SSY135 Wireless Communications

Department of Signals and Systems

Exam Date: March 18 2016, 14:00-18:00

Teaching Staff
Henk Wymeersch (examiner), 031 772 1765
Alireza Sheikh, 031 772 5748
Themistoklis Charalambous, 031 772 3392

Material Allowed material is

- Chalmers-approved calculator
- L. Råde, B. Westergren. Beta, Mathematics Handbook, any edition.
- One A4 page with your own handwritten notes. Both sides of the page can be used. Photo copies, printouts, other students' notes, or any other material is not allowed.
- A dictionary.

Grading A correct, clear and well-motivated solution gives a maximum of 12 points.

An erroneous answer, unclear, incomplete or badly motivated solutions give point reductions down to a minimum of 0 points. Answers in any other language than English are ignored.

Results Results are posted no later than April 1.

Grades To pass the course, all projects and the exam must be passed. The exam is passed by securing at least 12 points. The project is passed by securing at least 8 points (4 for the report and 4 for the oral exam) in each part of the project. The final grade on the course will be decided by the homework (max score 6), project (max score 40), quizzes (max score 6), and final exam (max score 48). The sum of all scores will decide the grade according to the following table.

Total Score	0–39	40–59	60–79	≥ 80
Grade	Fail	3	4	5

PLEASE NOTE THAT THE PROBLEMS ARE NOT NECESSARILY ORDERED IN DIFFICULTY.

Good luck!

Question 1: Rate and power adaptation

1. *[4 pt]* Consider designing an adaptive modulation system that may use up to 2 out of 3 available modulation schemes; namely BPSK, QPSK and 8-PSK for a target bit error rate (BER) $P_b = 10^{-3}$. When the target P_b cannot be achieved for any of the 2 modulation schemes used, then no data is transmitted. The channel gain is Rayleigh Fading with $\bar{\gamma} = 13$ dB. Which modulation scheme(s) out of the 3 available should you choose for the system to maximize the spectral efficiency? Justify.
Hints: For M-PSK, $P_b = \frac{2}{\log_2 M} Q(\sqrt{2\gamma}\sin(\pi/M))$, and Q can be estimated from Figure 1. The probability

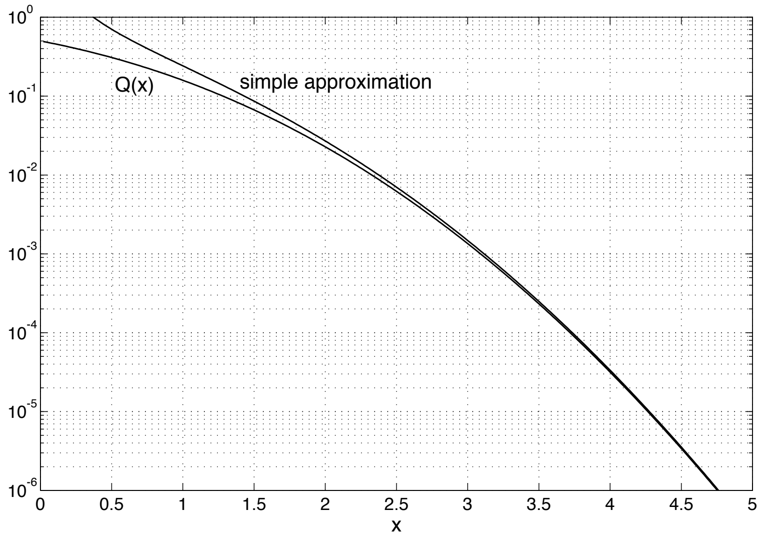

Figure 1: The Q function $Q(x)$.

density function of the Exponential distribution is given by $p(x;\gamma) = \exp(-x/\gamma)/\gamma$, defined only for $x \geq 0$.

2. *[4 pt]* Consider transmission over 3 parallel channels. We require a fixed SNR σ in all channels. The instantaneous SNR γ_i in channel i when the total power $\bar{P}$ is used, is given by $(\gamma_1, \gamma_2, \gamma_3) = (15, 6, 10)$ dB.

