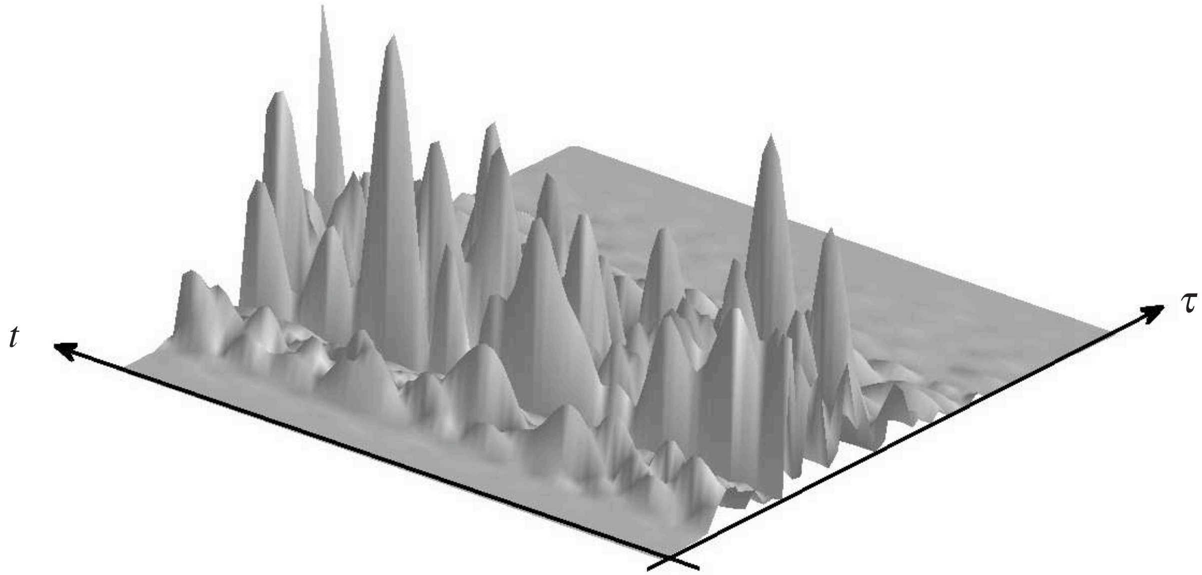
Wireless Communications SSY135 – Lecture 4

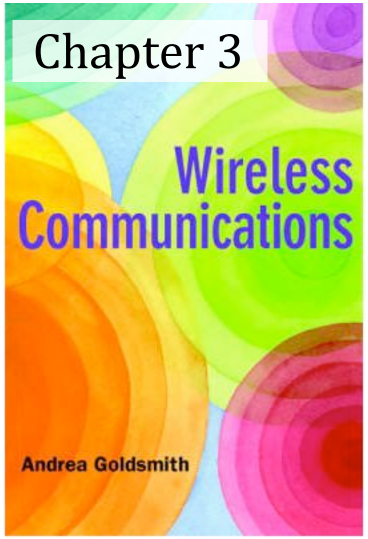
Henk Wymeersch
Department of Electrical Engineering
Chalmers University of Technology
http://tinyurl.com/hwymeers
email: henkw@chalmers.se



http://publik.tuwien.ac.at/files/PubDat_204518.pdf

Topics for today

- Lecture learning outcomes
- Time-varying convolution
- Wideband models: the WSS-US assumption
- Coherence bandwidth and delay spread
- Coherence time and Doppler spread



Suggested reading:
- Every section from Chapter 3, except 3.2.4
- No derivations from sections 3.2.3
- Not derivations (3.18)-(3.27)

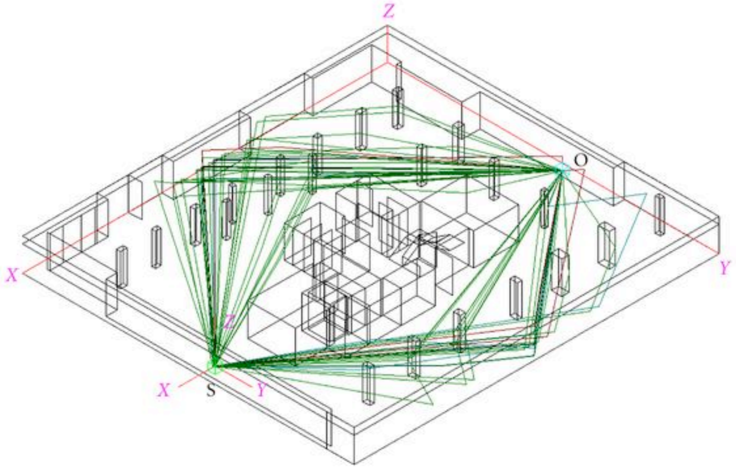
Today's learning outcomes

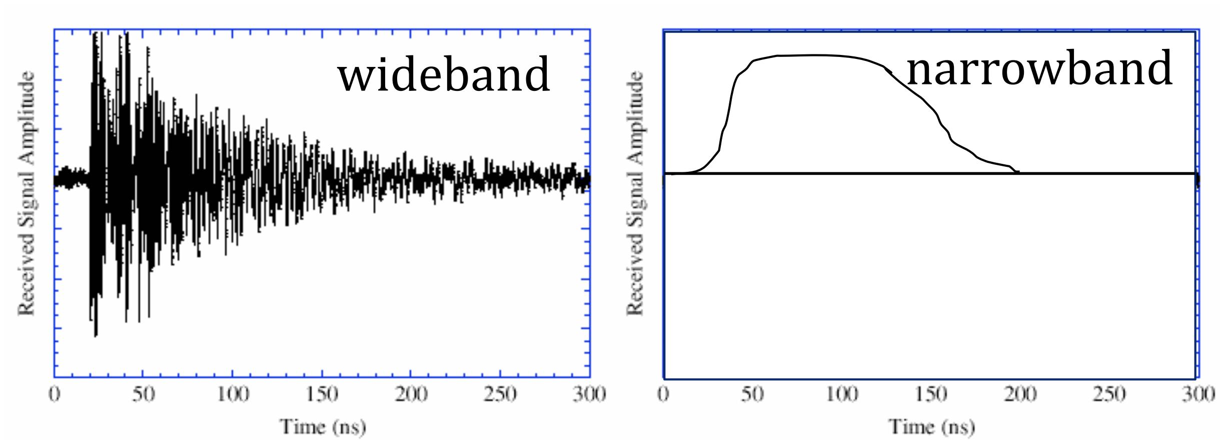
At the end of this lecture, you must be able to

- Describe the difference between
  - (a) delay spread, coherence time, Doppler spread and coherence bandwidth
  - (b) narrowband and wideband communication
  - (c) slow fading and fast fading
- Compute the 2D autocorrelation function of WSS-US processes
- Quantify (a) from the 2D autocorrelation function and its various transforms
- Design a communication system to satisfy (b)-(c)

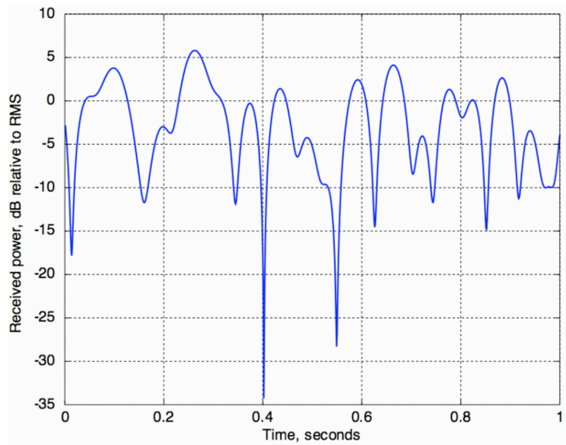
Last time

- Many signal paths add up incoherently





- Signal varies over time due to mobility
  - Modeled through **distribution** and **autocorrelation**
  - Distribution: Rayleigh, Rician
  - Autocorrelation (determines time variation): Jakes' model, related to coherence time



Time-varying impulse response

- Channel: physics

Delay spread [s] ≐ time between first arrival and last reflection (inverse = **coherence bandwidth** [Hz])

fix time, study delay

$c(\tau, t)$

fix delay, study time

Coherence time [s] ≐ time during which channel is fixed (inverse = **Doppler spread** [Hz])

- Communication over a channel: engineering, choose a bandwidth B (T=1/B)

Narrowband communication (also: frequency-flat)

YES

T >> delay spread?

