Fading Channels: Capacity, BER and Diversity

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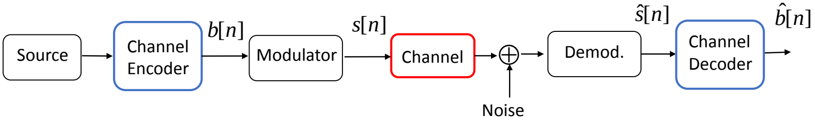
Contents

Introduction

- We have seen that the randomness of signal attenuation (**fading**) is the main challenge of wireless communication systems
- In this lecture, we will discuss how fading affects
  1. The capacity of the channel
  2. The Bit Error Rate (BER)
- We will also study how this channel randomness can be used or exploited to improve performance $\rightarrow$ **diversity**

General communication system model



- The channel encoder (FEC, convolutional, turbo, LDPC ...) adds redundancy to protect the source against errors introduced by the channel
- The capacity depends on the fading model of the channel (constant channel, ergodic/block fading), as well as on the channel state information (CSI) available at the Tx/Rx

Let us start reviewing the Additive White Gaussian Noise (AWGN) channel: no fading

AWGN Channel

- Let us consider a discrete-time AWGN channel

$$y[n] = hs[n] + r[n]$$

  where $r[n]$ is the additive white Gaussian noise, $s[n]$ is the transmitted signal and $h = |h|e^{j\theta}$ is the complex channel
- The channel is assumed **constant** during the reception of the whole transmitted sequence
- The channel is **known** to the Rx (coherent detector)
- The noise is white and Gaussian with power spectral density $N_0/2$ (with units W/Hz or dBm/Hz, for instance)
- We will mainly consider passband modulations (BPSK, QPSK, M-PSK, M-QAM), for which if the bandwith of the lowpass signal is W the bandwidth of the passband signal is $2W$

Signal-to-Noise-Ratio

- Transmitted signal power: P
- Received signal power: $P|h|^2$
- Total noise power: $N = 2WN_0/2 = WN_0$
- The received SNR is

$$\mathrm{SNR} = \gamma = \frac{P|h|^2}{WN_0}$$

- In terms of the energy per symbol $P = E_s/T_s$
- We assume Nyquist pulses with $\beta = 1$ (roll-off factor) so $W = 1/T_s$
- Under these conditions

$$\mathrm{SNR} = \frac{E_s T_s |h|^2}{T_s N_0} = \frac{E_s |h|^2}{N_0}$$

- For M-ary modulations, the energy per bit is

$$E_b = \frac{E_s}{\log(M)} \quad \Rightarrow \quad \mathrm{SNR} = \frac{E_b|h|^2}{N_0 \log(M)}$$

- To model the noise we generate zero-mean circular complex Gaussian random variables with $\sigma^2 = N_0$

$$r[n] \sim CN(0, \sigma^2) \quad \text{or} \quad r[n] \sim CN(0, N_0)$$

- The real and imaginary parts have variance

$$\sigma_I^2 = \sigma_Q^2 = \frac{\sigma^2}{2} = \frac{N_0}{2}$$

Average SNR and Instantaneous SNR

Sometimes, we will find useful to distinguish between the average and the instantaneous signal-to-noise ratio

$$\text{Average} \Rightarrow \mathrm{SNR} = \bar{\gamma} = \frac{P}{WN_0}$$

$$\text{Instantaneous} \Rightarrow \mathrm{SNR} = \bar{\gamma}|h|^2 = \frac{P}{WN_0}|h|^2$$

- **AWGN channel**: N_0 and h are assumed to be known (coherent detection), therefore we typically take (w.l.o.g) $h = 1 \Rightarrow \mathrm{SNR} = \bar{\gamma} = \frac{P}{WN_0}$, and the SNR at the receiver is constant
- **Fading channels**: The instantaneous signal-to-noise-ratio $\mathrm{SNR} = \gamma = \bar{\gamma}|h|^2$ is a random variable

Capacity AWGN Channel

- Let us consider a discrete-time AWGN channel

$$y[n] = hs[n] + r[n]$$

  where $r[n]$ is the additive white Gaussian noise, $s[n]$ is the transmitted symbol and $h = |h|e^{j\theta}$ is the complex channel
- The channel is assumed constant during the reception of the whole transmitted sequence
- The channel is known to the Rx (coherent detector)

The capacity (in bits/seg or bps) is given by the well-known Shannon's formula

$$C = W \log (1 + \mathrm{SNR}) = W \log (1 + \gamma)$$

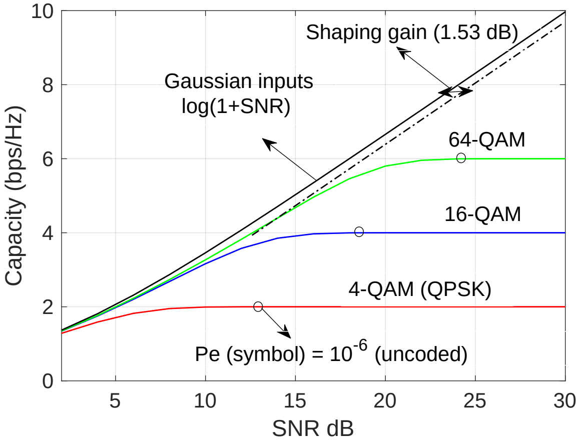
where W is the channel bandwidth, and $\mathrm{SNR} = \frac{P|h|^2}{WN_0}$; with P being the transmit power, $|h|^2$ the power channel gain and $N_0/2$ the power spectral density (PSD) of the noise

