Solution to Exercise 7

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1. **[G 12.6]**

(a) $T_m = 20$ us, $B_c = 1/T_m = 50$ KHz. $B_N = B_c/2 = 25$ KHz. By using raised cosine pulses, the minimum frequency separation required for subcarriers to remain orthogonal is B_N. Therefore, the total bandwidth occupied by a multicarrier system with 8 subcarriers is $8B_N = 200$ KHz. However, if overlapping subcarriers are used, then the total bandwidth needed will be (from eq.(12.4) of textbook) $B = (N+\beta+\epsilon)/T_N = 112.5$ kHz, when $\beta = 1$ and $T_N = (1+\beta)/B_N$. Therefore, bandwdith required for overlapping subcarriers is approximately 50% of the bandwidth required for non-overlapping case.

(b) $\gamma = 20\text{dB} = 100$, $M \leq 1 + 0.283\gamma = 29.3$. Hence, the maximum constellation size for MQAM is $M = 16$. The corresponding data rate on each subchannel is 4 bits/symbol. The symbol duration on each subchannel is $T_N = (1+\beta)/B_N = 80$ us. Therefore, the total data rate of the system is $\frac{4\times 8}{80\times 10^{-6}}$=400 Kbps

2. **[G 12.5]**

(a) If the baseband bandwidth is 100 Khz, then at the carrier frequency they have bandwidth of $B = 200$ KHz.

For flat fading we need the coherence bandwidth to be much greater than the bandwidth of the signal. Therefore $B_c \geq 10B = 2$ MHz.

For independent fading, we want the channel between two carriers to be uncorrelated. That is, we want $B_c < \triangle f = 200$ KHz.

(b) Since BER $\leq .2e^{-1.5\gamma/(M-1)}$, hence we have

$$M \leq 1 - \frac{1.5\gamma}{\ln(5\text{BER})} = 1 + 0.283\gamma. \tag{1}$$

The received SNR on the first subchannel is$\gamma_1 = 11\text{dB} = 12.589$. Substituting γ_1 into (**??**), we have $M_1 \leq 4.56$. Therefore, we use 4-QAM for the signals transmitted over the first subchannel. The corresponding data rate is 2 bits/symbol.

The received SNR on the second subchannel is$\gamma_2 = 14\text{dB} = 25.12$. Substituting γ_2 into (**??**), we have $M_2 \leq 8.11$. Therefore, we use 8-QAM for the signals transmitted over the second subchannel. The corresponding data rate is 3 bits/symbol.

The received SNR on the second subchannel is$\gamma_3 = 18\text{dB} = 63.10$. Substituting γ_3 into (**??**), we have $M_3 \leq 18.86$. Therefore, we use 16-QAM for the signals transmitted over the second subchannel. The corresponding data rate is 4 bits/symbol.

Therefore, at each symbol time we will transmit 9 bits in total. The symbol duarition on each subchannel is $T_s = 1/B_{\{baseband\}} = 10$ us. Therefore, the total data rate of the multicarrier signal is $9/T_s = 900$ Kbps.

(c) In order to achieve that same data rate, we will need 3 bits/symbol per subchannel, that is 8-QAM constellation. Plug $M = 8$ into (**??**), we have $\gamma \geq 24.73$. Hence, the transmit power required on the first subchannel is $\frac{24.73}{12.589} \times 100 = 196.4$ mW. For the second subchannel, the transmit power needed is $\frac{24.73}{25.12} \times 100 = 98.4$ mW. For the third subchannel, the transmit power needed is $\frac{24.73}{63.10} \times 100 = 39.2$ mW. The total transmit power is 334 mW. Hence, 34 mW needs to be increased over the 300 mW transmit power in part (b).

3. **[Design of an OFDM system]**

In order to avoid ICI and ISI, we must have

$$T_{cp} \geq T_m = \text{maximum delay spread} = 50 \text{ ns}$$

and

$$T + T_{cp} \ll T_c = \text{coherence time}$$

To find the coherence time, we want to find the smallest T_c such that $|J_0(2\pi 100 \triangle t)|$ is small for all $\triangle t \geq T_c$. Looking up the bessel function value for the first zero crossing happen$\Longrightarrow 2\pi 100 T_c = 0.4 \times 2\pi$, hence, $T_c = \frac{0.4}{100} = 0.004$ s.

In order to keep the cyclic prefix overhead as little as possible, here we chose $T_{cp} = T_m = 50$ ns. Note that the subcarrier spacing $\triangle f = 1/T$. In order to satisfy $T + T_{cp} \ll T_c$, we must have $\triangle f \gg \frac{1}{T_c - T_{cp}} \approx \frac{1}{T_c} = 250$ Hz. Here, we choose $\triangle f = 10$ KHz.

Since the data rate $R_b = \frac{N \log_2 M}{T + T_{cp}} = 100$ Mbit/s, with $M = 4$, $T = 1/\triangle f = 0.1$ ms, $T_{cp} = 50$ ns. It can be derived that the total number of subcarriers required for this data transmission is $N = \frac{R_b(T+T_{cp})}{\log_2 M} \approx 5003$. Hence, the approximate bandwidth of the transmitted signal is $B_{OFDM} \approx (N-1)\triangle f + 2 \times \frac{1}{T+T_{cp}} \approx (N+1)\triangle f \approx N \triangle f = 50.03$ MHz.

(b) The data rate is $R_b = \frac{N \log_2 M}{T + T_{cp}}$. Hence,

$$\bar{\gamma}_b = \frac{\overline{P}_r}{R_b} \frac{1}{N_0} = \frac{\overline{P}_r}{N_0 N \log_2 M}(T + T_{cp}) = \frac{\overline{P}_r T}{N_0 N \log_2 M} + \frac{\overline{P}_r T_{cp}}{N_0 N \log_2 M} \ ,$$

in which only $\frac{\overline{P}_r T}{N_0 N \log_2 M}$ is used for detecting data. Let $\bar{\gamma} = \frac{\overline{P}_r T}{N_0 N \log_2 M}$, then we have

$$\bar{\gamma}_b = \bar{\gamma} + \bar{\gamma} \frac{T_{cp}}{T} \ ,$$

thus, $\bar{\gamma} = \bar{\gamma}_b \frac{T}{T + T_{cp}}$.