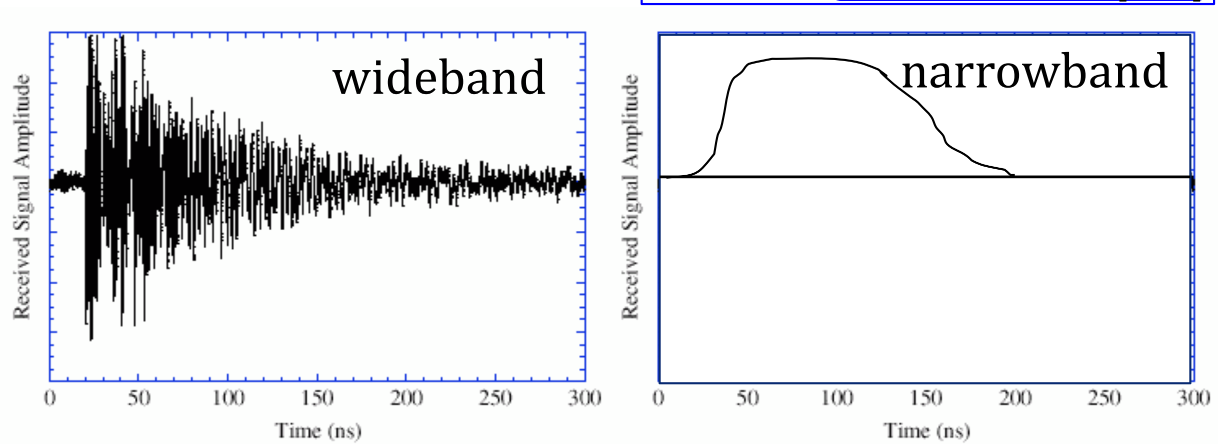
- Path loss: due to distance, deterministic $P_r = P_t K \left(\frac{d_0}{d}\right)^{\gamma}$
- Shadowing: due to obstacles, modeled as random $\psi_{\mathrm{dB}} \sim \mathcal{N}(0, \sigma^2_{\psi_{\mathrm{dB}}})$
- Total received power in dB: $P_t\ [\mathrm{dBm}] + K\ [\mathrm{dB}] - 10\gamma \log_{10} \frac{d}{d_0} + \psi_{\mathrm{dB}}$
- Multipath fading: due to mobility, reflections, modeled as random

Multipath fading

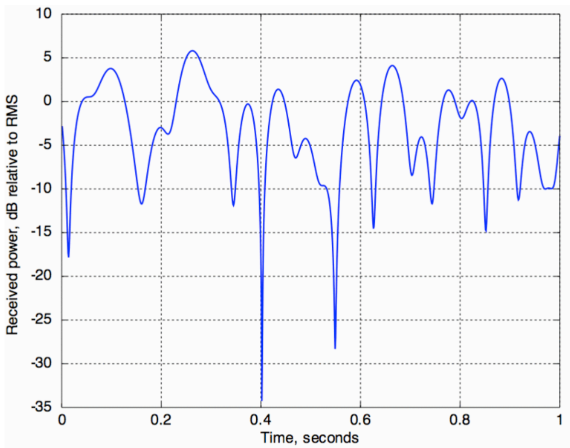
$$\begin{aligned} P_r[\text{dBm}] = P_t[\text{dBm}] \\ + \text{pathloss}[\text{dB}] \\ + \text{shadowing}[\text{dB}] \\ + \text{multipath}[\text{dB}] \end{aligned}$$

