Solution to Exercise 4

February 7, 2020

1 Tentative Solutions

1. [G-6.12]

   (a)

$$\begin{aligned} P_e &= 0.5e^{-\gamma_b} &\implies \gamma_b = 13.1224 \\ \frac{P_\gamma}{N_0 B} &= 13.1224 &\implies P_\gamma = 1.3122 \times 10^{-14} \\ \frac{P_\gamma}{P_t} &= \big[\tfrac{\sqrt{G_{TX} G_{RX}}\lambda}{4\pi d}\big]^2 &\implies P_t = 4.8488\,\mathrm{W} \end{aligned}$$

   (b) $x = 1.3122 \times 10^{-14} = -138.82$ dB

   $P_{\gamma,dB} \sim \mathcal{N}(\mu P_\gamma, 8), \quad \sigma_{dB} = 8$

   $P(P_{\gamma,dB} \geq x) = 0.9 \quad \implies P(\frac{P_{\gamma,dB} - \mu P_\gamma}{8} \geq \frac{x - \mu P_\gamma}{8})$
   $\implies Q(\frac{x-\mu P_\gamma}{8}) = 0.9 \implies \frac{x - \mu P_\gamma}{8} = -1.2816$
   $\implies \mu P_\gamma = -128.5672\,dB = 1.39 \times 10^{-13} \implies P_t = \big[\frac{4\pi d}{\sqrt{G_{TX}G_{RX}}\lambda}\big]^2 \mu P_\gamma = 51.36\mathrm{W}$

2. [G-6.16] For DPSK in Rayleigh fading, $\bar{P}_b = \frac{1}{2\bar{\gamma}_b} \implies \bar{\gamma}_b = 500$

   $N_0 B = 3 \times 10^{-12} mW \implies P_{\text{target}} = \bar{\gamma}_b N_0 B = 1.5 \times 10^{-9}\,\mathrm{mW} = -88.24\,\mathrm{dBm}$

   Now, consider shadowing: $P_{\text{out}} = P[P_r < P_{\text{target}}] = P[\psi < P_{\text{target}} - \bar{P}_r] = \Phi\big(\frac{P_{\text{target}} - \bar{P}_r}{\sigma}\big)$

   $\implies \Phi^{-1}(.01) = 2.327 = \frac{P_{\text{target}} - \bar{P}_r}{\sigma}$

   $\bar{P}_r = -74.28\,\mathrm{dBm} = 3.73 \times 10^{-8}\,\mathrm{mW} = P_t\big(\frac{\lambda}{4\pi d}\big)^2$

   $\implies d = 1372.4\mathrm{m}$

3. [G-6.10] $T_s = 15\,\mu\mathrm{sec}$

   At 1mph, $T_c = 1/B_d = 1/(v/\lambda) = 0.74s \gg T_s$. $\therefore$outage probability is a good measure.