

# Wireless Communications SSY135 - Lecture 1

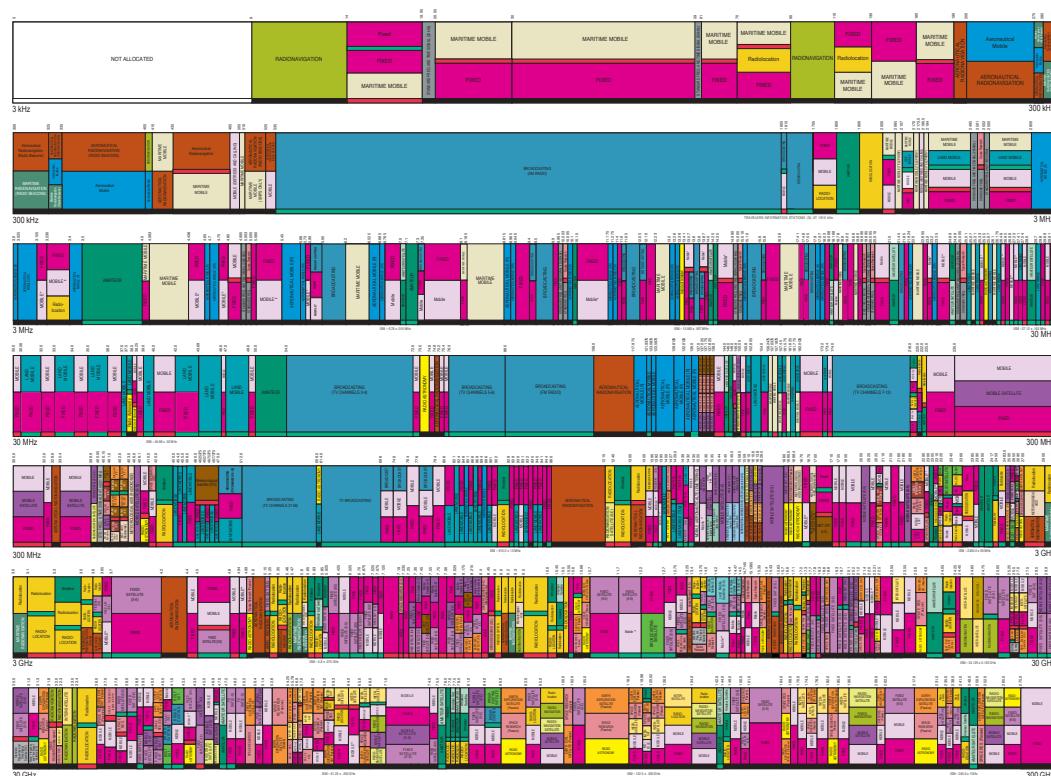
Henk Wymeersch

Department of Electrical Engineering  
Chalmers University of Technology

<http://tinyurl.com/hwymeers>  
email: henkw@chalmers.se



CHALMERS

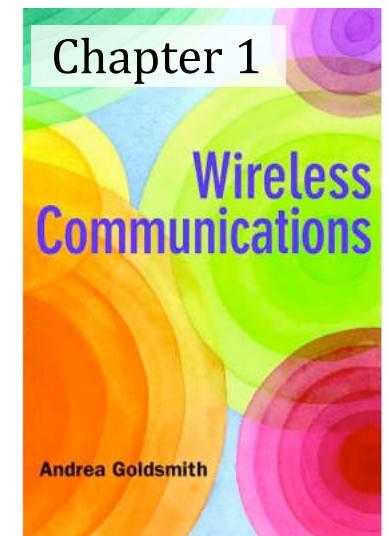


# Topics for today

- Welcome and practical information
- Course topics
- Lecture learning outcomes
- Math review
- Wireless propagation and challenges

Suggested reading:

- Chapter 1 entirely
- Appendices A, B, and C



# Course information

- Staff (all located at E2, 6<sup>th</sup> floor):



**Henk Wymeersch**  
henkw@chalmers.se



**Yasaman Ettefagh**  
ettefagh@chalmers.se



**Nima Hajiabdolrahim**  
nimahaj@chalmers.se

- **Office hours:** Thursdays, 9:00 – 10:00. Use these opportunities!
- **Course location:** ES51
- **Communication & details:** course website <https://chalmers.instructure.com/courses/8763>
- **Lectures & book:** A. Goldsmith, *Wireless Communications*, Cambridge University Press, 2005 (Cremona & e-book). Read the book!
- **12 Tutorials:** 6 quizzes (1 points each)
- **Projects:** 2 MATLAB group projects (8 p) + oral examination (15 p).
- **Exam:** closed-book, 4 questions
- **Grading:** Quiz (6), project (2\*23), Exam (48)

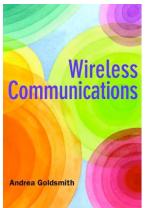
| Score | 0-39 | 40-59 | 60-79 | ≥80 |
|-------|------|-------|-------|-----|
| Grade | Fail | 3     | 4     | 5   |

