Section 2: Graph Pattern Matching

UNSW
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Never Stand Still

Faculty of Engineering

Computer Science and Engineering

Outline of Section 2

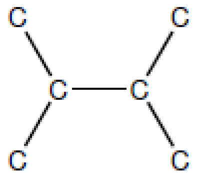
- Introduction
- Given a set of graphs and a pattern graph
  - ➢ **G-Index**
  - ➢ FG-Index (brief)
  - ➢ **QuickSI**
- All-matching
  - ➢ **TurboISO**
  - ➢ **CFL-Match**
- Distributed Algorithms
- Similarity All-matching

Introduction

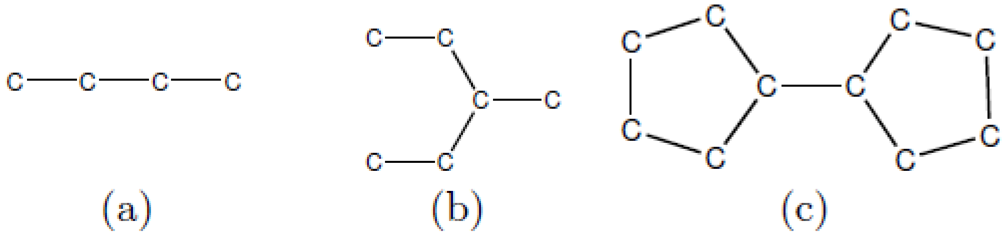
- **Graph Pattern Matching** is an important problem in the graph theory.
- Two categories:

  1. Graph Pattern Matching in Graph Database D

     Given a query pattern, find all graphs in the database D containing this pattern.



Query graph

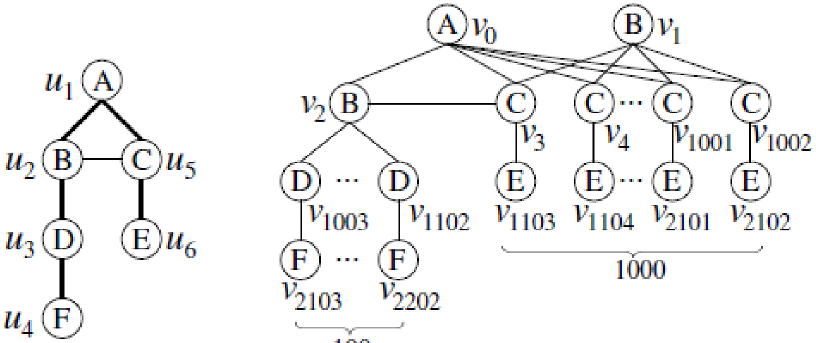


Graph Database D

Introduction

2. All-Matching

Given a query pattern, enumerate all subgraph embeddings of this pattern in the data graph G.



(a) Query q (b) Data graph G

Graph Pattern Matching in Graph Database

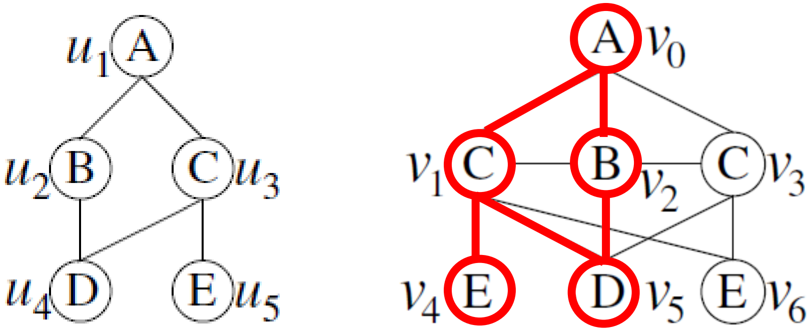
- Subgraph Isomorphism

An subgraph isomorphism is an injective function $f : V(g) \to V(g')$ such that

$$(1)\ \forall u \in V(g),\ l(u) = l'(f(u))$$

$$(2)\ \forall (u, v) \in E(g)$$

where l and l' are the label function of g and g', respectively.



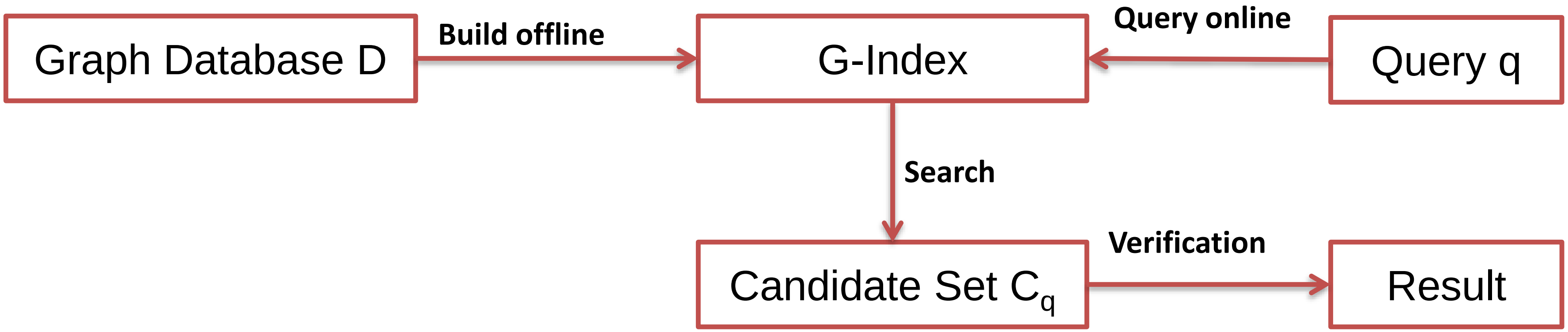
Graph Pattern Matching in Graph Database

- Naive Method
  - Verify all graphs in the graph database D for the given query
  - Infeasible: subgraph isomorphism testing is NP-complete.

