Wireless Communications SSY135 – Lecture 9

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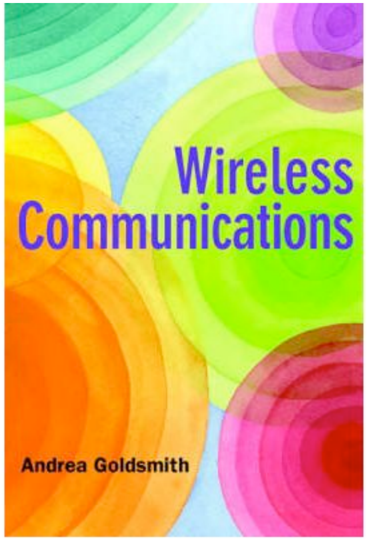
Today's learning outcomes

At the end of this lecture, you must be able to

- Describe uplink and downlink, multiple access and random access
- Mathematically describe TDMA, FDMA, and CDMA and list their properties
- Design and analyze a multiple access scheme for a given scenario and compute the SIR and SINR

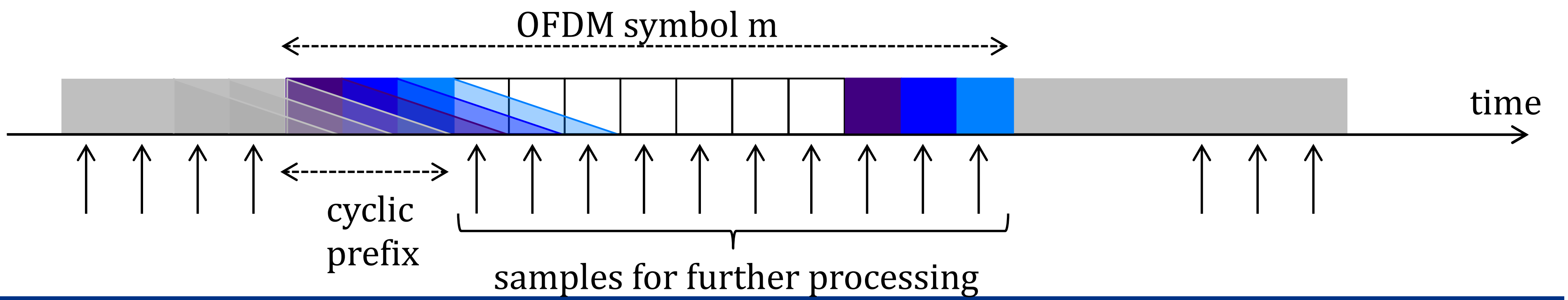
Outline

• Last lecture recap: OFDM	Chapter 12
• Multi-user uplink, downlink	Section 14.1
• Duplexing	Section 14.1
• Multiple access	Section 14.2
• Cellular networks	Section 15.1-15.2
• Power control	Section 15.5.3
• Ad-hoc networks: random access	Section 14.3 (no math)

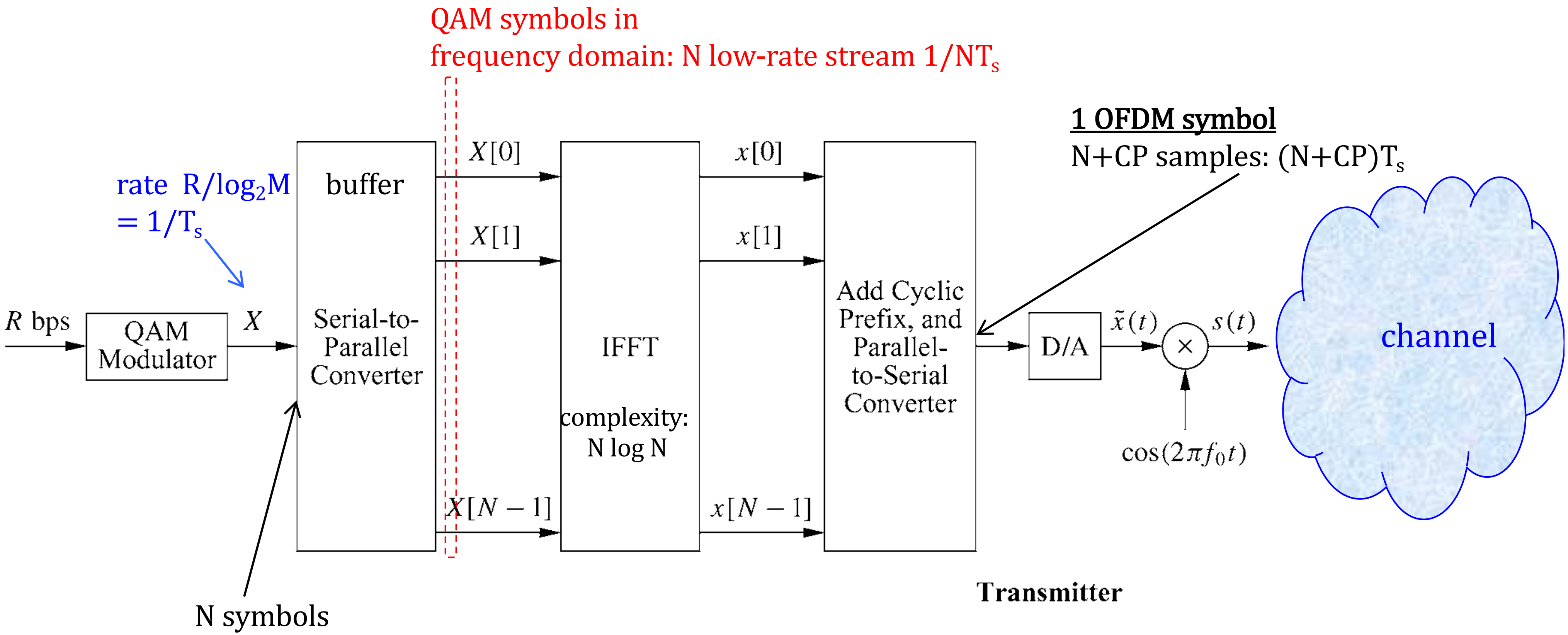


Last time: OFDM

- Increasing the data rate
  - Increase bits per symbol (limited by SNR)
  - Reduce duration per symbol (limited by ISI)
- Digital multicarrier: based on FFT and CP
- For fixed sample time
  - CP duration more than delay spread
  - OFDM symbol duration much less than coherence time
- Combine with waterfilling
- PAPR is important drawback

