SSY135 Wireless Communications

Department of Electrical Engineering

Exam Date: March 18 2019

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Material Allowed material is

- Chalmers-approved calculator
- L. Rade, B. Westergren. Beta, Mathematics Handbook, any edition.
- One A4 page with your own handwritten notes. Both sides of the page can be used. Photo copies, printouts, other students' notes, or any other material is not allowed.
- A dictionary.

Grading A correct, clear and well-motivated solution gives a maximum of 12 points.

An erroneous answer, unclear, incomplete or badly motivated solutions give point reductions down to a minimum of 0 points. Answers in any other language than English are ignored.

Results Results are posted no later than April 4^{th}. The grading review is on April 5^{th} in the E2 Room Landahlsrummet 7430, between 9:30 am and 11:30 am.

Grades To pass the course, all projects and the exam must be passed. The exam is passed by securing at least 12 points. The project is passed by securing at least 8 points (4 for the report and 4 for the oral exam) in each part of the project. The final grade on the course will be decided by the project (max score 46), quizzes (max score 6), and final exam (max score 48). The sum of all scores will decide the grade according to the following table.

Total Score	0–39	40–59	60–79	≥ 80
Grade	Fail	3	4	5

PLEASE NOTE THAT THE PROBLEMS ARE NOT NECESSARILY ORDERED IN DIFFICULTY.

Good luck!

1 Hints

Friis' Law is $P_r = P_t G_r G_t(\lambda/4\pi d)^2$. The SVD $\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H$ and SINR for an observation $y = s + i + n$, where s is the useful signal, i is the interference, n is the noise, is SINR $= \mathbb{E}\{|s|^2\}/(\mathbb{E}\{|i|^2\}+\mathbb{E}\{|n|^2\})$.You can approximate the Q-function using Chernoff bound, i.e., $Q(x) \approx \frac{1}{2}e^{-\frac{x^2}{2}}$. The BER upper bound of MQAM, is $P_b(\gamma) \leq 0.2e^{\frac{-1.5\gamma}{M-1}\cdot\frac{P(\gamma)}{P}}$. In waterfilling, the optimal power allocation to maximize the sum rate of parallel channels subject to total power constraint $\bar{P}$ is

$$\frac{P_i}{\bar{P}} = \begin{cases} \frac{1}{\gamma_c} - \frac{1}{\gamma_i} & \text{if } \gamma_i \geq \gamma_c \\ 0 & \text{if } \gamma_i < \gamma_c \end{cases},$$

where P_i, γ_i, and γ_c are the the power of i-th channel, the received SNR of i-th channel assuming $\bar{P}$ as the transmit power, and power allocation threshold, respectively. The MIMO channel capacity under CSIR is $C = \log_2 \det\left(\boldsymbol{I}_{N_r} + \rho \boldsymbol{H}\boldsymbol{H}^H\right)$, where $\rho = P/N_t$, for transmit power P and N_t transmit antennas. For a vector $\mathbf{h}$, we have that $\det \mathbf{h}\mathbf{h}^H = \|\mathbf{h}\|^2$.

Additional Definitions

1. Let $X_1, X_2, \cdots, X_K$ be independent random variables, then $\Pr\{X_1 < a, X_2 < a, ...X_K < a\} = \prod_{k=1}^{K} \Pr\{X_k < a\}$.

2. If $X \sim \mathcal{N}(0, \sigma^2)$ and $Y \sim \mathcal{N}(0, \sigma^2)$, then $Z = \sqrt{X^2 + Y^2} \sim \text{Rayleigh}(\sigma)$:

$$f_Z(z) = \frac{z}{\sigma^2}e^{-\frac{z^2}{2\sigma^2}} \qquad , \quad x \geq 0.$$

3. If $X \sim \exp(\lambda)$, then $\sqrt{X} \sim \text{Rayleigh}(\frac{1}{\sqrt{2\lambda}})$.

4. Let $X_1, X_2, \cdots, X_k$ be i.i.d standard Normal random variables (i.e. $X_i \sim \mathcal{N}(0,1)$), then $Y = \sum_{i=1}^{k} X_i^2$ will be a Chi-squared random variable with k degrees of freedom:

$$f_Y(y) = \frac{y^{\frac{k}{2}-1}e^{-\frac{y}{2}}}{2^{\frac{k}{2}}\Gamma(\frac{k}{2})} \qquad , \quad y \geq 0.$$