- Index-based methods
  - G-Index
  - FG-Index
  - ……

G-Index Framework

- Overview of G-Index framework



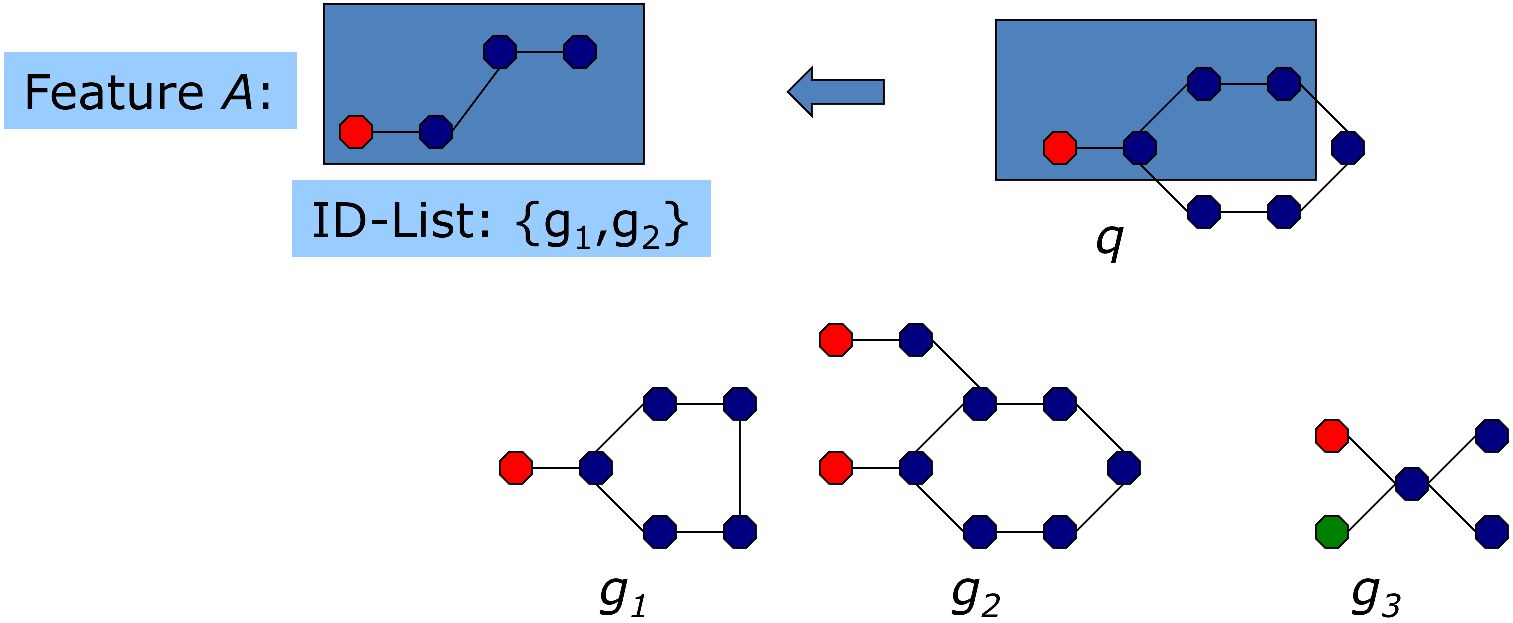
- Cost Analysis

$$\textbf{Query Response Time} = T_{search} + |C_q| * T_{iso\_test}$$

To improve the query response time, we need to minimize:

1) the size of graph feature set |F|
2) the size of candidate set C_q

gIndex Pruning



Feature A:

ID-List: $\{g_1,g_2\}$

q

g_1 g_2 g_3

Filtering:	**Pass** **(Feature A)**	**Pass** **(Feature A)**	**Pruned**
Verification:	**False Positive**	**Answer**	

Substructure Search: gIndex

- Index a set *F* of features from *D*.
- $\forall f \in F$, **D$_f$:** set of graph ids in *D* contain f

Filtering: $$C_q = \bigcap_{f \subseteq q \wedge f \in F} D_f$$

Verification: verify each data graph in C_q.

G-Index Overview

- Index Construction
  - build an inverted index on the graph feature set F

- Query Processing
  1) Search: query the index to compute the candidate set
  2) Verification: perform subgraph isomorphism test

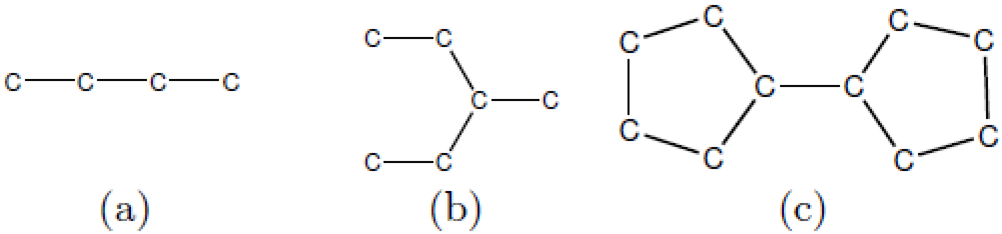
Frequent Fragment

- Frequency

  Given a graph database D, the *frequency* of g, denoted as

  $$\text{freq}(g) = |D_g|$$

  , is the number of graphs in D, which contain g as a subgraph.

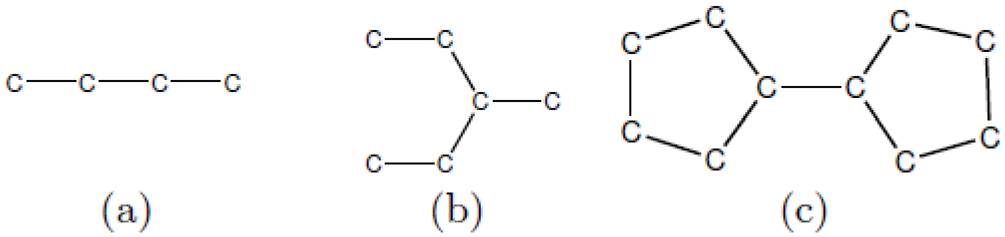


For the graph “c-c”, its frequency in the database consisting of the above three graphs, is 3.

Frequent Fragment

- Frequent graph

  A graph/pattern g is *frequent* if its occurrence frequency is no less then a minimum frequency threshold, *minFreq*.



(a) (b) (c)

If *minFreq* is set to 2, then pattern “c-c”, “c-c-c” and “c-c-c-c” are *frequent*.

Can you locate all frequent patterns with *minFreq*=2 in the above given database?

Frequent Fragment

- Uniform *minFreq* is infeasible
  - In a completely connected graph with 10 vertices, there are 45 1-edge subgraphs, 360 2-edge ones and more than 1,1814,400 8-edge ones.

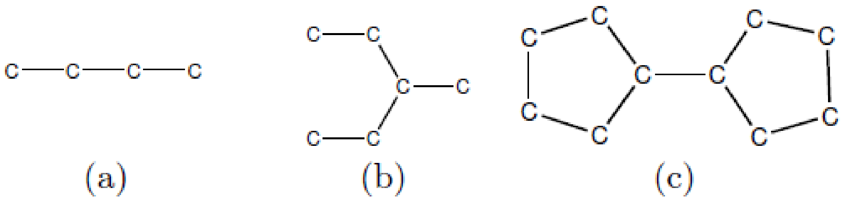
- It's more appropriate to have
  - Low minimum frequency (threshold) on small fragments (for effectiveness)
  - High minimum frequency (threshold) on large fragments (for compactness)

- Size-Increasing Frequency
  - Pattern g is *frequent* if and only if freq(g) ≥ f (size (g)), where freq is the occurrence frequency and f (x) is an increasing function.

Discriminative Fragment

- Do we need to index every frequent fragment?

  All the graphs in the sample database contain carbon-chains: c, c-c, c-c-c, and c-c-c-c. Fragments c-c, c-c-c, and c-c-c-c do not provide more indexing power than fragment c. Thus, they are useless for indexing.



