Tentative solutions for Exam in SSY135 Wireless Communications

Department of Signals and Systems

Exam Date: March 20 2015, 14:00-18:00

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Material Allowed material is

- Chalmers-approved calculator
- L. Råde, B. Westergren. Beta, Mathematics Handbook, any edition.
- One A4 page with your own handwritten notes. Both sides of the page can be used. Photo copies, printouts, other students' notes, or any other material is not allowed.
- A dictionary.

Grading A correct, clear and well-motivated solution gives a maximum of 12 points.

An erroneous answer, unclear, incomplete or badly motivated solutions give point reductions down to a minimum of 0 points. Answers in any other language than English are ignored.

Solutions Are made available at the earliest at 7 PM on March 24 2015, on the course web page.

Results Results are posted no later than March 30. The grading review is on March 31, noon–1PM in room 7430 in the EDIT-building.

Grades The final grade on the course will be decided by the projects (maximum score 46), quizzes (maximum score 6), and final exam (maximum score 48). The sum of all scores will decide the grade according to the following table.

Total Score	0–39	40–59	60–79	≥ 80
Grade	Fail	3	4	5

PLEASE NOTE THAT THE PROBLEMS ARE NOT NECESSARILY ORDERED IN DIFFICULTY.

Good luck!

Question 1: waterfilling and diversity combining

1. [*4 pt*] Consider a single-input single-output (SISO) communication system with a target BER of 10^{-3}. Suppose the system uses MQAM modulation and the wireless channel has four states. Assume that for a fixed transmit power $\bar{P}$, the received SNR associated with each channel state is $\gamma_1 = -10$ dB, $\gamma_2 = -20$ dB, $\gamma_3 = 6$ dB, and $\gamma_4 = 12$ dB, respectively. The probabilities associated with the channel states are $p(\gamma_1) = p(\gamma_2) = 0.1$ and $p(\gamma_3) = p(\gamma_4) = 0.4$. Find the optimal power $P(\gamma)$ and rate adaptation $M(\gamma)$ for continuous-rate adaptive MQAM on this channel to maximize the average throughput subject to an average power constraint.
   Hints:

   (a) *Use the following BER upper bound of MQAM:* $P_b(\gamma) \leq 0.2 \times \exp\left(-1.5\gamma P(\gamma)/((M-1)\bar{P})\right)$.

   (b) *Recall that under water-filling, the optimal power allocation is of the form*

   $$P(\gamma) = \begin{cases} c\bar{P}\left(\frac{1}{\gamma_{\text{cut}}} - \frac{1}{\gamma}\right) & \text{if } \gamma \geq \gamma_{\text{cut}} \\ 0 & \text{if } \gamma < \gamma_{\text{cut}} \end{cases}$$

   where c is a constant and γ_{cut} is a cut-off value. Note that you should set both c and γ_{cut} to appropriate values.

2. [*4 pt*] Suppose that the receiver is equipped with one additional antenna, so the system is changed to SIMO configuration. The receiver has two modes: i) using maximum ratio combining (MRC); ii) using selective combining (SC). Furthermore, suppose that the wireless channel model is changed as follows: the SNR at the first receive antenna is γ_1 with a uniform distribution between 0 and 5; the SNR at the second receive antenna is γ_2 with a uniform distribution between 0 and 10. Note that the SNR is in linear scale and that γ_1 and γ_2 are statistically independent. Calculate the distribution of the combined SNR for each combining mode of the receiver.
   Hint: For both MRC and SC, you should express the combined SNR as a simple function (e.g., sum, product, difference, ratio, minimum, maximum) of the SNRs on each receive antenna.

3. [*2 pt*] Calculate the average received SNR for both MRC and SC.

4. [*2 pt*] Suppose that the system quality of the service restricts us to have minimum receive SNR of 0 dB. Calculate the outage probability for MRC and SC. Which combining mode is better? Motivate your answer.

Question 1

(1) $P_b(\gamma) \leq 0.2 \exp(-1.5\gamma P(\gamma)/((M-1)\bar{P}))$ *

$$k = \frac{-1.5}{\ln(5P_b)}$$

$$* \Rightarrow \underset{(**)}{M(\gamma)} = 1 + k\gamma \frac{P(\gamma)}{\bar{P}}$$

$$\text{Maximize} \quad \sum_{i=1}^{4} \log_2(M(\gamma_i))\, p(\gamma_i)$$

$$\text{s.t.} \quad \sum_{i=1}^{4} P(\gamma_i)\, p(\gamma_i) = \bar{P}$$

Solution:

$$P(\gamma) = \begin{cases} \frac{\bar{P}}{k}\left(\frac{1}{\gamma_{cut}} - \frac{1}{\gamma}\right) & \gamma \geq \gamma_{cut} \quad (***) \\ 0 & \gamma < \gamma_{cut} \end{cases}$$

