SSY135 Wireless Communications

Department of Signals and Systems

Exam Date: March 16 2018, 14:00-18:00

Teaching Staff

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Material Allowed material is

- Chalmers-approved calculator
- L. Råde, B. Westergren. Beta, Mathematics Handbook, any edition.
- One A4 page with your own handwritten notes. Both sides of the page can be used. Photo copies, printouts, other students' notes, or any other material is not allowed.
- A dictionary.

Grading A correct, clear and well-motivated solution gives a maximum of 12 points.

An erroneous answer, unclear, incomplete or badly motivated solutions give point reductions down to a minimum of 0 points. Answers in any other language than English are ignored.

Results Results are posted no later than April 1. The grading review is on April 4, 11:30–12:30 in the Blue room in the E2 Department.

Grades To pass the course, all projects and the exam must be passed. The exam is passed by securing at least 12 points. The project is passed by securing at least 8 points (4 for the report and 4 for the oral exam) in each part of the project. The final grade on the course will be decided by the homework (max score 6), project (max score 40), quizzes (max score 6), and final exam (max score 48). The sum of all scores will decide the grade according to the following table.

Total Score	0–39	40–59	60–79	≥ 80
Grade	Fail	3	4	5

PLEASE NOTE THAT THE PROBLEMS ARE NOT NECESSARILY ORDERED IN DIFFICULTY.

Good luck!

1 Fading models and system design

Consider a fading channel with impulse response

$$c(\tau, t) = 2\beta_1\delta(\tau) + 2\sin(t + U)\delta(\tau - \tau_1) + 3\beta_1(t)\delta(\tau - \tau_2)$$

where $\tau_1 = 20$ ns, $\tau_2 = 50$ ns, $\beta_1(t)$, are 2 i.i.d. complex, zero-mean unit variance Gaussian random variables, and U is a random variable uniformly distributed between 0 to 2π, i.e., $U \sim \text{uniform}\,(0, 2\pi)$. Unless explicitly stated otherwise, all random variables are mutually independent. Figure 1 contains some useful relations.

1. [4 pt] Compute and draw the power delay profile. Recall that $A_c(\tau, \Delta t) = \mathbb{E}\{c(\tau, t + \Delta t)\, c^*(\tau, t)\}$.
   Hint:

$$\cos(a)\cos(b) = \frac{1}{2}(\cos(a+b) + \cos(a-b))$$
$$\sin(a)\sin(b) = \frac{1}{2}(\cos(a-b) - \cos(a+b))$$
$$\sin(a)\cos(b) = \frac{1}{2}(\sin(a+b) + \sin(a-b))$$
$$\cos(a)\sin(b) = \frac{1}{2}(\sin(a+b) - \sin(a-b))$$

   Answer:

$$A_c(\tau, \Delta t) = E\{c(\tau, t + \Delta t)c^*(\tau, t)\} =$$
$$4E\left\{|\beta_1(t)|^2\right\}\delta(\tau) + 4E\{\sin(t + U + \Delta t)\sin(t + U)\}\delta(\tau - \tau_1) + 9E\left\{|\beta_2(t)|^2\right\}\delta(\tau - \tau_2)$$
$$= 4 \times 1_{\Delta t=0}\delta(\tau) + 2\delta(\tau - \tau_1) + 9 \times 1_{\Delta t=0}\delta(\tau - \tau_2)$$
$$\lim_{\Delta t \to 0} A_c(\tau, \Delta t) = A_c(\tau) = 4\delta(\tau) + 2\delta(\tau - \tau_1) + 9\delta(\tau - \tau_2)$$

2. [2 pt] Compute the RMS of the delay spread,

$$\sigma_{T_m} = \sqrt{\frac{\int_0^{+\infty}(\tau - \mu_{T_m})^2 A_c(\tau)\, d\tau}{\int_0^{+\infty} A_c(\tau)\, d\tau}}$$

   where

$$\mu_{T_m} = \frac{\int_0^{+\infty} \tau A_c(\tau)\, d\tau}{\int_0^{+\infty} A_c(\tau)\, d\tau}.$$