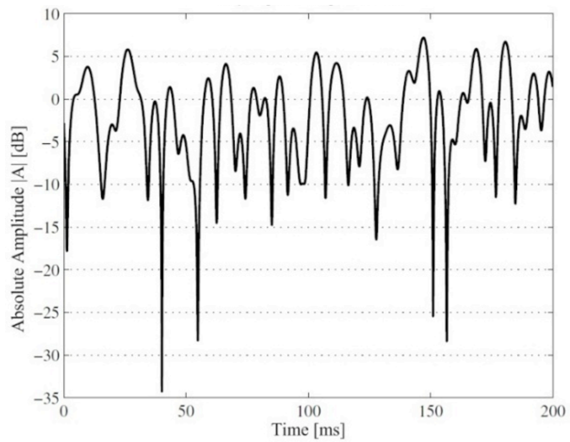
- Many signal paths add up incoherently



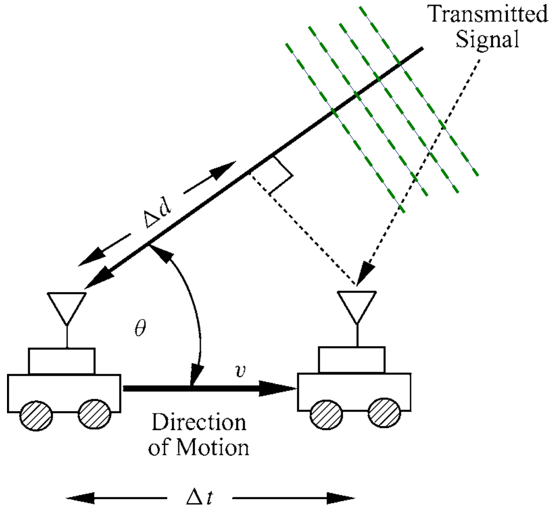


- Signal varies over time due to mobility





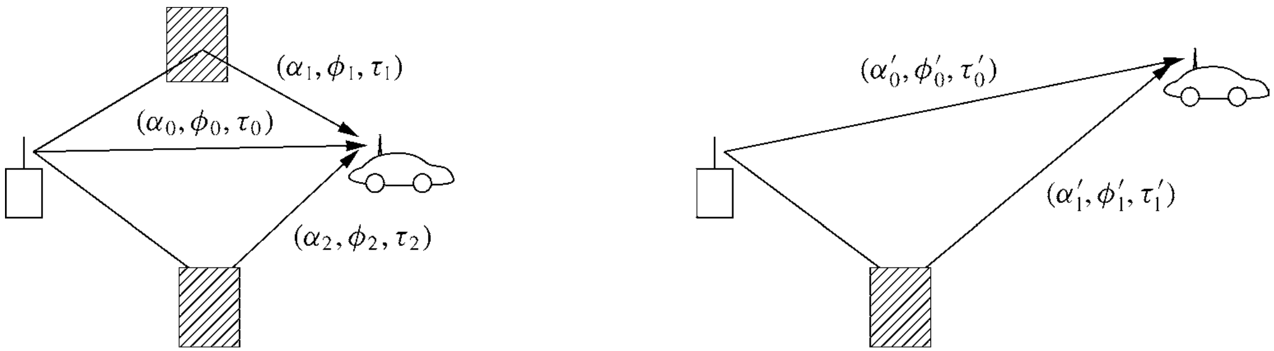
Doppler shift



- Moving transmitter or receiver, far away
- Transmitter frequency f_c
- Received frequency f_c+f_D

$$f_D = \frac{v f_c}{c}\cos\theta = \frac{v}{\lambda}\cos\theta$$

- Example: 1GHz carrier, 75km/h, what is Doppler?
- Mobility causes differences in channel over time



System at t_1 System at t_2

Time-varying impulse response

- Transmitted signal $s(t) = \Re\{u(t)e^{j2\pi f_c t}\}$
- Line of sight received signal

path loss, shadowing

Doppler

$$r(t) = \Re\left\{u(t - \tau(t))\alpha(t)e^{j2\pi(f_c+f_D)(t-\tau(t))}\right\} + n(t)$$

delay increases with distance

- Many paths, different angles

$$r(t) = \Re\left\{\sum_{n=1}^{N} u(t - \tau_n(t))\alpha_n(t)e^{j2\pi(f_c+f_{n,D})(t-\tau_n(t))}\right\} + n(t)$$

$$= \Re\left\{e^{j2\pi f_c t}\sum_{n=1}^{N} u(t - \tau_n(t))\alpha_n(t)e^{j\phi_n(t)}\right\} + n(t)$$

Baseband model: $\int_{-\infty}^{+\infty} c(\tau, t)u(t-\tau)\mathrm{d}\tau$

- Time-varying convolution

$$c(\tau, t) = \sum_{n=1}^{N} \delta(\tau - \tau_n(t))\alpha_n(t)e^{j\phi_n(t)}$$

Time-varying impulse response

- Channel: physics

Delay spread [s] $\risingdotseq$ time between first arrival and last reflection (inverse = **coherence bandwidth** [Hz])

fix time, study delay ← $c(\tau, t)$ → fix delay, study time

Coherence time [s] $\risingdotseq$ time during which channel is fixed (inverse = **Doppler spread** [Hz])

- Communication over a channel: engineering, choose a bandwidth B (T=1/B)

$c(\tau, t), T = 1/B$

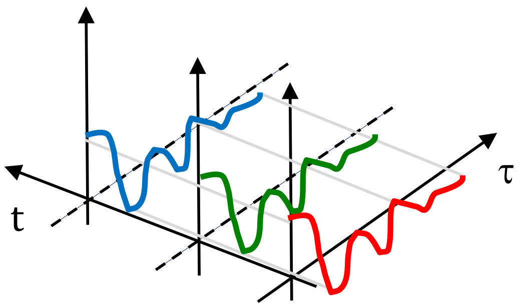
T >> delay spread?
- YES → **Narrowband** communication (also: frequency-flat)
- NO → **Wideband** communication (also: frequency-selective, dispersive)

T << coherence time?
- YES → **Slow** fading communication
- NO → **Fast** fading communication

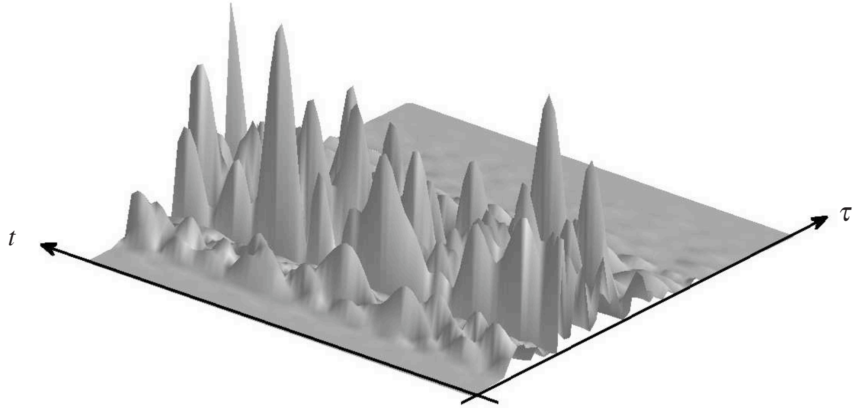
- Today: narrowband communication, both slow and fast fading

Extreme cases

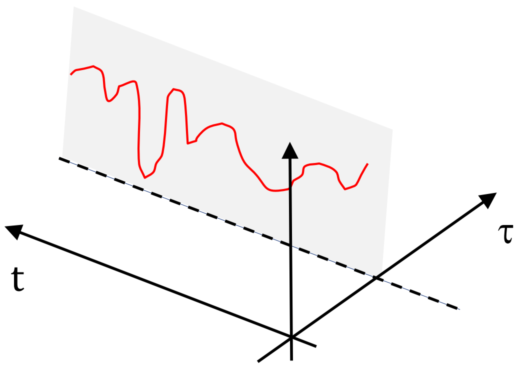
- Static: wideband (very) slow fading communication