NO

Wideband communication (also: frequency-selective, dispersive)

$c(\tau, t), T = 1/B$

Slow fading communication

YES

T << coherence time

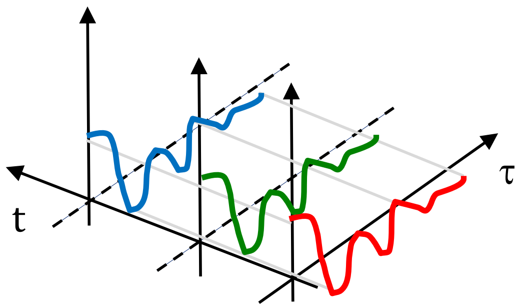
NO

Fast fading communication

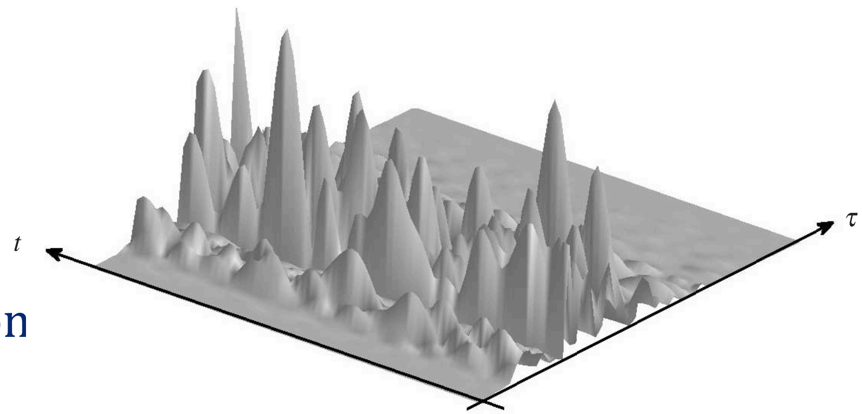
- Today: <u>wideband</u> communication, both slow and fast fading

Extreme cases

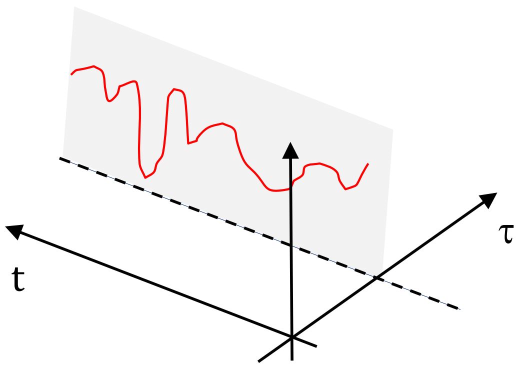
- Wideband (very) slow fading communication



Today: more general



- Narrow-band fast fading communication



Time-varying impulse response

- Many paths, different angles

$$r(t) = \Re\left\{\sum_{n=1}^{N} u(t-\tau_n(t))\alpha_n(t)e^{j2\pi(f_c+f_{n,D})(t-\tau_n(t))}\right\} + n(t)$$

$$= \Re\left\{e^{j2\pi f_c t}\sum_{n=1}^{N} u(t-\tau_n(t))\alpha_n(t)e^{j\phi_n(t)}\right\} + n(t)$$

Baseband model: $\int_{-\infty}^{+\infty} c(\tau,t)u(t-\tau)\mathrm{d}\tau$

- Time-varying convolution

$$c(\tau,t) = \sum_{n=1}^{N} \delta(\tau-\tau_n(t))\alpha_n(t)e^{j\phi_n(t)}$$

- Narrowband fading models: $c(\tau,t) \to c(t)$
  - distribution of envelope: Rayleigh, Rice
  - time-variability: Jakes spectrum; relied on WSS assumption
- Wideband fading models: $c(\tau,t)$

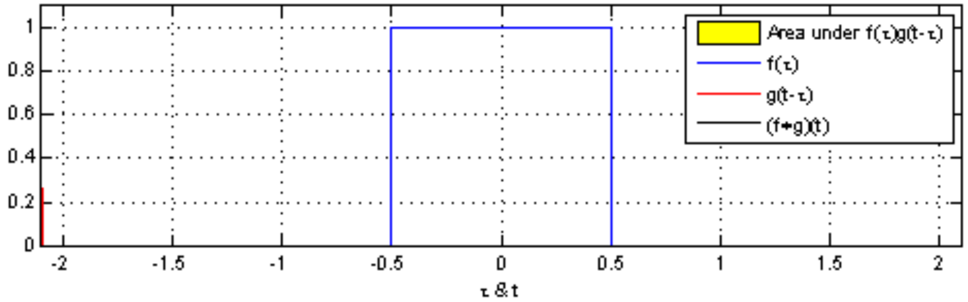
Recall: convolution

- Convolution: time t, delay τ

$$y(t) = \int_{-\infty}^{+\infty} c(\tau)u(t-\tau)\mathrm{d}\tau$$

$$Y(f) = C(f)U(f)$$

$$C(f) = \int c(\tau)e^{-j2\pi f\tau}\mathrm{d}\tau$$



- Time varying convolution: the filter changes over time (t)

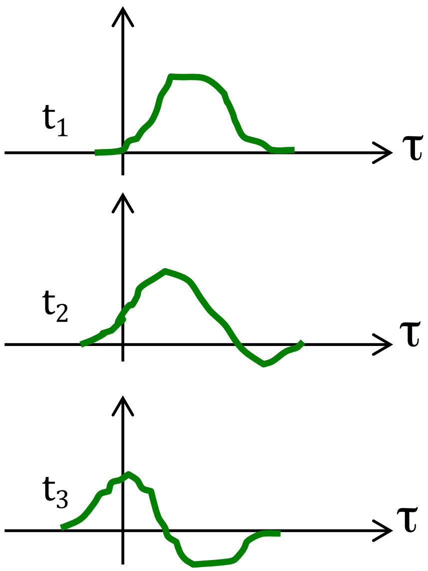
$$y(t) = \int_{-\infty}^{+\infty} c(\tau,t)u(t-\tau)\mathrm{d}\tau$$

