Exercise 9 in SSY135 Wireless Communications
Topic: MIMO

Feb. 27, 2020

1. [E2011-Mar]Suppose we want to compare two MIMO systems with channel matrices $\mathbf{H}_1$ and $\mathbf{H}_2$, where

$$\mathbf{H}_1 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \end{bmatrix}, \qquad \mathbf{H}_2 = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix},$$

   Which system will allow for the highest spectral efficiency (assuming that both systems use the same modulation and coding)? Motivate. Does the result change when both systems can use different modulation formats?

2. [E2010-Aug-Q.3] Consider a 4×4 narrowband MIMO system with channel matrix $\mathbf{H}$. The SVD of $\mathbf{H}$ is

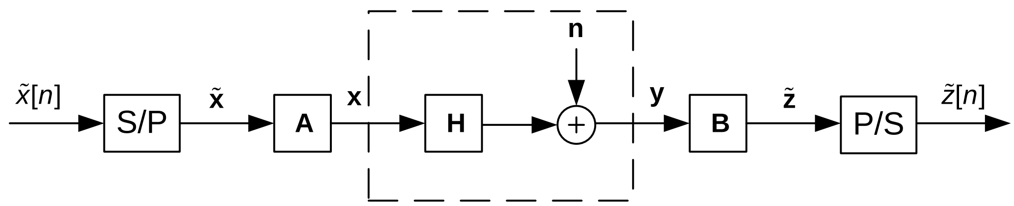
$$\mathbf{U} = \begin{bmatrix} -0.5490 - j0.4314 & 0.2819 - j0.1607 & 0.5305 + j0.0040 & 0.2050 - j0.2895 \\ 0.1777 - j0.1476 & -0.3890 + j0.6656 & 0.3482 - j0.3794 & -0.0749 - j0.2854 \\ -0.3466 - j0.2751 & -0.3435 + j0.0243 & 0.0463 - j0.2171 & -0.0048 + j0.7977 \\ 0.3261 + j0.3963 & 0.4264 + j0.0037 & 0.6120 - j0.1720 & 0.0800 + j0.3799 \end{bmatrix}$$

$$\mathbf{\Sigma} = \begin{bmatrix} 3.9987 & 0 & 0 & 0 \\ 0 & 2.7369 & 0 & 0 \\ 0 & 0 & 2.1914 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{V} = \begin{bmatrix} -0.4497 & 0.6578 & -0.2472 & -0.5513 \\ 0.4314 + j0.4758 & -0.1168 - j0.0283 & 0.2066 + j0.0283 & -0.5838 - j0.4345 \\ -0.2095 + j0.2227 & -0.3993 + j0.5607 & -0.2700 + j0.5083 & -0.1846 + j0.2594 \\ 0.3608 + j0.4020 & 0.2073 + j0.1898 & -0.6567 - j0.3646 & 0.2474 + j0.0620 \end{bmatrix}$$

   A block diagram for the transmission scheme is found below. The transmitted symbols, $\tilde{x}[n]$, are uncoded, Gray-mapped, unit-energy QPSK symbols. The vector $\tilde{\mathbf{x}}$ is made up by a number of consecutive QPSK symbols and the the transmitted vector is $\mathbf{x} = \mathbf{A}\tilde{\mathbf{x}}$, where $\mathbf{A}$ is the precoding matrix.

The MIMO channel, depicted inside the dashed lines, adds a noise vector $\mathbf{n}$, whose elements are iid, zero-mean, complex Gaussian random variables with variance N_0. At the receiver, the received vector $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$ is multiplied with the receiver matrix $\mathbf{B}$ to form the soft decision vector $\tilde{\mathbf{z}} = \mathbf{B}\mathbf{y}$. Hence, the decision on $\tilde{x}[n]$ is based on $\tilde{z}[n]$.



Suppose we can transmit one vector every 1μs.

(a) Specify $\mathbf{A}$ and $\mathbf{B}$ to achieve the maximum possible data rate. What is the data rate (in bits/second)? (4p)

(b) Specify $\mathbf{A}$ and $\mathbf{B}$ to achieve the MIMO beamforming. What is the data rate (in bits/second)? (4p)

(c) Compute the average bit error probability for the maximum data rate and MIMO beamforming cases. (4p)

3. **Different ways of using multiple antennas in MIMO**

Consider a 2×2 MIMO system with the following channel matrix

$$\mathbf{H} = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} -0.5257 & -0.8507 \\ -0.8507 & 0.5257 \end{bmatrix} \begin{bmatrix} 3.6 & 0 \\ 0 & 1.4 \end{bmatrix} \begin{bmatrix} -0.5257 & -0.8507 \\ -0.8507 & 0.5257 \end{bmatrix}$$

Note that $\mathbf{H}$ is written in terms of its singular value decomposition (SVD) $\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T$. While solving for data-rates, assume only rectangular MQAM is allowed.You should use the tight bound for BER $P_b \leq 0.2e^{-1.5\gamma/(M-1)}$. Total transmit power is 10 mW and noise power at each receive antenna is 1 mW. The system bandwidth is $B = 100$ kHz.

(a) We first use the MIMO channel for getting the highest possible data rate. Find the maximum MQAM size that can be sent on the two channels with equal power allocation on the two channels for an instantaneous P_b requirement of 10^{-3} on each channel. Find the total data-rate. Also provide the receive filter used, denoted by $B_{\text{SVD}}^{2\times 2}$ and the transmit filter being used denoted by $A_{\text{SVD}}^{2\times 2}$.

(b) Now suppose the transmitter wants to send a single information stream and for that uses only one transmit antenna (the first one) whereas the receiver uses both receive antenna and employs MRC. For the same data-rate as in part (a), what is the new BER value? Explain briefly how does this strategy of arbitrarily choosing a transmit antenna compare to beamforming.

(c) Now suppose the transmitter uses both the transmit antennas again, however the two receive antennas are geographically far apart. This situation may arise when the two receive antennas belong to two different users. Effectively, there is a single transmitter with two transmit antennas and two users each with a single receive antenna. The same channel matrix $\mathbf{H}$ describes the channel i.e. the channel to user 1 is given by the first row of $\mathbf{H}$ and the channel to user 2 is given by the second row of $\mathbf{H}$. Since the users are geographically far apart, no joint processing of received signals is possible i.e. neither receiver shaping nor MRC filtering can be done as it requires joint processing of the received signals. Thus in order to transmit to the two users independent information in a manner such that there is no interference between them, the transmitter has to premultiply the transmit signal by a transmit filter $A_{\text{BC}}^{2\times 2}$ to make the effective channel diagonal. Find the transmit filter $A_{\text{BC}}^{2\times 2}$. Using the transmit filter found, find the maximum MQAM size (and the corresponding data-rate) that can be sent to the two users for an instantaneous P_b requirement of 10^{-3} for each user, with equal power allocation to the two users.

Hints for part (c):

i. The transmitter can make the effective channel a (2×2) identity matrix by premultiplying with an appropriate matrix (transmit filter). Think in terms of matrix inverse.

ii. The transmit filter that you derive may not be unitary (unlike in part (a)), hence you have to take the gain provided by the tranmsit filter on each antenna into account when trying to allocate equal power to two users.