## Announcements

- Register for the course ASAP or send us an email with your name and request to be assigned to a project groups
- Changes and updates are found on the course web
- The schedule is available at Time Edit <https://se.timeedit.net/web/chalmers>
- Slides will be made available at the course web and hard copies will be handed out at lectures
- Short videos of the lectures are available on YouTube
- Student representatives for course evaluation, please see Henk in the break
- In general, include “[SSY135]” in email subject lines to avoid problems with spam filter
- Matlab:
  - You are expected to be fluent
  - If not go to:  
<https://trainingenrollment.mathworks.com/selfEnrollment?code=003L666N0Q5P>

Expensive course, offered free of charge to Chalmers students  
Gives you credits, good for your CV

## What we will cover from the book



- Not everything in detail, some material from other sources

| Chapter             | Lecture | Chapter             | Lecture |
|---------------------|---------|---------------------|---------|
| 1: Intro            | ✓       | 9: adaptation       | ✓       |
| 2: PL, shadowing    | ✓       | 10: MIMO            | ✓       |
| 3: multipath models | ✓       | 11: equalization    | ✗       |
| 4: capacity         | ✓       | 12: OFDM            | ✓       |
| 5: detection        | ✗       | 13: spread spectrum | reading |
| 6: error prob.      | ✓       | 14: multiuser       | ✓       |
| 7: diversity        | ✓       | 15: cellular        | ✓       |
| 8: coding           | ✗       | 16: ad-hoc          | reading |

- Note: Chapters 5, 8 are covered in SSY125

# Course learning outcomes

After the course, the students should be able to

- explain why small-scale and large-scale fading occurs
- describe the conditions under which the standard path loss and fading models accurately predicts real-world radio wave propagation
- define Doppler spread, delay spread, coherence time, and coherence bandwidth and explain how these parameters are related and affect the wireless physical layer design
- define the performance metrics instantaneous error probability, average error probability, and outage probability and understand which metric is appropriate for a given scenario
- define ergodic and outage channel capacity and explain under which conditions these concepts indicate the spectral efficiency of an optimum link design
- evaluate the performance of communication links over fading channels by analysis and computer simulations
- define the concepts of channel reuse, uplink, downlink, multiple access, multiplexing, frequency-division, time-division, code-division, and space-division
- define the concepts of time, frequency, and space diversity and explain how diversity can be achieved in practice
- explain the concept of spatial channels for multiple input, multiple output (MIMO) systems
- describe and compare complexity and performance of the following channel equalizations methods: zero-forcing, linear MMSE, maximum likelihood
- design and interpret power and rate allocation algorithm, including statistical and deterministic water-filling
- explain the effect of phase noise and power amplifier nonlinearities on the communication link
- describe the current knowledge of health effects of electromagnetic radiation and how this affects the design of wireless communication equipment via regulations, recommendations, and measurement methods for determining safe levels of exposure
- describe the foundations for ethical scientific research (e.g. related to dual use, data collection, plagiarism and authorship)

See material on Canvas from WHO and FCC



## Today's learning outcomes

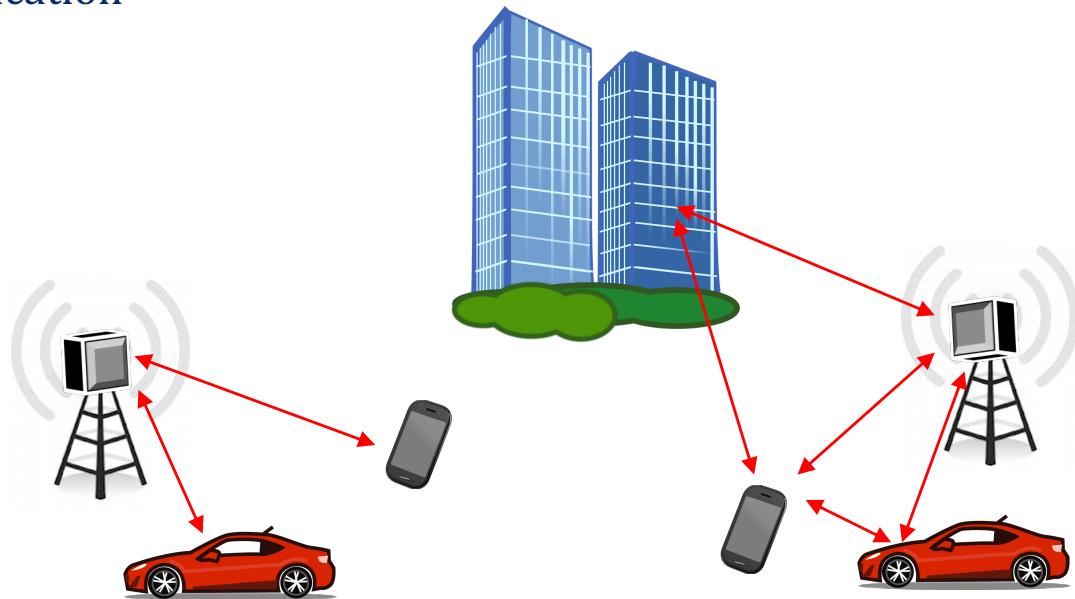
At the end of this lecture, you must be able to