- Sometimes, we will find useful to express C in bps/Hz (or bits/channel use)

$$C = \log\left(1 + \mathrm{SNR}\right)$$

A few things to recall

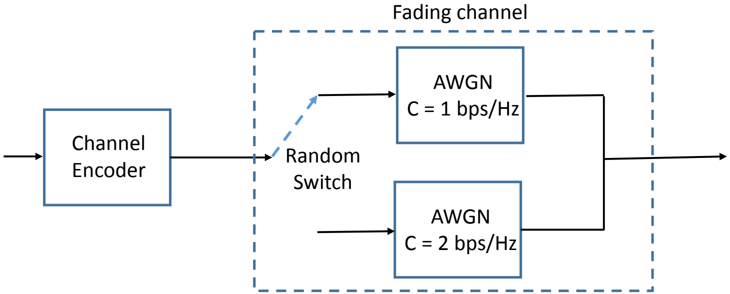
1. Shannon's coding theorem proves that a code exists that achieves data rates arbitrarily close to capacity with vanishingly small probability of bit error
   - The codewords might be very long (delay)
   - Practical (delay-constrained) codes only approach capacity
2. Shannon's coding theorem assumes Gaussian codewords, but digital communication systems use discrete modulations (PSK,16-QAM)



For discrete modulations, with increasing SNR we should increase the constellation size M (or use a less powerful channel encoder) to increase the rate and hence approach capacity: **Adaptive modulation and coding** (more on this later)

Capacity fading channels

- Let us consider the following example[1]



- The channel encoder is fixed
- The switch takes both positions with equal probability
- We wish to transmit a codeword formed by a long sequence of coded bits, which are then mapped to symbols
- What is the capacity for this channel?

[1] Taken from E. Biglieri, *Coding for Wireless Communications*, Springer, 2005

The answer depends on the rate of change of the fading

1. If the switch changes position every symbol period, the codeword experiences both channels with equal probabilities so we could use a fixed channel encoder, and transmit at a maximum rate

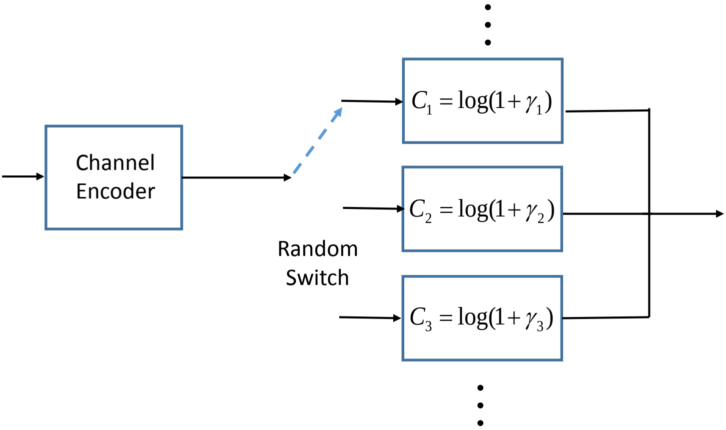
$$C = \frac{1}{2}C_1 + \frac{1}{2}C_2 = 1.5\, bps/Hz$$

2. If the switch remains fixed at the same (unknown) position during the transmission of the whole codeword, then we should use a fixed channel encoder, and transmit at a maximum rate

$$C = C_1 = 1\, bps/Hz$$

   or otherwise half of the codewords would be lost

What would be the capacity if the switch chooses from a continuum of AWGN channels whose SNR, $\gamma = \frac{P|h|^2}{WN_0}$, $0 \leq \gamma < \infty$, follows an exponential distribution (**Rayleigh channel**)



Fast fading (ergodic) channel

- If the switch changes position every symbol period, and the codeword is long enough so that the transmitted symbols experience all states of the channel (fast fading or ergodic channel)

$$C = E\left[\log(1+\gamma)\right] = \int_0^\infty \log(1+\gamma) f(\gamma) d\gamma \qquad (1)$$

where $f(\gamma) = \frac{1}{\overline{\gamma}} e^{-\gamma/\overline{\gamma}}$, with $\overline{\gamma}$ being the average SNR

- Eq. (1) is the **ergodic capacity** of the fading channel
- For Rayleigh channels the integral in (1) is given by

$$C = \frac{1}{2\ln(2)} \exp\left(\frac{1}{\overline{\gamma}}\right) E_1\left(\frac{1}{\overline{\gamma}}\right)$$

where $E_1(x) = \int_x^\infty \frac{e^-t}{t} dt$ is the Exponential integral function[2]

[2] y=expint(x) in Matlab

- The ergodic capacity is in general hard to compute in closed form (we can always resort to numerical integration)
- We can apply Jensen's inequality to gain some qualitative insight into the effect of fast fading on capacity