Next lecture: general case

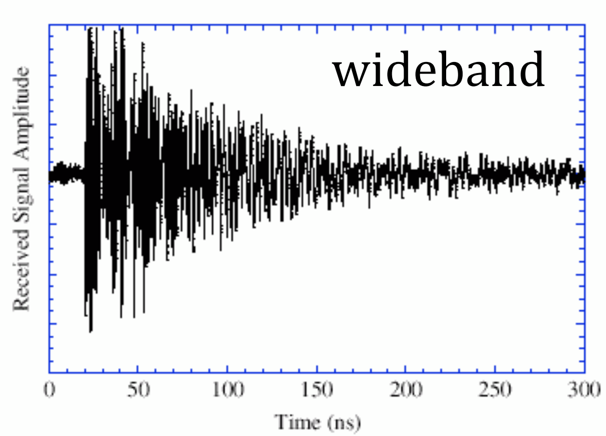


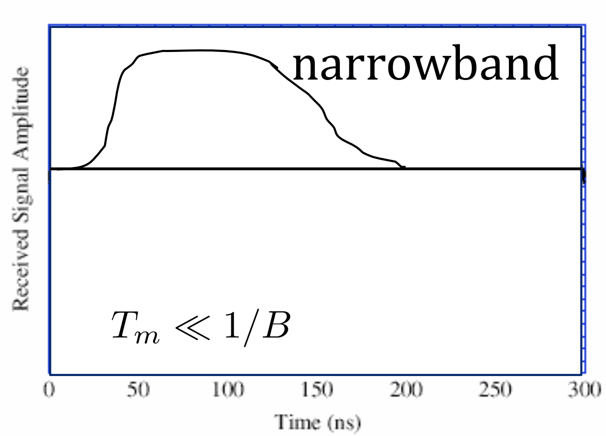
- Today: narrow-band communication

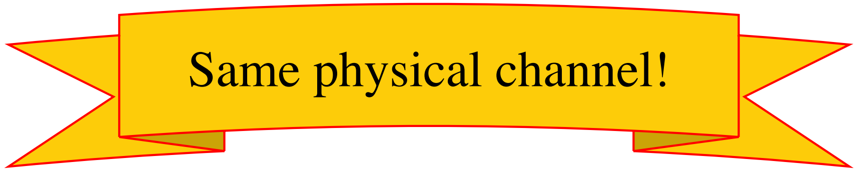


Resolvable paths

- Paths are resolvable when $|\tau_n - \tau_m| \gg 1/B$
- Otherwise they appear as $\tau_n \approx \tau_m$
- Note that everything varies over time!
- Delay spread $T_m = \max_n |\tau_n - \tau_0|$
- Delay spread causes inter-symbol interference







Resolvable paths

Given

- A channel with delay spread of 10 us
- Two systems with baud rates (i) 10 Mbaud and (ii) 10 kBaud

Task

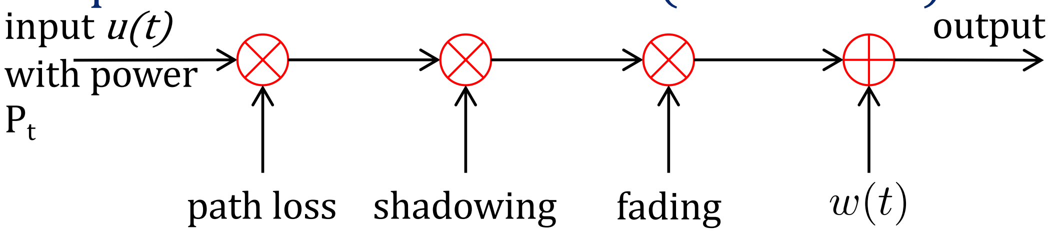
- What is propagation distance between first and last path?
- Determine the amount of inter-symbol interference

Narrowband fading models

- All paths arrive at same time

$$
\begin{aligned}
r(t) &= \Re\left\{ e^{j2\pi f_c t} \sum_{n=1}^{N} u(t-\tau_n(t))\alpha_n(t)e^{j\phi_n(t)} \right\} + n(t) \\
&= \Re\left\{ e^{j2\pi f_c t} u(t) \sum_{n=1}^{N} \alpha_n(t)e^{j\phi_n(t)} \right\} + n(t) \\
&= \Re\left\{ e^{j2\pi f_c t} u(t)\beta(t) + w(t) \right\}
\end{aligned}
$$

- Equivalent baseband model (linear scale)



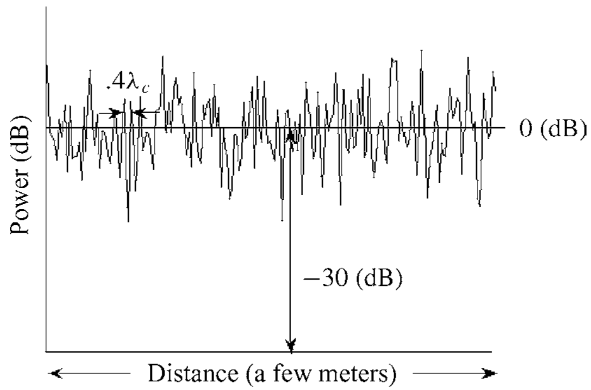


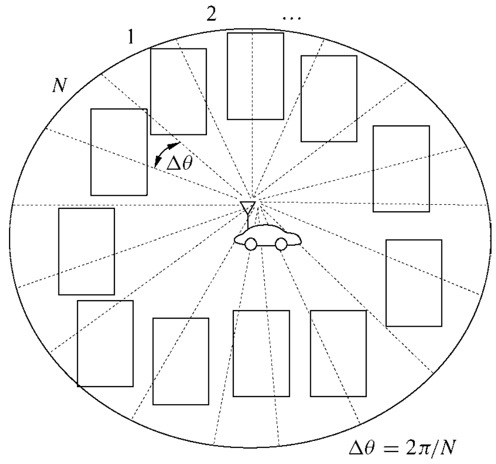
Figure 3.9: Narrowband fading.