- Describe the main properties and challenges of wireless communication systems



## The overall goal of this course

- At the end of the course, you should be able to design and analyze 5G massive MIMO communication with mobile users
- Requires knowledge of
  - The propagation channel and effect on communication
  - (Adaptive) signal design for high rate, low complexity
  - Sharing of resources among users
  - Multi-antenna communication



# What is a wireless communication system?



## Definitions

- The transmitted signal is an electromagnetic wave in the frequency band 10 kHz to 300 GHz
- We will mainly be concerned with systems in the 300 MHz to 10 GHz range

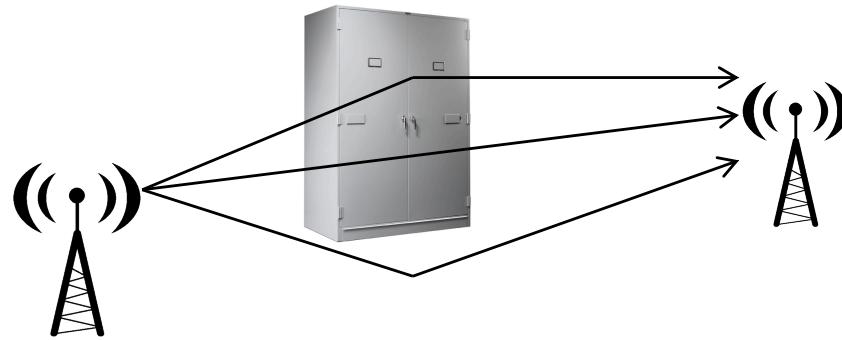
## Properties and challenges

- The wireless channel is
  - **shared** medium: interference is a problem
  - **time-varying** medium (user mobility): special demands on design
- The power and time-frequency occupancy of the transmitted signal must be carefully controlled to avoid disturbing other systems

# Basics of the wireless channel

- Before: the AWGN channel  $y_k = x_k + w_k$
- Now: 
$$\begin{aligned} y_k &= \sum_{l=-L_1}^{L_2} h_{l,k} x_{k-l} + w_k + i_k \\ &= h_{-1,k} x_{k+1} + h_{0,k} x_k + h_{1,k} x_{k-1} + w_k + i_k \end{aligned}$$
- Effects:
  - Inter-symbol interference / dispersive channel
  - Time-varying channel
  - Interference from other users
- This course
  - Understanding the channel statistics and impact on communication
  - Designing communication systems, single and multi-user over this channel
- Tools: linear systems (convolution, fft), linear algebra (SVD), statistics (mean, variance, autocorrelation, ML), Gaussian random variables, digital communications (modulation, pulse shaping, baseband, passband)

# Basics of the wireless channel

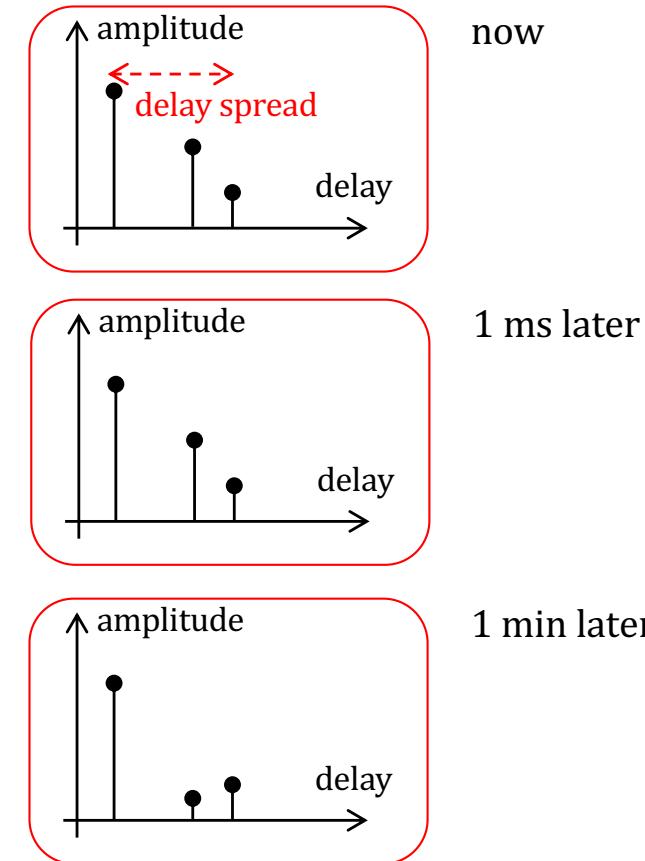


## Channel impulse response

- amplitudes depend on antenna pattern, environment
- delays depend on environment → delay spread (e.g., 100 ns)
- varies over time due to mobility of TX, RX, changes in environment → coherence time (e.g., 10 ms)
- Overall three effects: path loss, shadowing, multipath