Jensen's inequality

If $f(x)$ is a convex function and X is a random variable

$$E[f(X)] \geq f(E[X])$$

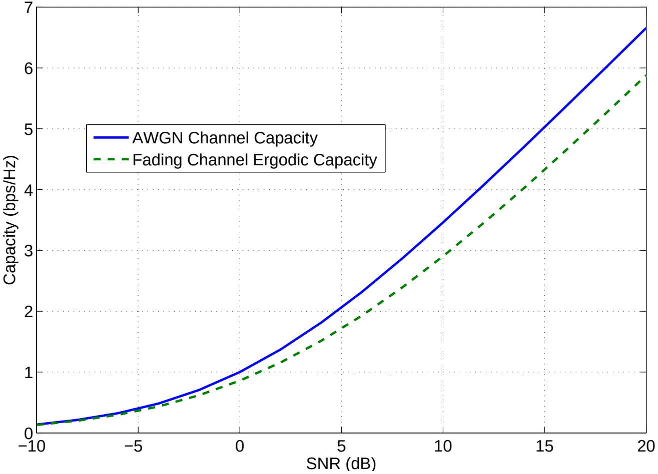
If $f(x)$ is a concave function and X is a random variable

$$E[f(X)] \leq f(E[X])$$

- $\log(\cdot)$ is a concave function, therefore

$$C = E[\log(1+\gamma)] \leq \log(1 + E[\gamma]) = \log(1+\overline{\gamma})$$

The capacity of a fading channel with receiver CSI is less than the capacity of an AWGN channel with the same average SNR



Block fading channel

- If the switch remains fixed at the same (unknown) position during the transmission of the whole codeword, there is no nonzero rate at which long codewords can be transmitted with vanishingly small probability or error
- Strictly, the capacity for this channel model would be zero
- In this situation, it is more useful to define the **outage capacity**

$$C_{out} = r \Rightarrow Pr(\log(1+\gamma) < r) = Pr(C(h) < r) = P_{out}$$

Suppose we transmit at a rate C_{out} with probability of outage $P_{out} = 0.01$ this means that with probability 0.99 the instantaneous capacity of the channel (a realization of a r.v.) will be larger than rate and the transmission will be successful; whereas with probability 0.01 the instantaneous capacity will be lower than the rate and the all bits in the codeword will be decoded incorrectly

- P_{out} is the error probability since data is only correctly received on $1 - P_{out}$ transmissions
- What is the outage probability of a block fading Rayleigh channel with average $\mathrm{SNR} = \overline{\gamma}$ for a transmission rate r?

$$P_{out} = Pr(\log(1+\gamma) < r)$$

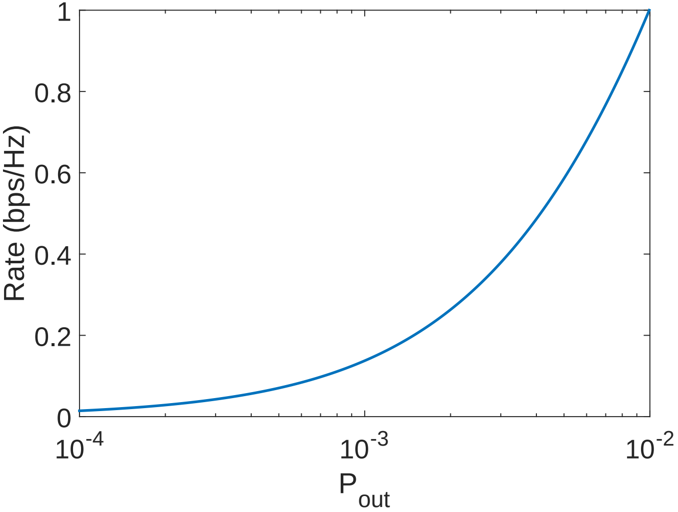
- The rate is a monotonic increasing function with the $\mathrm{SNR} = \gamma$; therefore, there is a γ_{min} needed to achieve the rate

$$\log(1+\gamma_{min}) = r \quad \Rightarrow \quad \gamma_{min} = 2^r - 1$$

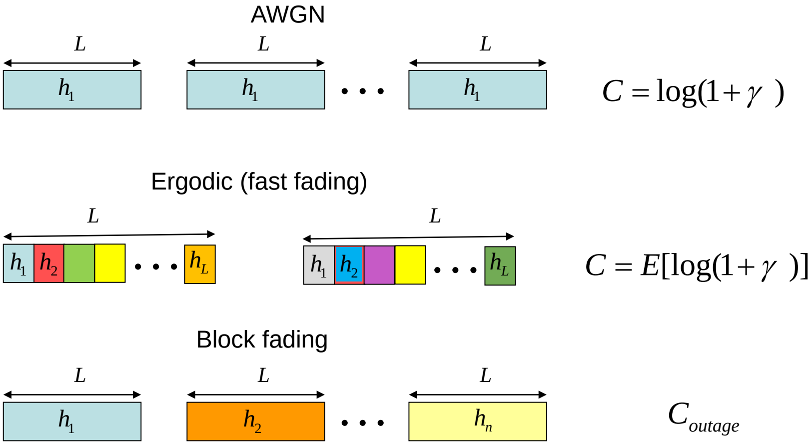
and P_{out} can be obtained as

$$\begin{aligned} P_{out} =& Pr(\gamma < \gamma_{min}) = \int_0^{\gamma_{min}} f(\gamma) d\gamma = \int_0^{\gamma_{min}} \frac{1}{\overline{\gamma}} e^{-\gamma/\overline{\gamma}} d\gamma = \\ =& 1 - e^{-\frac{\gamma_{min}}{\overline{\gamma}}} = 1 - e^{-\frac{2^r-1}{\overline{\gamma}}} \end{aligned}$$

