January 28, 2020

1 Tentative Solutions

1. [G-2.1] Solution:

$$\begin{aligned} P_r &= P_t[\frac{\sqrt{G_{TX}G_{RX}\lambda}}{4\pi d}]^2 \\ 10^{-3} &= P_t[\frac{\lambda}{4\pi 10}]^2 P_t = 4.39KW \\ 10^{-3} &= P_t[\frac{\lambda}{4\pi 100}]^2 P_t = 438.65KW \end{aligned}$$

2. Solution:

   (a) Consider

   $$\left[\overline{P}_r(R)\right] = [P_t]_{\text{dBm}} + [10\text{log}_{10}K]_{\text{dB}} - 10\gamma\text{log}_{10}(R/d_0)$$

   it is given that $\overline{P}_r(R) = -100\text{dBm}$, $P_t = 10\text{W} = 40\text{dBm}$.
   Need to find the $[10\text{log}_{10}K]_{\text{dB}} = K_{\text{dB}} = -[P_L(d_0)]_{\text{dB}}$
   $\therefore$, the pathloss at distance $d_0[P_L(d_0)]_{\text{dB}} = [P_L(d_0)]_{\text{dB},freespace} + 18\text{dB}$
   $[P_L(d_0)]_{\text{dB}} = 10\text{log}_{10}\left((\frac{4\pi d_0}{\lambda})^2 \frac{1}{G_t G_r}\right) + 18\text{dB}$
   we know that $d_0 = 1000\text{m}$ and $\lambda = \frac{c}{f} = \frac{3\times 10^8}{900\times 10^6}$.
   $[P_L(d_0)]_{\text{dB}} = 91.5266\text{dB} + 18\text{ dB} = 109.5266\text{ dB}$
   $\therefore$, we get $R = 6.662\text{kms}$

   (b) Cell coverage area, C: Evaluate eq. (2.60) from the text book, where $P_{min} = -110$, $\overline{P}_r(R) = -100$ and $\sigma_{\psi_{\text{dB}}} = 7$. This evaluates for $a = -1.4286$ and $b = 2.2956$. $\therefore$, $C = 0.9779$.

3. Solution:

   (a) $x + x' - l = \sqrt{(h_t + h_r)^2 + d^2} - \sqrt{(h_t - h_r)^2 + d^2} \simeq (d + \frac{(h_t+h_r)^2}{2d}) - (d + \frac{(h_t-h_r)^2}{2d}) = \frac{2h_t h_r}{d}$

   $$\Delta\phi = \frac{4\pi h_t h_r}{\lambda d}$$

   (b) With the assumption of $G_a = G_b = G_c = G_d = 1$ and $R = -1$ and by approximating $x + x\prime \approx l$, signal nulls occur when $\Delta\phi = 2n\pi$

   $$\begin{aligned} \frac{2\pi(x + x' - l)}{\lambda} &= 2n\pi \\ \sqrt{(h_t + h_r)^2 + d^2} - \sqrt{(h_t - h_r)^2 + d^2} &= \frac{\lambda}{2}(2n) \\ \text{Let } m = 2n & \\ \sqrt{(h_t + h_r)^2 + d^2} &= m\frac{\lambda}{2} + \sqrt{(h_t - h_r)^2 + d^2} \end{aligned}$$

   $x = (h_t + h_r)^2, y = (h_t - h_r)^2, x - y = 4h_t h_r$

   $$\begin{aligned} x &= m^2\frac{\lambda^2}{4} + y + m\lambda\sqrt{y + d^2} \\ d &= \sqrt{[\frac{1}{m\lambda}(x - m^2\frac{\lambda^2}{4} - y)]^2 - y} \\ d &= \sqrt{(\frac{4h_t h_r}{(2n)\lambda} - \frac{(2n)\lambda}{4})^2 - (h_t - h_r)^2}, n \in \mathbb{Z} \end{aligned}$$