Lecture-8 notes for SSY150: Multimedia and video communications

Network Modeling

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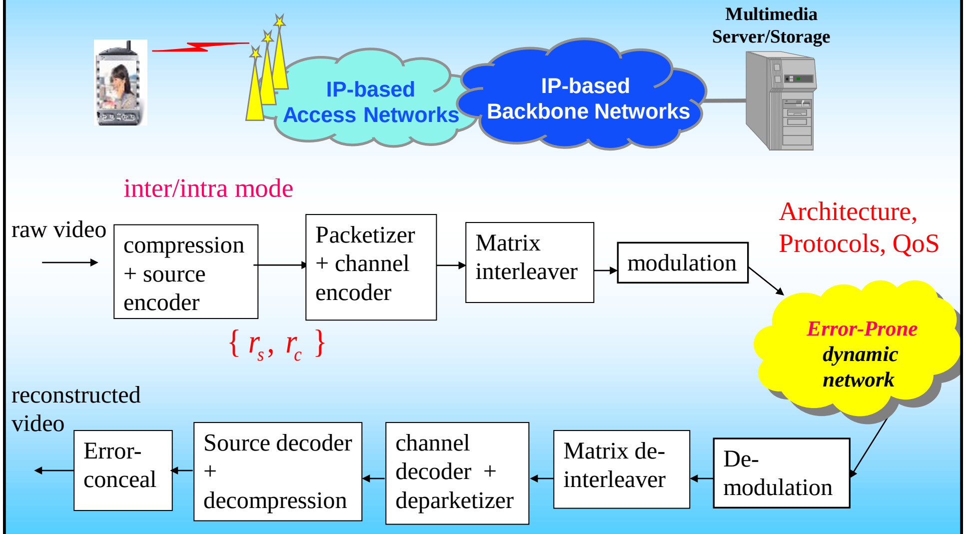
Content

1. Introduction
2. Network modeling for computing packet loss probability
3. FEC for error correction and detection, basics of error hiding (concealment)
4. Performance evaluation: Expected end-to-end distortions
5. Formulation: joint source-channel coding for end-to-end performance optimization

1. Introduction

Transport compressed video over error-prone networks



2. Network Modeling for Computing Packet Loss Probability

Estimate Packet Loss Rate

Packet loss can be estimated:

- **Empirically**, by computing at the receiver side:

$$\frac{\text{the number of lost packets}}{\text{the number of expected packets}}$$

- **Theoretically**, by using mathematical models (e.g. Erasure channels with random delays)

Network Modeling

- **Erasure network (packet losses in the network layer)**

Channel Modeling (in the physical layer):

- AWGN channel (bit/byte/symbol errors)
- Rayleigh fading channel (bit/byte/symbol errors)
- Rician fading channel (bit/byte/symbol errors)

Network Modeling

Models can be built in different layers

Internet: packet loss is modeled in the **network (IP) layer.** Error packets are discarded in the link layer (not forward to the network layer).

Wireless: bit errors are modeled as channel noise in the **physical layer**.

Note: bit errors in the physical layer may also impact the packets, could lead to packet losses in the network layer.

Modeling: Packet Losses

Packet losses: Packet loss + Packet truncation
Delay: Queuing delay in the network

Model: independent time-invariant packet erasure channel with random delay

Overall packet losses
= packet losses in the network + excessively delayed packets

$$\boxed{\rho_k = \varepsilon_k + (1-\varepsilon_k)\,\nu_k} \qquad (1)$$

where: ε_k probability of packet k is lost in the network layer
ν_k probability of packet k is lost due to excessive delay

$$\nu_k = \int_{\tau>\tau_0} p(\tau \mid \text{packet } k \text{ received})\, d\tau \qquad (p \text{ defined in (2)})$$

ε_k: from a 2−state Markov chain model (see (3)), or, a Bernoulli process

Modeling: Packet Delays

- Network delay τ varies randomly ~ a self-similar law.
- τ is heavy tail distributed, rather than Poisson distributed.
- A simple model*: shifted Gamma distribution

$$p(\tau \mid \text{packet received}) = \frac{\alpha}{\Gamma(n)}\left(\alpha\left(\tau-\gamma\right)\right)^{(n-1)} e^{-\alpha(\tau-\gamma)}, \quad \tau \geq \gamma \qquad (2)$$

n: number of routers, each being a M/M/1 queue with service rate α,
γ: total end-to-end processing time

