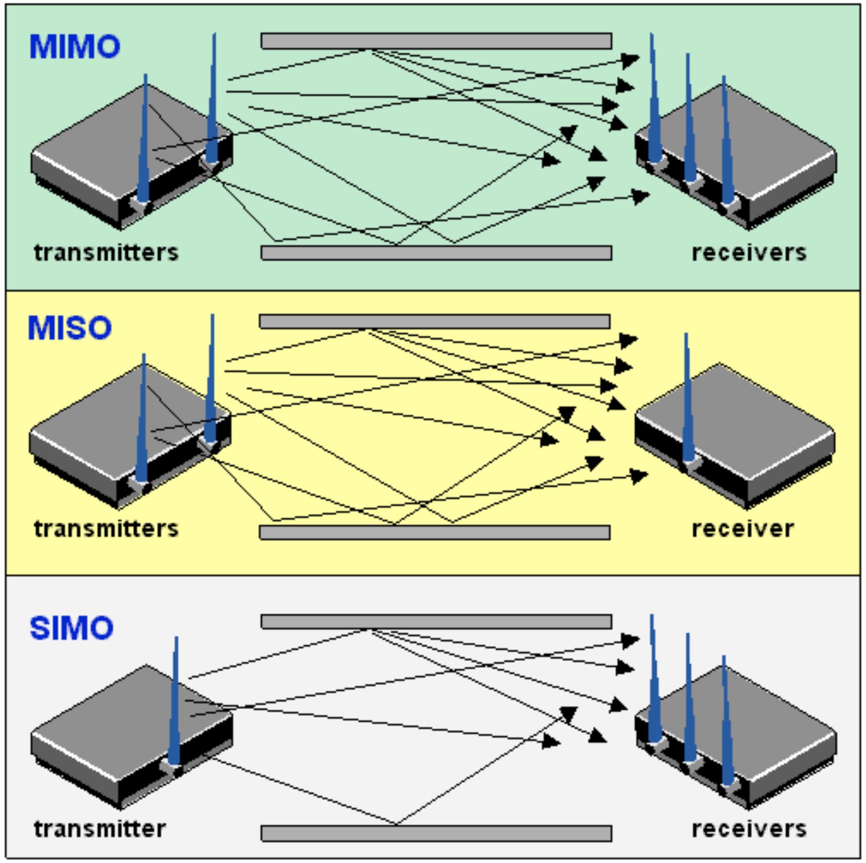
Wireless Communications SSY135 – Lecture 10

Henk Wymeersch
Department of Electrical Engineering
Chalmers University of Technology
http://tinyurl.com/hwymeers
email: henkw@chalmers.se

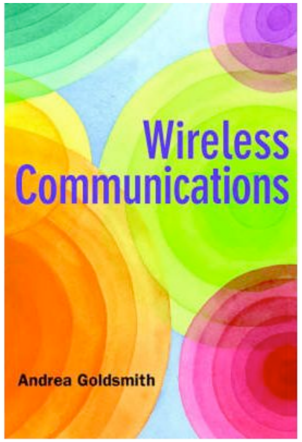


http://img.tfd.com/cde/MIMO.GIF

CHALMERS

Topics for today

- Lecture learning outcomes
- Channel model — Section 10.1
- ML, ZF, MMSE detection — (Chapter 11)
- Parallel decomposition with SVD — Section 10.2
- Beamforming — Section 10.4
- Capacity — Section 10.3



Suggested reading:
- Good time to review Chapter 4 (Capacity)

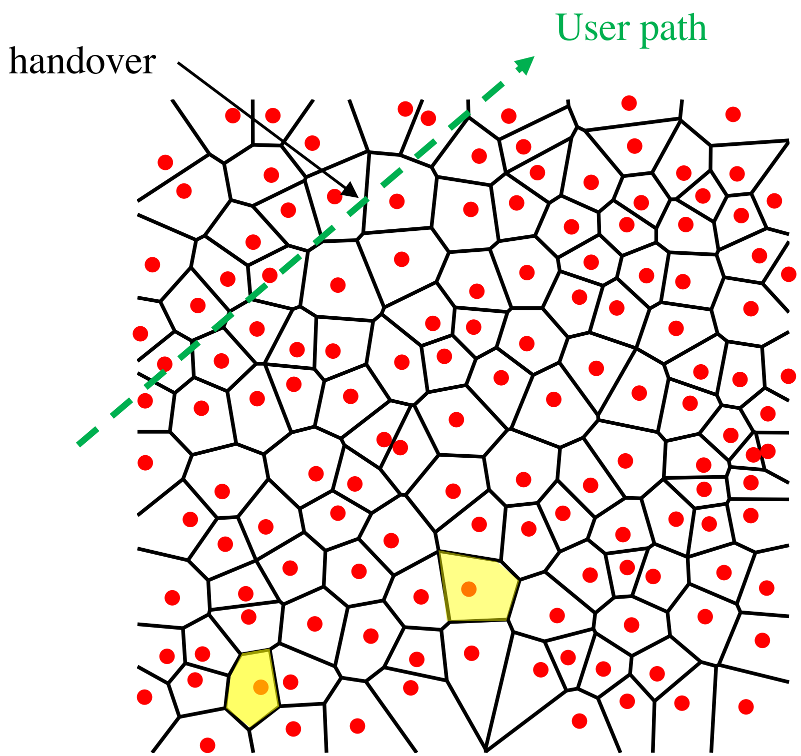
Today’s learning outcomes

At the end of this lecture, you must be able to

- Describe SIMO, SISO, MISO, and MIMO transmission
- Express the general narrowband MIMO model: geometric and Rayleigh
- Use ML, ZF, and MMSE detectors and describe their properties
- Perform parallel decomposition of a MIMO channel and combine with deterministic water-filling
- Perform beamforming