   If you are unable to answer the first question, use $A_c(\tau) = 3\delta(\tau) + 7\delta(\tau - \tau_1) + 2\delta(\tau - \tau_2)$.
   Answer:

$$\mu_{T_m} = \frac{2\tau_1 + 9\tau_2}{15} = \frac{2 \times 20 + 9 \times 50}{19} = 32.667 \text{ ns}$$
$$\sigma_{T_m} = \sqrt{\frac{(0-\mu_{T_m})^2 4 + (\tau_1 - \mu_{T_m})^2 2 + (\tau_2 - \mu_{T_m})^2 9}{15}} = 22.0504 \text{ ns}$$

3. [2 pt] Assume that the maximum delay spread can be approximated by 8 ns, compute the coherence bandwidth of the channel. Find the maximum data rate (in Mbit/s) where the system experiences flat fading if 16-QAM with raised cosine pulse shaping with roll of factor 1 is used?
   Answer

$$B_c \approx \frac{1}{\sigma_{T_m}} = 45.3507 \text{ MHz}$$
$$\frac{1+\beta}{T_s} < B_c$$
$$R_s < \frac{B_c}{1+\beta}$$
$$R_b < \frac{B_c \log_2 M}{(1+\beta)} = 136.0521 \text{ MHz}$$

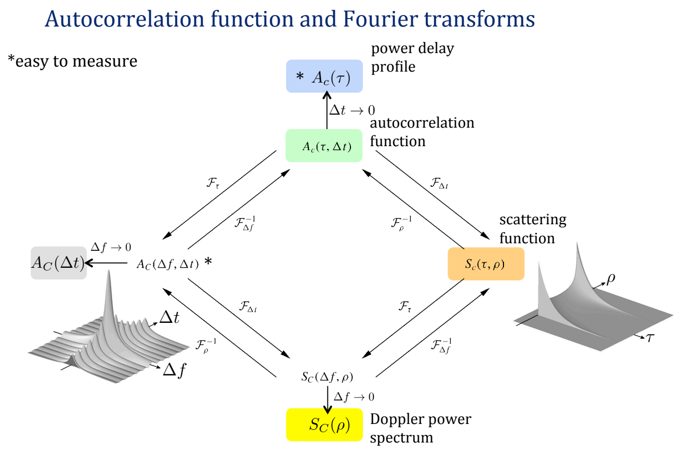

Figure 1: Fourier relationships

4. [4 pt] Consider a cell with log-distance pathloss, log-normal shadowing and Rayleigh fading. The base station uses 16-QAM modulation with 1Mbit/s and transmits with power 20W and carrier frequency 1800Mhz. The path loss exponent is 3.2 and path gain at the reference distance $d_0 = 1\,\text{km}$ is the same with free space path gain. Assume both transmit and receive antennas are omnidirectional. Suppose the shadow fading adds a fixed extra path loss of 10dB. Consider a vehicle traveling with 36km/h at a distance of 4 km from the base station. Assume that the small-scale fading has Clarke's power spectrum.
Compute the level crossing rate at 15dB below the average received power. Compute the average deep fade duration. Does the system experience burst errors?
Hint: The level crossing rate under Rayleigh fading is given by $L_Z = \sqrt{2\pi} f_D \rho \exp(-\rho^2)$, where $\rho = Z/\sqrt{\bar{P}_r}$, for a Doppler shift f_D, an average received power $\bar{P}_r$ and a target amplitude level Z. The average fade duration is given by $t_Z = (1 - \exp(-\rho^2))/L_Z$.
Answer:

$$f_D = \frac{v \times f_c}{c} = 60Hz$$
$$\bar{P}_r(dBm) = xdBm, P_0(dBm) = x - 15dBm,$$
$$\bar{P}_r = 10^{x/10}, P_0 = 10^{x/10\text{-}1.5}, \rho = \sqrt{\frac{P_0}{\bar{P}_r}} = 0.1778$$
$$L_Z = \sqrt{2\pi} f_D \rho e^{-\rho^2} = 25.9124$$
$$t_Z = (1 - e^{-\rho^2})/L_Z = 0.0012$$

