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***Abstract*—In this paper, the energy efficiency of multi-user non orthogonal multiple access (NOMA) systems in the presence of a backscatter device is investigated. The energy efficiency maximization problem is formulated as a tradeoff between the sum rate and the total power consumption and shown to be non-convex. We then derive a closed-form expression of the optimal reflection coefficient. Remarkably, the obtained expression allows the reformulation of the optimization in terms of the power allocation policy into a convex optimization problem that has recently been solved in closed form. This overall solution can then be exploited to reduce the computational complexity of Dinkelbach's algorithm for maximizing the ratio sum rate vs. total power. Simulation results show that the presence of backscatter devices significantly improve the energy efficiency of NOMA systems and reach up to 450% relative gains compared to OMA.**

***Index Terms*—multi-user NOMA, energy efficiency, ambient backscatter communications**

I. Introduction

The exponential growth in traffic volume and connected devices in the last decade has resulted in critical energy consumption and carbon emission [1]. Hence, energy consumption is expected to become among the major bottlenecks for future communication networks; the current roadmap for the next generation (6G) projects an energy efficiency improvement of 10 to 100 times compared to the recently standardized 5th generation (5G) network [2].

Non orthogonal multiple access (NOMA) has been recognized as a promising technology to increase the spectral and energy efficiency (EE) compared to traditional OMA [3], [4]. By employing superposition coding at the transmitter side and successive interference cancellation (SIC) at the receiver side, users can be multiplexed on the same radio resource.

In order to improve the performance of wireless links and multiple access schemes, techniques such as relaying, intelligent reflective surfaces and backscattering communication have been developed. Compared to backscattering communication, relaying forwards signals from source to destination which incurs additional power consumption, whereas intelligent reflective surfaces only reflect ambient signals without transmitting any message of their own. In this context, ambient backscatter communication has emerged as a means to enhance communications by modulating and reflecting ambient signals from base stations (BS) or Wi-Fi access points [5] to their intended receivers. The advantage of such backscattering devices is that they do not require local oscillators to generate carrier frequencies, and, hence, consume much less (by an order of magnitude) power than conventional devices [6]. Hence, we investigate the performance of NOMA in the presence of a backscatter, which can both send information and reflect the ambient RF signals without being power greedy.

Several recent works have combined NOMA with backscattering and have shown the benefits in terms of outage probability, network sum-rate and minimum user throughput performance [7]–[9]. In [10], the non-convex energy efficiency maximization problem for a two-user downlink NOMA system aided by a backscatter device has been investigated.

In this paper, we investigate the energy efficiency of a multi-user downlink NOMA system in the presence of a backscatter device. Remarkably, we derive a closed-form solution that introduces an attractive tradeoff between the sum rate and power consumption under a limited power budget and user QoS constraints for an arbitrary number of users. We first show that the expression of the optimal reflection coefficient can be obtained in closed form, resulting in a reformulated optimization with respect to the power allocation that is convex and has recently been solved in closed form in [4] without the backscatter device. Second, our obtained closed-form overall solution can then be exploited to maximize the ratio between sum rate and power consumption by reducing the iterative Dinkelbach's procedure to a simple line search of super-linear convergence.

Compared to [10], our novel contributions are two-fold: we solve the energy efficiency maximization problem for an arbitrary number of users ($K \geq 2$) as opposed to the particular case of only $K = 2$ users in [10]; we propose a low-complexity algorithm to solve the maximization of the

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overall rate vs. power ratio based on our closed-form solution as opposed to the iterative sub-gradient algorithm in [10].

II. System model and problem formulation

We consider a downlink system composed of one base station (BS) transmitting to $K \geq 2$ receivers or users in the presence of a single helping backscatter device.

At each time slot t, the BS employs superposition coding and broadcasts the signal $x(t) = \sum_{i=1}^{K} \sqrt{p_i} x_i(t)$, where $x_i(t)$ and p_i denote the message and the power allocated by the BS to user $i \in \{1, \ldots, K\}$ respectively. The helping device then modulates its signal $b(t)$ and backscatters to the K users the incident signal by adjusting its reflection coefficient $\rho \in [0, 1]$. As in [11], [12], we assume that both the processing and propagation delays, as well as the additional noise at the backscatter device are negligible.