P_{out} vs C_{out} (rate) for $\overline{\gamma} = 100$ (10 dB)



Summary


AWGN
L
h_1
$C = \log(1+\gamma)$
Ergodic (fast fading)
L
h_1 h_2 h_L
$C = E[\log(1+\gamma)]$
Block fading
L
h_1 h_2 h_n
C_{outage}

Final note

- All capacity results seen so far correspond to the case of perfect CSI at the receiver side (CSIR)
- If the transmitter also knows the channel, we can perform power adaptation and the results are different
- We'll see more on this later

Now, let's move to analyze the impact of fading on the Bit Error Rate (BER)

BER analysis for the AWGN channel (a brief reminder)

- **BPSK** $s[n] \in \{-1, +1\}$

$$P_s = P_b = Q\left(\sqrt{\frac{2E_s}{N_0}}\right) = Q\left(\sqrt{2\mathrm{SNR}}\right)$$

where $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} \exp -(x^2/2) \leq \frac{1}{2} \exp -(x^2/2)$

- **QPSK** $s[n] \in \{\frac{-1-j}{\sqrt{2}}, \frac{-1+j}{\sqrt{2}}, \frac{1-j}{\sqrt{2}}, \frac{1+j}{\sqrt{2}}\}$

$$P_s = 1 - \left(1 - Q\left(\sqrt{\mathrm{SNR}}\right)\right)^2$$

  - Nearest neighbor approximation $P_s \approx 2Q\left(\sqrt{\mathrm{SNR}}\right)$
  - With Gray encoding $P_e = P_s/2$

- **M-PAM** constellation $A_i = (2i - 1 - M)d/2$, $i = 1, \ldots, M$
  - Distance between neighbors: d
  - Average symbol energy

$$E_s = \frac{1}{3}(M^2 - 1)\left(\frac{d}{2}\right)^2 = \frac{(M^2 - 1)d^2}{12}$$

  - Symbol Error Rate

$$\begin{aligned} P_s &= \frac{M-2}{M} 2Q\left(\frac{d}{2\sigma_r}\right) + \frac{2}{M} Q\left(\frac{d}{2\sigma_r}\right) \\ &= \frac{2(M-1)}{M} Q\left(\sqrt{\frac{6\,\mathrm{SNR}}{M^2 - 1}}\right) \end{aligned}$$

  - With Gray encoding $P_e \approx P_s/M$

- **M-QAM**: the constellations for the I and Q branches are $A_i = (2i - 1 - M)d/2,\ i = 1, \ldots, M$
- At the I and Q branches we have orthogonal $\sqrt{M} - PAM$ signals, each with half the SNR
- Symbol Error Rate

$$P_s = 1 - \left(1 - \frac{2(\sqrt{M} - 1)}{\sqrt{M}} Q\left(\sqrt{\frac{3\,\mathrm{SNR}}{M-1}}\right)\right)^2$$

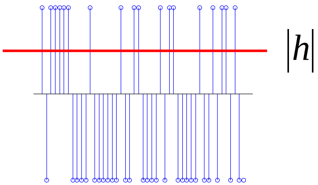
- Nearest neighbor approximation (4 nearest neighbors)

$$P_s \approx 4Q\left(\sqrt{\frac{3\,\mathrm{SNR}}{M-1}}\right)$$

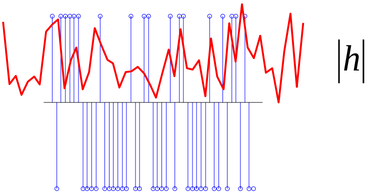
The impact of fading on the BER

- Let us consider for simplicity a SISO channel with a BPSK source signal
$$x[n] = hs[n] + r[n]$$
$s[n] \in \{+1, -1\}$, $r[n] \sim CN(0, \sigma^2)$ is the noise (additive, white and Gaussian)
- An AWGN channel or a fading channel behave also differently in terms of BER

AWGN channel: h is constant



Rayleigh fading channel: h ~ CN(0,1)



- The receiver knows perfectly the channel $\rightarrow$ coherent det.
- Since we are sending a BPSK signal over a complex channel the optimal coherent detector is

$$Re\left(\frac{h^* x[n]}{|h|}\right) \underset{-1}{\overset{+1}{\gtrless}} 0 \qquad (2)$$

- Assuming that the transmitted power is normalized to unity $E[|s[n]|^2] = 1$, the average SNR is

$$\mathrm{SNR} = \frac{1}{\sigma^2}$$

and the instantaneous Signal-to-Noise-Ratio is $\mathrm{SNR}|h|^2$
- For an AWGN channel, $\mathrm{SNR}|h|^2$ is a constant value and the BER is

$$P_e = Q\left(\frac{\sqrt{2}|h|}{\sigma}\right) = Q\left(\sqrt{2|h|^2\mathrm{SNR}}\right)$$