$$Y(f) \neq C(f)U(f)$$

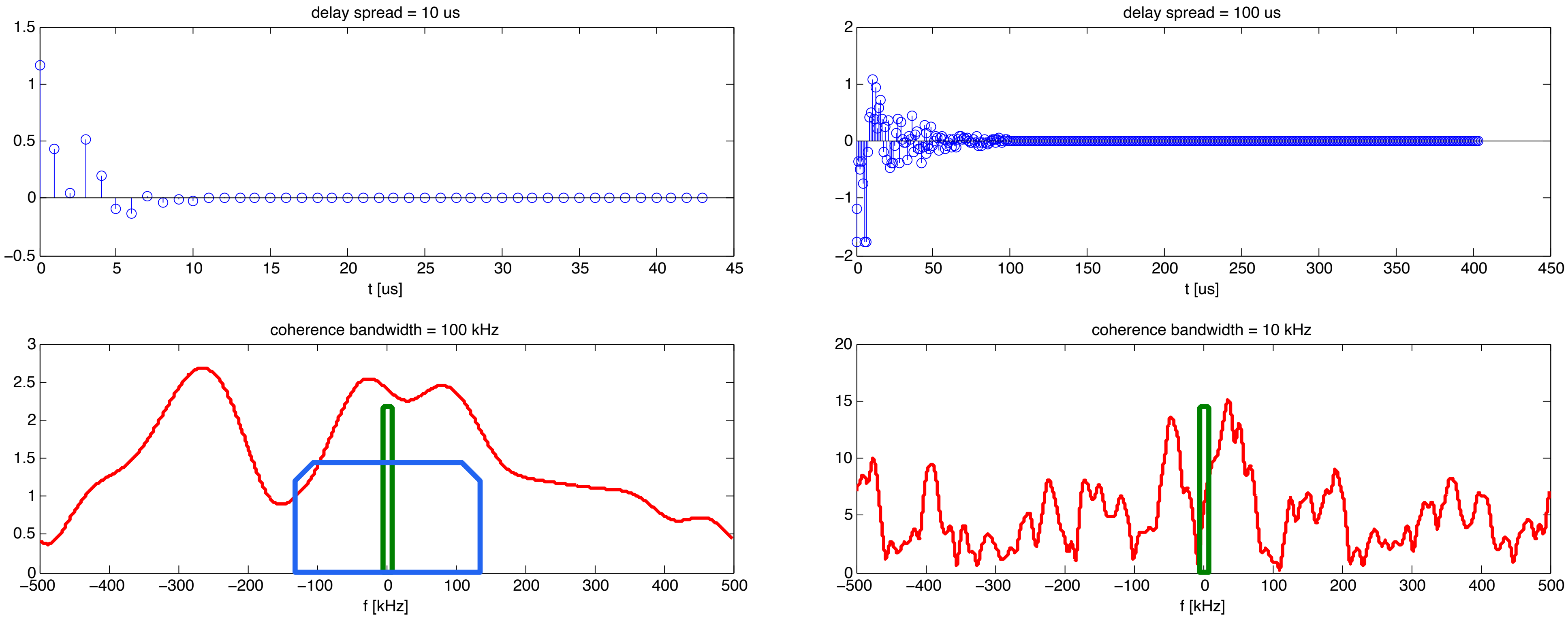
$$C(f,t) = \int c(\tau,t)e^{-j2\pi f\tau}\mathrm{d}\tau$$

$$u(t) = e^{j2\pi f_0 t}$$

$$y(t) = e^{j2\pi f_0 t}C(f_0,t)$$



Delay spread [s] and coherence bandwidth [Hz] example



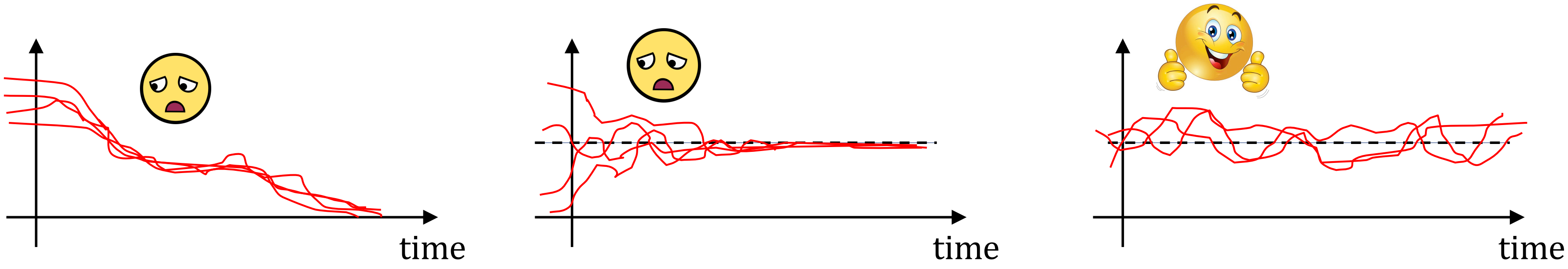
narrowband

wideband

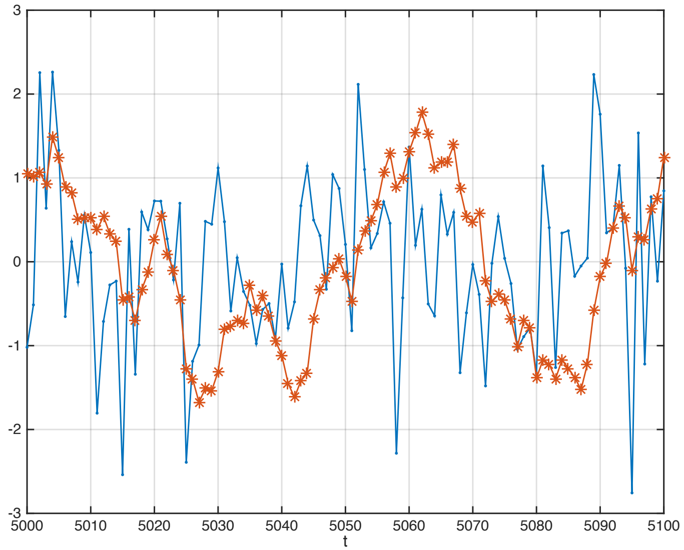
wideband

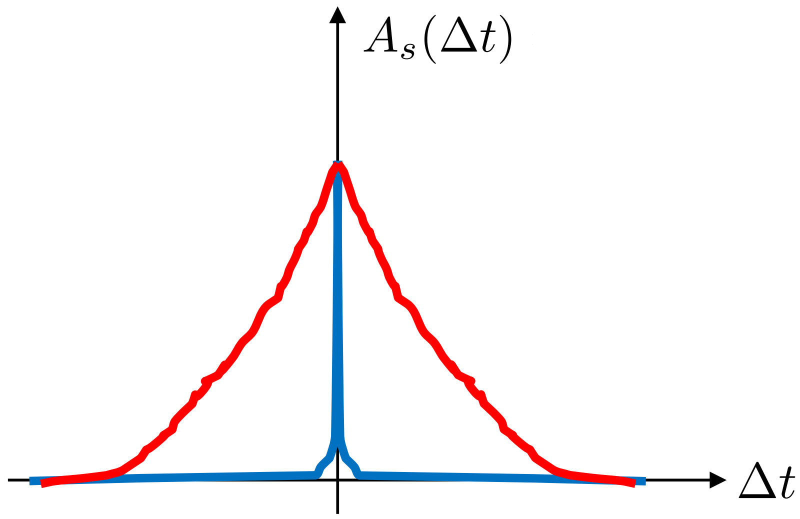
Autocorrelation function (ACF) for 1D signal