The received signal at each user k is expressed by

$$y_k(t) = h_k x(t) + \sqrt{\rho} g g_k x(t) b(t) + z_k(t), \tag{1}$$

where h_k, g and g_k denote the channel gain between the BS and user k, the channel gain between the BS and the backscatter device, and the channel gain between the backscatter device and user k, respectively. Since for backscatter communication systems the different links are usually assumed to have a strong line-of-sight (LOS) [13], [14], we consider a channel model with only a path loss of the type $d^{-\eta}$, where d is the distance between the transmitter and receiver nodes and η is the path loss exponent. The quantity $z_k(t)$, of variance σ_k^2, is the additive white Gaussian noise at receiver k and $b(t)$ is the message transmitted from the backscatter device. Since the backscatter device usually has a much lower data rate than the BS, we consider $b(t) = 1$ as constant for a sufficiently long period during which the message $x(t)$ is transmitted [15], [16]. Hence the received signal write as

$$y_k(t) = h_k x(t) + \sqrt{\rho} g g_k x(t) + z_k(t). \tag{2}$$

Throughout this paper, we assume that perfect channel state information is available at the BS[1] [7], [10] and, without loss of generality, that the channels are ordered as follows: $h_1^2/\sigma_1^2 \geq h_2^2/\sigma_2^2 \geq \ldots \geq h_K^2/\sigma_K^2$. Each user will then apply SIC to retrieve its message. In the following, the SIC order is fixed and based solely on the ordering of the BS regarding user channel gains. Hence, user k starts by decoding the interference from all the users j such that $K \geq j \geq k+1$, and suffers the interference from the users $1 \leq j \leq k-1$. Thus, the achievable rate of user k is [17], [18]

$$R_k(\rho, \mathbf{p}) = C\big(\min_i(\gamma_{k\to i})\big), \quad \forall i, k \in \{1, \ldots, K\}, \tag{3}$$

where $\mathbf{p} = (p_1, \ldots, p_K)$ is the power allocation vector, $C(x) = 1/2 \log_2(1 + x)$ denotes the Shannon capacity, and $\gamma_{k\to i}$ is the signal-to-interference-plus-noise ratio (SINR) to decode the message destined to user k at receiver i defined as

$$\gamma_{k\to i} = \frac{(h_i + \sqrt{\rho} g g_i)^2 p_k}{(h_i + \sqrt{\rho} g g_i)^2 (p_1 + \ldots + p_{k-1}) + \sigma_i^2}. \tag{4}$$

The BS is constrained by a total available power budget such that $\sum_k p_k \leq P_{\max}$ and each user has a minimum rate QoS constraint expressed as $R_k(\rho, \mathbf{p}) \geq R_{\min,k}$.

Notations: To simplify the presentation, the following notations will be used in the remaining of the paper: $A_k = 2^{2R_{\min,k}}, \theta_k = \sum_{i=1}^{k} p_i$, with $\theta_0 = 0$, $H_k = h_k/\sigma_k$, $G_k = g g_k/\sigma_k$, $\Gamma_k(\rho) = (H_k + \sqrt{\rho} G_k)^2$. The assumed channel order with these notations becomes $H_1^2 \geq H_2^2 \geq \ldots \geq H_K^2$.

Sum rate vs. power consumption tradeoff

To ensure the SIC decoding order described above, the SINR needs to meet the following constraints: $\gamma_{k\to i} \geq \gamma_{k\to k}, \forall k > 1, \forall i < k$. In this case, the achievable rate of user k simplifies to

$$R_k(\rho, \mathbf{p}) = C(\gamma_{k\to k}) = \frac{1}{2} \log_2 \left(\frac{1 + \Gamma_k(\rho)\theta_k}{1 + \Gamma_k(\rho)\theta_{k-1}} \right), \tag{5}$$

with $\Gamma_k(\rho)$ defined in the notations above.

As in [4], [10], the energy efficiency is measured via the scalarized tradeoff between sum rate and power consumption: $\sum_k R_k(\rho, \mathbf{p}) - \alpha(\sum_k p_k + P_c)$, where the parameter $\alpha \geq 0$ tradeoffs between the achievable network throughput and the network power consumption and P_c denotes the circuit power consumption.