Where $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1}e^{-x}\mathrm{d}x$ is the Gamma function. (*Note: If n is a natural value, then* $\Gamma(n) = (n-1)!$)

5. [*Jensen's inequality*] Let X be a random variable, then for any convex function like $g(.)$ it can be shown that

$$\mathbb{E}\{g(X)\} \geq g(\mathbb{E}\{X\}).$$

Similarly, $\sum_i g(x_i) \geq g\left(\sum_i x_i\right)$

2 MIMO and Massive MIMO

In this question, we will design and analyze MIMO and massive MIMO communication system. We consider a base station (BS) with M antennas and $N \ll M$ users each with 1 antenna. All the users use the same unit-energy constellation Ω and the same bandwidth. The noise variance is N_0. The channel is narrowband.

1. [2 pt] *Uplink communication.* Assume fixed channels.

(a) [1 pt] Express the observation at the BS in the the form

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}.$$

Explicitly write down the sizes of each matrix and vector.
Solution: The observation is of the form

$$\begin{aligned}\mathbf{y}_{M\times 1} &= \sum_{i=1}^{N}[\mathbf{h}_i]_{M\times 1}[x_i]_{1\times 1} + \mathbf{n}_{M\times 1} \\ &= [\mathbf{h}_1 \ \ldots \ \mathbf{h}_N]_{M\times N}[x_1 \ \ldots \ x_N]^T + \mathbf{n}_{M\times 1}\end{aligned}$$

(b) [0.5 pt] Assuming the BS knows the channel $\mathbf{H}$, which strategies can be BS use to recover $\mathbf{x}$? What is the complexity of each strategy (express as a function of $N, M, |\Omega|$, where $|\Omega|$ denotes the size of the constellation)?
Solution: The BS can apply maximum likelihood, zero forcing, or MMSE. The complexities are $O(|\Omega|^N)$, $O(M^3 + N|\Omega|)$, and $O(M^3 + N|\Omega|)$.

(c) [0.5 pt] If the users know their own uplink channel, can they adopt a strategy to reduce the complexity at the BS?
Solution: since the users have a single antenna, they cannot do any precoding. They could use TDMA or FDMA.

2. [8 pt] *Downlink communication.* Assume fixed channels and independent data for different users. The signal intended for user i is $s_i \in \Omega$, which is multiplied with a user-specific vector $\mathbf{w}_i$, yielding transmit vector $\mathbf{x}_i = \mathbf{w}_i s_i \in \mathbb{C}^{M\times 1}$, where $\|\mathbf{w}_i\| = 1$. A user only knows its own downlink channel.

(a) [3 pt] The BS transmits a vector $\mathbf{x} = \sum_{i=1}^{N} \mathbf{x}_i$ to all users over the same time-frequency resource. Express the observation at user i. Explicitly write down the sizes of each matrix and vector. Assuming that the users know their own downlink channel, which strategies can they use to recover their own data? What is the complexity of each strategy? What is the SINR at each user for fixed channels $\mathbf{h}_i$ and vectors $\mathbf{w}_i$?
Solution: The observation at user i is a scalar of the form

$$\begin{aligned} y_i &= \mathbf{h}_i^T \mathbf{x} + n_i \\ &= \mathbf{h}_i^T \mathbf{w}_i s_i + \sum_{j\neq i} \mathbf{h}_i^T \mathbf{w}_j s_j + n_i. \end{aligned}$$

The users can only do maximum likelihood, treating $\sum_{j\neq i} \mathbf{h}_i^T \mathbf{w}_j s_j$ as noise. The SINR is

$$\text{SINR} = \frac{|\mathbf{h}_i^T \mathbf{w}_i|^2}{\sum_{j\neq i} |\mathbf{h}_i^T \mathbf{w}_j|^2 + N_0}.$$