2 Water filling and Diversity combining

1. [5 pt] Consider a SISO system with a target BER of 10^{-3}. Suppose the system uses MQAM modulation and the wireless channel has four states. Assuming a fixed average transmit power $\bar{P} = 10$ mW, the received SNR associated with each channel state is $\gamma_1 = -10$ dB, $\gamma_2 = 0$ dB, $\gamma_3 = 5$ dB, and $\gamma_4 = 10$ dB, respectively. The probabilities associated with the channel states are $p(\gamma_1) = p(\gamma_2) = 0.2$ and $p(\gamma_3) = p(\gamma_4) = 0.3$. You will determine the optimal power for each channel state and the corresponding average spectral efficiency of the system. Assume that the modulation order can be continuous and use the BER upper bound of MQAM, $P_b(\gamma) \leq 0.2 \exp\left(\frac{-1.5\gamma}{M-1} P(\gamma)/\bar{P}\right)$.
Hint: If you express the constellation size as $M = 1 + K\gamma P(\gamma)/\bar{P}$, the optimum power allocation can be expressed as

$$P(\gamma) = \begin{cases} \frac{\bar{P}}{K}\left(\frac{1}{\gamma_K} - \frac{1}{\gamma}\right) & \text{if } \gamma \geq \gamma_K \\ 0 & \text{if } \gamma < \gamma_K \end{cases},$$

where γ_K is a threshold to ensure that the average power constraint is met.

(a) [2 pt] Determine the value of K and γ_K.

(b) [1 pt] Find the optimal power and M for each power level.

(c) [2 pt] Determine the average spectral efficiency.

Solution: We find that $K = \frac{-1.5}{\ln(5\times10^{-3})}$. Assuming that all channel states are used, then

$$\gamma_K = \frac{p(\gamma_1) + p(\gamma_2) + p(\gamma_3) + p(\gamma_4)}{K + \frac{p(\gamma_1)}{\gamma_1} + \frac{p(\gamma_2)}{\gamma_2} + \frac{p(\gamma_3)}{\gamma_3} + \frac{p(\gamma_4)}{\gamma_4}} = 0.3834,$$

which is not possible. Assuming channel state 1 is not used, then

$$\gamma_K = \frac{p(\gamma_2) + p(\gamma_3) + p(\gamma_4)}{K + \frac{p(\gamma_2)}{\gamma_2} + \frac{p(\gamma_3)}{\gamma_3} + \frac{p(\gamma_4)}{\gamma_4}} = 1.3159,$$

which is also not feasibly. If channel states 1 and 2 are not used, then

$$\gamma_K = \frac{p(\gamma_3) + p(\gamma_4)}{K + \frac{p(\gamma_3)}{\gamma_3} + \frac{p(\gamma_4)}{\gamma_4}} = 1.4704,$$

which feasible. Then

$$\frac{P(\gamma_3)}{P} = \frac{1}{K}\left(\frac{1}{\gamma_K} - \frac{1}{\gamma_3}\right) = 1.2848$$
$$\frac{P(\gamma_4)}{P} = \frac{1}{K}\left(\frac{1}{\gamma_K} - \frac{1}{\gamma_4}\right) = 2.0486$$
$$M(\gamma_3) = 1 + K\gamma_3\frac{P(\gamma_3)}{P} = 2.1502, M(\gamma_4) = 1 + K\gamma_4\frac{P(\gamma_4)}{P} = 6.7996$$
$$\bar{R} = \log_2(M(\gamma_3))\,p(\gamma_3) + \log_2(M(\gamma_4))\,p(\gamma_4) = 1.161$$