Last time: Multiuser communication

- Multiple users share the same channel, so communication needs to be coordinated
- Systems: cellular and ad-hoc
- Cellular:
  - Base station to mobile: downlink
  - Mobile to base station: uplink



Andrews, Jeffrey G., Abhishek K. Gupta, and Harpreet S. Dhillon. "A primer on cellular network analysis using stochastic geometry." *arXiv preprint arXiv:1604.03183* (2016).

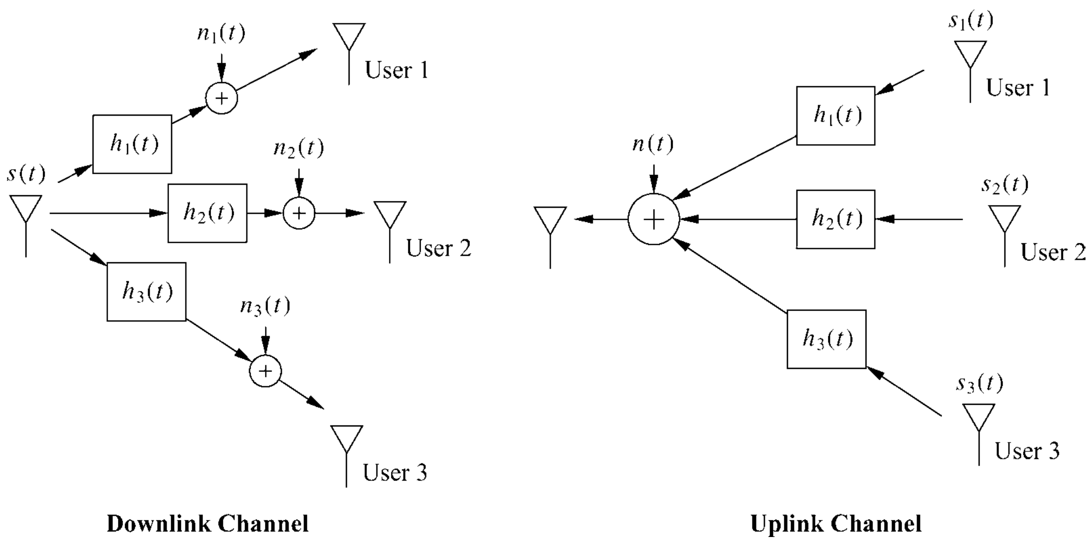
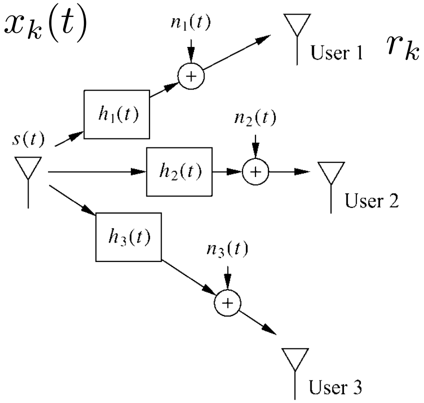


Figure 14.1: Downlink and uplink channels.

Cellular: Duplexing, multiple access

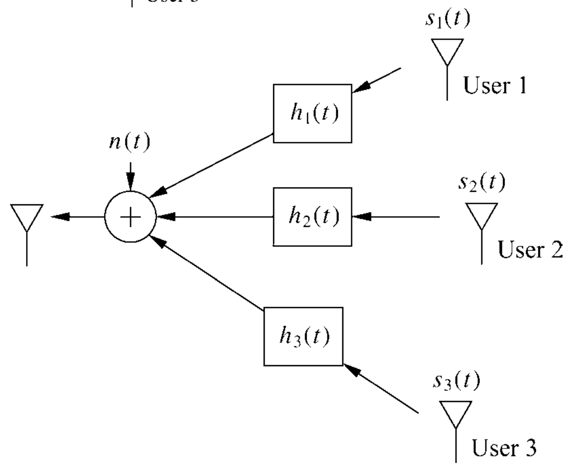
- Duplexing: separate uplink and downlink: FDD and TDD
- Multiple access: sharing of resources in uplink and downlink: TDMA, FDMA, CDMA, SDMA
- Downlink $x_k(t) = \sum_{l=-\infty}^{+\infty} a_{k,l} s_{k,l}(t)$ $\quad s(t) = \sum_{k=1}^{K} x_k(t)$