(b) [4 pt] Suppose that $M = 3$, $N = 2$ with downlink channels $\mathbf{h}_1^T = [1\,2\,3]$ and $\mathbf{h}_2^T = [4\,-2\,0]$ and $N_0 = 1$. If the BS knows the downlink channel, implement the zero-forcing precoder? What is the SINR and rate per user?
Solution: If the BS knows the overall downlink channel $[\mathbf{H}]_{N\times M}$, it can invert the channel by setting $\mathbf{W} = \mathbf{H}^H(\mathbf{H}^H\mathbf{H})^{-1}$.We find (2 pt)

$$\mathbf{W} = \begin{bmatrix} 1/9 & 2/9 & 2/9 \\ 1/5 & -1/10 & 0 \end{bmatrix}^T.$$

Each column should be normalized to ensure the power is not increased:

$$\mathbf{W} \approx \begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 0.9 & -0.45 & 0 \end{bmatrix}^T.$$

Then $\mathbf{HW} = diag(3, 4.47)$.

$$\begin{aligned} y_1 &= 3s_1 + n_1 \\ y_2 &= 4.47s_2 + n_2 \end{aligned}$$

from which the SINR and rate are easy to compute. (2pt for observation and SINR)

(c) [1 pt] Why can the BS and users in general not employ an SVD? What would be required to perform SVD?
Solution: An SVD requires a combining matrix of all the observations. This requires (i) each user knowing all the channels; (ii) each user having access to the observations of other users. The first requirement can be met with feedback. The second requirement needs user-to-user communications. (0.5 pt for each correct answer)

3. [2 pt] *Massive MIMO regime.* Consider a communication with a line of sight (LOS) path and multi-path. Let the uplink channel from user i to the BS be of the form

$$\begin{aligned} \mathbf{h}_i &= \mathbf{h}_i^L + \mathbf{h}_i^N \\ [\mathbf{h}_i^L]_n &= \sqrt{\rho_i}\exp(j\theta_{in}) \\ \mathbf{h}_i^N &\sim \mathcal{CN}(\mathbf{0}, (1-\rho_i)\mathbf{I}_N) \end{aligned}$$

where θ_{in} is uniform in $[0, 2\pi)$ and independent across i and n and $0 \le \rho_i \le 1$.

(a) [0.5 pt] no LOS: for the case $\rho_i = 0$, $\forall i$, write down and expression for $\mathbf{h}_i^H\mathbf{h}_{i'}/M$, for both $i = i'$ and $i \neq i'$ as $M \to \infty$. How can the BS use this information to perform precoding?
Solution: $\mathbf{h}_i \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_N)$. Then

$$\mathbf{h}_i^H\mathbf{h}_{i'}/M = \frac{1}{M}\sum_{k=1}^{M} h_{i,k}^* h_{i',k} \to \begin{cases} 0 & i \neq i' \\ 1 & i = i' \end{cases}$$

The BS can precode using $\mathbf{H}^H$.

(b) [1 pt] with LOS: for the case $0 < \rho_i < 1$, $\forall i$, write down and expression for $\mathbf{h}_i^H\mathbf{h}_{i'}/M$, for both $i = i'$ and $i \neq i'$ as $M \to \infty$. How can the BS use this information to perform precoding?
Solution: Now

$$\begin{aligned} \mathbf{h}_i^H\mathbf{h}_{i'}/M &= \frac{1}{M}\sum_{k=1}^{M} h_{i,k}^{N*} h_{i',k}^{N} + \frac{1}{M}\sum_{k=1}^{M} h_{i,k}^{L*} h_{i',k}^{L} \\ &\to (1-\rho_i)\delta_{i-i'} + \sqrt{\rho_i\rho_{i'}}\frac{1}{M}\sum_{k=1}^{M}\exp(j(\theta_{ik} - \theta_{i'k})) \\ &= (1-\rho_i)\delta_{i-i'} + \rho_i\delta_{i-i'} = \delta_{i-i'} \end{aligned}$$

The BS can precode using $\mathbf{H}^H$.

(c) [0.5 pt] only LOS: for the case $\rho_i = 1$, $\forall i$, write down and expression for $\mathbf{h}_i^H\mathbf{h}_{i'}/M$, for both $i = i'$ and $i \neq i'$ as $M \to \infty$. How can the BS use this information to perform precoding?
Solution: follows immediately from the previous case.