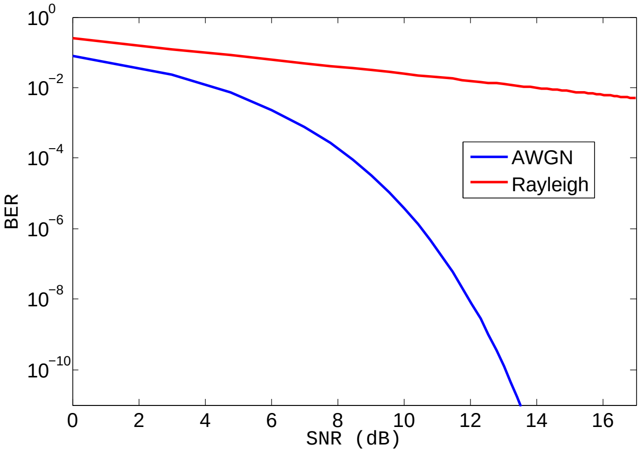
where $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} \exp -(x^2/2) \leq \frac{1}{2} \exp -(x^2/2)$

- For a fading channel $\mathrm{SNR}|h|^2$ is now a random variable
- We can therefore think of $P_e = Q\left(\sqrt{2|h|^2\mathrm{SNR}}\right)$ as another random variable
- The BER for the fading channel would be the average over the channel distribution $E[P_e]$
- This applies to a fast fading channel model

BER for a Rayleigh channel

$|h|$ is now a Rayleigh random variable ($z = |h|^2$ is exponential with pdf $f(z) = \exp(-z)$, the BER is given by

$$P_e = \int_0^\infty Q\left(\sqrt{2z\mathrm{SNR}}\right) e^{-z} dz = \frac{1}{2}\left(1 - \sqrt{\frac{\mathrm{SNR}}{1+\mathrm{SNR}}}\right) \approx \frac{1}{4\mathrm{SNR}}$$



At an error probability of $P_e = 10^{-3}$, a Rayleigh fading channel needs 17 dB more than an AWGN (constant) channel, even though we have perfect channel knowledge in both cases !!

- The main reason of the poor performance is that there is a significant probability that the channel is in a deep fade, and not because of noise
- The instantaneous signal-to-noise-ratio is $|h|^2\mathrm{SNR}$, and we may consider that the channel is in a deep fade when $|h|^2\mathrm{SNR} < 1$
  - In this event, the separation between constellation points will be of the order of magnitude of σ (noise standard deviation) and the probability of error will be high
- For a Rayleigh channel, the probability of being in a deep fade is

$$Pr(|h|^2\mathrm{SNR} < 1) = \int_0^{1/\mathrm{SNR}} e^{-z} dz = 1 - e^{-1/\mathrm{SNR}} =$$

$$= 1 - \left(1 - \frac{1}{\mathrm{SNR}} + \left(\frac{1}{\mathrm{SNR}}\right)^2 - \dots\right) \approx \frac{1}{\mathrm{SNR}}$$

which is of the same order as P_e

Remarks

- Although our example considered a BPSK modulation, the same result is obtained for other modulations

On the Rayleigh channel, all modulation schemes are equally bad and for all them the BER decreases as SNR^{-1}

- The gap between the AWGN and the Rayleigh channel in terms of BER is much higher than in terms of capacity

This suggests that coding can be very beneficial to compensate fading

Diversity

- The negative slope (at high SNR) of the BER curve with respect to the SNR is called the **diversity gain** or **diversity order**

$$d = -\lim_{\mathrm{SNR}\to\infty} \frac{\log(P_e(\mathrm{SNR}))}{\log(\mathrm{SNR})}$$

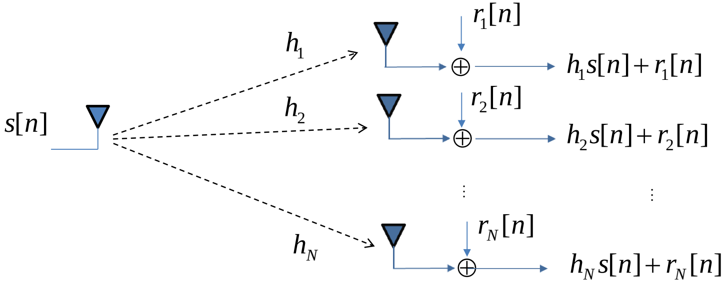
- In other words, in a system with diversity order d, the P_e at high SNR varies as

$$P_e = \mathrm{SNR}^{-d}$$

- For a SISO Rayleigh channel the diversity order is 1 (remember that $P_e \approx 1/(4\mathrm{SNR}) \propto \mathrm{SNR}^{-1}$!!