According to the Hint (b): $C = \frac{1}{k}$

Assumption 1 : Assigning Power to all channels

$$k = \frac{-1.5}{\ln(5 \times 10^{-3})} = 0.2831$$

$\gamma_1 = 0.1$, $\gamma_2 = 0.01$, $\gamma_3 = 3.98$, $\gamma_4 = 15.84$

$$\frac{\bar{P}}{k}\left(\frac{1}{\gamma_{cut}} - \frac{1}{\gamma_1}\right)p(\gamma_1) + \frac{\bar{P}}{k}\left(\frac{1}{\gamma_{cut}} - \frac{1}{\gamma_2}\right)p(\gamma_2) + \frac{\bar{P}}{k}\left(\frac{1}{\gamma_{cut}} - \frac{1}{\gamma_3}\right)p(\gamma_3)$$

$$+ \frac{\bar{P}}{k}\left(\frac{1}{\gamma_{cut}} - \frac{1}{\gamma_4}\right)p(\gamma_4) = \bar{P}$$

$\Rightarrow \gamma_{cut} = 0.087$

$\min(\gamma_1, \gamma_2, \gamma_3, \gamma_4) = \gamma_2 = 0.01 > 0.087$ ✗ $\longrightarrow$ Assumption 1 is not correct

Assumption 2 : Assigning Power to $\gamma_1, \gamma_3, \gamma_4$

$$\frac{\bar{P}}{k}\left(\frac{1}{\gamma_{cut}}-\frac{1}{\gamma_1}\right)p(\gamma_1)+\frac{\bar{P}}{k}\left(\frac{1}{\gamma_{cut}}-\frac{1}{\gamma_3}\right)p(\gamma_3)+\frac{\bar{P}}{k}\left(\frac{1}{\gamma_{cut}}-\frac{1}{\gamma_4}\right)\overset{p(\gamma_4)}{=}\bar{P}$$

$\gamma_{cut} = 0.6388$

$\min(\gamma_1, \gamma_3, \gamma_4) = \gamma_1 = 0.1 > 0.63$ X $\longrightarrow$ Assumption 2 is not correct

Assumption 3

$$\frac{\bar{P}}{k}\left(\frac{1}{\gamma_{cut}}-\frac{1}{\gamma_3}\right)p(\gamma_3)+\frac{\bar{P}}{k}\left(\frac{1}{\gamma_{cut}}-\frac{1}{\gamma_4}\right)p(\gamma_4)=\bar{P}$$

$\gamma_{cut} = 1.95$

$\min(\gamma_3, \gamma_4) = 3.98 > 1.95$ ✓

$$\frac{P(\gamma_3)}{\bar{P}} = \frac{1}{k}\left(\frac{1}{\gamma_{cut}}-\frac{1}{\gamma_3}\right) = 0.923$$

$$\frac{P(\gamma_4)}{\bar{P}} = \frac{1}{k}\left(\frac{1}{\gamma_{cut}}-\frac{1}{\gamma_4}\right) = 1.5884$$

(**) and (***)

$$M(\gamma_3) = 1+k\gamma_3\frac{P(\gamma_3)}{\bar{P}} = 2.04$$

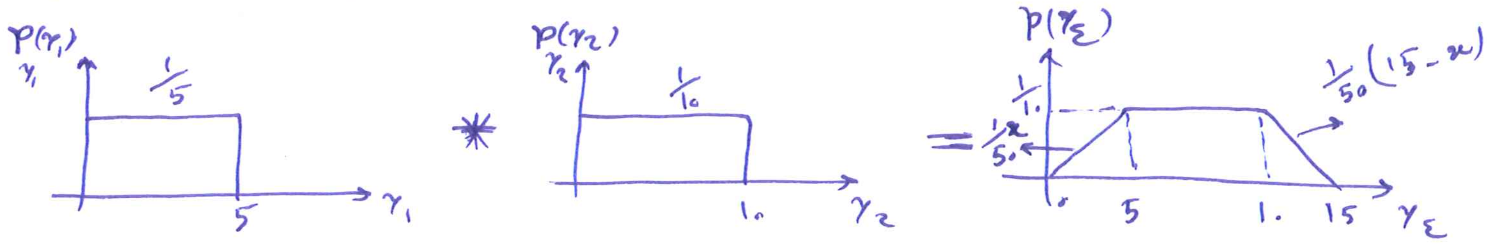
$$M(\gamma_4) = 1+k\gamma_4\frac{P(\gamma_4)}{\bar{P}} = 1.588$$