3 Fading and Waterfilling

1. [6 pt] *Fading*: Assume a wireless link between a base station and a car with log distance path loss. The signal is modulated with QPSK and the path loss exponent is 3. The path loss at reference distance $d_0 = 1$ km is 5 dB larger than the corresponding free space path loss. The transmitted signal has a carrier frequency of 900 MHz and a bandwidth of 10 KHz. Assume that the gain of the transmitter and receiver antennas is 0 dB and the base station transmit power is limited to 1 W. Furthermore, the receiver sensitivity, i.e., the minimum received power that can be detected by the car is -30 dBm.

(a) [3 pt] What is the maximum distance between transmitter and receiver that the car can always detect the transmitted signal from the base station?

$$\begin{aligned} &[P_L(d_0)]_{\text{dB}} = 10\log_{10}\left(\left(\tfrac{4\pi d_0}{\lambda}\right)^2 \tfrac{1}{G_t G_r}\right) + 5 = 96.52 \\ &[P_r(R)]_{\text{dBm}} = [P_t]_{\text{dBm}} + [10\log_{10}K]_{\text{dB}} - 10\gamma\log_{10}(d_0/d_0) \\ &[K]_{\text{dB}} = -[P_L(d_0)]_{\text{dB}} = -96.52 \\ &[P_r(R)]_{\text{dBm}} = [P_t]_{\text{dBm}} + [10\log_{10}K]_{\text{dB}} - 10\gamma\log_{10}(R/d_0) \\ &-30 = 30 - 96.52 - 30\log_{10}(R/d_0) \\ &R = 60.59 \text{ m} \end{aligned}$$

(b) [3 pt] Assume that the channel between base station and car changes such that a log-normal shadowing with zero-min and standard deviation of 5 dB is added on top of the log-distance pathloss. What is the maximum distance between base station and car that ensures the outage probability of 0.01?

$$
\begin{aligned}
&P_{\min} = -30 \text{ dBm} \\
&Q\left(\frac{P_r - P_{\min}}{\sigma_{\psi_{dB}}}\right) = 0.01, P_r = \sigma_{\psi_{dB}} Q^{-1}(0.01) + P_{\min} \\
&P_r = -18.36 \text{ dBm} \\
&[P_r(R)]_{\text{dBm}} = [P_t]_{\text{dBm}} + [10\log_{10} K]_{\text{dB}} - 10\gamma\log_{10}(R/d_0) \\
&-18.36 = 30 - 96.52 - 30\log_{10}(R/d_0) \\
&R = 24.815 \text{ m}
\end{aligned}
$$

2. [6 pt] Now assume that the car receives signal from 4 different base stations. The wireless channel from the car perspective is therefore a 4 parallel channels. The base stations are controlled from a central unit which tunes the transmit power of each base station. The channel gains from base station 1, 2, 3, and 4 to the car are 0.015, 0.022, 0.025, and 0.03, respectively. Also, assume that the receiver has a fixed bandwidth of 400 KHz and power spectral density of noise is $N_0 = 10^{-12}$ W/Hz. Furthermore, the total transmit power of the base stations ($\bar{P}$) is limited to 2 mW. MQAM modulation is used at the base stations and the target BER at the receiver is 10^{-3}.

   (a) [2 pt] If the central unit decides to assign the total power ($\bar{P}$) to only one base station, which one should be selected in order to maximize the spectral efficiency over this parallel channels? Find also the corresponding spectral efficiency with this power allocation approach.

$$
\begin{aligned}
&\alpha_1 = 0.015, \alpha_2 = 0.022, \alpha_3 = 0.025, \alpha_4 = 0.03, \\
&\gamma_1 = \frac{\bar{P}\alpha_1^2}{N_0 B} = 1.125, \gamma_2 = \frac{\bar{P}\alpha_2^2}{N_0 B} = 2.42, \gamma_3 = \frac{\bar{P}\alpha_3^2}{N_0 B} = 3.125, \gamma_4 = \frac{\bar{P}\alpha_4^2}{N_0 B} = 4.5, \\
&M = 1 - K \times \max_i\{\gamma_i\} = 1 - K\gamma_4 = 2.273, \text{ SE} = \log_2 M = 1.185
\end{aligned}
$$