To sum up, the energy efficiency (EE) maximization problem under investigation is cast as

$$(\mathbf{EE1}) \quad \max_{\rho, \mathbf{p}} \sum_{k=1}^{K} \frac{1}{2} \log_2 \left(\frac{1 + \Gamma_k(\rho)\theta_k}{1 + \Gamma_k(\rho)\theta_{k-1}} \right) - \alpha(\theta_K + P_c)$$

$$\text{s.t. } \theta_K \leq P_{\max}, \tag{C1}$$

$$\theta_k \geq A_k \theta_{k-1} + \frac{(A_k - 1)}{\Gamma_k(\rho)}, \ \forall k, \tag{C2}$$

$$\gamma_{k\to i} \geq \gamma_{k\to k}, \ \forall k \geq 2, \ \forall i \leq k-1, \tag{C3}$$

$$0 \leq \rho \leq 1, \tag{C4}$$

where $(C1)$ is the total power constraint, $(C2)$ are the K individual QoS constraints, $(C3)$ is the SINR SIC ordering constraints and $(C4)$ is the backscatter reflection coefficient constraint.

III. Closed-form energy efficient solution

The resulting optimization problem $(\mathbf{EE1})$ is non-convex because of the coupling between ρ and $\mathbf{p}$ as also discussed in [10] for the special two-user case $K = 2$. In [10], an iterative algorithm based on duality and sub-gradient descent is proposed. Here, we show that this problem can be solved in closed form and in the general multi-user case $K \geq 2$, without the need for an iterative procedure.

[1] We assume that the BS has full knowledge of h_k and g^d which can be directly obtained through pilot-based channel estimation, and g_k^d which can be backscattered to the BS.

A. Optimal reflection coefficient

As in [10], we start by optimizing the reflection coefficient ρ for an arbitrary power allocation vector $\mathbf{p}$ in the multi-user case.

Theorem. *The optimal reflection coefficient ρ^* for a fixed power allocation vector $\mathbf{p}$ is given by*

$$\rho^* = \begin{cases} \min\left(1, \min \mathcal{R}\right), & \text{if } \mathcal{R} \neq \emptyset \\ 1, & \text{if } \mathcal{R} = \emptyset, \end{cases} \tag{6}$$

where $\mathcal{R} \triangleq \left\{ \left(\frac{H_k - H_{k-1}}{G_k - G_{k-1}} \right)^2 \middle| k \in \{2, \ldots, K\}, G_k > G_{k-1} \right\}$.

The detailed proof is provided in the appendix.

Remark. *Note that the optimal ρ^* above is independent from $\mathbf{p}$, which means that decoupling the optimization problem with respect to ρ and $\mathbf{p}$ does not incur any optimality loss. Also, under ρ^* the constraint (C3) is satisfied and, as proven in the appendix, the channels are ordered as: $\Gamma_1(\rho^*) \geq \Gamma_2(\rho^*) \geq \ldots \geq \Gamma_K(\rho^*)$, which will be put to great use to find the optimal power allocation vector $\mathbf{p}^*$.*

B. Optimal power allocation

Given the optimal ρ^* and exploiting the resulting channel order above $\Gamma_1(\rho^*) \geq \Gamma_2(\rho^*) \geq \ldots \geq \Gamma_K(\rho^*)$, we can prove that the optimization problem $(\mathbf{EE1})$ is equivalent to the simpler convex problem below

$$(\mathbf{EE2}) \quad \max_{\mathbf{p}} \sum_{k=1}^{K} \frac{1}{2} \log_2 \left(\frac{1 + \Gamma_k(\rho^*)\theta_k}{1 + \Gamma_k(\rho^*)\theta_{k-1}} \right) - \alpha(\theta_K + P_{\mathrm{c}})$$

$$\text{s.t. } \theta_K \leq P_{\max}, \tag{C1'}$$

$$\theta_k \geq A_k \theta_{k-1} + \frac{(A_k - 1)}{\Gamma_k(\rho^*)}, \quad \forall k \tag{C2'}$$

The resulting convex problem can be solved in closed-form as shown in our previous work [4]. For the sake of completeness, we state the necessary and sufficient feasibility conditions as well as the closed-form expression of the optimal power allocation policy $\mathbf{p}^*$. We kindly refer the interested reader to [4] for the detailed proofs.