②

$\gamma_1 \sim U(0,5)$

$\gamma_2 \sim U(0,10)$

i) MRC $\quad \gamma_\Sigma = \gamma_1 + \gamma_2 \quad \Rightarrow \quad \gamma_1, \gamma_2$ independent $\Rightarrow \quad p_{\gamma_\Sigma}(\gamma_\Sigma) = p_{\gamma_1}(\gamma_1) * p_{\gamma_2}(\gamma_2)$



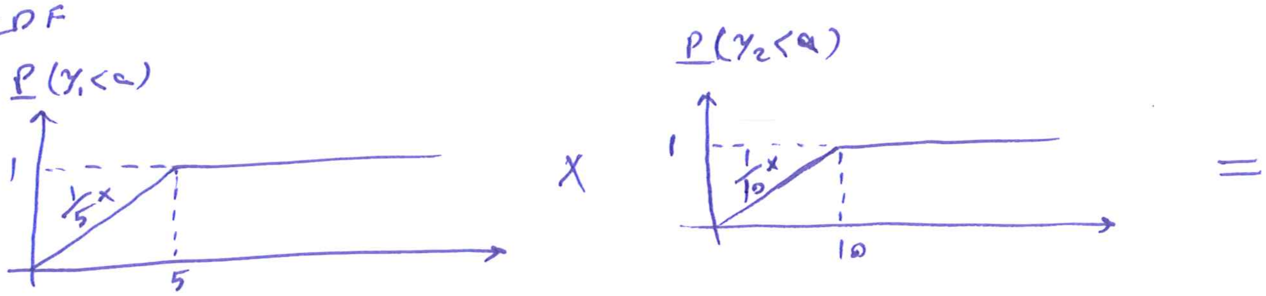
ii) SC

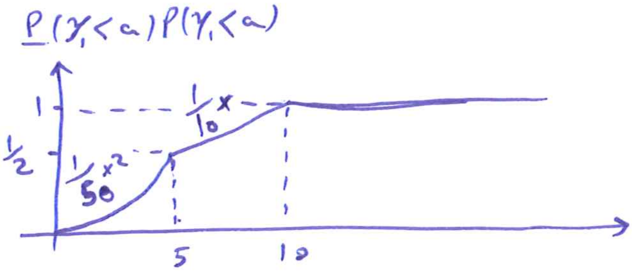
$\gamma_{sc} = \max(\gamma_1, \gamma_2)$

$P(\gamma_{sc} < a) = P(\max(\gamma_1, \gamma_2) < a) = P(\gamma_1 < a)\,P(\gamma_2 < a)$

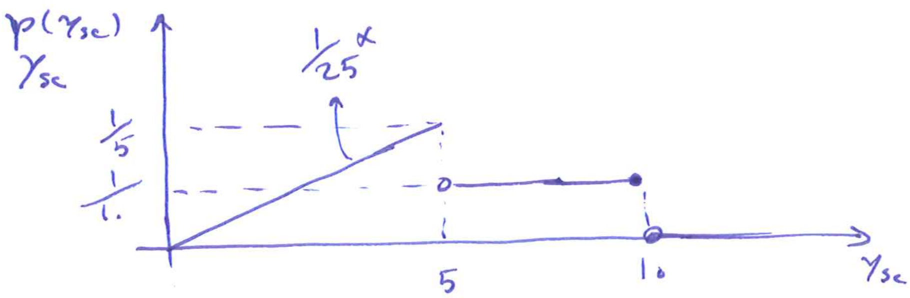
↓

CDF





$p_{\gamma_{sc}}(\gamma_{sc}) = \frac{d}{da} P(\gamma_{sc} < a)$



③

$$\bar{\gamma}_{\Sigma} = \int \gamma_{\Sigma} \, p_{\gamma_{\Sigma}}(\gamma_{\Sigma}) \, d\gamma_{\Sigma} \qquad x \triangleq \gamma_{\Sigma}$$

$$= \int_0^5 \frac{1}{50} x^2 \, dx + \int_5^{10} \frac{x}{10} \, dx + \int_{10}^{15} \frac{1}{50}(15x - x^2) \, dx$$

$$= \frac{x^3}{150} \Big|_0^5 + \frac{1}{20} x^2 \Big|_5^{10} + \left(\frac{15}{100} x^2 - \frac{x^3}{150} \right) \Big|_{10}^{15}$$

$$= \frac{125}{150} + \frac{1}{20}(100 - 25) + \left(\frac{15}{100}(225 - 100) - \frac{1}{150}(3375 - 1000) \right)$$