SIMO system

Consider now that the Rx has N antennas and that the resulting SIMO (single-input multiple-output) channel is spatially uncorrelated



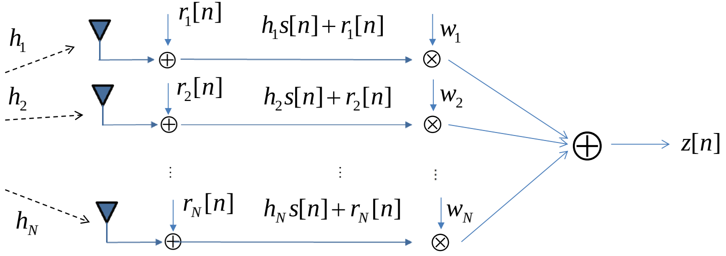
If the channels h_i are faded independently, the probability that the SIMO channel is in a deep fade will be approximately

$$Pr((|h_1|^2\mathrm{SNR} < 1)\& \ldots \&(|h_N|^2\mathrm{SNR} < 1)) = \left(Pr((|h|^2\mathrm{SNR} < 1)\right)^N \approx \frac{1}{\mathrm{SNR}^N}$$

- In consequence, the diversity order of a SIMO system with N uncorrelated receive antennas is N
  - Notice the impact of antenna correlation: if all antennas are very close to each other so that the channels become fully correlated, then $h_1 \approx h_2 \approx h_N$, and the diversity reduces to that of a SISO channel
- There are different ways to process the receive vector and extract all spatial diversity of the SIMO channel
- Assuming that the channel $\mathbf{h} = (h_1, h_2, \ldots, h_N)^T$ is known at the Rx, the optimal scheme called **maximum ratio combining (MRC)**

Maximum Ratio Combining

The MRC schemes linearly combines (with optimal weights w_i) the outputs of the signals received at each antenna



The signal $z[n]$ can be written in matrix form as

$$z[n] = \begin{pmatrix} w_1 & w_2 & \dots & w_N \end{pmatrix} \left(\begin{pmatrix} h_1 \\ h_2 \\ \vdots \\ h_N \end{pmatrix} s[n] + \begin{pmatrix} r_1[n] \\ r_2[n] \\ \vdots \\ r_N[n] \end{pmatrix} \right) = \mathbf{w}^T \left(\mathbf{h} s[n] + \mathbf{r}[n] \right)$$

- As long as $||\mathbf{w}|| = 1$ the noise distribution after combining does not change: i.e., $\mathbf{w}^T\mathbf{r}[n] \sim CN(0, \sigma^2)$
- It is easy to show that the optimal weights are given by

$$\mathbf{w} = \frac{\mathbf{h}^*}{||\mathbf{h}||},$$

which is just the matched filter for this problem!

- These are the optimal weights because they maximize the signal-to-noise-ratio at the output of the combiner:

$$\mathrm{SNR}_{MRC} = \frac{||h||^2}{\sigma^2} = ||h||^2\mathrm{SNR}$$

- For a BPSK transmitted signal, the optimal detector is

$$Re(z[n]) = Re\left(\frac{\mathbf{h}^H\mathbf{x}[n]}{||\mathbf{h}||}\right) \underset{-1}{\overset{+1}{\gtrless}} 0$$

where $(\cdot)^H$ denotes Hermitian (complex conjugate and transpose). Notice the similarity with the optimal coherent detector for the SISO case, which was given by (2)

- Similarly to the SISO case, the P_e can be derived exactly

$$P_e = Q\left(\sqrt{2||\mathbf{h}||^2\mathrm{SNR}}\right),$$

which is a random variable because **h** is random (fading channel)

- Under Rayleigh fading, each h_i is i.i.d $CN(0,1)$ and

$$x = ||\mathbf{h}||^2 = \sum_{i=1}^{N} |h_i|^2$$

follows a **Chi-square distribution with $2N$ degrees of freedom**

$$f(x) = \frac{1}{(N-1)!}x^{N-1}\exp(-x), \qquad x \geq 0$$

  - Note that the exponential distribution (SISO case with $N = 1$) is a Chi-square with 2 degrees of freedom

- The average error probability can be explicitly computed, here we only provide a high-SNR approximation

$$P_e = \int_0^{\infty} Q\left(\sqrt{2x\mathrm{SNR}}\right) f(x)dx \approx \binom{2N-1}{N}\frac{1}{(4\mathrm{SNR})^N}$$

The probability of being in a deep fade with MRC is

$$Pr(||\mathbf{h}||^2 \mathrm{SNR} < 1) = \int_0^{1/\mathrm{SNR}} \frac{1}{(N-1)!} x^{N-1} \exp(-x) dx \approx$$

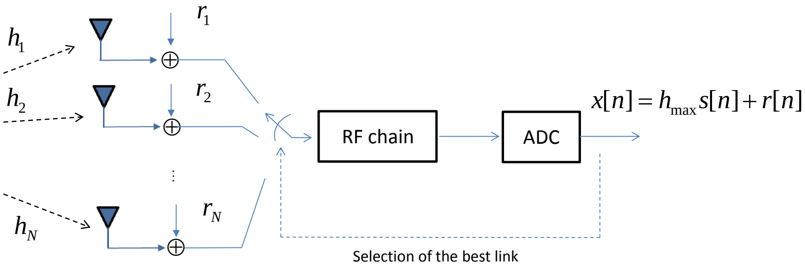
$$\text{(for } x \text{ small)} \qquad \approx \int_0^{1/\mathrm{SNR}} \frac{1}{(N-1)!} x^{N-1} dx = \frac{1}{N! \mathrm{SNR}^N}$$

Conclusions

- The P_e is again dominated by the probability of being in a deep fade
- Both (the P_e and the probability of being in a deep fade) decrease with the number of antennas roughly as $(\mathrm{SNR})^{-N}$
- MRC extracts all spatial diversity of the SIMO channel !!