   (b) [4 pt] As a second resource allocation approach, the central unit decides to maximize the total spectral efficiencies of the parallel channels by power allocation. Find the optimal power for each channel. Also, find the total sum rate after optimal power allocation.

$$
\begin{aligned}
&K = \frac{-1.5}{\ln(5\times10^{-3})}, \\
&A_1 \to \sum_i \frac{P_i}{\bar{P}} = 1, \gamma_c = \frac{1}{\frac{1}{4}\times\left(1+\frac{1}{\gamma_1}+\frac{1}{\gamma_2}+\frac{1}{\gamma_3}+\frac{1}{\gamma_4}\right)} = 1.406 > \gamma_1 \\
&A_2 \to \sum_i \frac{P_i}{\bar{P}} = 1, \gamma_c = \frac{1}{\frac{1}{3}\times\left(1+\frac{1}{\gamma_2}+\frac{1}{\gamma_3}+\frac{1}{\gamma_4}\right)} = 1.534 < \gamma_2, \\
&M_2 = 1 - K\gamma_2\frac{P_2}{\bar{P}} = 1.293 \\
&M_3 = 1 - K\gamma_3\frac{P_3}{\bar{P}} = 1.547 \\
&M_4 = 1 - K\gamma_4\frac{P_4}{\bar{P}} = 1.163 \\
&\text{SE} = \sum_i \log_2 M_i = 1.219
\end{aligned}
$$

4 Capacity Analysis

1. [6 pt] **SIMO System**

   Consider a SIMO system with 1 transmit antenna and N_R receive antennas. Independent $\mathcal{CN}(0,1)$ noise corrupts the signal at each of the receive antennas. The transmit signal has power constraint SNR$=\rho$. Additionally, the channel is assumed to be known to the receiver.

   (a) [2 pt] First consider a (maximum ratio combining) MRC receiver. Assuming a Rayleigh fading channel between the transmit antenna and each of the receive antennas (with parameter λ for Rayleigh distribution), find the outage probability for a target data transmission rate R bps/Hz in high SNR regime.

$$P_{\text{out}}(R) = \Pr\{\log(1+\rho\|\mathbf{h}\|^2) < R\} = \Pr\left\{\|\mathbf{h}\|^2 < \frac{2^R-1}{\rho}\right\}$$

$$f_{\|\mathbf{h}\|^2}(x) = \frac{1}{2^{N_R}(N_R-1)!}\left(\frac{x}{\lambda^2}\right)^{N_R-1} e^{-\frac{x}{2\lambda^2}}$$

$$\implies P_{\text{out}}(R) = \int_0^{\frac{2^R-1}{\rho}} \frac{1}{2^{N_R}(N_R-1)!}\left(\frac{x}{\lambda^2}\right)^{N_R-1} e^{-\frac{x}{2\lambda^2}}\,dx \approx \int_0^{\frac{2^R-1}{\rho}} \frac{1}{2^{N_R}(N_R-1)!}\left(\frac{x}{\lambda^2}\right)^{N_R-1} dx$$
$$= \frac{1}{N_R!}\frac{2^{N_R}(2^R-1)^{N_R}}{(2\rho)^{N_R}\lambda^{2N_R-2}}$$

   (b) [4 pt] Now consider a selection combining (SC) receiver, which selects the received signal at receive antenna with the highest channel gain as the input of the receiver. Again, assume a Rayleigh fading channel between the transmit antenna and each of the receive antennas with parameter λ and find the outage probability for a target data transmission rate R bps/Hz.

$$P_{\text{out}}(R) = \Pr\left\{\log(1+\rho\max(|h_i|^2) < R\right\} = \Pr\left\{\max(|h_i|^2) < \frac{2^R-1}{\rho}\right\}$$
$$= \Pr\left\{|h_1|^2 < \frac{2^R-1}{\rho}, ..., |h_{N_R}|^2 < \frac{2^R-1}{\rho}\right\}$$