$$= 7.5$$

$$\bar{\gamma}_{sc} = \int \gamma_{sc} \, p_{\gamma_{sc}}(\gamma_{sc}) \, d\gamma_{sc} \qquad x \triangleq \gamma_{sc}$$

$$= \int_0^5 \frac{1}{25} x^2 \, dx + \int_5^{10} \frac{1}{10} x \, dx$$

$$= \frac{x^3}{75} \Big|_0^5 + \frac{x^2}{20} \Big|_5^{10}$$

$$= \frac{5}{3} + \frac{15}{4} = 5.41$$

④ $\gamma_{min} = 0\,dB = 1$

MRC: $P_{out} = P(\gamma_{\Sigma} < \gamma_{min}) = \int_0^1 p_{\gamma_{\Sigma}}(\gamma_{\Sigma}) \, d\gamma_{\Sigma}$

$$= \boxed{0.01}$$

SC: $P_{out} = P(\gamma_{sc} < \gamma_{min}) = P_{\gamma_{sc}}(\gamma_{min}) = \frac{x^2}{50} \Big|_{x = \gamma_{min}}$

$$= \boxed{0.02}$$

Question 2: channel models and OFDM

Consider a WSS-US fading channel with scattering function

$$S_c(\tau,\rho) = \begin{cases} \frac{1}{T_1\rho_1} e^{-\frac{\tau}{T_1}} & \text{if } 0 \le \tau, \quad -\frac{\rho_1}{2} \le \rho \le \frac{\rho_1}{2} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

1. [*3 pt*] Find the power delay profile, and calculate the root mean square (rms) delay spread of this channel. *Hints:*

   (a) *You can use the figure below.*

   (b) *This is the Fourier relation between rectangular and sinc function*

   $$\operatorname{sinc}(2Wt) \xrightarrow{\mathscr{F}} \frac{1}{2W} \operatorname{rect}\left(\frac{f}{2W}\right)$$

2. [*3 pt*] Find the Doppler power spectrum, and calculate the Doppler spread of this channel. *Hint: You can use the figure below.*

3. [*2 pt*] Determine the interval for the symbol time which the transmitted signal over this channel experiences frequency-flat, fast fading.

4. [*4 pt*] Suppose an OFDM system is working in this wireless channel. Data rate is $R_b = 150$ Mbit/s and 64-QAM modulation is used. Assume that according to the exact values of T_1 and ρ_1, the delay spread of channel is 1 ms and Doppler spread of channel is 200 Hz. Suppose it is required to design an OFDM system to have minimum cyclic prefix overhead and maximum number of sub-carriers. Compute the cyclic prefix time, sub carrier spacing, number of sub-carriers, and bandwidth of the whole OFDM system.

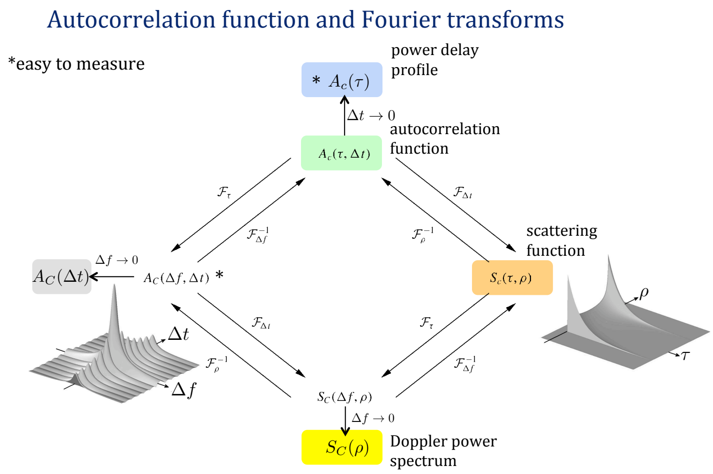


Figure 1: Relation between autocorrelation and various Fourier transforms.