$r_k(t) = \alpha_k s(t) + n_k(t)$

- Uplink $x_k(t) = \sum_{l=-\infty}^{+\infty} a_{k,l} s_{k,l}(t)$

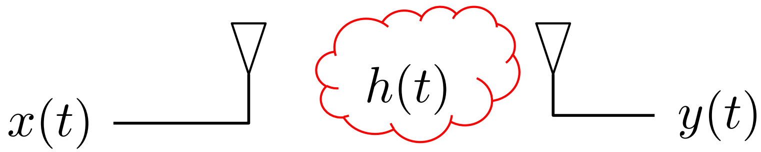
$$r(t) = \sum_{k=1}^{K} \alpha_k x_k(t - \tau_k) + n(t)$$



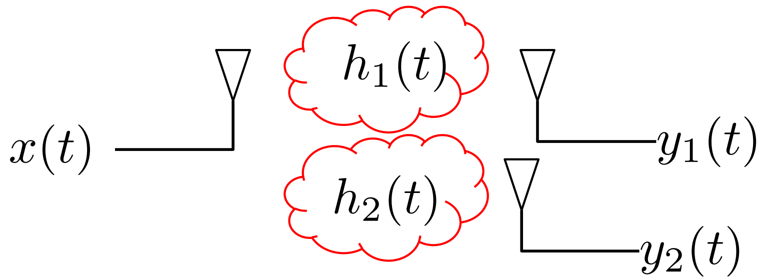
Uplink Channel

SISO, SIMO, MISO, and MIMO

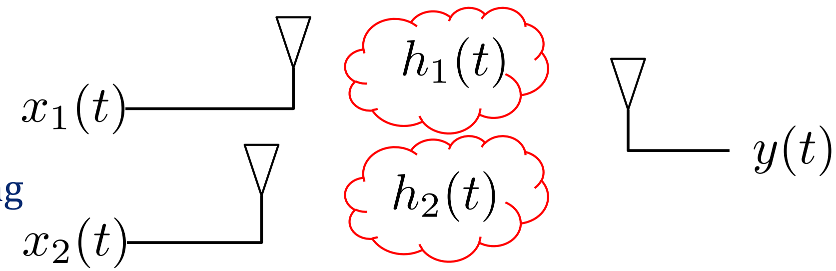
- **SISO**
  - no diversity gain and no array gain



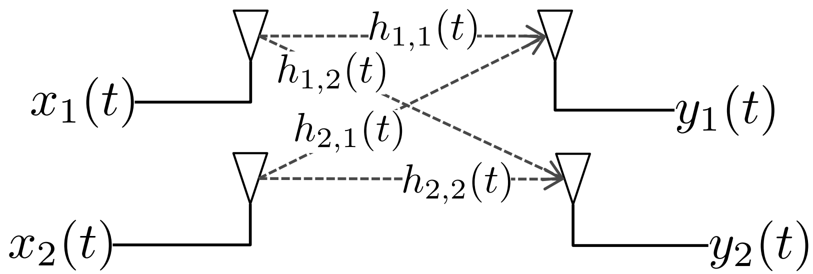
- **SIMO**
  - diversity gain and array gain



- **MISO**
  - TX power divided
  - diversity gain through CSIT
  - diversity gain through CSIR + space-time coding