2. [5 p] Suppose that the receiver is equipped with one additional receive antenna; so the system is changed to SIMO configuration. The receiver has two modes: 1) using maximum ratio combining (MRC), 2) using selective combining (SC). Furthermore, suppose that the wireless channel is changed and the received SNRs corresponding to each receiver antenna defined as γ_1, γ_2, respectively. The distribution of the received SNR for each receiver antenna is shown in Fig. 2. Calculate the distribution of the combined SNR for each combining mode of the receiver. Recall that MRC performs optimal combining of the branches, while SC only picks the best branch.
Hint: For independent random variables, $P(\max(X_1, X_2, \ldots, X_n) \leq \gamma) = \prod_{i=1}^{N} P(X_i \leq \gamma)$.

$$\gamma_{MRC} = \gamma_1 + \gamma_2$$
$$f_{\gamma_{MRC}}(\gamma_{MRC}) = f_{\gamma_1}(\gamma_1) * f_{\gamma_2}(\gamma_2)$$
$$f_{\gamma_{MRC}}(\gamma_{MRC}) = \left\{ \begin{array}{c} \frac{3}{200}x \; 0 \leq x \leq 5 \\ \frac{1}{200}x + \frac{1}{20} \; 5 \leq x \leq 10 \\ \frac{-3}{200}x + \frac{5}{20} \; 10 \leq x \leq 15 \\ \frac{-1}{200}x + \frac{2}{20} \; 15 \leq x \leq 20 \end{array} \right\}$$
$$F_{\gamma_{SC}}(\gamma_{SC}) = F_{\gamma_1}(\gamma_1)\,F_{\gamma_2}(\gamma_2) = \left\{ \begin{array}{c} \frac{3}{200}x^2 \; 0 \leq x \leq 5 \\ \frac{1}{200}x^2 + \frac{1}{20}x \; 5 \leq x \leq 10 \end{array} \right\}$$
$$f_{\gamma_{SC}}(\gamma_{SC}) = \frac{\partial}{\partial\gamma_{SC}}F_{\gamma_{SC}}(\gamma_{SC}) = \left\{ \begin{array}{c} \frac{6}{200}x \; 0 \leq x \leq 5 \\ \frac{1}{100}x + \frac{1}{20} \; 5 \leq x \leq 10 \end{array} \right\}$$

(c) [2 p] Suppose that the system quality of the service restricts us to have minimum receive SNR of 0 dB. Calculate the outage probability of each combining mode in part b. Which combining mode is better? Motivate your answer.

$$P_{out}^{\gamma_{MRC}} = p(\gamma_{MRC} \leq 1) = \frac{3}{400}$$
$$P_{out}^{\gamma_{SC}} = p(\gamma_{SC} \leq 1) = \frac{3}{200}$$

MRC provides lower outage, since it maximizes the combined SNR.

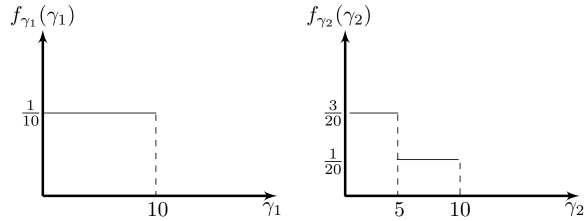

Figure 2: The distribution of the received SNR γ_1 and γ_2.

3 MIMO and Massive MIMO

In this question, we analyze MIMO and massive MIMO communication systems. We will consider communication of the form

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n},$$

where $\mathbf{n} \sim \mathcal{N}(0, 2\sigma^2 \mathbf{I}_{M_r})$ (i.e., real and imaginary part have variance σ^2). The communication bandwidth is 1 MHz and the noise power per receive antenna is 1 mW (real and imaginary part combined). The total transmit power $\mathbb{E}\{\|\mathbf{x}\|^2\}$ is limited to 60 mW. Transmitted data symbols are generated from M-QAM constellations ($M \in \{4, 16, 256\}$), for which the BER is approximated by $P_b \approx 0.2\exp(-1.5\gamma/(M-1))$, in which γ is the SNR.