Question 2: channel models and OFDM

1 scattering function

$$S_c(\tau,\rho) = \begin{cases} \frac{1}{T_1 \rho_1} e^{-\tau/T_1} & \tau \geq 0, \quad -\frac{\rho_1}{2} \leq \rho \leq \frac{\rho_1}{2} \\ 0 & o.w \end{cases}$$

According to Figure 1

$$\mathcal{F}_\rho^{-1}\{S_c(\tau,\rho)\} = A_c(\tau,\Delta t)$$

$$S_c(\tau,\rho) = \frac{1}{T_1} e^{-\tau/T_1} \left(\frac{1}{\rho_1}\right)\left(\text{rect}\left(\frac{\rho}{\rho_1}\right)\right) \quad \tau \geq 0$$

According to Hint b:

$$A_c(\tau,\Delta t) = \frac{1}{T_1} e^{-\tau/T_1} \text{sinc}(\rho_1 \Delta t) \quad \tau \geq 0$$

$$\text{sinc}(\rho_1 \Delta t)\Big|_{\Delta t \to 0} = 1$$

$$A_c(\tau) = \frac{1}{T_1} e^{-\tau/T_1} \quad \tau \geq 0$$

$$\mu_{T_m} = \frac{\int_0^\infty \tau A_c(\tau)\, d\tau}{\int_0^\infty A_c(\tau)\, d\tau} = T_1$$

$$\int_0^\infty \frac{1}{T_1} e^{-\tau/T_1} d\tau = -e^{-\tau/T_1}\Big|_0^\infty = 1$$

$$\int_0^\infty \frac{\tau}{T_1} e^{-\tau/T_1} d\tau = -\tau e^{-\tau/T_1} + \int e^{-\tau/T_1} d\tau$$

$$= -\tau e^{-\tau/T_1} - T_1 e^{-\tau/T_1}\Big|_0^\infty$$

$$= T_1$$

$$\sigma_{T_m}^2 = \frac{\int_0^\infty \frac{1}{T_1}(\tau - \mu_{T_m})^2 e^{-\tau/T_1} d\tau}{\int_0^\infty A_c(\tau)\, d\tau} = \cdots = T_1^2$$

$\sigma_{T_m} = T_1$

delay spread $\simeq \sigma_{T_m} = T_1$

(2) According to Figure 2:

$$\mathcal{F}_{\tau}\{S_c(\tau,\rho)\} = S_c(\Delta f,\rho) \Longrightarrow S_c(\rho) = S_c(\Delta f,\rho)\Big|_{\Delta f=0}$$

$$\mathcal{F}_{\tau}\left\{\frac{1}{T_1} e^{-\tau/T_1}\right\} = \int_0^{\infty} \frac{1}{T_1}\left(e^{-\tau/T_1} e^{-j\Delta f\tau}\right) d\tau$$

$$= \frac{1}{T_1}\left(-\frac{1}{T_1} - j\Delta f\right)^{-1} e^{-\tau/T_1} e^{-j\Delta f\tau}\Big|_0^{\infty}$$

$$= \frac{1}{T_1} \frac{1}{j\Delta f + \frac{1}{T_1}}\Big|_{\Delta f=0} = 1$$

$$\Longrightarrow S_c(\rho) = \begin{cases} \frac{1}{\rho_1} & -\frac{\rho_1}{2} \leq \rho \leq \frac{\rho_1}{2} \\ 0 & \text{o.w} \end{cases}$$

Doppler spread $\simeq \rho_1/2$

(3) T_s: symbol time

<u>According to (2) :</u>

frequency flat : $T_s > \sigma T_m \simeq T_1$

~~slow~~ fast fading : $T_s > T_c = \dfrac{1}{\text{Doppler spread}} = \dfrac{2}{\rho_1}$

$$\Longrightarrow \quad T_s > \min\left(T_1, \frac{2}{\rho_1}\right)$$

(4)

$T_{cp} \geq T_m$ = delay spread

$T + T_{cp} \ll T_c$ *

minimum cyclic perfix overhead: $\boxed{T_{cp} = T_m = 1\,ms}$

$$R_b = \frac{N \log_2 M}{T + T_{cp}} \Rightarrow N = \frac{R_b (T + T_{cp})}{\log_2 M}$$

T_{cp}, R_b, $\log_2 M$ $\rightarrow$ fixed $\Rightarrow$ max(N) $\Rightarrow$ max(T) **

(* and **) $\Rightarrow$ $T = T_c - T_{cp} = 5 - 1 = \boxed{4\,ms}$

Doppler spread $= 200\,Hz \Rightarrow T_c = \frac{1}{200} = 5\,ms$

$$N = \frac{150 \times 10^6 (5 \times 10^{-3})}{\log_2 64} = \boxed{125000}$$

$$\Delta f = \frac{1}{T} = \frac{1}{4 \times 10^{-3}} = \boxed{250\ Hz}$$

$$B_{OFDM} = (N-1)\Delta f + 2 \times \frac{1}{T + T_{cp}} = \boxed{31.25\ MHz}$$

