Exercise 7 in SSY135 Wireless Communications
Topic: OFDM

Feb. 20, 2020

1. [G-12.6] Consider a channel with impulse response
$$h(t) = \alpha_0 \delta(t) + \alpha_1 \delta(t - T_1) + \alpha_2 \delta(t - T_2) \quad (1)$$
Assume that $T_1 = 10$ µs and $T_2 = 20$ µs. You want to design a multicarrier system for the channel with subchannel bandwidth $B_N = B_c/2$. If raised cosine pulses with $\beta = 1$ are used and if the subcarriers are separated by the minimum bandwidth necessary to remain orthogonal, then what is the total bandwidth occupied by a multicarrier system with eight subcarriers? Assuming a constant SNR on each subchannel of 20 dB, find the maximum constellation size for MQAM modulation that can be sent over each subchannel with a target BER of 10^{-3}, assuming also that M is restricted to be a power of 2. Find the corresponding total data rate of the system.

2. [G-12.5] Consider a multicarrier modulation transmission scheme with three nonoverlapping subchannels spaced 200 kHz apart (from carrier to carrier) and with subchannel baseband bandwidth of 100 kHz.
   (a) For what values of the channel coherence bandwidth will the subchannels of your multicarrier scheme exhibit flat fading (approximately no ISI)? For which such values will the subcarriers exhibit independent fading?
   (b) Suppose that you have a total transmit power $P = 300$mW and that the noise power in each subchannel is 1mW. With equal power of 100 mW transmitted on each subchannel, the received SNR on each subchannel is $\gamma_1 = 11$ dB, $\gamma_2 = 14$ dB, and $\gamma_3 = 18$ dB. Assume the subchannels do not experience fading, so these SNRs are constant. For these received SNRs, find the maximum signal constellation size for MQAM that can be transmitted over each subchannel for a target BER of 10^{-3}. Assume the MQAM constellation is restricted to be a power of 2 and use the bound BER $\leq .2e^{-1.5\gamma/(M-1)}$ for your calculations. Find the corresponding total data rate of the multicarrier signal assuming a symbol rate on each subchannel of $T_s = 1/B$, where B is the subchannel bandwidth.
   (c) For the subchannel SNRs given in part (b), suppose we want to use precoding (to equalize the received SNR in each subchannel) and then send the same signal constellation over each subchannel. What size signal constellation is needed to achieve the same data rate as in part (b)? What transmit power would be needed on each subchannel to achieve the required received SNR for this constellation with a 10^{-3} BER target? How much must the total transmit power be increased over the 300-mW transmit power in part (b)?

3. [Design of an OFDM system] Consider a fading channel with impulse response
$$c(\tau, t) = 2\beta_0(t)\delta(\tau) + \beta_1(t)\delta(\tau - \tau_1)$$
where $\tau_1 = 50$ ns and $\beta_0(t)$ and $\beta_1(t)$ are iid complex, zero-mean Gaussian random processes with autocorrelation function $J_0(2\pi 100\Delta t)$, where $J_0(x)$ is the zeroth order Bessel function of the first kind.

   We want to design an OFDM system with QPSK modulation to transmit data with the data rate $R_b =$ 100 Mbit/s. The system should be designed to avoid inter-carrier interference (ICI) and intersymbol interference (ISI).
   (a) Compute and motivate reasonable values of the cyclic prefix duration and the subcarrier spacing to avoid ICI and ISI. What will be the (approximate) bandwidth of the transmitted signal?
   (b) Suppose the average received SNR/bit for a single-carrier system is $\bar{\gamma}_b$. What is the average SNR/bit that is used for detecting the bits in the OFDM system?