- Models: statistics of $\beta(t)$
  - mean power determined by path loss and shadowing:

$$
\mathbb{E}\{|\beta(t)|^2\} = P_t K \left(\frac{d_0}{d}\right)^{\gamma} \psi, \quad \psi[\mathrm{dB}] \sim \mathcal{N}(0, \sigma^2_{\psi_{\mathrm{dB}}})
$$

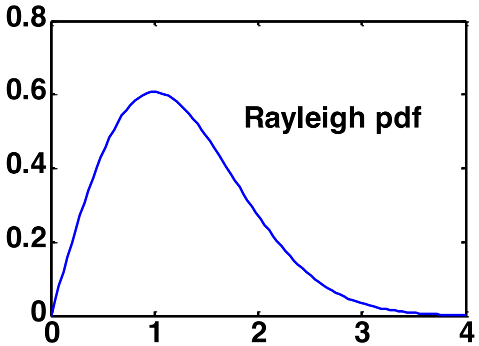
  - distribution: Rayleigh, Rice (others exist: e.g,., Nakagami)
  - autocorrelation: Jakes model, tells how quickly channel changes in time

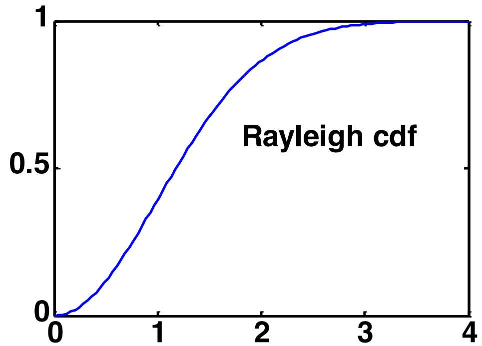
Distribution model 1: Rayleigh fading



- Large *N*: according to Central Limit Theorem without dominant path
  $\Re\{\beta(t)\}, \Im\{\beta(t)\} \sim \mathcal{N}(0, \sigma^2)$, i.i.d.
- Variance: due to shadowing + path loss

- Then $z(t) = |\beta(t)|$ has a *Rayleigh* distribution





Note:
- Gaussian channel coefficients
- Rayleigh envelope
- exponential power

$$p_z(x) = \begin{cases} \dfrac{x}{\sigma^2} \exp\left(-\dfrac{x^2}{2\sigma^2}\right), & x \geq 0 \\ 0, & x < 0 \end{cases}, \quad P_z(x) = \begin{cases} 1 - \exp\left(-\dfrac{x^2}{2\sigma^2}\right), & x \geq 0 \\ 0, & x < 0 \end{cases}$$

$$E[z] = \sqrt{\frac{\pi}{2}}\sigma, \qquad E[z^2] = 2\sigma^2, \qquad \mathrm{var}[z] = \left(2 - \frac{\pi}{2}\right)\sigma^2, \quad \text{median} = 1.177\sigma$$

Generate path-loss, shadowing, Rayleigh fading

1. Decide on parameters and P_t
2. Generate path loss: $PL = K(d_0/d)^\gamma$ → power averaged over shadowing [in dB domain] and fading [in linear domain] is $(P_t)(PL)$
3. Generate shadowing: SH [dB] ~ N $(0, \sigma^2[\text{dB}])$ → power averaged over fading is $(P_t)(PL)(10^{(0.1*SH)})$
4. Generate multipath fading power: MPFP ~ exp(1) → instantaneous power is equal to $P_r = (P_t)(PL)(10^{(0.1*SH)})(MPFP)$
5. Generate channel gain: $\beta \sim CN(0, (PL)(10^{(0.1*SH)}))$

```
N = 200;    % number of power realizations
K=8e-4; % linear scale
d0=1;    % [m]
d=150;   % [m]
gamma = 2; % path loss exponent
sigma2_psi=4; % shadowing variance [in dB scale]
P_TX=1;    % [mW]
path_loss = K*(d0/d)^gamma;
shadowing_dB = randn(1)*sqrt(sigma2_psi);
shadowing = 10^(0.1*shadowing_dB);
fast_fading = exprnd(1,1,N);
P_RX = P_TX*path_loss*shadowing*fast_fading;
plot(1:N,10*log10(P_RX),1:N,ones(1,N)*10*log10(P_TX*path_loss*shadowing))
xlabel('realization')
ylabel('P_RX [dB]')
```

Distribution model 1: Rayleigh fading

Given

- γ=3.71, d0=1 m, K = -31.5 dB, Pt = 1mW, shadowing variance [dB domain] of 13.3.

Task

- Assuming Rayleigh fading, how would you generate channel coefficients with for d=150 m and d=300 m?