   (a) *[2 pt]* What is the fixed SNR σ that can be obtained?

   (b) *[2 pt]* What is the minimum total power $\bar{P}_{\min}$ with respect $\bar{P}$ that is required to maintain a fixed SNR $\bar{\sigma}$ if the modulation scheme adopted is BPSK and the target BER is 10^{-3}?

3. *[4 pt]* Consider a single-input single-output (SISO) communication system with a target BER of 10^{-3}. Suppose the system uses M-QAM modulation and the wireless channel has four states. Assuming a fixed transmit power $\bar{P}$, the received SNR associated with each channel state is $\gamma_1 = -5$ dB, $\gamma_2 = 0$ dB, $\gamma_3 = 5$ dB, and $\gamma_4 = 10$ dB, respectively. The probabilities associated with the channel states are $q(\gamma_1) = 0.1$, $q(\gamma_2) = 0.2$, $q(\gamma_3) = 0.3$ and $q(\gamma_4) = 0.4$.

   (a) *[1 pt]* Express the joint rate and power allocation optimization problem for M-QAM in the form

$$\begin{aligned} \max_{x_i} \ & \sum_{i=1}^{n} f_i(x_i) \\ \text{s.t. } \mathbf{q}^{\mathbf{T}}\mathbf{x} &= b \\ x_i &\geq 0. \end{aligned}$$

   (b) *[3 pt]* If you express the constellation size as $M(\gamma_i) = 1 + k\gamma P(\gamma_i)/\bar{P}$, then the optimum power allocation can be expressed as

$$P(\gamma) = \begin{cases} \frac{\bar{P}}{k}\left(\frac{1}{\gamma_k} - \frac{1}{\gamma}\right), & \text{if } \gamma \geq \gamma_k, \\ 0, & \text{if } \gamma < \gamma_k, \end{cases}$$

   where γ_k is the threshold. Using the result above (you don't need to prove the result!), find the optimal power and rate adaptation for continuous-rate adaptive M-QAM on this channel.
   Hint: You can use the BER upper bound of M-QAM $P_b(\gamma) \leq 0.2\exp\left(\frac{-1.5\gamma}{M(\gamma)-1}\cdot\frac{P(\gamma)}{\bar{P}}\right)$ as an approximation to the M-QAM BER.

Question 2: MIMO and Multi-user Communications

You are given a 2×2 real MIMO channel of the following form:

$$\mathbf{H} = \begin{bmatrix} 1 & a \\ a & 1 \end{bmatrix},$$

where $a \in [0, 1]$. You communicate over this channel in discrete time, with an average transmission energy E_s and add complex Gaussian noise at the receiver with variance σ^2 for real and imaginary dimension. The observation model is thus of the form

$$\mathbf{y} = \mathbf{Hx} + \mathbf{n},$$

with $\mathbb{E}\{\|\mathbf{x}\|^2\} = E_s$ (with $\mathbb{E}\{|x_1|^2\} = \mathbb{E}\{|x_2|^2\} = E_s/2$) and $\mathbf{n} \sim \mathcal{CN}(0, 2\sigma^2)$. The energy of the channel is defined as $E_H = 2 + 2a^2$. The MIMO SNR is defined as $E_s E_H/(4\sigma^2)$.

1. *[3 pt]* Determine the MIMO SNR as a function of a. Applying a singular value decomposition (SVD) to $\mathbf{H}$ indicates that the singular values of the channel are given by $1+a$ and $1-a$ (so $\mathbf{H} = \mathbf{USV}^H$, in which $\mathbf{S} = [1+a\, 0;\, 0\, 1-a]$ and $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices). Using suitable precoding at the transmitter and shaping at the receiver, we can obtain two parallel, non-interfering channels of the form

$$z_i = x_i + w_i,\ i \in \{1, 2\},$$

   where w_i is an AWGN sample with variance σ_i^2 for real and imaginary dimension. Determine the SNR of each of the channels and relate to the MIMO SNR. Assume you are able to transmit at a rate of $\log_2(1+\text{SNR})$ bits per channel use on each of the channels, what is the total data rate? What is the total rate for $a = 0$ and $a = 1$? Is the rate maximized when the MIMO SNR is maximized (with respect to a)? Why or why not?