- Redundant fragment
  - Fragment x is redundant with respect to feature set F if

$$D_x \approx \bigcap_{f \in F \wedge f \subset x} D_f$$

  - c-c, c-c-c, and c-c-c-c are redundant in the above example.

Discriminative Fragment

- Discriminative fragment
  - Fragment x is discriminative with respect to feature set F if

$$D_x << \bigcap_{f \in F \wedge f \subset x} D_f$$

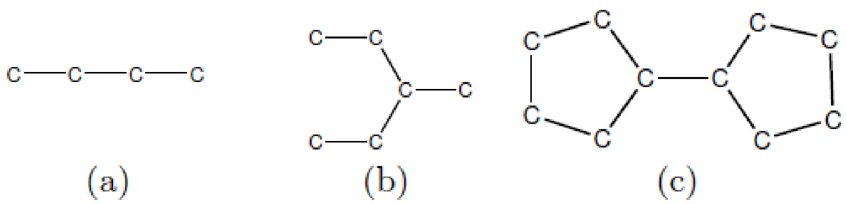
  - In the previous example, graph (a), graph (b) and the carbon ring in graph (c) are discriminative fragments.
- Discriminative ratio

$$\gamma = \frac{|\bigcap_i D_{f_{\varphi_i}}|}{|D_x|}$$

where D_x is the set of graphs containing x and
$\bigcap_i D_{f_{\varphi_i}}$ is the set of graphs which contain the proper subgraphs of x in the feature set

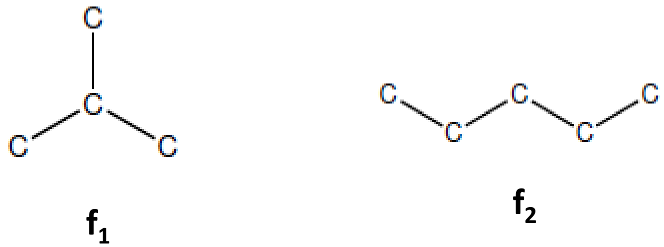
Discriminative Fragment

- Use γ_{min} to mine discriminative fragments
- Example with γ_{min} = 1.5



(a) (b) (c)

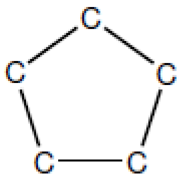
Graph database D



f_1 f_2

Current feature set F

Both f_1 and f_2 contain the subgraphs of this pattern

Given  , its discriminative ratio is $\frac{2}{1} = 2 > \gamma_{min}$

Thus, we add it into the feature set F.

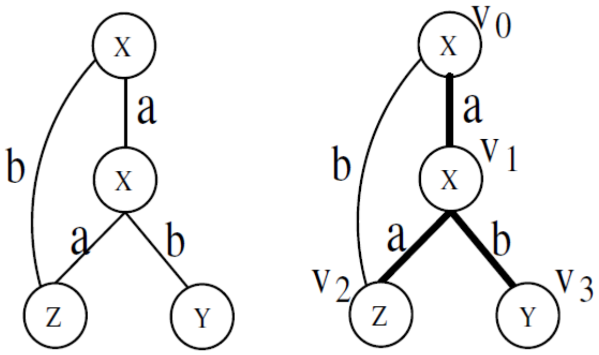
Only graph(c) contains this pattern

G-Index Construction

- Once discriminative fragments are selected, we construct G-Index using the following two steps:
  - Graph Sequentialization
    - Transfer each discriminative fragment into a sequence
  - Build G-Index Tree
    - Store all sequences of discriminative fragments into a prefix tree

Graph Sequentialization

- DFS Code Generation
  - DFS Coding translates a graph into an unique edge sequence, by performing a depth first search (DFS) in a graph.



(a) (b)

Bold edges are the edges of DFS tree originated from the node v_0.

The DFS Code corresponding to the DFS tree in (b) is $\langle (v_0, v_1), (v_1, v_2), (v_2, v_0), (v_1, v_3) \rangle$

Graph Sequentialization

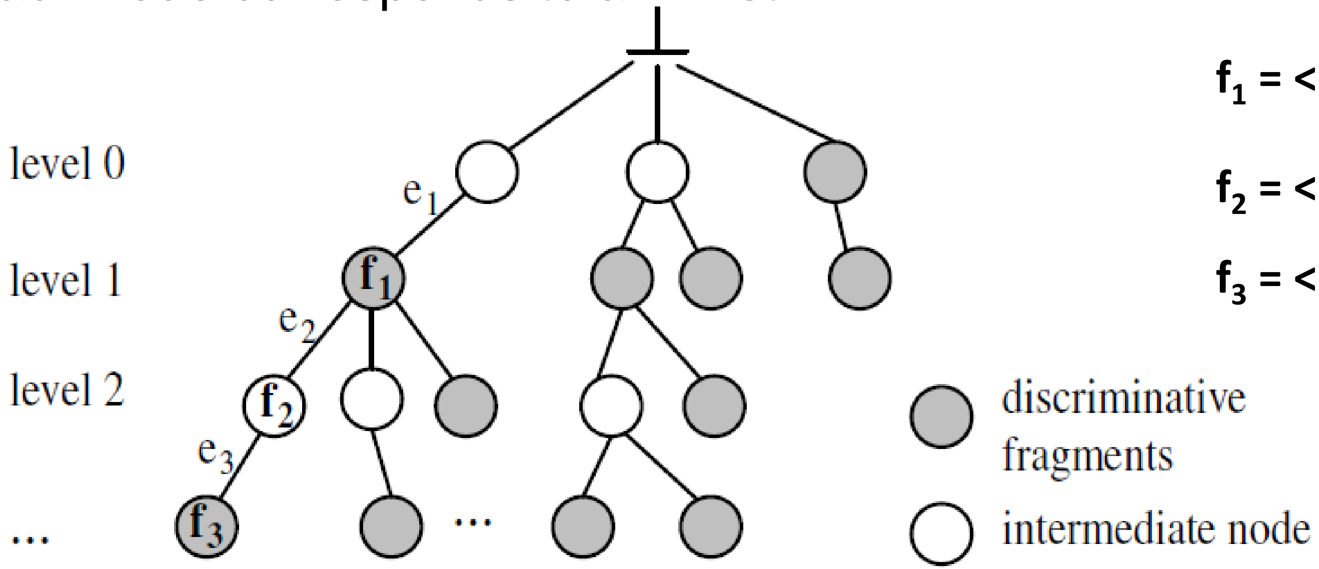
- DFS Code Generation
  - ➢ Represent each edge by a 5-tuple $(i, j, l_i, l_{(i,j)}, l_j)$
    - i and j and id of v_i and v_j
    - l_i and l_j are the labels of v_i and v_j
    - $l_{(i,j)}$ the label of the edge connecting v_i and v_j
  - ➢ The previous DFS code can be represented as

$$\langle\ (0,\ 1,\ X,\ a,\ X)\ (1,\ 2,\ X,\ a,\ Z)\ (2,\ 0,\ Z,\ b,\ X)\ (1,\ 3,\ X,\ b,\ Y)\ \rangle$$

- Canonical label
  - ➢ The minimum DFS code among all of g’s DFS code, denoted by dfs(g) based on the lexicographic order.
  - ➢ If two fragments are the same, they must share the same canonical label.