$$h_{\text{PB}}(t, \tau) = \sum_{k=1}^K A_k(t) \delta(\tau - \tau_k(t))$$

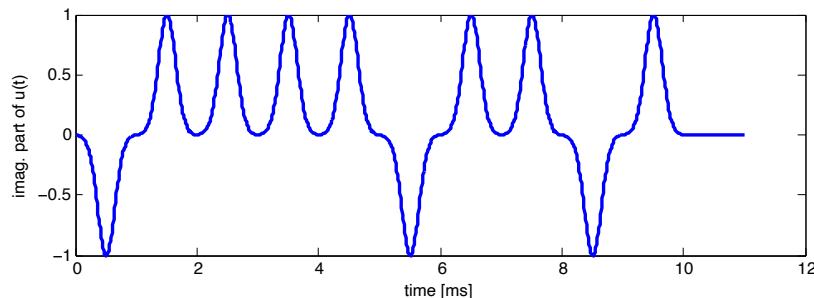
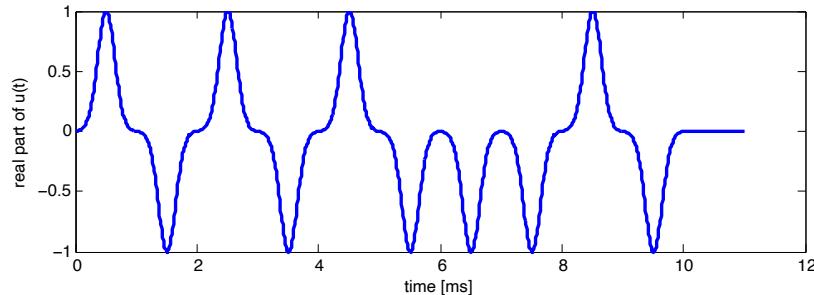
$$h_{\text{BB}}(t, \tau) = \sum_{k=1}^K A_k(t) \exp(-j2\pi f_c \tau_k(t)) \delta(\tau - \tau_k(t))$$



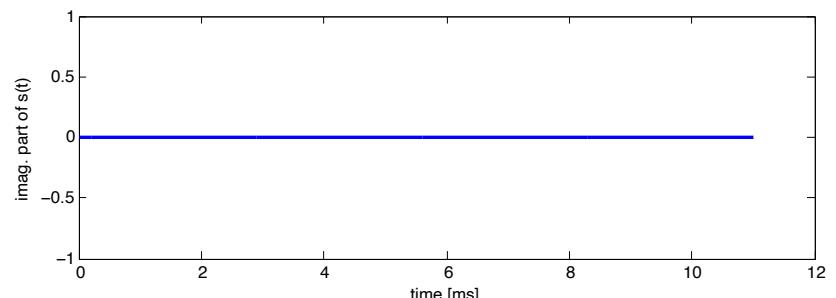
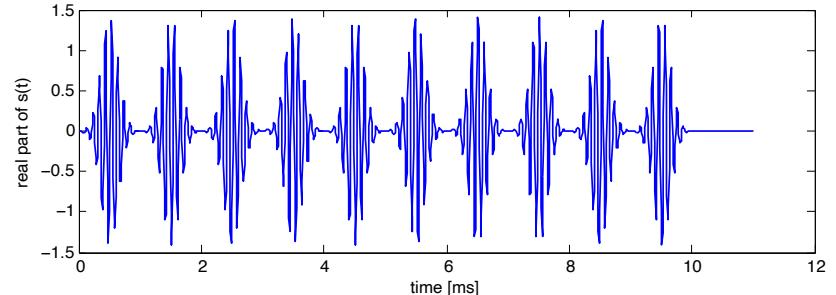
delays vary due to changing path lengths  
frequency varies due to Doppler  
amplitudes vary due to changes in environment

## Review of some basic concepts (see also Appendix B, C)

- Baseband  $u(t) = \sum_{k=-\infty}^{+\infty} a_k p(t - kT)$



- Passband  $s(t) = \Re\{u(t)e^{j2\pi f_c t}\}$



- Received (after filter)

$$r_{\text{BB}}(t) = u(t) \star c(t) + w(t)$$

- Received

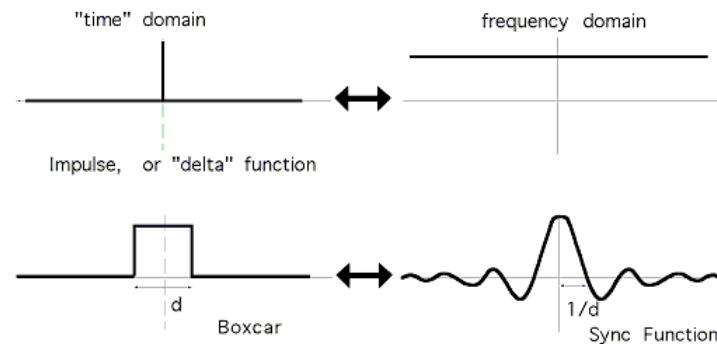
$$\begin{aligned} r_{\text{PB}}(t) &= s(t) \star c_{\text{PB}}(t) + n(t) \\ &= \Re\{(u(t) \star c(t))e^{j2\pi f_c t}\} + n(t) \end{aligned}$$