Question 3: MIMO

Gill Bates wanted to see the recent Orange Worldwide Developers Conference (OWDC) with the highest quality possible and see what are the latest products that are being released. Gill recalled that the Wifi access point (AP) he used last year was not able to provide video with highest quality. So Gill decided to replace this AP and found a new term called MIMO. Gill, being very curious to know what this is, reads about MIMO communication, and understood that optimally using different spatial dimensions is his best bet to get the best quality downloads. In particular, he knows that MIMO precoding is a technique that sits between full spatial multiplexing (using all spatial dimensions of the MIMO channel) and beamforming (using only the strongest spatial dimension). In this problem we investigate how Gill adapts both the use of different spatial dimensions and the modulation and power on each dimension to ensure each that he gets every detail of the OWDC event. Gill measures the channel from the WiFi AP to the WiFi system on his desk which also has three antennas and found the following channel matrix:

$$\mathbf{H} = \begin{bmatrix} 0.1 & 0.3 & 0.7 \\ 0.5 & 0.4 & 0.1 \\ 0.2 & 0.6 & 0.8 \end{bmatrix}$$

This can be written via the singular value decomposition (SVD) as:

$$\mathbf{H} = \begin{bmatrix} -0.555 & 0.3764 & -0.7418 \\ -0.3338 & -0.9176 & -0.2158 \\ -0.7619 & 0.1278 & 0.6349 \end{bmatrix} \begin{bmatrix} 1.3333 & 0 & 0 \\ 0 & 0.5129 & 0 \\ 0 & 0 & 0.0965 \end{bmatrix} \begin{bmatrix} -0.2811 & -0.7713 & -0.5710 \\ -0.5679 & -0.3459 & 0.7469 \\ -0.7736 & 0.5342 & -0.3408 \end{bmatrix}.$$

Assume the system bandwidth is $B = 1$ MHz and noise power spectral density is 1 mW / MHz. You can use the following BER approximation $P_b \approx 0.2 \times \exp(-1.5\gamma/(M-1))$ for your calculations, in which M is the size of the constellation and γ is the received SNR.

1. [*3 pt*] Find the transmit precoding and receiver shaping matrices used in the transmitter and receiver under spatial multiplexing (all 3 spatial dimensions used), beamforming (one spatial dimension used), and a combination of spatial multiplexing and beamforming called 2D precoding (two spatial dimensions used).

   $\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H$, *where* $\mathbf{U}$ *and* $\mathbf{V}$ *are unitary matrices and* $\mathbf{\Sigma}$ *is a diagonal matrix of rank* $R_H \leq min(M_r, M_t)$ *with* M_r *and* M_t *the number of receive and transmit antennas respectively. If* n *spatial dimensions are used, then* n *columns of* $\mathbf{V}$ *will form the precoding matrix and similarly* $\mathbf{U}(:, 1:n)^H$ *will form receiver shaping matrix. In this problem,* $n = 1, 2, 3$ *is for beamforming (one spatial dimension used), 2D precoding (two spatial dimensions used), and spatial multiplexing (all 3 spatial dimensions used) respectively.*

2. [*4 pt*] For a transmit power of 20 dBm, and M-QAM constellations with $M = 2^k, k = 1, 2, 3, 4, \ldots$, a target BER not exceeding 10^{-4} and power equally divided among all spatial streams, find the total data rate associated with all data streams under beamforming (1 spatial stream), 2D precoding (2 spatial streams), and spatial multiplexing (3 spatial streams). What is the best way for Gill to use his spatial dimensions to maximize rate?

   Signal power $\rho = 100\,mW$ *and since power is equally divided among all spatial streams, each spatial channel gets* ρ/n *power, where* n *is the number of spatial channels used.*

   Since the noise power is 1 mW, SNR of each spatial stream will be $\sigma_i^2\rho/n$*. The tight BER bound,* $P_b \leq 0.2e^{-1.5\gamma/(M-1)}$*, can be rewritten as* $M \leq 1 + \frac{1.5\gamma}{-ln(5P_b)}$*. Using these, we find the following:*

   <u>For beamforming</u> ($n = 1$*),* $\gamma_1 = (1.33)^2\frac{\rho}{1} = 177.68 \therefore M_1 \leq 36.06 \implies M_1 = 32 \implies k = 5$ *bits can be sent.*

   <u>For 2D precoding:</u> $\gamma_1 = (1.333)^2\frac{\rho}{2} = 88.84 \therefore M_1 \leq 18.53 \implies M_1 = 16 \implies k = 4$ *bits on first stream and*
   $\gamma_2 = (0.5129)^2\frac{\rho}{2} = 13.15 \therefore M_2 \leq 3.6 \implies M_2 = 2 \implies k = 1$ *bit on second stream*