However, MRC is not the only multiantenna technique that extracts all spatial diversity of a SIMO channel

Antenna selection



- In antenna selection the path with the highest SNR is selected and processed: $x[n] = h_{max} s[n] + r[n]$, where

$$h_{max} = \max\left(|h_1|, |h_2|, \ldots, |h_N|\right)$$

- In comparison to MRC, only a single RF chain is needed !!

- Intuitively, the probability that the best channel is in a deep fade is

$$Pr(|h_{max}|^2 \mathrm{SNR} < 1) = \prod_{i=1}^{N} Pr((|h_i|^2 \mathrm{SNR} < 1) \approx \frac{1}{\mathrm{SNR}^N}$$

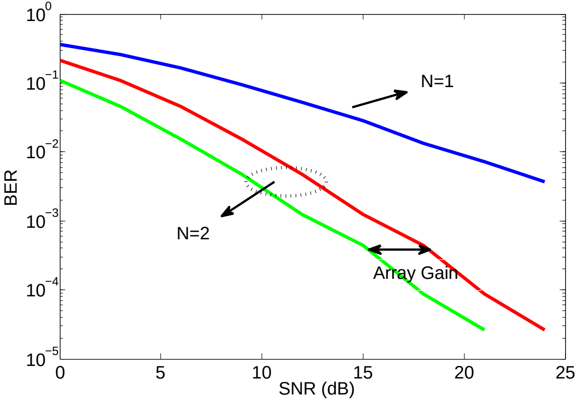
and, therefore, the diversity of antenna selection is also N

- The same conclusion can be reached by studying the P_e (we would need the distribution of $|h_{max}|^2$)
- However, the output SNR of MRC is higher than that of antenna selection (AS)

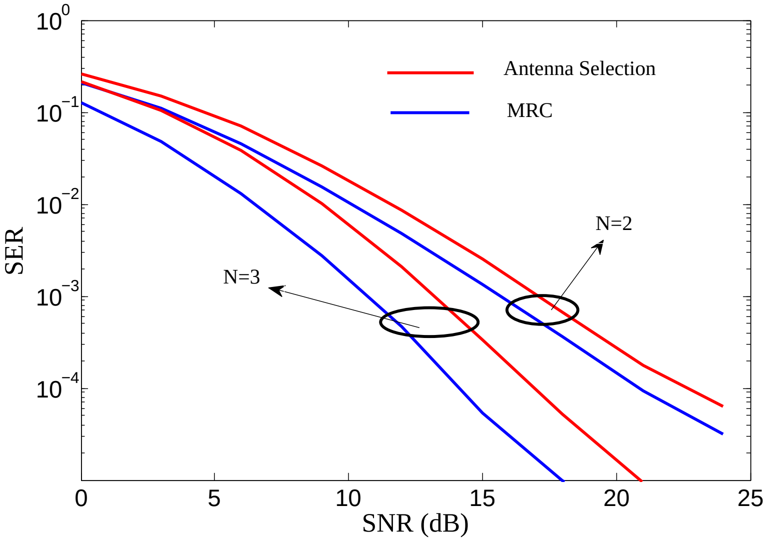
$$\begin{aligned} \mathrm{SNR}_{MRC} &= \frac{|h_1|^2}{\sigma^2} + \ldots + \frac{|h_N|^2}{\sigma^2} \\ \mathrm{SNR}_{AS} &= \frac{|h_{max}|^2}{\sigma^2} \end{aligned}$$

Array gain

- The SNR increase in a multiantenna system with respect to that of a SISO system is called **array gain**
- Whereas the diversity gain is reflected in slope of the BER vs. SNR, the array gain provokes a shift to the left in the curve

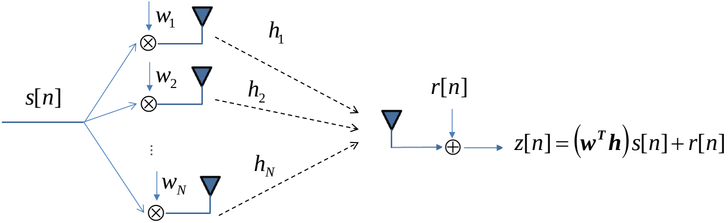


- For a receiver with N antennas, MRC is optimal because it achieves maximum spatial diversity and maximum array gain
- MRC achieves the maximum array gain by coherently combining all signal paths
- On average, the output SNR of MRC is N times that of a SISO system, so its array gain is $10 \log_{10} N$ (dBs)
- For instance, using 2 Rx antennas, MRC provides 3 extra dBs in comparison to a SISO system and in addition to the spatial diversity gain


Antenna Selection
MRC
N=2
N=3
SER
SNR (dB)
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0
5
10
15
20
25

Maximum Ratio Transmission

Similar concepts apply for a a MISO (multiple-input single-input) channel: the optimal scheme is now called **Maximum Ratio Transmission (MRT)**



Again, the optimal transmit beamformer that achieves full spatial diversity (N) and full array gain ($10 \log_{10} N$) is

$$\mathbf{w} = \frac{\mathbf{h}^*}{||\mathbf{h}||},$$

The main difference is that now the channel must be known at the Tx (through a feedback channel)