# Review of some basic concepts (see also Appendix B, C)

- dB scale
  - $X [\text{dB}] = 10 \log_{10} (X)$ , where X is dimensionless
  - $X [\text{dBm}] = 10 \log_{10} (X / 1\text{mW})$ , where X has dimension Watt
- Power and energy
  - Power [W]
  - Energy = power \* time [J]
- Fourier transform

$$S(f) = \int_{-\infty}^{+\infty} s(t) \exp(-j2\pi ft) dt,$$

$$s(t) = \int_{-\infty}^{+\infty} S(f) \exp(+j2\pi ft) df.$$



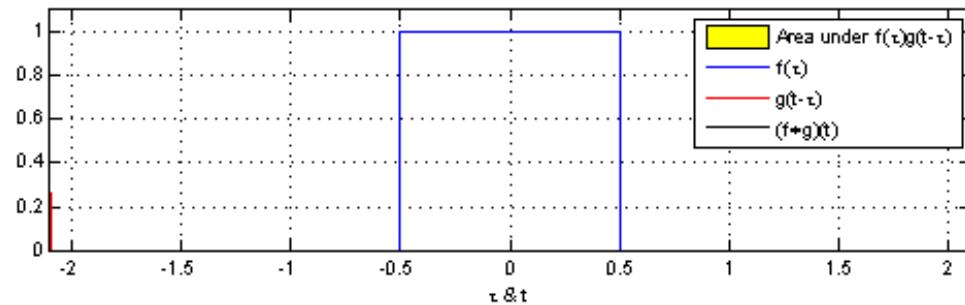
## Review of some basic concepts

- Convolution: time  $t$ , delay  $\tau$

$$y(t) = \int_{-\infty}^{+\infty} c(\tau)u(t - \tau)d\tau$$

$$Y(f) = C(f)U(f)$$

$$C(f) = \int c(\tau)e^{-j2\pi f\tau}d\tau$$

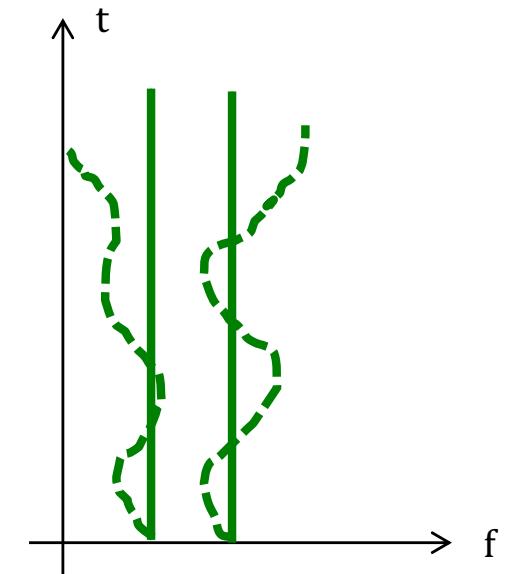
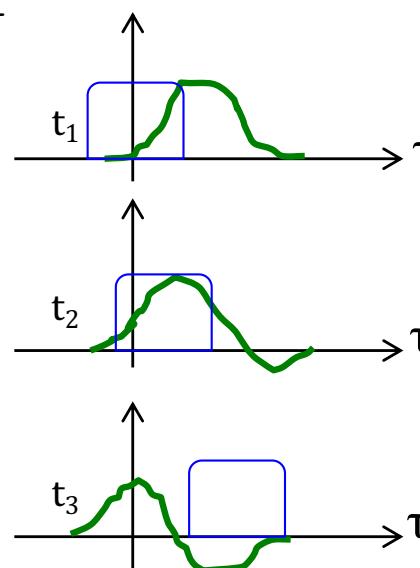


- Time varying convolution: the filter changes over time ( $t$ )

$$y(t) = \int_{-\infty}^{+\infty} c(\tau, t)u(t - \tau)d\tau$$

$$Y(f) \neq C(f)U(f)$$

$$C(f, t) = \int c(\tau, t)e^{-j2\pi f\tau}d\tau$$



# Review of some basic concepts



## Given

- Consider input [00100] for times [0 1 2 3 4]
- Channel1 [1234] for delays [-1 0 1 2]
- Channel2 [1234] for delays [-1 0 1 2] at times ... -1 0 1 2
- Channel2 [0123] for delays [-1 0 1 2] at times 3 4 ...

## Task

- Determine output for channel1 and channel2
- What is energy of input?
- What is energy of outputs?

