SSY135 Wireless Communications

Department of Electrical Engineering

Exam Date: March 16 2020

Teaching Staff

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Material Allowed material is

- Chalmers-approved calculator
- L. Rade, B. Westergren. Beta, Mathematics Handbook, any edition.
- One A4 page with your own handwritten notes. Both sides of the page can be used. Photo copies, printouts, other students' notes, or any other material is not allowed.
- A dictionary.

Grading A correct, clear and well-motivated solution gives a maximum of 12 points. An erroneous answer, unclear, incomplete or badly motivated solutions give point reductions down to a minimum of 0 points. Answers in any other language than English are ignored.

Results Results are posted no later than April 4^{th}. The grading review is on April 5^{th} in the E2 Room Landahlsrummet 7430, between 9:30 am and 11:30 am.

Grades To pass the course, all projects and the exam must be passed. The exam is passed by securing at least 12 points. The project is passed by securing at least 8 points (4 for the report and 4 for the oral exam) in each part of the project. The final grade on the course will be decided by the project (max score 46), quizzes (max score 6), and final exam (max score 48). The sum of all scores will decide the grade according to the following table.

Total Score	0–39	40–59	60–79	≥ 80
Grade	Fail	3	4	5

PLEASE NOTE THAT THE PROBLEMS ARE NOT NECESSARILY ORDERED IN DIFFICULTY.

Good luck!

1 MIMO and Massive MIMO

Consider a MIMO communication system with $N_t = 3$ transmit and $N_r = 2$ receive antennas. The channel matrix is

$$\mathbf{H} = \begin{bmatrix} 0.5431 & -1.0889 & 0.7241 \\ -0.4752 & 1.1738 & -0.6336 \end{bmatrix}$$

The matrix of right singular vectors is given by

$$\mathbf{V} = \begin{bmatrix} -0.36 & -0.48 & -0.8 \\ 0.8 & -0.6 & 0 \\ -0.48 & -0.64 & 0.6 \end{bmatrix}.$$

Communication is of the form $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$, where $\mathbb{E}\{\|\mathbf{x}\|^2\} = E_s$ and $\mathbb{E}\{\mathbf{n}\mathbf{n}^H\} = N_0\mathbf{I}$. We will consider the following communication strategies:

- *Scenario 1:* The transmitter and receiver know the channel, the transmitter and receiver apply SVD.
- *Scenario 2:* The transmitter and receiver know the channel, the transmitter applies zero-forcing precoding $\mathbf{W}$, where $\mathrm{trace}(\mathbf{W}^H\mathbf{W}) = 1$. (recall that the trace of a matrix is the sum of the diagonal elements).

1. [*2 pt*] Given $\mathbf{H}$ and $\mathbf{V}$, determine the SVD, i.e., $\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^{\mathrm{H}}$, where you should find $\mathbf{\Sigma}$ and $\mathbf{U}$. Verify that $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices. Show or verify that the singular values are 2 and 0.1.
2. [*7 pt*] We now consider scenario 1, where the transmitter and receiver use the SVD of the channel to create two parallel data streams.
   (a) [*3 pt*] Draw a block diagram of the transmitter and receiver with the relevant operations related to $\mathbf{U}$ and $\mathbf{V}$.
   (b) [*3 pt*] Given a power allocation $\alpha E_s/2$ to the first stream and $(1-\alpha)E_s/2$ to the second stream, express the sum-rate as a function of $\alpha \in [0, 1]$. Recall that for a single stream the rate is $\log_2(1 + \mathrm{SNR})$ and for two streams the sum-rate is $\log_2(1 + \mathrm{SNR}_1) + \log_2(1 + \mathrm{SNR}_2)$. Now evaluate the rate for $E_s/N_0 = 10$, for $\alpha = 0$ and $\alpha = 1$. Which choice is better?
   (c) [*1 pt*] Now optimize the rate as a function of α as a function of E_s/N_0. What is the optimal α when $E_s/N_0 \to 0$ and when $E_s/N_0 \to \infty$?
3. [*3 pt*] We now consider scenario 2, where the transmitter performs zero-forcing precoding.
   (a) [*1 pt*] Draw a block diagram of the transmitter and the receiver.
   (b) [*1 pt*] To determine the zero-forcing precoder, first find a matrix $\mathbf{W}$ than enables you to write

   $$\mathbf{H}\mathbf{W}\mathbf{x} = \mathbf{x}, \forall \mathbf{x} \in \mathbb{C}^2.$$