Corollary. *[4, Proposition 1, Theorem 1] The optimization problem $(\mathbf{EE2})$ is feasible if and only if the following condition holds: $P_{\max} \geq P_{\min} \triangleq \sum_{i=1}^{K} \frac{A_i - 1}{\Gamma_i(\rho^*)} \prod_{j=i+1}^{K} A_j$.*

When $(\mathbf{EE2})$ is feasible, the optimal power allocation is obtained in closed-form as follows:

$$\begin{aligned} p_k^*(\alpha) &= (A_k - 1)\Bigg(\frac{1}{\Gamma_k(\rho^*)} + p_1^*(\alpha) \prod_{i=2}^{k-1} A_i \\ &\quad + \sum_{i=2}^{k-1} \frac{A_i - 1}{\Gamma_i(\rho^*)} \prod_{j=i+1}^{k-1} A_j \Bigg), \quad \forall\, k \geq 2, \\ p_1^*(\alpha) &= \min\left(\max\left(\overline{p_1}(\alpha); \tfrac{A_1 - 1}{\Gamma_1(\rho^*)} \right); U_1 \right), \end{aligned} \tag{7}$$

where U_1 and $\overline{p_1}(\alpha)$ are expressed below

$$U_1 = \left(P_{\max} - P_{\min} + \frac{A_1 - 1}{\Gamma_1(\rho^*)} \prod_{j=2}^{K} A_j \right) \Bigg/ \prod_{i=2}^{K} A_i \,,$$

$$\overline{p_1}(\alpha) = 1 \Bigg/ \left(2 \ln 2 \; \alpha \prod_{i=2}^{K} A_i \right) - \frac{1}{\Gamma_1(\rho^*)}$$

C. Sum rate vs. power consumption ratio

Finally, our optimal closed-form solution can be used to maximize another very popular energy efficiency metric called global energy efficiency, which is defined by the ratio between the achievable sum rate and the total power consumption [1] given by

$$GEE(\rho, \mathbf{p}) = \frac{\sum_{k=1}^{K} R_k(\rho, \mathbf{p})}{\sum_{k=1}^{K} p_k + P_{\mathrm{c}}}. \tag{8}$$

Since only the numerator (sum rate) depends on the reflection coefficient ρ, it follows that ρ^* maximizing the objective in $(\mathbf{EE2})$ also maximizes $GEE(\rho, \mathbf{p})$ for all $\mathbf{p}$, and decoupling the problem's variables does not incur any optimality loss.

Now, in order to find the optimal power allocation policy, one has to maximize $GEE(\rho^*, \mathbf{p})$ with respect to $\mathbf{p}$, which is a fractional program since the sum rate $\sum_k R_k(\rho^*, \mathbf{p})$ is a concave function with respect to $\mathbf{p}$.

Using fractional programming, maximizing $GEE(\rho^*, \mathbf{p})$ reduces to searching the fixed point of the function $F(\alpha) = \sum_{k=1}^{K} R_k(\rho^*, \mathbf{p}^*) - \alpha \left(\sum_{k=1}^{K} p_k^* + P_{\mathrm{c}} \right)$, where $\mathbf{p}^*$ is the closed-form solution to $(\mathbf{EE2})$ explicited in the above Corollary. This search is commonly performed via a Dinkelbach's procedure that is reduced to a simple line-search thanks to our closed-form solution to $(\mathbf{EE2})$ (see Algorithm 1).

Algorithm 1 GEE maximization using Dinkelbach

```
Initialize ϵ > 0, α = 0
Compute ρ* via eq. (6)
repeat
    Compute p* via eq. (7)
    Update F(α) = Σ_{k=1}^K R_k(ρ*, p*) − α (Σ_{k=1}^K p*_k + Pc)
    Update α ← GEE(ρ*, p*)
until F(α) ≤ ϵ
```

Complexity analysis: Thanks to our closed-form expressions of the optimal reflection coefficient ρ^* and of the optimal power allocation $\mathbf{p}^*$, the complexity of the operations inside the repeat-loop of our Algorithm 1 scales as $\mathcal{O}(K)$. The optimal solution is reached after few iterations of the loop, due to the super-linear convergence rate of the Dinkelbach's procedure [19].