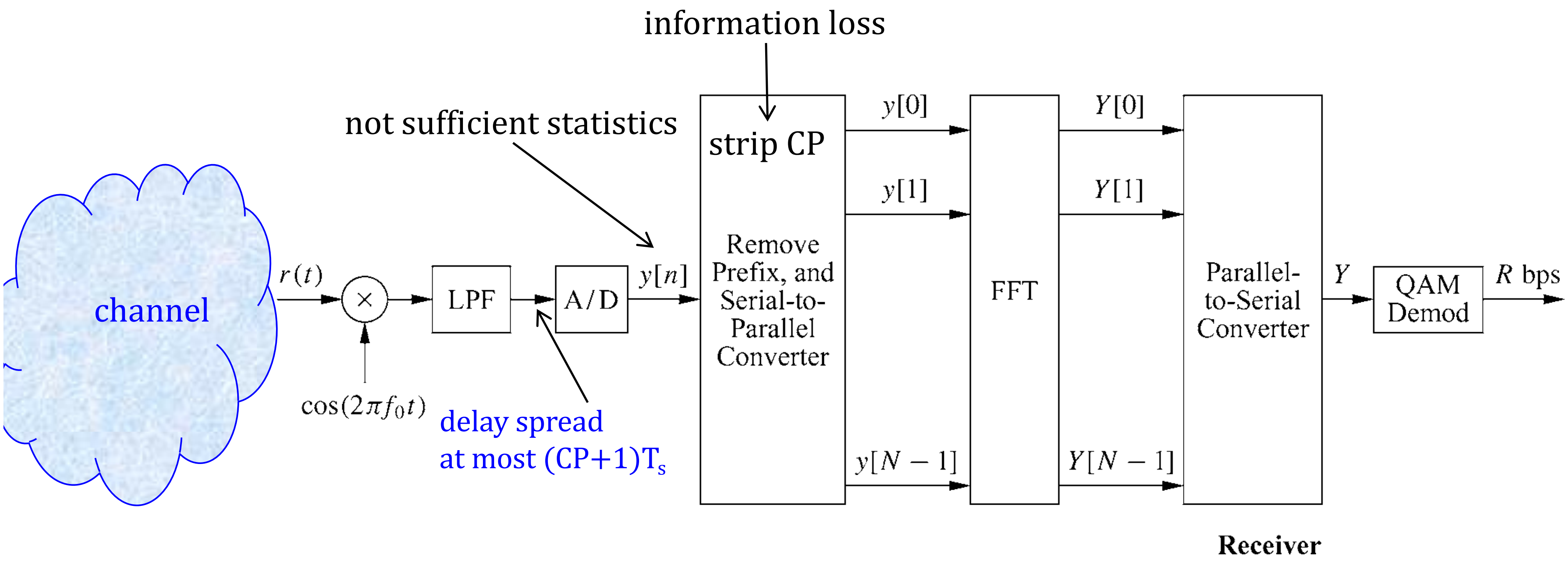


OFDM transmitter



- N should be power of 2 for low complexity FFT
- CP overhead based on the channel delay spread
  - Reduction in rate by CP/(N+CP)
  - Additional power
  - Trade-off between robustness and rate loss

OFDM receiver



- Channel must remain constant during each OFDM symbol
- In the absence of noise $Y[i] = \sqrt{N} \times X[i] \times H[i]$

Multiuser communication

- Multiple users share the same channel, so communication needs to be coordinated
- Systems: cellular and ad-hoc
- Cellular:
  - Base station to mobile: downlink
  - Mobile to base station: uplink

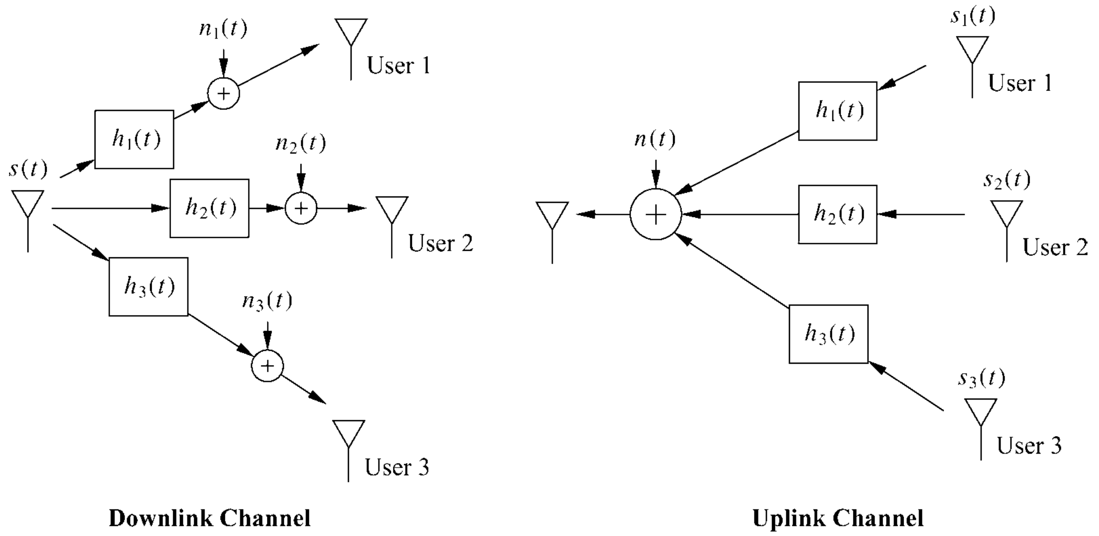


Figure 14.1: Downlink and uplink channels.