1. [9 pt] In this question, we look at receiver-side processing for a 2×2 MIMO system. The channel matrix is given by

$$\mathbf{H} = \begin{bmatrix} 1 & 0.1 \\ 1 & 0 \end{bmatrix}. \tag{1}$$

The transmitter does not know $\mathbf{H}$ so it sets $\mathbb{E}\{|x_1|^2\} = \mathbb{E}\{|x_2|^2\}$. The receiver knows the channel $\mathbf{H}$ and applies zero forcing.

(a) [2 pt] What will be the SNR for each of the two data streams after zero forcing?
Solution: The ZF equalizer is

$$\mathbf{W} = \begin{bmatrix} 0 & 1 \\ 10 & -10 \end{bmatrix},$$

so the receiver sees

$$\mathbf{z} = \mathbf{W}\mathbf{y} = \mathbf{W}\mathbf{H}\mathbf{x} + \mathbf{W}\mathbf{n} = \mathbf{x} + \mathbf{w},$$

where (in mW)

$$\begin{aligned} \mathbb{E}\{|w_1|^2\} &= \mathbb{E}\{|n_2|^2\} = 1 \\ \mathbb{E}\{|w_2|^2\} &= 100\mathbb{E}\{|n_1|^2\} + 100\mathbb{E}\{|n_2|^2\} = 200 \\ \mathbb{E}\{|x_1|^2\} &= \mathbb{E}\{|x_2|^2\} = 30 \end{aligned}$$

Hence, the SNR for stream 1 is 30 and for stream 2 is 3/20.

(b) [1 pt] Which constellation will the transmitter choose to minimize the BER? What will be the BER for each stream?
Solution: To minimize the BER, we should either maximize the SNR or minimize M. Since the transmitter does not know the SNR, it can only minimize M, i.e., $M = 4$. The BER for streams 1 and 2 will be

$$\begin{aligned} P_{b,1} &= 0.2\exp(-1.5 \times 30/3) \approx 6e^{-8} \\ P_{b,2} &= 0.2\exp(-1.5 \times 3/60) \approx 0.2. \end{aligned}$$

(c) [1 pt] Which constellation will the transmitter choose to maximize the data rate, given a target BER of 10^{-3}, but without knowing the channel?
Solution: Since the transmitter does not know the channel, it cannot choose a constellation to meet a target BER.

(d) [3 pt] Suppose the second transmit antenna is broken, how would you answer to (b) and (c) above change?
Solution: If the second transmit antenna is broken, the channel becomes

$$\mathbf{y} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} x_1 + \mathbf{n}.$$

The receiver can apply MRC $\mathbf{w}^T = [1\,1]$, leading to an observation $z = 2x_1 + w$, where $\mathbb{E}\{|w_1|^2\} = \mathbb{E}\{|n_1|^2\} + \mathbb{E}\{|n_2|^2\} = 2$ and $\mathbb{E}\{|x_1|^2\} = 60$ (since all power can be devoted to the first antenna). The SNR is thus $4 \times 60/2 = 120$. The transmission will thus have a BER of

$$P_{b,1} = 0.2 \exp(-1.5 \times 120/3) \approx 2e^{-27}.$$

It is still not possible to meet a certain BER target.

(e) [2 pt] Suppose that the transmitter now knows the channel and that the second transmit antenna is still broken. What is the maximum rate that can be achieved to meet a target BER of 10^{-3}?
Solution: In this case, the transmitter cannot affect the SNR but it can choose a good constellation, so

$$\begin{aligned} 10^{-3} &\approx 0.2 \exp(-1.5 \times 120/(M-1)) \\ M &= -1.5 \times 120/(\log(5 \times 10^{-3})) + 1 = 34, \end{aligned}$$

so $M = 16$ is the best option. Thus the rate becomes $4\,Mbit/s$.