The algorithm proposed in [10] for the special case of $K = 2$ users, differs from ours in the operations inside of the loop. In order to compute the optimal power allocation vector $\mathbf{p}^*$ in [10], a second iterative algorithm based on duality and subgradient descent was employed, which is much more complex than our closed-form $\mathbf{p}^*$. In fact, if one were to generalize the method in [10] to an arbitrary number of users $K \geq 2$, each individual iteration of the second algorithm inside of the loop would scale as $\mathcal{O}(K)$, whereas in our Algorithm 1 the overall calculations within the loop scale as $\mathcal{O}(K)$.

IV. Simulation results

In this section, we evaluate the energy efficiency defined by the ratio sum rate vs. power consumption (GEE) of NOMA and OMA with and without backscattering[2]. The path loss exponent is $\eta = 3$. We set the maximum distance between the BS and the backscatter to 10m, and between the BS and other receivers to 20m, $P_{\max} = 80$ dBm (unless stated otherwise), $P_{\mathrm{c}} = 30$ dBm, $\sigma_k^2 = \sigma^2 = -100$ dBm and $R_{\min,k} = R_{\min}, \forall k$. All results are averaged over 10^5 independent channel realizations satisfying the feasibility condition of the problem ($\mathbf{EE2}$). Note that under OMA, the K users are assumed to be served in a time sharing fashion with equal time slots and, for a fair comparison, the same minimum QoS constraints apply.

Fig. 1 plots the energy efficiency of NOMA and OMA with and without backscattering as a function of the number of users K for $R_{\min} = 1$bit/s. First, NOMA with backscattering always outperforms OMA (with or without backscattering) and conventional NOMA irrespective from the number of users K. Moreover, we see that for a large number of users, NOMA with backscattering greatly outperforms the other schemes in terms of energy efficiency.

One can notice that backscattering never decreases the energy efficiency under NOMA or OMA. More specifically, backscattering always improves the energy efficiency of both NOMA and OMA, and can still achieve a significant energy efficiency for a large number of users as opposed to NOMA and OMA without backscattering.

Furthermore, we see that when the number of users increases, the energy efficiency of NOMA with backscattering decreases. The intuition behind this follows from the expression of the optimal reflection coefficient in (6), which depends on the smallest difference between the channel gains: the larger the number of users K, the smaller the channel gap. When K increases, ρ^* eventually reaches zero, cancelling the backscatter effect and leading to a conventional NOMA.

This can also be observed in Fig. 2, in which the optimal reflection coefficient ρ^* is illustrated as a function of the number of users K, under both NOMA and OMA. Note that, under OMA, all users' rates are increasing functions of ρ, leading to a constant optimal value: $\rho^* = 1$. This means that the backscatter device reflects the entire ambient incident signal. With NOMA, only a fraction of the ambient incident signal is reflected ($\rho^* < 1$), which decreases with the number of users K. Nevertheless, this enables energy harvesting at the backscatter device, which can then be used for its circuit operation [15], [20].

[2]All our MatLab codes are available online: https://github.com/HajarElhassani/EE_NOMA_BD_Kusers

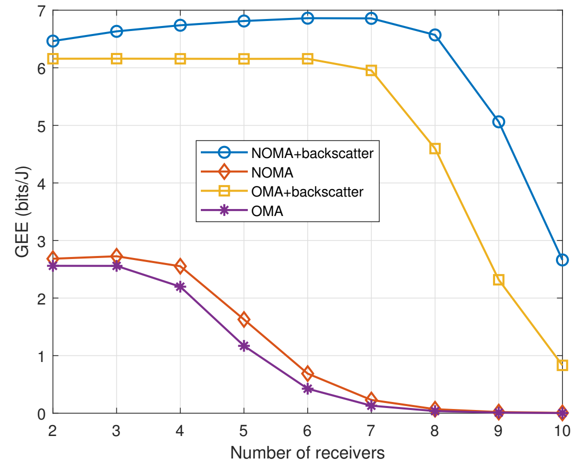

Fig. 1. Energy efficiency (GEE) as a function of the number of users K for $R_{\min} = 1$bit/s. Backscattering always improves the energy efficiency of both NOMA and OMA irrespective from the number of users K. When K grows large the backscattering advantage decreases.