## Review of some basic concepts

- Real random variable X

R.V.  $X$ , pdf  $p_X(x)$

$$\mu_X = \mathbb{E}[X] = \int x p_X(x) dx$$

$$\sigma_X^2 = \mathbb{E}[(X - \mu_X)^2]$$

- Complex Gaussian

$$Z = X + jY \sim \mathcal{CN}(\mu, \sigma^2)$$

$$X \sim \mathcal{N}(\Re(\mu), \sigma^2/2)$$

$$Y \sim \mathcal{N}(\Im(\mu), \sigma^2/2)$$

$X, Y$  independent

- Q-function:  $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{+\infty} \exp(-t^2/2) dt$

$$X \sim \mathcal{N}(\mu, \sigma^2)$$

$$p(X > \alpha) = Q\left(\frac{\alpha - \mu}{\sigma}\right)$$

- Wide sense stationary random process

$X(t)$  mean :  $\mu_X = \mathbb{E}\{X(t)\}$

autocorr. :  $A_X(\tau) = \mathbb{E}\{X(t)X(t + \tau)\}$

$$\text{PSD} : S_X(f) = \int A_X(\tau) e^{-j2\pi f \tau} d\tau$$

- Gaussian noise process

mean :  $\mu_X = 0$

autocorr. :  $A_X(\tau) = \delta(\tau)N_0/2$

$$\text{PSD} : S_X(f) = N_0/2$$

# Review of some basic concepts



## Given1

- Input WGN process  $n(t)$  with PSD  $N_0$
- A filter  $h(t)$

## Task1

- Determine the output PSD

---

## Given2

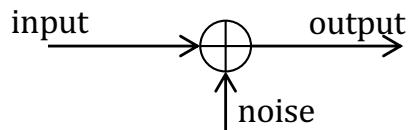
- $X$  is zero mean Gaussian with variance  $s$
- Let  $Y = \exp X$ .

## Task2

- What is distribution of  $Y$ ?
- Plot the pdfs of  $X$  and  $Y$

## Review of some basic concepts

- Discrete-time signaling: 1 symbol every T seconds



$$r_k = \sqrt{E_s} s_k + n_k, \quad n_k \sim \mathcal{CN}(0, N_0)$$

$$\mathbb{E}\{|s_k|^2\} = 1$$

$$\gamma_s = E_s/N_0 \quad (\text{SNR per symbol})$$

$$\gamma_b = E_s/(N_0 \log_2 M) = E_b/N_0 \quad (\text{SNR per bit})$$

Max. likelihood detector:

$$\hat{s}_k = \arg \max_s p(r_k|s)$$

- Performance depends =f(signal type, SNR)

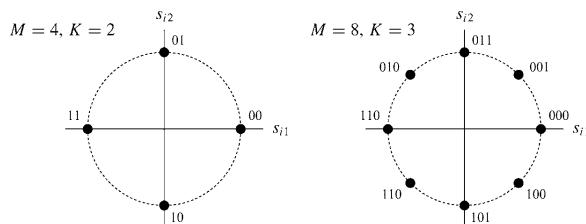


Figure 5.15: Gray encoding for MPSK.

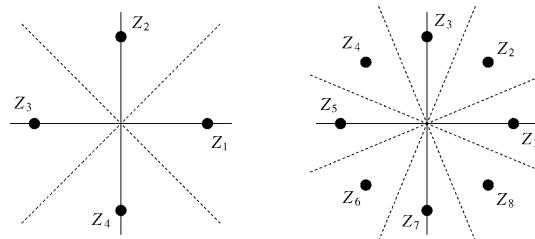
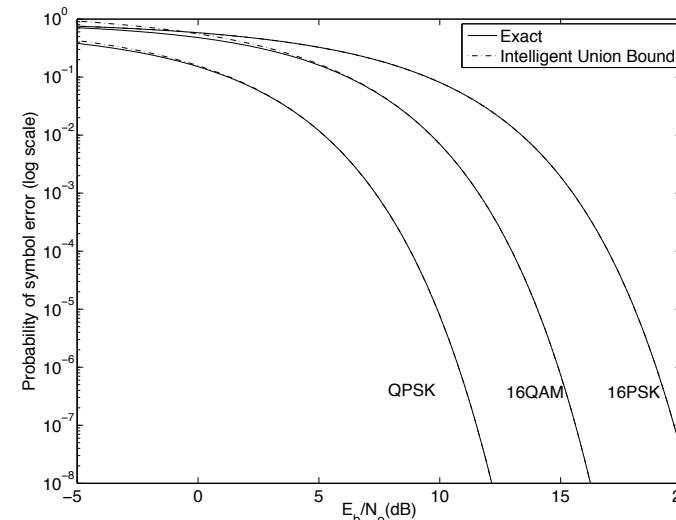


Figure 5.16: Decision regions for MPSK.