2. [3 pt] In this question, we look at massive MIMO communication under rich scattering conditions. We consider a base station with M antennas and K users, each with a single antenna. The uplink channel between the k-th user and the base station is a $M \times 1$ vector $\mathbf{h}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$ (i.e., the entries are independent zero-mean, unit variance complex Gaussian). The channels of different users are independent. The base station applies a matched filter precoder, leading to the downlink observation model

$$\mathbf{y} = \mathbf{H}^T \mathbf{W} \mathbf{s} + \mathbf{n},$$

where $\mathbf{s}$ is the $K \times 1$ vector of data symbols for all users (with $\mathbb{E}\left\{\|\mathbf{s}\|^2\right\} = KE_s$) and the k-th column of $\mathbf{W}$ is given by $\mathbf{w}_k = \mathbf{h}_k^*/\sqrt{M}$. The noise $\mathbf{n}_k \sim \mathcal{CN}(\mathbf{0}, N_0 \mathbf{I}_M)$.
Hint: In all questions below, you can make use of

$$\lim_{M \to \infty} \frac{1}{M} \sum_{m=1}^{M} f(x_m) \to \mathbb{E}\{f(x)\},$$

for independent $x_1, \ldots, x_M$.

(a) [1 pt] Let $\mathbf{x} = \mathbf{W}\mathbf{s}$. Show that $\mathbb{E}\left\{\|\mathbf{s}\|^2\right\} = \mathbb{E}\left\{\|\mathbf{x}\|^2\right\}$, so that the precoder does not amplify the signal, when $M \to \infty$.
Solution:

$$\mathbb{E}\left\{\|\mathbf{x}\|^2\right\} = \mathbb{E}\left\{\mathbf{x}^H \mathbf{x}\right\} = \mathbb{E}\left\{\mathbf{s}^H \mathbf{W}^H \mathbf{W} \mathbf{s}\right\}$$

in which

$$\begin{aligned} [\mathbf{W}^H \mathbf{W}]_{kl} &= \frac{1}{M} [\mathbf{H}^H \mathbf{H}]_{kl} = \frac{1}{M} \mathbf{h}_k^H \mathbf{h}_l = \frac{1}{M} \sum_{m=1}^{M} h_{k,m}^* h_{l,m} \approx \mathbb{E}\{h_k^* h_l\} \\ &= \begin{cases} 0 & k \neq l \\ 1 & k = l \end{cases} \end{aligned}$$

so $\mathbb{E}\left\{\|\mathbf{s}\|^2\right\} = \mathbb{E}\left\{\|\mathbf{x}\|^2\right\}$.

(b) [1 pt] Express the observation at user k as the sum of three components: a useful signal intended for user k (of the form $\alpha_k s_k$), interference corresponding to signals intended for other users (of the form $\sum_{k' \neq k} \alpha_{k'} s_{k'}$), and noise n_k. The data for different users is independent with $\mathbb{E}\{|s_k|^2\} = E_s$, $\forall k$. User k divides the observation with $\sqrt{M}$, so it uses $y_k/\sqrt{M}$.

i. For the useful signal $\alpha_k s_k/\sqrt{M} \to s_k$ when $M \to \infty$

ii. The interference signal $\sum_{k' \neq k} \alpha_{k'} s_{k'}/\sqrt{M}$ tends to 0 when $M \to \infty$

iii. The noise $n_k/\sqrt{M}$ tends to zero when $M \to \infty$.