- Recall single user communication

$$x(t) = \sum_{l=-\infty}^{+\infty} a_l s(t - lT_s)$$

$$= \sum_{l=-\infty}^{+\infty} a_l s_l(t)$$

$$y(t) = \alpha x(t) + n(t)$$

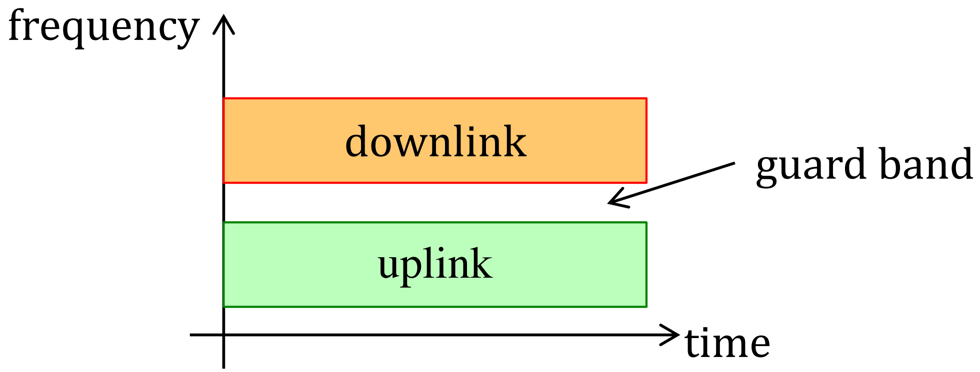
$$z_l = \int y(t) s_l^*(t) \mathrm{d}t$$

$$= \alpha \sum_{l'=-\infty}^{+\infty} a_l' \underbrace{\int s_l'(t) s_l^*(t) \mathrm{d}t}_{=0, l \neq l'} + \int n(t) s_l^*(t) \mathrm{d}t$$

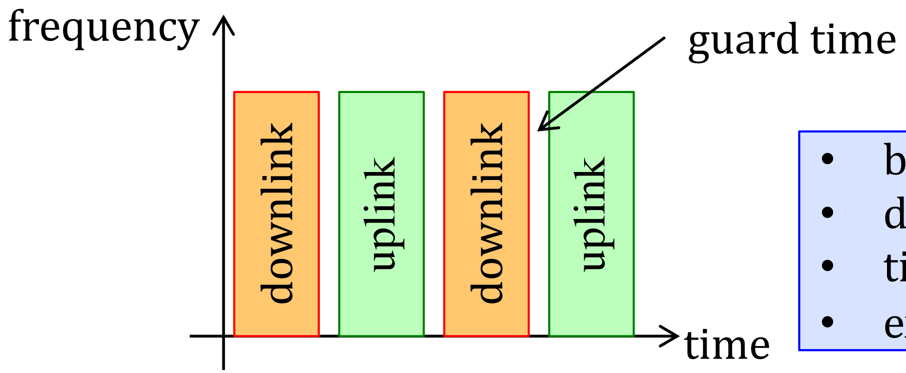
$$= \alpha a_l + n_l$$

Duplexing

- Uplink and downlink can not be in same time-frequency
- Frequency division duplexing



- Time division duplexing

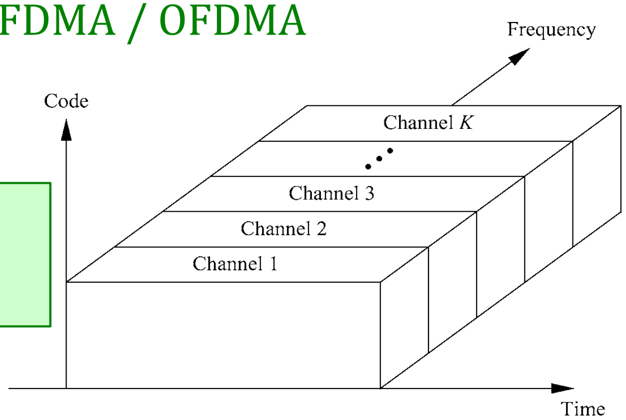


- better for asymmetric traffic
- delays
- tight synchronization
- exploit UL/DL channel symmetry

Multiple access

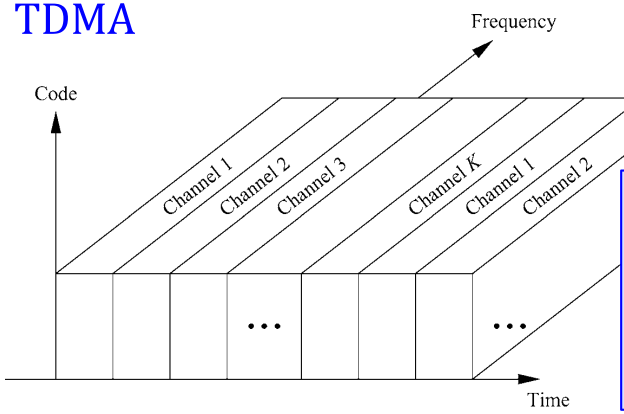
- Create *dedicated* channels for users
- Dedicated: divide resources over frequency, time, code (also space: see later)

FDMA / OFDMA



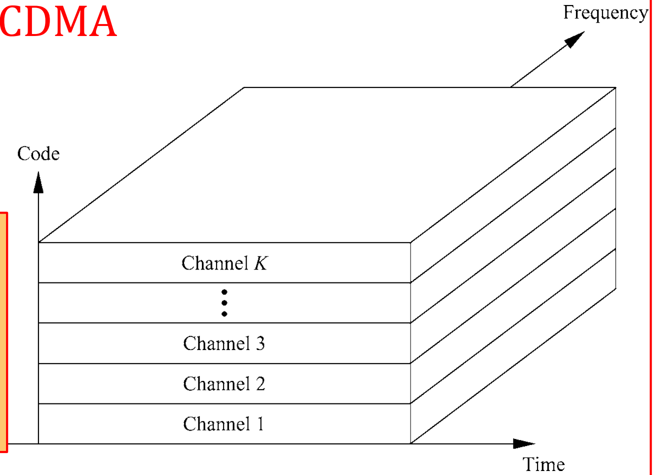
- guard band
- FDMA: analog
- OFDMA: digital