Distribution model 2: Rician fading

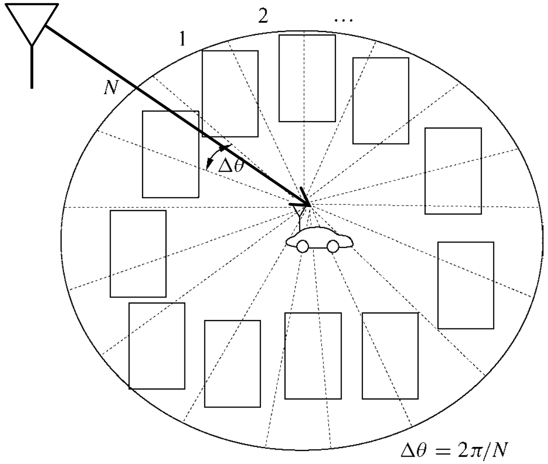
- One dominant path: $z(t) = |\beta(t)|$ has *Rice* distribution

$$\beta(t) = \alpha_0(t)e^{j\phi_0(t)} + \sum_{n=1}^{N} \alpha_n(t)e^{j\phi_n(t)}$$

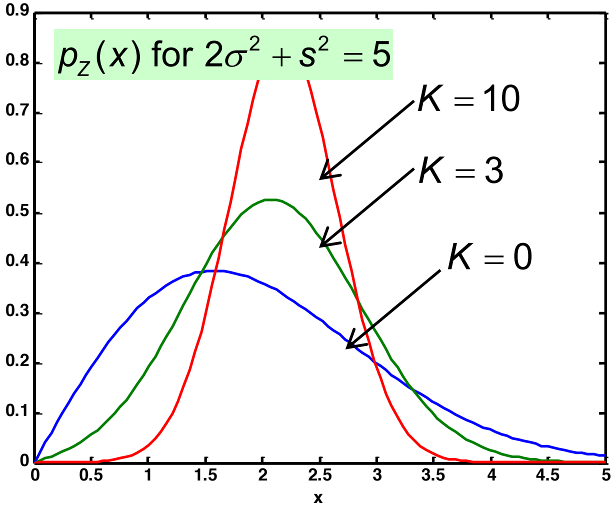
Rayleigh

$$|\alpha_0(t)|^2 = s^2, \sum_{n=1}^{N} \mathbb{E}\{\alpha_n(t)^2\} = 2\sigma^2$$

$$K = s^2/(2\sigma^2)$$ "K-factor"



- Rice distribution



$$p_z(x) = \frac{x}{\sigma^2}\exp\left(-\frac{(x^2+s^2)}{2\sigma^2}\right)I_0\left(\frac{sx}{\sigma^2}\right)$$

$2\sigma^2 + s^2 =$ total power — path loss and shadowing

$2\sigma^2 =$ power of scattered MPCs

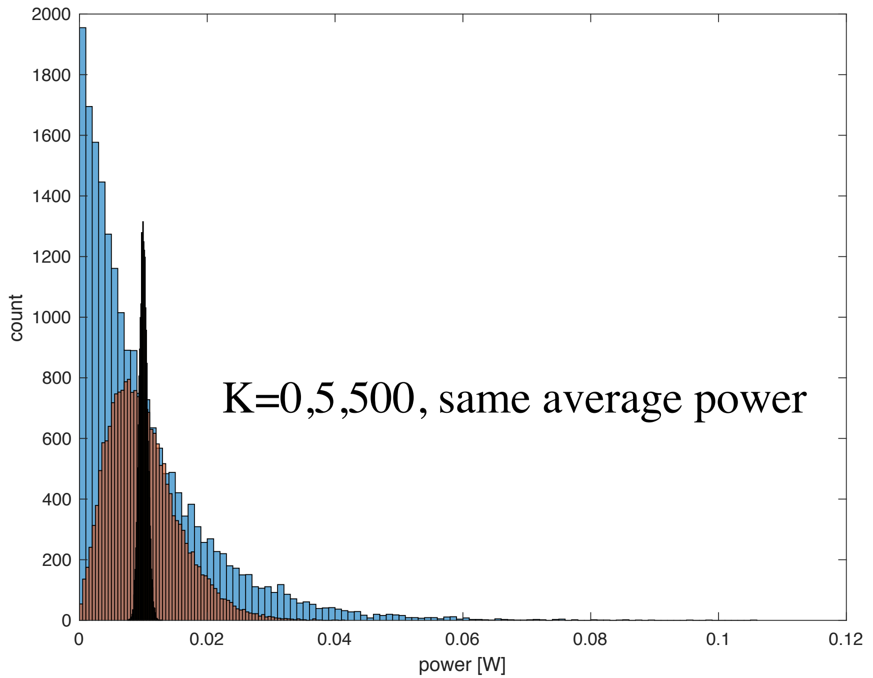
$s^2 =$ power in LOS component

$$K = \frac{s^2}{2\sigma^2} = \text{Ricean } K\text{-factor}$$

$$= \begin{cases} 0, & \text{Rayleigh fading (no LOS)} \\ \infty, & \text{no fading (no scattered MPCs)} \end{cases}$$

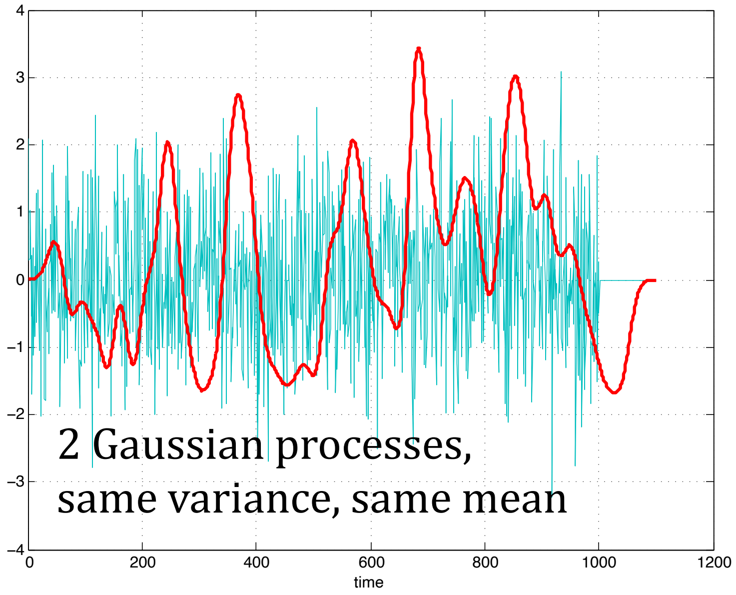
Generate Rician fading

```
N = 200;    % number of power realizations
P_RX_avg = 0.01; % average power (PL/SH)
K_factor=500;
sigma2=P_RX_avg/(2*(K_factor+1));
s2=K_factor*2*sigma2;
chLOS=sqrt(s2)*exp(j*rand(1,N)*2*pi);
chNLOS=sqrt(sigma2)*(randn(1,N)+j*randn(1,N));
channel=chLOS+chNLOS;
P_RX = abs(channel).^2;
histogram(P_RX)
mean(P_RX)
```