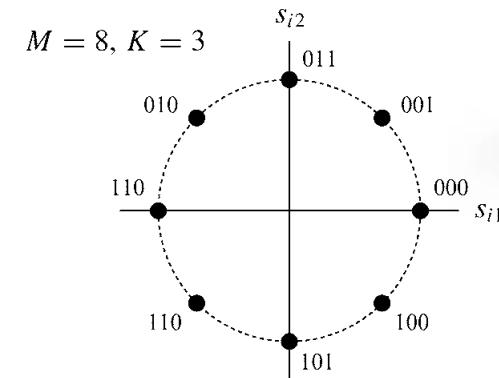
# Review of some basic concepts

## Given

- Constellation as shown
- $r = a + n$ , where  $a = 1$ ,  $n \sim \mathcal{CN}(0, 2\sigma^2)$

## Task

- Formulate the ML detector
- What is probability of confusing “1” for  $\exp(j\pi/2)$ ?



## Review of some basic concepts

- Optimization

$$\text{minimize} \quad f(\mathbf{x})$$

$$\text{s.t.} \quad g(\mathbf{x}) \leq 0$$

$$h(\mathbf{x}) = 0$$

- Solution (for convex  $f, g$ , linear  $h$ ) procedure\*:

1. Write down Lagrangian  $\mathcal{L}(\mathbf{x}, \lambda, \mu) = f(\mathbf{x}) + \lambda g(\mathbf{x}) + \mu h(\mathbf{x}), \lambda \geq 0$
2. Set partial derivative to zero

$$\partial \mathcal{L}(\mathbf{x}, \lambda, \mu) / \partial \mathbf{x} = \mathbf{0}$$

$$\partial \mathcal{L}(\mathbf{x}, \lambda, \mu) / \partial \lambda = 0$$

$$\partial \mathcal{L}(\mathbf{x}, \lambda, \mu) / \partial \mu = 0$$

3. Solve for  $(\mathbf{x}, \lambda, \mu)$

\*Other technical conditions may apply

# Review of some basic concepts



## Given

- minimize  $x^2 + y^2$ , with  $x+y=0.5$

## Task

- What is the optimal  $(x,y)$ ?

## Vector and matrix operations

- Linear algebra is your best friend. Download the matrix cookbook



- Singular value decomposition (SVD)
  - $X = U * S * V'$  ( $[U, S, V] = \text{svd}(X)$  in matlab)
  - $S$  is a non-negative diagonal matrix
  - $U$  and  $V$  are unitary matrices ( $U' * U = U * U' = I$ )
- Discrete Fourier transform (FFT)
  - `a=fft(b)`
  - `c=ifft(b)`

## Today's learning outcomes

At the end of this lecture, you must be able to

- Describe the main properties and challenges of wireless communication systems



# Review of some basic concepts



## Given

- Consider input [00100] for times [0 1 2 3 4]
- Channel1 [1234] for delays [-1 0 1 2]
- Channel2 [1234] for delays [-1 0 1 2] at times ... -1 0 1 2
- Channel2 [0123] for delays [-1 0 1 2] at times 3 4 ...

## Task

- Determine output for channel1 and channel2
- What is energy of input?
- What is energy of outputs?

## Solution

- For channel 1:  $y[0,1,2,3,4] = [0,1,2,3,4]$
- For channel 2:
  - For  $n < 3$ :  $y_n = u_{n+1} + 2u_n + 3u_{n-1} + 4u_{n-2}$ :  $y[0,1,2] = [0,1,2]$  as before
  - For  $n > 2$ :  $y_n = u_n + 2u_{n-1} + 3u_{n-2}$ , so  $y[3,4] = [2,3]$
- Energy of the input is 1
- Energy of the output:
  - For channel 1:  $1^2 + 2^2 + 3^2 + 4^2 = 30$
  - For channel 2:  $1^2 + 2^2 + 2^2 + 3^2 = 18$



# Review of some basic concepts



## Given1

- Input WGN process  $n(t)$  with PSD  $N_0$
- A filter  $h(t)$

## Task1

- Determine the output PSD

---

## Given2

- $X$  is zero mean Gaussian with variance  $s$
- Let  $Y = \exp X$ .

## Task2

- What is distribution of  $Y$ ?
- Plot the pdfs of  $X$  and  $Y$

## Solution

- Output noise is  $w(t) = \int h(\tau)n(t - \tau)d\tau$
- Autocorrelation

$$\begin{aligned}\mathbb{E}\{w(t)w^*(t + u)\} &= \int \int h(\tau)h(\tau')\mathbb{E}\{n(t - \tau)n(t + u - \tau')\}d\tau d\tau' \\ &= N_0 \int \int h(\tau)h(\tau')\delta(t + u - \tau' - t + \tau)d\tau d\tau' \\ &= N_0 \int h(\tau)h(u + \tau)d\tau \\ &= N_0 g(u)\end{aligned}$$

where  $g(u)$  is the convolution of  $h(\cdot)$  with itself. Applying a Fourier transform gives the PSD  $S(f) = N_0|H(f)|^2$