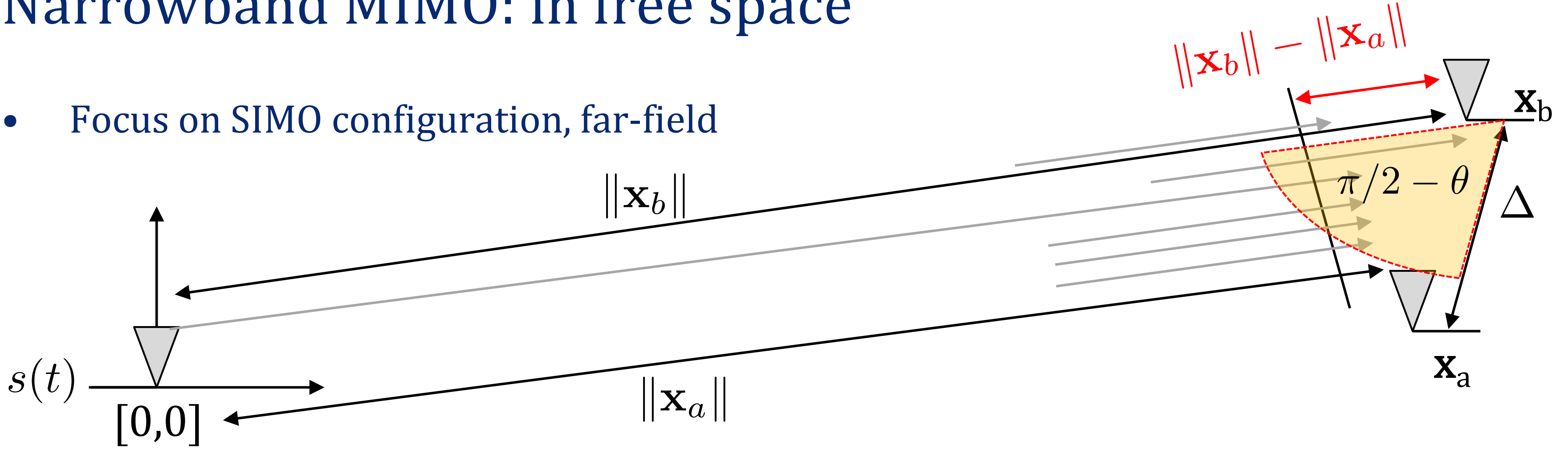


- **MIMO**
  - TX power divided
  - diversity gain
  - array gain



Narrowband MIMO: in free space

- Focus on SIMO configuration, far-field



- At antenna a: $\frac{\lambda}{4\pi\|\mathbf{x}_a\|} s(t-\|\mathbf{x}_a\|/c) e^{-j2\pi\|\mathbf{x}_a\|/\lambda} \approx \frac{\lambda}{4\pi d} s(t-d/c) e^{-j2\pi\|\mathbf{x}_a\|/\lambda}$
- At antenna b: $\frac{\lambda}{4\pi\|\mathbf{x}_b\|} s(t-\|\mathbf{x}_b\|/c) e^{-j2\pi\|\mathbf{x}_b\|/\lambda} \approx \frac{\lambda}{4\pi d} s(t-d/c) e^{-j2\pi\|\mathbf{x}_b\|/\lambda}$

- The narrowband channel is

$$\mathbf{h} = e^{j\phi}\frac{\lambda}{4\pi d}\begin{bmatrix} 1 \\ e^{-j2\pi(\|\mathbf{x}_b\|-\|\mathbf{x}_a\|)/\lambda} \end{bmatrix} = \alpha \begin{bmatrix} 1 \\ e^{-j2\pi\Delta/\lambda\cos(\pi/2-\theta)} \end{bmatrix} = \underbrace{\alpha}_{\text{Channel gain}} \underbrace{\begin{bmatrix} 1 \\ e^{-j\pi\sin\theta} \end{bmatrix}}_{\text{Array response}}$$

- In general $\mathbf{H} = \alpha\mathbf{a}(\theta)\mathbf{a}^T(\psi)$
- Depends on angle of arrival and angle of departure

Narrowband MIMO: the MIMO Rayleigh model

- Each channel becomes a scalar: TX antenna i, RX antenna j: $h_{i,j} \in \mathbb{C}$
- Overall, for M_t TX antennas and M_r RX antennas

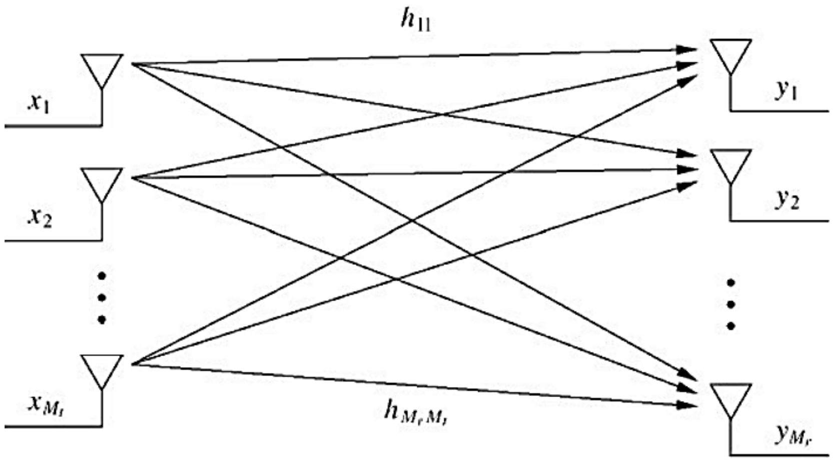


Figure 10.1: MIMO systems.

- Model:

$$\begin{bmatrix} y_1 \\ \vdots \\ y_{M_r} \end{bmatrix} = \begin{bmatrix} h_{11} & \cdots & h_{1M_t} \\ \vdots & \ddots & \vdots \\ h_{M_r 1} & \cdots & h_{M_r M_t} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_{M_t} \end{bmatrix} + \begin{bmatrix} n_1 \\ \vdots \\ n_{M_r} \end{bmatrix}$$

- More compact:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

Narrowband MIMO

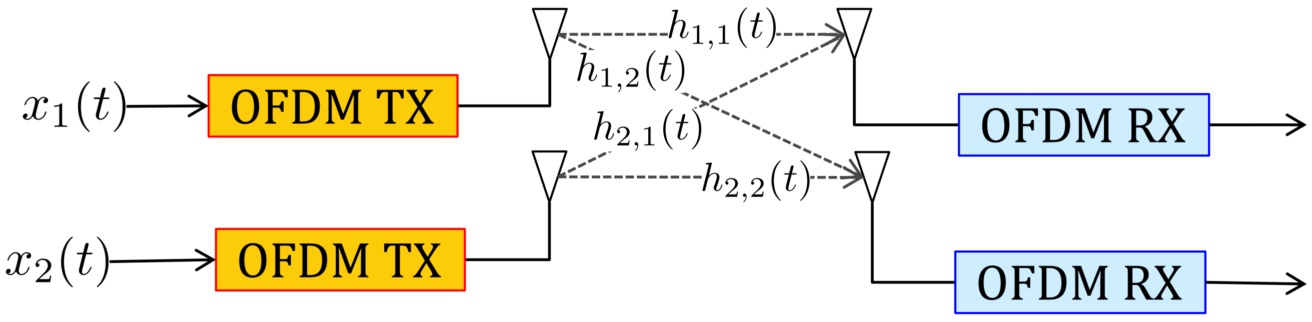
- Each channel becomes a scalar: TX antenna i, RX antenna j: $h_{i,j} \in \mathbb{C}$
- Overall, for M_t TX antennas and M_r RX antennas:

$$\mathbf{y} = \underbrace{\boxed{\mathbf{H}}}_{M_r \text{ x } M_t \text{ channel matrix}} \underbrace{\boxed{\mathbf{x}}}_{M_t \text{ x 1 column vector of symbols}} + \underbrace{\boxed{\mathbf{n}}}_{M_r \text{ x 1 column vector of AWGN}}$$

- **Models**
  - Input power does not increase with M_t: $\mathbb{E}\{\mathbf{x}^{\mathrm{H}}\mathbf{x}\} = \rho$
  - Noise is zero-mean Gaussian
  - Noise is white in time and space: $\mathbb{E}\{\mathbf{n}\mathbf{n}^{\mathrm{H}}\} = \mathbf{I}_{M_r}$
  - Zero-mean spatially white Rayleigh channel: $h_{i,j} \sim \mathcal{CN}(0,1), \quad \text{i.i.d.}$
  - SNR measure: ρ
  - CSI availability: usually at receiver (CSIR), sometimes also transmitter (CSIT)

Wideband MIMO: MIMO-OFDM

- MIMO can be combined with OFDM for frequency selective channels



- Model on carrier k

$$\mathbf{z}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{n}_k$$

M_r x 1 M_r x M_t M_t x 1

CSIR detection methods

Goal: recover $\mathbf{x}$ from $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$

1. ML detector:

$$\hat{\mathbf{x}} = \arg\max_{\mathbf{x}} p(\mathbf{y}|\mathbf{x}, \mathbf{H}) = \arg\min_{\mathbf{x}} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2$$

high complexity

1. Zero-forcing (ZF) detector ($M_r \geq M_t$) equalizer with per-antenna detector:

noise enhancement

$$\mathbf{z} = (\mathbf{H}^{\mathrm{H}}\mathbf{H})^{-1}\mathbf{H}^{\mathrm{H}}\mathbf{y} = \mathbf{x} + \mathbf{w}$$

1. Minimum mean squared error (MMSE) equalizer with per antenna detector:

$$\mathbf{z} = (\mathbf{H}^{\mathrm{H}}\mathbf{H} + \mathbf{I}_{\mathrm{M_t}}/\rho)^{-1}\mathbf{H}^{\mathrm{H}}\mathbf{y} \approx \mathbf{x} + \mathbf{w}$$

Notes:

- ZF and MMSE require a (simple) decision step based on $\mathbf{z}$
- MMSE relies on Gaussian model for $\mathbf{x}$