TDMA



- guard time
- synchronization in uplink
- buffers
- channel estimation

CDMA



- DS-CDMA
- FH-CDMA
- near-far effect
- orthogonal or non-orthogonal

Multiple access waveforms: mathematical model

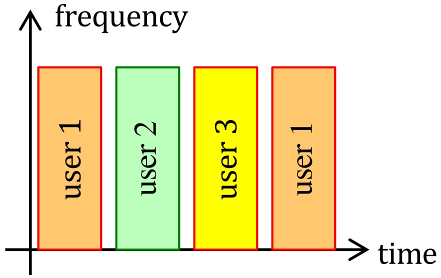
- Basic idea:
  - each user has its own waveform
  - waveforms are (nearly) orthogonal: $\int s_{k,l}(t)s^*_{k',l'}(t)\mathrm{d}t \approx \begin{cases} 0 & k \neq k' \\ 0 & l \neq l' \\ 1 & k = k' \text{ and } l = l' \end{cases}$
- Mathematically, signal for user k:

$$x_k(t) = \sum_{l=-\infty}^{+\infty} a_{k,l} s_{k,l}(t)$$

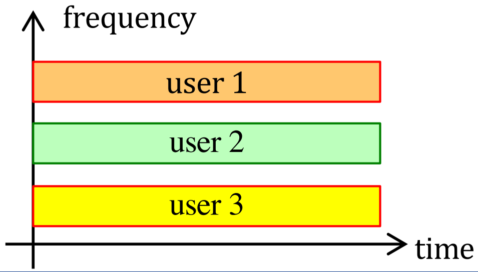
to reduce interference between users

to reduce interference between symbols

- TDMA:



- FDMA:



TDMA and FDMA

Given

- downlink 25 MHz bandwidth, divided in 125 channels in the frequency domain
- Each channel operates in TDMA. Each TDMA slot contains up to 8 user time slots
- Each user sends with 16-QAM and can use only one frequency at a given time.
- Maximum channel delay spread of 10 us, coherence time of 10 ms.

Task

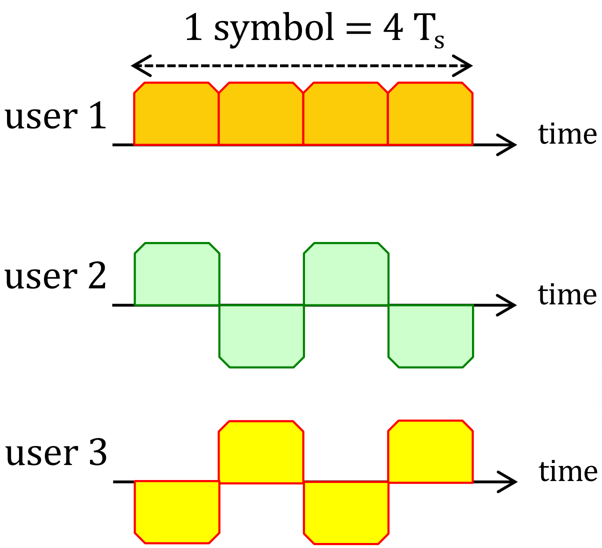
- How many users can be supported? What is the total data rate? What is the minimum guaranteed rate per user? What is the maximum rate per user?
- Show that equalization will be required. How would you modify the system to remove the need for equalization if only 200 users need to be supported? What would be the rate per user?

CDMA

- We can use same frequency and time for all the users, provided

$$\int s_{k,l}(t)s^*_{k',l'}(t)\mathrm{d}t \approx \begin{cases} 0 & k \neq k' \\ 0 & l \neq l' \\ 1 & k = k' \text{ and } l = l' \end{cases}$$

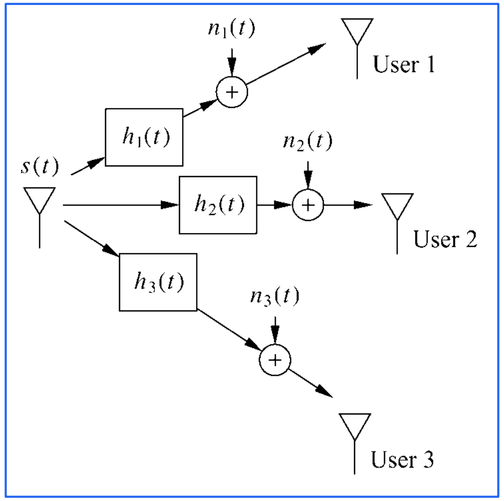
- CDMA assigns orthogonal "codes" to users
- Basic idea of direct sequence (DS) CDMA: "spread out the symbols"



Show that the codes of user 1 and 2 are orthogonal
Show that the codes of user 2 and 3 are not orthogonal
How is the data rate related to the code length

- Alternative: frequency hopping
- Generally not orthogonal in uplink, due to delays

Downlink for TDMA, FDMA, CDMA



- Transmitter

$$x_k(t) = \sum_{l=-\infty}^{+\infty} a_{k,l} s_{k,l}(t) \qquad s(t) = \sum_{k=1}^{K} x_k(t)$$

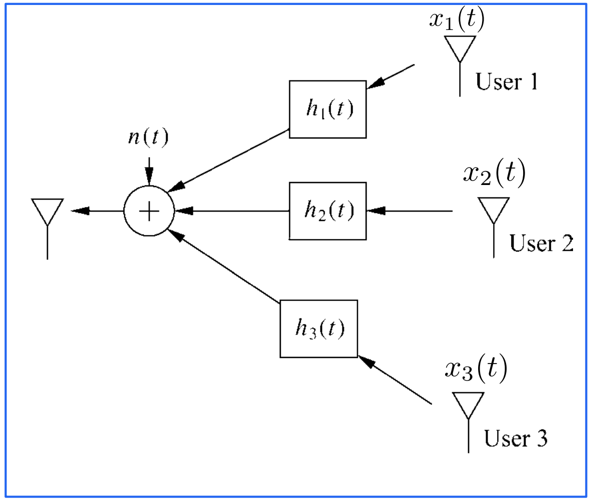
- Receiver: user k

$$r_k(t) = \alpha_k s(t) + n_k(t)$$

$$\begin{aligned} r_{kl} &= \int r_k(t) s^*_{k,l}(t)\mathrm{d}t \\ &= \alpha_k \sum_{l'} a_{kl'} \int s_{k,l'}(t) s^*_{k,l}(t)\mathrm{d}t + \alpha_k \sum_{k' \neq k} \int x_{k'}(t) s^*_{k,l}(t)\mathrm{d}t + w_{kl} \\ &= \alpha_k a_{kl} + \text{M.U.I.} + w_{kl} \end{aligned}$$

- MUI = multiuser interference: zero for TDMA, FDMA, CDMA with orthogonal codes
- General CDMA: cross-correlation function $\rho_{k,k'}(\tau) = \int s_k(t) s^*_{k'}(t-\tau)\mathrm{d}t$