- For a Gaussian RV  $X$ ,  $\exp(X)$  has a log-normal distribution:  
<http://en.wikipedia.org/wiki/Lognormal>
- The derivation of the lognormal distribution follows from the transformation of variables rules.

$$p_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2\sigma^2}(x - \mu)^2\right)$$

$$\begin{aligned}p_Y(y) &= f_X(\log y) \times \left| \frac{d \log y}{dy} \right| \\ &= \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2\sigma^2}(\log y - \mu)^2\right) \frac{1}{y}\end{aligned}$$



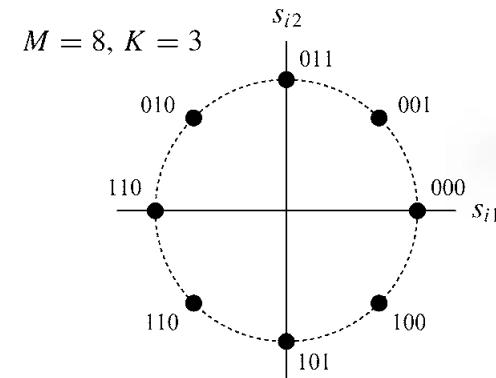
# Review of some basic concepts

## Given

- Constellation as shown
- $r = a + n$ , where  $a = 1$ ,  $n \sim \mathcal{CN}(0, 2\sigma^2)$

## Task

- Formulate the ML detector
- What is probability of confusing “1” for  $\exp(j\pi/2)$ ?



## Solution

The ML detector is

$$\hat{a} = \arg \max_a p(r|a)$$
$$= \arg \min_a |r - a|^2$$



The probability of sending 1 and deciding j is found through the Q-function:

$$\begin{aligned} P(|r - 1|^2 > |r - j|^2 | 1 \text{ is transmitted}) &= P(|r|^2 + 1 - 2\Re\{r^*\} > |r|^2 + 1 - 2\Re\{r^*j\} | 1 \text{ is transmitted}) \\ &= P(\Re\{r\} < \Re\{r^*j\} | 1 \text{ is transmitted}) \\ &= P(\Re\{r\} < \Im\{r\} | 1 \text{ is transmitted}) \\ &= P(\Re\{r\} - \Im\{r\} < 0 | 1 \text{ is transmitted}) \\ &= P(1 + n_I - n_Q < 0 | 1 \text{ is transmitted}) \\ &= P(n_I - n_Q < -1 | 1 \text{ is transmitted}) \\ &= Q\left(1/\sqrt{2\sigma^2}\right) \end{aligned}$$

where  $n_I$  and  $n_Q$  are the in-phase and quadrature noise components

# Review of some basic concepts



## Given

- minimize  $x^2 + y^2$ , with  $x+y=0.5$

## Task

- What is the optimal  $(x,y)$ ?

## Solution

$$\mathcal{L}(x, y, \mu) = x^2 + y^2 + \mu(x + y - 5)$$

$$\frac{\partial \mathcal{L}}{\partial x} = 2x + \mu = 0$$

$$\frac{\partial \mathcal{L}}{\partial y} = 2y + \mu = 0$$

$$\frac{\partial \mathcal{L}}{\partial \mu} = x + y - 5 = 0$$

$$\rightarrow \mu = -5, x = y = 2.5$$

## Review of some basic concepts

- Any passband signal  $s(t)$  can be written as

$$\begin{aligned}s(t) &= s_I(t)\cos(2\pi f_0 t) - s_Q(t)\sin(2\pi f_0 t) = \operatorname{Re}\{\tilde{s}(t)\exp(j2\pi f_0 t)\} \\ &= |\tilde{s}(t)|\cos(2\pi f_0 t + \arg \tilde{s}(t))\end{aligned}$$

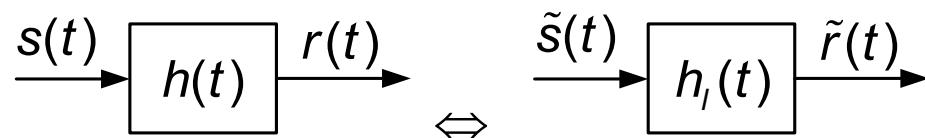
$f_0$  = any fixed frequency, normally the mid-band frequency

$s_I(t)$  = in-phase component;  $s_Q(t)$  = quadrature component

$\tilde{s}(t) = s_I(t) + js_Q(t)$  = complex envelope (or equivalent lowpass signal)

$|\tilde{s}(t)|$  = envelope;  $\arg \tilde{s}(t)$  = phase

$$S(f) = F[s(t)] = \frac{1}{2}[\tilde{S}^*(-f - f_0) + \tilde{S}(f - f_0)]; \quad \tilde{S}(f) = F[\tilde{s}(t)]$$



$$r(t) = s(t) * h(t) \quad \Leftrightarrow \quad \tilde{r}(t) = \tilde{s}(t) * h_I(t) \text{ where } h_I(t) = \frac{1}{2}\tilde{h}(t)$$