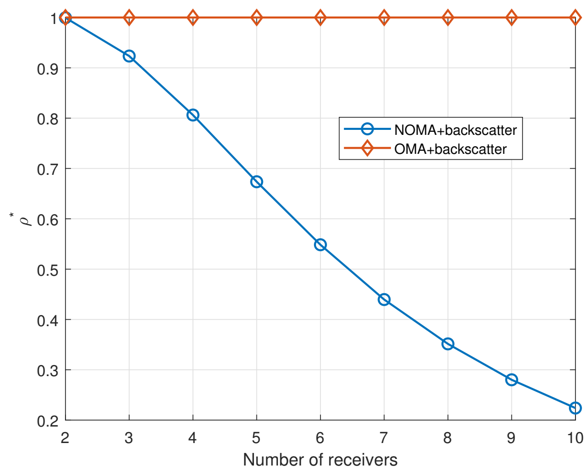

Fig. 2. Optimal reflection coefficient ρ^* as a function of the number of users K for $R_{\min} = 1$ bit/s. For OMA, the ambient incident signal from the BS is completely reflected and $\rho^* = 1$, while for NOMA, only a part of the incident signal is reflected ($\rho^* < 1$), the other part being exploited for energy harvesting.

Fig. 3 depicts the relative gain of 2 schemes: OMA with backscattering and NOMA with backscattering, compared to conventional OMA, which is defined as $\left(GEE^{\text{scheme}} - GEE^{\text{OMA}}\right)/GEE^{\text{OMA}}$, as a function of the noise variance σ^2 for $K = 3$ users and $P_{\max} = 50$ dBm. We see that for small values of σ^2, as considered in our setting, the gain is of 150%, whereas for larger values of σ^2, the gain remarkably increases up to 450% and 350% for NOMA with backscattering and OMA with backscattering respectively.

The impact of the minimum QoS constraint $R_{\min}$ is illustrated in Fig. 4 for $K = 3$ users and $P_{\max} = 50$ dBm. For all schemes, the larger the minimum user rate, the smallest the energy efficiency: indeed, more power has to be consumed at the BS to reach larger values of $R_{\min}$. Furthermore, backscatter-aided NOMA always outperforms the other transmission strategies.

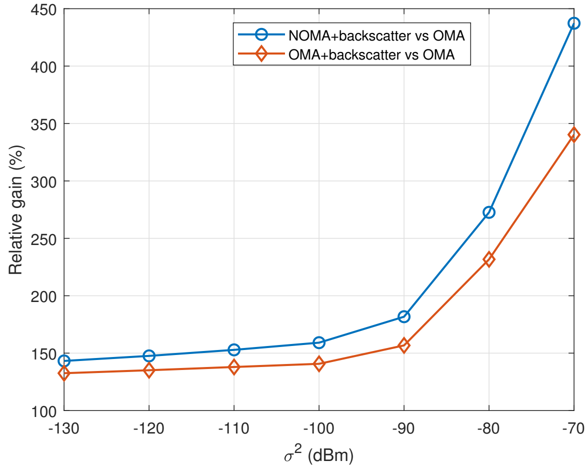

Fig. 3. Relative energy efficiency gain of backscatter-aided NOMA and backscatter-aided OMA compared to conventional OMA as a function of σ^2 for $K = 3$ users, $R_{\min} = 1$bits/s and $P_{\max} = 50$ dBm: the relative gain of NOMA with backscattering goes up to 450%; whereas the one of OMA with backscattering reaches 350%.

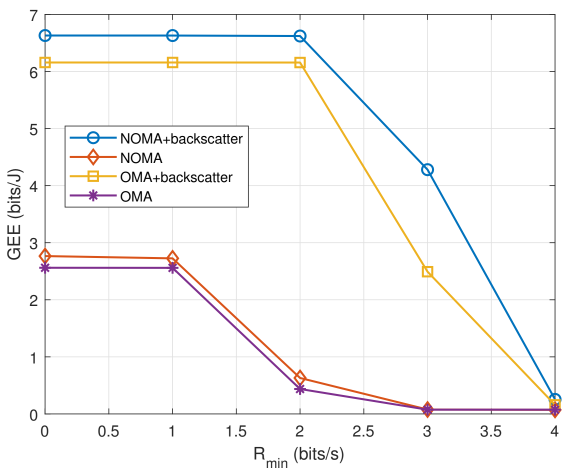

Fig. 4. Energy efficiency as a function of $R_{\min}$ for $K = 3$ users and $P_{\max} = 50$ dBm. Backscatter-aided NOMA outperforms all other schemes. The larger the required QoS, the more power is consumed at the BS, decreasing the energy efficiency.