Uplink for TDMA, FDMA, CDMA



- Transmitter $x_k(t) = \sum_{l=-\infty}^{+\infty} a_{k,l} s_{k,l}(t)$
- Receiver $r(t) = \sum_{k=1}^{K} \alpha_k x_k(t-\tau_k) + n(t)$

$$r_{kl} = \int r(t) s^*_{k,l}(t-\tau_k) \mathrm{d}t$$

$$= \alpha_k \sum_{l'} a_{kl'} \underbrace{\int s_{k,l'}(t-\tau_k) s^*_{k,l}(t-\tau_k)\mathrm{d}t}_{\delta_{l-l'}} + w_{kl}$$

$$+ \sum_{k' \neq k} \alpha_{k'} \sum_{l'} a_{k'l'} \int s_{k',l'}(t-\tau_{k'}) s^*_{k,l}(t-\tau_k)\mathrm{d}t$$

- MUI
  - FDMA: zero
  - TDMA: zero under proper *timing advance*
  - DS-CDMA: always residual MUI, harmful under near-far conditions
- Base station can resort to multi-user detection (MUD)

Performance metrics

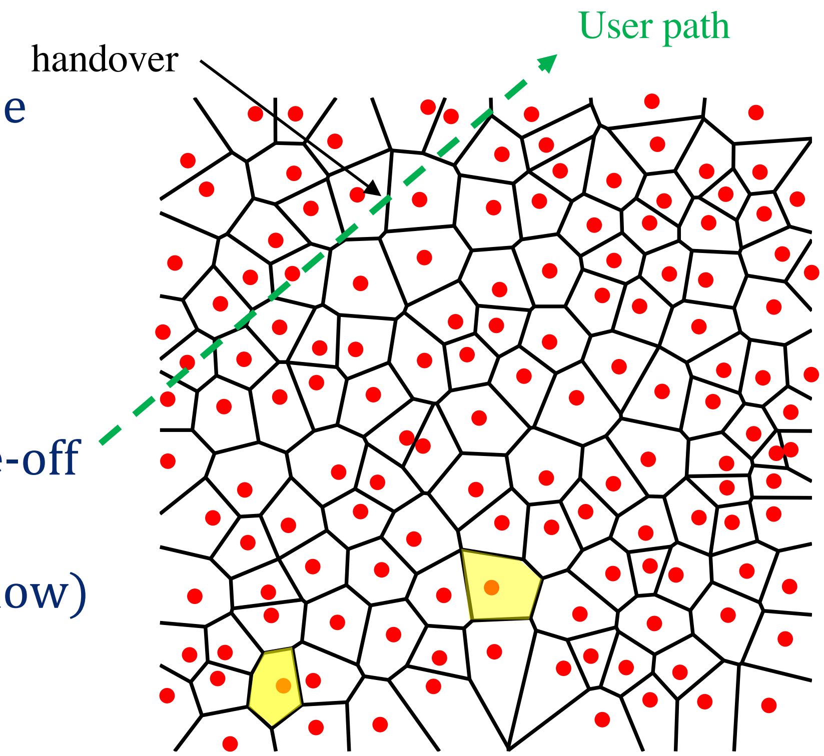
- Observation model

$$r_{kl} = \boxed{\alpha_k a_{kl}} + \boxed{\underbrace{\alpha_k \sum_{l' \neq l} a_{kl'} g_{kll'}}_{\text{ISI}} + \underbrace{\sum_{k' \neq k} \alpha_{k'} \sum_{l' \neq l} a_{k'l'} g_{kk'll'}}_{\text{MUI}}} + \boxed{w_{kl}}$$

- Performance metrics
  - SNR $\dfrac{\boxed{\text{signal}}}{\boxed{\text{noise}}}$ (for noise-limited regime)
  - SINR $\dfrac{\boxed{\text{signal}}}{\boxed{\text{N}} + \boxed{\text{I}}}$ (replaces SNR in error probabilities)
  - SIR $\dfrac{\boxed{\text{signal}}}{\boxed{\text{interference}}}$ (for interference-limited regime)

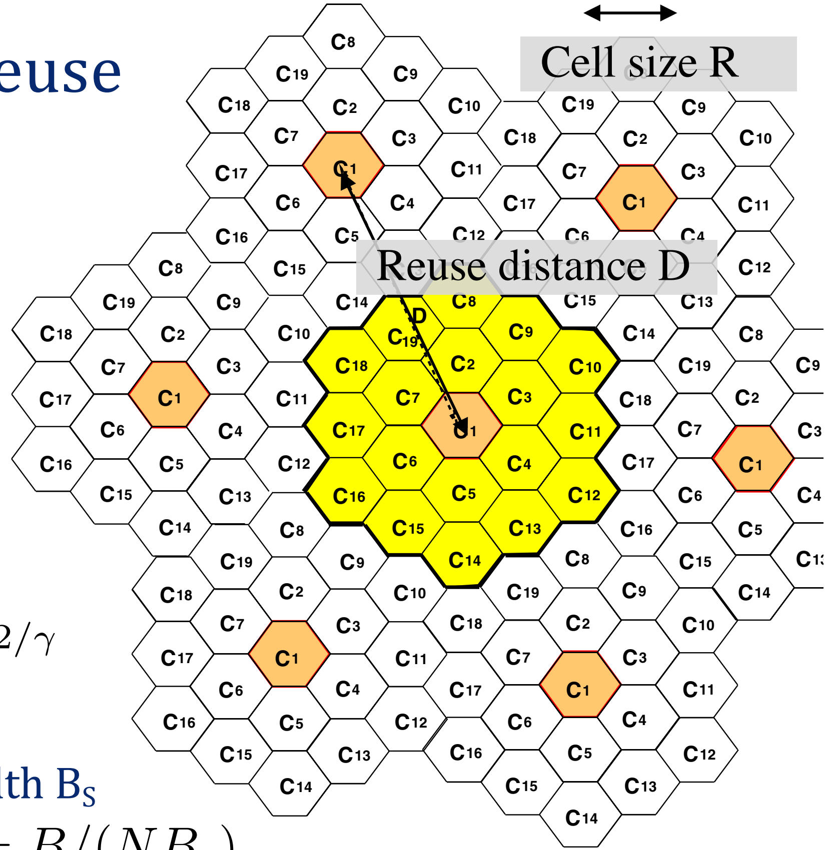
Cellular networks

- Communication between user (UE) and base station (BS)
- Central controller
- BS serves a cell, UE associated with 1 BS
- UE monitors multiple BS for handover
- Placement and density are important: trade-off between interference and rate
- Frequencies can be reused far-away (in yellow)
- Abstraction: hexagonal cells, periodic reuse



Andrews, Jeffrey G., Abhishek K. Gupta, and Harpreet S. Dhillon. "A primer on cellular network analysis using stochastic geometry." *arXiv preprint arXiv:1604.03183* (2016).