   Verify that $\mathbf{W} = \mathbf{V}\mathbf{\Sigma}^T(\mathbf{\Sigma}\mathbf{\Sigma}^T)^{-1}\mathbf{U}^H$ meets the condition $\mathbf{H}\mathbf{W}\mathbf{x} = \mathbf{x}, \forall \mathbf{x}$. Why is $\mathbf{W}$ not a suitable precoder?
   (c) [*1 pt*] In order to have a suitable precoder, consider a structure of the form

   $$\mathbf{W} = \mathbf{H}^H(\mathbf{H}\mathbf{H}^H)^{-1}\mathbf{P}$$

   where $\mathbf{P}$

   $$\mathbf{P} = \begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix}$$

   Determine a_1 as a function of a_2 so that $\mathbf{W}$ becomes a valid precoder, i.e., $\mathrm{trace}\{\mathbf{W}^{\mathrm{H}}\mathbf{W}\} = 1$. What is the minimum and maximum value for a_1?
   Hint: first show that for $\mathbf{H}^\dagger = \mathbf{H}^H(\mathbf{H}\mathbf{H}^H)^{-1}$,

   $$(\mathbf{H}^\dagger)^{\mathrm{H}}\mathbf{H}^\dagger = \begin{bmatrix} 50.1250 & 49.8750 \\ 49.8750 & 50.1250 \end{bmatrix}.$$

   (d) [*Bonus 1 pt*] Compute the sum rate as a function for $a_1 = 0$ and compare with the SVD approach.

2 Fading and Diversity

1. [*6 pt*] Consider a cellular system with log-distance path loss model with the path loss exponent of 3. The transmitted signal has a carrier frequency of 900 MHz, bandwidth of $B = 100$ KHz and the modulation type is DPSK. The noise spectral density is $N_0 = -160$ dBm/Hz, so the total noise is $N_0 \times B$. Assume that the path loss at the reference distance $d_0 = 1$km is 10 dB larger than the corresponding free space path loss. Also suppose that the transmitter and receiver antennas have unit gain.

   (a) [*3 pt*] Assume that the service provider must support users at BER of 10^{-6}. Find the maximum cell size if the transmit power is limited to 100 mW.
   Hint: For DPSK modulation: $P_b = \frac{1}{2}e^{-\gamma_b}$
   Q function:

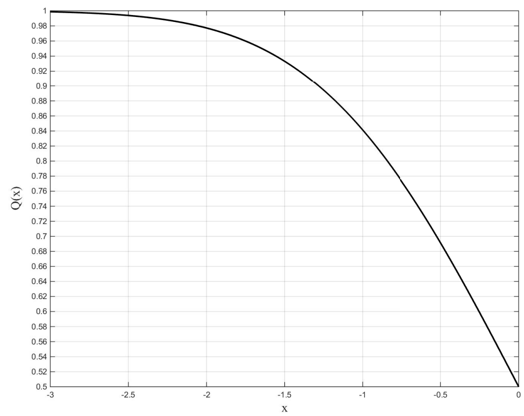

Figure 1: Part of Q function plot

   (b) [*3 pt*] Now assume that the transmitted signal faces log-normal shadowing with 0 dB mean and standard deviation of 5 dB as well. With the same transmit power, find the maximum cell size if the outage probability is 1%. Use the Q-function plot provided below.

2. [*6 pt*] Now assume that the users also experience fading as well as log-normal shadowing and path loss. To mitigate the effect of fading, the receiver is further equipped with another antenna, with proper antenna spacing such that the received signals at the antennas experience independent fading. Suppose that the wireless channel is changed such that the SNR at both antennas has a uniform distribution between 0 and 5. The receiver performs selection combining and maximum ratio combining.

   (a) [*2 pt*] Calculate the distribution of the combined SNR for both combining methods.
   Hint: For selection combining the cumulative density (CDF) function of the output SNR is the product of the CDFs of the input SNRs. For maximum ratio combining the probability density function (PDF) of the output SNR is the convolution of the PDFs of the input SNRs.

   (b) [*2 pt*] Calculate the average received SNR for both MRC and SC.

   (c) [*2 pt*] Suppose that outage happens if the received SNR is less than 0 dB. Calculate the outage probability for 2 combining methods and compare them.

3 Fading Channel and Adaptation Techniques

In this problem we will assume a specific wireless communication link between a base station and a moving user. The base station uses the carrier frequency of $f_c = 9$ GHz for data transmission, while it can switch between 4-QAM and 16-QAM schemes. Moreover, duration of the pulses used for pulse shaping in base station equals to $T_s = 10^{-6}$ seconds. The user is equipped with a channel estimator, and can transmit the estimated channel taps to the base station with a processing delay of approximately 150 symbol times.

1. [*3 pt*] *High-speed movement.* Assume that the user is moving with the speed of $v = 180\ \frac{km}{h}$ at the average distance of $30\,km$ from base station, while the delay spread of different paths from base station to the user is $T_d = 0.3\,\mu s$.