CSIR detection methods

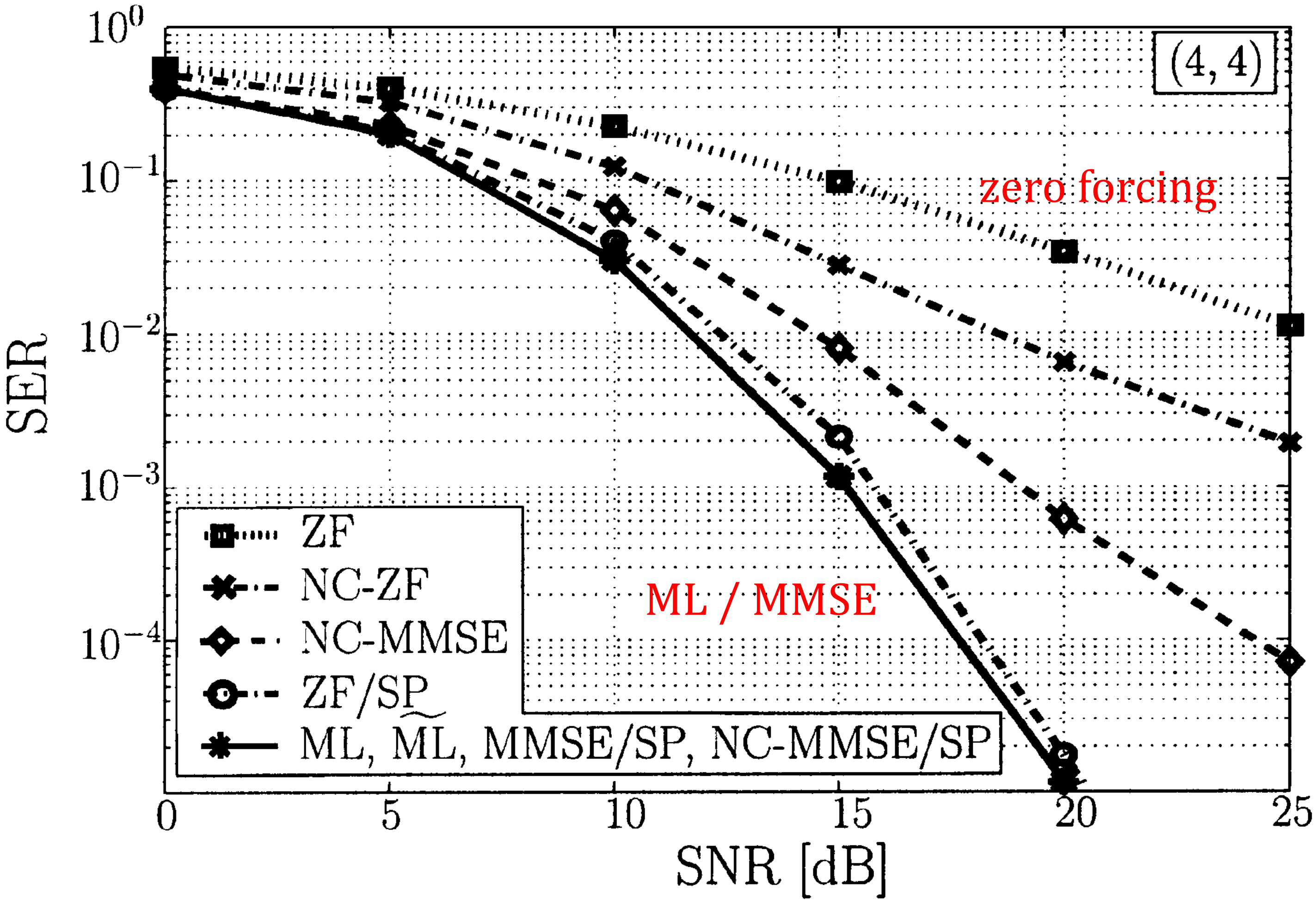
Given

- Channel H = [1 2; 1 2+ε]
- BPSK signaling {+1,-1}
- ρ=50 (17 dB)

Task

- Determine inv(H)
- Determine the ML estimator for ε=0.01. What is its complexity? Do you think it will perform well? Why/why not?
- Determine the ZF estimator for ε=0.01. What is its complexity? Do you think it will perform well? Why/why not?
- Determine the MMSE estimator for ε=0.01. What is its complexity? Do you think it will perform well? Why/why not?

Detection methods: comparison



Artés, Harold, Dominik Seethaler, and Franz Hlawatsch. "Efficient detection algorithms for MIMO channels: A geometrical approach to approximate ML detection." *Signal Processing, IEEE Transactions on* 51.11 (2003): 2808-2820.

Matrix decomposition: an essential tool

Contents [hide]

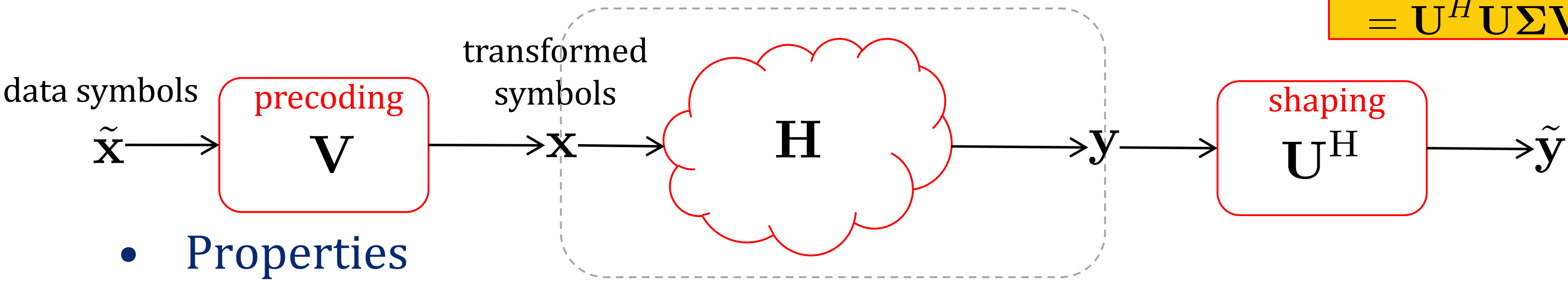
CSIT: parallel decomposition

- Main idea: *singular value decomposition* (SVD)
- Any complex M_r x M_t matrix can be expressed as $\mathbf{H} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathrm{H}}$ where
  - **U** is a M_r x M_r unitary matrix
  - Σ is a M_r x M_t diagonal matrix of rank $R_H \leq \min(M_r, M_t)$
  - **V** is a M_t x M_t unitary matrix

equality: rich scattering environment

- Parallel decomposition:

$$\boxed{\begin{aligned}\tilde{\mathbf{y}} &= \mathbf{U}^H\mathbf{H}\mathbf{V}\tilde{\mathbf{x}} + \mathbf{n}\\ &= \mathbf{U}^H\mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H\mathbf{V}\tilde{\mathbf{x}} + \mathbf{n}\end{aligned}}$$