Build G-Index Tree

- G-Index Tree
  - a prefix tree that store the canonical labels of discriminative fragments
  - Each node corresponds to a ID-list



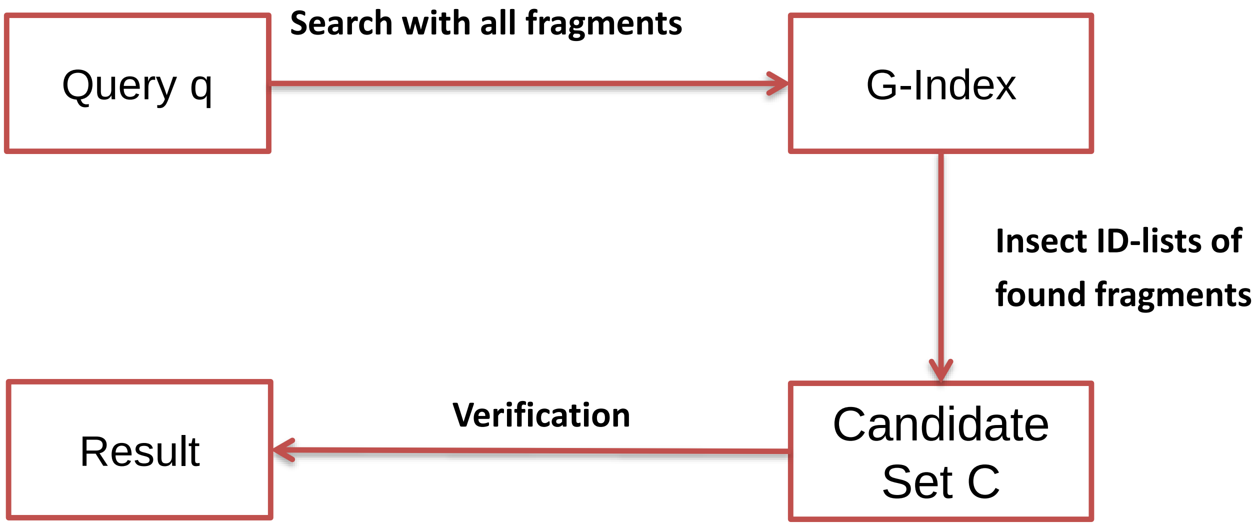
$f_1 = <e_1>$

$f_2 = <e_{1,} e_2>$

$f_3 = <e_1, e_2, e_3>$

  - Some redundant fragments are also stored in the G-Index Tree as intermediate nodes (white nodes).

Query G-Index



1) Enumerate all its fragments of q up to maximum size and locate them in the G-Index.

2) Intersect the ID-lists associated with found fragments to obtain candidate set C.

3) Verify the candidate set C.

Two Rules for Query G-Index

- Apriori Pruning
  - ➢ If a fragment is not in the G-Index tree, we need not check its super-graphs any more.

- Maximum Discriminative Fragments
  - ➢ If a query q has two fragments, $f_x \subset f_y$, it is not necessary to intersect C_q with D_{f_x}, as

$$C_q \bigcap D_{f_x} \bigcap D_{f_y} = C_q \bigcap D_{f_y}$$

  - ➢ Less intersections of ID lists

QuickSI (VLDB2008)

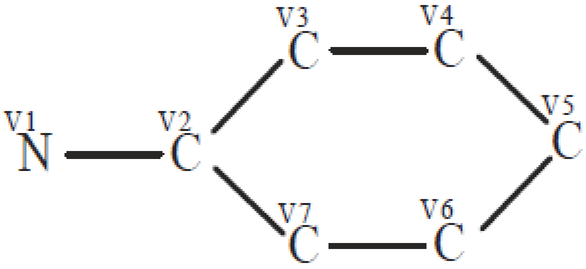
- An Efficient algorithm for Testing Subgraph Isomorphism
  - Proposed for taming verification hardness (NP-complete)
  - A synchronized depth-first traversal technique
  - Three novel pruning techniques

Related Work

- Ullmann Algorithm
  - ➢ First proposed subgraph isomorphism testing.
  - ➢ Random matching order + Backtracking

QI-Sequence

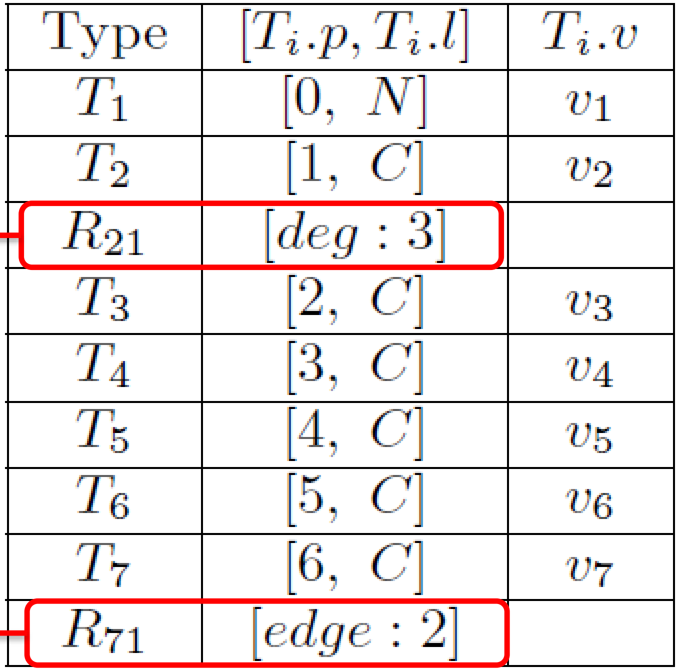
- A sequence that represents a rooted spanning tree for q
  - Basic spanning entries, T_i records basic information
  - Extra entries, R_{ij} records degree/back edges.



q

Check Degree

Check Back Edge



Type	$[T_i.p, T_i.l]$	$T_i.v$
T_1	$[0,\ N]$	v_1
T_2	$[1,\ C]$	v_2
R_{21}	$[deg : 3]$	
T_3	$[2,\ C]$	v_3
T_4	$[3,\ C]$	v_4
T_5	$[4,\ C]$	v_5
T_6	$[5,\ C]$	v_6
T_7	$[6,\ C]$	v_7
R_{71}	$[edge : 2]$	

A possible QI-sequence for q

Three pruning techniques in QI-sequence :

