Lecture notes for SSY150: Multimedia and video communications

End-to-end performance optimization:

Cross-layer design and
Joint source-channel coding

Mathematical formulation

(lectures 10)

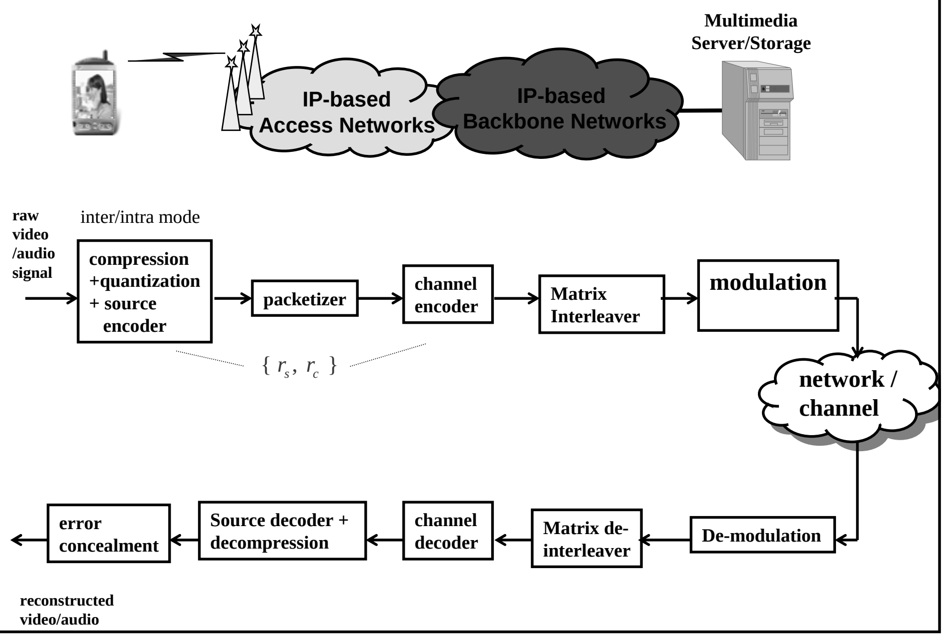
Irene Y.H. Gu, Dept. of Signals and Systems,
Chalmers Univ. of Technology, Sweden
May 7, 2009

Contents

1. End-to-end video communication system and adjustable parameters: revisit
2. Joint source and channel coding for end-to-end video optimization:
   the mathematical formulations
   (constrained optimization using Lagrange multiplies;
   equivalent expressions by using (a) the rate constraint,
   or (b) the delay constraint)
3. Resource-distortion optimization for end-to-end video optimization
4. Other ways of formulations: Utility-Cost-Based

1. End-to-end video communication system and adjustable parameters: revisit

Multimedia communications



Adjustable parameters related to error control and rate control

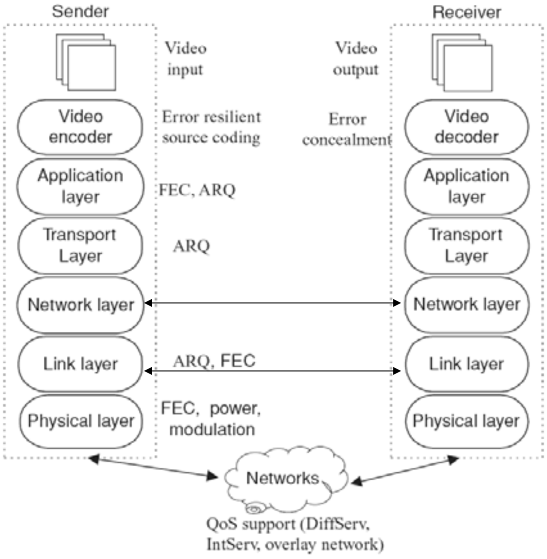


Figure: components that error and rate control can be applied for improving end-end performance

2. Joint source and channel coding for end-to-end video optimization

The mathematical formulation: (using the rate constraint)

Let: source coding parameters: $\mathbf{S}=\{s_1 \cdots s_M\}$

channel coding parameters: $\mathbf{C}=\{c_1 \cdots c_M\}$

M packets in each frame/ group of frames

Let: the bit rate constraint for an image frame: R_0

The criterion:

minimize the total expected distortion:	$\min_{s\in\mathbf{S},c\in\mathbf{C}} E[\ D(s,c)\ ]$	**(A)**
subject to the rate constraint:	$R(s,c) \le R_0$	

Constrained optimization and Lagrange multiplies

minimize the total expected distortion:	$\min_{s\in\mathbf{S},c\in\mathbf{C}} E[\ D(s,c)\ ]$
subject to the rate constraint:	$R(s,c) \le R_0$

$$\Updownarrow$$

Constrained optimization:

Minimize: $L(s,c,\lambda) \ = \ E[D(s,c)] + \lambda[R_0 - R(s,c)]$

=> $(s^*(\lambda), c^*(\lambda)) = \arg\min_{s\in S,c\in C} L(s,c,\lambda)$

where: $\lambda > 0$ is a Lagrange multiplier

Another equivalent mathematical formulation
(using the delay constraint, if the flow-rate is specified)

