A Simplified Helicopter UAV Modeling

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1. Preliminary Assumptions

This study presents a simplified model of a helicopter UAV. To make the analysis tractable, the following assumptions are adopted:

1. The main rotor provides lift and forward/backward thrust.
2. The tail rotor prevents yaw rotation.
3. Fuselage drag, side forces, and complex aerodynamic interactions are neglected.
4. The UAV is considered a rigid body.
5. Small-angle approximations are applied for translational motion: main rotor tilt angles remain within $\pm 10^\circ$, allowing $\sin(\theta) \approx \theta$.

These assumptions focus on the essential UAV dynamics while keeping the model suitable for control and simulation.

2. Simplified UAV Dynamics with Pitching Moment

Based on the assumptions above, the UAV is modeled as a rigid body with three primary degrees of freedom: vertical motion, forward/backward motion, and yaw rotation.

2.1. Force and Moment Decomposition

Let F_{rotor} be the main rotor thrust, θ the tilt angle, and d the distance from the center of mass to the rotor hub along the forward axis. Then:

$$\begin{aligned} F_x &= F_{\text{rotor}} \sin\theta && \text{(forward force)} \\ F_L &= F_{\text{rotor}} \cos\theta && \text{(vertical lift)} \\ M_y &= d \cdot F_x = d \cdot F_{\text{rotor}} \sin\theta && \text{(pitching moment)} \end{aligned}$$

2.2. Equations of Motion

Dynamics Systems:

Translational dynamics:

$$m\dot{u} = F_x = F_{\text{rotor}} \sin\theta, \quad m\dot{w} = F_L - mg = F_{\text{rotor}} \cos\theta - mg \tag{1}$$

Pitch rotational dynamics:

$$I_y\, \dot{q} = M_y = d \cdot F_{\text{rotor}} \sin\theta \tag{2}$$

Yaw rotational dynamics:

$$I_z\,\dot{r} = M_z = d_{\text{tail}} \cdot F_{\text{tail rotor}} \tag{3}$$

Kinematics in the inertial frame:

$$\dot{X} = u, \quad \dot{Z} = w, \quad \dot{\psi} = r \tag{4}$$

2.3. Control Inputs

$$\mathbf{u} = \begin{bmatrix} F_{\text{rotor}} \\ \theta \\ F_{\text{tail rotor}} \end{bmatrix}$$

- F_{rotor} controls vertical motion.
- θ controls forward/backward motion and induces pitching moment.
- $F_{\text{tail rotor}}$ controls yaw rotation.

3. Linearized UAV Dynamics and State-Space Matrices

3.1. State and Input Vectors

We define the state and control input vectors as:

$$\mathbf{x} = \begin{bmatrix} X \\ Z \\ \psi \\ u \\ w \\ q \\ r \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} F_{\text{rotor}} \\ \theta \\ F_{\text{tail rotor}} \end{bmatrix} \tag{5}$$

3.2. Hover Equilibrium

At hover, the UAV should have no linear or angular motion. The main rotor balances gravity, and the tail rotor cancels the main rotor torque:

$$u = w = q = r = 0, \quad \theta = 0, \quad F_{\text{rotor0}} = mg, \quad F_{\text{tail rotor0}} = F_{\text{tail hover}}$$

where $F_{\text{tail hover}}$ is the tail rotor force required to cancel the main rotor torque at hover.

3.3. Linearization

Linearizing around hover using first-order Taylor expansion:

$$\begin{aligned} m\dot{u} &\approx mg\,\theta \\ m\dot{w} &\approx \Delta F_{\text{rotor}} \\ I_y\dot{q} &\approx d\,mg\,\theta \\ I_z\dot{r} &\approx d_{\text{tail}}\,\Delta F_{\text{tail rotor}} \end{aligned}$$

3.4. State-Space Form

Define the state vector and input perturbations:

$$
\mathbf{x} = \begin{bmatrix} X \\ Z \\ \psi \\ u \\ w \\ q \\ r \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} \Delta F_{\text{rotor}} \\ \theta \\ \Delta F_{\text{tail rotor}} \end{bmatrix}
$$

Here, X and Z are horizontal and vertical positions, ψ is the yaw angle, u and w are body-frame velocities, and q and r are pitch and yaw rates. The control inputs are the main rotor thrust F_{rotor}, main rotor tilt θ, and tail rotor thrust $F_{\text{tail rotor}}$. Δ represents the deviation of a variable from its equilibrium (hover) value.

The linearized system can be written as:

$$
\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} \tag{6}
$$

with matrices:

$$
A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{m} & 0 & 0 \\ 0 & \frac{d\,mg}{I_y} & 0 \\ 0 & 0 & \frac{d_{\text{tail}}}{I_z} \end{bmatrix}
$$

Here, A captures kinematics (positions depend on velocities), and B captures dynamics (velocities and yaw rate respond to control inputs).

4. LQR Control Design

4.1. State-Space Representation

Based on the linearized 3-DOF UAV dynamics, the system can be expressed as:

$$
\mathbf{x} = \begin{bmatrix} X \\ Z \\ \psi \\ u \\ w \\ q \\ r \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} \Delta F_{\text{rotor}} \\ \theta \\ \Delta F_{\text{tail rotor}} \end{bmatrix}, \quad \dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}.
$$

4.2. LQR Controller

The Linear Quadratic Regulator (LQR) seeks the control input $\mathbf{u}$ that minimizes the cost function:

$$J = \int_0^\infty \left(\mathbf{x}^T Q\mathbf{x} + \mathbf{u}^T R\mathbf{u}\right) dt,$$

where $Q \succeq 0$ penalizes state deviations and $R \succ 0$ penalizes control effort. The optimal state feedback law is

$$\mathbf{u} = -K\mathbf{x},$$

with K obtained by solving the continuous-time algebraic Riccati equation (CARE):

$$A^T P + PA - PBR^{-1}B^T P + Q = 0.$$

4.3. Implementation Remarks

- LQR automatically coordinates vertical, forward, pitch, and yaw control through the coupled state dynamics.
- The weighting matrices Q and R can be tuned to prioritize position tracking, attitude stabilization, or minimization of control effort.
- The computed LQR outputs $[F_{\text{rotor}}, \theta, F_{\text{tail rotor}}]^T$ are then mapped to the actual rotor collective and cyclic inputs.
- Linearization is valid near the hover state; for large maneuvers, gain scheduling or nonlinear control may be required.

5. Further Comments on Last Interview

5.1. Testing the Cause of Instability

The idea of testing the force generated by the tail rotor and comparing it with the force required to deform the connecting boom is reasonable. This is because the forces interact with each other, and the resulting motion will be dominated by the smaller of the two forces—either from the tail rotor or the main rotor.

5.2. Possibility of Refining the Current PID Control Strategy

Based on the interview, I did not hear any specific details about the current PID control design. There are multiple possible implementations of PID control, so it is unclear whether the instability was caused by the controller design or some other factor. One thing that can be said with confidence is that PID control cannot provide instantaneous and highly precise control results in the way that a model-based strategy can.