V. Time-varying backscattering message

We discuss here the case in which the backscattering signal $b(t)$ is no longer constant. In several existing works [9], [10], [21], the backscattering signal $b(t)$ is assumed to be a unit-variance random variable, which is to the case in which the backscattering device has a message of its own. In this case, the achievable rate is no longer equal to $\log_2(1 + \text{SINR})$ as suggested in [9], [10], [21].

Indeed, a closer inspection of the channel capacity

$$C = \max_{p_{X_i}} \{I(X_i; Y_i)\}, \mathbf{E}[(X_i)^2] = P_i\}, \tag{9}$$

assuming perfect channel state information at the receiver and transmitter and where the received signal is

$$Y_i = \left(h_i \sum_{j=1}^{K} X_j + \sqrt{\rho} g g_i \sum_{j=1}^{K} B X_j \right) + Z_i, \tag{10}$$

reveals a non trivial issue because of the products BX_j, which are random variables following continuous distributions that are no longer Gaussian ones[3]. Hence, computing the mutual information via $I(Y_i, X_i) = h(Y_i) - h(Y_i|X_i)$, where $h(\cdot)$ denotes the differential entropy, is much more involving than in the standard Gaussian input case and will be tackled in our future work.

Similarly, in [22], the case of a discrete Bernoulli random message B for the backscatter device (a BPSK-type of message) is investigated from an information-theoretic perspective.

VI. Conclusions

In this paper, we have investigated the energy efficiency of a multi-user downlink NOMA system aided by an ambient backscattering device. By formulating the optimization problem as the sum rate vs. total power consumption tradeoff, and – even though not convex – we have provided the optimal closed-form solution for the joint reflection coefficient and power allocation policy for an arbitrary number of users. This closed-form solution leads to a simplified Dinkelbach's algorithm with a low complexity (i.e., one line search) to maximize the ratio sum rate vs. power consumption. Simulation results show that the proposed backscatter-aided NOMA scheme always outperforms both OMA (with and without backscattering) and conventional NOMA, especially in the high noise regime, irrespective from the number of users K. Furthermore, in the case of NOMA, the backscatter device is shown to reflect part of the incoming signal from the BS to the users and to simultaneously harvest the remaining part, which can be used for its circuit operation. Future work may include considering backscattering messages and deriving new capacity expressions, multiple carriers, multiple backscatter devices, harvested energy constraints, etc.

VII. Appendix: Proof of the main theorem

Since only the sum rate (or the first) term in the energy efficiency objective depends on the reflection coefficient ρ, we can only focus on optimizing the sum rate. Let $f(\rho)$ denote the sum rate as a function of the reflection coefficient ρ for a fixed power allocation vector $\mathbf{p}$:

$$f(\rho) = \sum_{k=1}^{K} \frac{1}{2} \log_2 \left(\frac{1 + (H_k + \sqrt{\rho} G_k)^2 \theta_k}{1 + (H_k + \sqrt{\rho} G_k)^2 \theta_{k-1}} \right).$$

The first-order derivative of $f(\rho)$ is given by

$$\frac{\partial f(\rho)}{\partial \rho} = \frac{1}{2 \ln 2} \sum_{k=1}^{K} \frac{2 H_k G_k (\rho)^{-1/2} + G_k^2}{\left(\frac{1}{\theta_k} + \left(H_k^2 + 2 H_k G_k \sqrt{\rho} + G_k^2 \rho \right) \right)} \frac{-2 H_k G_k (\rho)^{-1/2} + G_k^2}{\left(\frac{1}{\theta_{k-1}} + \left(G_k^2 + 2 H_k G_k \sqrt{\rho} + G_k^2 \rho \right) \right)} \geq 0.$$

Since $\theta_k \geq \theta_{k-1}$ by construction, one can prove that the sum rate $f(\rho)$ is increasing in ρ. Investigating the second order

[3]Here, we have used the more information theoretic notations: X_i, B, and Y_i, to denote the random messages intended for each user i, the message of the backscatter device and the received message at user i, respectively.

derivative, $f(\rho)$ can be shown to be concave in ρ. Hence, in order to maximize the sum. rate, the reflection coefficient ρ must be chosen as large as possible while meeting the constraints of $(\mathbf{EE1})$. In particular, the constraints (C2)-(C4) depend on ρ.