   <u>For spatial multiplexing</u>, $\gamma_1 = (1.333)^2\frac{\rho}{3} = 59.23 \therefore M_1 \leq 12.68 \implies M_1 = 8 \implies k = 3$ *bits on first stream,*
   $\gamma_2 = (0.5129)^2\frac{\rho}{3} = 8.76 \therefore M_2 \leq 2.73 \implies M_2 = 2 \implies k = 1$ *bit on second stream, and*
   $\gamma_3 = (0.0965)^2\frac{\rho}{3} = 0.31 \therefore M_3 \leq 1.06 \implies M_3 = 1 \implies k = 0$ *bit on third spatial stream. (We cannot transmit on this stream)*
   So the rate in beamforming, 2D precoding, and spatial multiplexing is $R = 5$ *Mbps,* $(4+1) = 5$ *Mbps, and* $(3+1) = 4$ *Mbps resåectively.*
   So, in this case, Gill can either use beamforming or 2D precoding to maximize rate.

3. [*2 pt*] Gill would like to also test data rates by sending the data from the WiFi system on his desk to the WiFi AP. When using the same antennas and assuming the environment is not changed, what will be the channel matrix between the transmitter (now the system on his desk) and the receiver (the AP). How does this affect precoding and shaping? How is the data rate affected? Motivate your answers.
Hint: If $svd(\mathbf{H}) = \mathbf{U\Sigma V^T}$, *then* $svd(\mathbf{H}^T) = \mathbf{V\Sigma}^T\mathbf{U}^T$.

   What we are doing is exchanging the transmitter and receiver. When this happens, we still have the same channel $\mathbf{H}$ *and it's only the precoding and post shaping matrices that are swapped. This can be seen by writing down the equation* $\mathbf{y}_{Rx} = \mathbf{H}\mathbf{x}_{AP} + \mathbf{n}$ *when AP is the transmitter. When AP is the receiver, we have* $\mathbf{y}_{AP} = \mathbf{H}^T\mathbf{x}_{Rx} + \mathbf{n}$. *Since* $SVD(\mathbf{H}^T) = \mathbf{V\Sigma}^T\mathbf{U}^T = \mathbf{V\Sigma U}^T$, *the precoding and shaping matrices will be interchanged. Data rate is still the same as the singualr values (diagonal entries of* $\mathbf{\Sigma}$*) of* $\mathbf{H}$ *and* $\mathbf{H}'$ *are the same.*

4. [*1 pt*] Suppose that the third antenna of the transmitter and the third antenna of the receiver are not working due to a software bug. What will be the channel matrix now? How would you perform zero-forcing equalization in this case?

   We need to discard last row and column of $\mathbf{H}$ *if it is the last antennas that is not working in both the cases.* $\mathbf{H_{new}} = \begin{bmatrix} 0.1 & 0.3 \\ 0.5 & 0.4 \end{bmatrix}$,*ZF equalizer* $\mathbf{H}_{ZF} = \mathbf{H}_{new}^{-1} = \begin{bmatrix} -3.6 & 2.7 \\ 4.5 & -0.9 \end{bmatrix}$

5. [*2 pt*] Besides the non-functional third antennas at the transmitter and receiver, suppose the second antenna at the receiver also stopped working effectively making the whole system 2 antennas at transmitter and 1 antenna at the receiver. Assume Alamouti scheme is used, explain mathematically how the receiver can decode information symbols s_1 and s_2 sent by the two antennas at the transmitter? What is the diversity order and array gain of this system? Motivate your answers
Hint: In the Alamouti scheme, if symbols s_1 *and* s_2 *are sent in time slot 1, then* $-s_2^*$ *and* s_1^* *are sent in the second time slot.*

   In first time slot $y_1 = h_1s_1 + h_2s_2 + n_1$ *and in the second time slot, we have* $y_2 = -h_1s_2^* + h_2s_1^* + n_2$,
$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} = \mathbf{H}_A\mathbf{s} + \mathbf{n}$. *The receiver can perform* $\mathbf{H}_A^H\mathbf{y}$, *which is*
$= (|h_1|^2 + |h_2|^2)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} \tilde{n}_1 \\ \tilde{n}_2 \end{bmatrix}$, *where* $\tilde{\mathbf{n}} = \mathbf{H_A}\mathbf{n}$.
The receiver can $\mathbf{H}_A^H\mathbf{y}/(|h_1|^2 + |h_2|^2)$ *and get an estimate of* s_1 *and* s_2.
Since we have $(|h_1|^2 + |h_2|^2)\mathbf{I}_2$, *we have the diversity of order 2. There is no array gain as the ransmit power is divided between two antennas at the transmitter and there is only one receiver antenna.*

Question 4: various topics

Below is a selection of smaller questions. There is often no unique answer, so provide a short, but well-motivated answer for each question.