Hexagonal tessellation and spatial reuse



- All C_1 cells uses the same band
- Induces a clustering
- Reuse factor: cells per cluster $N \propto D^2/R^2$
- Cell edge SIR

$$\mathrm{SIR} = \frac{P_t d^{-\gamma}}{\sum_{m=1}^{6} P_t d_i^{-\gamma}} \approx \frac{1}{6}\left(\frac{R}{D}\right)^{-\gamma}$$

- Minimum reuse to maintain target SIR: $N \geq \mathrm{SIR}^{2/\gamma}$
- **User capacity:** active users per cell
  - Given N, total bandwidth B, per user bandwidth B_S
  - User capacity: channels (users) per cell $N_c = B/(NB_s)$
- **System capacity:** channels per unit area $\propto N_c/R^2 = B/(R^2 N B_s)$
  - Reduce R: smaller cells
  - Reduce N: more reuse, more interference

Assumptions
- (Gaussian interference)
- Same TX power for all cells
- Interference dominated by first tier
- Path loss only, fixed PL exponent

Reuse

Given

- System with hexagonal cells
- 20 MHz total system bandwidth
- 100 kHz signal bandwidth per user
- Path loss exponent 2, SIR target of 10 dB

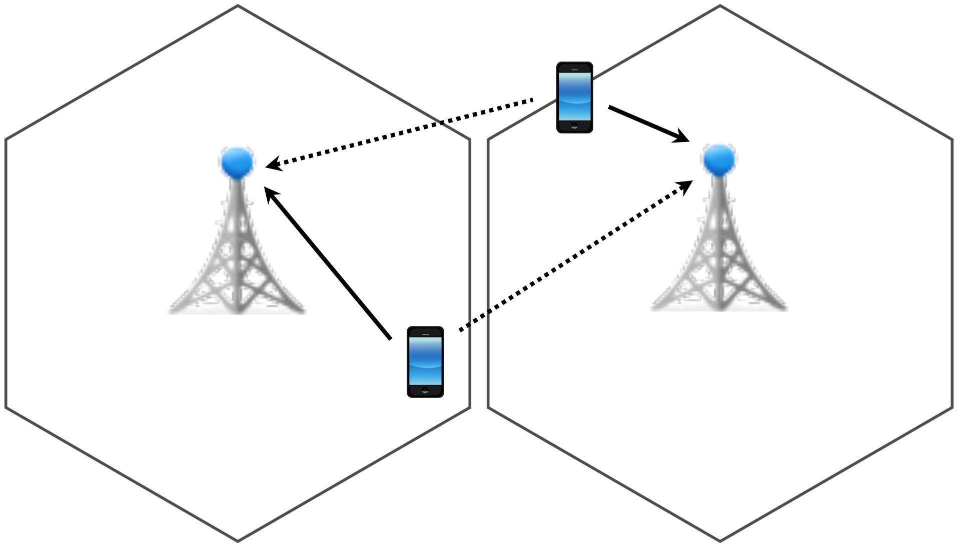
Task

- What is the reuse factor, R=200 m?
- What is the user capacity, R=200 m?
- What is the system capacity, R=200 m?
- How would you design the system given a certain number of users per square meter (say 1 per 100 m^2)?

Power control

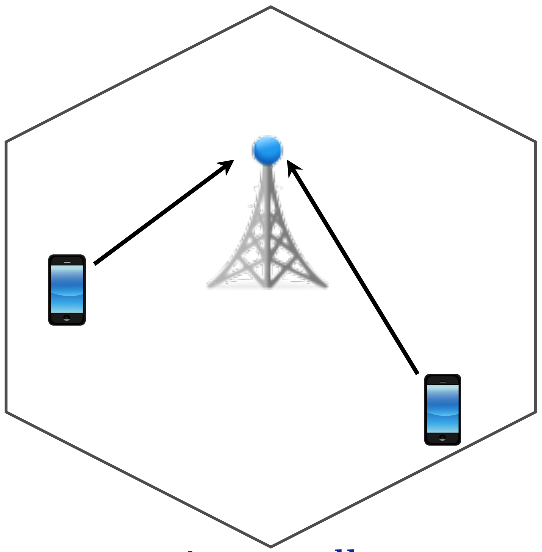
Higher transmit power on a communication link...

- ✓ In wireless fading channels, more protection against a signal fade
- ✓ Higher signal-to-noise ratio (SNR) - reduces the bit error rate (BER) of the link
- ✓ Use link adaptation to transmit at a higher data rate - greater spectral efficiency
- ✗ Higher power consumption - battery lifetime reduced considerably
- ✗ Interference to other users in the same frequency band is increased



inter-cell interference

e.g., in FDMA systems such as GSM



intra-cell

e.g., cellular spread-spectrum systems

Signal-to-Interference-and-Noise ratio

A transmission is successful (error free), if the SINR at the receiver is greater than the capture ratio, γ_i :