   (a) [*1 pt*] Compute the coherence bandwidth and coherence time at the user side.

   (b) [*2 pt*] Find the optimal adaptive transmission strategy at the base station with average power constraint of $\bar{S} = 20$ dB to achieve average bit error rate of $\bar{P_b} = 10^{-3}$.

2. [*9 pt*] *Low-speed movement.* Now assume that the user is moving with the speed of $v = 18\ \frac{km}{h}$ at the average distance of $10\,km$ from base station, while the delay spread of different paths from base station to the user is $T_d = 0.3\,\mu s$.

   (a) [*2 pt*] If the instantaneous transmit power is fixed to $S = 20$ dB at the base station, then, find the optimal rate adaptation strategy to maximize spectrum efficiency and meeting the instantaneous bit error rate of $P_b = 10^{-3}$. Make use of the fact that for M-QAM, $P_b\,(\gamma) \leq 0.2e^{\frac{-1.5\gamma}{M-1}}$.

   (b) [*7 pt*] Suppose that the average transmit power is constrained by $\bar{S}$ dB, and according to Rayleigh fading model for the channel between base station and user, the average received SNR at the user side is $\bar{\gamma} = 10$ dB. Furthermore, assume that the received SNR is quantized in 4 levels of 0 dB, 5 dB, 10 dB and 15 dB (for example if the received SNR is between 5 dB and 10 dB, we assume SNR = 5 dB at the receiver, and if the received SNR is less than 5 dB we assume SNR = 0 dB at the receiver).

   - [*2 pt*] First, find the probability mass function of the feedback values of received SNR, transmitted from the user to the base station.
   - [*1 pt*] In the next step, assume that we want to adapt both power and rate at the base station to meet the fixed bit error rate of $P_b = 10^{-3}$ and maximize the average spectrum efficiency, which is defined as $\mathbb{E}\{\log_2(M(\gamma))\}$. Formulate the maximization problem need to be solved to find the optimal allocation scheme.
     [*Hint: If we define the constellation size of transmitted symbols with respect to the received SNR of* γ *by* $M(\gamma) = \gamma/\gamma^*$ *, then according to the water filling solution, the optimal power allocation scheme is as follows*

     $$\frac{S(\gamma)}{\bar{S}} = \begin{cases} (M_j - 1)\frac{1}{\gamma K} & 0 < M_j \leq M(\gamma) < M_{j+1} \\ 0 & M_j = 0 \end{cases}$$

     Where, M_j *is the* j*-th constellation size that is allowed to be used at the base station and* K *is a constant defined by* $K = \frac{-1.5}{ln(5P_b)}$. *Then, it's sufficient to find the* γ^* *value that maximizes the average spectrum efficiency.*]
   - [*3 pt*] Now find the optimal rate and power adaptation scheme at the base station, by solving the optimization problem you found in previous part.
     [*Hint: It's noticeable that you should not exceed the power constraint at the base station, however it's possible to save energy and consume less power.*]
   - [*1 pt*] Is the proposed strategy in previous part power efficient? Why?

4 Small Questions

- [*4 pt*] Identify at least three ethical issues that could have happened while performing the projects. How would you deal with these issues? Answer in max 300 words.
- [*1 pt*] Consider a channel with a Doppler spread of 50 Hz. Assume a DPSK modulated signal with pulse duration of a) 1ms and b) 1μs is transmitted over this channel. Which signal would experience larger error floor and why?
- [*1 pt*] Consider a channel that has a Doppler spread of 100 Hz. Assume a 16-QAM modulated voice signal is transmitted over this channel at 2 Kbps. Is outage probability or average probability of error a better performance metric and why?
- [*1 pt*] Consider a system for which the transmitted signal experiences narrowband fast fading. Among time diversity and frequency diversity, which one would you choose and why?
- [*1 pt*] Give an example of a probability distribution that is typically used to model:
  a) pathloss b) shadowing
- [*1 pt*] In a wireless communication link hit by significant phase noise, which method is more likely to improve the average bit error rate?
  - Increasing the transmit power by 20 dB.
  - Using a better code with higher error correction ability.
- [*1 pt*] Consider a wireless communication link, transmitting data at the rate of $R = 0.1\,\frac{Mbit}{sec}$. The receiver experiences a fading channel with coherence time of $T_c = 0.1\,^{ms}$. For both of 4-QAM and 256-QAM schemes, determine which performance metric is more useful, outage probability or ergodic capacity?
- [*1 pt*] Describe the difference between array gain and diversity gain.
- [*1 pt*] What are the main benefits and drawbacks of OFDM, as compared to single-carrier transmission?