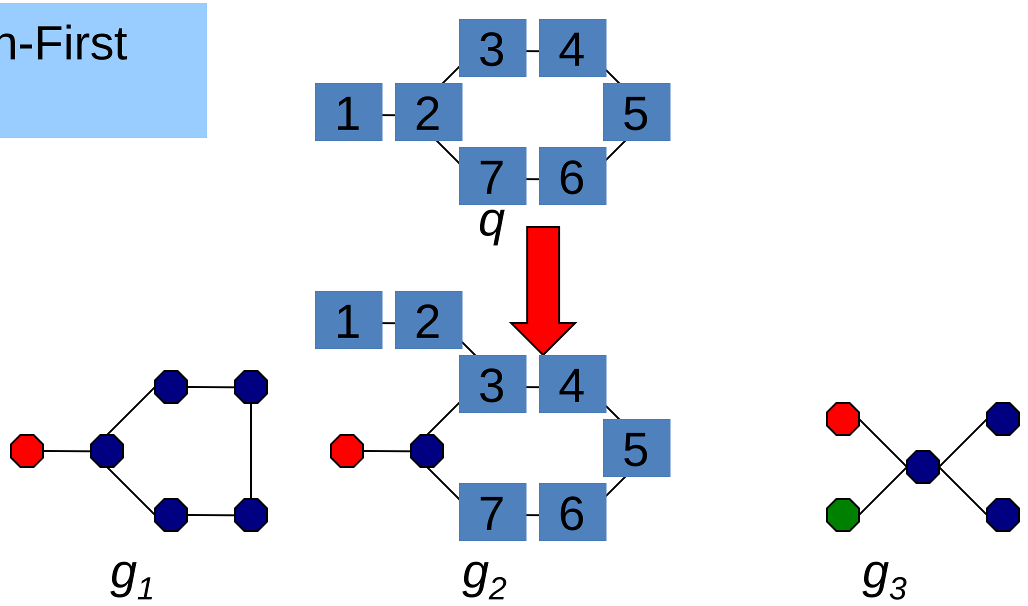
1. **Connected search order**
2. **Degree constraint**
3. **Extra(back) edge constraint**

QuickSI Example

Synchronized Depth-First Traversal

Forwarding

Backtracking



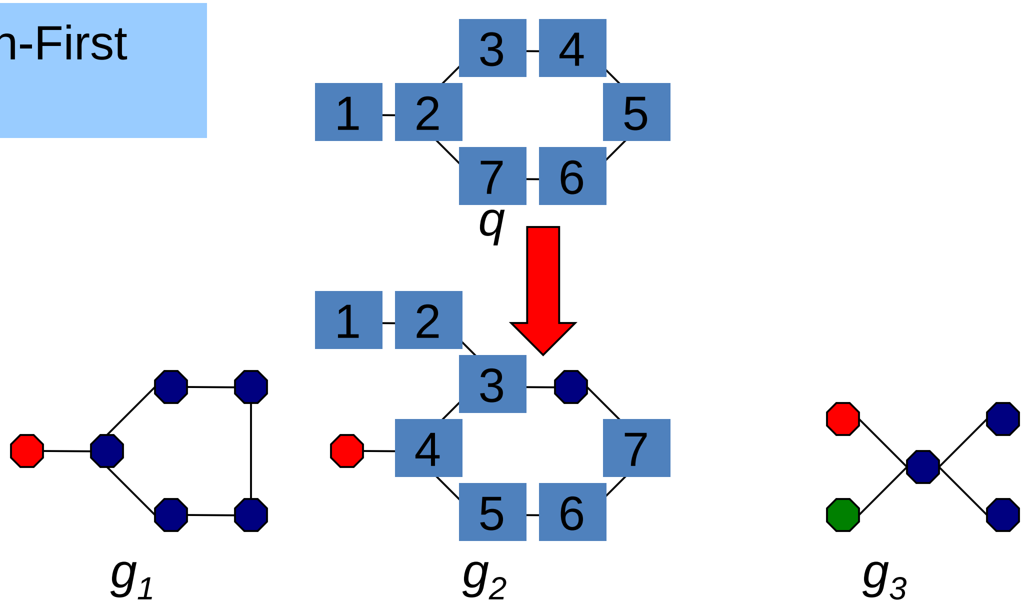
1. Determine the access order in *q*
2. Detect corresponding subgraphs in g_1, g_2 which can be mapped to the currently traversed vertices.

QuickSI Example

Synchronized Depth-First Traversal

Forwarding

Backtracking



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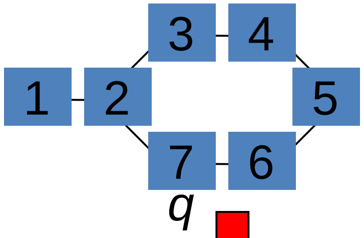
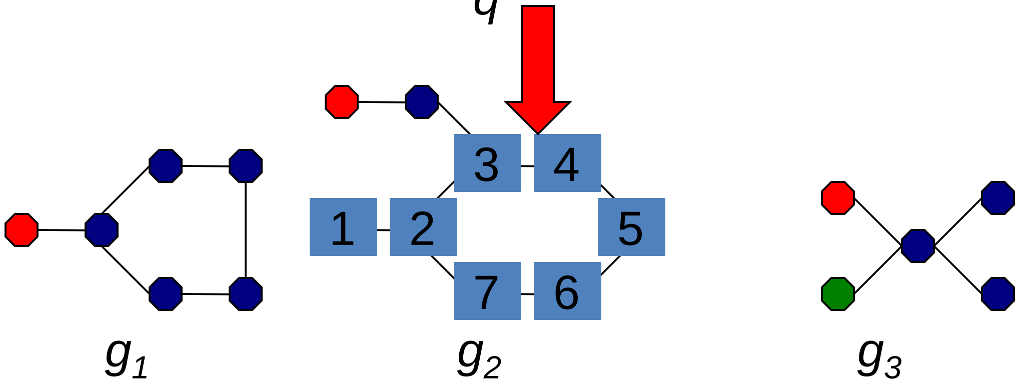
QuickSI Example