- Wide-sense stationary random signal s(t): 1st and 2nd moment not vary in time



- Autocorrelation does not depend on t: $A_s(\Delta t) = \mathbb{E}\{s^*(t)s(t+\Delta t)\}$



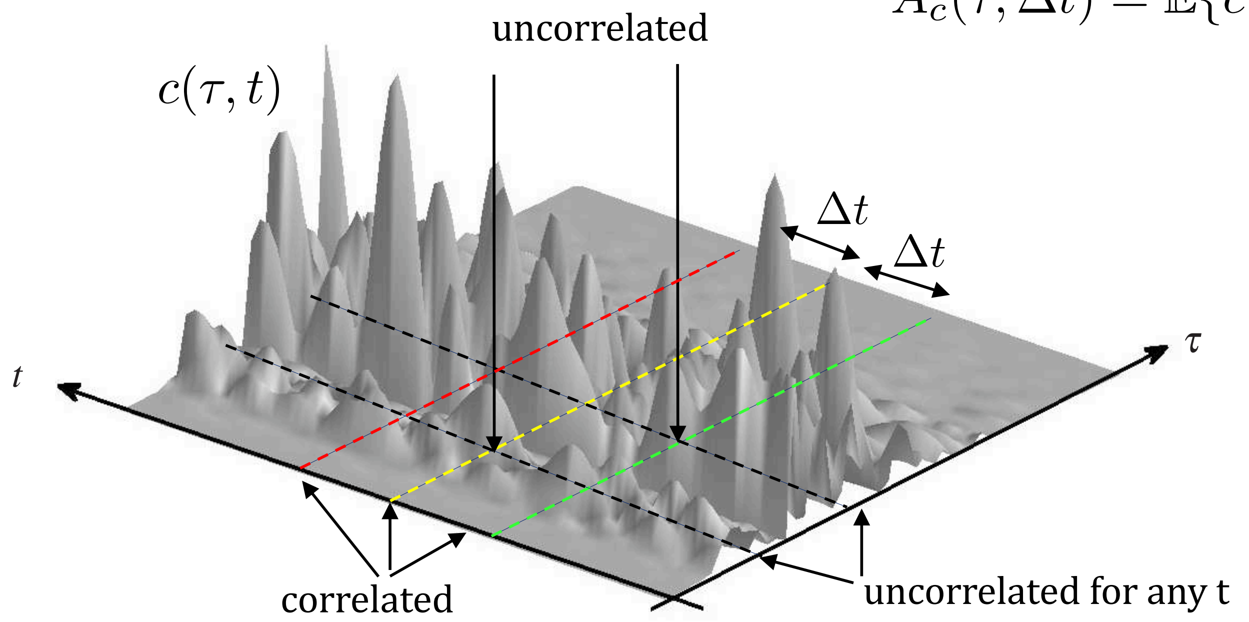


ACF for 2D signal

- Channel is special case
- In general: 4D autocorrelation function $A_c(\tau_1, \tau_2; t, t+\Delta t) = \mathbb{E}\{c^*(\tau_1, t)c(\tau_2, t+\Delta t)\}$

- We consider "simple" model: only 2 dimensions remain in ACF:

$$A_c(\tau, \Delta t) = \mathbb{E}\{c^*(\tau, t)c(\tau, t+\Delta t)\}$$



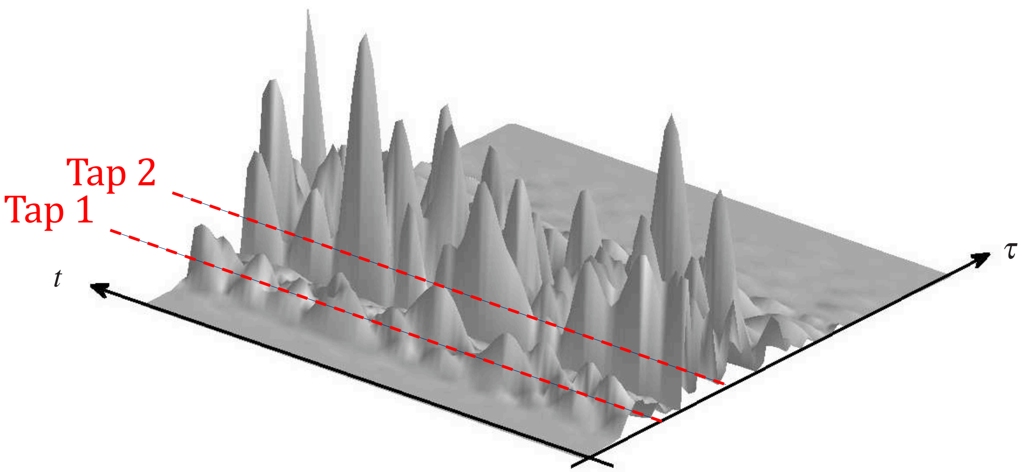
Wideband fading models

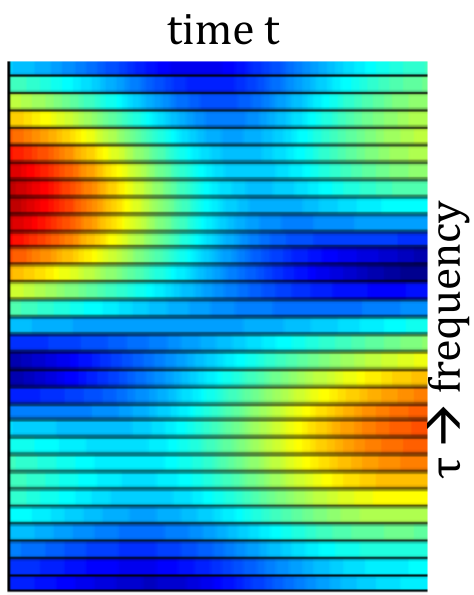
- Key assumptions (ignoring path loss, shadowing, only NLOS)
  1. $c(\tau, t)$ is a 2D complex Gaussian process: characterized by mean, autocorrelation and cross-correlation (I/Q)
  2. phases of multipath component uniform, independent: mean = 0, cross-correlation = 0
- Autocorrelation $A_c(\tau_1, \tau_2; t, t+\Delta t) = \mathbb{E}\{c^*(\tau_1, t)c(\tau_2, t+\Delta t)\}$
- Additional assumptions: WSS-US: $A_c(\tau, \Delta t) = \mathbb{E}\{c^*(\tau, t)c(\tau, t+\Delta t)\}$
  1. Wide-sense stationary (WSS): no dependence on t
  2. Uncorrelated scattering (US): no correlation for different delays (tau)

- Important quantities derived from autocorrelation function
  1. RMS delay spread σ_{T_m}
  2. Coherence bandwidth $B_c \approx 1/\sigma_{T_m}$
  3. Doppler spread B_D
  4. Coherence time $T_c \approx 1/B_D$

Tapped delay line model: satisfies WSS-US

- M time-varying taps, spaced Δ apart (Δ smaller than 1/B)
- M Δ > delay spread
- Each tap (m) is *independent* random process
  - Average power derived from $A_c(\tau, 0)$
  - Autocorrelation derived from $A_c(\tau, \Delta t)$
  - Statistics: Rayleigh (i.e., Gaussian values), Ricean, Nakagami, …