Spatial diversity of a MIMO channel

An $n_R \times n_T$ MIMO channel with i.i.d. entries has a spatial diversity $n_R n_T$, which is the number of independent paths offered to the source signal for going from the Tx to the Rx

- Some schemes extract the full spatial diversity of the MIMO channel
  - Optimal combining at both sides (MRT+MRC)
  - Antenna selection at both sides
  - Antenna selection at one side and optimal combining at the other side
  - Repetition coding at the Tx side (same symbol transmitted through all Tx antennas) + optimal combining at the Rx side
- But others might not: think of a system that transmits independent data streams over different transmit antennas (more on this later)

Time and frequency diversity

- We have mainly focused the discussion on the spatial diversity concept, but the same idea can be applied to the time and frequency domains
- **Time diversity**:
  1. The same symbol is transmitted over different time instants, t_1 and t_2
  2. For the channel to fade more or less independently: $|t_1 - t_2| > T_c = \frac{1}{D_s}$
- **Frequency diversity**:
  1. The same symbol is transmitted over different frequencies (subcarriers in OFDM), f_1 and f_2
  2. For the channel to fade more or less independently: $|f_1 - f_2| > B_c = \frac{1}{\tau_{rms}}$
- To achieve time or frequency diversity, **interleaving** is typically used

Interleaving

- The symbols or coded bits are dispersed over different coherence intervals (in time or frequency)
- A typical interleaver consists of an $P \times Q$ matrix: the input signals are written rowwise into the matrix and then read columnwise and transmitted (after modulation)
- Example: A 4×8 interleaving matrix

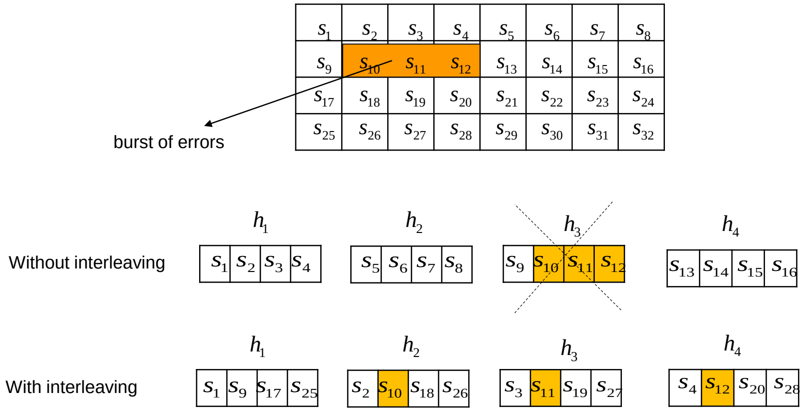
from source

$s_1 s_2 s_3 s_4 s_5 \cdots s_{28} s_{29} s_{30} s_{31} s_{32}$ $\longrightarrow$

s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8
s_9	s_{10}	s_{11}	s_{12}	s_{13}	s_{14}	s_{15}	s_{16}
s_{17}	s_{18}	s_{19}	s_{20}	s_{21}	s_{22}	s_{23}	s_{24}
s_{25}	s_{26}	s_{27}	s_{28}	s_{29}	s_{30}	s_{31}	s_{32}

to channel

$s_1 s_9 s_{17} s_{25} s_2 \cdots s_{31} s_8 s_{16} s_{24} s_{32}$ $\longrightarrow$


s1 s2 s3 s4 s5 s6 s7 s8
s9 s10 s11 s12 s13 s14 s15 s16
s17 s18 s19 s20 s21 s22 s23 s24
s25 s26 s27 s28 s29 s30 s31 s32
burst of errors
Without interleaving
h1 h2 h3 h4
s1 s2 s3 s4
s5 s6 s7 s8
s9 s10 s11 s12
s13 s14 s15 s16
With interleaving
h1 h2 h3 h4
s1 s9 s17 s25
s2 s10 s18 s26
s3 s11 s19 s27
s4 s12 s20 s28

Remarks:

- Doppler spreads in typical systems range from 1 to 100 Hz, corresponding roughly to coherence times from 0.01 to 1 sec.
- If transmissions rates range from 2.10^4 to 2.10^6 bps, this would imply that blocks of length L ranging from $L = 2.10^4 \times 0.01 = 200$ bits to $L = 2.10^6 \times 1 = 2.10^6$ bits would be affected by approximately the same fading gain
- Deep interleaving might be needed in some cases
- But notice that interleaving involves a delay proportional to the size of the interleaving matrix
- In delay-constrained systems (transmission of real-time speech) this might be difficult or even unfeasible

Conclusions

- We have analyzed the effect of fading from the point of view of capacity (only CSIR) and BER
- Depending of the channel model (ergodic or block fading) we use ergodic capacity or outage capacity
- With CSIR only, the capacity of a fading channel is always less than the capacity of an AWGN with the same average SNR
- The effect of fading is more significant in terms of BER than in terms of capacity (power of coding gain + interleaving)
- Diversity: slope of the BER wrt SNR at high SNRs
- Diversity techniques with multiantenna systems: MRT, MRC, antenna selection, etc
- We can extract the time and frequency diversity of the channel through interleaving