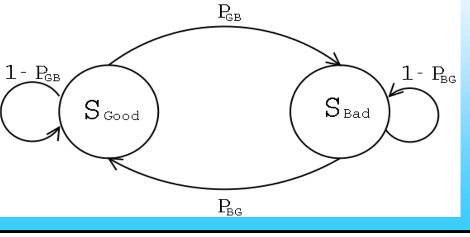
(Chou & Niao '06, IEEE multimedia)

Erasure network modeling:
Gilbert-Elliot (or, 2-state Markov chain) model

Network states:'good','bad'~ success/fail delivering a packet

Transition matrix $A = \begin{bmatrix} 1-P_{GB} & P_{GB} \\ P_{BG} & 1-P_{BG} \end{bmatrix}$

State probabilities: $P_G = \dfrac{P_{BG}}{P_{BG}+P_{GB}}, \quad P_B = \dfrac{P_{GB}}{P_{BG}+P_{GB}}$



Two-State Markov Chain Model

- **Probability of packet losses:**

$$\varepsilon_k = P_B = \frac{P_{GB}}{P_{GB} + P_{BG}} \qquad (3)$$

- **Average burst length:**

$$L_B = \frac{1}{P_{BG}} \qquad (4)$$

The above model is used in the network layer
Sometimes it is also used to describe the success/failure of the link layer packets for estimating the UDP throughput R_T

Modeling IP-based wireless channels

Wireless channel modeling is in the physical layer:

fading channels and AWGN channels mainly causing bit/byte errors → symbol error

need to convert to the errors to the IP level ! → erasure network

✓ A packet k is lost, if errors in any codewords cannot be recovered.

Probability of k-th packet lost due to codeword errors (assume C codewords/packet)

$$\varepsilon_{k|cw} = 1 - \left(1 - p_{cw|s}\right)^{C} \qquad (5)$$

✓ A codeword c is erroneous, if errors in any its symbols cannot be recovered

Probability of codeword errors due to symbol errors (assume N symbol/packet)

$$\varepsilon_{cw|s} = 1 - \left(1 - p_{s|b}\right)^{N} \qquad (6)$$

✓ A symbol is erroneous, if at least one bit in the symbol cannot be recovered

Probability of symbol errors due to bit error (assume B bits /symbol)

$$p_{s|b} = 1 - (1 - p_b)^{B} \qquad (7)$$

In such a case, the probability of kth packet loss in (1) is modified to:

$$\rho_k = \varepsilon_k + (1 - \varepsilon_k)\upsilon_k + (1 - \varepsilon_k)(1 - \upsilon_k)\varepsilon_{k|cw} \quad (8)$$

Remarks:

- Errasure packet loss, and codeword/symbol error-induced packet loss occur in different layers !
- Probability of packet loss is also a function of:
  - transmission power used for sending each packet
  - packet length
  - channel coding rate

4. Performance Evaluation: Expected end-to-end distortions

4.1. Expected end-to-end distortion for packet *k* (using pixel-based computation)

$$E(D_k) = \frac{1}{N_k}\sum_{j=1}^{N_k} E\left[d\left(f_j, \tilde{f}_j\right)\right]$$

where: N_k total number of pixels in *k*-th packet

f_j *j*-th pixel value in the original image packet

$\tilde{f}_j$ *j*-th pixel value from the reconstructed packet at the receiver

MSE as the distortion: $$E(D_k) = \frac{1}{N_k}\sum_{j=1}^{N_k}(f_j - \tilde{f}_j)^2$$

PSNR as the distortion: $$E(D_k) = 10\log_{10}\left(\frac{255^2}{MSE_k}\right) \quad (dB)$$

4.2. Expected end-to-end distortion for packet *k* (using packet-based computation):

$$E(D_k) = (1-\rho_k)E[D_{R,k}] + \rho_k E[D_{L,k}]$$

ρ_k: probability of *k*th packet lost R/L: packet received/lost

Example: in an error concealment scheme,
assume: if packet *k* is lost, one can replace it by the *(k-1)* packet

$$E(D_k) = (1-\rho_k)E[D_{R,k}] + \rho_k(1-\rho_{k-1})E[D_{C_R,k}] + \rho_k\rho_{k-1}E[D_{C_L,k}]$$

$D_{C_R,k}$: distortion after concealment when previous packet is received
$D_{C_L,k}$: distortion after concealment when previous packet is also lost

5. Formulation: Cross-layer design for end-to-end performance optimization (e.g. joint source-channel coding)

Joint source-channel coding is formulated mathematically:

- Constrained optimization using Lagrange multipliers;
- Two equivalent expressions by using
  a) rate constraint
  b) delay constraint

