Exercise 3 in SSY135 Wireless Communications

February 6, 2020

1. [G-3.17] Let a scattering function $S_c(\tau, \rho)$ be nonzero over $0 \leq \tau \leq 0.1$ ms and $-0.1 \leq \rho \leq 0.1$ Hz. Assume that the power of the scattering function is approximately uniform over the range where it is nonzero.

   (a) What are the multipath spread and the doppler spread of the channel?

   (b) Suppose you input to this channel two identical sinusoids separated in frequency by $\triangle f$. What is the minimum value of $\triangle f$ for which the channel response to the first sinusoid is approximately uncorrelated of the channel response to the second sinusoid.

   (c) For two sinusoidal inputs to the channel $u_1(t) = \sin(2\pi f t)$ and $u_2(t) = \sin(2\pi f(t + \triangle t))$, what is the minimum value of $\triangle t$ for which the channel response to $u_1(t)$ is approximately uncorrelated of the channel response to $u_2(t)$.

   (d) Will this channel exhibit flat fading or frequency-selective fading for a typical voice channel with a 3 KHz bandwidth? How about for a cellular channel with a 30 KHz bandwidth?

   Hint: $S_c(\tau; \rho) = F_{\Delta t}(A_c(\tau; \Delta t))$

2. [Exam Q - 2015] Consider a WSS-US fading channel with scattering function

$$S_c(\tau, \rho) = \begin{cases} \frac{1}{T_1 \rho_1} e^{-\tau/T_1} & \text{if } \ 0 \leq \tau, \quad -\rho_1/2 \leq \rho \leq \rho_1/2 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

   (a) Find the power delay profile, and calculate the root mean square (rms) delay spread of this channel.

   (b) Find the Doppler power spectrum, and calculate the Doppler spread of this channel.

   (c) Determine the interval for the symbol time which the transmitted signal over this channel experiences frequency-flat, fast fading.

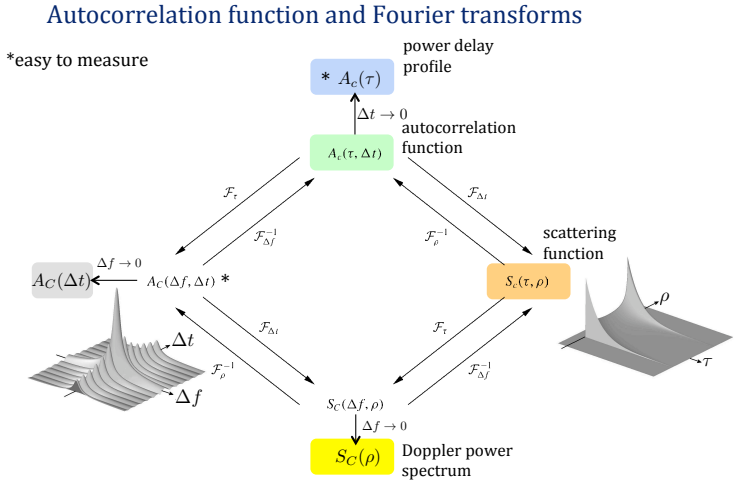

Figure 1: Relation between autocorrelation and various Fourier transforms.

3. [Exam Q - 2017] Consider a fading channel with impulse response

$$c(\tau, t) = 4\cos(t + U)\delta(\tau) + 3\beta_1(t)\delta(\tau - \tau_1) + 2\sin(t + U)\delta(\tau - \tau_2)$$

where $\tau_1 = 25$ ns, $\tau_2 = 50$ ns, $\beta_1(t)$, is an i.i.d. complex, zero-mean unit variance Gaussian random variable, and U is a random variable uniformly distributed between 0 to 2π, i.e., $U \sim \text{uniform}\,(0, 2\pi)$. Unless explicitly stated otherwise, all random variables are mutually independent.

(a) Compute and draw the power delay profile.

(b) Compute the RMS of the delay spread.