Synchronized Depth-First Traversal



Forwarding

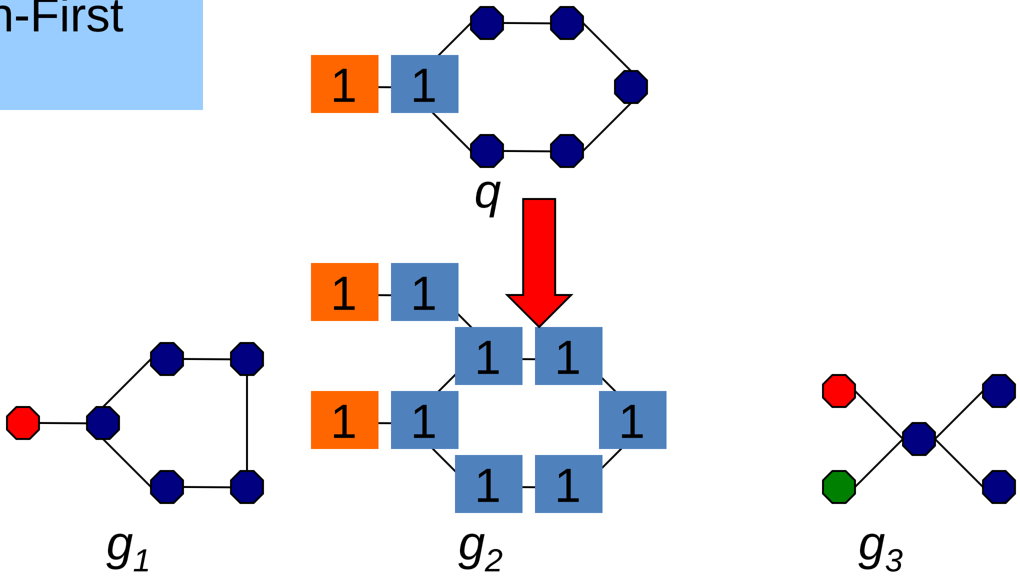


1. Determine the access order of *q*
2. Detect corresponding subgraphs in g_1, g_2 which can be mapped to the currently traversed vertices.

QuickSI Example

Access infrequent labels as early as possible

Synchronized Depth-First Traversal



1. Determine the access order for *q.*
2. Detect corresponding subgraphs in g_1, g_2 which can match the currently traversed vertices.

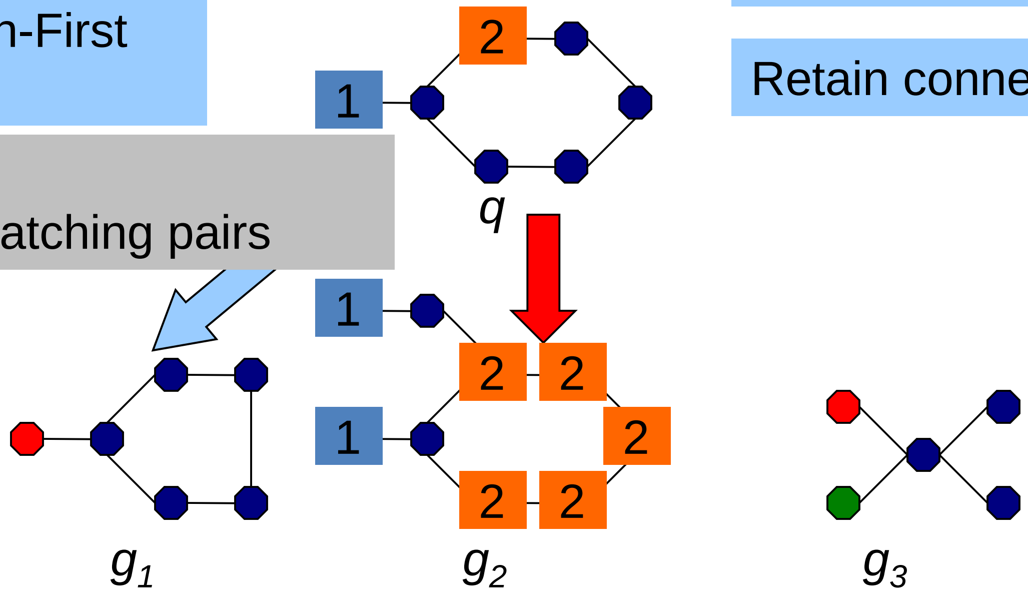
QuickSI Example

Access infrequent labels as early as possible

Retain connectivity

Synchronized Depth-First Traversal

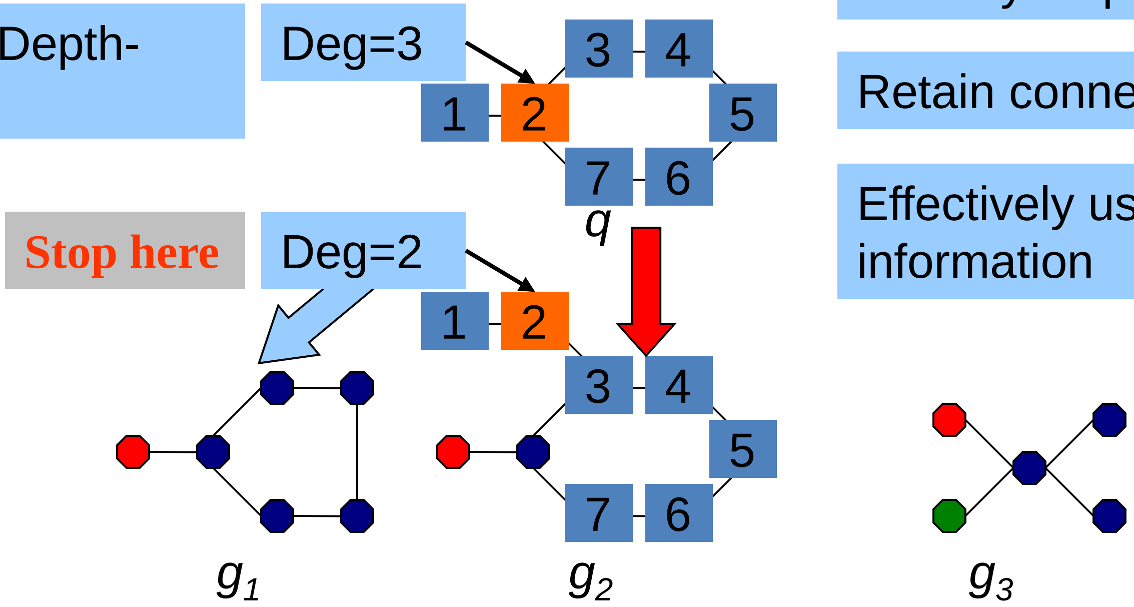
Sparse Graph!
2x5=10 possible matching pairs



1. Determine the access order for *q*.
2. Detecting corresponding subgraphs in g_1, g_2 which can match the currently traversed vertices.