2. *[3 pt]* Derive the zero-forcing (ZF) receiver. Show that after applying the ZF receiver, you again obtain 2 parallel channels of the form

$$z_i = x_i + w_i,\ i \in \{1, 2\},$$

   where w_i is an AWGN sample with variance σ_i^2 for real and imaginary dimension. Determine the SNR of each of the channels and relate to the MIMO SNR. Assume you are able to transmit at a rate of $\log_2(1 + \text{SNR})$ bits per channel use on each of the channels, what is the total data rate? Comment on the differences between the SVD approach. What is the total rate for $a = 0$ and $a = 1$? What are the benefits that ZF has over SVD?

3. *[3 pt]* Now we associate the two transmit antennas with different users. We also consider a different channel

$$\mathbf{H} = \begin{bmatrix} 1 & a_2 \\ a_1 & 1 \end{bmatrix},$$

   where $a_1, a_2 \in \mathbb{R}$ with $a_1 > a_2$. So there is a channel $\mathbf{h}_1 = [1\, a_1]^T$ between user 1 and the receiver and a channel $\mathbf{h}_2 = [a_2\, 1]^T$ between user 2 and the receiver. We consider TDMA transmission, where only one user is allowed to transmit at a time and they alternate in using the channel. Assume each user is able to transmit data symbols with energy E_s at a rate of $\log_2(1 + \text{SNR})$ bits per channel use. What is the *effective* per-user rate and sum rate, under maximum ratio combining? Which user has the highest rate?

4. *[3 pt]* We now use a slightly more sophisticated version of TDMA, where user 1 transmits with a probability $p \in [0, 1]$. If user 1 transmits, user 2 is quiet and vice versa. What is the per-user rate and the sum rate? Can you recover the effective per-user rate and the sum rate for TDMA from the previous sub-question? Suppose we want to maximize the sum rate, how would you choose p? What can you say about the fairness of this solution? Finally, consider a_1 and a_2 to be independent random variables uniformly distributed over the interval $[0, 1]$. User i now no longer coordinates with the other user, but transmits when $a_i > 0.5$. When both users transmit, there is a packet collision and the information is lost. What is the expected effective per user rate, expressed as an integral?

Question 3: Propagation, impact on communication, and diversity

Consider a cellular system with circular cell shape and log-distance path loss which operates on 900 MHz carrier frequency. The base station is located at the middle of cell and transmits 10 mW power. Assume path loss exponent is 3 and the reference distance (d_0) is 1 km. The path loss for users inside the cell is 10dB larger than free-space path loss using unit gain transmitter and receiver antennas.

1. *[2 pt]* If cell radius is 1 km, find the received power at cell boundary. If user A receives two times larger power than user B which is located at cell boundary, find the distance between user A and base station. *Hint: The relation between received power (P_r) and transmit power (P_t) in free-space path loss model is* $P_r/P_t = \sqrt{G_{TX}G_{RX}}\lambda^2/(4\pi d)^2$.

2. *[3 pt]* Assume the users inside the cell experience log-normal shadowing on top of the path loss with 1 dB mean and 3 dB standard deviation. If -80 dBm received power is required for each user, find the cell radius where the users have at most 2% outage probability.

3. *[3.5 pt]* Assume the users inside the cell experience fading in addition to log-normal shadowing and path loss. Also assume that a user is equipped with two antennas and the signal-to-noise ratio (SNR) of the antenna 1 is uniformly distributed between 0 and 5 and SNR of the antenna 2 is uniformly distributed between 0 and 15. Find the probability density function (pdf) of the received SNR under maximum ratio combining (MRC). If minimum acceptable SNR is 3 for each user, find the outage probability of MRC. *Hints: Recall that under MRC the output SNR is the sum of the input SNRs.*

4. *[3.5 pt]* Repeat part (c) using selection combining (SC).