   But we know that in a Rayleigh fading channel $|h_i|^2$'s are i.i.d random variables and $|h_i|^2 \sim exp(\alpha)$ with $\alpha = \frac{1}{2\lambda^2}$. So, it can be concluded that

$$P_{\text{out}}(R) = \prod_{i=1}^{N_R} \Pr\left\{|h_i|^2 < \frac{2^R-1}{\rho}\right\} = \prod_{i=1}^{N_R}\left[\int_0^{\frac{2^R-1}{\rho}} \frac{1}{2\lambda^2}e^{-\frac{x}{2\lambda^2}}\,\mathrm{d}x\right] = \left[\int_0^{\frac{2^R-1}{\rho}} \frac{1}{2\lambda^2}e^{-\frac{x}{2\lambda^2}}\,\mathrm{d}x\right]^{N_R}$$
$$= \left[1 - e^{-\frac{2^R-1}{2\rho\lambda^2}}\right]^{N_R}$$

2. [6 pt] **MIMO System**

   (a) [4 pt] Consider a MIMO system with N_t transmit antennas, N_r receive antennas and the channel matrix $\mathbf{H}$ (not necessarily full-rank) with dimensions $N_r \times N_t$. The transmitted signal has the total power constraint of P and there is an additive white Gaussian noise at each receive antenna with distribution $\mathcal{CN}(0,1)$. Vector $\boldsymbol{\sigma} \in \mathbb{R}^{r\times 1}$ is defined as a column vector containing all non-zero singular values of $\mathbf{H}$. Assuming that there is only CSI at the receiver and $||\boldsymbol{\sigma}||^2 = 1$, find an upper bound for the capacity of data transmission in bps/Hz. What does this upper bound change into if we assume that $\mathbf{H}$ describes the channel in a Massive MIMO system with few number of antennas for each user?

Using SVD decomposition, $\boldsymbol{H} = \boldsymbol{U\Sigma V}^H$. Due to unitary characteristic of $\boldsymbol{U}$ and $\boldsymbol{V}$, one can easily show that

$$\boldsymbol{HH}^H = \boldsymbol{U}\Sigma\boldsymbol{V}^H\boldsymbol{V}\Sigma^H\boldsymbol{U}^H = \boldsymbol{U}\begin{bmatrix} \sigma_1^2 & 0 & 0 & 0 & 0 \\ 0 & . & 0 & 0 & 0 \\ 0 & 0 & . & 0 & 0 \\ 0 & 0 & 0 & \sigma_r^2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}\boldsymbol{U}^H.$$

In other words,

$$\det\left(\boldsymbol{I}_{N_r} + \frac{P}{N_t}\boldsymbol{HH}^H\right) = \prod_{l=1}^{r}(1 + \frac{P}{N_t}\sigma_l^2). \tag{1}$$

Where $r = rank(\mathbf{H})$. Substituting (1) in MIMO channel capacity leads to the following

$$C = \log_2\det\left(\boldsymbol{I}_{N_r} + \frac{P}{N_t}\boldsymbol{HH}^H\right) = \sum_{l=1}^{r}\log_2\left(1 + \frac{P}{N_t}\sigma_l^2\right) = r\sum_{l=1}^{r}\frac{1}{r}\log_2\left(1 + \frac{P}{N_t}\sigma_l^2\right). \tag{2}$$

Now (2) can be rewritten as

$$C = r\mathbb{E}\left\{\log_2\left(1 + \frac{P}{N_t}\sigma_l^2\right)\right\}, \tag{3}$$

where $E.$ is the expectation function over uniformly distributed random variable σ_l^2. Since logarithm is a concave function, with applying the Jensen's inequality, we have

$$C \leq r\log_2\left(1 + \frac{P}{N_t}\mathbb{E}\{\sigma_l^2\}\right) = r\log_2\left(1 + \frac{P}{N_t \times r}\sum_{l=1}^{r}\sigma_l^2\right) = r\log_2\left(1 + \frac{P}{rN_t}\right). \tag{4}$$

Under the assumption of Massive MIMO system we can say that $rank(\mathbf{H}) = N_r$, so the upper bound will be changed into $N_r\log_2\left(1 + \frac{P}{N_rN_t}\right)$.

3. [2 pt] If CSI would be also available at the transmitter, then how will the capacity expression change? What about the computed upper bound in previous part, how does it change?