- Properties
  - creates R_H parallel channel: no data on M_t-R_H channels
  - does not affect noise
  - can be combined with rate and power adaptation
  - simple detector

MIMO with CSIT

Given

- Channel H = [1 2; 1 2+ε]
- BPSK signaling {+1,-1}
- ρ=50, ε=0.01
- [U,S,V]=svd(H) yields
  - U = [-0.7057 -0.7085; -0.7085 0.7057];
  - S = [3.1686 0; 0 0.0032],
  - V =[-0.4463 -0.8949; -0.8949 0.4463];

Task

- Verify that U and V are unitary
- How would you design the MIMO communication system?
- What is the SVD of a geometric LOS MIMO channel?

MIMO SVD

```
Mt=10;                          % TX antennas
Mr=10;                          % RX antennas
Ns=9;                           % number of streams
Ns=min(min(Mt,Mr),Ns);          % should be less than min(Mr,Mt)
sigma=0.1;                      % noise variance
H=randn(Mr,Mt)+j*randn(Mr,Mt);% generate channel
[U,S,V] = svd(H);               % channel SVD

% transmitter
x_tilde=2*round(rand(Ns,1))-1+j*(2*round(rand(Ns,1))-1);
precoding=V(:,1:Ns);            % TX needs to know V
x_tr=precoding*x_tilde;

% observation
y=H*x_tr+sigma*(randn(Mr,1)+j*(randn(Mr,1)));

% receiver
shaping=U(:,1:Ns)';             %RX needs to know U
y_tilde=shaping*y;              %now compare y_tilde with x_tilde
```

Beamforming: CSIT to increase diversity gain in low SNR

- Send one symbol over all M_t antennas

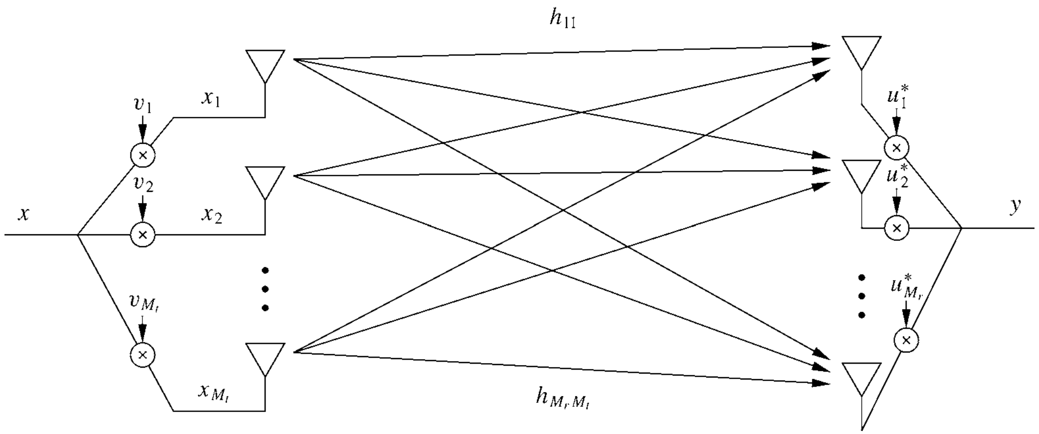


Figure 10.7: MIMO channel with beamforming.

- Scale by **v** and **u** (unit-energy)

$$y = \mathbf{u}^{\mathrm{H}}\mathbf{H}\mathbf{v}x + n$$

- Choose **u** and **v** along largest singular value.
- Leads to high diversity, low rate

Capacity of the MIMO channel

- Model $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \mathbb{E}\{\mathbf{x}\mathbf{x}^{\mathrm{H}}\} = \mathbf{R}_{\mathbf{x}}, \mathrm{tr}(\mathbf{R}_{\mathbf{x}}) = \rho, \mathbb{E}\{\mathbf{n}\mathbf{n}^{\mathrm{H}}\} = \mathbf{I}_{M_r}$
- Channel: $\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^{\mathrm{H}}$ with $\mathbf{\Sigma} = \mathrm{diag}(\sigma_1^2, \sigma_2^2, \ldots, \sigma_{R_H}^2, 0, \ldots, 0)$
- Channel capacity for fixed channel (without proof)

$$C = \max_{\mathbf{R}_{\mathbf{x}}:\mathrm{Tr}(\mathbf{R}_{\mathbf{x}})=\rho} B \log_2 \det[\mathbf{I}_{M_r} + \mathbf{H}\mathbf{R}_{\mathbf{x}}\mathbf{H}^H],$$

- Channel *unknown* at the transmitter (no CSIT): $\mathbf{R}_{\mathbf{x}} = \rho / M_t \mathbf{I}_{M_t}$

$$C = B \sum_{i=1}^{R_H} \log_2 \left(1 + \frac{\rho \sigma_i^2}{M_t}\right)$$

- Channel *known* at the transmitter (CSIT): waterfilling over parallel channels

$$C = \max_{\rho_i : \sum_i \rho_i \leq \rho} \sum_{i=1}^{R_{\mathbf{H}}} B \log_2(1 + \sigma_i^2 \rho_i),$$