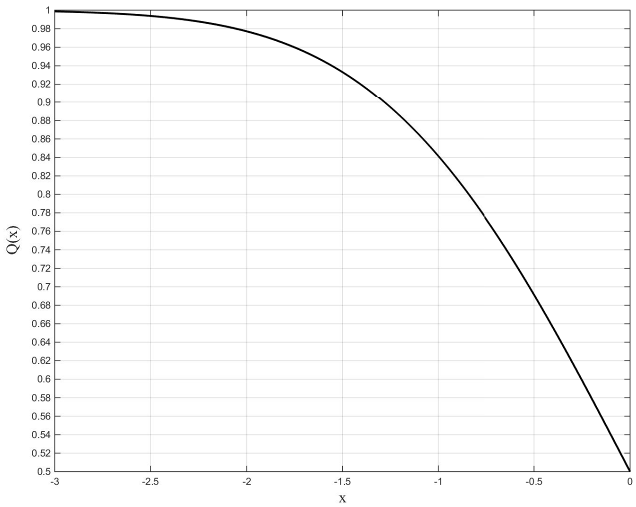

Figure 2: Q function zoomed in for the range of interest in this question.

Hints: Recall that under SC, the output SNR is the maximum of the input SNRs.

Question 4: various topics

Below is a selection of smaller questions. There is often no unique answer, so provide a short, but well-motivated answer for each question.

Questions

1. *[1 pt]* Assume M-PSK modulation is used at transmitter. Assume also a noise free receiver where the received signal is corrupted only by phase noise. If the receiver does not use any compensation for phase noise, express the average probability of symbol error as a function of constellation size.
2. *[1 pt]* Suppose a receiver is moving in a certain direction with respect to the transmitter with Doppler frequency f_d. If the receiver rotates 180 degree and keeps moving with the same speed, how much does the Doppler frequency change due to its rotation?
3. *[1 pt]* CDMA communication is known to suffer from the so-called near-far effect. What is it and how is it mitigated in cellular communications?
4. *[1 pt]* Given the following observation model

$$\mathbf{y} = h\mathbf{a} + \mathbf{n},$$

   where $\mathbf{y} \in \mathbb{C}^N$ is an observation, $h \in \mathbb{C}$ is a channel, $\mathbf{a} \in \mathbb{C}^N$ is a vector of N unit-energy M-PSK symbols, and $\mathbf{n} \sim \mathcal{CN}(0, \sigma^2\mathbf{I})$ is a vector of N i.i.d. AWGN samples. Write down a closed-form expression for the maximum likelihood estimate of h.
5. *[1 pt]* Suppose you generate channel gains $h_1, \ldots, h_N$ according to a $\mathcal{CN}(0; \sigma^2)$ distribution. How would you modify these channel gains, resulting in new gains $g_1, \ldots, g_N$ so that their absolute values would have a Ricean distribution? Justify your answer.
6. *[1 pt]* According to the WHO, is there any evidence of correlation between use of cell phones and brain cancer? Is there any change in user behavior that would increase or decrease exposure, compared to before 2005?
7. *[1 pt]* What is the role of the federal communication commission in the context of health of effects of electromagnetic radiation? When do you have more exposure when using a cell phone, close to a base station or far away?
8. *[2 pt]* There are two notions of channel capacity relevant for wireless communications.
   (a) What are those channel capacity concepts?
   (b) When would each concept be applicable?
9. *[2 pt]* What are the main benefits and drawbacks of OFDM, as compared to single-carrier transmission?
10. *[1 pt]* Consider a FDMA cellular system with hexagonally shaped cells and with path-loss exponent $\alpha = 2$ for all signal propagation in the system. Find the user capacity (users per cell) for a target SIR of 10 dB, assuming a total system bandwidth of 20 MHz and a required signal bandwidth of 100 kHz.