Generating a Rayleigh wideband channel

Given signaling band B, delay spread D, total power P_{RX}

1. Set L channel taps 1/(2B) apart, with $L = D \times 2B$
2. Choose variance for each tap according to a profile $A_c(\tau, 0)$ (e.g. exponential) $\sigma_i^2 = \alpha \sigma_{i-1}^2, \alpha \leq 1$ such that power is [more dense taps, same total power]
$$\sum_{i=1}^{L} \sigma_i^2/(2B) = P_{RX}$$
3. Generate channel tap i: $c_i = \mathcal{CN}(0, \sigma_i^2)$
4. To model time-evolution $A_c(\tau, \Delta t)$ of c_i, use a proper filter with bandwidth equal to Doppler spread

Comments:

- Instead of P_{RX} in step 2, use "1" for a normalized PDP
- For Ricean fading: add constant to first tap, redistribute energy based on K-factor

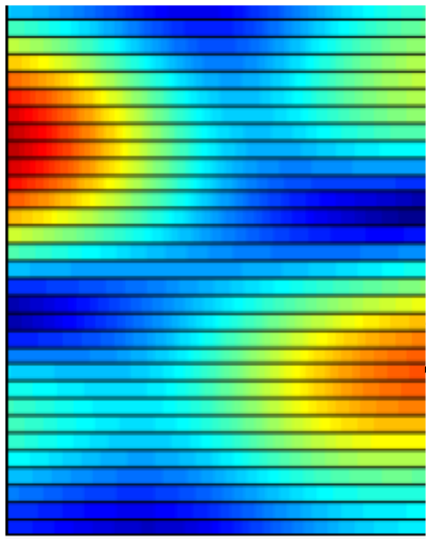
What can be derived from autocorrelation function?

- Example: correlation of channel at time t for two different frequencies?

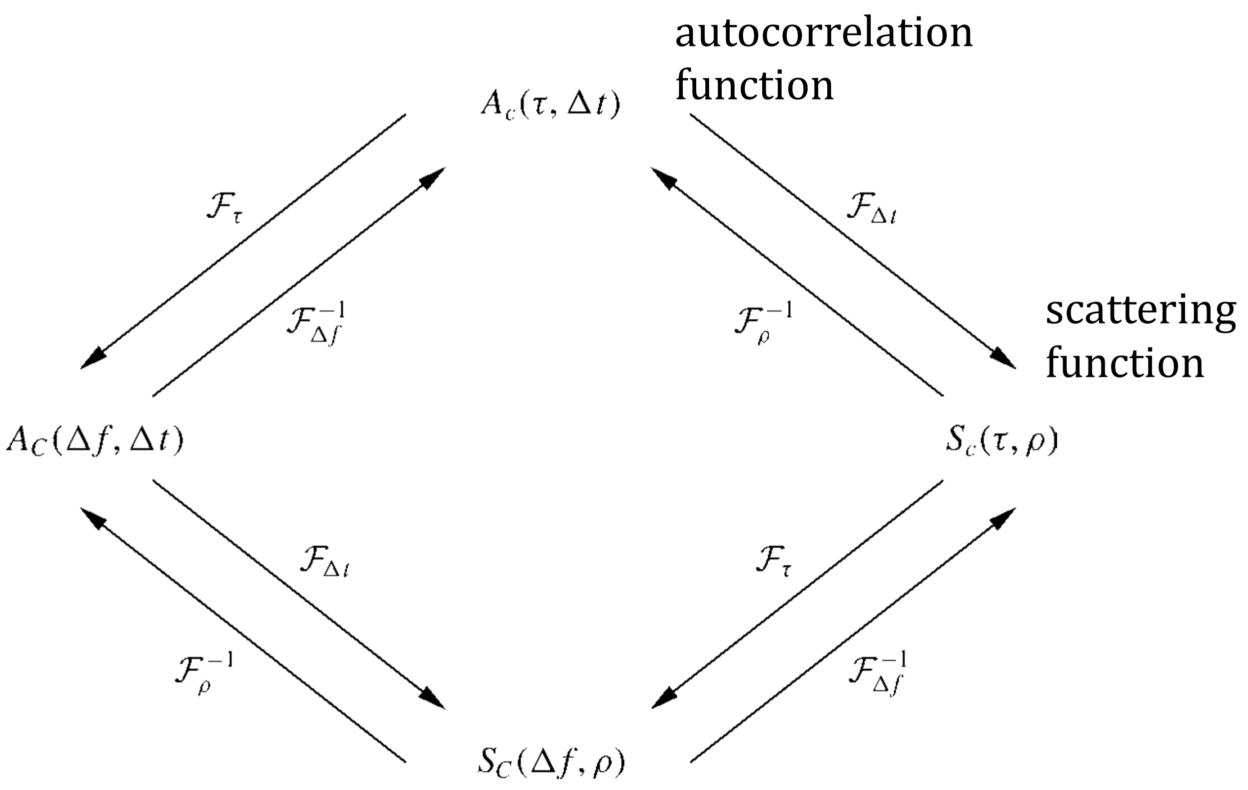
$$\mathbb{E}\{C^*(f_1,t)C(f_2,t)\}$$

- Use US property:

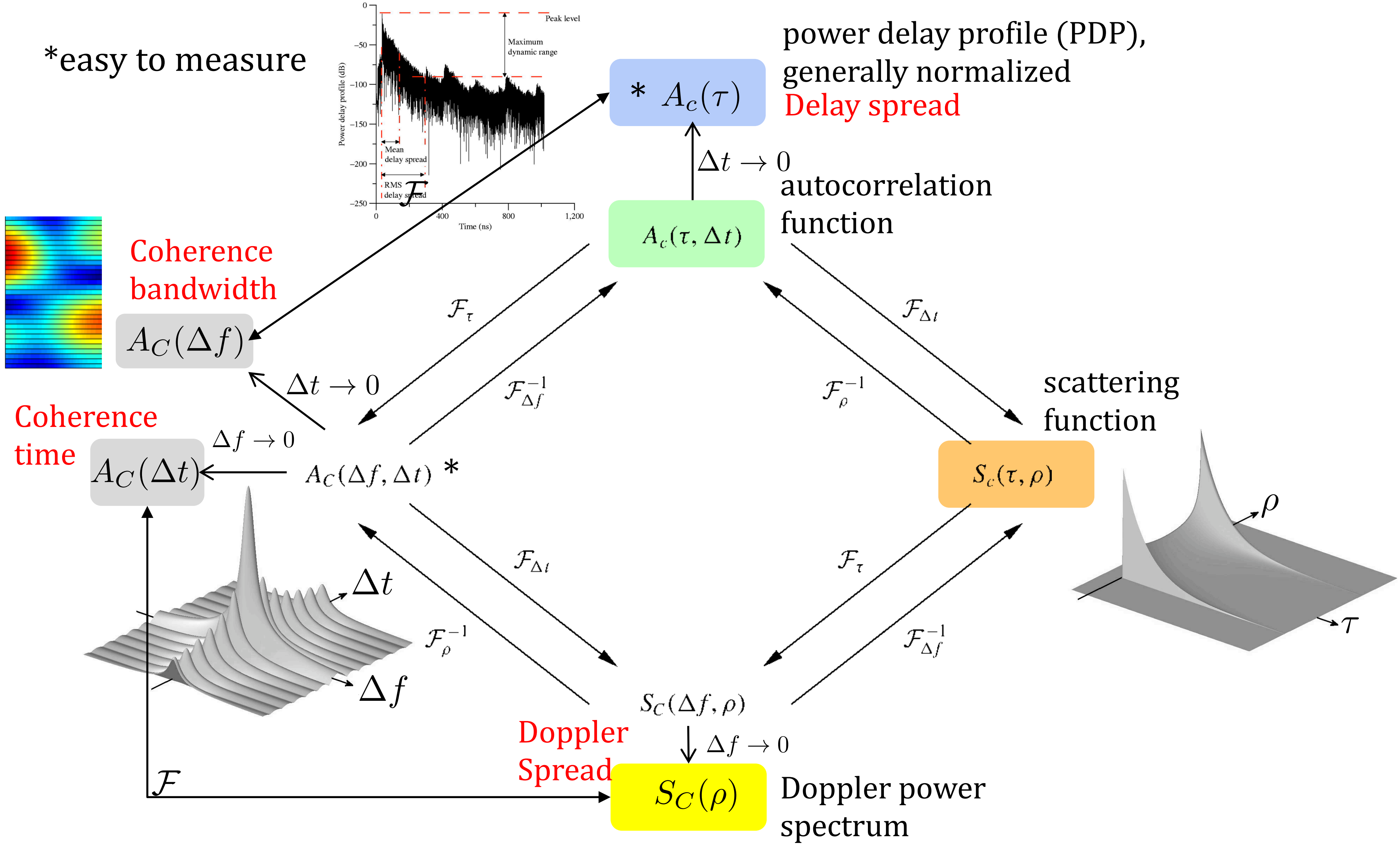
$$\begin{aligned}\mathbb{E}\{C^*(f_1,t)C(f_2,t)\} &= \mathbb{E}\{\int c^*(\tau_1,t)e^{-j2\pi f_1\tau_1}d\tau_1 \int c\ \ (\tau_2,t)e^{j2\pi f_2\tau_2}d\tau_2\} \\ &= \int\int \mathbb{E}\{c^*(\tau_1,t)\quad c\ \ (\tau_2,t)\}e^{-j2\pi f_1\tau_1}d\tau_1 e^{j2\pi f_2\tau_2}d\tau_2 \\ &= \int \mathbb{E}\{c^*(\tau,t)\quad c\ \ (\tau,t)\}e^{-j2\pi f_1\tau}e^{j2\pi f_2\tau}d\tau \\ &= \int A_c(\tau,0)e^{j2\pi(f_2-f_1)\tau}d\tau \\ &\doteq A_C(\Delta f)\end{aligned}$$