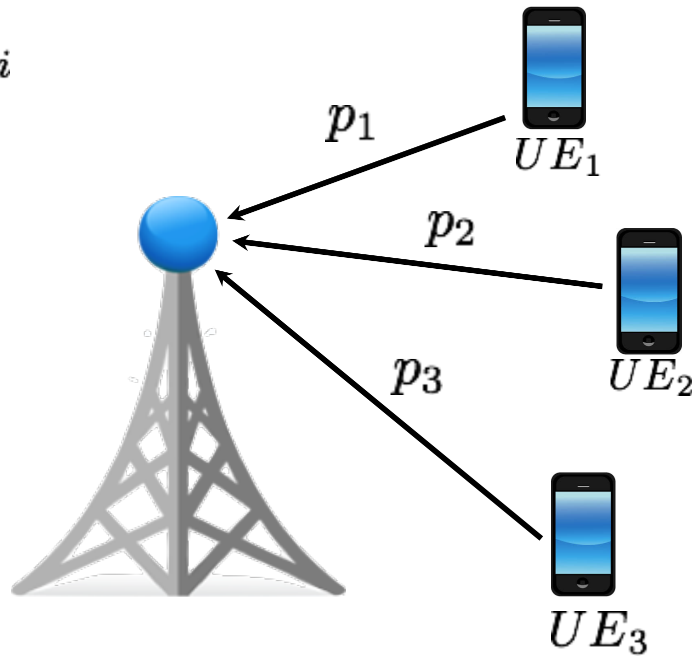
$$\frac{g_{ii}p_i}{\sum_{j\neq i, j\in\mathcal{T}} g_{ji}p_j + \nu_i} \geq \gamma_i$$

where

g_{ij} the channel gain between transmitter i and receiver j

p_i the power level chosen by transmitter i

ν_i the variance of thermal noise at receiver i

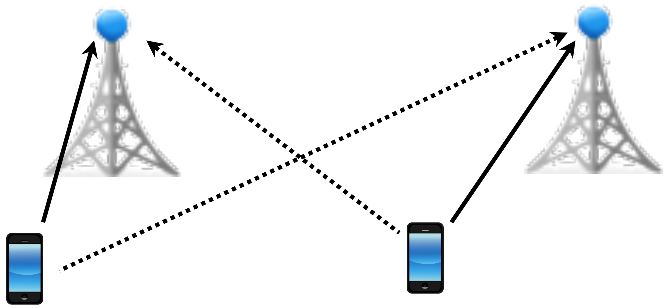


Optimal transmit powers satisfy

$$\frac{g_{ii}p_i}{\sum_{j\neq i, j\in\mathcal{T}} g_{ji}p_j + \nu_i} = \gamma_i$$

2-link example

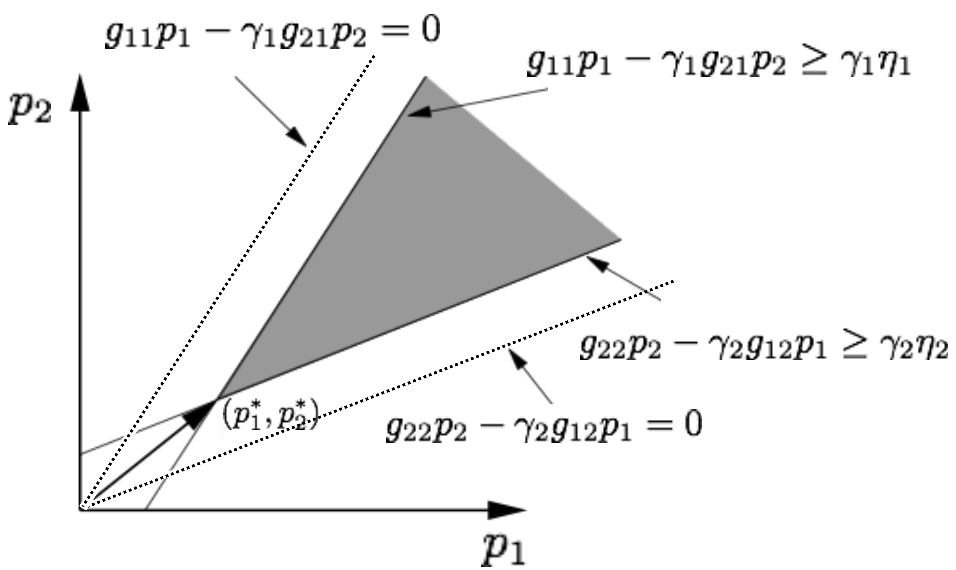
Consider the network below.

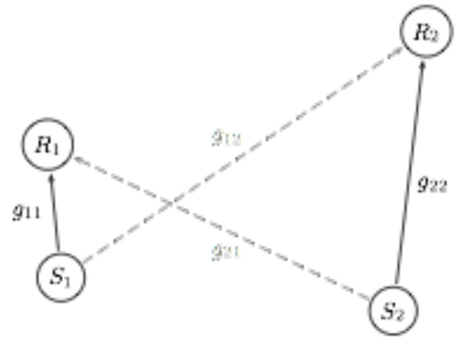


Task

- Write the SINR inequalities for each of the links.
- Plot the inequalities in coordinates (p_1, p_2) and indicate on the graph which are the optimal power levels.

Solution





Courtesy of Prof Toumpis (from his lecture notes on Wireless Networks)

SINR for each communication pair is given by:

$$\frac{g_{11}p_1}{g_{21}p2 + \eta_1} \geq \gamma_1 \qquad \frac{g_{22}p_2}{g_{12}p_1 + \eta_2} \geq \gamma_2$$

The point (p_1^*,p_2^*) is optimal because it minimizes energy consumption.

Feasibility of a network & distributed implementation

The transmission condition can be written as

$$p_i \geq \gamma_i \left(\sum_{j \neq i, j \in \mathcal{T}} \frac{g_{ji}}{g_{ii}} p_j + \frac{\nu_i}{g_{ii}} \right).$$

In matrix form, for a network consisting of n communication pairs, $\mathbf{p} \geq \Gamma G \mathbf{p} + \eta$

where

$$\Gamma = diag(\gamma_i)$$

$$\mathbf{p} = \begin{pmatrix} p_1 & p_2 & \dots & p_n \end{pmatrix}^T$$

$$G_{ij} = \begin{cases} 0 & \text{, if } i = j, \\ \frac{g_{ji}}{g_{ii}} & \text{, if } i \neq j, \end{cases}$$

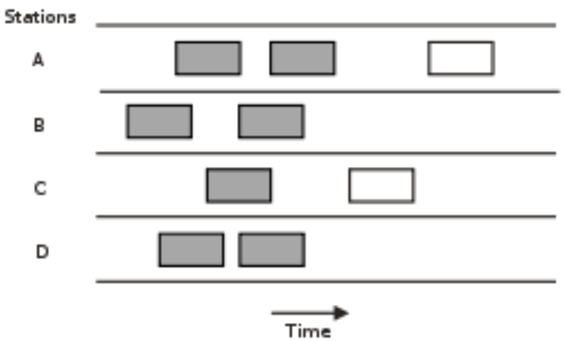
$$\eta_i = \frac{\gamma_i \nu_i}{g_{ii}}.$$