Solution: The signal for user k is the k-th entry in $\mathbf{y}$

$$\begin{aligned} y_k &= \mathbf{h}_k^T \mathbf{W}\mathbf{s} + n_k \\ &= \mathbf{h}_k^T \left(\sum_{k'=1}^{K} \mathbf{w}_{k'} s_{k'} \right) + n_k \\ &= \underbrace{\mathbf{h}_k^T \mathbf{w}_k}_{\alpha_k} s_k + \underbrace{\sum_{k' \neq k}^{K} \mathbf{h}_k^T \mathbf{w}_{k'}}_{\alpha_{k'}} s_{k'} + n_k. \end{aligned}$$

The useful signal is

$$\mathbf{h}_k^T \mathbf{w}_k s_k/\sqrt{M} = \frac{1}{M} \sum_m |h_{k,m}|^2 s_k = E\{|h_{k,m}|^2\} s_k = s_k$$

The interfering signal is

$$\mathbf{h}_k^T \left(\sum_{k' \neq k}^{K} \mathbf{w}_{k'} s_{k'} \right) /\sqrt{M} = \sum_{k' \neq k} \frac{1}{M} \sum_m h_{k,m} h_{k',m} s_{k'} \to 0$$

(c) [1 pt] How would the overall design change when the transmitter does not know the channel? Assuming the base station sends to a single user with precoder, $\mathbf{w} = 1/\sqrt{M}$, what is the distribution of the effective downlink channel for that user?

Solution: When the transmitter does not know the channel, it cannot perform precoding. So the only way to communicate would be to use TDMA (or another orthogonal scheme). Then the received signal at user k would be

$$y_k = \mathbf{h}_k^T \mathbf{w}_k s_k + n_k,$$

where $\mathbf{w}_k = \mathbf{1}/\sqrt{M}$, so

$$y_k = \underbrace{\frac{1}{\sqrt{M}} \sum_m h_{k,m}}_{g_k} s_k + n_k,$$

with $g_k \sim CN(0, 1)$.

4 Smaller questions

1. [2 p] Describe one role that the FCC (Federal Communications Commission) plays in the reduction of harmful effects of electro-magnetic radiation? Answer is 20 words or less.
   Answer: FCC sets minimum guidelines for safe human exposure from wireless devices.

2. [1 p] Describe at least three ways in which ethics can play a role in research.
   Answer (examples): authorship, data collection, military use.

3. [2 p] Consider a channel that has a doppler spread of 5 Hz. Assume a QPSK modulated voice signal is transmitted over this channel at 30 Kbps. Is outage probability or average probability of error a better performance metric and why?
   Answer: Symbol duration $T_s = 2/30k = 0.067ms$. Coherence time $T_c \approx 1/B_D = 0.2s$. Since $T_s \ll T_c$, outage probability is a better performance measure as a single deep fade can affect a lot of symbols.

4. [2 p] In which two scenarios ergodic capacity has greater value? a. A fading channel with CSIR and a fading channel with CSIR and CSIT with no power adaptation.
   Answer: with no power adaptation, both capacity values are the same

5. [1 p] What is channel reuse in cellular systems? What are the trade-offs involved in setting a reuse factor?
   Answer: Number of cells per cluster, lower N leads to more efficiency, but with more interference

6. [1 p] Describe the difference between array gain and diversity gain.

   Answer: Array gain is the increase in average received SNR, while the diversity gain is the decrease of the variance of the normalized received SNR which leads to increase the slop of the BER.

7. [1 p] Consider a flat fading channel where the complex envelope of the received signal is $r(t) = \alpha(t)s(t) + n(t)$, where $n(t)$ is complex white Gaussian noise with variance σ^2 per real dimension, and $s(t)$ is the complex envelope of the transmitted signal. For Rayleigh-fading, what is the distribution of $\mathbb{E}\{n(t)\}$, $\mathbb{E}\{|n(t)|^2\}$?
   Answer: both are delta's, one at zero, one at $2\sigma^2$

8. [2 p] Assume a cellular system with dedicated 160 MHz bandwidth for each cluster where the pathloss is the dominated fading. If the pathloss exponent is 2, the required SINR of each cell is 12 dB, and the each user has 50 KHz requested bandwidth, find the maximum user capacity (active users per cell) of the cellular system? Hint: The reuse factor (the number of cells per cluster is $N \geq (SIR)^{2/\gamma}$).
   Answer: $N \geq 16$, $N_s = \frac{B_{cluster}}{N_{\min} B_{user}} = 200$.