Autocorrelation function and Fourier transforms



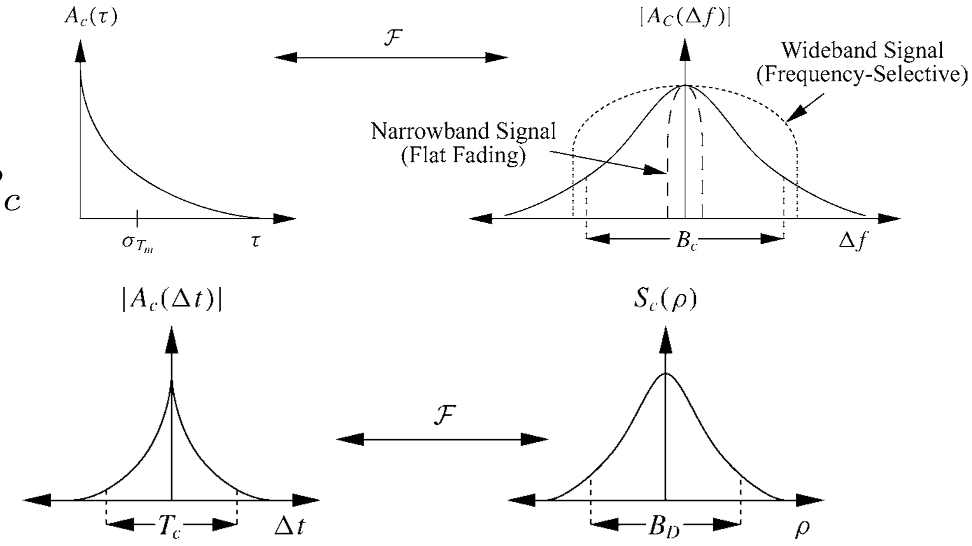
Autocorrelation function and Fourier transforms



Matz, Gerald, and Franz Hlawatsch. "Fundamentals of time-varying communication channels." *Wireless Communications over Rapidly Time-Varying Channels* (2011): 1-63.

Flat, Dispersive, Fast, Slow

- Flat fading: $B \ll B_c$
- Frequency-selective fading: $B \gg B_c$



- Slow fading: $T \ll T_c$
- Fast fading: $T \approx T_c$

(RMS) delay spread [s]: fix t, how fast does channel decay with τ?
Coherence bandwidth [Hz]: fix t, over which bandwidth is the channel flat?

Coherence time [s]: how long does it take for the channel to decorrelate?
Doppler spread [Hz]: at what rate does a frequency component change?

Autocorrelation function: special cases

Given

- A time-invariant channel
- A frequency-flat time-varying channel

Task

- What are the expressions for the 2D autocorrelation function?
- How would you determine the 4 important quantities?

Coherence time and coherence bandwidth

Given

- A communication system with a data rate of R_s
- A channel with coherence time 12.5 ms and a delay spread of 50 ns

Task

- What is the coherence bandwidth?
- What is the Doppler spread?
- For what values of R_s is there only negligible ISI?
- For a baud rate of 1MBaud, is it reasonable to assume that the channel remains fixed for around 100 consecutive symbols?

Today's learning outcomes

At the end of this lecture, you must be able to

- Describe the difference between
  - (a) delay spread, coherence time, Doppler spread and coherence bandwidth
  - (b) narrowband and wideband communication
  - (c) slow fading and fast fading
- Compute the 2D autocorrelation function of WSS-US processes
- Quantify (a) from the 2D autocorrelation function and its various transforms
- Design a communication system to satisfy (b)-(c)

Solutions

Autocorrelation function: special cases

Given

- A time-invariant channel
- A frequency-flat time-varying channel

Task

- What are the expressions for the 2D autocorrelation function?
- How would you determine the 4 important quantities?

Solution

Time-invariant channel $c(\tau, t) = c(\tau)$

with autocorrelation function $A_c(\tau, \Delta t) = A_c(\tau) = \mathbb{E}\{|c(\tau)|^2\}$ from which we can determine delay spread. The coherence time is infinite (the channel is constant).

Frequency-flat time-varying channel $c(\tau, t) = c(t)$

with autocorrelation function $A_c(\tau, \Delta t) = \mathbb{E}\{c^*(t)c(t + \Delta t)\} = A_c(\Delta t)$ from which we can compute the coherence time. The coherence bandwidth is infinite (the channel is flat).

Coherence time and coherence bandwidth

Given

- A communication system with a data rate of R_s
- A channel with coherence time 12.5 ms and a delay spread of 50 ns

Task

- What is the coherence bandwidth?
- What is the Doppler spread?
- For what values of R_s is there only negligible ISI?
- For a baud rate of 1MBaud, is it reasonable to assume that the channel remains fixed for around 100 consecutive symbols?