If CSI is also available at the transmitter then it's possible to allocate appropriate power to each transmit antenna in a way that data transmission capacity maximises (water-filling problem), so the new capacity expression would be as follows

$$C = \max_{\sum p_l = P} \sum_{l=1}^{r}\log_2\left(1 + p_l\sigma_l^2\right).$$

Therefore, the calculated upper bound in the previous part is no longer an upper bound.In general the new upper bound can be expressed as follows

$$C \leq \max_{\sum p_l = P} r\log_2\left(1 + \frac{1}{r}\sum_{l=1}^{r}p_l\sigma_l^2\right) = r\log_2\left(1 + \frac{1}{r}\left[\max_{\sum p_l = P}\sum_{l=1}^{r}p_l\sigma_l^2\right]\right)$$

However, if we assume that $\tilde{p}_i$ is the allocated power to singular value σ_i (after solving the maximisation problem), the new upper bound will be

$$C \leq r\log_2\left(1 + \frac{1}{r}\sum_{l=1}^{r}\tilde{p}_l\sigma_l^2\right).$$

5 Small Questions

- [4 pt] Identify at least three ethical issues that could have happened while performing the projects. How would you deal with these issues? Answer in max 300 words.

- [1 pt] Consider a channel that has a Doppler spread of 600 Hz. Assume a 16-QAM modulated voice signal is transmitted over this channel at 2 Kbps. Is outage probability or average probability of error a better performance metric and why?

  Answer: Since $T_c = 0.0016$ and $T_s = 0.002$ are in the same order, therefore, average probability of error is the appropriate metric.

- [2 pt] Consider two scenarios:

  1. a rural area with flat terrain, limited vegetation, and few mobile phone users per square meter.
  2. an urban area with many buildings and many mobile phone users per square meter.

  Argue for which type of small-scale fading we are likely to experience when making a mobile phone call in scenario (a) and (b).

  Answer: (a) a strong line-of-side (LOS) with some scattering component: Racian fading (b) we do not expect LOS: Rayleigh fading

- [1 pt] Consider a wireless channel with log-distance path loss and log-normal shadowing with zero-mean and standard deviation of σ_1. At the distance d from transmitter, the outage probability of 0.02 is garanteed. If the variance of log normal shadowing increased to σ_2 ($\sigma_2 > \sigma_1$), to maintain the same outage probability at the same distance, how does the transmit power change?

  Answer: The received power should be increased, therefore for a fixed distance, the transmit power should be increased.

- [1 pt] Consider a wireless transmitter that can transmit data in the range of $[1, 10]$ Mbps using a QAM modulation scheme with the order of 4,16 or 64. If its desired to use this transmitter in a channel with coherence time $T_c = 50\,^{ns}$ and coherence bandwidth $B_c = 10\,^{kHz}$, then which one is a more suitable metric for evaluating system performance, outage capacity or ergodic capacity?

  Answer: The minimum possible symbol time in the transmitter is $T_s = 2 \times 10^{-7} = 200\,^{ns}$ which is larger than the channel coherence time, so the system will experience fast fading channel and the suitable evaluation metric would be ergodic capacity.

- [2 pt] Suppose we need to use a hardware platform that has significant phase noise. Which of the following modulation schemes would be preferable to use: OFDM with a large number of subcarriers, OFDM with a long cyclic prefix, single carrier BPSK with square pulses with same total data rate as multi carrier system? (*Hint: Assume that the higher the symbol rate, the lower the degradation effect due to phase noise.*)

  Answer: Since the symbol rate in OFDM is usually lower than the equivalent single carrier system with the same total bit-rate, the OFDM scheme shows a worse performance. Hence, (i) OFDM with more subcarriers could only increase ICI but with CP, this could be mitigated. While in (ii) OFDM with long CP, will make the actual symbol rate much smaller hence, worsening the phase noise effects. Therefore, a single carrier system such as BPSK with square pulses is the best choice for the hardware with significant phase noise.

- [1 pt] What is channel reuse in cellular systems? What are the trade-offs involved in setting a reuse factor?

  Answer: Number of cells per cluster, lower N leads to more efficiency, but with more interference.