Since $\Gamma_k(\rho)$ is an increasing function of $\rho \in [0,1]$, we can see that the higher the value of ρ the less constrained (C2) becomes, implying a larger possible set for the other variables of the problem: $\theta_k,\ \forall k$. Hence, choosing the largest value of ρ is optimal in terms of (C2).

Now, to compute the largest possible value of ρ, we can focus only on the constraints (C3) and (C4) with no loss of optimality. These constraints require that $\gamma_{k\to i} \geq \gamma_{k\to k}, \forall\ k \geq 2, \forall i \leq k-1$ and $0 \leq \rho \leq 1$.

First, note that for any user $k \geq 2$ and any $i \leq k-1$, we have the following equivalencies

$$\begin{aligned} &\gamma_{k\to i} \geq \gamma_{k\to k} && \Leftrightarrow \\ &\frac{\Gamma_i(\rho)p_k}{\Gamma_i(\rho)\sum_{j=1}^{k-1}p_j+1} \geq \frac{\Gamma_k(\rho)p_k}{\Gamma_k(\rho)\sum_{j=1}^{k-1}p_j+1} && \Leftrightarrow \\ &\Gamma_i(\rho) \geq \Gamma_k(\rho), \end{aligned} \tag{11}$$

where the first equivalence follows from our notations and the definitions of $\gamma_{k\to i}$ and $\gamma_{k\to k}$ in (4); and the second equivalence is obtained after some simple derivations. Hence, constraint (C3) is equivalent to the following set of $k-1$ constraints:

$\Gamma_1(\rho) \geq \Gamma_k(\rho), \ldots,$ and $\Gamma_{k-1}(\rho) \geq \Gamma_k(\rho)$, for any user $k \geq 2$. For example, for user $k=2$, the only constraint writes as $\Gamma_1(\rho) \geq \Gamma_2(\rho)$. For user $k=3$, the two constraints write as $\Gamma_1(\rho) \geq \Gamma_3(\rho),$ and $\Gamma_2(\rho) \geq \Gamma_3(\rho)$. All these constraints (for user 2 and 3) simplify to $\Gamma_1(\rho) \geq \Gamma_2(\rho) \geq \Gamma_3(\rho)$. The same reasoning can be extended up to user K, and all the constraints reduce to $\Gamma_1(\rho) \geq \Gamma_2(\rho) \geq ... \geq \Gamma_K(\rho)$.

Let us now focus on a single inequality of the form $\Gamma_{k-1}(\rho) \geq \Gamma_k(\rho)$ in the inequality chain above. The aim is to find the largest value of $\rho \in [0,1]$ fulfilling this constraint, which can be equivalently expressed as

$$\begin{aligned} &\Gamma_{k-1}(\rho) \geq \Gamma_k(\rho) && \Leftrightarrow \\ &(H_{k-1} + \sqrt{\rho}G_{k-1})^2 \geq (H_k + \sqrt{\rho}G_k)^2 && \Leftrightarrow \\ &(G_k - G_{k-1})\sqrt{\rho} \leq (H_{k-1} - H_k) \end{aligned} \tag{12}$$

where the first equivalence follows from the definition of $\Gamma_{k-1}(\rho)$ and $\Gamma_k(\rho)$ and the second one by simply rearranging the terms.

Two cases can arise based on the order of G_k and G_{k-1}: a) If $G_k > G_{k-1}$, since $H_{k-1} \geq H_k$ by assumption, the constraints (12) and $C(4)$ of the optimization problem lead to the following upper bounds

$$\rho = \min\left\{1, \left(\frac{H_{k-1} - H_k}{G_k - G_{k-1}}\right)^2\right\}, \forall k > 1. \tag{13}$$

b) If $G_k \leq G_{k-1}$, then (12) becomes trivial since $H_{k-1} \geq H_k$ by assumption. In this case, we only have the constraint $C(4)$ to be met: $0 \leq \rho \leq 1$. Taking all this into account, the optimal reflection coefficient ρ^* can be expressed as in the main theorem, which completes the proof.

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