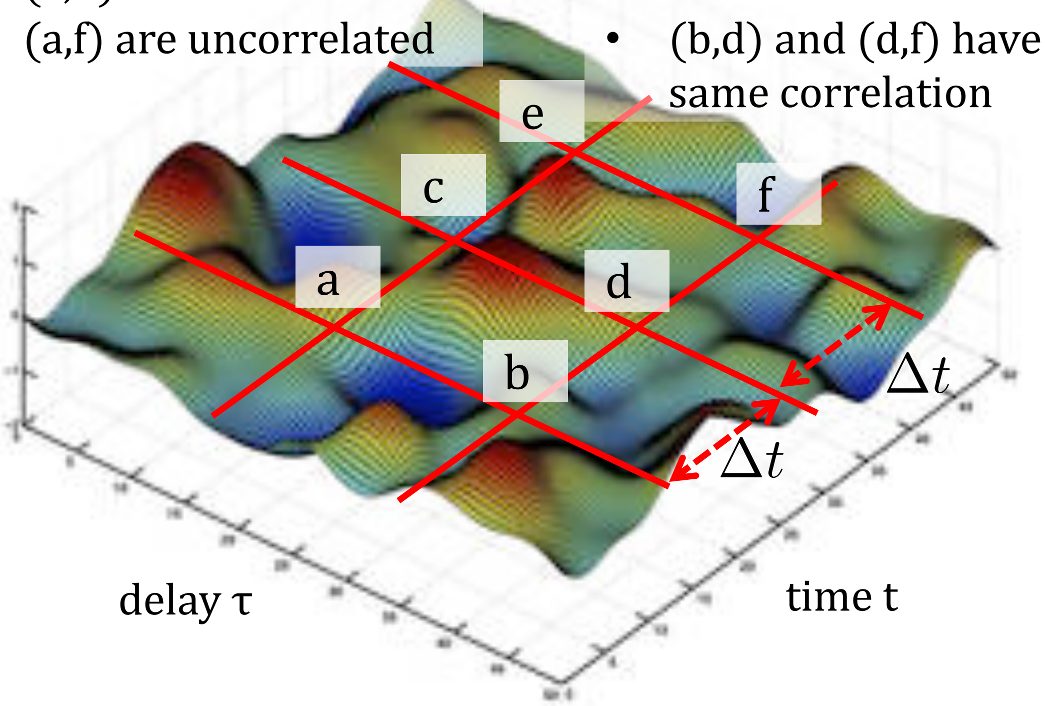
Solution

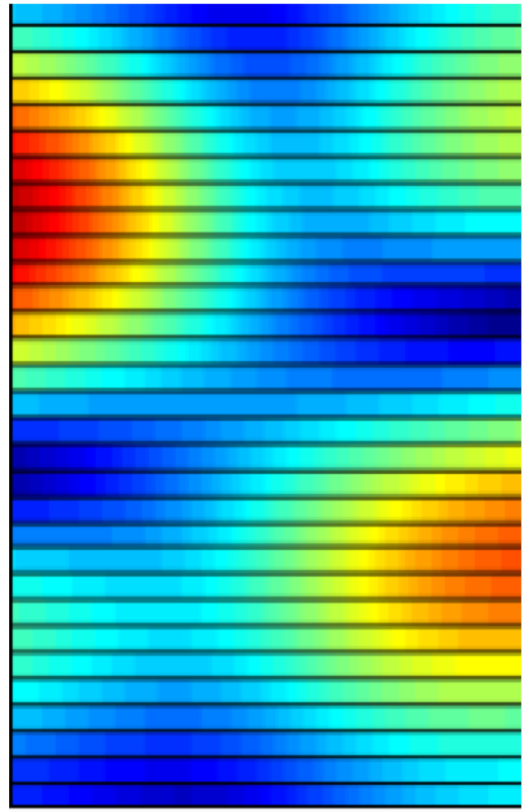
- Coherence bandwidth = 1/(50ns) = 20 MHz
- Doppler spread = 1/(12.5ms) = 80 Hz
- Negligible ISI: $R_s \ll 20$ MHz, so one should transmit at a rate significantly smaller than 20 Mbaud
- 1Mbaud: 1 symbol = 1 us, so 100 symbols is 0.1ms. Since the coherence time is 12.5ms≫0.1ms, the channel will be approximately constant during 100 symbols.

Two-dimensional Gaussian process

Uncorrelated scattering
- (a,b) are uncorrelated
- (a,d) are uncorrelated
- (a,f) are uncorrelated

Wide-sense stationary
- (a,c) and (c,e) have same correlation
- (b,d) and (d,f) have same correlation





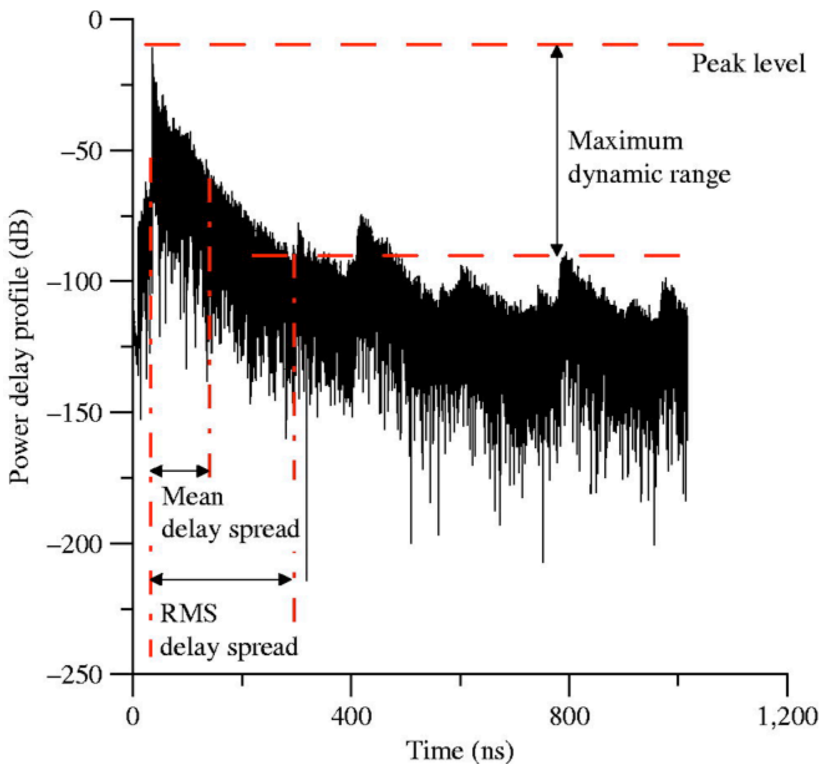
RMS delay spread [s]: fix t, how fast does channel decay with τ?
Coherence bandwidth [Hz]: fix t, over which bandwidth is the channel flat?
Doppler spread [Hz]: at what rate does a frequency component change?
Coherence time [s]: over what time window does a frequency component remain flat?