Autocorrelation

- Variation in time $A_\beta(t, t+\Delta t) = \mathbb{E}\{\beta^*(t)\beta(t+\Delta t)\}$



high correlation
low correlation

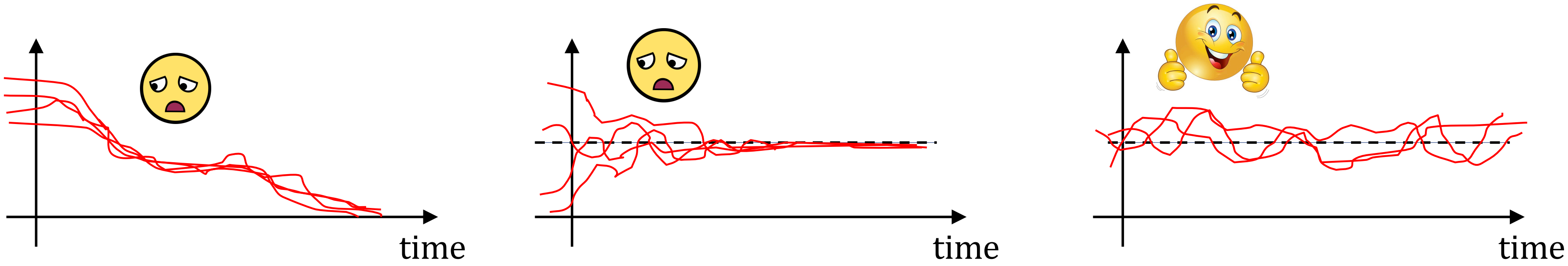
- Assume wide-sense stationary: no dependence on t

$$A_\beta(\Delta t) = \mathbb{E}\{\beta^*(t)\beta(t+\Delta t)\}$$

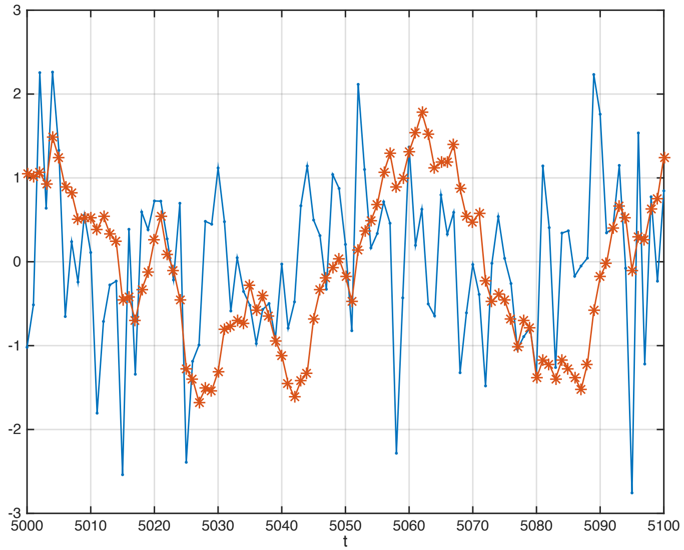
$$S_\beta(f) = \int A_\beta(\Delta t)\exp(-j2\pi f\Delta t)\mathrm{d}\Delta t$$

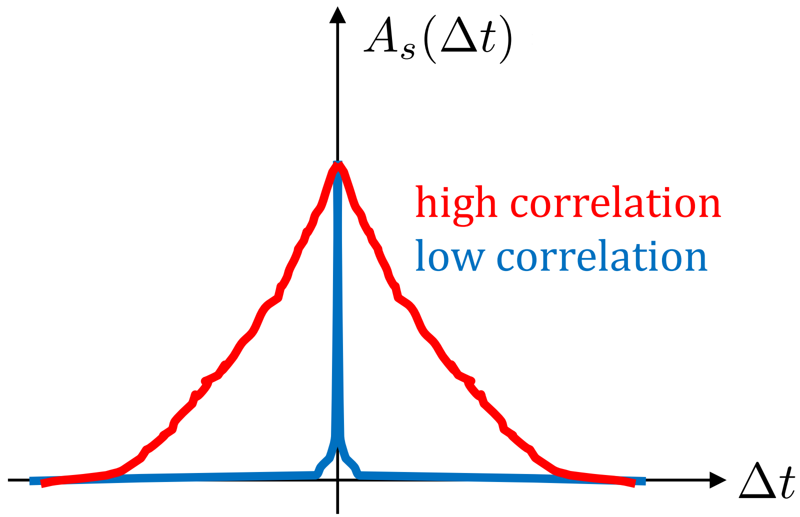
Autocorrelation function for 1D signal

- Wide-sense stationary random signal s(t): 1st and 2nd moment not vary in time



- Autocorrelation does not depend on t: $A_s(\Delta t) = \mathbb{E}\{s^*(t)s(t+\Delta t)\}$





