1. 复合求导

(1) 设 $u(x,y)$ 有连续的一阶偏导数，又设 $x=r\cos\theta$,$y=r\sin\theta$, 证明:

$$(\frac{\partial u}{\partial x})^2+(\frac{\partial u}{\partial y})^2=(\frac{\partial u}{\partial r})^2+\frac{1}{r^2}(\frac{\partial u}{\partial \theta})^2$$

(2) 设 $w=f(x,y,z),x=u+v,y=u-v,z=uv$, 求 $\frac{\partial w}{\partial u}$ 和 $\frac{\partial w}{\partial v}$
(3) 设 $w=f(x,u,v),u=g(y,z),v=h(x,y)$, 求 $\frac{\partial w}{\partial x},\frac{\partial w}{\partial y},\frac{\partial w}{\partial z}$

2. 隐函数求导

(1) 设 $z=z(x,y)$ 是由方程

$$f(x+\frac{z}{y},y+\frac{z}{x})=0$$

所确定的隐函数. 计算 $\frac{\partial z}{\partial x},\frac{\partial z}{\partial y}$
(2) 设 $z=z(x,y)$ 是由方程

$$\mathrm{e}^z-xyz=0$$

所确定的隐函数. 计算 $\frac{\partial z}{\partial x},\frac{\partial z}{\partial y}$
(3) 设 $F(x,y,z)=0$. 求证:$\frac{\partial x}{\partial y}\frac{\partial y}{\partial z}\frac{\partial z}{\partial x}=-1$
(4) 设 $x=x(u,v),y=y(u,v)$ 是由方程组

$$xu-yv=0,yu+xv=1$$

所确定的隐函数, 求 $\frac{\partial x}{\partial u},\frac{\partial y}{\partial u}$

3. 高阶微分

(1) 设 $u=f(r,r\cos\theta)$ 有二阶连续偏导数, 求 $\frac{\partial u}{\partial r},\frac{\partial u}{\partial \theta},\frac{\partial^2 u}{\partial r\partial\theta}$.(这个题极其容易做错)
(2) 设方程 $x^2+y^2+z^2=4z$ 确定 z 为 x,y 的函数, 求 $\frac{\partial^2 z}{\partial x^2},\frac{\partial^2 z}{\partial x\partial y}$

3.Taylor 公式

证明: 当 $|x|$ 和 $|y|$ 充分小时，有近似式

$$\frac{\cos x}{\cos y}\approx 1-\frac{1}{2}(x^2-y^2)$$

答案

1.(2) $\frac{\partial w}{\partial u}=\frac{\partial f}{\partial x}+\frac{\partial f}{\partial y}+v\frac{\partial f}{\partial z}$

$\frac{\partial w}{\partial v}=\frac{\partial f}{\partial x}-\frac{\partial f}{\partial y}+u\frac{\partial f}{\partial z}$

1.(3) $\frac{\partial w}{\partial x}=\frac{\partial f}{\partial x}+\frac{\partial f}{\partial v}\frac{\partial h}{\partial x}$

$\frac{\partial w}{\partial y}=\frac{\partial f}{\partial u}\frac{\partial g}{\partial y}+\frac{\partial f}{\partial v}\frac{\partial h}{\partial y}$

$\frac{\partial w}{\partial z}=\frac{\partial f}{\partial u}\frac{\partial g}{\partial z}$

2.(1)

$$\frac{\partial z}{\partial x}=\frac{\frac{z}{x^2}f_2'-f_1'}{\frac{f_1'}{y}+\frac{f_2'}{x}}$$

$$\frac{\partial z}{\partial y}=\frac{\frac{z}{y^2}f_1'-f_2'}{\frac{f_1'}{y}+\frac{f_2'}{x}}$$

2.(2) $\frac{\partial z}{\partial x}=\frac{yz}{\mathrm{e}^z-xy}$, $\frac{\partial z}{\partial y}=\frac{xz}{\mathrm{e}^z-xy}$

2.(4) $\frac{\partial x}{\partial u}=-\frac{ux+vy}{u^2+v^2}$, $\frac{\partial y}{\partial u}=\frac{vx-uy}{u^2+v^2}$

3.(1) $\frac{\partial u}{\partial r}=f_1'+f_2'\cos\theta$

$\frac{\partial u}{\partial \theta}=-rf_2'\sin\theta$

$\frac{\partial^2 u}{\partial r\partial\theta}=-rf_{12}''\sin\theta-f_2'\sin\theta-rf_{22}''\sin\theta\cos\theta$

3.(2) $\frac{\partial^2 z}{\partial x^2}=\frac{(2-z)^2+x^2}{(2-z)^3}$

$\frac{\partial^2 z}{\partial x\partial y}=\frac{xy}{(2-z)^3}$