The discrete-time Foschini-Miljanic algorithm ($0<k_i\leq 1$) is given by

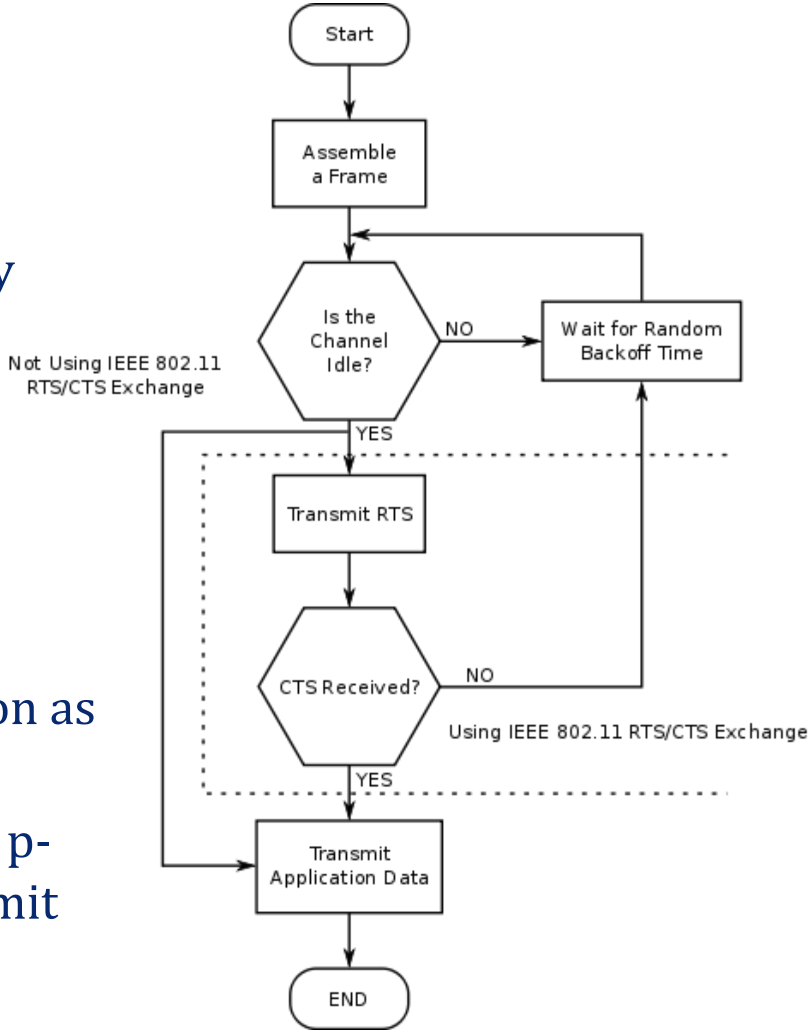
$$p_i(n+1) = (1-k_i)p_i(n) + k_i\gamma_i \left(\sum_{j \neq i, j \in \mathcal{T}} \frac{g_{ji}}{g_{ii}} p_j(n) + \frac{\nu}{g_{ii}} \right)$$

Random access

- Suitable for bursty traffic: e.g., Poisson packet arrivals
- More users than channels
- Common protocols
  1. *Aloha:* transmit as soon as packet ready



  2. *Slotted Aloha:* transmit next slot, as soon as packet is ready
  3. *Carrier sense multiple access:* example p-persistent (sense until free, then transmit with probability p) and CSMA/CA



Today's learning outcomes

At the end of this lecture, you must be able to

- Describe uplink and downlink, multiple access and random access
- Mathematically describe TDMA, FDMA, and CDMA and list their properties
- Design and analyze a multiple access scheme for a given scenario and compute the SIR and SINR

TDMA and FDMA

Given

- downlink 25 MHz bandwidth, divided in 125 channels in the frequency domain
- Each channel operates in TDMA. Each TDMA slot contains 8 user time slots
- Each user sends with 16-QAM and can use only one frequency at a given time.
- Maximum channel delay spread of 10 us, coherence time of 10 ms.

Task

- How many users can be supported? What is the total data rate? What is the minimum guaranteed rate per user? What is the maximum rate per user?
- Show that equalization will be required. How would you modify the system to remove the need for equalization if only 200 users need to be supported? What would be the rate per user?

Solution

- We have 125 channels, meaning 200 kHz per channel
- The total data rate is 4bits/symbol x 200 ksymbols x 125 = 100 Mbit/s
- Each TDMA slot serves 8 users, so 1000 users are supported. The rate per user is 100 kbps.
- If 1000 is the maximum numbers of users than can be supported, the guaranteed rate per user is 100 kbps.
- If we assign a complete channel to a single user, the user can get a rate of 800 kbps
- Since each channel has a bandwidth of 200 kHz, corresponding to a symbol slot of 5 us, a delay spread of 10 us will lead to ISI. So equalization is required.
- We can design an OFDM system as follows: Suppose we expect to service 200 users, we can divide the band in 25 bands of 1 MHz, and use TDMA for 8 users per band. For a given band, the cyclic prefix length should be 10 symbols. We can send one OFDM symbol per 1 ms (due to the coherence time), so we can have an FFT of length 512. The rate per channel is then 512x4bits/522 us = 3.9Mbit/s, or 500 kbps per user.

Reuse

Given

- System with hexagonal cells
- 20 MHz total system bandwidth
- 100 kHz signal bandwidth per user
- Path loss 2, SIR target of 10 dB

Task

- What is the reuse factor, R=200?
- What is the user capacity, R=200?
- What is the system capacity, R=200?
- How would you design the system given a certain number of users per square meter (say 1 per 100 m^2)?

Solution

- Reuse factor $N \geq \mathrm{SIR}^{2/\gamma}$ so N=10 would work

- User capacity: 20 active users cell
- System capacity: 20 /(pi*400 m^2) < 0,02 user per square meter

- To increase the system capacity to 1 user / 100m^2, we need much smaller cells, around 20 – 25 m radius