Questions

1. [*2 pt*] Consider two scenarios:

   (a) a rural area with flat terrain, limited vegetation, and few mobile phone users per square meter.

   (b) an urban area with many buildings and many mobile phone users per square meter.

   Argue for which type of small-scale fading we are likely to experience when making a mobile phone call in scenario (a) and (b).

   In (a), we expect a strong line-of-sight (LOS) component with limited scattering for non-LOS components. Hence, Ricean fading is likely to occur. In (b), we do not expect a LOS component. Scattering will probably be significant and we therefore expect Rayleigh fading.

2. [*2 pt*] You are seated at Café Linsen and you are connected with your laptop to the Chalmers wireless network called NOMAD which is based on IEEE 802.11b.

   (a) What channel capacity concept would be applicable?

   (b) During lunch time, there are a lot of people walking around in Café Linsen. Motivate what channel capacity would be applicable, given that your laptop is still connected to the same NOMAD network?

   At Café Linsen, (i) the laptop being stationary, there is no doppler, due to which the coherence time can be treated as innte. Therefore, outage capacity is a good metric. (ii) With people moving around during lunch, can only give rise to the multipaths and again as there is no doppler, outage capacity is still a good metric

3. [*1 pt*] Consider a channel that has a doppler spread of 10 Hz. Assume a BPSK modulated voice signal is transmitted over this channel at 30 Kbps. Is outage probability or average probability of error a better performance metric and why?

   Symbol duration $T_s = 1/30k = 0.03ms$. *Coherence time* $T_c \approx 1/B_D = 0.1s$. *Since* $T_s \ll T_c$, *outage probability is a better performance measure as a single deep fade can affect a lot of symbols.*

4. [*2 pt*] Consider the downlink of a cellular system with hexagonal cells of the same size, equal transmitted downlink powers, and channel reuse. Assume log-distance path loss and neglect shadowing and small-scale fading effects. Describe the effect on the signal-to-interference ratio at the cell border and the system capacity (measured in the number of available channels per unit area) as

   (a) the cell radius, R, is decreased.

   (b) the cluster size, N, is decreased.

   $SIR = \frac{1}{6}(3N)^{\gamma/2}$, *where* γ *is the path loss exponent and* N *is the cluster size.*

   System capacity $= \frac{1}{NR^2}K$, *where* R *is the cell radius and* K *is a constant, which does not depend on* N *and* R

   (a) *As R decreases, SIR is unchanged whereas capacity increases*

   (b) *As N decreases, SIR decreases and capacity increases*

5. [*1 pt*] Suppose you are talking on a GSM phone while approaching the base station. Will your exposure to microwave radiation increase or decrease? Motivate carefully.

   Typically, the mobile phones attempt to radiate isotropically (as beamforming is an advance feature). The radiation to the human body is from the mobile phone, as it is closer to the human body. While walking closer to the base station, the radiation decreases, as the lesser power is required to communicate with the base station due to the decrease in distance. In short, in GSM, uplink power control kicks in steps of 2 dB reduction, as the mobile moves closer to the base station.

6. [*1 pt*] The question if there exists long-term health effect from low-level microwave exposure is still open. Several epidemiological studies have been performed on mobile phone users. Give a short overview of the research results from these studies.

   In three years exposure studies show that there is no elevated risk of tumors. However, in a ten year period, several studies showed that there is a risk of hearing nerve or brain tumors.

7. [*2 pt*] Suppose we need to use a hardware platform that has signficant phase noise. Which of the following modulation schemes would be preferable to use: OFDM with a large number of subcarriers, OFDM with a long cyclic prefix, single carrier BPSK with square pulses with same total data rate as multi carrier system? Motivate.
Hint: Assume that the higher the symbol rate, the lower the degradation effect due to phase noise.

   Since the symbol rate in OFDM is usually lower than the equivalent single carrier system with the same total bit-rate, the OFDM scheme shows a worse performance. Hence, (i) OFDM with more subcarriers could only increase ICI but with CP, this could be mitigated. While in (ii) OFDM with long CP, will make the actual symbol rate much smaller hence, worsening the phase noise effects. Therefore, a single carrier system such as BPSK with square pulses is the best choice for the hardware with significant phase noise.

8. [*1 pt*] In OFDM systems, what are the two main components that are distorted due to phase noise? Explain the two components briefly.

   Two main components distorted by phase noise in OFDM systems are common phase error (CPE) and intercarrier interference (ICI). CPE is the constant phase rotation experienced by all the subcarriers within one OFDM symbol duration. ICI is when the neighbouring carriers interfer.