Resource-distortion optimization

Another way of formulation: utility cost-based approaches

The mathematical formulation by using the rate constraint

Let: source coding parameters: $\mathbf{S}=\{s_1 \cdots s_M\}$

channel coding parameters: $\mathbf{C}=\{c_1 \cdots c_M\}$

M packets in each frame / group of frames

Let: the bit rate constraint for an image frame: R_0

The criterion:

minimize the total expected distortion: $\min_{s\in\mathbf{S}, c\in\mathbf{C}} E[D(s,c)]$

subject to the rate constraint: $R(s,c) \le R_0$ **(A)**

An equivalent mathematical formulation by using delay constraint, if flow-rate is specified

Let: The transmission rate (e.g. UDP throughput) that a channel/network allows: R_T (physical limitation)

➔ maximum delay constraint: $T_0 = R_0 / R_T$

➔ transmission delay: $T(s,c) = R(s,c)/R_T$

Rate constraint: $R(s,c) \leq R_0$ $\Leftrightarrow$ Delay constraint: $T(s,c) \leq T_0$

The criterion:

Minimize the total expected distortion: $\min_{s \in \mathbf{S}, c \in \mathbf{C}} E[\,D(s,c)\,]$

subject to the delay constraint: $T(s,c) \leq T_0$

(B)

where, the expected distortion can be chosen as:

MSE distortion: $$E[D_{(\mathbf{f},\hat{\mathbf{f}})}] = \frac{1}{N}\sum_{j=1}^{N} E[(f_j - \tilde{f}_j)^2]$$

PSNR distortion: $$E(D_{(\mathbf{f},\hat{\mathbf{f}})}) = 10\log_{10}\left(\frac{255^2}{MSE}\right) \quad (dB)$$

Example 1: Joint source-channel coding with a possibility of retransmission

- Assume:
  a) Packets up to one frame in sender's buffer are eligible for retransmission;
  b) Lost packets in *(n-1)*th frame are resent during sending packets the *n*th frame.
- RS codec is used for channel coding:
  There are q different RS coding modes: $\mathrm{RS}(n_i, k),\ i = 1, \cdots, q$
  Then: $\boldsymbol{C} = \{ \mathrm{c}_i = k/n_i \,, i = 1 \cdots q\}$
- Video compression and source coding:
  $\boldsymbol{S}$={ prediction modes in MC (B,P prediction), quantization step size}
- Retransmission:
  - probability of *k*th packet in *(n-1)*th frame is $\sigma_k^{(n-1)} = \begin{cases} 1, & \text{Lost} \\ 0, & \text{received} \end{cases}$
  retransmission of *k*th packet of *(n-1)*th frame, if $\sigma_k^{(n-1)} = 1$
- For simplicity, we use the time delay constraint T_0

Problem:

Formulate the criterion that minimizes the expected distortion for the *(n-1)*th frame.

Solution: (mathematical formulation)

$\boldsymbol{\sigma_k^{(n-1)}} = \mathbf{0}$: probability of *k*th packet is received

$\boldsymbol{D}^{(\boldsymbol{n}-\mathbf{1})}$: distortion in *(n-1)*th frame

$$\min_{S,C} E\left[D^{(n-1)}(s,c)\right]$$

$$E\left[D^{(n-1)}(s,c)\right] = \sum_{k \in I(n-1)} (1-\sigma_k^{(n-1)}) E\left[D_{k,R_{n-1}}^{(n-1)}\right] + \\ + \sum_{k \in \boldsymbol{I}(n-1)} \sigma_k^{(n-1)} \left((1-\sigma_k^{(n)}) E\left[D_{k,R_n}^{(n-1)}\right] + \sigma_k^{(n)} E\left[D_{k,L_n}^{(n-1)}\right]\right)$$

s.t.: delay: $\sum_k \sigma_k^{(n-1)} T_k^{(n)} + \sum_k T_k^{(n-1)} \le T_0$

where $T_k^{(j)}$: delay time for the kth packet in the jth frame

assume: probability of packet loss in each frame is a constant

Formulation: resource-distortion optimization

More general: joint design of error-resilient source coding, cross-layer resource allocation, and error concealment.

Let: $\mathbb{k}_0$ the maximum allowed total cost
$\mathbb{k}$ the set of network adaptation parameters

The criterion:

Minimize the total expected distortion: $\min_{s \in \mathbf{S}, c \in \mathbf{C}} E\,[D(s,c)]$

subject to: the delay constraint $T(s,c) \le T_0$

other cost constaints $\mathbb{k}(s,c) \le \mathbb{k}_0$

(C)