Solution to Exercise 3

February 6, 2020

Tentative Solutions

1.
   (a) $T_m \approx 0.1$ msec, $B_d \approx 0.2$ Hz
   (b) $B_{coh} \approx \frac{1}{T_m} = 10$ kH, $\Delta f > 10$kHz for uncorrelated response
   (c) $T_{coh} = 10$ s, $\Delta t > 10$s for uncorrelated response
   (d) 3kHz$< B_{coh} \Rightarrow$ flat, 30 kHz$> B_{coh} \Rightarrow$ frequency selective

2.
$$S_c(\tau,\rho) = \begin{cases} \frac{1}{T_1\rho_1}e^{-\tau/T_1} & \text{if } 0 \le \tau, \quad -\rho_1/2 \le \rho \le \rho_1/2 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

(a)
$$A_c(\tau,\Delta t) = F_\rho^{-1}\{S_c(\tau,\rho)\} = \int_{-\rho_1/2}^{\rho_1/2} \frac{1}{T_1\rho_1}e^{-\tau/T_1}e^{j2\pi\rho\Delta t}d\rho = \frac{1}{T_1}\text{sinc}(\rho_1\Delta t)e^{-\tau/T_1}, \ \tau \ge 0$$

So, power delay profile is: $A_c(\tau) = A_c(\tau,0) = \frac{1}{T_1}e^{-\tau/T_1}, \ \tau \ge 0$

$$\mu_{T_m} = \frac{\int_0^{+\infty} \tau A_c(\tau)\,d\tau}{\int_0^{+\infty} A_c(\tau)\,d\tau} = T_1$$

So,

$$\sigma_{T_m} = \sqrt{\frac{\int_0^{+\infty} (\tau-\mu_{T_m})^2 A_c(\tau)\,d\tau}{\int_0^{+\infty} A_c(\tau)\,d\tau}} = T_1$$

(b)
$$S_C(\Delta f,\rho) = \int_0^{\infty} S_c(\tau,\rho)e^{-j2\pi\tau\Delta f}d\tau = \frac{1}{\rho_1}\frac{1}{1+j2\pi\Delta f T_1}, \quad \rho_1/2 \le \rho \le \rho_1/2$$

Doppler power spectrum: $S_C(\rho) = S_C(\rho) = \frac{1}{\rho_1}, \ \rho_1/2 \le \rho \le \rho_1/2$ and 0 other wise.
Doppler spread: $B_D = \rho_1$

(c) The transmitted signal experiences frequency flat fading when: $B_u \approx \frac{1}{T_s} < B_{coh} \approx \frac{1}{\sigma_{Tm}} = \frac{1}{T_1}$. So it happens when $T_s > T_1$
The transmitted signal experiences frequency fast fading when: $T_s > T_{coh} \approx \frac{1}{B_D} = \frac{1}{\rho_1}$. So to satisfy both, $Ts > \max\{T_1, \frac{1}{\rho_1}\}$

3.

$$\begin{aligned} A_c(\tau,\Delta t) &= E\{c(\tau,t+\Delta t)\,c^*(\tau,t)\} \\ &= 16E\{\cos(t+U+\Delta t)\cos(t+U)\}\,\delta(\tau) + 9E\left\{|\beta_1(t)|^2\right\}\delta(\tau-\tau_1) + 4E\{\sin(t+U+\Delta t)\sin(t+U)\}\,\delta(\tau-\tau_2 \\ &= 8\delta(\tau) + 9\times 1_{\Delta t=0}\delta(\tau-\tau_1) + 2\delta(\tau-\tau_2) \end{aligned}$$

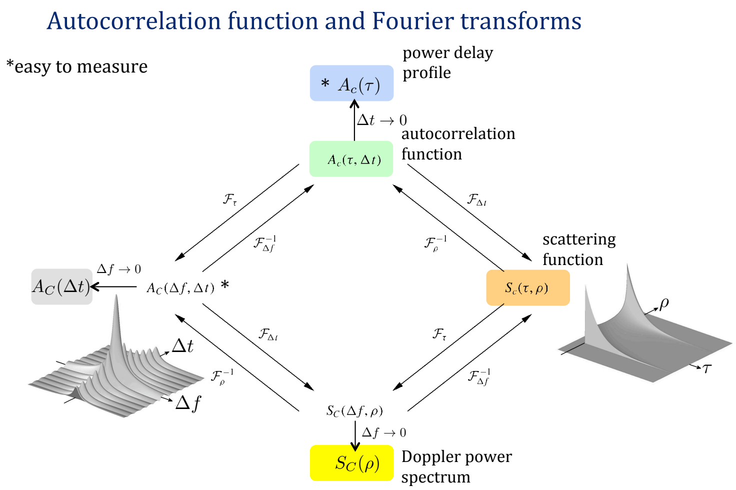

Figure 1: Relation between autocorrelation and various Fourier transforms.

$$\lim_{\Delta t \to 0} A_c\left(\tau, \Delta t\right) = A_c\left(\tau\right) = 8\delta\left(\tau\right) + 9\delta\left(\tau - \tau_1\right) + 2\delta\left(\tau - \tau_2\right)$$

$$\mu_{T_m} = \frac{9\tau_1 + 2\tau_2}{19} = \frac{9 \times 25 + 2 \times 50}{19} = 17.1053 \text{ ns}$$
$$\sigma_{T_m} = \sqrt{\frac{\left(\tau_1 - \mu_{T_m}\right)^2 \tau_1 + \left(\tau_2 - \mu_{T_m}\right)^2 \tau_2}{19}} = 7.7669 \text{ ns}$$