Exercise 10 in SSY135 Wireless Communications
Topic: 5G Communication (Massive MIMO)

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1. (a) Show that the capacity of MIMO channel with the assumption that receiver has perfect knowledge of the channel matrix, and the noise with complex Gaussian distribution with unit variance is upper bounded as

$$C \leq \min\left(N_t, N_r\right) \log_2 \left(1 + \frac{P \times \ Tr\left(\boldsymbol{H}\boldsymbol{H}^H\right)}{N_t \times \min(N_t, N_r)}\right),$$

where $Tr\left(\boldsymbol{H}\boldsymbol{H}^H\right) = \sum_{l=1}^{\min(N_t,N_r)} \lambda_l^2$ and λ_l is the lth singular value of the channel matrix $\boldsymbol{H}$ and also we have assumed that $\boldsymbol{H}$ is full-rank. Note that $\mathbb{E}\{\lambda_l^2\} = \sum_{l=1}^{\min(N_t,N_r)} \frac{\lambda_l^2}{\min(N_t,N_r)}$ holds, if you assume that λ_l^2 are distributed uniformly.

(b) Find the condition for channel matrix $\boldsymbol{H}$, in which the upper bound is changed into the following

$$C \leq \min\left(N_t, N_r\right) \log_2 \left(1 + \frac{P \times \ \max(N_t, N_r)}{N_t}\right).$$

(c) Evaluate the upper bound in part (b), when $N_t \gg N_r$.

Hints:

- The capacity of MIMO channel with the assumption that receiver has perfect knowledge of the channel matrix, $\boldsymbol{H}$, and noise with complex Gaussian distribution with unit variance is computed as

$$C = \log_2 \det\left(\boldsymbol{I}_{N_r} + \frac{P}{N_t}\boldsymbol{H}\boldsymbol{H}^H\right),$$

where P is the transmit power, N_t is the number of transmit antennas, N_r is the number of receive antennas, $\boldsymbol{I}_{N_r}$ and $(.)^H$ are the $N_r \times N_r$ identity

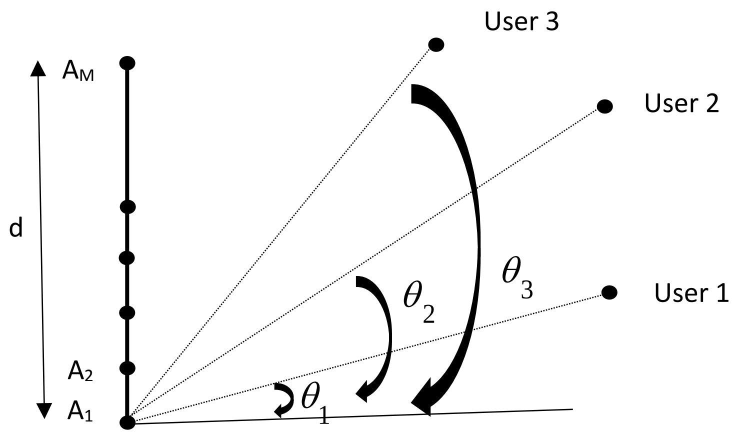

Figure 1: Cellular system with base station with linear array antenna with M antennas and 2 users.

matrix and Hermitian operation, respectively. Apply SVD decomposition and Jensen's inequality to prove the upper bound.

- Jensen's inequality: For a concave function $f(x)$, we have

$$\mathbb{E}\{f(x)\} \leq f(\mathbb{E}\{x\}),$$

while for a convex function $g(x)$, we have

$$\mathbb{E}\{g(x)\} \geq g(\mathbb{E}\{x\}).$$

where $\mathbb{E}\{.\}$ is the expectation function.

2. Assume that in a cellular system with three users, the base station is equipped with a uniform linear array antenna with M antennas and total length d, shown in Fig. 1. Consider LoS only, so g_1, g_2, and g_3 are complex numbers that depend on path loss only, but has a random phase. Furthermore, the users are sufficiently far from the array antenna, i.e., the rays from each element of the array antenna to each user are parallel. The received vector at the two users can be written as

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n},$$

where $\boldsymbol{H}$ is the channel matrix, $\boldsymbol{x} = [x_1, x_2, x_3]^T$ is the transmit vector, $\boldsymbol{y} = [y_1, y_2, y_3]^T$ is the received vector, and $\boldsymbol{n} = [n_1, n_2, n_3]^T$ is the received noise.

(a) Show that the channel matrix can be written as [assume that $d = \frac{\lambda}{2}$, where λ is the wavelength.]

$$\mathbf{H} = \begin{bmatrix} g_1 & g_2 & g_3 \\ g_1 e^{-(j\pi/(M-1))\sin\theta_1} & g_2 e^{-(j\pi/(M-1))\sin\theta_2} & g_3 e^{-(j\pi/(M-1))\sin\theta_3} \\ g_1 e^{-(j2\pi/(M-1))\sin\theta_1} & g_2 e^{-(j2\pi/(M-1))\sin\theta_2} & g_3 e^{-(j2\pi/(M-1))\sin\theta_3} \\ \cdot & \cdot & \cdot \\ g_1 e^{-(j\pi)\sin\theta_1} & g_2 e^{-(j\pi)\sin\theta_2} & g_3 e^{-(j\pi)\sin\theta_3} \end{bmatrix}^T.$$

(b) Now assume that the array has 30 antennas with $d = \lambda/2$, working on carrier frequency 900 MHz. The users are located such that $\theta_1 = \pi/6$, $\theta_2 = \pi/4$, $\theta_3 = \pi/3$ $g_1 = 1$, $g_2 = 2$, and $g_3 = 3$. Find the zero-forcing beamforming at the transmitter to remove the interference from one user to the other ones, i.e., find the matrix $\boldsymbol{A}$ in order to precode the data as $\boldsymbol{Ax}$. Note that due to power constraint at the transmitter, the norm of matrix $\boldsymbol{A}$ should be equal to 1, i.e., $||\boldsymbol{A}||_F = \sqrt{Tr\left(\boldsymbol{A}\boldsymbol{A}^H\right)} = 1$.

(c) Now assume that after precoding with matrix $\boldsymbol{A}$, the output power for each data streams should be equal to 5 mW. Find the powers P_1, P_2, and P_3 corresponding to the transmit power of x_1, x_2, and x_3.

(d) Repeat part (b) and (c), assuming that user one and two become closer, in which $\theta_1 = \pi/6$, $\theta_2 = \pi/6 + \pi/35$, and $\theta_3 = \pi/3$.