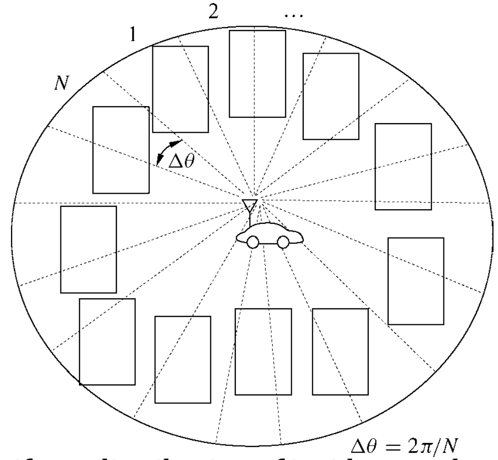
Jakes model / Clarke's spectrum

- Under "standard Rayleigh" assumptions

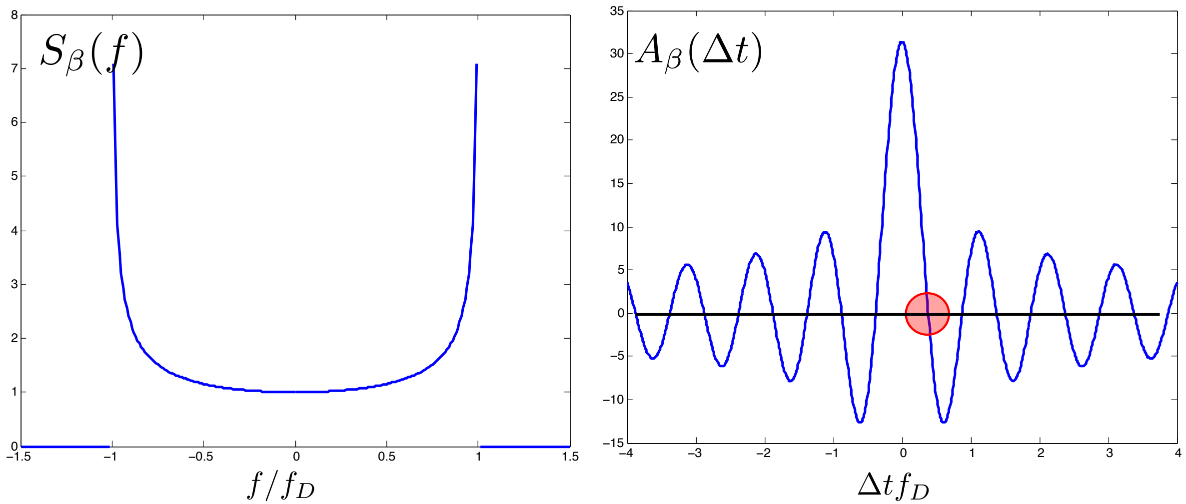
$$S_\beta(f) = \frac{C}{\sqrt{1-(f/f_D)^2}}, \quad |f| < f_D$$

$$A_\beta(\Delta t) = J_0(2\pi f_D \Delta t)\pi f_D C$$

C: received power (path loss, shadowing)



uniform distribution of incident angles, equal average power per multipath component, same delay for each component, 2 dimensional propagation



zero-crossing at

$f_D \Delta t \approx 0.4$

$v\Delta t \approx 0.4\lambda$

- Both autocorrelation and PSD can be used for simulation
- Mobility causes spectral broadening (Jakes) or spectral shifting (LOS)
- Channel approximately constant for $\Delta t \ll 0.4/f_D \approx 1/f_D$
- Careful with coherence time: $0.4/f_D$ (time to decorrelate) or $0.04/f_D$ (time to stay coherent)

Jakes model / Clarke's spectrum

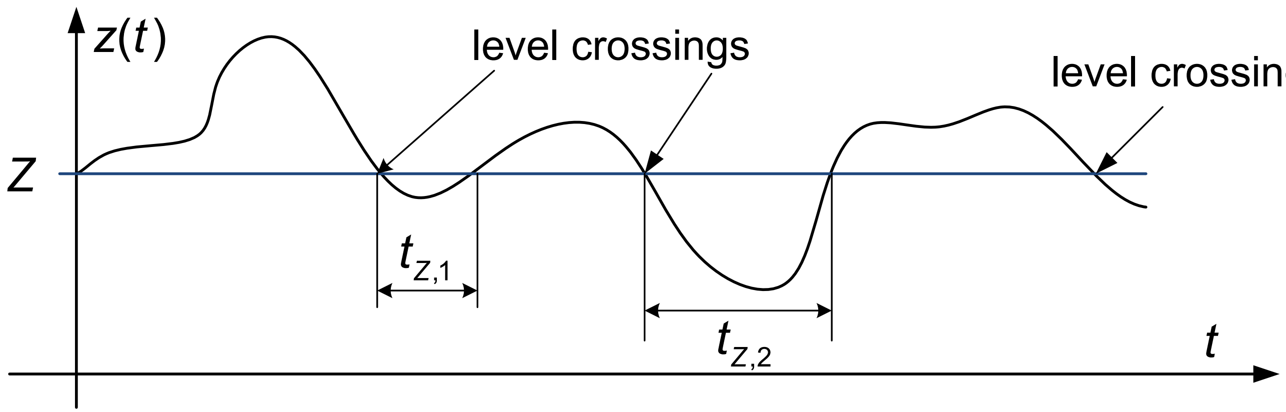
Given

- Mobile at v km/h, communication over a 1 GHz carrier and 1 MBaud transmission
- Packets containing 2000 symbols

Task

- How fast can the mobile move but still experience a near-constant channel for each packet?