- What is the capacity for beamforming?
- Coding: across space and time
- Capacity for fading channel is random: average capacity / outage capacity (see capacity lectures)

Today's learning outcomes

At the end of this lecture, you must be able to

- Describe SIMO, SISO, MISO, and MIMO transmission
- Express the general narrowband MIMO model: geometric and Rayleigh
- Use ML, ZF, and MMSE detectors and describe their properties
- Perform parallel decomposition of a MIMO channel and combine with deterministic water-filling
- Perform beamforming

CSIR detection methods

Given

- Channel H = [1 2; 1 2+ε]
- BPSK signaling {+1,-1}
- ρ=50 (17 dB)

Task

- Determine inv(H)
- Determine the ML estimator for ε=0.01. What is its complexity? Do you think it will perform well? Why/why not?
- Determine the ZF estimator for ε=0.01. What is its complexity? Do you think it will perform well? Why/why not?
- Determine the MMSE estimator for ε=0.01. What is its complexity? Do you think it will perform well? Why/why not?

Solution

Channel inverse

$$\mathbf{H}^{-1} = \begin{bmatrix} 1+2/\epsilon & -2/\epsilon \\ -1/\epsilon & 1/\epsilon \end{bmatrix}$$

ML detector: given a y, the ML detector will compute the Euclidean distance between y and Hx, for all possible x in {√ρ[1 1], √ρ[1 -1], √ρ[-1 1], √ρ[-1 -1]}. The complexity is low since there are only two transmit antennas and 2 points in the constellation. We expect the performance of ML to be good, since the received points are well separated.

ZF equalizer: The ZF matrix is

$$\mathbf{H}^{-1} = \begin{bmatrix} 201 & -200 \\ -100 & 100 \end{bmatrix}$$

After multiplication with y, the noise will have covariance matrix $\mathbf{H}^{-1} \times \mathbf{H}^{-1}$ which means the noise will be very large

MMSE: The MMSE matrix is $\begin{bmatrix} 0.2 & 0 \\ 0.15 & 0.25 \end{bmatrix}$. After multiplication with y, the noise will have less power, but there will be residual interference. So both ZF and MMSE will have poor performance.

MIMO with CSIT

Given

- Channel H = [1 2; 1 2+ε]
- BPSK signaling {+1,-1}
- $\rho=50$, $\varepsilon=0.01$
- [U,S,V]=svd(H) yields
- U = [-0.7057 -0.7085; -0.7085 0.7057]; S = [3.1686 0; 0 0.0032], V =[-0.4463 -0.8949; -0.8949 0.4463];

Task

- Verify that U and V are unitary
- How would you design the MIMO communication system?

Solution

Try this Matlab code:

```
>>H= [1 2; 1 2+0.01]
>>[U,S,V]=svd(H)
>> U‘*U
>> V‘*V
```

To communicate over this MIMO channel, we note that only one singular value is large, so we will only send one stream. We generate one data symbol x and perform the following operations:

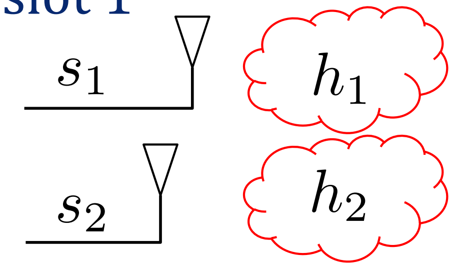
```
>>v=V(:,1)
>>u=U(:,1)
>>x_precoded=x*v
>>y=H*x_precoded + randn(2,1)*sqrt(0.5)
>>y_shaped=u'*y
```

Now it is easy to see that `y_shaped = sigma_max * x + w`, where w is a noise term with distributed as $\mathcal{N}(0,0.5)$

Alamouti code: increase diversity gain in low SNR without CSIT

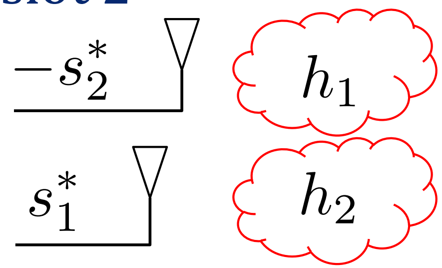
Space-time code for M_t=2, M_r=1, achieves full diversity order of 2 with only CSIR. Sends 2 symbols (s_1,s_2) in 2 time slots

1. Time slot 1



$$y_1 = h_1 s_1 + h_2 s_2 + n_1$$

1. Time slot 2



$$y_2 = -h_1 s_2^* + h_2 s_1^* + n_2$$

1. Combine: time slot 1 + conjugate of time slot 2: $\mathbf{y} = \mathbf{H}_A \mathbf{s} + \mathbf{n}$ with $\mathbf{H}_A \mathbf{H}_A^{\mathrm{H}} = (|h_1|^2 + |h_2|^2)\mathbf{I}_2$. So diversity order 2, but no array gain

Concepts can be generalized: space-time codes