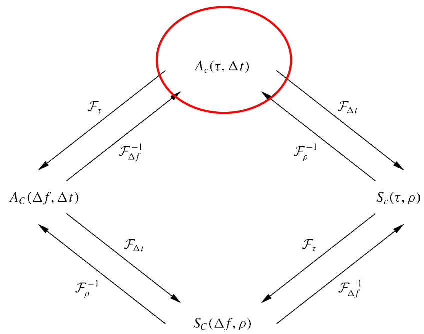
Delay spread

Derivations, starting from $A_c(\tau, \Delta t)$

- Set $\Delta t = 0$ to obtain PDP: $A_c(\tau,0) = \mathbb{E}\{|c(\tau,t)^2\} = \mathbb{E}\{|c(\tau,0)^2\}$
- Then

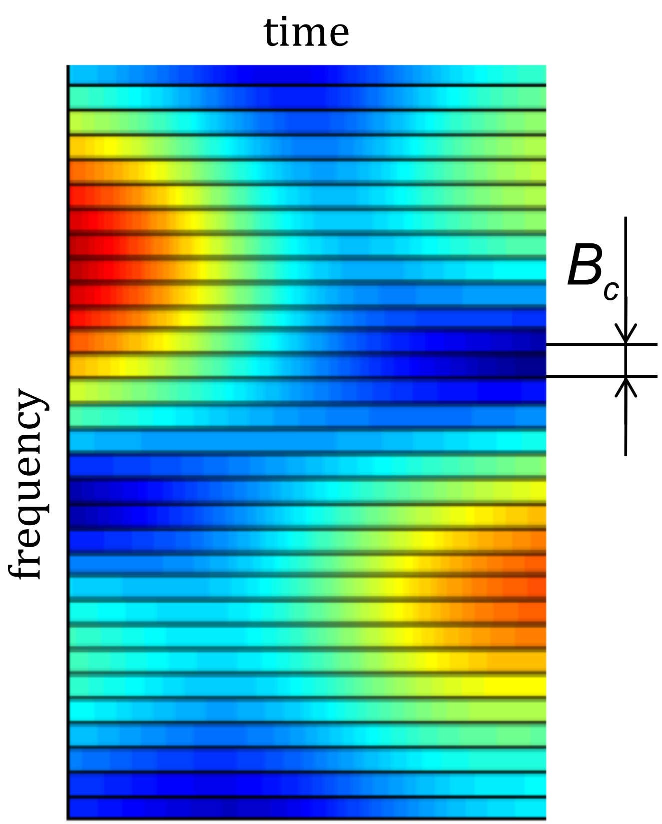
$$\sigma_{T_m} = \sqrt{\frac{\int_0^\infty (\tau - \mu_{T_m})^2 A_c(\tau)\, d\tau}{\int_0^\infty A_c(\tau)\, d\tau}}.$$





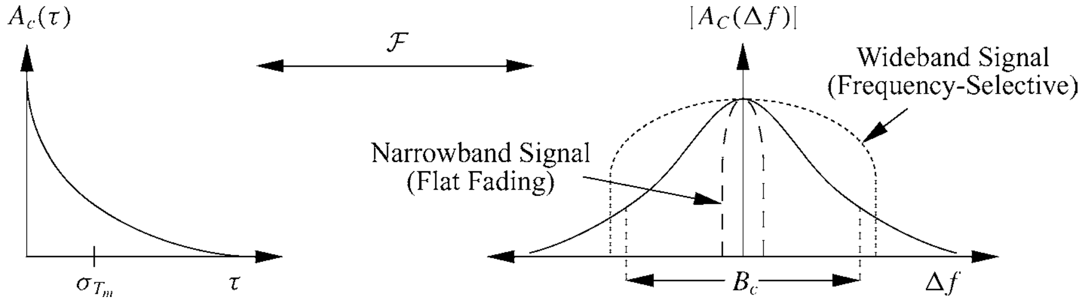
Coherence bandwidth

Realization of channel

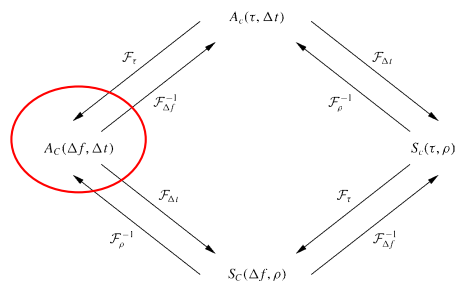


Derivations, starting from $A_c(\tau, 0)$

- **Coherence bandwidth:** from Fourier transform of PDP

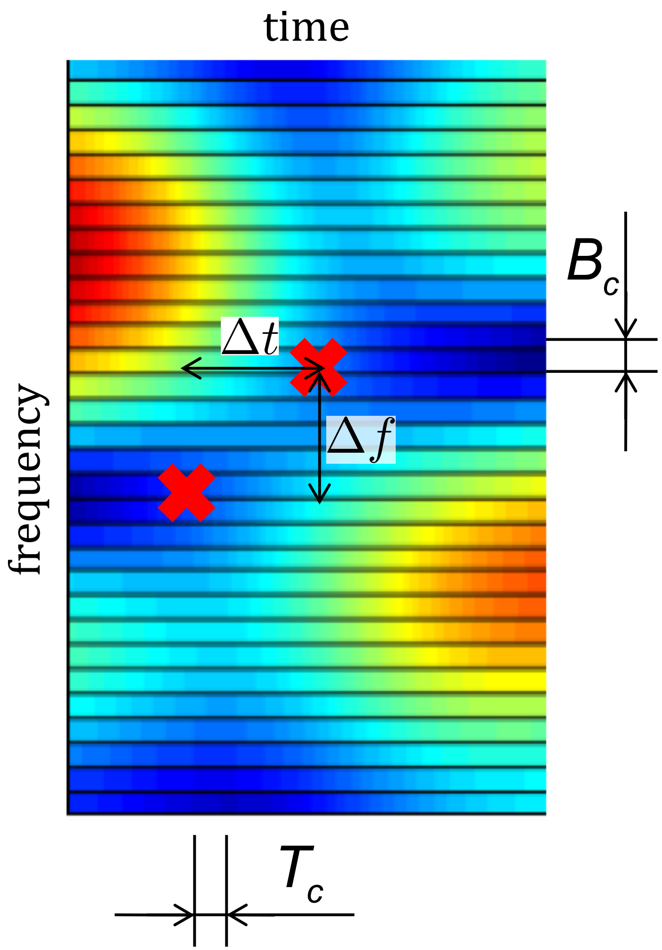


- Flat fading: $B \ll B_c$
- Frequency-selective fading: $B \gg B_c$



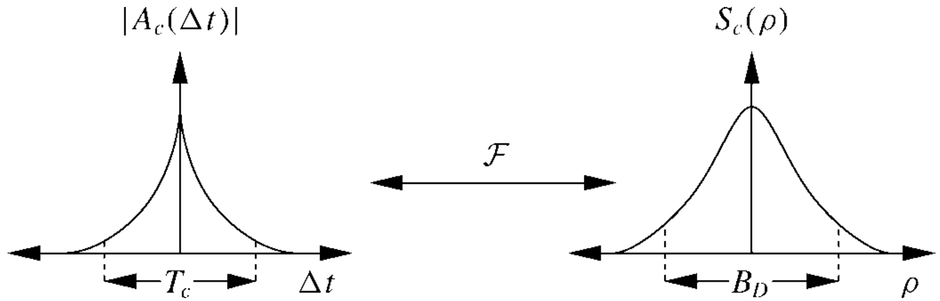
Coherence time and Doppler spread

Realization of channel



Derivations, starting from $A_c(\tau, \Delta t)$

- **Coherence time, Doppler spread:** fixed frequency
  1. go to frequency domain
     $$A_C(\Delta f, \Delta t) = \int_{-\infty}^{+\infty} A_c(\tau, \Delta t) e^{-j2\pi\Delta f \tau} \mathrm{d}\tau$$
  1. Let $\Delta f \to 0$ to obtain $A_C(\Delta t)$, describing overall time variation of channel impulse response
  2. Fourier transform of $A_C(\Delta t)$ is Doppler power spectrum $S_c(\rho)$



- Slow fading (w.r.t. symbol rate)
- Fast fading (w.r.t. symbol rate)