Level crossing rate and average fade duration



- The level crossing rate L_Z is the average number of times per second the envelope $z(t) = |\beta(t)|$ crosses the level Z in the downward direction
- The average fade duration $\bar{t}_Z$ is the average time the fading envelope is below a certain level Z
- For Rayleigh fading with Clarke's spectrum and $\mathbb{E}\{z^2(t)\} = 2\sigma^2$

$$L_z = \sqrt{2\pi} f_{D,\max} \frac{Z}{\sqrt{2\sigma^2}} \exp\left(-\frac{Z^2}{2\sigma^2}\right); \qquad \bar{t}_Z = \frac{\sigma}{Z f_{D,\max}\sqrt{\pi}}\left[\exp\left(\frac{Z^2}{2\sigma^2}\right) - 1\right]$$

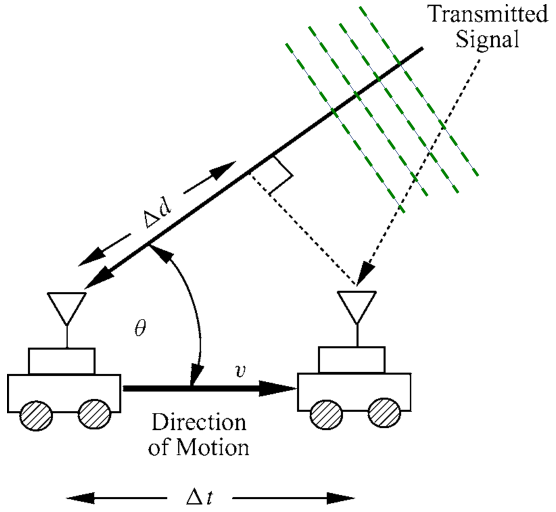
Today's learning outcomes

At the end of this lecture, you must be able to

- Distinguish between wideband and narrowband communication
- Derive Doppler shift due to mobility
- Define what resolvable paths are
- Express the general formula for time-varying channels
- Identify Rayleigh and Rician fading
- Define the autocorrelation and PSD of a WSS fading process

Solutions

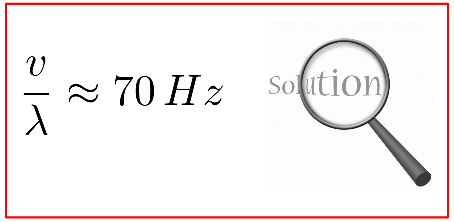
Doppler shift

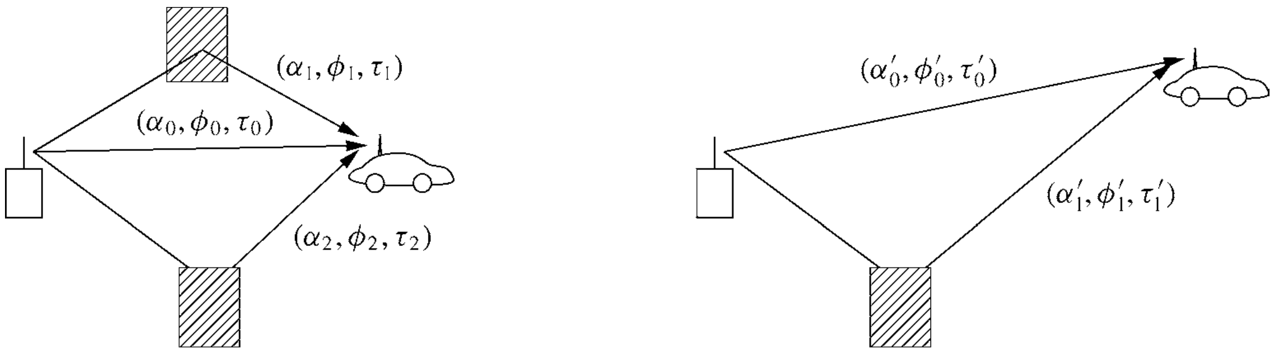


- Moving transmitter or receiver, far away
- Transmitter frequency f_c
- Received frequency f_c+f_D

$$f_D = \frac{v f_c}{c}\cos\theta = \frac{v}{\lambda}\cos\theta$$

- Example: 1GHz carrier, 75km/h (=21m/s), what is Doppler?
- Mobility causes differences in channel over time





System at t_1 System at t_2

Resolvable paths

Given

- A channel with delay spread of 10 us
- Two systems with baud rates (i) 10 Mbaud and (ii) 10 kBaud

Task

- What is propagation distance between first and last path?
- Determine the amount of inter-symbol interference

Solution

10 us = 3 km

For 10 Mbaud, symbol slots are 0.1us, so the ISI would last of 100 symbols

For 10 kbaud, symbol slots are 100us, so the ISI would be very small

Distribution model 1: Rayleigh fading

Given

- γ=3.71, d0=1 m, K = -31.5 dB, Pt = 1mW, shadowing variance [dB domain] of 13.3.

Task

- Assuming Rayleigh fading, how would you generate channel coefficients with for d=150 m and d=300 m?

Solution

For 150 m:

P_t = 0 dBm

Path loss in dB = -31.5 – 10*3.71*log10(150) = -112.2330 dB

Shadowing in dB = randn(1)*13.3 (say the results is "SH")

P_{tot} = 0 dBm -112.2 dB + SH

Rayleigh fading channels ~ CN(0, P_{tot})

For 300m: replace 150 by 300.

Jakes model / Clarke's spectrum

Given

- Mobile at v km/h, communication over a 1 GHz carrier and 1 MBaud transmission
- Packets containing 2000 symbols

Task

- How fast can the mobile move but still experience a near-constant channel for each packet?

Solution

- For a baud rate of 1Mbaud, the symbol slots are 1us.
- For 1 GHz, λ=0.3 m
- 2000 symbols = 2 ms
- so 2ms * v << λ/2, so v << 15 cm / 2ms = 270 km/h
- So velocities up to 30-60 km/h should be fine