Let: The transmission rate (e.g. UDP throughput) that the channel/network allows: R_T (physical limitation)

=> Transmission delay: $T(s,c) = R(s,c)/R_T$

=> maximum delay constraint: $T_0 = R_0 / R_T$

Rate constraint: $R(s,c) \le R_0$ $\Leftrightarrow$ Delay constraint: $T(s,c) \le T_0$

The criterion:

minimize the total expected distortion:	$\min\limits_{s\in\mathbf{S},c\in\mathbf{C}} E[\ D(s,c)\ ]$	**(B)**
subject to the delay constraint:	$T(s,c) \le T_0$	

$\Downarrow$

Constrained optimization:

Minimize: $L(s,c,\lambda) = E[D(s,c)] + \lambda[T_0 - T(s,c)]$

=> $(s^*(\lambda), c^*(\lambda)) = \arg\min\limits_{s\in S, c\in C} L(s,c,\lambda)$

Example 1: Joint source-channel coding with possibility of retransmission

- Assume: up to A frames in the sender's buffer are eligible for retransmission.
- RS codec is used for channel coding:
  There are q different RS coding modes: $\mathrm{RS}(n_i, k), \quad i = 1, \cdots, q$
  Then: $C = \{ c_i = k / n_i \ , i = 1 \cdots q \}$
- For video source coding:
  S={prediction modes for MC (B,P prediction), quantization step size}
- Retransmission parameter of *k*-th packet in *n*-th frame: $\sigma_k^{(n)} \in \{0,1\}$
  retransmission parameter vector for n-th frame: $\sigma^{(n)} = \{\sigma_k^{(n)}, k = 1, \cdots M\}$
  Parameter vector for retransmission: $\{\sigma^{(j)}, j = n - A, \cdots, n\}$
- For simplicity, the time delay constraint is T_0 obtained from the rate controler

Problem:

Formulate the criterion that minimize the expected distortion.

Solution (mathematical formulation):

denote $\sigma_k^{(n)} = 0$: no re-transmission for *k*th packet at *n*th frame

$$\min_{S,C} \sum_{i=0}^{A} E\left[D^{(n-i)}\right] = \min_{S,C} \left(E\left[D^{(n)}\right] + \sum_{i=1}^{A} E\left[D^{(n-i)}\right] \right)$$

$$= \min_{S,C} \left(\sum_k E\left[(1-\sigma_k^{(n)}) D_k^{(n)}(s,c)\right] + \sum_{i=1}^{A} \sum_k E\left[\sigma_k^{(n-i)} D_k^{(n-i)}(s,c)\right] \right)$$

s.t.: $$\sum_{i=1}^{A} \sum_k \sigma_k^{(n-i)} T_k^{(n-i)} + \sum_k T_k^{(n)} \le T_0$$

(where $T_k^{(j)}$ Denotes the *k*-th packet delay at *j*-th frame)

$$L(S,C,\lambda) =$$

$$\sum_k E\left[(1-\sigma_k^{(n)})D_k^{(n)}\left(s,c\right)\right] + \sum_{i=1}^{A}\sum_k E\left[\sigma_k^{(n-i)}D_k^{(n-i)}(s,c)\right]$$

$$+\lambda\left[T_0 - \sum_{i=1}^{A}\sum_k \sigma_k^{(n-i)}T_k^{(n-i)} - \sum_k T_k^{(n)}\right]$$

$$\min_{S,C,\lambda} L(S,C,\lambda), \qquad \lambda > 0$$

3. Resource-distortion optimization for end-to-end video optimization

More general: Joint design of error-resilient source coding, cross-layer resource allocation, and error concealment.

Let: $\Bbbk_0$ the maximum allowed total cost
$\Bbbk$ the set of network adaptation parameters

The criterion:

minimize the total expected distortion:	$\min_{s\in\mathbf{S},c\in\mathbf{C}} E\ [D(s,c)]$	
subject to: the delay constraint	$T(s,c) \le T_0$	**(C)**
the cost constaint	$\Bbbk(s,c) \le \Bbbk_0$	

$$\Downarrow$$

Constrained optimization:

Minimize:

$$L(s,c,\lambda) \;=\; E[D(s,c)] + \lambda_1[T_0 - T(s,c)] + \lambda_2[\Bbbk_0 - \Bbbk(s,c)]$$

$$=> \quad (s^*(\lambda), c^*(\lambda)) = \arg\min_{s\in S, c\in C} L(s,c,\lambda)$$

4. Other ways of formulations: Utility-Cost-Based formulation

- **Define: A Set of Utility Functions (U):**
  represents: connections, preferences, ...
- **Define A Set of Cost Functions** $C=\{C_j, j=1,\cdots,q\}$ **:**

represents: services
(e.g. flow constraint in network layer, rate constraint in the link layer, mobile power constraint ...)
provider's preferecens (fairness to all users, but could be constraints to individual customers)

- **Solution: objective function**

Maximize the sum of untility functions : $\sum_{i,k} U_{ik};$

Subject to constraints of cost functions: $C_j, j=1,\cdots,q$