QuickSI Example

Synchronized Depth-First Traversal



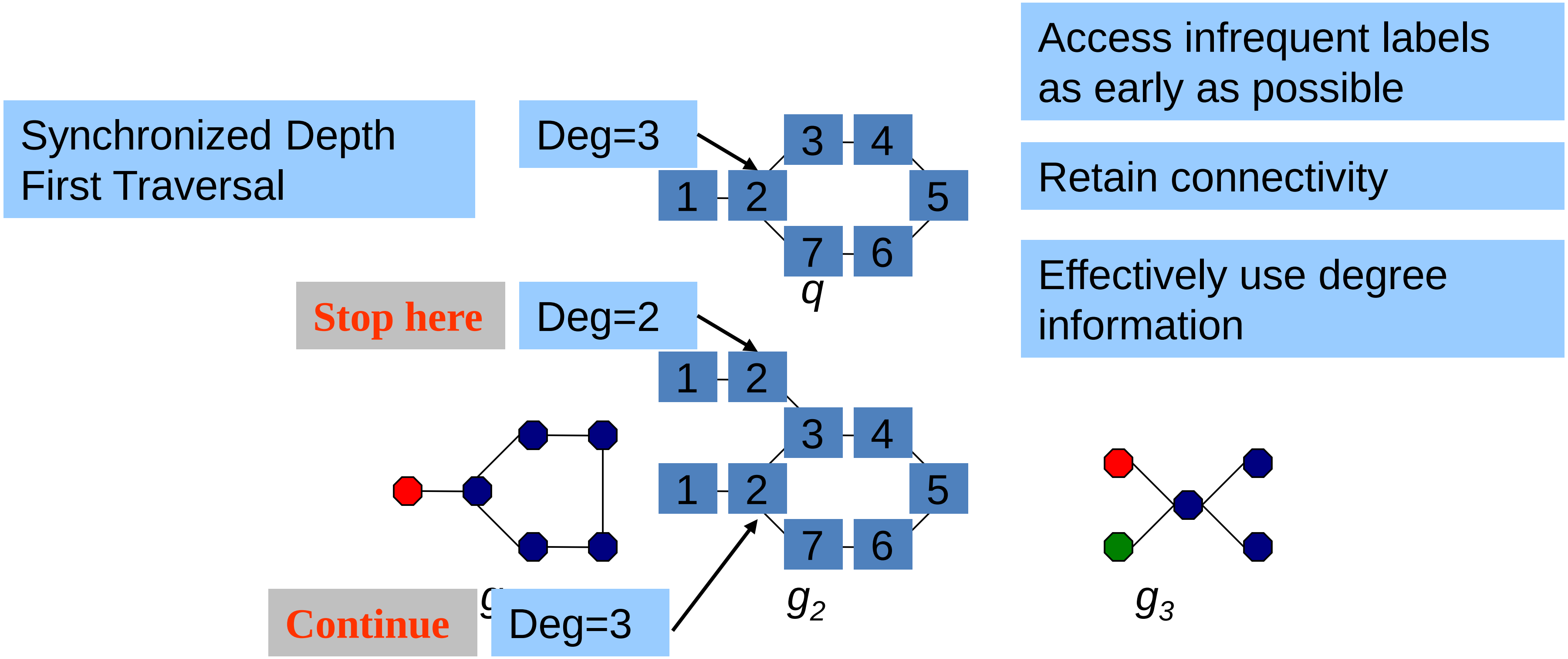
Access infrequent labels as early as possible

Retain connectivity

Effectively use degree information

1. Determine the access order for *q.*
2. Detecting corresponding subgraphs in g_1, g_2 which can match the currently traversed vertices.

QuickSI Example



Synchronized Depth First Traversal

Deg=3

Access infrequent labels as early as possible

Retain connectivity

Effectively use degree information

Stop here

Deg=2

Continue

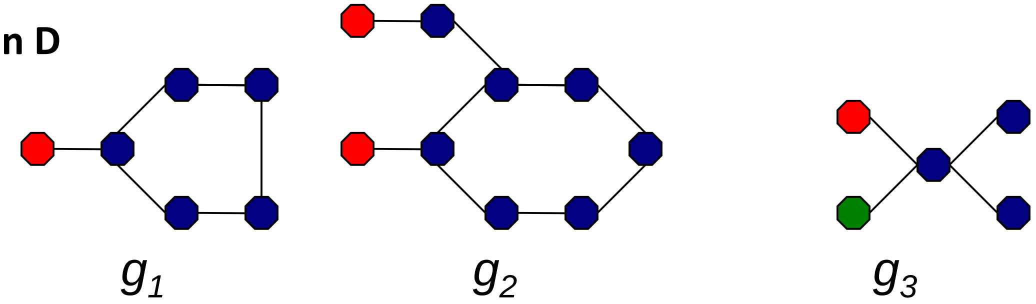
Deg=3

1. Determine the access order for *q.*
2. Detecting corresponding subgraphs in g_1, g_2 which can match the currently traversed vertices.

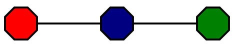
Swift Index

- A New Filter Approach
  - Precompute **QI-Sequence** for all indexed features(fragments)
    - Only **tree** features are indexed for lower cost of feature mining
  - Index all QI-Sequences in a prefix tree, called **Swift Index**

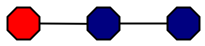
Given three graphs in D



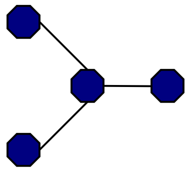
Mined Features and their ID-List:



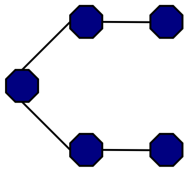
ID-List: $\{g_3\}$



ID-List: $\{g_1,g_2,g_3\}$



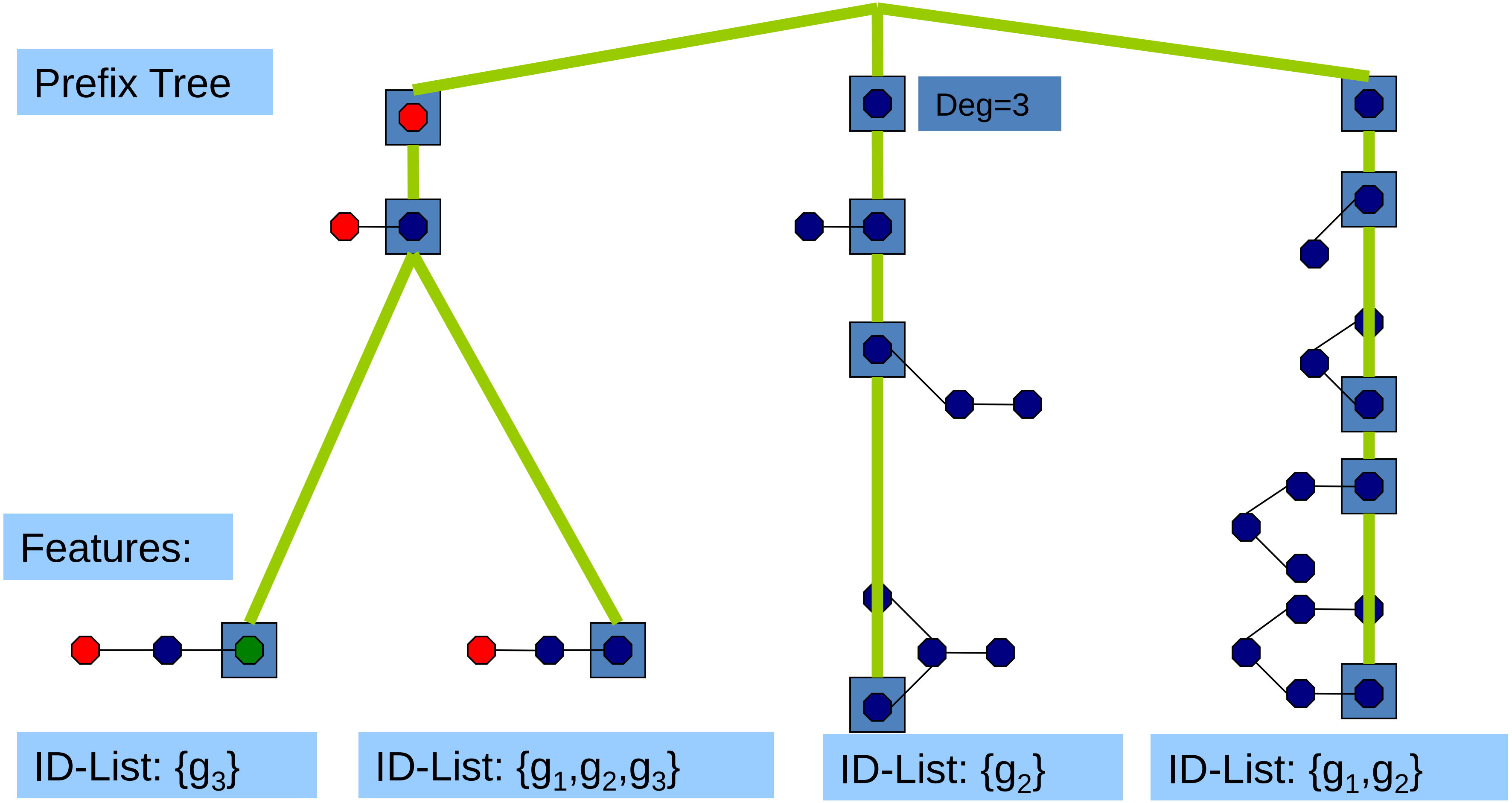
ID-List: $\{g_2\}$



ID-List: $\{g_1,g_2\}$

Swift Index

The constructed Swift Index is



Prefix Tree

Deg=3

Features:

ID-List: $\{g_3\}$

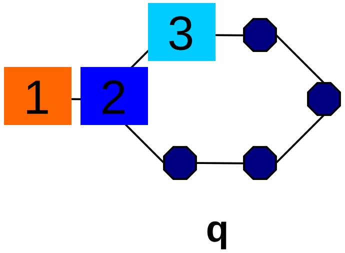
ID-List: $\{g_1,g_2,g_3\}$

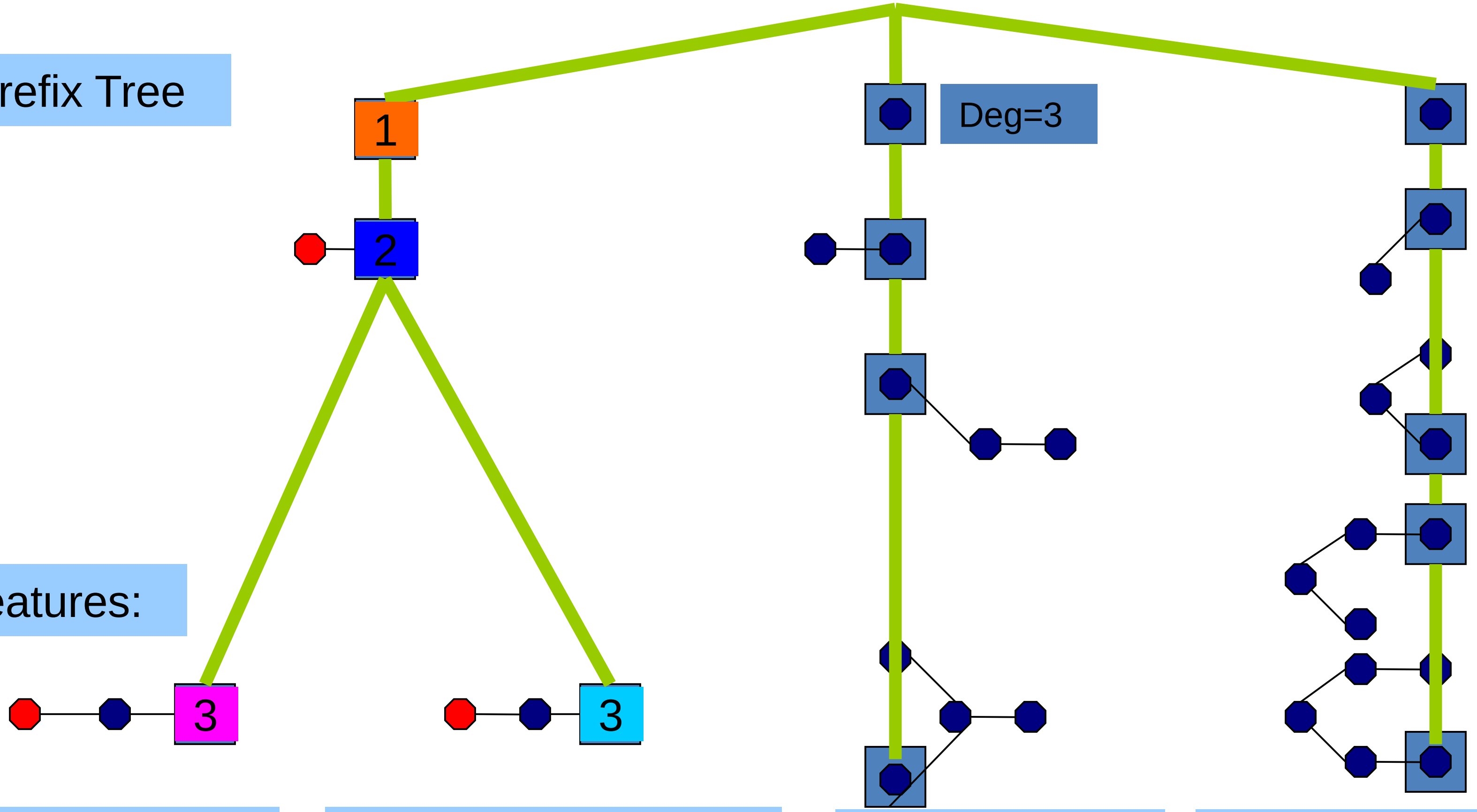
ID-List: $\{g_2\}$

ID-List: $\{g_1,g_2\}$

- Query Swift Index

Given a query q, traverse the prefix tree from the top to the bottom in the depth-first fashion, to obtain candidate set.





Prefix Tree

Features:

ID-List: $\{g_3\}$

ID-List: $\{g_1,g_2,g_3\}$

ID-List: $\{g_2\}$

ID-List: $\{g_1,g_2\}$

Candidate set for q is $\{g_1,g_2,g_3\}$

Thank you!

Questions?